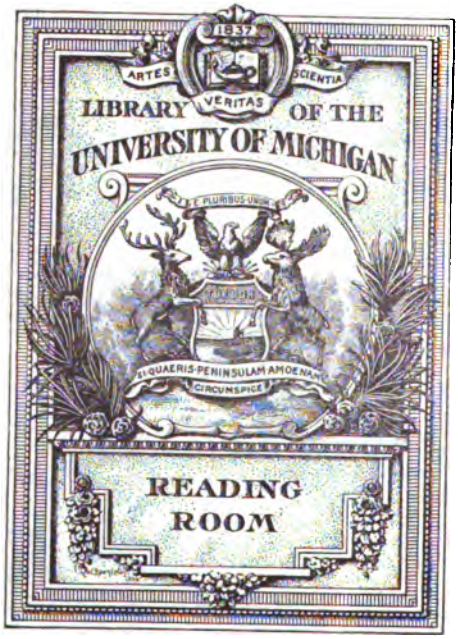

A TREATISE

ON

CIVIL ENGINEERING.

BY

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FIRST EDITION.

FIRST THOUSAND.

NEW YORK:

JOHN WILEY & SONS,

58 EAST TENTH STREET.

1895.

ROBERT DRUMMOND, ELECTROTYPER AND PRINTER, NEW YORK.

DEDICATED

TO

My Beloved Wife and Daughters,

THROUGH WHOSE LABORS AND SELF-SACRIFICES AND

UNDER GOD'S BLESSING I HAVE BEEN ENABLED

TO WRITE THIS BOOK.

AUTHOR.

PREFACE.

My first thought was to call this work a Treatise on Engineering, which is a more pretentious title than the one finally adopted.

Notwithstanding the scope of the volume, it has fallen short somewhat, both from the theoretical and practical point of view, of the encyclopædic character which that designation would imply.

I have therefore inscribed the rather confusing words Civil Engineering on the title-page. The prefix "Civil" has entirely lost its significance by the introduction and substitution of such terms as Railway, Bridge, Hydraulic, Mining, Sanitary, and Electrical, each presumably signifying an engineer who is a specialist in one of those branches of engineering. We have many eminent experts in hydraulics, practical geology, metallurgy, electricity, pneumatics, steam, heat, and sanitation, and many of these are accomplished and skilful engineers. It should, however, be recognized that those well versed in these subjects need not be and are not necessarily engineers; on the contrary, an engineer (civil) should know enough of all these special branches to adapt his designs and constructions to their intended purposes.

In treatises on civil engineering there has been a tendency to devote very much the larger space to the theory of stress and strain in bridge and roof trusses and to general railroad construction, to the exclusion of many equally important subjects, or at most the cursory presentation of only a few facts in regard to them.

I have endeavored to present in this volume the theory and practice connected with all the more important branches of engineering, without giving undue prominence to any one or treating each exhaustively, but to impress on the mind of the reader the essential and fundamental principles and practical facts with which the engineer should be familiar, leaving him to pursue his researches in any department which he may select. This is an

age of specialists and specialisms. There are special treatises on almost all branches of engineering, to which the reader is referred for a more thorough development of the subjects discussed in this volume.

While recognizing the value and importance of an extended training in pure and applied mathematics, this training should be given prior to or concurrent with the course of engineering, but a treatise such as the present one should not be used for this purpose. The important principles of engineering as a practical science should not be lost sight of, in wearisome struggling with the intricacies of a purely mathematical problem. I have therefore endeavored to ease the way of the student by removing the mathematical difficulties by which his mind would be diverted from the proper channel.

From a practical point of view little benefit is to be derived from the simple ability to solve theoretical problems and evolve formulæ, unless the relations and value of the quantities contained are fully appreciated and the limitations of its applications are thoroughly understood, together with the acquisition of a facility in handling and applying them. I have therefore taken great pains to direct attention to these matters, and to give a number of practical examples, showing the proper application of scientific principles to practical purposes.

Numerous works have been consulted, a partial list of them being given on page xviii. Where quotations from any of them have been made due credit has been given.

It is a simple act of justice to acknowledge my indebtedness to my daughter, who, during her leisure hours out of school, prepared nearly all the drawings in this volume.

W. M. Patton.

Chicago, Ill., July 15, 1895.

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A TREATISE ON CIVIL ENGINEERING.

ART. I.

SURVEYING.

1. In all engineering works of whatever kind, surveying, in the most general acceptation of the term, is the first actual work that has to be performed, and it is no mean or unimportant part of the work. The principles and practice of surveying, in all departments of engineering, should not only be understood in a general way, but there should be an ease and a facility in handling whatever tools or instruments may be necessary, and a readiness in knowing how to go about it. However strange it may seem, it is nevertheless true that students can make diagrams and drawings from models, explain how a structure should be laid out, make all the necessary calculations of base-lines, angles, or sides of triangles, yet give them the ground-plans of a house, a pier, or abutment with instructions to transfer the same to the ground in order to guide and direct builders, and the awkward, uncertain, and slow way in which they will go about it, is simply astonishing, even if they can do the work at all. While these things are noticeable even in structures of simple forms of section, they are painfully so in the higher and more complicated types of surveying, such as geodetic surveying, railway and canal surveying, location of piers in wide rivers, and the ordinary and simple astronomical determinations. The necessary familiarity with the essential principles and their applications can only be secured by constant drilling in

these subjects; and while it may not be necessary for the civil engineer to be a profound master in all the branches of surveying, yet he should clearly understand and be able to apply readily and with confidence in himself certain simple and elementary principles in all of them, whatever may be the special line of engineering selected.

2. While the writer must refer the student to works on the several branches of surveying for full information, it will not be out of place to call attention to a few of the simpler principles in the several branches of surveying and their applications to a few common and useful problems under these principles.

In a work on Civil Engineering but few of the principles of geodesy and astronomy are essential. These are special sciences, and require special and thorough instruction, more delicate construction of the instruments, more powerful telescopes, and steadier and firmer supports upon which to rest the instruments, and more care, skill, and delicacy in adjustments, graduations, observations, and in handling the instruments.

In mining surveying, while the instruments, tools, and appliances are somewhat different from those required in surface surveying, and in some respects and under some conditions greater care and accuracy may be required, yet the general principles and their applications are not materially different.

3. In what will be said in this article on the subject of surveying, it will be assumed that the student is familiar with the adjustments and use of instruments, their general construction, and the manner of making such simple repairs as an ordinary engineer may have to perform in the field; he will also be supposed to have perfect familiarity with the elements of mathematics, such as ordinary arithmetical calculations in fractions common and decimal, powers and roots, simple ratios and proportions; the solution of algebraic expressions and equations; the solutions of triangles and circles; the ready use of tables containing sines, tangents, latitudes and departures, logarithms and logarithmic sines and tangents, etc. While the higher and more complicated expressions, relations, and solutions in mathematics may be and are of great value, and the student may be better off the more he may know of these things, yet a thorough understanding of the elementary relations in mathematics will be alone necessary

for the solution of a very large majority of engineering problems. But familiarity, readiness, and promptness in expressing these relations, solving these problems, and using the tables must be so thorough that no study or reading of instructions will be necessary after the problem is presented and the necessary conditions and relations are understood. The want of this familiarity and readiness in these elementary matters is a great source of loss and waste of time, besides often resulting in inaccurate or erroneous results which entail more delay, waste of time and money, inextricable confusion, and want of confidence than if absolute ignorance was acknowledged and known in the beginning; and it is not infrequently the experience of engineers to find young men, well versed and even handy with the expressing and solution of problems in the higher mathematics and mechanics, lamentably ignorant of the elementary relations and solutions in these subjects.

4. Much valuable knowledge and experience can be acquired and many useful problems can be solved without any tools or instruments. Distances can be determined by pacing and stepping, directions determined and lines ranged by the eye, and from these angles and distances, unknown distances can be determined both in vertical and horizontal planes. These serve excellently for rough approximations, and when these are compared with actual instrumental measurements and repeatedly compared a great degree of accuracy in distances, angles, and directions can be attained by the use of the legs and the eye. The construction and use of many such problems are made and described in books, but the tools of the engineer are the chain, hatchet, and stakes, as well as the ruler, dividers, and pencil: they must be required to use all if it is desired to develop useful, practical, and intelligent engineers, and should use them together, while the student is young and impressionable. Study and practice in these methods are of great value, but must necessarily be confined to those cases where rough approximations are alone required, and when rendered necessary by the want of more accurate tools and instruments, or where, in connection with the use of these, certain approximate data can be advantageously employed.

5. In surveying for or laying out any ordinary structure there are three distinct operations to be provided for: 1st, to mark the

limits of the excavation; 2d, to mark the limits of the footing-or foundation-courses; 3d, to mark the limits of the neat work or body of the structure. The last is the only one calling for absolute accuracy; as a rule there is a margin allowable in the first two. Ordinarily the surveyor's work can be limited to the third operation. By simply placing stakes in the prolongations of all bounding lines of the neat work, and in positions and at distances from the structure where they are not likely to be disturbed by the necessary building operations, the intersection of strings connecting the two stakes on each line will mark the corners and angles of the structure; or temporary stakes may be placed at or near these angle-points, with reference to which the excavation can be commenced at proper distances to allow for the

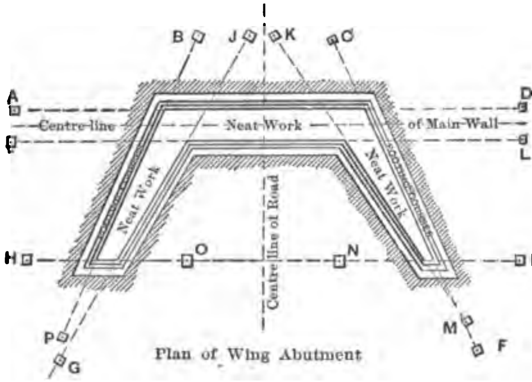


FIG. 1.

footing-courses, which can also be commenced by reference to the same stakes. But greater accuracy is required for commencing the neat work, the bounding lines of which must be accurately determined by the transit or by the intersection of strings connecting permanent stakes. In the case of piers, abutments, and usual structures the laying out is easy and simple, but in structures of complicated outline the operation requires much labor, calculation, and skill, and a careful study of the easiest and shortest way of doing the work is time well spent. The instruments usually required are a transit, a level, a steel tape, and the transit and level rods. Fig. 1 shows the ground-plan of an ordinary wing-

abutment for the extreme end-rests of a bridge structure, showing outlines of the excavation, two footing-courses, and neat work. The lettered rectangles represent the stakes placed on the prolongations of the lines of the neat work; the dotted lines, the strings which mark at their intersections the corners of the neat work. With such a system of stakes the builders can do all the laying out. Often, however, engineers merely give roughly the limits of the excavation, and subsequently lay out the neat work on the footing-courses by means of the instruments, using the centre line of the roadway as a base and line of reference. The principles involved are the same. The latter method when time is available will generally be more satisfactory and will avoid possible errors and misunderstanding.

6. The proper location of piers and abutments probably requires more skill, patience, and accuracy on the part of civil engineers than any other operation in this kind of surveying. Wide, deep rivers with rapid currents cause him many days and weeks of hard labor, and he awaits anxiously the ultimate swinging and completion of the superstructure to fully satisfy himself that no mistake has been made. The locating of floating structures is necessarily attended with some uncertainty and ultimate inaccuracy, and until the foundation-structures rest hard and firm on the beds of the streams no absolute certainty can exist; but with proper care the error will be insignificant. There is or should be some margin in the size of foundation: too small margins will often lead to trouble and expense. Even where, as in open caissons, provisions are made for floating the structure by pumping the water out, there is no certainty that a second effort will prove more successful. A reasonable excess in horizontal dimensions is wise. When resting firmly the exact location of the neat work can be made. Nothing but uselessly expensive staging, beams, and rods with screw-threads can be relied upon—and these are at best uncertain—to hold a heavy structure exactly in position while sinking it to the bed of a stream. When such structures as large open cribs and pneumatic caissons are once landed, it is a slow and expensive operation to lift them again. The writer's practice has been to leave a margin of from 2 to 4 feet on the caisson. This, while enlarging the base of the structure, also provided a margin to go upon in the case of error in sinking the caisson. His greatest error in sinking 90 feet through

water and soil was only 18 inches at one end of the caisson, usually not over 3 to 6 inches. When the caisson rested firmly, the exact centre of the masonry could be located on it; also the centre line from which, by turning right angles and measuring proper distances, the first course of masonry could be accurately laid out. To locate any of these structures requires the use of two transits, or one transit and an accurately-measured steel wire. Both should be used as checks. Holding rods, one on each side of the caisson, on the proper line, or driving nails through narrow straight-edges placed at these points, and keeping these with the transit on the centre line, will keep the caisson square with the line; and measuring with a wire by holding one end at the geometrical or masonry centre of the caisson will give the proper distance, thereby locating it in its true position. In locating a cofferdam, where practicable it is better to drive a pile at the proper centre of the space to be enclosed. Measurements can be conveniently made from this pile. The upper and lower ends of the dam can be lined by a transit from the shore, adjusted over points at the proper distances above or below the centre line. Having driven spikes accurately in line, a large frame composed of two pieces at right angles to each other can be used to get the piles in the sides of the dam in the proper line, and when two or three are driven the remaining ones can be aligned by these, or the piles in the sides of the dams can be located by the transit and measurements with the wire or steel tape.

7. When two transits are used a base-line is accurately measured on one or both banks as nearly at right angles to the centre line of the road as practicable. The entire length of this base-line should be equal, if practicable, to the width of the stream, and points should be marked by stakes on this line at distances measured from its intersection with the centre line equal to the distances of the respective piers from this same point. Having the right angle and the two including sides, the other angles can be calculated in each triangle having the centre of the pier at one of its apices. One transit sighted along the centre line and the other on a line making the calculated angle with the base will intersect at the centre of the pier; the rod being placed at this intersection locates the centre of the pier. These conditions and relations are shown in Fig. 2, in which AE is the base-line; 1, 2, 3, 4, and 5, the positions of the piers; AB and

AC, *AD* and *AE* distances on the base-line as nearly equal as possible to the distances *A*2, *A*3, *A*4, and *A*5. The angle at *A* to be measured, and the angles at *B*, *C*, *D*; and *E* to be calculated. With one transit placed at *A* and sighting along the centre line *AG*, which should be permanently marked by several large stakes or hubs with tacks driven in them exactly in the prolongation of the centre line *AG*, and another transit placed at any of the points *B*, *C*, *D*, or *E*, with which turning the calculated angle at these points and bringing the rod simultaneously in the two lines of sight, the centres of the piers may be located successively. Or if the steel wire (a piano-forte wire No. 10 is a good size) is used, on which are marked the distances *A*2, 2–3, 3–4, 4–5, by tarred strings, as determined from an accurately-measured base on the ground, when pulled to a certain tension by means of a spring-balance, say from 15 to 20 pounds, then rolled upon a reel. The wire is then stretched with the same tension, with the end of one string at *A* and the end of the other string brought into line at pier 2, and similarly from 2 to 3.

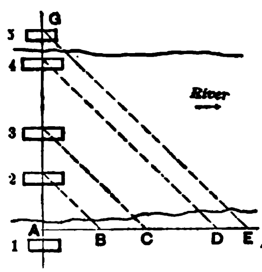


FIG. 2.

Another and simple method is as shown in Fig. 3. A base-line is laid off in any desired direction on one shore, such as *AE*, and certain distances measured on, it *AB*, *AC*, *AD* *AE*, of lengths equal

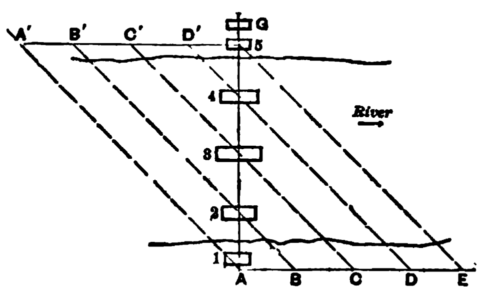


FIG. 3.

or nearly so to the distances *A*1, *A*2, *A*3, etc. Also measure any base-line on the opposite shore and in the opposite direction to the first base-line. Piers 5 and 1 are commonly located at conven-

ient and suitable points on or near the banks, and the distances between them determined by triangulation from a suitable base-line, the proper distances between the intermediate piers and between them and the end piers being determined by the required conditions or an act of Congress. Then the distances $5D'$, $5C''$, $5B'$ can be determined, so that the lines of sight DD', CC'', BB' will intersect the centre line AG at the proper position for the centres of the piers 2, 3, and 4. For in the triangle $AB2$ the angle at A and the sides AB and $A2$ are known, hence the angle at 2 can be calculated; then in the triangle $G2B'$ the angle at 2 has been calculated, the angle at point 5 measured, and the distance 2–5 known, from which $5B'$ can be calculated.

All triangles formed should be well conditioned; that is, no angle, especially that one at the piers 2, 3, 4, or 5, should be too small: the larger the better, as the smaller will be the possible error in locating that point. Should any piers be built above the line of sight, both the centre line and any oblique line intersecting the pier should be marked on both faces of the pier, both to enable the centre of the pier itself to be located, as well as the centres of the other piers.

8. Base-lines are often measured by means of the 50- or 100-foot steel tapes. These may or may not be absolutely accurate in total lengths or intermediate divisions, and in addition require corrections for changes in temperature, which are both uncertain and troublesome, and the measurements may not correspond with those made by the bridge companies in measuring the lengths of the chord bars, which in bridge construction is the important consideration, as it is the lengths they use rather than the absolute length that is essential. But if the engineer decides to use the United States standard, it will not be his fault if the bridge company does not use the same. Therefore it is best to secure a U. S. standard steel bar three feet long, which can be obtained from the chief of the Coast Survey department, and with this measure at the standard temperature at least three bars made of some light timber, tipped with small pyramidal-shaped brass points, the extreme end being not more than $\frac{1}{16}$ to $\frac{1}{8}$ inch square. The bars should not be more than 12 to 15 feet in length. Base-lines should then be ranged and large square hubs driven in line and at distances apart equal to the length of the base-line bars. With three

base-line bars placed end to end on these hubs, after they have been sawed off to the same level (or as many to the same level as the nature of the ground will admit of, and at the points of change in height of hubs the ends of the bars are to be adjusted by the plumb-line), then the rear bar is moved to the front and accurately adjusted to the end of the former front bar, and so on until the entire length of the span or spans is measured. This measurement should be repeated several times, and also checked by an accurate steel tape. The extreme end hubs should be large timber or stone monuments, firmly set or driven into the ground, and the proper points for distances indicated by tacks or small pins, if timber monuments are used, or by holes when the hubs are of stone. These monuments should reach from two to three feet above the ground, and should be at the same level. All intermediate hubs should then be removed. The steel wire then should be tested by stretching it with the proper tension between these points, and should not touch the ground at any point. The exact distances can then be marked by wrapping tarred strings on the wire covering a distance of from 1 to 2 inches on the wire, the inner ends of the string indicating the exact distances apart, equal to the length of the span; the string

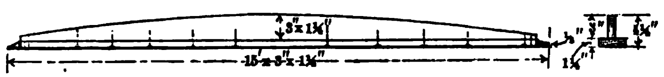


FIG. 4.—BASE-LINE BAR, LONGITUDINAL SECTION. CROSS-SECTION AT CENTRE.

nearest the end being common to all spans, and any number of other strings placed to correspond with the different lengths of span. The wire should be tested both before stretching it to measure the span and also afterwards, merely as a precaution against any slipping. This method has been successfully used by the writer in locating piers from 150 to 525 feet apart, and he much prefers it to triangulation, though the triangulation method was often resorted to merely as a check or when, owing to the excessive distance or other cause, the wire could not be used. And on some forty or more spans measured in this way there has never been any appreciable error, or discrepancy in the lengths. The greatest error he now recalls was about 2½ inches in a 250-foot span, which was evidently the result of carelessness somewhere. The following diagram, Fig. 4, shows

the form of base-line bar used by the writer. It was made of pine, was stiff, and deflected to no appreciable extent. The length was 15 feet. It was measured and graduated while suspended between two points of support, in order to take into consideration any deflection that might exist.

Each pier after completion should have its exact centre both as to line and distance marked by a short bolt or pin of small diameter before being abandoned by the builders. It may serve several important and useful purposes.

These principles, rules, and experiences may have many other useful applications.

ART. II.

RAILWAY RECONNOISSANCES, SURVEYS AND LOCATIONS.

9. It having been determined to connect two points called *termini* with a railway, canal, or highway, with possibly either the necessity or desirability of passing through one or more intermediate points of importance, the engineer has about all the information possible to be given, and upon such meagre data he must develop the balance and bring about satisfactory results.

The writer has seen but few good locating engineers. Much can be acquired by practice and experience, and to some extent by certain fixed principles and rules; yet without a certain natural talent and adaptability, they will fall short of reaching any high standing in this, one of the most, if not the most, valuable and important departments of civil engineering. The importance of which is but poorly understood or appreciated by capitalists and business men, as evidenced by the practice of employing almost any one who can handle an instrument. The result is often a poor location, requiring unnecessary cost in construction, maintenance, and operation; especially when but little time is given the engineer to make the reconnoissance, preliminary surveys, and locations. The moment they begin to pay out money they want the construction to begin, and as soon as this is fairly started they want the track laid and trains to commence running. This policy is often followed to save the first cost of construction, but it

often defeats the very purposes intended. The only thing certainly gained is the early operation of the road bringing in some money; but a very great interest charge on the money spent in changes of location, repairs, and renewals of defective work and structures, increased cost of operation and maintenance, will do away with the so-called earnings of a badly-located and poorly-constructed road. Thousands upon thousands of dollars are spent or wasted, whereas a few hundreds spent in a careful location and better construction would avoid most if not all of the waste. It is true that there are matters with which the engineer has little to do, as very often his instructions are simply to push the road to completion in some sort of way: and obedience to properly-constituted authority is one of the most important elements in the training of an engineer, even if in so doing he must run the risk of sacrificing his professional reputation. Where questions of honesty, justice, and square dealing do not enter, he must simply obey; but he should be careful to put himself properly on the record that it may be understood that he is simply obeying orders and doing the best possible in the time and under the conditions existing.

10. The maps generally available are unreliable and vague, but, notwithstanding, general directions of streams, ridges, and low lands can be obtained; and the names of streams, rivers, gaps, or low divides, as well as the names of parties living adjacent to the proposed route, are given. There can thus be found the residences of people who are acquainted with the general topographical features, who may give information and act as guides. The county or township maps can be obtained from time to time.

11. The main objects of the reconnoissance are to become acquainted with the general topographical features, to determine the accuracy of the maps in regard to the directions of roads, streams, and ridges, and to make corrections in these where maps are found to be defective or in error, and to obtain all useful information. The only instruments that can be carried or used will be the pocket compass and the aneroid barometer.

The most difficult problems are frequently limited to a few points, and from two to five miles on each side of these points. The general actual and relative elevations of these can be determined by the barometers, their directions from each other by the com-

pass. The intermediate portions of the route are determined by the direction of the lines joining the important points on the route, to be modified by the topographical features determined during the preliminary survey. There are frequently several sets or series of important features, either of which may be a practicable route, the one offering advantages of one kind, and another those of a different kind. All should be examined and that one selected which seemingly offers the greatest number of advantages. A preliminary survey may, however, be necessary to settle the point satisfactorily, and herein often lies the importance of the element of time.

12. If the line of a road is of necessity to follow watercourses, many of the greatest difficulties of location disappear, as the route is confined to a narrow strip on one or both sides of the stream. In this case a reconnoissance is necessarily confined to the selection of that one of several streams which offers the greatest number of advantages, such as the straightness of the stream, rapidity of fall, steepness and proximity of the confining ridges, and the general trend or direction of the streams; also the best tributaries to follow when nearing the main divide. This latter can better be determined by preliminary surveys. Frequently there is no selection to be made so far as the main stream or watercourse is concerned, as it will often be evident that there is but one practicable route.

13. On a route of this kind a rapid preliminary survey should be made, using only the compass and chain, in order that a map of the valley may be drawn showing the sinuosities, widths, and depths of the stream and the foot-lines of the adjacent hills on either side. In such cases preëminently is a careful and full topographical survey necessary from foot-hill to foot-hill. The line should be ranged on the most available ground for rapidity of progress. The levels should generally be taken at the usual distance of 100-foot intervals, though often only turning-points need be taken. Good judgment should here be exercised. The width, depth, and direction of all tributaries should be noted; also the relative value of property on both sides of the main stream. With such data and maps it will not be difficult to plot the proper line to be located with a reasonable degree of accuracy, including the amount and degree of curvature required. Several lines thus plotted can be compared as to cost of construction for earthwork, culverts, trestles, and bridges.

When the stream is small it will generally be good practice to cross it often in order to maintain a good alignment; the ultimate advantages gained will fully offset the additional cost of the increased number of bridges. If, however, the stream is large, wide, and deep, the location will of necessity be confined to the one side or the other for considerable distances; and only a few crossings at long intervals will be allowable.

The preliminary survey may be confined to the one side, but the nature of the survey both for line and topography should be the same on that side of the stream upon which the line is to be located. The above conditions are represented in the following sketch, Fig. 5. A straight line from A to C, though the shortest line, would

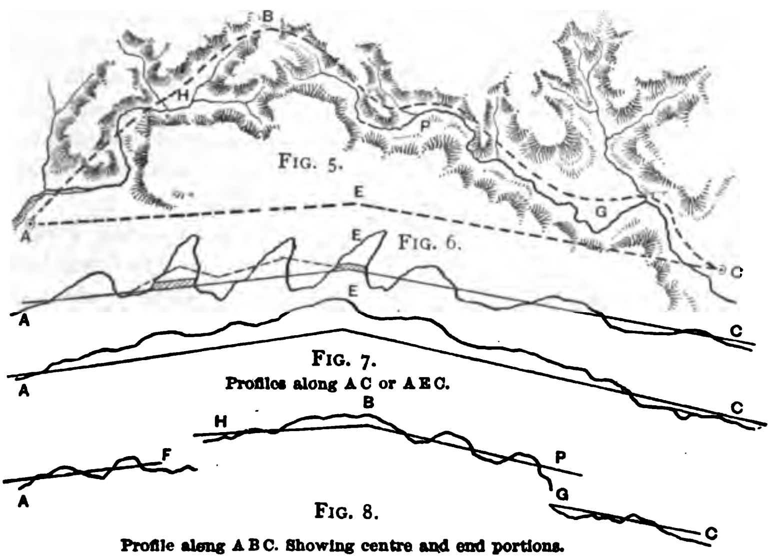


FIG. 5.

FIG. 6.

FIG. 7.
Profiles along A C or A E C.

FIG. 8.
Profile along A B C. Showing centre and end portions.

have to cross the intervening high ground, the vertical section or profile over which would be exceedingly irregular, as shown in Fig. 6, cut up by a number of ridges and hollows, requiring either very steep grades or an immense amount of cutting and filling, possibly tunnels and high and costly bridges; or if at all regular in rise, either very steep grades or very costly excavation and tunnelling, as shown on Fig. 7. Whereas following the streams from

A to *B* and *B* to *C*, the profile Fig. 8 shows a less total elevation to overcome and a longer distance in which to overcome it, thereby giving easier grades and lighter work. In Fig. 5 the line from *A* to *B* has a better alignment by crossing the stream several times, whereas the alignment from *B* to *C*, remaining on one side of the stream, has a greater amount and higher degree of curvature.

14. In a cross-country location greater difficulties are encountered. The more nearly the streams are at right angles to the line of the road, the more difficult will it be to select a route. Valleys and ridges have to be crossed alternately; the grades will be broken into alternate ascending and descending grades for relative short lengths; or the amount and degree of curvature increased, thereby increasing the length of the line. The only thing to be done is to run the line so as to pass through the lowest points of the divides or ridges, and to cross the streams where the banks are high, thereby reducing the rises and the falls. And even then a considerable development of the line may be necessary to prevent excessive rate of grades. Such conditions not only require good judgment; a clear understanding of the lay of the ground, but will usually necessitate the running of a number of trial lines.

The sketch Fig. 9 represents roughly, but not to scale, a somewhat difficult piece of location. The distance from *A* to *D* was from 130 to 140 miles, according to the route taken. Both of the large rivers were bordered by swampy and low lands for a considerable portion of the distance, and extending to the foot-hills for a varying width on both sides. Starting at *A*, the line ran northward along the large river formed by the junction of the two rivers, on ground much of which was overflowed in times of high water, and all of it low, flat land, for a distance of about 60 miles, where it crossed one of the large streams, thence following a tributary of considerable size, rising gradually to the point *G* over low and sometimes swampy ground. This much of the route was practically located by the necessities of the case. The total distance from *A* to *G* was about 80 miles. The next section, from *G* to *K* on one route about 25 miles, and from *G* to *F* on the lower route about 15 miles, with any number of intermediate routes between the two that could be chosen, presented the greatest uncertainty and difficulties. The profile along the first line *ABCHD* is shown in Fig. 10, and that along the line *ABGFD* is shown in Fig. 11. The first has a

greater total difference of level to overcome, but a much longer distance in which to overcome it, thereby reducing the rate of grade; but owing to the greater number of high ridges and deep depressions in the section from *C* to *H* the cost of the work would be greatly increased. Along the line from *G* to *F* the grades would necessarily be much greater, but the cost of the work would necessarily be less. If the line is developed, that is, lengthened, by holding to the hillsides, in order to descend into the valley to the right of *H*, the line would lose the benefit arising from the

FIG. 9.

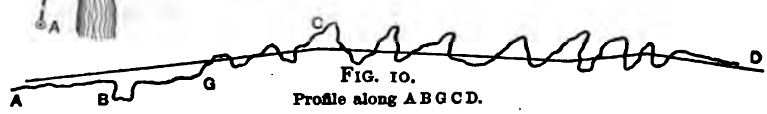


FIG. 10.
Profile along A B G C D.

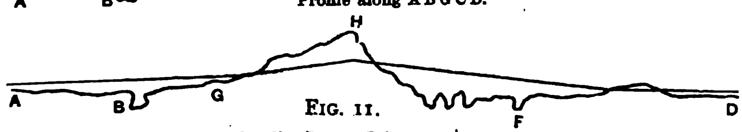


FIG. 11.
Profile along A B G H F D

shorter length and the cost of construction would be considerably increased, as in no case must a certain maximum rate of grade be exceeded. That route must be adopted which within the limit of maximum grade and degree of curvature will cost the least in construction. If the line is carried too high upon the spurs from the main ridges, both grades and cost of construction will be increased; and on the contrary, although grades and cost of construction may be kept in reasonable limits on the lower lines, there is always danger of locating a large portion of the line where it may be overflowed in times of floods, or the cost of the work may be greatly increased to

avoid this contingency. The higher line was finally adopted in the above case. The remaining distance of 25 to 35 miles from *K* to *D* and from *F* to *D* was on good level and high ground, with no special choice between the two routes.

15. Volumes might be written and have been written on the subject of location. The above is only given as a general outline of the rules and principles applied in practice.

It may be stated that unless the engineer has specific instructions to the contrary, it is his duty to select the best line, that is, the one which will prove the least expensive to maintain and operate. In this expense should also be included the interest on the first cost. But often he is compelled to seek the route requiring the least first cost. As much, however, of the best line as practicable should be included in the cheapest route confining the deviations to certain portions in order to avoid especially difficult and expensive portions of the work; or steeper grades may be temporarily adopted, leaving the questions of improvement in the alignment and reduction of the rate of grades for some subsequent time.

16. Having decided upon the best route from the reconnoissance and preliminary surveys, considered in reference to economy in grades and curves, cost of construction, the present and prospective traffic,—all of which should be carefully determined from the surveys and from information obtained in regard to the agricultural products in kind and quantities, timber, minerals, and the advantages offered for building towns, establishing mills, furnaces, and manufacturing establishments of all kinds,—and the line to be located fairly well established and plotted on the maps of the preliminary surveys, the real work of the location can then commence; though as a rule it is not infrequent, if not common, to find reconnoissance, preliminary surveys, location, construction, track-laying, and even the running of trains, all going on at the same time on different portions of the route: to which there can be no objection, if the line can be divided into sections of greater or less lengths, each independent of the other, and fortunately such is often the case. But frequently the push to commence the running of trains leads to confusion, expense, bad locations, poor general construction, and the building of inferior structures, with largely-increased operating expenses and large expenditure in renewals and repairs

of structures, lowering grades, and reducing the amount and degree of curvature.

A locating party is composed of: one chief of party, one transitman, two chainmen (the front chainman also front rodman), one back rodman, one stake-carrier, marker and driver, two to four axemen (these latter can generally make stakes, and also carry them when the material for making them is only found at long intervals), one leveller, and one level-rodman. A good topographical party, consisting of one topographer and one or two assistants, will always pay for the additional expense, but from a penny-wise, pound-foolish policy this party is often deemed unnecessary and of no importance or value.

If the preliminary survey has been made with great care and thoroughness in respect both to the alignment and the topographical work (which is rarely the case owing to the rapidity required, in order to gain information as to cost, time to complete the construction, and the selection of the best route) the topographical party may be dispensed with on location, but in this case the work must usually partake both of the nature of a preliminary survey and final location, running trial lines for more or less great distances and then returning and locating the same, as it will be usually found that the plotted tangents will not lay well or properly on the ground, and the plotted curves will also have to be changed for the same reason. Much valuable time will be saved by following up the trial lines closely with accurate topographical work on many portions of the line.

In general terms it is the duty of the chief of party to indicate to the transitman the general direction and alignment; secure all information as to crops, timber, and minerals; ascertain the present and prospective value of these and the value of the lands; enlist the interest of the owners; get promises of the right of way, and, when the exact location is determined, secure the right either by gift or by purchase at evidently low prices; determine proper crossings of streams; the cost of culverts, trestles, and bridges; fix boundary lines and ascertain owners' names when practicable, as well as the time, extent, and duration of floods, high and low water marks; to take such topographical features as may be desirable and practicable, thereby familiarizing himself with the general and detailed features of the country. He arranges for lodging the

men at night, or indicates positions for the camps. He should get all practicable information from farmers, business men, woodmen, and miners. He should not believe all that he hears, but confirm the information by personal observation when practicable. He should especially protect all rights of property-owners, be polite and courteous to all, and make as many friends as possible: they will be needed and useful in many ways.

The transitman takes the bearing of the lines when using the compass alone, and the bearings and intersecting angles of the tangents or straight portions of the line when using the transit. He has the supervision of the chainmen, seeing that they do their work carefully and rapidly; locates the rod in the proper line; takes bearing of roads, streams and boundary lines, and such topographical features as time and opportunity will permit; directs the axemen when clearing out brush and trees, so as to avoid delay by wild and unnecessary clearing; selects points in advance upon which to direct the instrument in running long straight lines; indicates proper position for transit turning-points. An intelligent, reliable front chainman can relieve him of much of his labor in regard to many of the above matters.

Both chainmen should be intelligent men, but especially so for the front chainman; it is a great mistake to think that anybody that can be picked up will be suitable for the front chainman. He should be able to plant his rod almost exactly in line; should see that the chain is stretched straight and horizontal; that the links are not bent or in kinks; should place the stakes with the number facing to the rear, or in the direction opposite to that which is called the direction of the line; when up with the axemen, he should plant his rod in line and, going forward in the forest and undergrowth, he should keep them in line. The rear chainman should hold his end of the chain exactly at the last stake, calling out the number of the stake to prevent the front chainman making an error. On open ground they should move rapidly. The front chainman should select turning-points which can readily be seen from the instrument and at the same time admit of ranging a long stretch of line in advance. The rapid progress of the survey will depend to a great extent upon his energy, carefulness, and good judgment.

All that is really required in a back rodman is watchfulness

and patience; he should hold his rod vertical, and unless the movements of the transitman can be clearly distinguished he should stand erect and hold his rod vertically and on the tack continuously until signalled to move forward. He should also as he moves forward see that no errors in stake-numbering have been made.

The stake-maker, whether axeman or man employed for the purpose, should point the stakes carefully and blaze the two sides near the top to a smooth surface. The station-stakes should be fifteen to eighteen inches long. Split stakes are preferred, as they will be lighter.

The stake-carrier should mark the stakes neatly and clearly with red chalk; select the proper number and drive the stake at the point indicated by the front chainman, and drive it in a vertical position. He should also be provided with a few round or square hubs one foot or more in length, which should be driven for transit points, the tops being flush with the ground, and when the exact point is given on the hub drive a tack in it. He then places a long stake in an inclined position about two feet from the hub and marks it with the station number if at an even station, such as Sta. 1521, or if between this and the next full station-point, Sta. 1521 + 52.2. He should have on hand stakes, properly numbered, in order to avoid delay.

Though the work of the transitman is important, yet that of the leveller is equally or more important, and especial accuracy and care are required, as his errors cannot be discovered as readily as those of the transitman. He should read the rods to hundredths or even to thousandths on turning-points and bench-marks, but on the regular and intermediate stations it is only necessary to read the rod to the nearest tenth. Bench-marks should be made on trees near the bottom, generally on an exposed root. These consist of small pyramidal-shaped projections formed by cutting the material from around it; the tree should be blazed above on the side facing the line. The bench marks should be from 50 to 100 feet from the line. The level should be kept in good adjustment, and when practicable the turning-points should be at nearly equal distances on the two sides of the instrument, as this, to a great extent, will prevent errors arising from the imperfections in the adjustments. The readings are only taken as a rule at the regular

stations; but they should also be taken at any intermediate high or low points, the banks and bottom of ditches, small ravines, and on the banks of streams and, when practicable, in the centre of the streams. The elevation of high and low water marks should be taken when known.

The level-rodman should learn to hold his rod in a vertical position. He should call out at each station the number on the stake, and without being told should hold his rod at such intermediate points as above mentioned, either pacing or estimating the distances from the last station and calling out the number, such as Station 125 + 30. He should carry a hatchet to cut out brush, small limbs of trees, and to drive the pegs for turning-points; he should always carry with him a few such pegs five to six inches long.

The instruments used and the organization of the parties are practically the same on preliminary surveys and locations. On the former either a compass or the needle of the transit is used, and there is no need in this case of a back rodman.

The topographer has a tape, a straight-edge about ten or twelve feet long with a bubble attachment to keep it horizontal. This straight-edge is graduated to feet and tenths. He also has a rod five to six feet long, likewise graduated in the same way. With these he can determine the rate of slope of the ground, or he can use a clinometer from which he determines the slope of the ground in degrees of arc, which can be reduced to rise or fall in feet. He should have a tape and Locke level. He runs cross-lines, from the regular stations or intermediate points, perpendicular to the main line to such distances as may be necessary. On ground sloping at a regular rate across the line it is only necessary to measure the slope for a single length of straight-edge. But on irregular and rolling ground this must be extended for considerable distances on both sides of the line ; or he can use a tape and Locke level, or pace the distances unless great accuracy is required.

The instruments then required, are the transit, two transit-rods, one 100-foot steel chain, the level, the level-rod, one or two metallic tapes, axes and hatchets, Locke level, straight-edges or clinometers.

The monthly cost of such a party, though varying in different parts of the country with the demand for the services of engineers

and laborers, is about as follows: 1 chief, $200; 1 transitman, $100; 1 leveller, $75; 1 front chainman, $45; 1 rear chainman, $30; 1 back rodman, $15; 1 level-rodman, $50, 2 axemen together, $60—or a total of $575. With a topographer at $75 and an assistant at $30 the total will be $680. Some companies make an allowance of three to four dollars per week for board for each man; this runs the total to $840, and without the topographical party to $735. It is a much better plan to supply tents and camp-outfits and board the men. It will really prove less expensive on the monthly pay-rolls. The men will be better satisfied, and much time can be put in useful work both day and night that will otherwise be necessarily lost.

Although the above description of organization and division of duties and responsibilities may apply more strictly to preliminary surveys when fully and carefully conducted, the same remarks apply to locating parties; the main difference being that on location greater care and time are consumed in the work, and the simple angle-line is replaced by carefully-run tangents connected by curves; a more careful measurement of the widths and depths of streams and rivers, more detailed examinations of the banks and beds of rivers, and more accurate information on all matters of importance obtained and recorded and the line is run with closer reference to proper grades and easy curves and favorable crossings of rivers. The same general principles and rules are, however, applicable.

17. All notes should be kept neatly and intelligibly, in such a manner that any other engineer may take hold, unstanding de clearly what has been done and be able to proceed with the work.

The line run on the preliminary survey is shown in Fig. 12,

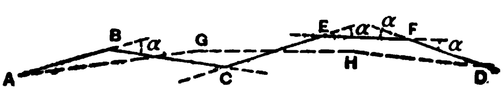


Fig. 12.

and Fig. 13 shows a located line based upon this, also shown by the dotted lines Fig. 12 with a better alignment, fewer in number and easier curves, longer tangents, and better grades;

A, C, and D being points on low ground, B, E, and F, points on higher ground.

The tangents being run to intersection and a little beyond, the angle between one tangent and the prolongation of the other, or the exterior angle, is measured; this is called in America the angle

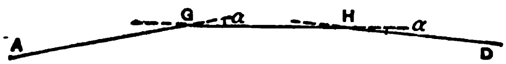


FIG. 13.

of intersection and is commonly indicated by the letter α. In England the interior angle between the tangents, or its equal between the prolongations of the tangents, is used. They are the supplements of each other. This being done, the transitman proceeds to make the necessary calculations and to run in the curves. The following diagram (Fig. 14) shows the necessary relations

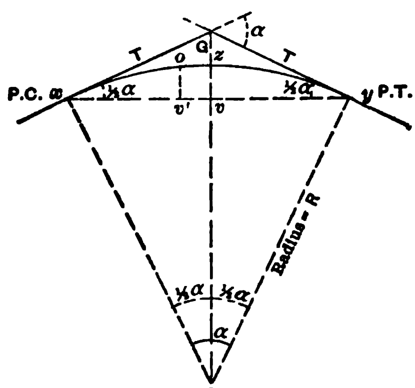


FIG. 14.

existing between angles and distances to be considered, and the necessary equations expressing these relations. These are invariable and true for all circular curves, long or short, and under all conditions, and should be clearly understood in all of their relations, and should be strongly impressed on the student's mind, as they are fundamental and of the utmost importance. If the line is supposed to be running from left to right, as indicated by the numbers on the stakes which are supposed to be placed at intervals of 100 feet, the point of intersection is determined by running

the two tangents to intersection at G, which is determined by running these lines a little beyond the point G and driving two hubs on each tangent in such positions that the point will be between the two. Tacks being driven in each hub, strings are stretched between the tacks, a hub driven under the intersection, and a tack driven accurately at the point of intersection. Then the angle α is measured with the transit. Take the portion AGH of the line Fig. 13 and make the construction as shown in Fig. 14, but on a different and larger scale. We can readily understand and express these relations. AG and GH are the tangents; α, the angle of intersection; $Gx = Gy$ is the tangent length for any particular curve; the tangent point x is the beginning of the curve, called the P.C., and the point y is the end of the curve, called the P. T.; $xC = yC$ is the radius; zv is the versine; Gz, the distance from the curve measured on the prolongation of the radius Cz to the point of intersection of the tangents; the whole angle at the centre C, $xCy, = \alpha$; $GCy = GCx = \frac{1}{2}\alpha$; $xzy =$ length of curve measured on the inscribed polygon; $xy =$ length of chord.

From the right-angled triangle GyC we have

$$Gy = yC \text{ tang } \tfrac{1}{2}\alpha, \quad \text{or} \quad T = R \text{ tang } \tfrac{1}{2}\alpha. \quad . \quad . \quad . \quad (1)$$

Any two of these quantities being known, the third can be found. Usually α is measured on the ground, and R is assumed, or calculated from the assumed degree of curvature, which has a specific meaning in engineering. A 1° curve is defined as one in which an angle of 1° at the centre is subtended by a chord of 100 feet on the circumference. As there are 360° of arc, the length of the full circumference of a 1° curve is 36,000 feet; hence $2\pi r = 36{,}000$, from which we find that the radius of a 1° curve $r = 5729.57$ feet, commonly taken at 5730 feet. A 2° curve is one in which 100 feet of chord subtend 2° of arc; hence the radius of a 2° curve is $\frac{5730}{2} = 2865$ feet; and so on. Then from equation (1) T can be found; or if T is assumed, R can be found.

The length of the chord xy

$$= R \times 2 \sin \tfrac{1}{2}\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The length of the centre ordinate vz

$$= R - R \cos \tfrac{1}{2}\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The length of the curve $xzy = 100$ feet $\times$ number of degrees in the angle α for a $1°$ curve, 100 feet $\times$ one half the number of degrees in the angle α for a $2°$ curve, and 100 feet $\times$ one third the number of degrees in the angle α for a $3°$ curve, and so on; and in general the length of the curve $= 100$ feet $\times$ number of degrees in $\alpha \div$ the degree of curve. . . . . (4)

From the triangle Gvx,

$$Gv = \sqrt{T^2 - \overline{xv}^2} = T \times \sin \tfrac{1}{2}\alpha \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$Gz = Gv - zv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Any other ordinate of the curve, or in fact any ordinate between the chord and the curve, can be found exactly, but the operation is tedious. They can be found, however, with a sufficient degree of accuracy for any purpose by the following rule: Any ordinate is equal to the product of the segments of the chord divided by twice the radius; viz.,

$$\text{the ordinate } ov' = \frac{xv' \times yv'}{2yC(= 2R)}; \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{centre ordinate } zv = \frac{xv \times yv}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

These equations will enable us to determine the lengths and magnitude of the quantities connected with the location of simple curves. It is, however, convenient sometimes to locate curves by offsets from the tangents and chords. The following diagrams (Figs. 15 and 16) will serve to illustrate these methods, which are mainly applicable to short curves and where a great

degree of accuracy is not required; but with care any degree of accuracy may be attained.

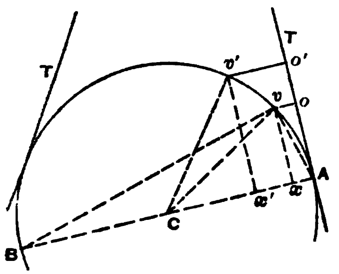


FIG. 15.

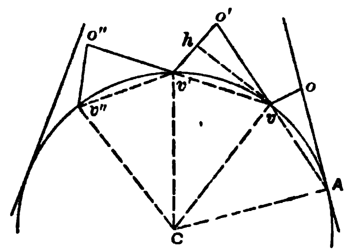


FIG. 16.

In Fig. 15 points v, v', etc., on the curve are determined by laying off perpendicularly to the tangents the calculated offsets ov, $o'v'$, etc. The construction readily gives the following proportions:

$$BA : vA :: vA : Ax, \quad \therefore Ax = vo = \frac{\overline{vA}^2}{BA} = \frac{C^2}{2R};$$

and similarly,

$$BA : v'A :: v'A : Ax', \quad \therefore Ax' = v'o' = \frac{\overline{v'A}^2}{2R} = \frac{C_1^2}{2R}, \quad . \quad (9)$$

and the distances from the tangent point A to offset point on the tangent

$$\left.\begin{aligned} Ao = vx = Cv \sin vCA = R \sin vCA,\\ \text{and for} \qquad\qquad\qquad\qquad \\ Ao' = v'x' = R \sin v'CA. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (10)$$

Where chords and subtended angles are equal the calculations are simplified, though the principles remain the same. That is, chord vA = chord vv' = C = 25, 50, or 100 feet.

In Fig. 16 the first offset is a tangent offset and is found from equation (9), and vA and vv' are equal. It is seen that $vo = \frac{1}{2}v'o'$;

hence for same chord or length of arc the offset from a chord produced is equal to twice the tangent offset for same length of arc. The triangle $o'vv'$ is similar to the triangle vCv', since the angle $vCv' = 180 - (Cvv' + Cv'v)$, $Cv'v = AvC$, by construction; hence $vCv' = 180 - (Cvv' + AvC) = o'vv'$. Both of the triangles $o'vv'$ and vCv' being isosceles and having one angle equal, the remaining angles will also be equal; hence the homologous sides are proportional:

$$vC : vv' :: vv' : o'v'; \quad \therefore o'v' = \frac{\overline{vv'}^2}{vC} = \frac{C^2}{R}. \quad . \quad . \quad . \quad (11)$$

Draw vh perpendicular to $o'v'$, bisecting the angle $o'vv'$. Then the right-angled triangle $o'vh$ and oAv will be equal, since $vA = vo'$, and the angle $o'vh = oAv$; hence

$$ov = \tfrac{1}{2}o'v' = \frac{C^2}{2R}, \text{ same as equation (9).} \quad . \quad . \quad . \quad (12)$$

It may happen that vA is shorter than vv'; in that case C in equation (11) is not equal to C in equation (12). The proper values must be substituted and the results used whatever their relative values may be. First lay off oA, and from o the ordinate ov, then prolong Av to o', and laying off the offset $o'v'$, the points v and v' on the curve will be located. Usually the remaining offsets, except the last one, will be equal.

ART. III.

LOCATION OF SIMPLE, COMPOUND, AND REVERSED CURVES.

18. To locate simple curves we can use either the transit and chain or tape, or the chain and tape alone.

First, to locate a circular curve between two tangents by the usual method of deflections, by means of the transit and chain. Referring to Fig. 17, we will assume that it is desired to run in a 2° 32′ curve. As the radius of a 1° curve is 5729.57 feet, and

$2° 32' = 2\frac{32}{60}° = 2.533°$, the radius of the curve desired is $= \frac{5729.57}{2.533} = 2261.97 = 2262.0$ feet, nearly.

Having measured the angle of intersection $\alpha = 35° 30'$, we find $T = R \times \text{tang } \frac{1}{2}\alpha = 2262.0 \times 0.3201 = 724.07$ feet.

Since the point of intersection G is at station $58 + 70$, and the length of the tangent is 7.24 stations (of 100 ft. each) long, the P. C. of the curve will be at station $58.70 - 7.24 = 51 + 46$, and station $58.70 + 7.24 = 65.94$ will be the P. T. of the curve. Marking these points with a tack driven in a hub at M and N respectively, the instrument is then set up over the point M. As the instrument is placed on the circumference, the angle between the

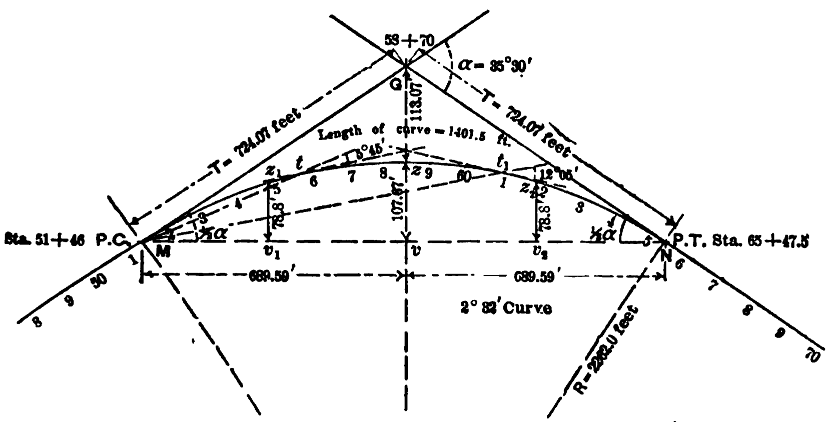


FIG. 17.

line of sight and the tangent will only be equal to one half the angle subtended at the centre by the same arc or chord. Therefore the angle to be deflected for any given length of chord is only one half of that indicated by the degree of curvature for the same distance on the chord or arc. As the degree of curve is $2° 32'$, for each 100 ft. the deflection would be $1° 16'$; and since the P. C. is at station $51 + 46$, station 52 on the curve is 54 ft. from the P. C. Then to find the deflection for 54 ft. we have $100 : 54 :: 1° 16' : x$, the required deflection; we find $x = 41.04'$ of arc. Having sighted the instrument along the tangent from M towards G, with the vernier reading zero of arc, then deflecting an angle of $41.04'$ to the right, the line of sight will intersect the curve at a point 54 ft. from M,

the P. C. The rear chainman holding one end of the chain at *M*, the front chainman holds the rod at the 54-foot link, and moving until the rod is in the line of sight, a stake driven at the point of the rod marks the first full station on the curve; then deflecting 1° 16′ more, the rear chainman goes to the point first marked, and the front chainman, holding the rod at the other end of the chain when straight and taut, moves until the rod is again in line, and has another stake driven, and so on, repeating the same operation, which is continued until the line of sight is obstructed. When, however, four or five hundred feet have been run, it will conduce to the accuracy of the work to place a tack-point on the line and move the instrument to this point. The ordinary transits being graduated to read only to minutes and by estimation to half or at best quarter minutes, rapidly-increasing errors in the position of the points will accrue as the distance from the instrument increases. Having moved the instrument to the turning-point, it is only necessary to sight back to the P. C. having first adjusted the vernier to the zero of the limb, then turning off an angle equal to the sum of the deflections up to this point, the line of sight will be in the direction of the new tangents; or what is still better, after having fixed the point on the curve let the rodman move forward, and, the transitman place him in the prolongation of the first line of sight, it will not be then necessary to sight backwards after moving the instrument, turning off the proper angle, and reversing the telescope, as in the second case the telescope will be pointing in the right direction along the new tangent after turning off an angle equal to the sum of the deflections. In whatever manner the direction of the new tangent is determined, points on the curve beyond are located from it precisely as from the original tangent, and continued to a new turning-point or to the end of the curve. It matters not how many intermediate points are taken; and if at the last point the vernier is so adjusted that when turning off the proper angle to sight along the new tangent, the reading of the vernier shall be the sum of all of the deflections from the beginning to that point, the ending point or P. T. of the curve will be indicated by the reading of the vernier being made equal to one half of the intersecting angle α, in this case 17° 45′. Taking the first turning-point at 500 feet from *M*, or better at station 56, which would be 454 feet from *M*, the sum of the deflections to that point

FORM OF NOTES.

	Station.	Distances.	Deflection	Reading.	Bearing.	Remarks.
	71					
	70					
	9				Tangent	
	8					
	7					
	6				S. 74° 15′ E.	
P. T.	65 + 47.5	47.5	0° 36′	17° 45′		
	5	100	1° 16′	17° 09′	Curve 2° 32′ Rt. $R = 2262′$.	
	4	"	"	15° 53′		
	3	"	"	14° 37′	Angle intersection, 35° 30′.	
	2	"	"	13° 21′		
	1	"	"	12° 05′	Length of curve, 1401.5′.	
	60	"	"	10° 49′		
	9	"	"	9° 33′	" " tangent, 72407.	
	8	"	"	8° 17′		
	7	"	"	7° 01′		
	⊙6	"	"	5° 45′		
	5	"	"	4° 29′		
	4	"	"	3° 13′		
	3	100′	1° 16′	1° 57′		
	2	54	0° 41.04′			
	51 + 46					
	P. C. ⊙				N. 70° 15′ E., tangent.	

would be 5° 45′; and at this point making the reading zero, sighting along the prolongation of the last chord, and turning off 5° 45′, the line of sight will be on the tangent at that point; and if the next transit point is 500 feet farther, or at station 61, the new deflections would sum up 6° 20′ and the reading on the instrument would be 12° 05′. Then moving up to this point, if it is convenient sight back to the P. C. after adjusting the reading to zero, and turn 12° 05′ to get on the new tangent; or sighting to the last transit-point or on the prolongation of the chord between the two last transit-points, the instrument, instead of being adjusted to zero, should read 5° 45′; then turning off 6° 20′, the line of sight will be on the tangent to the curve at the second transit-point, t_1, the first being marked t. From t_1 the deflections and measurements may be continued to the end of the curve, the final reading being 17° 45′, when the P. T. is reached. But to locate this point the closing distance must be known, and consequently the length

of the curve, as indicated in equation (4), $= 100 \times \frac{35.5}{2^\circ\ 32'\ \text{curve}}$ $= 1401.50$ feet.

And as the transit-point was at station 61 and 954 from the P. C., the distance from it, i.e., t_1 to the P. T., is $1401.50 - 954 = 447.50$ to the end of the curve, which will then be station $65 + 47.5$, or the last deflection will be for 47.5 feet. When the rod is held at this link of the chain and swung around in the line of sight, the instrument reading $17^\circ\ 45'$, the rod should be at the P. T. marked N in Fig. 17. The other equations as a rule need not be solved. It may sometimes be well to determine the distance Gz, if it is likely that the curve might lie on unfavorable ground, such as reaching out into a stream, a steep precipice, or any obstacle that ought to be avoided.

From equation (3),

$$vz = R - R \cos \tfrac{1}{2}\alpha, \quad \text{and} \quad Gz = Gv - zv,$$

in which $Gv = T \times \sin \frac{1}{2}\alpha = \sqrt{T^2 - \overline{mv}^2}$. Substituting values, we find $Gv = 220.74$ feet, $vz = 107.67$ feet; consequently $Gz = 113.07$ feet. We can then determine approximately whether the curve will be on good ground by measuring the centre ordinate vz from the chord, or the distance Gz from the point of intersection of the tangents. If not, the degree of the curve can be changed to suit.

From equation (2) the length of the chord is

$$mN = R \times 2 \sin \tfrac{1}{2}\alpha = 1379.18 \text{ feet},$$

and from equation (8) the centre ordinate, Fig. 17, is

$$zv = \frac{mv \times Nv}{2R} = \frac{689.59 \times 689.59}{2 \times 2262} = 105.1 \text{ feet},$$

from which it is seen by the approximate rule that the error in centre ordinate is only $107.67 - 105.1 = 2.57$ feet, for a very long chord, and for chords not over 100 to 200 feet in length there will be practically no error. It will seldom be necessary to find the ordinates for any but short chords, no matter how sharp or how long the curve may be.

From equation (7) any other ordinate may be found: for either quarter point the segments would be 344.79 and 1034.39, and the ordinate

$$z_1v_1 = z_2v_2 = \frac{mv_1 \times Nv_1}{2R} = \frac{344.79 \times 1034.39}{2 \times 2262} = 78.8 \text{ feet},$$

and similarly for any other ordinate.

It is to be noted that the station numbers are carried on around the angle of intersection: this is only for convenience of measuring the length of the tangent and locating the P. C. and the P. T. The proper station number for the P. T. is 65 + 47.5.

It is often the case in locating curves that it is not necessary to run tangents to intersection at all. The tangent is simply run to the P. C. and prolonged forward a short distance, then either a predetermined degree of curve is run from the P. C., or by a few trial deflections a degree of curve is found that will fit the ground; this is then simply run to some point from which a suitable tangent can be ranged. The instrument is then set up at this point, the P. T., a sight taken back to the last turning-point or along the prolongation of this chord, and turning off an angle equal to the sum of the deflections between the two points, the line of sight will be along the forward tangent which is run to the next P. C. To determine the degree of curve from deflections, it is only necessary to remember that if the deflection is 30 minutes for a distance of 100 feet, the curve will be a 1° curve; if 1° deflection for 100 feet, the curve is a 2° curve; and if in a distance of 400 feet the deflection is 3° 30′, the deflection for 100 feet will be 52½ minutes of arc, and 52½′ × 2 = 105′ = 1° 45′ curve.

19. It is often convenient to compound a curve, that is, to run a curve for a certain distance as a 1°, 2°, 2° 30′ curve, then to change to a 30′ or 1° 30′, 2° 30′ or a 2° 15′ or 2° 45′ curve. Having run the curve to the proper point, called the point of compound curve, the P. C. C., the instrument is moved to this point, the direction of the tangent determined as in simple curves, and the curve completed with the proper deflections for the new degree of curve. Fig. 18 represents this condition. All of the relations and equations previously found apply to each of the curves considered separately, and if run without limitations as to the relative positions of the principal tangents AG and GH, the curves would be run as

just explained, and the direction of the final tangent GH determined as already explained. If, however, the two tangents have been run to intersection at G and the angle α measured, then there will exist certain interdependent relations between the lengths and the degree of curves. One or two of the simpler relations will be discussed. Whether passing from one main tangent to another by a simple or a compound curve, the sum of the deflections will be the same and equal to α. The following problems are easily solved.

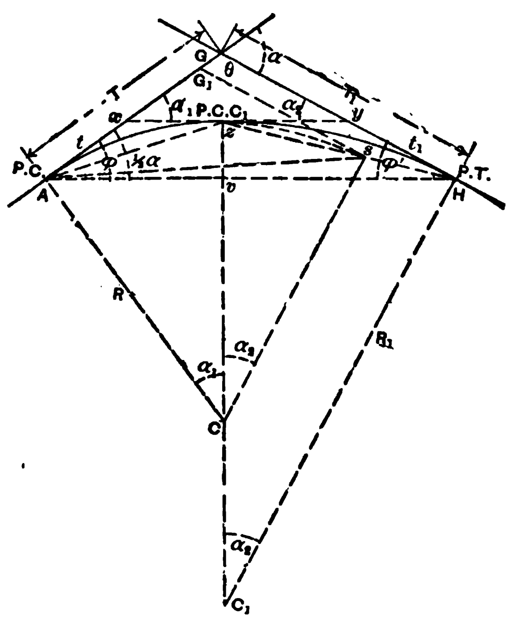


FIG. 18.

Let $AG = T$; $GH = T_1$; radius $AC = R$; $HC_1 = R_1$; $\alpha = GAH + GHA$
$= Gxy + Gyx = \alpha_1 + \alpha_2$; $Ax = xz$; $Hy = yz$.

1st. Given: the tangents AG and GH, the degree of the curve from A to z, with the shorter radius R in consequence, and also the angle of intersection at $G = \alpha$, to find α_1 and α_2, and the radius R_1, and degree of curve for the flatter curve zH.

In the triangle AGH we have the sides AG and GH, and the angle $AGH = \theta = 180 - \alpha$; $\alpha = GAH + GHA = \phi + \phi'$. Hence

$$T_1 + T : T_1 - T :: \text{tang } \tfrac{1}{2}(\phi + \phi') : \text{tang } \tfrac{1}{2}(\phi - \phi'), \quad (13)$$

from which ϕ and ϕ' can be found. Then, in the same triangle,

$$\sin \phi : \sin \theta :: GH : AH; \quad \therefore\ AH = GH \frac{\sin \theta}{\sin \phi}. \quad . \quad (14)$$

If then the curve from A to z be extended to s, a point at which $GAs = \frac{1}{2}(\alpha_1 + \alpha_2) = \frac{1}{2}\alpha = GsA$, then s will be a tangent point for a line G_1s parallel to the tangent GH, since

$$\phi + \phi' = \alpha = G_1As + G_1sA = GG_1s.$$

Hence, since AG and AG_1 coincide, GH and G_1s must be parallel; consequently the lines sC and HC_1, being radii and perpendicular to parallel tangents G_1s and GH, must themselves be parallel. And since z, the P. C. C., is a point common to both branches of the curve, the two triangles zCs and zC_1H, being isosceles since $zC = sC$ and $zC_1 = HC_1$ and the angles at C and C_1 equal by construction, must have their bases zs and zH parallel, and having one point z in common, they must coincide; therefore the tangent point s must be found on the chord of one branch of the compound curve. Then in the triangle sAH we have, from equations (13) and (14), the line AH, and from equation (13) ϕ and ϕ'; consequently $sAH = \phi - GAs = \phi - \frac{1}{2}\alpha$. The line As being the chord of the simple curve Azs with radius R, we have $As = R \times 2 \sin \frac{1}{2}\alpha$. Having two sides and the included angle in sAH, by applying similar equations to (13) and (14) we can find the angles AsH and sHA and the side sH. The chord $zs = R \times 2 \sin \frac{1}{2}\alpha_2$, and the chord $zH = R_1 \times 2 \sin \frac{1}{2}\alpha_2$; $zH = zs + sH$. Substituting in this last equation the values of zs and zH, we have

$$R_1 \times 2 \sin \tfrac{1}{2}\alpha_2 = R \times 2 \sin \tfrac{1}{2}\alpha_2 + sH. \quad . \quad . \quad . \quad (15)$$

Hence

$$R_1 = R + \frac{sH}{2 \sin \frac{1}{2}\alpha_2}. \quad . \quad . \quad . \quad . \quad (16)$$

R is known, sH has been found as explained above, and

$$\alpha_2 = 2zHG = 2 \times (\phi' - sHA), \quad . \quad . \quad . \quad . \quad (17)$$

and

$$\alpha_1 = \alpha - \alpha_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Then by the equations $t = R \text{ tang } \frac{1}{2}\alpha_1$, and $t_1 = R_1 \text{ tang } \frac{1}{2}\alpha_2$, the length of tangents $Ax = xz = t$ and $zy = yH = t_1$ can be found, also the lengths of the two curves from A to z and from z to H.

20. The following is an example under one set of conditions: Given $AG = T = 450$ feet; $GH = T_1 = 515$ feet; $\alpha = 15° \; 00'$. Degree of curve from A to z, 1°48'; then radius $R = 3183$. Applying equation (13) to the triangle AGH,

$$T_1 + T : T_1 - T :: \tan \tfrac{1}{2}(\phi + \phi') : \text{tang } \tfrac{1}{2}(\phi - \phi');$$
$$965 : 65 \quad :: \text{tang } 7° \; 30' \quad : \text{“} \quad \text{“} \quad \text{“} \; ;$$

from which, either using table of natural tangents or applying logarithms,

$$\tfrac{1}{2}(\phi - \phi') = 0° \; 30' \; 29'', \quad \tfrac{1}{2}(\phi + \phi') = 7° \; 30';$$

hence

$$\phi = 8° \; 00' \; 29'' \quad \text{and} \quad \phi' = 6° \; 59' \; 31''; \quad \phi + \phi' = \alpha = 15° \; 00'.$$

Then, from equation (14),

$$AH = GH\frac{\sin \alpha \text{ or } \theta}{\sin \phi} = 515\frac{\sin 15°}{\sin 8° \; 00' \; 29''} = 956.78 \text{ feet};$$
$$As = R \times 2 \sin \tfrac{1}{2}\alpha = 3183 \times 2 \sin 7° \; 30' = 830.93 \text{ feet}.$$

Then, in the triangle sAH, angle $sAH = \phi - \frac{1}{2}\alpha = 0° \; 30' \; 29''$; the sum of the angles $AsH + sHA = 180° - 30' \; 29'' = 179° \; 29' \; 31''$; AH and As already found. Then

$$AH + As : AH - As :: \text{tang } \tfrac{1}{2}(AsH + sHA) : \text{tang } \tfrac{1}{2}(AsH - sHA);$$
$$1787.71 : 125.85 \quad :: \text{tang } 89° \; 44' \; 45'' \quad : \text{“} \quad \text{“} \quad \text{“} \quad \quad \text{“}$$

$\therefore \frac{1}{2}(AsH - AHs) = 86° \; 22' \; 28°$; $AsH = 176° \; 7' \; 13''$; and $AHs = 3° \; 22' \; 17''$. Then, in the same triangle,

$$sH = As\frac{\sin 0° \; 30' \; 29''}{\sin 3° \; 22' \; 17''} = 125.33 \text{ feet}.$$

The centre angle of the curve zsH

$$= \alpha_2 = 2(\phi' - sHA) = 2(6^\circ\,59'\,31'' - 3^\circ\,22'\,17'') = 7^\circ\,14'\,28'' = \alpha_2.$$

The centre angle of the curve Az

$$= \alpha_1 = \alpha - \alpha_2 = 7^\circ\,45'\,32''.$$

From equation (16) the radius of the curve zsH

$$= R_1 = R + \frac{sH}{2 \sin \frac{1}{2}\alpha_2};$$

$R_1 = 3183 + \dfrac{125.33}{2 \sin 7^\circ\,14'\,28''} = 4175.33$ feet, and the degree of the curve $= \dfrac{5729.57}{4175.33} = 1^\circ\,22'$.

The length of the curve from A to $z = 100\dfrac{7^\circ\,45\frac{1}{2}'}{1^\circ\,48'} = 431.02$ feet;

" " " " " " z to $H = 100\dfrac{7^\circ\,14\frac{1}{2}'}{1^\circ\,22'} = 529.88$ feet;

and the total length on the curves from A to $H = 960.90$ feet.

The tangents $Ax = xz$ and $zy = yH$ can now be found by applying equation (1). The chords Az and zH can be found from equation (2).

In the above example the length of the tangents and the position of the tangent points were fixed, and the position of the P. C. C. had to be found. In the following example the position of one tangent point and length of one tangent is fixed, and it will be required to find the length of the second tangent and the point of tangency.

Given the length of the tangent AG, the angle α, the angle α_1, and the radius R. Required to find the radius of the second curve zsH and the length of the tangent GH.

Referring to Fig. 18, from the data given we can find at once the length of the tangent $Ax = xz$, the length of the chord Az, and the length of the curve Az.

$$Ax = xz = t = R \text{ tang } \tfrac{1}{2}\alpha_1; \quad Az \text{ (the chord)} = 2R \sin \tfrac{1}{2}\alpha_1;$$

$$\text{the length of the curve } Az = 100\frac{\alpha_1}{\text{degree of curve}}. \quad . \quad (19)$$

Then, in the triangle Gxy, $Gx = AG - Ax$; α_1 is known and $\alpha_2 = \alpha - \alpha_1$; hence all of the angles and one side are known, and we can find Gy and xy. The length of the tangent line

$$zy = yH = t_1 = xy - xz \quad \text{and} \quad GH = Gy + yH. \quad . \quad . \quad . \quad (20)$$

From equation (20) we fix the point of tangency H and the length of the tangent $yH = t_1$. Then

$$R_1 = t_1 \cot \tfrac{1}{2}\alpha_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$\left.\begin{array}{r} \text{Length of chord } zH = 2R_1 \sin \tfrac{1}{2}\alpha_2; \quad . \quad . \quad . \quad . \\ \text{Length of curve } zH = 100 \text{ ft.} \dfrac{\alpha_2}{\text{degree of curve}}. \quad . \end{array}\right\} (22)$$

Given $GA = T = 1091.12$ ft.; $\alpha = 74°$; $R = 955.37$; $\alpha_1 = 42° 54'$; $\alpha_2 = 74° - 42° 54' = 31° 06'$.

Then from equation (19),

$$Ax = xz = t = R \text{ tang } \tfrac{1}{2}\alpha_1 = 375.66 \text{ feet};$$
$$Gx = AG - Ax = 1091.12 - 375.66 = 715.46 \text{ feet}.$$

Then, in the triangle Gxy,

$$xy = Gx\frac{\sin \alpha}{\sin \alpha_2} = 1331.5 \text{ feet} \quad \text{and} \quad Gy = Gx\frac{\sin \alpha_1}{\sin \alpha_2} = 942.9 \text{ feet};$$
$$t_1 = yH = zy = xy - xz = 1331.5 - 375.66 = 955.84 \text{ feet},$$

which is the length of the tangent for the second curve.

Then the length of the main tangent $GH = Gy + yH = 942.9 + 955.84 = 1898.74$ feet; $yH = t_1 = R_1 \tan \frac{1}{2}\alpha_2$;

$$\therefore R_1 = t_1 \cot \tfrac{1}{2}\alpha_2 = 955.84 \times \cot 15° 33' = 3435.00 \text{ feet}.$$

$$\text{Length of chord } Az = 2R \sin \tfrac{1}{2}\alpha_1 = 698.74 \text{ feet};$$
$$\text{" \quad " \quad " } zH = 2R_1 \sin \tfrac{1}{2}\alpha_2 = 1841.71 \text{ feet};$$
$$\text{Length of curve } Az = 100 \text{ ft.}\frac{42° 54'}{6°} = 715.00 \text{ feet};$$
$$\text{" \quad " \quad " } zH = 100 \text{ ft.}\frac{31° 06'}{1° 40'} = 1866.00 \text{ feet}.$$

The degree of the first curve is $\dfrac{5729.57}{955.37} = 5° 59' 50''$, or practi-

cally a 6° curve; and of the second curve, $\frac{5729.57}{3435} = 1° \ 40' \ 10''$, nearly a 1° 40′ curve.

This last example is applied when a simple curve, if run all the distance between the tangents, would rest partly on good ground and partly on bad ground, in which case a 6° curve would be run to a certain point resting on good ground; then by changing to a 1° 40′ curve the line would be well adapted to the ground.

The following is a useful application of compound curves (see Fig. 19):

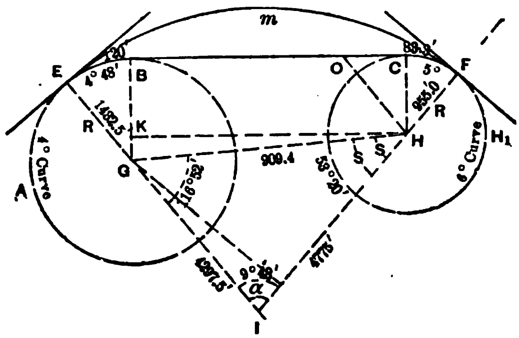


FIG. 19.

21. Having run any two curves, such as *AEB*, a 4° curve, and CFH_1, a 6° curve, which are connected by a straight tangent *BC*, required to find the points *E* and *F* of tangency for any curve, say 1° curve, thereby connecting the curves by an easy curve instead of the straight tangent.

Given, then, *AEB*, a 4° curve, and CFH_1, a 6° curve, $BC = 774.0$ feet, to find the distances *BE* and *CF* in order to locate the P. C. C.'s *E* and *F*. We have by construction $KG = BG - BK = BG - CH = R - R_1 = 1432.5 - 955 = 477.5$ feet. From the triangle *KGH*,

$$KH = KG \times \text{tang}\, KGH; \quad \therefore \ \text{tang}\, KGH = \frac{KH}{KG} = \frac{774}{477.5} = 1.62094;$$

hence

$$KGH = 58° \ 20', \quad \text{and} \quad GH = \frac{KH}{\sin KGH} = 909.4 \text{ feet.}$$

Since EI and IF are the radii of the substitute curve, which is a 1° curve in this case, they are always known, and in this case equal to 5730 feet; hence

$$GI = 5730 - 1432.5 = 4297.5 \text{ feet};$$
$$IH = 5730 - 955 = 4775 \text{ feet};$$
$$GH = 909.4 \text{ feet}.$$

All three sides in the triangle GHI are known, consequently the angles can be found. Applying the proper formulæ, we find

$$GIH = \alpha = 9^\circ\ 48'; \qquad GHI = s = 53^\circ\ 20';$$

and
$$HGI = 116^\circ\ 52';$$

$$EGH = GIH + GHI = \alpha + s = 63^\circ\ 08'.$$

Then

$$EGB = EGH - KGH = 63.08 - 58.20 = 4^\circ\ 48',$$

and the length of arc BE for a 4° curve $= 100 \text{ ft.} \frac{4.8}{4} = 120$ feet. And similarly

$$CHF = OHF - OHC = \alpha - EGB = 9^\circ\ 48' - 4^\circ\ 48' = 5^\circ\ 00',$$

and the length of the arc CF for a 6° $= 100\frac{5}{6}$ ft. $= 83.3$ feet. The points of tangency E and F being found, the simple curve EmF can be run in. By similar application of the above principle, the joining of a curve and a straight line can be effected by flattening the curve for a short distance from the P. C. and P. T., rendering an easier and more gradual change from a straight line to a sharp curve.

The preceding examples are selected as well representing the principles involved in compound curves. There may be and are simpler solutions of these problems; and while simplicity and rapidity of solving such problems are of the first importance, the writer has adopted the above forms as clearly setting forth the essential and constant relations existing between the quantities. The first two examples were worked out accurately, carrying distances to hundredths, and angles to seconds of arc. In the last, distances are

only carried to tenths, and arcs to minutes. All the computations should be accurately made, but it must not be expected that on the field curves and tangents will come together as accurately as they are plotted on paper and scaled from the computations. The transit is ordinarily graduated to read only to minutes, and by estimates to half or quarter minutes; the chain is not as accurate as a steel tape; the ground is rough and broken: consequently absolutely accurate results cannot be expected. In running a 2° curve for a distance of 1500 to 2000 feet and missing closing on the tangent with 6 inches or 1 foot of the exact point is doing about as well as if you should waste a week's time, as is often done by young engineers, in the vain effort to get exactly accurate results. The curvatures can be slightly changed near the P. T. to make the stakes come in properly. After the roadbed has been constructed, with clear lines of sight, using a steel tape on the smooth surface of the roadbed, any such slight inaccuracies can be adjusted; and if accuracy is required in the beginning, accurate instruments and tapes should be used. But such care is not necessary in locating lines, as it would cause too much waste of time. Resident engineers on construction will have the time, and should accurately adjust the alignment where necessary. This subject will be further discussed under the head of Track-laying.

REVERSED CURVES.

22. In compound curves both branches lie on the same side of the common tangent, and they necessarily have unequal radii. If, on the contrary, the two branches of the curve having a common tangent lie on different sides of the tangent, the one must turn to the right and the other to the left, and they form what is termed a reversed curve. The common tangent point is called the P. R. C., the point of reversed curve.

A reversed curve may connect two parallel lines, or lines inclined to each other, and the two radii may be unequal or equal. The P. R. C. of a reversed curve connecting two parallel lines or tangents will be on the line connecting the tangent points.

Fig. 20 represents the case of a reversed curve connecting two parallel tangents, AB and CD.

Given the perpendicular distance between the parallel tan-

gents AB and $DC = BF = EC = x$, and $BC = y$, the distance between the tangent points B and C. Required the point G, the P. R. C., and the equal radii $OG = O'G = R$. From the construction it is evident that the angles of intersection of the tangents, and consequently the centre angles BOG and GO_1C, are equal, $\alpha = \alpha_1$, and, as the sides of the triangles BO and OG are equal and equal to the GO_1 and O_1C, the triangles BOG and GO_1C are equal; hence $BG = GC = \frac{1}{2}BC$, and the P. R. C. is at the centre

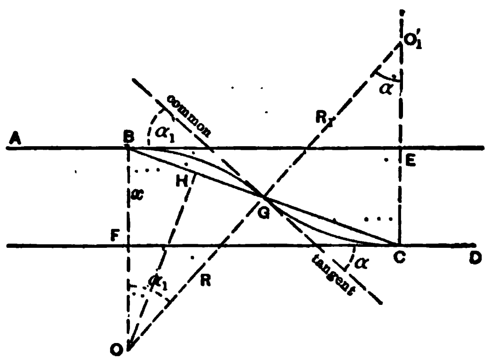


FIG. 20.

of BC. In the triangle BOG draw OH perpendicular to BG. Then the triangles BOH and CBE are similar, having equal angles; hence

$$BO : BH :: BC : EC, \quad \text{or} \quad R : \tfrac{1}{4}y(\tfrac{1}{4}BC) :: y : x; \quad \therefore R = \frac{y^2}{4x}. \quad (23)$$

Assuming $y = 300$ feet, $x = 14$ feet, $\therefore R = 1607.1$, and the curves will be 3° 34′ curves. If R and x are given, y can be found; or if R and y are given, then x can be found.

$$BG = 2R \sin \tfrac{1}{2}\alpha_1, \quad \therefore \sin \tfrac{1}{2}\alpha_1 = \frac{BG}{2R} = \frac{150}{3214.2} = 0.0467;$$

$$\therefore \tfrac{1}{2}\alpha_1 = 2^\circ\ 41'; \qquad \therefore \alpha_1 = \alpha = 5^\circ\ 22'.$$

Given $EC = x$, $BC = y$, radius $BO = R$ of a reversed curve connecting two parallel tangents (see Fig. 21). Find the point G, the P. R. C., and the radius O_1G of the second curve.

Drawing the perpendiculars OH and O_1K, from the similar triangles BOH and BCE we have, from Fig. 21,

$$EC : BC :: BH : BO;$$

$$\therefore\ BH = \frac{EC \times BO}{BC} = \frac{xR}{y}, \text{ and } 2BH = BG = \frac{2Rx}{y}. \quad (24)$$

$$GC = BC - BG = y - \frac{2Rx}{y}; \quad (25)$$

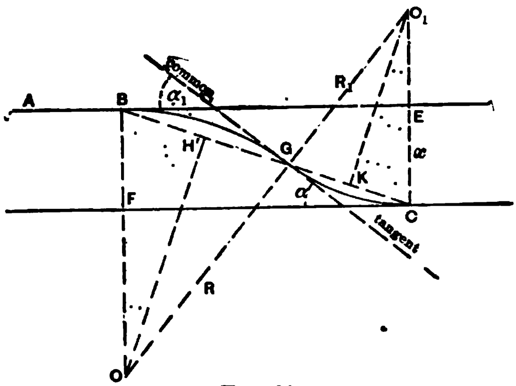


Fig. 21.

and from similar triangles BCE and KO_1C,

$$EC : BC :: KC : O_1C;$$

$$\therefore\ R_1 = O_1C = \frac{BC \times KC}{EC} = \frac{y \times \frac{1}{2}GC}{x} = \frac{y \times GC}{2x}. \quad (26)$$

Given $y = BC = 200$ feet, $x = EC = 14$ feet, $R = 1000$ feet. The first portion BG then is a 5° 44′ curve. Then

$$BG = \frac{2 \times 1000 \times 14}{200} = 140 \text{ feet.}$$

Hence $GC = 200 - 140 = 60$ feet. The radius of the second curve $R_1 = O_1C = \frac{200 \times 60}{28} = 428.6$ feet, corresponding to a 13° 22′ curve.

$$\sin \tfrac{1}{2}\alpha = \frac{BG}{2R} = \sin \tfrac{1}{2}\alpha_1 = \frac{GC}{2R_1} = \frac{140}{2000} = \frac{60}{857.2} = 0.07;$$

$$\therefore\ \tfrac{1}{2}\alpha = \tfrac{1}{2}\alpha_1 = 4^\circ\ 1'; \qquad \therefore\ \alpha = \alpha_1 = 8^\circ\ 2'.$$

Required to find the common radii for a reverse curve to connect two tangents not parallel. (See Fig. 22.)

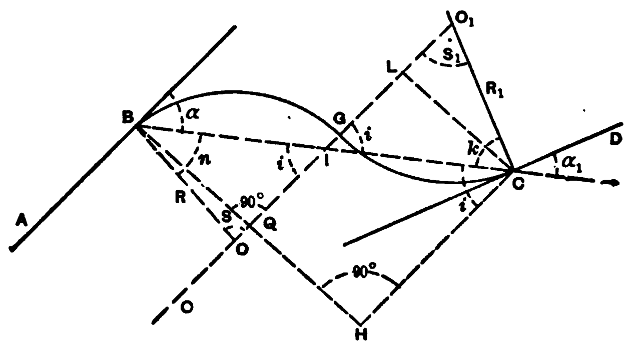


FIG. 22.

Given the angles α and α_1, and $BC = y$, the line joining the tangent points on the tangent lines AB and CD.

$$BO : OI :: \sin i : \sin n; \qquad O_1C : O_1I :: \sin i : \sin k;$$
$$BO \sin n = OI \sin i, \quad \text{and} \quad O_1C \sin k = O_1I \sin i;$$

hence

$$BO \sin n + O_1C \sin k = (OI + O_1I) \sin i;$$
$$BO = O_1C = R; \qquad OI + O_1I = OO_1 = 2R;$$
$$R \sin n + R \sin k = 2\,R \sin i; \qquad \sin i = \frac{\sin n + \sin k}{2};$$
$$\sin n = \cos\alpha; \ \sin k = \cos\alpha_1; \ \therefore \ \sin i = \frac{\cos\alpha + \cos\alpha_1}{2}. \quad . \quad (27)$$
$$BH = BC \sin i; \qquad BH = BQ + QH; \qquad BQ = R \sin S;$$
$$QH = CL = R \sin S_1; \ \therefore \ BC \sin i = R(\sin S + \sin S_1).$$
$$\therefore R = \frac{BC \sin i}{\sin S + \sin S_1}; \ S = 180 - (n + i); \ S_1 = 180 - (k + i); \ (28)$$
$$n = 90 - \alpha; \ k = 90 - \alpha_1. \quad . \quad . \quad . \quad . \quad . \quad (29)$$

If $\alpha = 30°$, $\alpha_1 = 20°$, and $BC = 1400$ feet, then, from equation (27), .

$$\sin i = \frac{\cos 30 + \cos 20}{2} = 0.902\,86; \ \therefore \ i = 64° \ 32';$$

$n = 90 - 30 = 60$, $k = 90 - 20 = 70$, from equation (29); and from equation (28)

$$S = 180 - (60 + 64.32) = 55° \ 28'$$

and

$$S_1 = 180 - (70 + 64.32) = 45° \ 28'.$$

Substituting in value of R,

$$R = R_1 = \frac{BC \sin i}{\sin S + \sin S_1} = \frac{1400 \times 0.90286}{0.82380 + 0.71284} = 822.6 \text{ feet.}$$

In these examples, the tangents and curves represent the centre line of tracks, and are the more common and simplest forms of passing from one track to another either parallel with or inclined to the other. A great many problems based upon many conditions can and do arise, but the principles are the same as those given in the above examples. The main tracks may be straight or they may be curved. All of these matters are very elaborately discussed and problems for nearly all possible conditions are worked out and illustrated in the works of Wellington, Searles, Henck and others.

ART. IV.

THE THEORY OF MAXIMUM ECONOMY IN GRADES AND CURVES.

23. Although the solution of the problems in regard to grades and curves is of prime importance, and, if correct values of the terms which enter into the formulæ are known or can be found, some reliance may be placed on them, just in proportion as the facts and data may be accurate in the same proportions will the results be trustworthy. The writer will not, therefore, undertake to discuss this subject, and will only give a few statements and partly empirical formulæ.

The selection of the best line having been made, either from considerations of economy in construction, from the greater amount of present business, or from the subsequent development of the country, the question of greatest importance is what are the best grades and curves to be adopted, as they both affect

the first cost of construction and the subsequent and continuous expense of operating the road. The grades and curves which reduce the sum of these two elements of expense to a minimum will be the best on any given line.

The annual expense per mile of operating a road depends, first, upon the interest on the cost of equipment, such as engines and cars, engine-houses, depot buildings, and all those items of cost which require the expenditure of large sums of money in movable or fixed property and structures; and secondly on the actual expenses annually of all employees, costs of repairs and renewals, and the expense for fuel, oil, waste, and the thousand other incidentals. Such data can only be obtained from the experience of other roads.

24. The tractive force of an engine is a force with which it can pull a load or train, and is limited by the reaction of the drivers against the rail. The reaction depends upon the weight on the drivers, the number of drivers, and the coefficient of friction or adhesion. Some locomotives have 4 drivers, some 6, and some 8, the weight on each being from 10,000 to 12,000 lbs., and in many later types a still greater weight. The coefficient of friction varies between 0.09 and 0.37, the average being between 0.15 and 0.25.

An engine, then, with four pairs of drivers, or eight drivers with a weight on each of 12,000 lbs. and a coefficient of 0.20, will have a tractive force of $12,000 \times 8 \times 0.20 = 19,200$ lbs.

25. The resistances to be overcome are, first, the friction in the moving parts of the engine and train, impacts and oscillations, and the resistances of the atmosphere. These elements of resistance vary with the condition of the road and of the rolling machinery, and the state of the weather. That part of the resistance due to friction may be taken as constant at all speeds of the train; the remaining items increase with the speed. The actual amount of resistance can be determined by means of dynamometers placed between an engine and a train, and is usually expressed as a certain number of pounds per ton.

While there are many formulæ given to express the resistance due to the above causes, some much more complicated than others, but probably no more accurate, the following formula for these resistances on a level, straight track, which is assumed to be in a

fairly average condition, with good rolling-stock and reasonably calm weather, is found in Vose's Civil Engineering, page 34:

$$r = \frac{v^2}{171} + 8; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

in which r is the resistance in pounds and v the velocity in miles per hour. With a velocity of 25 miles per hour the resistance will be $r = 11.66$ pounds per ton of the entire train weight, and this under unfavorable conditions should be increased from 25 to 50 per cent. Under these conditions an engine exerting a tractive force of 19,200 lbs. could haul, at a speed of 25 miles per hour on a straight and level track, $\frac{19200}{11.66} = 1647$ tons, weight of train and its load.

26. The second cause of resistance is that arising from the grades, and the force necessary to overcome it is simply expressed by the weight multiplied by the ratios of the vertical rise to the length of the slope or grade. Then in Fig. 23 the resistance is to

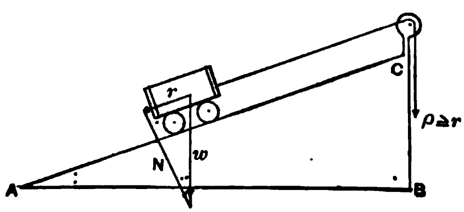


FIG. 23.

the load as the vertical CB is to the length of the slope AC, or, in symbols,

$$r : w :: BC : AC; \quad \therefore r = w\frac{AC}{BC} \quad . \quad . \quad . \quad . \quad (31)$$

If, then, $w = 1$ ton of 2000 lbs., $BC = 1$ mile $= 5280$ feet, and the rise or grade per mile $BC = 26.4$ feet, the resistance per ton, $r = 2000 \cdot \frac{26.4}{5280} = 10.0$ pounds. The power, then, required to run a mile at the above rate of grade and a speed of 25 miles per hour would be $11.66 + 10.0 = 21.66$ pounds per ton of load, or the

same power would haul the train at the same speed on a level for nearly two miles.

The resistance due to curvature of the track arises from the friction of the wheels upon the top of the rail and that of their flanges upon the sides of the rail. This resistance has been taken, as the result of experiments, at from one half to one pound per ton multiplied by the degree of curvature. If r_2 is the resistance per ton, and the degree of curvature D, then $r = 0.5$ to $1 \times D = 0.5$ to 1 pound for a 1° curve and from 5 to 10 pounds for a 10° curve.

We would then have for the resistance in pounds per ton on a 10° curve on a grade of 26.4 feet per mile, running at the rate of 25 miles per hour, $11.66 + 10.0 + 10.0 = 31.66$ pounds, or with a tractive force of 19,200 pounds the locomotive could haul only 606 tons.

It should, however, be the object of an engineer to avoid as far as practicable locating curves on grades, or at any rate to reduce or lighten the grade, so that the combined resistance should not exceed that on the ruling grades on the straight portions of the line. As the resistance due to grade is expressed by $r_1 = 2000 \frac{G_1}{100}$ for a load of one ton on a rate of grade G_1, and the resistance due to curvature is $r_2 = D$, in which D is the degree of curvature, then placing $D = 2000 \frac{G_1}{100}$; $\therefore G = \frac{D}{20}$, which gives the grade on a straight line which offers the same resistance as a given curve. If, then, the ruling grade on the line is 0.5 foot in 100 feet, the curve of an equivalent resistance is $D = 0.5 \times 20 = 10°$. If, therefore, in establishing a grade line on the profile a grade of 0.5 foot per 100 feet should fall on a 10° curve, in order to maintain a uniform resistance on the line the grade should be taken out and that portion of the line occupied by the curve should be level, or the grade should be reduced to 0.25 foot per 100 feet, and the degree of curvature also reduced from a 10° to a 5° curve.

Having fixed upon the ruling gradients and maximum degrees of curvature, the rate of grade varies from 0 to 52.8 feet per mile, and in some exceptional cases as high as from 80 to 100 feet per mile, and the degree of curvature from 0° to 10°, with a usual maximum of 6°. The most important duty of an engineer is to lay

down the grades upon the profile to the best advantage. This requires the careful comparison of several rates of ascending and descending grades. The only points on roads that are fixed by considerations other than the cost of construction and keeping within the maximum established rate of grade are the crossing of swamps and rivers, where the grade-line must be above high water; the crossing of other roads, which may be on the same level, or high enough above to allow the passage of vehicles or trains underneath, the clearance in either case varying from 18 to 25 feet; or such fixed grade heights are determined by considerations of convenience or are fixed and regulated by law. The law regulating the crossing of navigable streams requires a height of about 80 to 100 feet above low water or about 40 feet above high water.

After these requirements and conditions economy of construction requires that grade-lines shall ascend towards the tops of high points and descend towards the low points, by which both the amounts of excavation and embankments are lessened, as is also the cost of bridges and trestles.

It is also desirable to so locate the grade-lines that the amounts of excavations and embankments of a given portion or section of the line may be as nearly equal as possible.

A fine thread of silk stretched along the surface line of the profile will enable the engineer to observe the effect of changing the grade-line by shifting the position of the thread, and to make comparisons which will greatly facilitate the final and proper adjustment of the grade-line to suit the required conditions, by the lines representing respectively the vertical and horizontal distances. A careful study of the profile and careful comparisons of the effects of many different grades will prove full compensation for the time consumed.

ART. V.

LOCATION OF HIGHWAYS AND COUNTRY ROADS.

27. The same general principles which have been explained in regard to the location of railroads apply to the location of highways. The relations between the tractive forces and the resistances are not so simple or capable of as close calculation and adjustment as on railways, for the reasons that the wheels do not run on smooth, hard surfaces, nor do these as a rule rest on as firm and carefully constructed and carefully drained foundations. Although not desirable, very steep grades for short distances can be overcome by the exercise of great efforts for short periods of time, followed by breathing-spells to the animals furnishing the power. Very sharp curves may be used, the vehicles being turned through great angles in short distances; and although small and large bridges are desirable, they are not so necessary on highways as they are on railways. Small streams can be forded as often as may be necessary in order to keep the road on the best ground. Large streams can be crossed on ferry-boats, the cost of maintaining and operating being small. Traffic may be interrupted for a few days during periods of floods. In thinly settled districts this may not be a matter of very great importance, though often causing much inconvenience to individuals and communities. In thickly settled communities these matters are of the greatest importance; easy grades and curves; firm roadbeds, and these well drained; many bridges across both small and large streams; in such cases highways should be located and constructed more nearly on the principles and under the conditions required for railways, and the best lines, grades, curves, and character of structures and construction should be adopted as the cost of construction and maintenance will justify, compared not with the pecuniary benefits accruing to private corporations, but as measured by the pecuniary benefits and conveniences derived by the community as a whole.

In a great many respects good highways are of more benefit to

a given district or to a community of districts than railways have ever been or will probably ever be.

Fortunately, builders of railways are compelled, by the relations between the power employed and the resistances to be overcome; by the great cost of construction, equipment, and operating; by the necessity of securing a tangible and obvious return for the money invested, to select in the first place only such lines as are the best, without, except in a few instances, regard to the wants or needs of particular communities, and trusting to a great extent to future developments being forced upon the communities by the superior advantages offered for transportation to large and important centres of commerce.

Unfortunately for the builders of highways, the roads are usually constructed in a large and extended and thinly settled country. All roads constructed by individuals have only the object of benefiting some particular community or district, and when these are connected as the interchange of trade demands, they are simply connected in some haphazard way, without regard to the greatest conveniences and benefit of either. There are in this country a few and only a few exceptions to the above general statement. Such roads when once established are rarely changed. Moreover the roads in any particular district are neither located nor constructed with reference to the best lines or character of construction. The roads are forced to be placed along boundary-lines between neighboring landowners, or they to a great extent become boundary-lines and cannot be altered without trouble and litigation.

The highways of the relatively small European states or governments have been brought to a high degree of perfection, both as regards location and construction; and having them, the great benefits derived have been fully realized, and the people are willing to be taxed to maintain them in a high state of perfectness.

Hundreds of thousands of dollars have been expended in this country annually for the last half-century on ill-located and badly constructed roads, with the result that at this time, except in a few States, and frequently only in a few districts of these States, the country is no better off than in the beginning in respect to decent roads.

The demand now is and will be for the next half-century, for

better roads. New roads must be located and built on sound engineering principles and practice, and old roads must be relocated and constructed on the same considerations. There is no more important subject in the whole range of engineering than that of highway construction.

When engineers know how to locate and build highways and the people are aroused, as is now becoming the case, proper highway facilities will be provided. This much-desired end can and will be gradually attained, but a beginning should now be made. The location, construction, and maintenance of roads, the relations between power and resistance, and the cost will now be considered.

The same instruments, the same principles and rules, within certain limits, must be used and followed as are required in railway construction.

28. The best location for a highway should be determined by the following principles, leaving out the consideration of the cost:

1st. A straight and level line, and the nearest approach to these conditions the more perfect the road.

2d. A straight and uniformly inclined road.

3d. A route which affords the easiest grades, remembering that the easiest grade for any given road will depend upon the material used for its surface.

4th. With the established grades select the shortest and most direct route between the main termini.

5th. Alternate ascents and descents should be avoided.

6th. Cross railroads either high enough above or deep enough below to allow ample head-room or clearance for the traffic on the lower road, which ever it may be.

It is not the absolute cost that must be considered in modifying the above rules and principles, but the cost as compared with the actual value of the present traffic, and a fair allowance for prospective traffic resulting from the increase of agricultural, timber, and mineral development and the establishment of manufactories and other industrial enterprises. The considerations of costs require:

1st. So placing the line on the surface of the ground as to reduce the cost of construction, with the established curves and grades, to a minimum.

2d. To cross streams, rivers, and other lines of communication as nearly as practicable at right angles.

3d. To pass through the lowest divides or passes practicable.

Where the cost enters as a controlling factor, a straight line may still be adopted, following the vertical undulations of the surface of the country, or a line either level or gradually inclined may be laid following the sinuosities of the surface formed by projecting spurs and intermediate receding valleys or depressions. A compromise between the last two lines will commonly be found satisfactory and economical. These require a balancing of the cost of construction on the shorter and more direct line caused by increased excavation and embankments, and higher bridges, resulting from the limiting rate of grades, and that on the longer line caused by an increased length and the determination of what increase in the rate of grade would be equivalent to a given decrease in its length. This requires a comparison of the power required to draw various vehicles with given loads upon a level road and roads with different rates of grade.

29. If there are intermediate towns or centres of industry it will often be found advisable to deviate from a direct line more or less in order to pass by these points. If, however, this should lead to much increase of cost, it may frequently be advisable to keep the more direct route between the principal termini, and at some convenient point on this line construct a branch road to the town.

The above remarks apply mainly to the location of roads in what might be called a rolling or undulating country. In locating mountain roads the same latitude of adjusting grades and distances and quantities of work is not afforded.

Mountain Roads.—In locating a mountain road it is usually necessary to hold somewhat closely to the maximum grade for a large portion of the distance between low ground at the entrance to the mouth of the gorge and the summit or divide, else a long development of the line must be made to gain length, usually increasing both the length and the amount of work; or, since the rise is more gentle near the mouth of the gorge, acute-angled zigzags have to be located near the summit for the same reasons; and, in addition, having reached a point near the summit on somewhat easy grades, the maximum established rate of grade will frequently have to be excluded owing to want of room in which to zigzag or

develop the line. It will therefore generally be advisable, as the summit is the only fixed point either on the alignment or grade, to begin the location at that point; and instead of locating the line and then running the levels over it, which will inevitably lead to confusion and malposition of the line, it will be better to determine the position of points at intervals of 100 feet or more, which will be on the descending maximum grade-line, this can be done by means of the level, one end of the chain being held at the instrument and the rod being held at a link or at the end of the chain, and moved about on the surface of the ground until the reading of the rod indicates the fall for that distance; or the transit may be used, the rate of grade being converted into degrees of arc, and the reading on the vertical circle made equal to that angle; the axis of the telescope will be parallel to the grade-line, and the height of the axis above the ground will be the vertical distance between the two. A rod planted on the ground, with a point marked on it equal to that distance in the line of sight or prolongation of the axis, will mark a point on the ground which will be also on the grade-line. Stakes should be placed at these points and marked in some special manner. The bearing of this line could be taken and the distance between the points measured, the instrument moved forward to this point, and the same process repeated; or, more simply still, a transit with stadia fixtures and rods can be used, by which the position, direction, and distance of the point can be determined without actual measurements on the ground. These data being plotted on a map, a line can be plotted to fit the ground, which can then be transferred to the ground, over which levels are run, and profile and estimates of quantities made. If there is sufficient length of line, the grades can be eased for the lower portion of the distance, or a uniform grade-line falling at a less rate than the maximum can be adopted. If a line is located on the maximum descending grade, or any other uniform grade, an ascent towards the top of a ridge or spur crossing the line causes what is called loss of height, and produces an increase in the length of the whole line equal to the horizontal distance from this summit to a point on the regular grade, as shown in the following diagram (Fig. 24).

Taking the summit at *B*, and *AB* the road descending from *B* to *A* at the maximum rate of grade. At *C*, owing to the

intervening ridge D, the grade is broken and a short ascending grade CD is introduced to save excavation. From D the grade must not descend more rapidly than allowed by the maximum rate, hence DH must be parallel to AB, reaching the level of A at H. In addition to increasing the amount of embankment from K to H, the original length required, AB, must be increased by the distance $AH = DG$, as above stated, or an excessively steep grade must be used between D and A, combined also with an increase of length. If the ascending grade CD is at

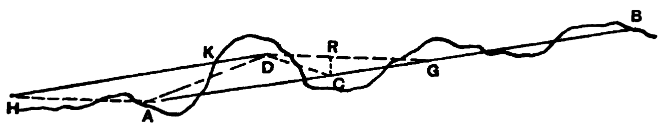


FIG. 24.

the same rate as AB, the increased distance DG = twice that due to the vertical rise RC. If the rise is 10 ft. and the rate of grade 1 in 20, then the distance $RG = RD = 200$ ft. and $DG =$ 400 ft. If the rate of the grade DC is greater than the maximum $RG = 10$ feet; and DC is 1 in 15 then $RD = 150$ feet and $DG = 350$ feet. And finally, if DC is an easier grade, say 1 in 25, then $DR = 250$ feet and $DG = 450$ feet, which shows the increase of length required by breaking a uniform grade, and this will be repeated at every intervening ridge where an ascending grade is adopted.

30. Grades are designated in several ways: 1st, as the vertical rise in any given number of feet measured horizontally, usually 100 feet, as $\frac{1}{2}$, 1, 5, or 10 feet in 100 feet, or as so many feet to the mile of 5280 feet. The corresponding values to the above would be $\frac{1}{2} \times 52.8 = 26.4$ feet, 52.8 feet, 264.0 feet, or 528 feet per mile. Or grades are spoken of as so many per-cents; as, $\frac{1}{2}$ of a foot in 20 feet, equivalent to $2\frac{1}{2}$ in 100, called a $2\frac{1}{2}$-per-cent grade; 1 in 20, a 5-per-cent; 2 in 20, or 10 in 100, a 10-per-cent grade; and so on. All of the above designations are used in the United States. In England a grade of $\frac{1}{2}$ foot in 100 feet is indicated as 1 in 200, a 1-foot grade as 1 in 100, a 5-foot grade as 1 in 20, and a 10-foot as 1 in 10; and sometimes in terms of the angle which the grade-line makes with the horizon, which is the angle whose tangent is the rate of grade di-

vided by the corresponding length; a grade of ½ foot in 100, the tangent $= \frac{\frac{1}{2}}{100} = 0.005$, and the corresponding angle or inclination is 0° 17′ 10″ nearly, commonly taken to the nearest minute; 1 in 100, tang $= \frac{1}{100} = 0.010$, angle 0° 34′ 20″; 5 in 100, angle 2° 51′ 40″; 10 in 100, angle 5° 42′ 40″.

31. The determination of the proper gradients or the rate of ascent or descent, is determined by the power expended in ascending and the acceleration in descending. This latter is an especially important consideration where animal power is employed.

A level surface over long stretches of road, though favorable so far as tractive force alone is concerned, is unfavorable for good drainage, upon which largely depends the perfectness and permanency of the road-bed. Gentle undulations of 1 in from 100 to 150 are desirable; this may be taken as a minimum rate of grade. As an ascending grade in one direction is a descending one in the opposite direction, the question of maximum rate of grade will depend on both, and cannot be fixed with reference simply to the actual power of the animals.

If the gradients are steeper than the angle of repose, which varies with the material forming the surface and the perfectness of the surface, the vehicles will press upon the animals, and their power is expended wastefully in holding or pushing back. A horse only moves rapidly downhill when pressed to do so. That grade which will admit of high speed in descending should practically regulate the maximum rate. It must not be assumed that there is as much gain of power in descending as there is lost power on ascending. Of necessity the same number of animals are used in descending as are required in ascending, but the gain of power cannot be utilized. If the grades are less steep than the angle of repose, tractive force is required on both ascending and descending grades—as much more in the first as it is less in the second; and the average is the same as the force required on a level road.

The limiting gradient is then the same as the angle of repose for any given surface, which is that slope on which the vehicle would be on the point of sliding or rolling down from the action of gravitation alone, and the application of any force however feeble would start it. As the component of the weight parallel to

the surface of the road is the force any excess over which would cause motion, it is equal to the force required to overcome the resistance on a level, the condition of the surfaces being the same. If this force is $\frac{1}{50}$ of the weight, the same fraction expresses the angle of repose for that surface. For all grades from zero to 3 in 100 the normal pressure of the weight is practically equal to the weight or load itself, and the rate of grade in the above case would be 1 in 50 or 2 in 100, which would correspond with the angle of repose. Both men and animals can ascend steeper slopes than they can descend safely.

32. The maximum grade for a given road will depend upon the character of the surface-covering material; upon whether traffic is to be light and fast, heavy and slow, or both; and upon the cost. The questions of convenience, cost of motive power, and cost of construction must be adjusted by careful comparisons in order to decide what grades and what kind of paving shall be adopted.

For fast and light traffic and mixed 3 feet in 100 can be adopted; for slow and heavy, 5 feet in 100.

The French adopt 5 feet in 100 for macadamized roads, Telford recommended $3\frac{1}{3}$ feet in 100.

Grades as high as 7 to 10 in 100 are often found on country roads, especially mountain roads; but both location and construction are bad.

Without regard to descending grades, the maximum ascending grade is limited by the tractive power of a horse. The total resistance on a level, expressed as a fraction, being taken as t, and the resistance to ascending the grade as $g = \frac{\text{rise}}{\text{distance on the slope}}$ = sine of the angle of slope, expressed also as a fraction of the load, then if $W =$ total load, the total resistance to be overcome in ascending the grade will be $R = (t + g) W$; and if P be tractive power of the horse, $(t + g) W < P$, and $g < \frac{P}{W} - t$ (to avoid excessive acceleration in descending, g should always be less than t). The following are some of the values of t, the resistance in terms of the load: For stone pavements, $t = \frac{1}{68}$; for broken-stone, $t = \frac{1}{45}$; for gravel roads, $t = \frac{1}{15}$; for soft sand or loose gravel, $t = \frac{1}{7}$. Where grades form more or less sharp angles, as at B, C, D, and A (Fig. 24) the angle should be rounded by a vertical curve.

33. While, theoretically, the smoother the surface of the road the more gentle the gradient required, and the rougher the surface the steeper the allowable grades, practically it is not strictly true. Animals get a better foothold on rougher roads, and consequently can exert a greater traction than on the smoother roads, but at the same time the resistances are greater in the first than in the second case. The only thing to be done, then, is to get as smooth a surface as practicable consistent with a good foothold for the animals, such as stone pavements. In establishing a grade-line on the profilé, if a continuous grade-line not exceeding the maximum rate can be laid down, it will at least be satisfactory. If, however, the surface rise is greater, more excavation will be required, or the line must be developed to give a greater length and lower rate. At intervals on a continuous grade, short horizontal resting-places are introduced, especially if it can be done without increasing the rate of grade on other portions of the road.

ART. VI.

LOCATION OF LINES OF COMMUNICATION BY TOPOGRAPHICAL MAPS.

34. In the United States but little attention, comparatively speaking, has been given to full and complete topographical surveys; nor have their value and importance as a means of securing good lines for railways and highways been fully appreciated. When embracing an extensive territory the operations are slow and expensive, and it has been the custom to rely absolutely upon the judgment of the engineer to decide upon the best route. Frequently, however, a small and insignificant stream, in a mountainous country, leads to a low divide, of which no information is obtainable, and any engineer who may walk or ride along it, with no road or even path to follow, through thick forests and undergrowth, and having frequently to make long detours to pass around what may prove an insignificant obstacle, knows the difficulties of forming a correct estimate of the distance and consequent rate of ascent, even with the use of a pocket compass and aneroid barometer. The time occupied in the ascent is the only absolutely

reliable fact, and frequent stops, if not noted, render this sometimes doubtful; whereas travelling along another stream on a more or less travelled roadway or path may lead to forming an entirely erroneous estimate as to the relative merits of the two or more routes examined. In view of these considerations the importance of topographical surveys cannot be overestimated.

The quickest method of making such surveys is undoubtedly with the stadia-transits and rods; and whatever may be said or believed as to the accuracy and reliability of surveys made by this method, it can be safely said that they are accurate enough for the purposes now under consideration.

The next method, though more accurate and reliable, consists in running a series of transit or compass lines along the streams and the roads, and in addition such a number of lines, radiating from the intersection of the main streams and their tributaries, along the spurs and ridges included between them, as may be considered necessary. This method is slow and costly. The levels are then run over these lines, the readings, however, being only taken at such points as indicate abrupt changes in the slope. These points should be selected by the transit or compass man, and should be marked by stakes.

And, lastly, these lines may be run by the use of the hand or Locke level, and by determining distances by stepping or pacing. Whatever may be said as to the rapidity of progress by this method, the degree of accuracy attained is at least uncertain. Heights can be checked by using a barometer. Engineers should be drilled in all of these methods, and have facility in applying any of them.

35. In whatever manner the survey has been made, the contour-map is plotted in the following manner. This will be clear by referring to Fig. 25, which represents a territory included between two large rivers, in somewhat the form of a large triangle, showing the dividing ridges and the tributary streams.

First plotting the main and tributary streams as determined by the line notes, points are noted on these at those positions whose elevations are known, whether on streams or roads—the points whose elevations were determined either by stadia measurements or by the level on radial lines. This gives a series of numbers, and the relative heights, with respect to some datum surface, scattered irregularly over the paper. Lines are then drawn with a free hand

through points of the same elevation. These lines are called contours, or lines of equal elevation. Every point on any one of these lines is, or is assumed to be, in the same horizontal surface intersecting the face of the ground. All such lines will either close upon themselves after meandering over a small or large portion of the paper, or will simply run to the lines forming the outer edges of the map and there terminate abruptly, giving a series of irregular lines, often approaching each other closely or receding from each other, *but never crossing each other,* these lines simply extending from one edge of the plot to another.

If in directing a line through one elevation to another point of the same elevation it would have to cross other lines of different elevations, the line must simply be turned off and run to the edge of the plot and stopped. To properly trace in contour-lines requires a somewhat accurate mental picture of the lay of the ground, in addition to the elevation of the isolated points scattered over the plot. Without numbers or lines indicating the direction of the higher ground, it is not practicable to distinguish spurs from ravines or peaks from closed depressions, as contour-lines enclose alike points of highest and lowest elevations. If streams are plotted in, of course their obvious direction of flow enable the directions of the higher grounds and summits to be determined. A sufficient number of contours should be numbered, and the numbers placed on the higher side.

Since each contour-line on a map is the horizontal projection on a common plane of the intersection of a series of horizontal planes with the natural surface of an extended portion of the earth, and since the vertical distances between the planes are usually the same, or rather equal to each other, it follows that the steeper the ground the closer the contours are to each other, and on the face of a precipice they would all be projected into the same contour, whereas on very gently sloping ground the contours are farther apart. If the map is constructed to a scale of 400 feet to one inch and the vertical intersecting planes are 10 feet apart, and two contours are found at any point half an inch apart, at that place the ground has a slope of 10 feet in 200 feet; if they are one and one half inches apart, the ground has a slope of 10 feet in 600 feet, or a much more gentle slope than in the first case. If a distance of 4 inches is scaled in such a direction that one end is on one

contour and the other on an adjacent one, the fall along this line will be only 10 feet in $4 \times 400 = 1600$feet, or 1 in 160. If the four inches reach from one contour to the second one on either side, then the slope would be 20 in 1600, or 1 foot in 80. Hence it is an easy matter to lay down a line on a contour-map so that the rate of grade shall not exceed 1 in 10, 1 in 20, 1 in 100, etc.; or with any given line traced on the map it is easy to construct a vertical section or profile along this line and determine a suitable grade-line for the road..

Contour-lines cut all lines of steepest declivity, as well as all ridge and valley lines, at right angles. They are designated as 10, 20, 40, or 50 foot contours, according as they are 10, 20, 40, or 50 feet above the datum-plane, but the vertical distance between any two planes is the difference between the contour numbers.

It will be noticed that no contour is traced directly across a stream or ravine, but turns up stream and disappears in the outer stream-line. For small streams it is generally indicated as crossing and then turning down stream on the other side.

Between any two contours the slope is assumed to be uniform, no matter how far apart they may be.

36. The map Fig. 25 represents approximately the topographical features of four cases arising in the writer's experience. The upper portion of the map is equally applicable to any of the four cases. The lower portion only applies to one case, namely, the district lying between the Ohio and Kanawha, where near the junction of the two rivers there are well-defined foot-hills, between which and the rivers fairly level and arable bottom-lands are found; these are flooded to depths varying from four to ten feet in the highest floods, and it is only necessary to lay the line on the foot-hills high enough to be above the flood-line, and at the same time avoid excessive damages for right of way if the line lay on the valuable bottom-lands. The line also connects two main termini in the same valley. In this case the line marked $A\ B\ D'\ K\ O\ N$ in Fig. 25 and the corresponding profile marked $A\ B\ D'\ A'\ K'\ K\ O\ N$ in Fig. 27 would of course be adopted, as giving easy grades, light work, good alignment, and the shortest distance between the main termini A and N.

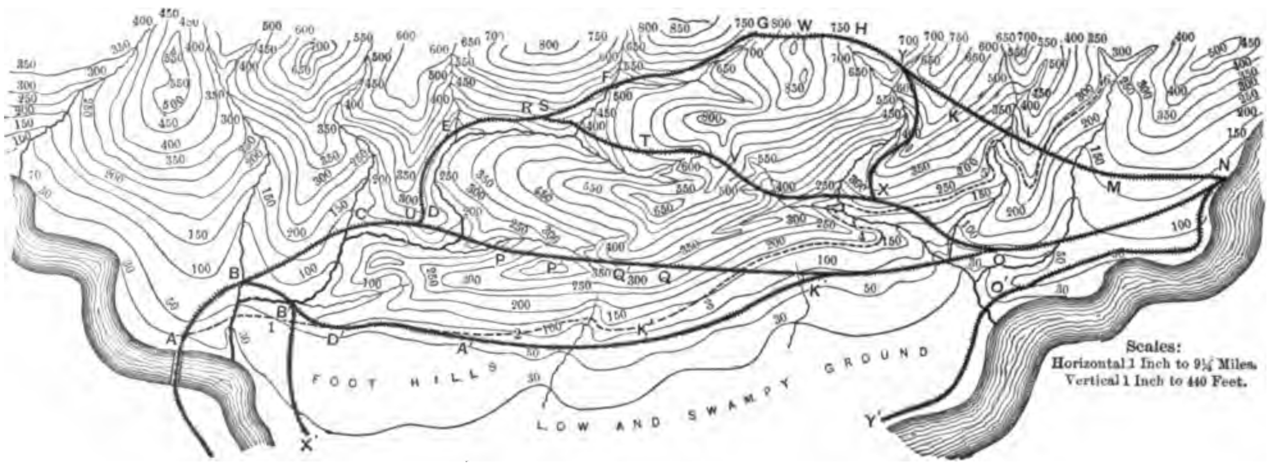
Such favorable conditions do not often arise. A common case is where the ridges and high hills extend all of the way to the junction

TOPOGRAPHICAL MAP AND LINES OF COMMUNICATION.

FIG. 25.

Railways
Contours — Light full lines.
Streams — Heavy full lines
Lines of Uniform Rates of Grade Lines

Numbers Indicating Elevations of Contours are placed above the Contour lines or at their ends.

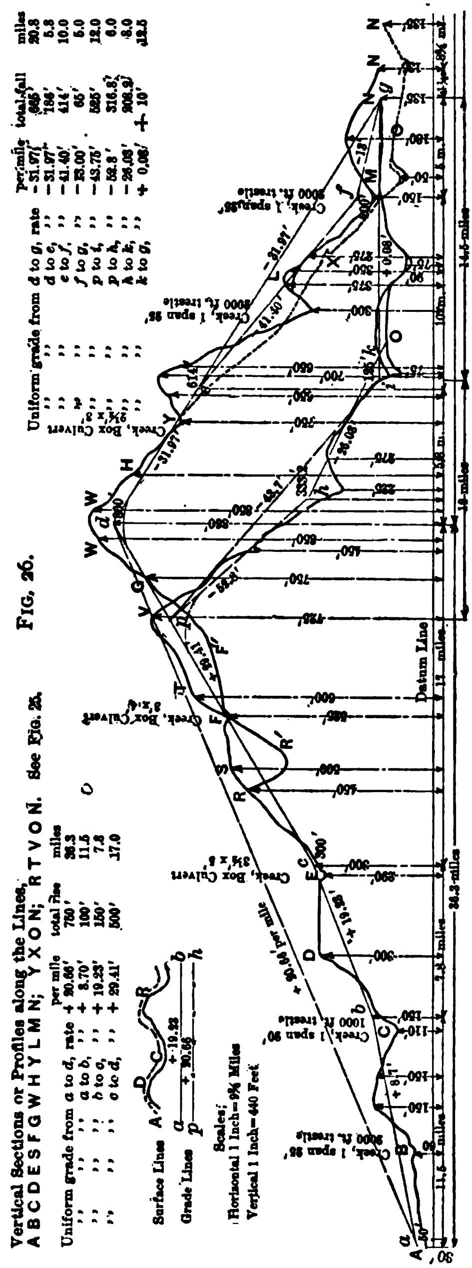


of the two rivers. In such cases—for instance, the North River and the James in Virginia—it would be necessary in developing the line, in order to avoid excessively high grades and quantities of work, to adopt one of the upper lines; or, after crossing the river at A, to turn down the river, following the line $ABB'X' \ldots Y'O'N$ and keeping close to the banks of the rivers. This would insure easy grades and usually light work per mile, but would, by greatly increasing the length of the line, add to the total cost of first construction, especially as all tributaries would have to be crossed where largest, that is, widest and deepest. This was, however, the line adopted. It was a bad location; a shorter and better line could have been secured by following $ABCDPP'Q'QQKON$.

In the next case, in the district included between the Alabama and Tombigbee rivers, neither of the two lines above mentioned could be adopted. First, because there were no well-defined foot-hills. A very large area near the junction of the rivers was a low swamp, formed of unknown depths of alluvial silt; a number of lagoons or sloughs extended well up in the interior between spurs of considerable rise, requiring either a long development of the line, meandering around these spurs and the adjoining low places, or very expensive work cutting through the spurs and costly bridges and pile-trestles over the intervening swamps. It became necessary to adopt, then, one of the two upper routes, $ABCDERT$ $VXON$, or $ABCDESFGWHYLN$, or after reaching Y to turn down the stream and come into the first line at X. This would, however, increase greatly the length of the line, and unless the intervening hills were excessively high this deviation from the one or the other routes would hardly be justifiable; this line, then, from Y to X will not be further considered. Its profile is shown between Y to N through X and O in Fig. 26.

37. All of the profiles are made by sealing the distance on the lines, Fig. 25, from the assumed fixed point A to the higher and lower points along the lines. These distances are laid down on a horizontal line to any desired scale, which in Figs. 26, 27, and 28 is 1 inch to 4 miles. At these respective distances verticals are drawn, upon which are laid off the heights indicated by the contour-lines at their intersection with the railway or highway lines to any desired scale; in Figs. 26, 27, and 28 this is taken at 1 inch to 200 feet. The upper extremities of these lines mark points on the

FIG. 26.


Vertical Sections or Profiles along the Lines,
A B C D E S F G W H Y L M N; Y X O N; R T V O N. See FIG. 25.
per mile total rise miles
Uniform grade from a to d, rate + 20.66' 750' 36.3
" " " a to b, " + 8.70' 100' 11.5
" " " b to c, " + 19.23' 150' 7.8
" " " c to d, " + 29.41' 500' 17.0
Surface Lines
Grade Lines
Scales;
Horizontal 1 Inch=9¾ Miles
Vertical 1 Inch=440 Feet
per mile total fall miles
Uniform grade from d to g, rate - 31.97' 665' 20.8
" " d to e, " - 31.97' 186' 5.8
" " e to f, " - 41.40' 414' 10.0
" " f to g, " - 13.00' 65' 5.0
" " p to i, " - 43.75' 525' 12.0
" " p to h, " - 52.8' 316.8' 6.0
" " h to k, " - 26.08' 208.2' 8.0
" " k to g, " + 0.08' + 10' 12.5
+ 20.66' per mile
Creek, 1 span 25' 2000 ft. trestle
Creek, 1 span 20' 1000 ft. trestle
Creek, Box Culvert 3½' x 3'
Creek, Box Culvert 3' x 4'
Creek, Box Culvert 2½' x 3'
Creek, 1 span 25' 2000 ft. trestle
Creek, 1 span 25' 2000 ft. trestle
Datum Line
11.5 miles
7.8 miles
17 miles
36.3 miles
18 miles
14.5 miles

surface of the ground; between these points, such as *A, B, C, D*, etc., lines are traced following the undulations of the ground, as indicated at the intersections of the lines and contours. This can be done with sufficient accuracy for ordinary practical purposes. Where greater accuracy is required the horizontal distance on the lines from contour to contour must be scaled off, and vertical heights drawn to scale for each contour as it is crossed.

This irregularly rising and falling line marks the surface-line on the profile. The positions, widths, and depths of streams must be noted in the field-books and marked on the profiles as indicated, together with the necessary bridges, trestles, culverts, etc.

38. Having thus established the profile which shows the vertical undulations of the surface of the ground along the line, the next step is the establishment of the grade-lines. The principles controlling this have already been explained. On the profiles several grade-lines are drawn, in order to show the effect on the quantity of work necessitated by the positions of these lines with respect to surface-lines. It is to be noted that where the grade-line is above the surface-line a fill or embankment is required, the height of which is the distance on a vertical line between the surface and the grade line; where the grade-line is below the surface-line it indicates a depth of cutting or excavation equal to the difference of the two elevations. Referring to Fig. 26, the uniform slope from *A* to *d*, which rises $800 - 30 = 750$ feet in 36.3 miles, gives a rate of grade (ascending) of $\frac{750}{36.3} = +20.66$ feet to the mile. This grade, although not very steep, does not suit the profile along either of the two lines, as it gives practically all embankment. At *E* this would be $\frac{11}{16}$ inch $\times 200 = 130.8$ feet in height, and at *C* $\frac{3}{4} \times 200 = 150$ feet, and from these heights down with a slight excavation at *V* on one of the lines and from *G* to *W* on the other. By breaking the grade as shown on the line *a b c d* the grade-lines with their respective rates are found as above; that is, from *a* to *b*, total rise 100 feet, distance 11.5 miles, rate of grade $= \frac{100}{11.5} = + 8.7'$; from *c* to *d*, rate $\frac{500}{17} = +29.41$ feet; and so on between other points of change in grade. We see that the line fits fairly well the surface-line, giving neither excessive cuts nor fills, with no

grade exceeding 30 feet per mile, which is an easy grade. Still further, breaking the grade-line into shorter lengths, ascending towards the high points *D*, *R*, *T*, *V*, and *W*, and descending towards *C*, *R'*, *F'*, etc., the amounts of cuts and fills would be still further reduced; and if the rate of grade on any one distance does not exceed the maximum allowed, say 52.8' per mile or a 1-per-cent grade, it should be done, unless such grades occur on curves, when the proper reduction in rate should be made. These remarks apply to all of the grades shown on both sides of the summit. It will be noticed that in all of the grades shown on the profiles there is only one grade-line—namely, from *p* to *h*, which is 52.8 feet per mile—that is not well within this maximum rate. Since this is the case, the simple question would be which of the two main lines shown on Fig. 26 should be adopted. The length of the upper line measured horizontaily is 1¼ miles the longer, as shown by the distance between *N* and *N* on the two lines. With the exception of the one heavy grade, which could also be used on a portion of the upper line, the grades do not materially differ. The cost of construction apparently would not be materially different. The difference in length is not material; and unless the alignment of one of the lines, having fewer and easier curves, is much better than the other, there would seem to be no special reason for selecting the one over the other. The upper of the two lines was adopted. It admitted of somewhat easier grades for the same amount of excavation and embankment, and the lower line after reaching the bottoms near the point *o* required longer trestles and bridge spans, as it crossed the creeks nearer their mouths, and much of the bottom-lands along the creeks were flooded in high water. The upper line shown on Fig. 27 is from two to three miles shorter than either of those shown on Fig. 26; it also has more uniform and easier grades, and less amount of work, unless the lower portion of the line is so depressed as to require high trestles and bridges, with greater lengths in order to place the line above high-water mark.

The foregoing map, profiles, and descriptions, though made from memory and general impressions of conditions, represent fairly well those actually existing, and fully illustrate the principles involved and their usual practical applications.

Fig. 28 is drawn to show the method of locating on a contour-map a line that shall have a uniform grade; the rate adopted in

Vertical Sections or Profiles along the Lines, A B C U P P' Q Q' K O N: A B D' K O N. See Fig. 25.

FIG. 27.

Uniform grade from	rate per mile.	Total rise.	Total miles.
a to b,	+ 12.55'	330'	26.5
a to f,	+ 1.92'	70'	36.5
b to c,	− 22.92'	275' Fall	12.0
b to d,	− 31.25'	187.5'	6.0
d to c,	− 14.58'	87.48'	6.0
f to g,	− 4.17'	25.00'	6.0
c to k,	0.00'	0'	8.75
c to g,	0.00'	0'	5.0
g to N,	+ 5.58'	60' Rise	10.75
k to N,	+ 7.06'	60'	8.5

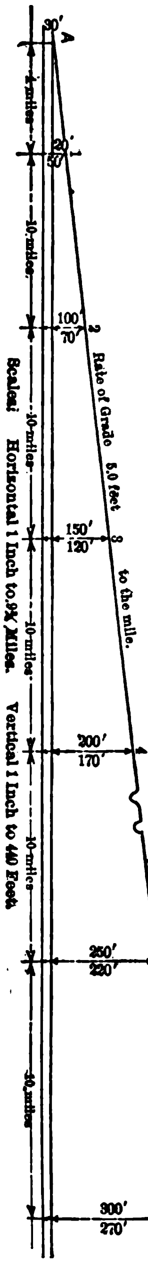


FIG. 28.

Vertical Section or Profile along Line of Uniform Rate of Grade. 1,2,3,4,5,6 and 7. Rate of Grade 5 feet to the mile. See Fig. 25.

Scales: Horizontal 1 inch to 9¼ Miles. Vertical 1 inch to 440 Feet.

this case is 5 feet per mile, or 20 feet per each 4 miles, or 1 inch on the scale adopted. The dotted line commencing at *A*, Fig. 25, corresponding to this condition, is traced out as follows: as *A* is taken on the 30-foot contour, and the next is the 50-foot contour, the difference being 20 feet, it is only necessary to move a scale pivoting at the point *A* until the 1-inch point or division intersects the 50-foot contour at the point 1; then draw a line from *A* to 1. As the next contour is the 100-foot one, a difference of level of 50 feet exists, which corresponds to 2½ inches on the scale. It will not be usually practicable to draw a straight line of this length that will not cross the 100-foot contour. It will therefore be necessary to trace a line with the free hand, gradually rising towards the contour, but not crossing it. This will give a meandering line of 2½ inches in length, every portion of which is between the two contours, the rear extremity 1 resting on the 50-foot and the forward on the 100-foot contour at the point 2. Similarly for each 2½ inches, extending from the 100-foot at 2 to the 150-foot contour at 3, from 3 to 4 on the 200-foot contour, and so on to the point 7 near the top and right-hand corner, as shown by the broken line *A*, 1, 2, 3, 4, 5, 6, 7. This simply gives a line on the surface, and necessarily gives a great length of line.

If a straight line 2½ inches be made to rest with its extremities on adjacent contours, it will usually cross other contour-lines, and to bring it to a uniform grade or slope a certain amount of excavation and embankment will be required. A combination of these two processes is usually resorted to when it is required to locate a highway or railway on a given rate of grade. It is a mere question of balancing the cost of construction, operating, and maintenance on a long line with small amounts of work, and a shorter line with greater amounts of excavation, embankments, bridges, etc.

The light grade of 5 feet per mile was adopted to explain the methods. Grades for railways go as high as from 20 to 80 feet per mile, or from 0.379 to 1.51 feet per 100 feet; and for highways from 1 to 5 feet or more in 100 feet, equivalent to from 52.8 to 264.0 or more feet per mile.

The usual scale adopted in constructing profiles for railways is, for the horizontal distances, 400 to 500 feet to the inch, and for the vertical scale, 20 feet to the inch.

39. Making a profile simply from the level-notes as taken along

FORM OF FIELD-LEVEL NOTES.

Stations.	Distances.	Back-sights +.	Height of Instruments.	Fore-sights −.	Surface Heights.	Grade Heights.	Centre. Cuts and Fills. − +	Remarks.
B.M.		1.21	106.21		105.00			On oak-tree 100 ft. to right of station 50 + 25.
50				5.0	101.2	105.21	+4.0	
						+0.5		
1				4.2	102.0	105.71	+3.7	
2				4.0	102.2	106.21	+4.0	
3				3.5	102.7	106.71	+4.0	
4				3.0	103.2	107.21	+4.0	
						+0.25		
+15				5.0	101.2	107.25	+6.0	Left bank creek.
+25				10.0	96.2	107.28	+11.1	Centre, bed.
+35				5.0	101.2	107.32	+6.1	Water surface, foot of slope.
+60				2.5	103.7	107.38	+3.7	Right bank.
⊙5				0.51	105.70	107.46	+1.8	On peg; turning-point.
		10.23	115.92					
6				9.1	106.8	107.71	+0.9	
						+1.0		
7				8.5	107.4	108.71	+1.3	
8				5.0	110.9	109.71	−1.2	
9				3.0	112.9	110.71	−2.2	
60				2.2	113.7	111.71	−2.0	
1				1.1	114.8	112.71	−2.1	
⊙		3.85	119.08	0.24	115.68			
+50				6.5	112.5	113.21	+0.7	Small ditch, 3 ft. wide.
2				2.2	116 8	113.71	−3.1	
3				3.5	115.5	114.71	−0.8	
						−0.6		
4				5.0	114.0	114.11	+0.1	
5				3.5	110.5	113.51	+3.0	
6				3.5	110.5	112.91	+2 4	
7				9.0	110.0	112.31	+2.3	
						+0.8		
+20				9.5	109.5	112.41	+2.9	Left bank stream.
+25				11.5	107.5	112.51	+5.0	Foot of slope, water surface.
+40				9.0	110.0	112.61	+2.6	" " "
+50				4.0	115.0	112.71	−2.3	Right bank.
						+1.0		
8				1.0	118.0	113.21	−4.8	
⊙9		9.21	127.71	0.58	118.50	114.21	−4.3	On peg.
70				8.0	119.7	115 21	−4.5	
1				7 5	120.2	116.21	−4.0	
2				0.2	127.5	117.21	−10.3	
3				6.3	121.4	118.21	−3.2	
B.M.				9.0	118.7			On chestnut stump 150 ft. to right of station 73+40.

Cuts are indicated by the − sign; fills by the + sign. Sign ⊙ indicates a turning-point.

any given line is similar in every respect to the above. Horizontal distances equal to the intervals from point to point are first sealed on a horizontal line, and verticals are drawn at the points of division to the adopted scale representing the surface elevations at these points. A line traced through their upper extremities gives the undulations of the ground along the line.

The regular distances horizontally are 100 feet. Any abrupt changes in elevation, such as rises or depressions, ditches, banks of streams and their beds, when practical, are noted, and placed at their proper positions with respect to the regular stations. Grade-lines are then tried and established according to the principles and requirements already explained. The following are examples of the usual methods of keeping the field-notes. The more common form is that known as the height-of-instrument method.

It will be noticed that the first entry is the elevation of the bench-mark, 105.00, in the column headed Surface Heights. The instrument being set up at some suitable point, so that the line of sight, when the axis of the telescope is horizontal, will pass above the bench-mark, the reading of the rod on this point 1.21 foot, the line of sight may be forward or rearward with respect to the line. It is, however, usually called a back-sight, more properly a plus sight. This reading is entered in the column of back-sights. Adding 1.21 to the elevation of the bench-mark, it gives 106.21, which is called the height of the instrument. This means that the axis of the telescope is 106.21 feet above a fixed datum-plane, which remains fixed with respect to any point of the line. The rod is then held at stations 50, 51, 52, 53, 54, and at the points + 15, + 25, + 35, + 60, marking the banks and bed and surface of the creek. All of these readings on the rod are recorded in the column of Fore-sights, or more properly *Minus sights*, as the direction in which the sights are taken is immaterial. These rod readings deducted from the height of the instrument give the surface heights of the ground above the same datum-plane. These differences are recorded in the column of Surface Heights. The last reading, being only 0.51 foot, shows that the line of sight which must be maintained in a horizontal direction is about to intersect the ground surface, and no further readings of the rod can be taken, as the ground is still rising. It then becomes necessary to move the instrument forward. To keep up a continuous line with reference to the datum-plane, a

turning-point is taken, that is, a stake or peg is driven into the ground firmly; this is really a temporary bench-mark. As in this case it is driven at station 5, its top being flush with the surface of the ground, the reading is the same as before, 0.51. This gives for the elevation of the turning-point 105.70 feet. The instrument can now be moved forward to any convenient point, set up, and levelled.

The first reading taken with the rod resting on the turning-point is 10.22, which, added to the height of the turning-point, gives $105.70 + 10.22 = 115.92$ feet above the same datum-plane for a new height of instrument. Foresights are now taken to other stations; the readings subtracted from the new height of instrument give the surface heights as before. When necessary another turning-point is taken, the instrument moved forward, and the same operations repeated. Even when the rise or fall of the ground does not necessitate a turning-point, they should be taken at intervals of 400 to 600 feet from the instrument for accurate work. The readings on the turning-points are always taken to hundredths of a foot, and often to thousands, as any error or inaccuracy made on these points will be carried throughout the length of the line. The readings on the intermediate points are taken only to tenths of a foot, as this reading only affects the one point. Just here, however, is the only objection to this method of levelling. There is no method of correcting the error in readings, and incorrect elevations are often obtained. Consequently some engineers prefer the method of keeping notes known as by the column of differences, as follows: Each sight is a fore-sight to the preceding and a back-sight to the following one, and treated alternately as a plus and minus sight; for instance, between stations 50 and 55. $+ 1.21 - 5 = - 3.79$; $+ 5 - 4.2 = + 0.8$; $+ 4.2 - 4 = + 0.2$; $+ 4 - 3.5 = + 0.5$; $+ 3.5 - 3 = + 0.5$; $+ 3.0 - 5 = - 2$; $+ 5 - 10 = - 5$; $+ 10 - 5 = + 5.0$; $+ 5 - 2.5 = + 2.5$; $+ 2.5 - 0.51 = + 1.99$.

Now it is evident that the algebraic sum of these differences, added to the first elevation, 105.00, must give the last 105.70, or $- 3.79 + - 2 + - 5 = - 10.79$, and $+ 0.8 + 0.2 + 0.5 + 0.5 + 5.0 + 2.5 + 1.99 = + 11.49$. $+ 11.49 - 10.79 = + 0.70$ and $105.00 + 0.70 = 105.70$ feet. This prevents either error on the turning-points or on any intermediate station. This check can easily be applied, and should be done at reasonable intervals. The following

diagram, Fig. 29, shows the profile made from the preceding table of level notes. The diagram shows the usual form of profile-paper. The vertical lines are 100 feet apart horizontally, and the station numbers are marked at the bottom; every tenth vertical is a heavy line to facilitate keeping station numbers consecutively. Perpendicular to these are a series of horizontal lines spaced 1 foot vertically, every fifth line made heavy and every twenty-fifth line still heavier to aid in counting off feet of elevation. The bottom line may be either the datum-plane or any plane parallel and above it; for economy of space it is taken at an elevation of 95 feet above the datum in the diagram. At station 50 the elevation is 101.2; a dot is made on the 101 line above station 50. At station 51 it is 102; a dot is made on the 102 line. At station 3 it is 102.7; a dot is made between lines 102 and 103; and at station 60 a dot between lines 113 and 114, to indicate an elevation of 113.7 feet. And so on for each station, and such ravines, ditches, or rises as are seen between stations 54 and 55, 61 and 62, 67 and 68, and 72 and 73. No matter how long the line, the profile is constructed as shown above. By the aid of the lines on the profile-paper the work is carried on rapidly. The grade elevation at station 50 is 105.21; it rises then on a 0.5 grade to station 54, where it is 107.21; then on a 0.25 grade to station 56, where it is 107.71; then on a 1-foot grade to 63, where it is 114.71; then on a descending grade of 0.6 to station 67, at which it is 112.31; then on a rising 0.8 grade for 50 feet, and from there to the end on a + 1.0-foot grade. The proper elevations at these points being marked, straight lines joining them gives the grade-lines. The distances between these lines and the surface-lines show the cuts or fills, as recorded in the last column of the notes. The grade-line as established does very well, except between *c* and *b*. This gives some heavy cutting, relatively speaking. The horizontal scale being small as compared with the vertical scale, makes the profile appear more undulating than really exists. A uniform grade from *a* to *b* would diminish the cutting, but would increase the filling and the heights and cost of the trestle and bridge required between *c* and *d*; which is the better to adopt is mainly one of cost. The better grades would be obtained by adopting that direct from *a* to *b*. Grade heights must always be calculated from point to point, and not scaled from the profile. To find the grade height at station 61, Fig. 29: At the point of change in that grade-

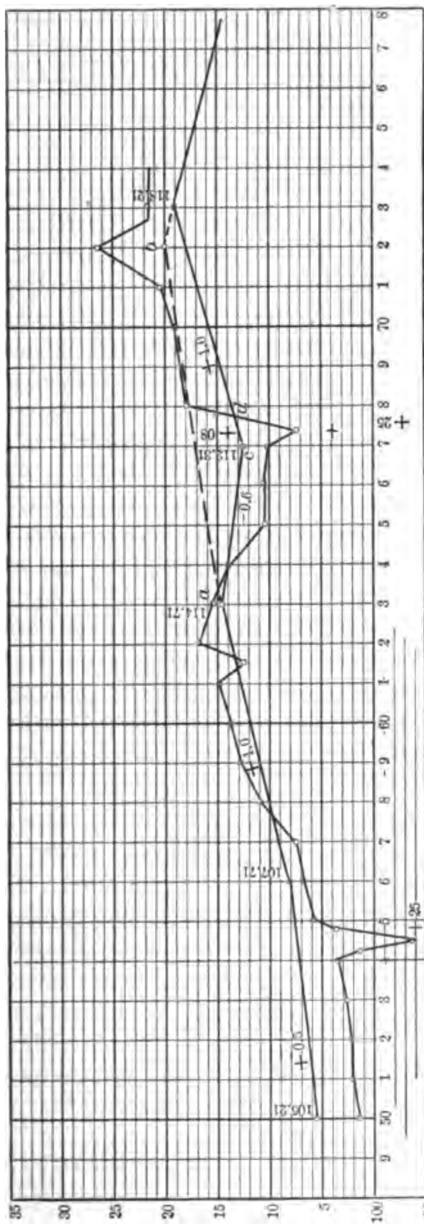


FIG. 29.

line, the elevation is 107.71 at station 56, the distance is 500 feet and the rate of grade is 1 foot per 100 feet; hence 5 feet added to 107.71 gives 112.71 for elevation of grade at station 61.

PRACTICAL EXAMPLES.

40. So much depends upon the determination of the proper grades and curves, that the success of the enterprise depends to a great extent upon their proper application in the location of a railway or a highway. On the one hand, we may expend so much on securing light grades and easy curves that the business of the road will never pay the interest on the first cost; and, on the other hand, with excessively steep grades and sharp curves the cost of operating will consume the entire income. These are the extreme conditions. A proper location provides for curves, grades, and costs of constructing and operating between these extremes. Although we may not be able to exactly reconcile and balance these, in many respects, conflicting conditions and requirements, we can in some degree reach an approximately satisfactory result, and determine the bearing and influence of the one upon the other, based upon a set of conditions approximately true. Assuming that the line is straight and level, weather favorable, and track and rolling machinery in fair order, the resistance on a level increases with the square of the speed. For different velocities we have from equation (30) the following resistances:

$$R = \frac{v^2}{171} + 8;$$

and from equation (31) the resistances arising from grades:

$$r = W\frac{BC}{AC}.$$

From this we can find the rate BC per mile necessary to double any resistance on a level at any given velocity:

$$BC = \frac{r \times AC(= 1 \text{ mile} = 5280')}{W(= 1 \text{ ton} = 2240 \text{ lbs.}}.$$

TABLE I.

Velocity, v, in miles per hour.	Resistance, R, in lbs. per ton.	Rise per mile, BC, in feet to double resistance ($r = R$).
15	9.32	21.97, $r = 9.32$
20	10.34	24.37, $r = 10.34$
25	11.65	27.46, $r = 11.65$
30	13.26	31.25, $r = 13.26$
40	17.36	40.92, $r = 17.36$
50	22.62	53.32, $r = 22.62$
60	29.17	68.75, $r = 29.17$

From this table it is seen that with a speed of 25 miles per hour it requires a grade of 27.46 feet per mile to double the resistance, or about a 0.5-per-cent grade; but with a speed of 60 miles per hour it requires a grade of 68.75 feet, or a 1.3-per-cent grade. In other words, the greater the velocity the steeper is the grade to require a double expenditure of power. If, then, we find the power required to move a given train one mile on a level road, and the equivalent ascent which requires an equal expenditure of power, and divide this into the total rise on the line, we find additional length allowable in order to avoid the grade, or the equivalent length on a level. If, as in Figs. 25 and 27, we have a total rise of $350 - 30 = 320$ feet in 25.5 miles, we could at a speed of 30 miles per hour afford to increase the length of the line, so far as power consumed is concerned, by $\frac{320}{31.25} = 10.24$, or $10\frac{1}{4}$ miles, or lengthen the line to $35\frac{3}{4}$ miles, if by so doing we could avoid altogether the grade. This is usually called Equating for Grades.

41. Although this gives a length proportionate to the power expended, it does not give a length proportionate to the money expended in developing that power. As the operating expenses are not directly proportional to the power expended, the fuel used may be taken as proportional to the power exerted. The wear and tear is greater upon grades than on a level. The fixed charges, such as salaries of officers and laborers, cost of station-houses, and the like, are not materially affected whether the road is level or has grades.

It is therefore necessary in equating for grades to know the portion of the total cost properly chargeable to the grades. So long as the

grades do not necessitate a reduction in the number of cars hauled, or in the usual lengths of the trains, this proportion of the total cost has been taken at about one sixth, and the allowable increase of length of line would in this case be only $\frac{1}{6}$ of $10\frac{1}{4} = 1.71$ miles, which is equivalent to multiplying the numbers in the third column of the preceding table by six before dividing into the total rise; that is, for a speed of 30 miles and rise of 320 feet we have $\frac{320}{6 \times 31.25} = 1.71$ miles, and similarly for other velocities. But if the grades are over, say, 70 feet, and it is desirable to maintain an average speed of 30 miles per hour, a larger percentage of the total cost would be chargeable to grades, say one half; then the increase of length would be $\frac{1}{2} \times 10\frac{1}{4} = 5\frac{1}{8}$ miles $= \frac{320}{2 \times 31.25} = 5.12$ miles.

These proportions of one sixth and one half the total costs are merely used to explain the principles. In any given case their proper values should be found from roads already running on similar conditions to the one under consideration. A descending grade reduces the engine expense, and if it is on the same slope as the angle

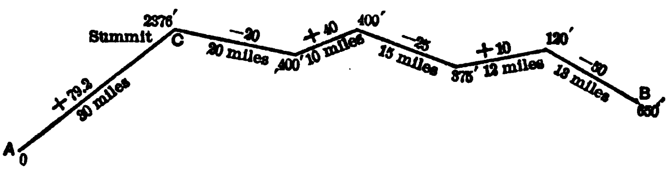


FIG. 30.

of repose—which may be taken at $\frac{1}{280}$, see Par. 31, or about 18.9 feet per mile, or rather somewhat greater, say 22 feet, the steam being practically shut off entirely, the train descending by gravitation alone—there is saved fuel sufficient to haul the train one mile upon a level. We might therefore deduct, say, $\frac{1}{8}$ mile from the measured length for each mile, and for grades less than 22 a proportionate amount, assuming that one eighth of the charge arises from the grade of 22 feet per mile or over; that is, any grade over 22 feet is simply considered as if reduced to 22 feet. But for a grade under 22 feet, say 20 feet, the deduction would be $\frac{1}{8} \times \frac{20}{22}$. But, considering the increased wear and tear, it is as well not to allow anything at all on descending grades. The following example, however, allows for both ascending and descending grades.

At the speed of 30 miles per hour the divisors for the total rises going from A to B would be 2×31.25, 4×31.25, and 6×31.25 for the 79.2, 40, and 10 feet rates of grade, respectively. We then have

$$100 + \left(\frac{2376}{2 \times 31.25} + \frac{400}{4 \times 31.25} + \frac{120}{6 \times 31.25}\right)$$
$$- \left(\frac{1}{8} \times \frac{20}{22} \times 20 + \frac{1}{8} \times 15 + \frac{1}{8} \times 13\right)$$
$$= 100 + 41.84 - 5.77 = 136.07 \text{ miles}$$

for the equated length going from A to B. And in going from B to A,

$$100 + \left(\frac{650}{3 \times 31.25} + \frac{375}{4 \times 31.25} + \frac{400}{6 \times 31.25}\right)$$
$$- \left(\frac{1}{8} \times \frac{10}{22} \times 12 + \frac{1}{8} \times 10 + \frac{1}{8} \times 30\right)$$
$$= 100 + 11.06 - 4.68 = 106.38,$$

and the mean of the two is 121.22 miles. And, leaving out of consideration any saving or benefit from descending grades, the equated length = 126.45 miles.

42. When practicable, the grades should be so arranged that they shall be as easy as possible on that slope of the dividing ridge facing the direction of the heaviest traffic, and they may be much steeper on the opposite. This is indicated in Fig. 30 by assuming the heaviest traffic to be carried from B to A; the length of the line is increased between B and the summit C, and between C and A the line is shorter, but has steeper grades.

Assuming the tractive force of an engine to be 19,200 pounds, and the resistance on a level to be 10 pounds per ton. Then the total resistance on the 50-foot grade going from B to C will be

$$r = 2240 \times \frac{50}{5280} + 10 = 21.2 + 10 = 31.2,$$

and on the 79.2 grade from C to A

$$r' = 2240 \times \frac{79.2}{5280} + 10 = 33.8 + 10 = 43.8 \text{ pounds per ton},$$

or 31 and 44, respectively.

The same engine, then, could carry $\frac{19200}{31} = 620$ tons from B to C and only $\frac{19200}{44} = 440$ tons from A to C. If, then, we assume that the train weight, or dead load, is 440 tons, the engine could carry a live or paying load of $620 - 440 = 180$ tons going from B to A, but could only carry the empty cars back from A to B.

The tractive power of a locomotive depends both upon its steam-producing capacity and the adhesion arising from the weight on the drivers. Even with ample weight and adhesion long steep grades might be beyond the steam-producing capacity of the engine, and it may therefore be better to put steep grades on short lengths of road than overcome the same differences of level by less steep grades on greater lengths. Where the steeper grades can be concentrated on certain divisions stronger and more powerful engines can be used on those portions. It has been found that on descending grades of 1 in 100, down which a train is running freely, the maximum velocity acquired will not be much over 40 miles per hour.

The power developed in the steam-cylinders and transferred to the circumference of the wheel is usually called traction; its amount depends upon the diameter of the cylinder, the mean pressure of the steam on the piston, and the length of the cylinder or the stroke of the piston, and the diameter of the drivers.

Drivers of large diameter are generally used for light loads and high speeds. The engine should be capable of producing large quantities of steam of little density, and the stroke of the piston should be short.

With slow speed and heavy trains steam of greater density and long stroke, with small wheel diameters, are required.

RESISTANCE FROM CURVATURE.

43. The resistance produced by curves to the motion of a railway train depends upon the length of the radius, the gauge of the track, the size of the wheels and form of tread of the tires, the coning of the wheels, and elevation of the outer rail.

We have already seen that the resistance from curvature is

inversely as the radius; in other words, the resistance in running one mile of a 1° curve is the same as in running one half a mile of a 2° curve; and also, that the resistance on a 10° curve, having a radius of 573 feet, at a speed of 20 miles per hour is about double that upon a straight line, or about equal to that on a 24-foot grade; that is, as much power is consumed in running over a 10° curve one mile long as is necessary to haul the same train two miles upon a level straight line. The complete circumference of a 10° curve is $2\pi r = 2 \times 3.1416 \times 573 = 3600$ feet, which contains 360° of arc; hence $3600' : 5280' :: 360° : 528°$, which is the number of degrees of arc in the length of a mile, and is the number of degrees of arc consuming power enough to draw a train one mile on a straight and level road. Theoretically this is independent of the length of the radius, that is, the flatness or sharpness of the curve, and of the length of the curve.

Practically, however, it is evident that a number of long easy curves distributed over a long line of road may oppose an inappreciable resistance, whereas the same number of degrees of arc placed in short sharp curves, simple or compound or reversed, might oppose very great resistances of a momentary nature. In equating for curvature, therefore, on any given line such conditions should be weighed and considered.

Owing to the greater wear and tear of machinery on curves as compared with the same on grades, we increase the cost of operating more in doubling the resistance on curves than we do on grades. If, then, a mile of road contains 528° of curvature doubling the power required and involving 25 per cent more cost of maintenance and operating, the equating number instead of being 528° will be $\frac{100}{25} \times 528 = 2112°$ for curvature at a speed of 20 miles per hour. Since the resistance on a straight line varies with the speed, a different radius or degree of curve will be necessary to double the resistance. $R : R' :: r' : r$; that is, the radii will be inversely as the resistances. From this the column 3, headed Radii, Table II, are obtained. The speed and the resistances, columns 1 and 2, are the same as in the preceding table for equating for grades. Column 4 headed is obtained from R (column 3) $\times 2\pi : 5280 :: 360° : x$, the corresponding number of degrees. The equating numbers in column 5 are found by multiplying the numbers in column 4 by $\frac{100}{25} = 4$. We then construct the following table:

TABLE II.

Speed in miles per hour.	Resistance in lbs. per ton.	Radii of Curves to Double Resistance.	Number of Degrees per mile.	Equating number in Degrees.
15	9.32	636	476	1904
20	10.34	573	528	2112
25	11.65	508	595	2380
30	13.26	447	677	2708
40	17.36	341	888	3552
50	22.62	262	1155	4620
60	29.17	203	1490	5960

For 20 miles per hour on a 10° curve $R' = 573$ feet, and for 15 miles per hour $R : R' :: 1034 : 9.32$; $\therefore R = 636$, nearly; and for 25 miles $R : R' :: 10.34 : 1165$, $\therefore R = 508$, nearly; and similarly for other speeds.

$$R \times 2\pi : 5280 :: 360° : x = \frac{5280 \times 360}{2 \times 3.1416R} = \frac{302521}{R} = \text{number of}$$

degrees per mile, $\therefore x = \frac{302521}{636} = 476°$ for speed of 15 miles per hour and $x = \frac{302521}{573} = 528°$ for 20 miles, and so on. If, then, a line of 100 miles in length has 7000 degrees of curvature, and the speed is to be 30 miles per hour, the equated length is $100 + \frac{7000}{2708} = 102.58$ miles. To make the comparison between the cost of construction, maintenance, and operating of two roads, it is only necessary to capitalize the annual cost of operating and maintenance at 6 per cent and add it to the cost of construction. If the equated lengths of two roads, respectively 90 and 100, are respectively 120 and 110 miles, and the cost of construction is, respectively, \$50,000 and \$30,000 per mile, and the operating and maintaining charges are \$4000 per mile, annually, for the equated lengths, the equivalent outlay on the two lines will be

$$90 \times 50000 + 120 \times 4000 \times \frac{1}{6/100} \text{ p.c.} = \$12,500,000;$$

$$100 \times 30000 + 110 \times 4000 \times \frac{1}{6/100} \text{ p.c.} = \$10,233,333.$$

Assuming that the most economical grades and curves have been adopted for any number of lines or routes between the two termini, that one will be the best on which the sum of the cost of construction and annual expense of operating and maintaining are the least.

To find the value of saving a mile on any route, divide the sum of the annual operating expenses and interest on cost of construction by the rate of interest, and this quotient by the length of the line in miles. This value varies so greatly with the amount of traffic, that the result determined at one period would not apply to another. Mr. Vose states that when the Pennsylvania Railway was built a mile of distance saved was valued at $53,000, or $10 a foot; it has come to be valued under the traffic of the year 1883 at $433,000. The values of grades and curves would also increase. Assuming that a grade of 24 feet per mile and a curve of 10° cause an equal resistance of 10 pounds per ton, then $\frac{24}{10} = 2.4$ feet per mile per degree of curvature gives the proper reduction of grade in order that the resistance may be uniform. Then for a 2° curve or radius of 2865 feet the reduction in grade on such a curve should be 4.8 feet per mile; for a 10°, radius 573 feet, the reduction should be 24.0 feet per mile; for a 4°, and 6° curve, radius 1432 and 955 feet, the reduction in rate of grade per mile would be 9.6 and 14.4 feet, respectively.

Although the above practical application of principles may not be accurate or reliable under varying conditions, yet their accuracy will be reliable in proportion to the accuracy of the data upon which they are based. They, however, indicate the lines upon which such determinations are to be made, and are therefore of great importance. The haphazard way in which many of our most important roads are located and constructed always eventuates in large expenditures to relocate and reconstruct roads improperly located and constructed in the beginning. It is true that it has often been the case that the roads could not have been constructed at all if considerations of economy, motive power, grades, and curves had been carefully balanced and adjusted in the beginning, cheapness and rapidity alone having been considered; adjustment of these conflicting relations and conditions being left until the business of the road had been developed and become

remunerative. Such methods of location and construction are doubtless inevitable and necessary in the earlier enterprises, but as conditions are settled down to normal and uniform relations it becomes a matter of great importance to weigh them all and carefully and to approximate in the beginning to the best locations and methods of construction.

By spending a little more money in surveys on lines and in careful and extended topographical surveys, and in more careful comparisons as to cost of constructing and operating different routes between the same terminals, we may locate on the best route, and even where temporarily steep grades and sharp curves are adopted, from economical considerations, on a few short stretches of the line, a good portion of it may be located on the best route, by due consideration of, and balancing, the foregoing conditions.

ART. VII.

RESISTANCE TO TRACTION ON HIGHWAYS.

44. The following discussion on the subject of resistance to traction is taken substantially from Mr. Austin T. Byrne's (C.E.) most interesting and instructive work on Highway Construction, to which the student is referred for a more full discussion of this important subject.

The resistance to traction on highways is caused: 1st. By the want of uniformity in the surface of the road. The weight has to be lifted over inequalities, projecting points, and out of ruts: these diminish the effective load which the horse can draw to that which it can lift.

2d. By the want of strength and firmness in the road-bed. If its foundation yields under the pressure of the wheels, the horse is constantly attempting to lift the load out of the hollow or depression; but the fulcrum, instead of being fixed as in the first case, is yielding and varying in position.

The first case is shown in Fig. 31 and the second in Fig. 32. In the first case the wheel is mounting the fixed obstruction *D*, and in the second it has sunk into the soil, compressing it forwards and sideways as the wheel moves forward.

The solution of the first problem is simply one of the balance of moments. Let $P =$ the force just sufficient to balance the weight on the point D, $W =$ the weight; their respective lever-arms

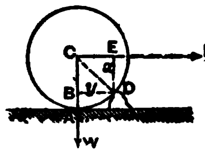


FIG. 81.

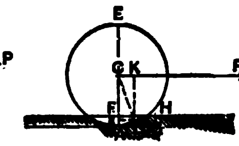


FIG. 82.

with respect to an axis at D being $ED = BC = x$, and $CE = BD = y$. Then

$$Px = Wy; \quad \therefore P = \frac{Wy}{x} = W\frac{\sqrt{R^2 - x^2}}{R - AB}. \quad . \quad . \quad (32)$$

Let the radius $R = 26$ inches, the vertical height of the obstacle $= AB = 4$ inches, and the total weight $W = 500$ pounds. Then

$$P = 500\frac{\sqrt{676 - 484}}{26 - 4} = 314.8 \text{ pounds.}$$

The pressure at the point D is the resultant of P and W. Representing P by CE, W by CB, then the pressure F will be represented by CD; hence

$$BC : CD :: W : F; \quad \therefore F = W\frac{CD}{BC} = 500\frac{26}{22} = 591 \text{ pounds.}$$

The direction of the draught is not exactly horizontal and the diameters of the wheel and axle are not considered. The solution is, however, practically accurate. Both the pressure F, striking suddenly against the stone or other obstruction, and the weight dropping from the stone to the road-bed, tend to destroy the road-bed, and the tendency is also to break the wheel or bend the axle. The power P is lessened by the use of springs, by increasing the diameter of the wheels, or by inclining upwards the line of draught.

The resistance caused by sinking into and compressing the soil does not admit of as close and accurate computation, owing to the unknown and indeterminate conditions existing, and certain assump-

tions as to the nature, amount, and directions of the resistances have to be made. Referring to Fig. 32, the arc AH is submerged; assuming this equal to the chord AH, and that the pressure is distributed and uniformly varying over the surface AH, its intensity at any point being the ordinate of a triangle whose base is AH, and being zero at H, the surface, and greatest at A, the point of deepest penetration, the resultant pressure would act at D, a point one third of the distance AH from A. The point D is the centre of resistance and pressure, CD will be the resultant of the load CB and the tractive force CK. CB may be taken without sensible error as equal to $CA = CD = R$; and $BD = \frac{1}{3}FH = \frac{1}{3}$ of the semi-chord of submersion. Then

$$CB : BD :: W : P; \quad \therefore P = W\frac{BD}{CB} = W\frac{\frac{1}{3}FH}{R};$$

$$FH = \sqrt{EF \times FA}; \quad \therefore P = \tfrac{1}{3}W\frac{\sqrt{EF \times FA}}{R}. \quad . \quad (33)$$

Assuming radius of wheel = 26 inches, diameter 52 inches, and penetration $FA = 4$ inches, then

$$EF = 52 - 4 = 48 \text{ inches}; \qquad W = \text{one ton} = 2240 \text{ pounds.}$$

Substituting in equation,

$$P = \frac{1}{3}2240\frac{\sqrt{48 \times 4}}{26} = 397.1 \text{ pounds per ton.}$$

Such formulæ are necessarily only roughly approximate even when applied to homogeneous substances, such as clay and sand.

45. Mr. Morin's conclusion that the resistance varies inversely as the diameter of wheel, that is, to halve the resistance double the diameter of wheel, does not seem to be reliable. M. Dupuit placed a number of wheels at the summit of an inclined plane, at the foot of which was a horizontal plane. The wheels rolled down the inclined plane, and were allowed to come to a state of rest after expending the energy acquired. From these experiments he drew the following conclusions:

On macadamized roads in good condition, and on uniform surfaces generally—

1st. The resistance to traction is directly proportional to the pressure. 2d. It is independent of the width of the tire. 3d. It is inversely as the square root of the diameter. 4th. It is independent of the speed.

On paved roads causing constant concussion he admits that the resistance increases as the speed, and that it is diminished by increasing the width of the tire up to a certain limit, say from 3 to 4 inches wide.

Friction.—The resistance of friction arises from the rubbing of the wheels against surfaces with which they come in contact. This resistance can only be determined by experiment. Friction of the axles and resistance of the air may generally be neglected: their effects are constant, and are independent of the imperfections of the road.

46. The destruction of the road is greater as the diameter of the wheels are less, and is greater with vehicles without than it is with springs.

47. Upon soft roads of earth, sand, turf, or roads freshly and thickly gravelled, the resistance to traction is independent of the velocity.

48. The comparative ease of draught depends on the amount of foothold afforded. The tractive force is influenced by the diameter of the wheels, the friction of the wheels on the axles, the speed, as well as by the resistances of the road surface.

49. All estimates on the tractive power of horses must to a certain extent be vague. The draught or pull which a good average horse weighing 1200 pounds can exert an a level smooth road at a speed of $2\frac{1}{2}$ miles per hour is 100 pounds, equivalent to 22,000 foot-pounds per minute, or 13,200,000 foot-pounds per day of 10 hours.

The tractive power diminishes as the speed increases, and approximately from $\frac{3}{4}$ to 4 miles per hour—nearly in the inverse proportion to it.

A horse on a level is as strong as five men; on a grade of 15 per cent is less strong than three men. A horse can exert for a short time twice the average tractive force throughout a day's work. If, then, the resistance on a short grade does not exceed twice that on a level, he can draw his full load. But even a single excessive grade on a comparatively level road will limit the load over the

entire distance. The surfaces on grades are more worn by the horses' feet than on a level, requiring thicker covering and increased cost of construction. In winter the surfaces become slippery, causing danger in descending and more labor in ascending. The surfaces are more injured by running water, causing increased cost of maintenance.

50. Tires only 2½ inches wide will cause twice the wear that 4½-inch tires will. The narrow tires cut into the road surface, forming and deepening ruts. For light vehicles 2½ inches will be sufficient. The heavy vehicles in France have tires from 3 to 10 inches wide. Usually from 4 to 6 inches should be required. The rear axles are 14 inches longer than the front ones; the wheels therefore do not follow the same exact line.

Size of Wheels.—Wheels diminish the friction on the ground by transferring it from the circumference to the nave and axle, and they serve to lift the vehicle more easily over obstacles.

The friction is diminished in the ratio of the circumference of the axle to the circumference of the wheel; hence the larger the wheel and smaller the axle the less is the friction. Wheels, however, should not have so great a radius that the line of traction will incline downwards, as the tendency would be to press the wheel against the ground. Wheels of large diameter do less damage to the road and cause less draught for horses.

The best average diameter is about 6 feet.

A two-wheel cart does far more damage than one with four wheels with the same load, caused by sudden and irregular twisting motions.

51. The harder and smoother materials used for road coverings, the greater will be the economy in horse-power. For example, if it requires 40 forty horses to draw a certain load on a sandy road, it will require 20 on an earth road, 13 on a cobblestone, 7 on the best-laid cobblestone, 3½ on the best Belgian-block pavement, 1½ horses on asphalt, and 1 horse to draw it on iron rails, on a level roadbed.

Much interesting information can be obtained from a paper on Haulage by Horses, read by Mr. Thomas H. Brigg at the World's Engineering Congress, Chicago. If Mr. Brigg's device for increasing the efficiency of animal power, increasing the animal's endurance and adding to his comfort, is of any value, it should certainly be adopted.

52. *Alignment.*—The line should be as straight as possible consistent with easy grades and the least cost of construction. These will usually necessitate a greater or less deviation from the straight line. The curves should have as long radii as practicable, and should never be less than 50 feet in length, this corresponding to a 114° 36′ curve.

53. TABLE III.

TABLE OF RESISTANCE TO TRACTION.

Character of Road.	Resistance— In Fractions of Load.	Resistance— In Pounds per Ton.
Sand	1/5	448
Sandy road	1/12	187
Gravel (loose)	1/7	320
" (hard rolled)	1/30	75
Turf (wet)	1/8	280
" (dry and hard)	1/25	90
Earth (ordinary)	1/10	224
" (dry and hard)	1/22 to 1/30	101 to 75
Clay (hard)	1/20	112
Cobblestones	1/8 to 1/30	280 to 75
Macadam (bad)	1/14	160
" (ordinary)	1/25 to 1/35	90 to 64
" (best)	1/50 to 1/70	45 to 30
Belgian block (ordinary)	1/40	56
" " (good)	1/45 to 1/87	50 to 26
Granite block (ordinary)	1/35	90
" " (good)	1/17 to 1/50	132 to 45
Plank-road	1/40 to 1/57	56 to 40
Asphalt	1/133	17
Granite tramway	1/130 to 1/166	17 to 14
Iron "	1/200	11
Sleighs on snow 3 inches thick, ¾-inch runners, at 26° Fahr.	1/30	75

The above table gives the results of experiments by many persons. They can only be taken as rough guides. The use of the table is simple. On a sand road to haul 2 tons at an ordinary pace or trot from 3 to 12 feet per second requires an expenditure of over 896 pounds of tractive force, whereas on asphalt it would require only 34 pounds; or the same power would haul on the latter nearly 27 times as much as on the former, or it would take 27 horses to haul on a sand road what 1 horse could haul on asphalt: and similarly for other conditions and character of road surfaces.

TABLE IV.

RESISTANCE ON GRADES.

Grade resistance is due to the force of gravity, and is the same on good as on bad roads.

Grade of 1 foot in......	20,	25,	50,	75,	100,	150,	200,	300,	400 ft.
Grade, feet per mile...,	264,	211,	106,	70,	53,	35,	26,	18,	13
Resistance, lbs. per ton.	112,	90,	45,	30,	22½,	15,	11½,	7½,	5⅝.

The resistance being $= 2240 \times \frac{264}{5280} = \frac{2240}{20} = \frac{2240}{\text{rate of grade}} = 112$; and similarity for the other rates of grade.

Taking from Table III the resistance on a Belgian-block pavement the resistance of 50 pounds on a level road, the resistance on a grade of 1 in 20, or a 5-per-cent grade, would be $50 + 112 = 162$ pounds per ton; or a horse could only haul about $\frac{50}{162}$ of the load on a level—something less than $\frac{1}{3}$. Table V shows the load that a horse can draw on a level and on grades of 5 and 10 per cent, the horse being a good average and weighing 1200 pounds:

TABLE V.

Character of Surface.	On a Level. Pounds.	On a Grade of 5 per cent, or 1 in 20. Pounds.	On a Grade of 10 per cent, or 1 in 10. Pounds.
Asphalt....	13,216	not used on such steep grades.	
Broken stone (best)	6,700	1,840	1,060
" " (bad)	1,840	1,040	740
Earth (best)	3,600	1,500	930
" (moist, not muddy)	1,100	780	600
Stone block (dry and clean)	8,300	1,920	1,090
" " (muddy)	6,250	1,800	1,040
Sand (wet)	1,500	675	390
" (dry)	1,087	445	217

The decrease in load which a horse can draw on inclines depends not only upon gravity, but upon the foothold afforded by the road surface, as seen by the following:

TABLE VI.

	Level.	1 in 100.	2 in 100.	3 in 100.	4 in 100.	5 in 100.	10 in 100.
Earth, per cent.......	1.00	.80	.66	.55	.47	.41	.26
Broken stone........	1.00	.66	.50	.40	.33	.29	.16
Stone block.........	1.00	.72	.55	.44	.36	.30	.14
Asphalt....	1.00	.41	.25	.18	.13	.10	.04

54. In Table VI the load that a horse can haul on a level at a given speed is taken equal to unity for either of the materials used on the road surface. The other numbers in the same line show the percentages that can be hauled on grades of different rates with the same road-covering material and at the same speed. These percentages also show the effects of rough and smooth surfaces, being relatively higher on those materials affording good foothold for the horses.

The tractive power of horses decreases rapidly as the speed increases. This is shown in the following table:

TABLE VII.

Miles per hour............	0.75,	1,	$1\frac{1}{2}$,	2,	$2\frac{1}{2}$,	$3\frac{1}{2}$,	4.
Tractive force in pounds...	333,	250,	167,	125,	100,	$71\frac{1}{2}$,	$62\frac{1}{2}$.

Within the limits of $\frac{3}{4}$ and 4 miles the power increases in inverse proportion to the speed, and can be interpolated for any intermediate speed. For instance, at 2 miles per hour the tractive force is 125 pounds; hence for 3 miles per hour

$$3 : 2 :: 125 : 85.33 \text{ pounds.}$$

Table VIII shows some useful relations. Column 1 is the rate of grade in feet per 100 feet. Column 2 is the rate of grade in feet per mile, found by multiplying the numbers in column 1 by 52.8. In column 3 are the weights resting on the inclines and taken at 1 ton = 2240 pounds. In column 4 are the components of these weights normal to the incline. $N = W \cos \alpha$, which produces a pressure on the plane. The friction F caused by this pressure is $F = fN$, in which f is the coefficient of friction and determined by experiment, and which varies with the nature of the bodies and the condition of their surfaces. Column 5 contains the components of the weights acting parallel to the inclined surface,

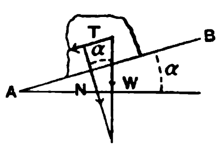


FIG. 33.

$$T = W \sin \alpha = N \frac{\sin \alpha}{\cos \alpha} = N \operatorname{tang} \alpha.$$

This tangential component T tends to produce motion down the incline.

Stability of position requires that T shall not exceed F; hence

$$T = F = fN = N \text{ tang } \alpha = W \sin \alpha,$$

the angle α being the greatest angle of inclination consistent with stability, and is usually indicated by the letter ϕ. Hence

$$F = N \text{ tang } \phi = fN.$$

From which we see that the coefficient of friction $f = \text{tang } \phi$, or the tangent of the angle of repose, which is defined as the steepest inclination of a plane to the horizon at which a block of a given material will remain in equilibrio upon it. So long as T is less than fN, or α is less than ϕ, T will be balanced by the friction, which will be equal and opposed to it; and since fN is the maximum value of the friction, motion will occur if T is greater than fN.

(Table IX contains some usual values of f, the coefficient of friction, and the corresponding values of ϕ.)

Column 6 contains the power required to haul 1 ton up the inclines, that on a level being taken at 45 pounds, which may apply to macadam, Belgian, or granite block roads or pavements. Column 7 gives some equivalent lengths of level roads in miles, and simply in the ratio of the resistances or powers in column 6; for instance, where the resistances are 45 and 67.40 we have

$$45 : 67.4 :: 1 \text{ mile} : x = 1.5 \text{ miles},$$

the equated length. Column 8 gives average maxima loads that the horse can draw on a level and on the several inclines.

It will be noticed that the normal pressure N, column 4, is practically the same as the weight until a grade of 2 in 100 is reached, and decreases at nearly a uniform rate to 3 in 100; from that point to 10 feet in 100 feet it decreases at an increasing rate. The tendency down the plane T increases in the proportion of the rate of grade throughout the table, as the angle corresponding to 10 in 100 is very small, being about 5° 43′ of arc, and the arc nearly equal to its sine.

TABLE VIII.

Rate of Grade in feet per 100 feet.	Rate of Grade in feet per mile.	Weights resting on the Inclines in pounds.	Normal Pressure on the Inclined Surfaces, $N = W \cos \alpha$, in lbs. per Ton.	Tendency down the Inclined Surfaces, $T = W \sin \alpha$, in lbs. per ton.	Power required to haul one ton = 2240 lbs. on the Inclines.	Equivalent Lengths of Level Road in miles.	Maximum Loads in lbs. which a Horse can draw on the Inclines.
0.0	0.0	2240	2240	0.0	45.0	1.000	6270
0.25	13.2	"	"	5.6	50.6	1.124	5376
0.50	26.4	"	"	11.2	56.2	1.249	4973
0.75	39.6	"	"	16.8	61.8	1.373	4490
1.00	52.8	"	"	22.4	67.4	1.500	4145
1.25	66.0	"	"	28.0	73.0	1.622	3830
1.50	79.2	"	"	33.6	78.6	1.747	3584
1.75	92.4	"	"	39.2	84.2	1.871	3290
2.00	105.6	"	2239.55	44.8	89.8	1.995	3114
2.25	118.8	"	2239.41	50.4	95.4	2.120	2935
2.50	132.0	"	2239.27	56.0	101.0	2.444	2725
2.75	145.2	"	2239.13	61.6	106.6	2.369	2620
3.	158.4	"	2239.0	67.2	112.2	2.493	2486
4.	211.2	"	2238.2	89.6	134.6	2.991	2083
5.	264.0	"	2237.2	112.0	157.0	3.490	1800
6.	316.8	"	2236.	134.4	179.4	3.987	1568
7.	369.6	"	2234.5	156.8	201.8	4.485	1367
8.	422.4	"	2232.8	179.2	224.2	4.982	1235
9.	475.2	"	2231.0	201.6	246.6	5.480	1125
10.	528.	"	2229.	224.0	269.0	5.977	1030

TABLE IX.

	ϕ	f	$1/f$
Dry masonry and brickwork.... ...	31° to 35°	0.6 to 0.7	1.67 to 1.43
Masonry and brickwork, mortar damp	36½°	0.74	1.35
Timber on stone....	22°	0.4	2.5
Iron " "	35° to 16⅔°	0.7 to 0.3	1.43 to 3.33
Timber on timber	26½° to 11⅓°	0.5 to 0.2	2 to 5
" " metals..	31° to 11⅓°	0.6 to 0.2	1.67 to 5
Metals on metals..................	14° to 8½°	0.25 to 0.15	4 to 6.67
Masonry on dry clay........	27°	0.51	1.96
" " moist clay..............	18¼°	0.33	3
Earth on earth.....................	14° to 45°	0.25 to 1.0	4 to 1.
Earth on earth, dry sand, clay, and mixed earth........	21° to 37°	0.38 to 0.75	2.63 to 1.33
Earth on earth, damp clay...........	45°	1.0	1.0
" " " wet "	17°	0.31	3.23
" " " shingle and gravel...	39° to 48°	0.81 to 1.11	1.23 to 0.9

Table IX gives the more useful values of the angles of repose ϕ, the values of the coefficient of friction f equal to the tangent of the angles of repose, and $1/f$ equal to the cotangent of the angle of repose.

The first column being the angle of repose, the second, f, is the length of the tangent to radius unity, and the third is the length of the cotangent to the radius unity. This table will also be useful in connection with masonry construction.

TRACTION ON HIGHWAYS.

54½. In the preceding paragraphs the subject of traction on highways was fully discussed, and tables given showing the tractive power required to move vehicles on streets and roads, taken mainly from old experiments made in France and England.

The following table is taken from recent experiments made by Studebaker Brothers' Manufacturing Company, of South Bend, Indiana.

The tests were made by attaching a Fairbanks' dynamometer to the doubletree and making the pull through this instrument.

TABLE IX*a*.

RESULTS OF TRACTION EXPERIMENTS.

Wagon Type	Thimble-skein 3¼ in.							California Wide Track, Iron Axle.		Narrow Track, Iron Axle.
Diameter of wheels	3′ 8″ and 4′ 6″		3′ 6″ and 3′ 10″					3′ 8″ and 4′ 6″		3′ 8″ and 4′ 6″
Width of tire	4 in.	1½ in.	4 in.	4 in.	1½ in.	1½ in.	3 in.	3 in.	1¾ in.	1¾ in.
Total weight, pounds	4845	4235	4260	3700	4050	3500	4590	5280	5280	5130
Weight per inch of tire	272	706	266	231	675	583	383	440	754	733
Pull to start on: Block pavement	350	300	600	....	500	....	..	600	500	400
Pull to start on: Good sand roads	700	725	700	...	700	....	700	800	700	900
Pull to start on: Good gravel "	600	650	750	...	...	....	...	700	550	500
Pull to start on: Muddy "	800	900	1000	..	900	....	....	1050	1250	1000
Pull to move on: Block pavement	100	75	150	90	125	90	125	100	75	100
Pull to move on: Good sand roads	275	300	350	300	350	350	350	300	300	325
Pull to move on: Good gravel "	175	175	250	....	200	....	....	200	175	150
Pull to move on: Muddy "	550	500	650	450	550	....	....	600	700	700

Many important conclusions may be drawn from the study of the above table. (1) The small influence that the width of the tire has

on the draught necessary. Taking the third and fifth cases, where the weights per inch of tire were 266 and 675 pounds respectively, the percentages of total load required to start the wagon on block pavements, sand, and mud were 14, 16½, and 23½ in the former case, and 12½, 17¼, and 22¼ in the latter; while the percentages of load required to keep the wagon moving when started on these surfaces were 3½, 8¼, and 15¼, and 3, 8½, 13½. There is not as much variation as 2 per cent, even on muddy roads.

(2) With the wide iron tire, iron axle California wagon, excluding muddy roads, the narrow tire was found to be the better, and on muddy roads the difference was less than 5 per cent. (3) The value of large wheels in diminishing draught in starting is shown, but there is but little difference in the traction required to keep the motion up. (4) The average proportion of load required as a pull to start a wagon on a block pavement was 10 per cent, 16 per cent on good hard sand roads, 18 per cent on good level gravel roads, and 21 per cent on muddy roads; and to keep the vehicle moving on the same, 2½, 7¼, 4, and 13 per cent of the load was required.

It was also found, but not tabulated, that with the weight and tire given in the first case a pull of 1050 pounds was required to start the load in mud from 12 to 14 inches deep, and 550 to keep it moving. Under the conditions of the third tests from 900 to 1600 pounds were required to move the load on muddy dirt roads filled with ruts. On sandy roads, where the wagon given in the fourth case cut into the road to an average depth of 3 inches, the pull required was 650 pounds; and on level sandy roads, where the tires cut into the sand to an average of 4 inches, 1100 pounds were required to start the wagon.

In order to ascertain the effect of narrow and wide tires in driving across fields the wagon and load of the seventh test was chosen, with the tire cutting into the sod 1½ inches. 1250 pounds were required to start the wagon with a 1½-inch tire, and 1100 pounds with a 3-inch tire, and to move at a dead pull 650 and 550 pounds respectively. On good hard roads the wagon with a 1½-inch tire was started with a pull of 850 pounds and kept moving with 350 pounds, while with a 3-inch tire 700 and 300 pounds respectively were required.

To start the California wagon with a 3-inch tire on roads with deep ruts a pull of 1500 pounds was required, and to keep it going

450 pounds; to start it on good sod, where the tire cut in about half an inch, 900 pounds, and to keep it going 600 pounds; while with a $1\frac{3}{4}$-inch tire 1000 and 500 pounds were required in the same field. The narrow tire did not cut deeper than the wide one.

The experimenters concluded from their tests that, on hard roads and pavements, there is no special advantage to be gained by the use of the wide tire, and on such surfaces a narrow tire gives less resistance and friction. Neither in soft mud or slush do they find any advantage in wide tires as regards draught; but where the narrow tire will cut through and a wide one will not, the latter is to be preferred. The general conclusion being that for farm use the wide tires are better, but for streets and pavements the narrow tires are preferable.

ART. VIII.

CITY SURVEYING.

55. The conditions determining the site of a city depend largely upon (if they are not controlled by) considerations outside of the lay of the ground as affecting lines, curves, and grades; such as natural advantages arising from navigable streams; watercourses suitable for feeding canals; good centres for the junction and crossing of highways and railways, as governed by the topographical features of the country; and the agricultural, mineral, and general industrial conditions and capacities of the community. Frequently cities simply start and grow to considerable size without surveys or considerations governing such questions as the alignment of its streets, easy grades, advantages of drainage and sewerage, proper widths of streets and pavements, or any of those conveniences and sanitary requirements arising from a number of parks and resting-plots, and often even without reference to the sufficiency and healthfulness of a water-supply. In such cases, when inconveniences, conflicts, and necessities require, the considerations of proper surveys, sewerage, drainage, water-supply, etc., are first brought into prominence. From this period the surveys are directed, as far as practicable, to correct the errors already made, often at a very large expense, or they are left as they are found, and the surveys are mainly directed

to properly provide for the future requirements which other growing towns have shown to be necessary or desirable. It is mainly this latter view of city surveying that will be considered in this article, especially as in this country the probable future requirements are well known from the experiences of other cities, barring alone the uncertainties in regard to the future growth of the proposed city. This latter question should not, however, be considered, and the city should be laid out carefully on the basis of some assumed number of inhabitants and a corresponding commercial and industrial development.

56. City surveying therefore differs from road surveying in two very important points.

1st. The laying out of the site of the city must be adapted to the ground as found, as the ground cannot be selected to suit the site of the city. Hills, ravines, watercourses, etc., must be dealt with in the main as they are found.

2d. As we are to deal not with miles and acres, but with small blocks and lots, the value of which may be enormous per front foot or even front inch, absolute accuracy in the survey is necessary. As this may be impracticable, every care should be taken that it may be obtained within very small limits of error in directions and distances, otherwise ultimate confusion, trouble, and litigation will be inevitable.

In land surveying an error of not more than 1 in 300 may be allowable. In city surveying it should be not more than 1 in 1000, and should average less than 1 in 5000, and a standard of average observed difference in one surveying party's work may not exceed 1 in 20,000. Two parties working to reach the same standard may err in opposite directions, making the apparent error greater than that of either party.

57. As the magnetic needle at best is unreliable and seldom correct, its use in city surveying should never be allowed. Transits without needles and compass-box can be constructed affording more steadiness, accuracy, and convenience in reading and handling.

The steel tape is almost universally used for measuring distances. These tapes are commonly 50 to 100 feet long, and may be used as long as 200 to 300 feet.

It is not necessary that such tapes should be graduated, as is usually the case, to feet and tenths.

Every subdivision in the measurement introduces an element of error, and the number varies inversely as the length of the measure. Measuring on an inaccurate alignment, or with the ends of the tape not in a precisely the same horizontal line, also introduces errors which are cumulative in proportion to the number of times the tape is stretched in measuring any given distance. These errors are called minus errors, since the measured distance between any two points is greater than the true distance. In using long tapes, the natural sag due to the length introduces a minus error; if pulled to any given tension the minus error is reduced, and moreover the elongation introduces a plus error. If a tape is standardized at a certain temperature, an increase in temperature causes a plus error, whereas a decrease causes a minus error.

All such errors may to a certain extent be self-compensating on long lines, but to what extent and at what points is uncertain.

There is, then, a limit to the length of a tape that will give the best results. Probably the 100-foot tape is more commonly used.

The accuracy with which angles can be measured depends upon the character of the instrument. With ordinary transits reliance cannot be placed on reading nearer than to one half minute or 30 seconds. Lines making an angle of 1 minute with each other would separate in a mile by 1.53 feet or 18.36 inches, and with angle of 30 seconds by 0.765 foot or 9.18 inches, and proportionately for any other distance. The error in distance on this line would be inappreciable either for a minute or half minute of angle, but the error in alignment would be appreciable. It is probably about as easy to keep the angular error in measuring angles within one half minute as it is to keep the error in measuring distances within 1 in 10,000.

58. The correction for temperature may be taken for each degree Fahr. at 0.0000065 of the length, which for the distance of a mile is about 0.41 of an inch for change of 1° Fahr. Such corrections should be applied on the field.

With a constant pull or tension the sag increases nearly as the cube of the length. With any given length the sag can be reduced to any degree by the increase of the tension, but this tends to unsteadiness and want of uniformity in the pull, even with only a slight wind blowing. It is necessary, however, to keep the sag uniform. This to a great extent can only be determined by the

expeirenced eye. The eye must also be mainly relied upon to determine the horizontality of the ends of the tape.

The catenary is defined as the curve assumed by a cord, tape, or chain of uniform cross-sectional area and of uniform material when loaded with its own weight alone. The weight of any part of it is simply proportional to its length. A full discussion of this curve is found in Rankine's Applied Mechanics, page 177. For the present purpose it will be sufficient to assume the following equations:

$$y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}} - 2\right) = \sqrt{s^2 + m^2} - m = \frac{H}{2w}\left(e^{\frac{wx}{H}} + e^{-\frac{wx}{H}} - 2\right); \quad (33\tfrac{1}{2})$$

$$s = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) = \frac{H}{2w}\left(e^{\frac{wx}{H}} - e^{-\frac{wx}{H}}\right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

in which $m = \frac{H}{w} = \frac{\text{horizon tension}}{\text{weight per unit of length}}$; e = base of Naperian logarithms; s = length of curve measured from centre or lowest point taken as the origin; l = the horizontal distance between points of support; $x = \frac{1}{2}l$; y the ordinates of the curve; $W = wl$ = weight of the entire tape; $w = \frac{W}{l}$; $\therefore \ \frac{H}{w} = \frac{Hl}{W}$. If $\frac{H}{w}$ is constant, that is, if the horizontal pull is constant, then for the same tape both y and s will be constant, but will vary with different tapes; but if $\frac{Hl}{W}$ is constant, both y and s are constant.

If the chain is 100 feet long, then $x = 50$ feet; let $H = 10$ pounds, $w = 0.0125$ pounds, then $\frac{H}{w} = 800$; y becomes 1.56 feet; $s = 50.0325$, $\therefore 2s = 100.0650$ and $2s - l = 0.065$ feet; and from the formula for elongation (see Johnson's Surveying, page 379), $\lambda = \frac{HL}{Ek}$, and making the section $k = \frac{w}{3.4}$, $E = 27500000$, then $\lambda = 0.01$. With $H = 20$ pounds, $y = 0.78$ feet; $2s - l = 0.014$ feet; $\frac{H}{W} = 16$; $\lambda = 0.02$ feet. With $H = 30$ pounds, $y = 0.52$ feet;

$2s - l = 0.007$; $\frac{H}{W} = 24$; $\lambda = 0.03$ feet. And similarly for any other ratio of $\frac{H}{w}$ and $\frac{H}{W}$ for a 100-foot tape. We then have, for the three relations taken,

	$2s - l$ — error.	λ + error.	Resultant error in 100 feet ± error.	In 1000 feet ± error.
1st,	0.065 foot	0.01 foot	− 0.055 foot	− 0.55 foot
2d,	0.014 "	0.02 "	+ 0.006 "	+ 0.06 "
3d,	0.007 "	0.03 "	+ 0.022 "	+ 0.22 "

From which we see that the error of sag diminishes rapidly, and the plus error of elongation increases rapidly with an increase in the pull. With care and practice experienced men can make a pull of 15 to 20 pounds with a reasonable degree of uniformity. The pull should be steady and without jerk, and the pins should be stuck quickly in the ground. From a complete table, calculated as above indicated from 10 to 30 pounds pull, for either 50 or 100 foot tapes, the proper pull is determined by the tension at which the tape is tested. Call this h; then with the determined weight W of the tape we have $\frac{hl}{W} = \frac{h}{w}$. Seek the plus error from elongation for this value of $\frac{h}{w}$; then find the same plus error between the curve for that length of tape and the straight line: the corresponding $\frac{h}{w}$ is right for field use. This means the combined correction for sag and elongation. Mr. Johnson illustrates this principle by the following example: A 50-foot tape weighs six ounces, and the pull when tested was five pounds, $\frac{h}{w} = \frac{5 \times 50}{\frac{6}{16}} = 666$, and the elongation $= 0.083$. The curve for a 50-foot tape marked − error from sag is distant from the line marked + error from pull the same amount when $\frac{h}{w} = 1233$. Whence $P = 1233 \times \frac{6}{16} \div 50 = 9\frac{1}{4}$ pounds and the sag $= 0.25$.

When a tape is to be suspended freely in use, the tension, h, at the test should not be such that the working tension H will be too

great. Diagram Fig. 34 is plotted from tables calculated as above indicated for a 50 and a 100 foot tape, in which is shown the curves for errors in sag, and also for plus errors in pull, in full

Errors for 1000 feet $\frac{P}{w}$ variable

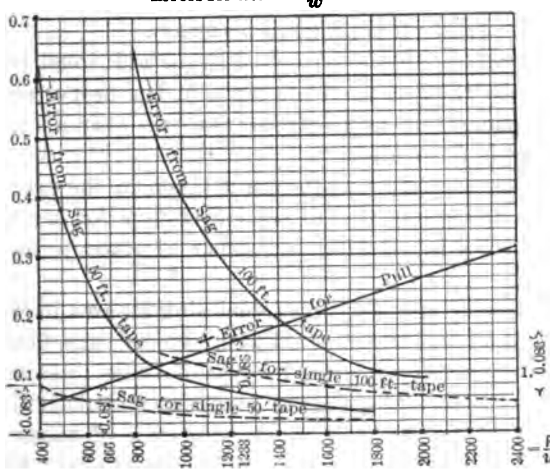


FIG. 34.

lines for 1000 feet and in dotted lines for single lengths of 50 and 100 feet. The equal errors of 0.083 and their corresponding ratios of $\frac{P}{w} = 666$ and 1233 respectively are marked on the diagram.

59. While such niceties in calculation of and applying errors for sag and elongation are of great importance in measuring long lines, and may be properly applied in city surveying, it would seem, as accuracy is what is required, better to secure a standard U. S. steel bar, and having carefully measured a base-line 50 or 100 feet long, the ends marked with strong hubs extending well above ground, all that is necessary to correct for sag and elongation is to determine the pull necessary to make the two ends of the tape have the proper distance between them when suspended between the fixed hubs. The sag and elongation will then be the same as when used in measuring distances on the ground, and whether they be much or little the measured distances will be correct, excepting temperature errors, which must be applied for the actual tempera-

ture, as compared with the standard temperature stamped on the U. S. standard bar at the time of measurement, if the change is material. Such a base-line could be measured near the camp or office, and, if desired, at two or more widely separated situations within the boundaries of the survey, and the tape could be tested once a day, or oftener if considered necessary.

Wind errors can only be corrected by an additional intensity of pull, judging by the eye when the sag is of the proper amount and in a vertical plane. To avoid errors from this source it is better to stop work in windy weather.

It is unwise, no matter what care is taken in the execution of a survey and in the application of the corrections for errors, to trust to direct accuracy in running a number of separate and distinct lines.

Outline lines or boundary lines, enclosing an area in the form of a quadrilateral, should be measured with especial care, so as to secure a high degree of accuracy in the lengths of these lines and in the angles between them. These serve as resting and check lines. The actual length of any division line joining two of these boundary lines at fixed points can be accurately calculated in length and direction, and measuring these division lines, usually street lines, they should tie up on these boundary lines within the allowable limits of error in distance and angles, and the lines should be retraced until they do.

60. *Contour Maps.*—An accurate topographical survey as a preliminary to laying out a town may or may not be of any immediate advantage, if accurately executed with 5-foot contours, and time is given to make a careful study of it, in order to lay out the streets to the best advantage in regard to grades and quantities of work required, and so adjusting streets in suburban districts as to make them easily accessible on streets with easy grades, and adjusting them at the lot lines to the undulations of the ground. Much expense may be avoided and considerable areas divided into desirable building sites, which otherwise would be either undesirable or actually wasted. But if such use is not made of these maps, and the streets are laid out at right angles to each other, which is usually the case, without reference to the above considerations, such surveys are a waste both of time and money. The cost of laying out 1000 acres into streets and blocks will depend upon

the roughness of the surface, the proportion of cleared land and of that in timber and brush, and to a very great extent upon the topographical work, and will vary from two to five dollars per acre. The more topography taken the higher the cost. As a matter of fact, but little use is ordinarily made of the contour maps in the first laying out of a town site. And when required, either from inaccuracy in the maps, or on account of the maps being lost or misplaced, with no reliable and fixed points on the ground, the work will have to be done over by running lines and levels usually along both sides of the streets.

ARRANGEMENT OF CITY STREETS.

61. Whatever method may be adopted in surveying the town sites in order to secure accuracy, the first questions requiring consideration are:

1st. The proper width of the streets and the dimensions of the blocks and lots.

2d. Upon what system the streets shall be laid out—following straight lines at right angles to each other over as large districts as practicable, or a system of curved streets similarly arranged.

3d. A combination of these two systems, or finally an entirely irregular system, with street lines inclined at all angles with each other. This latter is intricate, inconvenient, and expensive.

The circular system, except as connecting two districts the main axes of which are inclined to each, as in the space between two streams coming together with an acute angle between them, is evidently defective, as the chords are shorter than the arcs ; one important consideration being to secure the most direct route between two points. The rectangular system causes loss of time and distance. So it will in general be found advantageous to combine the rectangular and diagonal systems. There may be one central point from which the diagonals radiate, or there may be several such points distributed over the town site. The diagonal streets open up new building lines. But with too many and too wide streets there will be an unnecessary loss of building area. In Fig. 35 is shown a combination of the square and diagonal systems. A variety of combinations can be made, which on a good topographical

map can be arranged to fit to a greater or less degree the undulations of the ground.

Mr. Lewis M. Haupt, Professor of Civil Engineering in the University of Pennsylvania, has made an elaborate investigation and discussion of this subject, and gives at length the explanations and formulæ showing the relations between block areas, widths of streets, the benefits accruing both from shortening of distance from point to point and the additional frontage furnished, with a small displacement of the inhabitants. In the case of a perfect square with n

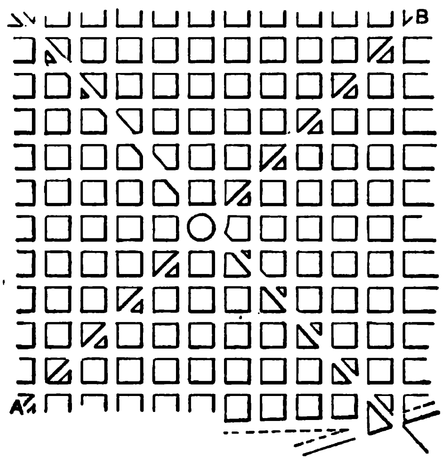


FIG. 35.

blocks on a side or n^2 blocks in all, and with a width of street w, L the length of the side of the entire square, and l one side of a square block in feet, l^2 = area of block. Then, from Fig. 35, the distance from A to B with square blocks must be equal to $2L$ by any route followed, whereas with the diagonal the distance $AB = L\sqrt{2}$, and the ratio is $\frac{L\sqrt{2}}{2L} = \frac{1.41.42}{2} = \frac{1.41.42}{200} = \frac{70}{100}$, or a saving in distance of about 30 per cent, the maximum possible. The percentage of street to property area is $\frac{w'c + aw + ww'}{ac} \times 100$, in which w and w' are the widths of the streets, a and c the sides of the blocks when rectangular. If the widths are the same, then $w = w'$, and if the blocks are square then also $a = c = l$, and the

above expression becomes $\frac{2wl + w^2}{l^2} \times 100$. If the streets are 60 feet wide, $l = 400$ feet, then $\frac{2 \times 60 \times 400 + 3600}{160000} \times 100 = 36$ per cent of the area consumed by streets; with the width of the streets $w = 50$, and the sides of the blocks $l = 500$ feet, then the percentage will be 21 only. Similar formulæ can be obtained for the additional area consumed by diagonals and for the increase in frontage.

62. There is a great diversity of opinion in regard to the proper width of streets. An important commercial thoroughfare should be at least 120 feet, 80 feet for the carriageway and 20 feet on each side for the sidewalks. For streets to be occupied by residences, the carriageway need not be over 32 to 35 feet, and the sidewalks from 10 to 12, or a width between building lines from 52 to 65 feet or more. A wider system of streets in one direction than in the other is sometimes adopted, and rectangular blocks instead of square blocks are used, with an alley of 15 or 20 feet running parallel to the longer side; such a block containing 16 lots, 8 on each long side, 50 feet wide, and making the length of the lot 140 feet and alley 20 feet. The block, not including any portion of the street would be 400 × 300 feet, containing 120,000 square feet. Or from centre to centre of streets 60 feet wide, a block would contain 460 × 360 = 165,600 square feet. 165,600 ÷ 43,560 = 3.8 acres nearly, or about 4.21 lots per acre.

63. *Monuments.*—Permanent marks should always be provided. There is much difference of opinion in regard to the material, number, and positions of such monuments. Stone is beyond doubt the best material. They should be dressed to a square cross-section for from 3 to 6 inches in length from one end, the cross-section being 4 × 4 to 6 × 6 inches; the rest of the stone can be left rough, and should be, as furnishing broader base and better bearing. Their lengths should be from 3 to 5 feet. Two lines, intersecting in the centre of their upper ends, should be cut; in this case the stones must be set with this intersection of cross-cuts exactly in line. This is best, but makes it difficult to set and adjust. Otherwise the stones are set first and the cross-cuts made to intersect at the proper points. This is convenient and sufficient so long as the cross-cuts remain distinct. Otherwise holes can be cut in the stone and filled with lead, the exact intersection marked by small holes punched in the lead.

Yellow Locust makes a hard, durable monument. The exact intersection's marked with a small nail. These are less expensive than the stone.

It would be better probably to use, say one third or one half stone and the remainder timber.

As to the proper number and positions, opinions and practice differ. One is sometimes set at the intersection of the centre lines of streets. Again, two may be used on diagonal opposite corners; and again, four may be used at each intersection, set about the middle of the pavements on the diagonal lines. In every case they should be set with their tops either flush with the ground or, better, a few inches below. The character, position of points on top, number and positions of the monuments, should be recorded on the sectional maps.

GRADES ON STREETS.

64. With time and a good topographical map the street-lines and their grades can be adjusted to the best advantage. This is seldom done, as in a rolling district a large amount of excavation and embankment would be required for the streets, and a large number of builders would have to provide basement stories or cut to great depths. Grades are usually adjusted to the ground in order to keep both fills and excavations within reasonable limits.

The following are a few examples of the widths and grades of streets of some large cities:

TABLE X.

City of—	Width of Streets between Building-lines.		Max. Grades Feet per 100 Feet.	Average Widths of Sidewalks in Feet.
	Max. Feet.	Min. Feet.		
New York	100	60	13	15
Syracuse, N. Y.	120	33	20	
Lowell, Mass.	70	30	16	14
Trenton, N. J.	80	30	8	16 to 6
Richmond, Va	118	30	8	
Washington, D. C.	160	80		
Providence, R. I.	225	10	19	$\frac{1}{6}$ the width of the street.
St. Paul, Minn.	200	50		
Milwaukee, Wis.	100	60	9	25 to 12
London, Eng.	80	12	4	15 to 3

The proper adjustment at street intersections, where the grades are different, will be alluded to under Street Construction.

ART. IX.

HYDROGRAPHY AND HYDROGRAPHIC SURVEYING.

65. Canals are simply artificial water-channels. They may be constructed for navigation, irrigation, drainage, or sewerage, and to supply cities with water.

As the rainfall is the original source of water-supply for any purpose, it becomes necessary to know the drainage area of any watercourse or district, and the amount, periods, and duration of the rainfall upon it.

66. *Drainage Area.*—The surface of any continent is divided into a series of districts, which are inclosed on all sides but one by a ridge-line, or watershed line as it is called. A large proportion of the water on one side of this ridge-line finds its exit either directly or indirectly at the lower and open side of such a district. This district constitutes a drainage area or catchment-basin. It may be thousands of miles in length and hundreds in width. It is traversed by one main stream having its source or head on the main ridge and its mouth in some bay, gulf, or ocean. This river gives the name to the district, and it is generally spoken of as the Mississippi, or the James River Valley, or some other similar appellation.

Such a main stream follows a sinuous, meandering course around the spurs or secondary watersheds, which project out from the main ridge-line, and divide the main drainage area into a series of secondary basins, each of which has its own stream—large or small according to the area it drains. These streams flow into the main stream and are called tributaries. A survey of such a district will enable its area to be determined.

67. The second important factor in a water-supply is the amount of rainfall on any given district, as the drainage area is measured in square miles or acres, according to its dimensions. The rainfall is measured as the depth to which this area would be covered by water, this depth being usually measured in inches.

In any large district the annual rainfall not only varies from year to year, but also in different parts of the same district. In some cases also the rainfall may be distributed throughout the year—a little each month, less or more in the several months, or it may be concentrated in two or three months, followed by periods of drought with little or no rainfall. And often a large percentage of the annual rainfall may be measured by the fall in a few hours or days. It becomes necessary therefore to determine: 1st. The maximum annual rainfall. 2d. The mean annual rainfall. 3d. The least annual rainfall. 4th. The general distribution throughout the year; and, finally, the greatest rainfall in any short period.

It is also important to know what proportion of the rainfall escapes by evaporation from the surface; what proportion is absorbed by or sinks in the ground, all or a portion of which finds its way into the streams at some lower level; and, finally, what proportion flows directly and rapidly into the tributaries and main water-courses. These latter proportions are all more or less indefinite and indeterminate.

68. The amount of rainfall in any of its phases can be found approximately by the use of a number of rain-gauges distributed over the drainage area and by repeated examinations of the gauge. A rain-gauge, or pluviometer, consists of three parts: the collector *A*, the receiver *B*, and the overflow attachment *C*. The overflow or encasing vessel is simply a hollow cylinder, 23½ inches long and 6 inches diameter. The collector is a funnel-shaped arrangement, the upper 2½ inches being cylindrical and 8 inches in diameter; at its bottom the receiver, 20 inches long and 2.53 inches diameter, is connected. Its cross-sectional area is exactly 0.1 of the area of the collector, so that the depth of the water measured is ten times that of the rainfall. The collector rests on top of the overflow cylinder into which the receiver extends. The whole is then placed in a close-fitting box, which is sunk into the ground in some clear open space, and at least 30 feet from any building or other obstruction. The upper rim of the collector should be about one foot above the surface, and its plane should be horizontal. The depth of water in the receiver is measured by a graduated rod, or the receiver may be of glass, and the graduations of inches and fractions of an inch can be made on it, from which the depth can be determined directly.

To measure the snow, the collector and receiver can be removed, and the snow collected in the overflow cylinder and melted; or the cylinder may be inverted and pressed through the snow to the surface on some space indicating an average fall, and a piece of tin slipped under it. It is then lifted and melted.

69. The following Table XI shows the mean annual precipitation in some of the more important river-basins.

In Italy the average annual precipitation is about 40 inches. In India in some places it is as high as from 100 to 300 inches per annum.

TABLE XI.

	Altitude in Feet.	Mean Annual Precipitation in Inches.	Mean Precipitation April to August in Inches.
Rio Grande River, Summit....	11,800	29.0	
" " " "	5,082	7.19	4 to 8
Gila River..................	6,022	14.72	
" "	1,068	7.38	3 to 5
" "	141	2.81	1 to 3
Platte River................	14,134	28.65	
" "	4,500	8.08	7 to 10
Missouri River..............	5,480	16.0	5
" "	1,955	10.50	
PRECIPITATION BY STATES.			
Arizona (Fort Apache)........	5,050	21.04	10.27
" (Texas Hill.........	355	3.47	0.66
New Mexico (Las Vegas)......	6,418	22 08	12.70
" " (Albuquerque)...	5,026	7.19	4.22
California (Summit)..........	7,017	43.56	....
" (Sacramento)........	64	19.80	2.78
Nevada (Pike's Peak).........	14,134	28 65	....
" (Monte Vista).........	7,765	6.91	4.18
Utah (Nephi).................	5,550	18.19	7.40
" (St. George).............	2,880	6.74	1.32
Idaho (Eagle Rock)....... ...	4,781	18.67	4.69
" (Boisé).................	1,198	14.74	4.11
Wyoming (Cheyenne).........	6,105	1.32	5.55
" (Fort McKinney)...	....	9.60	4.45
Montana (Helena)	4,266	14.26	4.48
" (Fort Benton)........	2,730	13.30	5.45

It will be observed that throughout the western portions of the United States the precipitation in the high mountains is much

greater than in the adjacent low valleys. In winter the rainfall is relatively low in the valleys and high in the uplands. Often a large proportion of the rainfall occurs during the early spring months.

The above are only a few of the records in these States. For fuller information the reader is referred to Mr. H. M. Wilson's work on Irrigation Engineering, from which much information on matters pertaining to this subject has been obtained.

Great Rainfalls.—In Yuma, Arizona, the average annual rainfall is about 3 inches. In February, 1891, 2½ inches fell in 24 hours. In San Diego, California, the average annual fall is about 12 inches. At the above date 13 inches fell in 23 hours, and 23½ inches in 54 hours. As much as 17 inches in 24 hours was registered in California. Such storms may increase an average flood discharge of 10,000 second-feet (cubic feet per second) to from 140,000 to 350,000 second-feet. The precipitation is not unfrequently as high as from 3 to 5 inches per hour.

70. *Evaporation and Absorption.*—An important question is what becomes of the rainfall. Many experiments have been made to determine the evaporation and percolation in different soils, with no very satisfactory results, however. Evaporation is greatest when the atmosphere is dryest, and when the water is warm and its surface agitated by brisk winds. In summer the cool surfaces of deep waters condense the moisture from the warm air passing over them, and thereby gain moisture, the difference between the two being the resultant loss by evaporation.

Evaporation is greater in the desert regions of the Southwest than in the high mountains. It may vary, according to altitude, temperature, etc., from 0.6 to 1 in winter and from 6 to 14 inches in the summer months. The evaporation from snow may amount to 0.02 of an inch per day or 2½ inches in a season, and from ice 0.06 inch per day or 7 inches in a season. The evaporation from ordinary soils is about 3 inches less than from water surfaces, but it is much less from sandy soils. From some observations where the mean evaporation from water was 20.4 inches, from earth 19.7 inches, it was only 3.7 inches from sand.

Percolation.—The losses due to percolation in canals and storage reservoirs are considerable, and added to evaporation may amount to from 25 to 100 per cent, according to the character of the soil. In some soils the evaporation may greatly exceed the percolation.

But in sandy soils, or rather sand, this condition will be reversed. Loss from percolation is small on steep and rocky slopes. In dense forests, with gentle slopes and a covering of earth, leaves, etc., the percentage of percolation is high. From observations made in 1872, with a rainfall of 23.8 inches the evaporation from water was 20.4 inches, percolation in earth 4 inches, and in sand 20.1 inches. And from observations made in Bavaria it was found that on open bare ground the percolation was 11 per cent, but in forests it was 36 per cent, of the rainfall.

71. It is usual to combine the losses by percolation and evaporation under the head of loss by absorption. In ordinary canals in India the loss by absorption has been found to be about 1 second-foot per linear mile. In new canals the loss is greater, and in sandy soils it will amount to from 40 to 60 per cent of the volume entering the head, on long lines. In shorter lengths it may not exceed 25 to 30 per cent of the volume, the lengths being about 40 miles. Where canals and reservoirs have sandy beds the losses from evaporation and percolation may be about equal; but if the bottom is silted, clayey or lined with clay, the loss from percolation will be relatively small. These losses are often to a great extent compensated for by seepage-water from the hillsides, or by a return of the water lost by percolation and drainage to the canals at lower levels. In fact the actual quantity may be more than that originally supplied to the canals. It is a well-known fact that in wells from 60 to 80 deep to the water surface originally, the water surface has been raised to within 10 to 15 feet below the ground surface after long-continued irrigation in the adjacent districts.

The greater the absorbing area the greater will be the loss. It is then bad practice to let the water spread over large areas by simply building one bank across a depression. It saves some expense in construction, but entails loss from absorption.

DISCHARGE OF STREAMS.

72. The available and useful water-supply must be measured by the quantity flowing in streams and rivers.

This includes not only that which flows directly on the surface of the drainage area into the watercourses, but also that portion

which after disappearing in the surface reappears in springs and seepage water, and ultimately reaches the watercourses. This quantity is known as the run-off, and is expressed either as so many second-feet, or as so many inches in depth over the entire drainage area, or as so many acre-feet. Many formulæ have been proposed for calculating the run-off; none of them is reliable or satisfactory. Mr. J. T. Fanning gives the following:

$$D = 200(M)^{\frac{5}{6}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

in which M is the area of the watershed in square miles, D the volume of discharge in second-feet. In India the two following formulæ are used:

$$D = C\sqrt[4]{M^3} \quad \text{and} \quad D = C\sqrt[3]{M^2}. \quad . \quad . \quad . \quad . \quad (36)$$

Where the maximum recorded rainfall was from 3 to 6 inches in 24 hours the following are some of the values of the coefficient C: For 3.5 to 4 inches in flat countries, $C = 200$; mixed country, $C = 250$; hilly country, $C = 300$; and for a maximum rainfall of 6 inches C varies between 300 and 350 for the first of the two formulæ; for the second C varies between 400 and 500 in flat countries to 650 in hilly countries with maximum rainfall.

In the Eastern rivers, where the basins are comparatively flat, and covered with earth and forests, the streams are not liable to excessively high or low water, as a large portion is absorbed by the ground and returned gradually to the streams. In the Western, with barren and rocky surfaces and steep slopes, a large quantity of the water flows direct into the streams, causing very high floods for short periods, subsiding in a few hours or days to insignificant rivulets. The run-off is found to be from 50 to 80 per cent of the rainfall, according as the slopes are flat and timbered, or steep and barren. These may be taken as mean discharges of streams.

73. Wells and springs usually derive their water-supplies from shallow gravel, sand, or marl formations, which find an outlet at the surface, or, if excavated into, at no very great depths below the surface. They are more or less affected by recent rains, and their temperature by that at the surface. If, however, such water-bearing strata have no outlets except at elevated points well above their lower portions, being encased in impervious strata, an artesian

basin is formed. If pipes or shafts are sunk from the surface into them the water will rise to great elevations in them and often spouting out above the surface, due to the hydrostatic pressure of the water in the strata. These are called artesian wells.

Artesian wells may have any depth. Some of the deepest are from 3000 to 5000 feet. Up to 1890 there were 8097 artesian wells in the arid regions of the West and Southwest. Their average depth is 210 feet, average cost $245, and average discharge 54.4 gallons per minute. Sometimes tunnels are run under and near the dry beds of old streams; these are often run for a half to one mile into the water-bearing strata. Sometimes, also, dams are constructed below the surface and extended below the water-bearing strata, thereby cutting off the flow and bringing the water to the surface.

THE FLOW AND MEASUREMENT OF WATER IN OPEN CHANNELS.

74. The motion or flow of water is due to the action of gravity on its molecules, the general phenomenon being the same as the falling or rolling down an inclined plane of solid bodies. The particles composing a mass of water, however, flow or roll over and among each other with an inappreciable frictional resistance.

The weight of distilled water at its maximum density, 39.2° Fahr., weighs 62.425 pounds per cubic foot. Both above and below this temperature the weight of a given volume decreases. At 32° Fahr., the freezing-point, it weighs 62.417 pounds, and at 60° Fahr. it weighs 62.367 pounds, per cubic foot. At the boiling-point, 212°, it only weighs 59.707 pounds per cubic foot. A cubic foot of ice weighs 57.2 pounds. A United States gallon weighs 8.3799 pounds, equivalent to $\frac{62.425}{8.3799} = 7.45$ gallons. A cubic foot of water is usually taken at 62.5 pounds, and $7\frac{1}{2}$ gallons to the cubic foot. Ordinary river, spring, and sea-water are slightly heavier than pure water on account of the salts and solid materials contained in them.

Owing to the almost perfect freedom of motion or want of friction amongst the particles of water, the angle of repose ϕ for water is zero, and its natural slope is horizontal; and the action of gravity will in consequence cause motion with the slightest inclination of the bed of the channel. In one of the tunnels mentioned

the cross-section is 1.7 × 3 feet, the discharge about 9 second-feet, and slope of bed 3 in 1000 or 0.003 of a foot per foot of length.

75. Primarily the flow of water is governed by the same laws as falling bodies,

$$v = gt, \quad t = \sqrt{\frac{2h}{g}}, \quad \text{and} \quad v = \sqrt{2gh}, \quad . \quad . \quad . \quad (37)$$

in which g is the acceleration of gravity equal to 32.2 feet per second, t is any number of seconds, v is the velocity acquired in the time t, h is the height of fall necessary to produce any given velocity:

$$h = \frac{gt^2}{2} = \frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The above value of the velocity v is necessarily modified by the relations between the area of cross-section A in square feet and the wetted perimeter p in linear feet, and $\frac{A}{p} = r$ is called the hydraulic mean depth or radius; also by the character of the bed and sides of the channel or the surfaces along which frictional resistances to flow are developed, which not only varies with the condition as to roughness and materials forming the bed and sides of the channels, but with the presence of weeds, stones, bowlders, etc.; and finally upon the surface slope expressed as the ratio of fall h to length l, that is, sine of angle of slope $= \frac{h}{l} = i$. These conditions varying so greatly render any formula unreliable and uncertain; nor does the velocity at any point correspond strictly with the slope, as the inertia of the water tends to produce uniform motion on varying slopes. Therefore the simple difference of level between water surface at any two points will not necessarily determine the velocity of flow at the lower section. It would be necessary for the slope to be uniform for a considerable distance above and below the section in question. A simple formula representing the mean velocity of flow is

$$v = \sqrt{\frac{2g}{m}} \times \sqrt{ri}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

in which g, r, and i have the values given above, and m is a variable coefficient varying with the other conditions modifying the velocity. m varies between 0.05 for a value of hydraulic mean radius $r = 0.25$, to 0.0298 for $r = 1$, and then diminishes constantly to 0.0074 for $r = 10$, and 0.002 for $r = 25$.

D'Arcy's formula is

$$v = r\sqrt{\frac{1000i}{0.08534r + 0.35}}. \quad . \quad . \quad . \quad . \quad . \quad (40)$$

$i = \frac{h}{l}$ as before, and $r = \frac{A}{p}$.

The formula now more commonly indorsed is known as Kutter's formula, and is as follows:

$$v = \left\{ \frac{\frac{1.811}{n} + 41.6 + \frac{0.00281}{i}}{1 + \left(41.6 + \frac{0.00281}{i}\right) \times \frac{n}{\sqrt{r}}} \right\} \times \sqrt{ri} = C\sqrt{ri}. \ (41)$$

v = velocity of flow in open channels; r = hydraulic mean radius equal to area of cross-section in square feet divided by wetted area in feet; i = sine of angle of slope $= \frac{h}{l}$, the fall divided by the length in the same units; and n the coefficient of roughness. Some of the values of n are as follows:

$n = 0.009$ for well-planed timbers, and 0.012 for rough timber;

$n = 0.01$ for plaster in cement, glazed-iron pipes, and glazed-stoneware pipes;

$n = 0.013$ to 0.017 for ashlar masonry, tuberculated-iron pipes, and brickwork, according to smoothness of the surface and its condition;

$n = 0.02$ for rubble in cement, coarse rubble of all kinds, also coarse gravel carefully laid and rammed, or for rough rubble where the interstices have become filled with silt;

$n = 0.0225$ for good earth canals; and from 0.025 to 0.03 for those having tolerably uniform cross-section and slopes to those in rather bad condition having some stones and weeds obstructing the channels;

$n = 0.035$ to 0.05 for canals and rivers with earth beds in bad

condition and obstructed by stones, etc., to torrents covered with all varieties of detritus.

It is usually better to use the higher values of $n = 0.0225$ and 0.035, with these values and the $\sqrt{r} = 0.4$, 1.8, and 4.0, and $i = \frac{1}{1000}, \frac{1}{5000}, \frac{1}{10000}$.

$C = 35.7$	80.3	99.9	for 1 in	1000	and	$n =$	0.0225;
$C = 33.0$	80.3	103.7	" "	5000	"	"	";
$C = 30.5$	80.3	107.9	" "	10000	"	"	";
$C = 19.7$	51.6	69.2	" "	1000	"	"	0.035;
$C = 18.3$	51.6	72.2	" "	5000	"	"	";
$C = 17.1$	51.6	75.4	" "	10000	"	"	".

Such tables are worked out in many books with the different values of i, r, n, and are useful in saving the labor of calculation. They are found in such works as Wilson's Irrigation and Johnson's Surveying.

76. It is interesting to understand the methods adopted in evolving such formulæ. Assuming a uniform bed and slope,

Let A = area of cross-section; p = wetted perimeter;

$r = \frac{A}{p}$ = hydraulic mean radius;

v = velocity in feet per second ($= l$ for one second);

i = surface slope $= \sin \alpha = \frac{h}{l}$; h = fall in length l;

Q = quantity discharged in one second;

S = wetted surface in length $l = pl$;

f = coefficient of friction per unit of area S;

w = weight one cubic foot of water = density or heaviness.

Then, since the friction varies with f, w, S and the square of v, the total frictional resistance

$$R = Swfv^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

the work K performed in overcoming this in one second of time

$$= K = Rv = fwSv^3; \quad . \quad . \quad . \quad . \quad . \quad (43)$$

and if the velocity is uniform, as assumed, K is also the work per-

formed by gravity on this same mass of water in falling through the difference of level h. Hence

$$K = \text{weight} \times \text{fall} = Qwh = fwSv^2 = vAwh, \quad . \quad . \quad (44)$$

as the quantity is equal to velocity by the area $= Q = vA$.

Then

$$h = \frac{fwSv^2}{vAw} = \frac{fS}{A}v^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$S = pl$, $r = \frac{A}{p}$, and $\frac{h}{l} = \sin \alpha = i$. Hence

$$h = il = \frac{fpl}{rp}v^2 = \frac{fl}{r}v^2; \quad \therefore \; i = \frac{fv^2}{r}; \quad . \quad . \quad . \quad (46)$$

and

$$v = \sqrt{\frac{1}{f} \times ri}, \quad \text{but} \quad \sqrt{1/f} = C; \quad \therefore \; v = C\sqrt{ri}. \quad (47)$$

This is known as the Chezy formula, and Kutter's formula is an adaptation of this to all cases of constant flow as expressed in the somewhat complicated coefficient equation (41).

Cross-section of Least Resistance.—From equation (47) it is seen that for channels formed by a given material with surfaces in a given condition the coefficient C is constant, and with a uniform slope of surface i is constant. Under these conditions the velocity varies as the square root of the hydraulic mean radius $r = \frac{A}{p}$. The maximum value of r will exist when with a given area A the value of the wetted perimeter p is the least. This corresponds to a circular cross-section for a closed conduit and a semicircular for an open channel. In either case

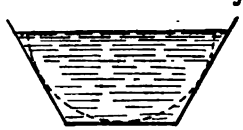


FIG. 36.

$$r = \frac{\pi R^2}{2\pi R} = \frac{R}{2},$$

in which R is the radius of the circle. It is ordinarily inconvenient to give a circular form to the cross-section of an open channel; it is usually polygonal, and should conform as closely as possible to the circle of the required area as seen in Fig. 36.

If the slope is great and it is desirable to reduce the velocity of flow, the channel should be made wide and shallow.

THE DISCHARGE OR VOLUME OF FLOW.

77. The volume of discharge in a river or canal is the product of the area of cross-section, usually in square feet, and the velocity of flow in feet per second or per minute. This product gives the volume of water passing any section in cubic feet per second or minute, and is usually expressed as

$$Q = vA, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

as in certain stretches or lengths of streams the quantity Q of flow is the same, regardless of the form and dimensions of the cross-section and also of the velocity of flow at any given section. It is usual to select a point on a stream where the cross-section is fairly uniform and approximating some regular geometric figure, and where the velocity is moderate and fairly uniform for a distance of 100 feet or more above and below the point selected. To determine the area of cross-section it is necessary to make a number of accurate soundings on a straight line as nearly perpendicular to the direction of the current as possible. This forms a series of triangles, trapezoids, or rectangles whose areas are determined from the soundings and their distances apart. The area of the entire section is the sum of these smaller areas. This is a simple operation on small streams with low velocities; and is a difficult one on large streams with even low velocities, but especially so with high velocities. As it is difficult to locate and hold the exact positions of the soundings, which have to be done by the use of two transits at the ends of a base-line on the shore, or by the sextant, locating the positions of the soundings from the boat or position itself, this operation involves the well-known three-point problem. And when the position is satisfactorily located, it is difficult on account of the force of the current to make the sounding accurate, either in position or in the depth of the water. These difficulties have to be encountered and overcome.

78. The determination of the velocity is simple, but also involves great care and a large number of determinations, as it is the mean velocity over the entire cross-section that is desired and necessary to be known. It may be stated that the surface velocity is, as a rule, a little slower than that some distance below the surface,

owing to the resistance of the atmosphere. The velocity close to the beds and sides of the channels is greatly reduced by friction, which varies with the nature of the material and its condition as to roughness and smoothness. It has been found that, in an average trapezoidal cross-section, the mean velocity of the entire channel is near the centre of the channel and at a point about one third of the depth below the surface. It has also been found that the mean velocity is from 0.9 to 0.8 of the surface velocity at the centre of the stream. If, therefore, a number of small floats be thrown into a stream some distance above the required point, say 100 or 200 feet, fixed by measurement, and the time taken to pass over this distance, the velocity $v = \frac{\text{distance}}{\text{time}}$ gives the mean surface velocity, and about 0.8 of this mean surface velocity will give the mean velocity of the entire section.

79. The mean velocity can also be approximately determined by the use of floats from 4 to 8 feet in length, so weighted at their lower extremities as to float in a vertical position and with their upper extremities only an inch or two above the surface of the water.

Such floats can be made of hemlock or other light timber from 4×4 to 6×6 inches cross-section and in lengths from 4 to 8 feet, which should be immersed in water a day or more before required. Weights are then attached to the bottom in amounts required to make them sink deep in the water and stand upright. These are deposited in the water at certain intervals across the stream, the lengths used varying with the depth of the water. The time taken to pass over measured distances will give directly the mean velocity of the entire stream. This operation is one of the prerequisites to obtaining authority from the Secretary of War for constructing bridges across navigable streams in the United States. It will not be inappropriate in this place to explain the manner of complying with this requirement, which is, that such floats shall be placed at a sufficient number of points in the width of a river, say 50-feet intervals, and allowed to float down the stream from points a mile above the proposed bridge site to points one-half-mile below. Observations from two transits at the ends of a base-line are to be taken every minute or half-minute, according to the velocity of the stream. This ob-

viously requires the location of the float at each starting-point. If the river is not over 1000 to 1500 feet in width, a strong cord or wire, with small floats attached at 50-feet intervals, can be stretched across the river a little above the starting-line. The float can then be dropped at the proper positions. The transit-men then take simultaneous observations, the time being called out or signalled by an assistant holding a watch in his hand. It is difficult to get a straight base of sufficient length to prevent the use of very small angles, and it may be necessary to stop the float after passing over about one half of the required distance, carry it back to the starting-line and let it float down the same distance, and continue this operation until the entire width of the river has been covered. Then from a new base the lower half of the work can be carried on in the same manner as described; or, better, a bent base could be measured and three transits used, the upper and the middle one for the upper half of the observations, and at a given signal the middle and the lower transit used. This will give continuous lines of current over the entire distance, giving both the velocity and direction of the current along them. With this system of observations the exact positions of the float at each minute or half-minute is located, the distances between them calculated from the known base and measured angles, and, with the time known, the velocity is determined over any part or over the entire distance. It may or may not be uniform.

80. *Current-meters.*—There are a number of current-meters in use. They consist principally in wheels with vanes attached, upon which the force of the current acting, causes a certain number of revolutions per minute or hour. This number is recorded by some mechanical contrivance. These revolutions are then converted into velocity, for which purpose the meters have to be rated or standardized: this is done by determining the number of revolutions when the meter is drawn through quiet water over a measured distance, and noting the time. The distance through which the meter is drawn divided by the time gives the velocity of the meter through the water. The number of revolutions of the wheel divided by the time gives the rate of motion of the wheel. The ratio of these two quotients gives a coefficient by which the number of revolutions in any case can be converted into velocity of current. This coefficient is not constant, there being a different co-

efficient for each rate of speed of the meter. If the number of registrations of the meter per second be taken as abscissæ, called x, and the velocity in feet per second as ordinates, called y, then $y = ax + b$. a and b are constants for the given meter.

MEASURING-WEIRS.

81. The discharge from canals and streams of moderate size can be accurately and readily measured by means of weirs. The three forms of weir which are more commonly used are the rectangular, trapezoidal, and triangular. In both the trapezoidal and triangular the inclined sides have slopes of one fourth horizontal to one vertical. The sides of the rectangular weir are vertical.

A weir may be defined as a dam constructed across a stream, the entire discharge passing over its crest, and commonly confined to only a part of its length. This may be effected by leaving a notch or depression of the proper figure and dimensions in the crest or top of the dam, or by cutting such a notch in the crest of the dam. These would be permanent, and constitute what are known as waste-weirs, the object of which is to prevent the water running over the top of the dam proper. For measuring purposes the weirs are temporary dams of timber which are provided with a notch, the dimensions of which in reference to area and discharge of the stream are determined by experience as giving the most accurate results. The crest of a weir may be sharp and well defined, as shown in Fig. 37. This is entirely cleared by the water. The wide crest is shown in

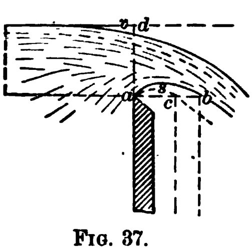


FIG. 37.

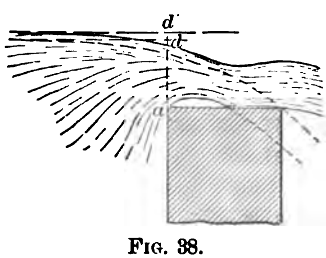


FIG. 38.

Fig. 38. It has the effect of increasing depth of water on the weir for a given discharge, as shown at ad'; whereas with a narrow crest, the flow being the same, the depth on the weir would be ad.

If in Fig. 37 the width of the crest, instead of being sharp, as shown by the full lines, has a thickness *ab* equal to the horizontal clearance over the sharp edge, the depth on the weir will not be altered; but if less than *ab*, say *ac*, there will be a tendency to a vacuum in the space *s* which may reduce the depth of overflow, as shown by the dotted lines of flow.

For ordinary purposes a timber bulkhead built across the stream at right angles to the direction of the current, well braced against the water-pressure, and having a notch or weir, as it is sometimes called, cut in it, will answer every purpose.

This construction is shown in plan, elevation, and cross-section in Fig. 39. If the notch or opening over the crest is the full width of the stream, there will be no side or end contractions; if shorter than the width of the stream, there will be contractions. This is shown in the plan.

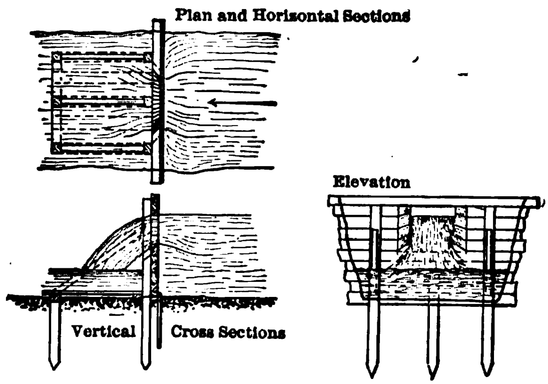


FIG. 39

The above figures show a weir of rectangular notch. For other forms the general construction would be the same, the form of the opening or notch being varied to suit the figure adopted. The crest and sides should be chamfered to an angle of not less than 30°, as shown in the drawings.

The dimension of the notch should be sufficient to carry the entire discharge, with a depth of water on the crest of not less than 5 inches. The sectional area of the jet should not exceed one fifth that of the approaching stream. In order to maintain the proper

proportion between the area of the notch and that of the jet, intermediate partitions may be introduced, dividing the weir into several orifices. In any case the quantity of flow is the mean velocity multiplied by the area of the prism of water or notch $Q = Av$.

The mean velocity of discharge is $v = \frac{2}{3}\sqrt{2gh}$, h being the vertical distance between the crest and a horizontal line coinciding with the surface of the water above the weir at a point where it begins to lower, and is represented by the line ad, Fig. 37. If l' is the effective length of the notch (l' and h both in feet), then the area is

$$l' \times h \quad \text{and} \quad Q = l' \times h \times \tfrac{2}{3}\sqrt{2gh}. \quad . \quad . \quad . \qquad (48\tfrac{1}{2})$$

Equation ($48\frac{1}{2}$) is easily understood by remembering that the horizontal film of water flowing through an orifice or over a weir is flowing under a head increasing from 0 to h, and that the mean velocity bears the same ratio to the greatest as the area of a parabolic segment does to its circumscribing rectangle; in other words, two thirds the velocity due to the head h. From equation ($48\frac{1}{2}$)

$$Q = l' \times \tfrac{2}{3}h^{\frac{3}{2}}\sqrt{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

It has been found that for one contraction the effective length of the weir l' is equal to the total length l minus $0.1h$; for two contractions, as in Fig. 39, $0.2h$ must be subtracted; and with any number n contractions $0.1nh$ must be subtracted. Owing to the falling away at the crest it is necessary to introduce another constant m, when formula (49) becomes

$$Q = \tfrac{2}{3}m\sqrt{2g}(l - 0.1nh)h^{\frac{3}{2}} = cl'h^{\frac{3}{2}}. \quad . \quad . \quad . \quad . \qquad (50)$$

The factor $\frac{2}{3}m\sqrt{2g}$ was found to be 3.33; $m = 0.622$; and equation (50) becomes

$$Q = 3.33(l - 0.1nh)h^{\frac{3}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (51)$$

By imposing the conditions that the water shall not be less than 4 nor more than 24 inches in depth; that the depth on the crest of the weir shall not exceed one third its length; that there shall be complete contraction and free discharge; and that the water shall

approach without perceptible velocity or cross-currents. We may use equation (52) to secure these conditions the distance from the side walls must be at least equal to the depth on the weir, and the distance of the crest above the bottom of the channel must be equal at least to twice the depth of water on the crest. Air should have free access under the falling water, and the approaching channel should be much larger than the weir opening.

$$Q = 3.33lh^{\frac{3}{2}}. \qquad (52)$$

For trapezoidal weirs under similar conditions to the above, the distance of the sill of the weir from the bottom of the canal must be at least three times the depth of the weir, and its length must be at least three times the depth of water flowing over it. Then

$$Q = 3.36^{\frac{3}{8}}lh^{\frac{3}{2}}. \qquad (52\tfrac{1}{2})$$

For triangular weirs

$$Q = 2.65th^{\frac{5}{2}}, \qquad (53)$$

in which t is the tangent of half the angle in the notch of the triangle. If the triangle is right-angle the formula used is

$$Q = 0.317h^{\frac{5}{2}}. \qquad (54)$$

This gives excellent results on small streams.

The exact determination of the value of the height h involves some difficulties. At a distance not less than $2\frac{1}{2}$ times the height of the weir from the weir on the up-stream side a rod can be steadied against a post, with its lower end just touching the water surface; or, for greater accuracy, a hollow column made of plank, with holes through the plank at mid depth, can be set up in the water. This will insure an undisturbed surface in the box. A graduated rod with a hook at its lower end is connected with the box. This rod can be raised until the hook just touches the surface. A reading with a level can be taken on this rod, giving the height of instrument with respect to the water surface. A reading is then taken with a rod held on the crest of the weir. The difference gives an accurate value of h, provided the velocity of approach is imperceptible. If there is a velocity of approach the head h' due

to this mean velocity can be determined by surface floats, recollecting that the mean velocity is about 0.8 of the surface velocity.

It can be readily shown that under these conditions we should make the head h in the preceding formulæ $h'' = \{(h + h')^{\frac{3}{2}} - h'^{\frac{3}{2}}\}^{\frac{2}{3}}$. See Fanning's Water-supply Engineering.

An understanding of the foregoing principles in hydrography and hydrographic surveying is essential, no matter for what purposes a water-supply is required.

The author of the following formulæ is unknown to the writer of this volume. In Fig. 39½ is indicated the flow of a stream over a weir. The cross-section of the issuing stream is determined from h. $h' - h$ is the head imparting the velocity of flow.

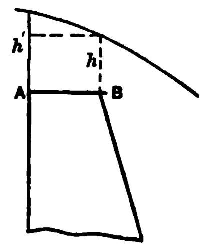


FIG. 39½.

Let Q = quantity of flow per unit-length of weir. Then

$$Q = h\sqrt{2g(h' - h)}. \quad . \quad . \quad . \quad (54a)$$

Q must be a maximum with respect to h; $\frac{dQ}{dh} = 0$.

$$Q^2 = h^2[2g(h' - h)]; \quad \therefore \; Q\frac{dQ}{dh} = 2gh(h' - h) - gh^2. \quad (54b)$$

Hence

$$2ghh' = 3gh^2; \quad \therefore \; h = \tfrac{2}{3}h'. \quad . \quad . \quad . \quad . \quad (54c)$$

Therefore $\frac{2}{3}h'$ is taken in determining area of cross-section of stream, and $\frac{1}{3}h'$ is expended in generating velocity. Then

$$Q = \tfrac{2}{3}h'\sqrt{2g\frac{h'}{3}} = 3.09h'\sqrt{h'}. \quad . \quad . \quad . \quad (54d)$$

In formulæ for overflow of ideal weir

$$Q = 3.33h'\sqrt{h'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (54e)$$

Example.—Let $h' = 15$ feet; $h = 10$ feet; $h' - h = 5$ feet; $v = \sqrt{5 \times 2g} = 17.9$ feet per second. Then discharge per unit of length along crest of weir $= 17.9 \times 10 = 179$ cubic feet per second; and for a weir 1150 feet long, $179 \times 1150 = 205,850$ cubic feet per second.

The formula would not be applicable when h' much exceeds the width of the top of the dam AB.

CANAL LOCATION.

82. Canals for navigation usually follow more or less closely the main watercourses of drainage districts to which they are confined if the terminal points are in the same valley, but passing over the divides or watershed lines if connecting two points in different valleys.

Where canals simply connect two points in the same valley the termini are usually so located that the natural discharge of the stream will supply a sufficiency of water under all circumstances; otherwise expensive storage-reservoirs must be provided above and beyond the upper or interior terminus, or for certain portions of the year navigation must be suspended over a part of the route. In either event the important question to be determined is the least annual rainfall and the longest droughts.

If the fall of the stream is not rapid, so as to require a great number and great height of dams, which are not only expensive but may cause overflow of valuable lands either at ordinary stages of water or in time of floods, it is simply necessary to build a series of dams at such distances apart that the least depth in any level or reach shall be sufficient to easily float boats of the greatest draughts required, the water backing up from each dam to the next one above. Locks are then required at each dam in order to raise or lower the boats from one level to another. Locks are simply formed by side walls—usually of good masonry—with the proper space between them for the easy entrance of the boats or ships. These side walls have at each end head or wing walls, and as they are usually placed at or near one shore, one head wall connects with the dam at the upper end and the other is simply curved off normally to the side walls in order to give stability and ease of access. Both head and tail wall of the other side-wall is well run into or connected with the bank of the stream.

Gates, usually of timber, are constructed from side wall to side wall, and sufficiently far apart to inclose he longest boat or vessel. Each of these gates is usually composed of two halves turning on

a pivot, and coming together with a slight angle pointing up-stream at the centre. In such locks no lift-walls are necessary, as the beds of the two levels are nearly on the same plane, the upper and lower gates being constructed of the same heights.

A lift-wall may be used, in which case the upper gates have proportionately less height. If the dams are very high, it will be necessary to provide a flight of locks, the one opening into the other. Such canals would evidently have to follow all of the sinuosities of the stream, and would consequently be very expensive, and usually impracticable, for the reasons stated.

83. It is therefore usually necessary or convenient to combine this system with channels excavated in the earth or enclosed in one or two embankments, and extending from one dam to another at a greater or less distance apart. Such a canal practically follows a contour-line from the upper dam to the lower dam, where it again usually enters the backwater of some still lower dam, from which it again departs, crossing the country. Along these cross-country stretches or levels there may be one or more intermediate locks, with stretches between them before re-entering the river. Each stretch from dam to dam or lock to lock is practically level. It is sometimes economical, instead of following around spurs and up ravines in order to keep on a contour-line, to cut across the point of the spur, and pass over the ravines on embankments with culverts under them, or to construct aqueducts in case of large streams, these passing under the canal. Such diversion canals shorten the distance, decrease the cost, and avoid questions of damage to property from overflow. Canals for the passage of small boats drawn by horses were very common, useful, and convenient before the days of railways, which have to a large extent, if not entirely, stopped the construction of such canals and caused the abandonment of many of those formerly constructed.

In some instances, where business between the main terminals is large in amount and diverse in kind, it has been found advisable to enlarge the old canals or adapt them to the use of steam-power; and the question of using electrical power is now being discussed and experimented upon. These remarks apply notably to the Erie Canal, in the State of New York.

84. If it becomes desirable or necessary to extend the navigation beyond that point at which the least annual flow is sufficient

for navigation, it is then necessary to provide storage-reservoirs. These are usually single dams constructed across some narrow valley or gorge, of sufficient height to hold water in large enough quantities to supply the deficiency in the stream during periods of drought. The same conditions apply to canals with summits, or where they pass over divides from one valley into another. If a very low divide can be found the canal may be constructed passing over it, provided a sufficient drainage area can be found above to supply the requisite water on both sides of the divide until points are reached in the streams on either side where a sufficiency of water is found at all times. If such a divide cannot be found the summit-level of the canal must be lowered by excavation and tunnelling so as to leave ample drainage area above. These questions evidently require ample and careful surveys, and full knowledge of the rainfall of the district, and character and topography of the country.

85. For a single lock each boat ascending or descending draws a lockful of water from the upper level or pond. If a close calculation is required, allowance must be made for the displacement of the boat when it enters the lock, forcing this amount back in the upper pond, requiring less than a lockful for the descending boat, and for an ascending boat more than a lockful by the same amount. If a train of boats follow each other in one direction the same rule holds. If, however, trains alternate—one boat descending and one ascending—less water is required. Single locks are more favorable to economy of water than flights of locks. On the contrary, at a flight of locks boats in train cause a less expenditure of water than equal numbers of boats ascending and descending alternately. The question of water is also affected as the locks may be found empty or full. Such calculations are easily made, but are more interesting than useful, as it would be unwise not to allow for the maximum quantities to pass boats through locks, especially as the loss by leakage through sides and beds and gates of canals and evaporation are all unknown and uncertain quantities. The maximum results as obtained by experience and experiment for all of these losses should be liberally allowed for before projecting and locating the expensive works required. The waste of water by leakage of the channels, repairs, and evaporation = area of surface of the canal × $\frac{1}{8}$ of a foot; leakage at lock-gates from 10,000 to 20,000

cubic feet per day; and maximum expenditure of water for lockage would give a rough basis for calculation.

86. The general width of a canal should be sufficient for two boats to pass each other. Expensive portions, such as aqueducts, and where bridges cross the canal, can be reduced to that required for one boat. The usual form of cross-section is trapezoidal; least breadth at bottom $= 2 \times$ greatest breadth of boat; least depth $=$ $1\frac{1}{2}$ feet $+$ maximum draught of boat; least area of waterway $=$ $6 \times$ greatest section of boat. When the sides are of earth the slopes are usually $1\frac{1}{2}$ to 1. If of masonry the sides may be vertical, but additional width will be required to give sufficient area of waterway.

The general design and construction of locks are the same as for ship-canals,—which will be explained in another article,—differing mainly in dimensions and the crude way provided for opening gates and filling and emptying the locks. Few canals of the dimensions and for the purposes above described are likely to be constructed in this country in the near future.

SHIP-CANALS.

87. Large ship-canals have been constructed of late years. Many have been projected and commenced, and some of these have resulted in gigantic failure after the expenditure of large sums of money—notably the Panama Canal across the isthmus of Panama, connecting the Atlantic and Pacific oceans. This was originally intended as a sea-level canal, by tunnelling under the high mountain backbone of the isthmus, but was changed to a system of locks passing over the divide. The Suez Canal, connecting the Mediterranean and the Red Sea, has proved both an engineering and a financial success. The Welland Canal, connecting lakes Erie and Ontario, and giving access to the great lakes above, has long been completed; there are many similar constructions along the rivers of the United States—notably the Ohio and Kanawha. And, finally, the Nicaragua Canal between the Atlantic and Pacific oceans has been much discussed, large sums of money have been spent in surveys, examinations, and reports, and some work has been done. At present this enterprise is at a standstill, but with great hopes of its early completion. This will be a lock and dam canal for a good part of its course.

The conditions governing the location of such canals are the same as for the smaller canals. There must at all times be a sufficiency of water from the highest to the lowest points. Where the natural flow of streams is insufficient the deficiency must be provided for at proper points by storage-reservoirs. The dimensions of these, of the locks, and of the level stretches must be in proportion to the size and number of the vessels transported.

On many of our most important rivers the improvements by means of locks and dams are not relatively very expensive, and do not involve any great engineering difficulties. Such rivers are rendered unnavigable at a few points by the formation of gravel and sand bars. Above and below these long stretches of deep water are found, with comparatively low current velocities, whereas over the shallow portions the current is usually very rapid. To promote the navigation of such rivers these bars must be dredged out, with the probability of re-forming (and in some cases they are solid rock ledges); or dikes and jetties must be constructed so as to force large quantities of water at high velocities through them, scouring the bar to a proper depth, with the almost certainty of the material being deposited at some point not far below the ends of the jetties, resulting in simply shifting the position of the bar. Or at these points dams, locks, and short stretches of open canal must be constructed for the passage of river steamers or of vessels. As such locks are only required and intended for low-water navigation, there is danger of the dams forming obstructions to the navigation during periods of what is known as good boating stages—that is, in rises from 5 to 15 feet or more. At these times the locks are certainly disadvantageous to navigation, as are also the dams unless provision is made for lowering or opening them. Such constructions are now used, and will be described in another article.

Such means of improvement would not be usually adopted unless the least flow of the stream, coupled with the incidental storage caused by the necessary dams, were sufficient to supply all necessary water.

IRRIGATION-CANALS.

88. Navigable canals usually are placed on as low levels as possible, and are constructed so as to have as little current through

them as practicable. Their efficiency is based upon the amount of water available during the lowest stages and greatest droughts. On the contrary, irrigation-canals are located at the highest levels, in order to supply as large areas as possible. And as the irrigation periods occur simultaneously, as a rule, with the periods of low water and droughts, it becomes necessary to store up the flood waters in sufficient quantities to properly irrigate large tracts of land during one season and frequently to provide for possible droughts through a couple of seasons. It is therefore necessary to know the extent, duration, and periods of greatest rainfall, as arrangement must be made not only to store sufficient water for use at the proper time, but also to make provisions for discharging freely the flood-waters without damage to dams, weirs, head-works, or any portions of the canal.

Again, irrigation-canals must be so located, with such relative dimensions of sections and slope of surface, that the velocity may be practically uniform throughout its length, and great enough to prevent silting, or the growth of weeds.

89. The first important points to be determined are the lay and area of the ground to be irrigated, and the quantity of water required —which depends upon the climate, the kind of soil, and the character of the crops. The first is determined by proper surveys. The second is determined by observation and experience. The duty of water is defined as the ratio between a given quantity of water and the amount of land that it will properly irrigate. For ordinary purposes the usual unit of standing water is the cubic foot; for running water the unit of measurement is the cubic foot per second, or, as it is called, the second-foot. The number of second-feet flowing in a canal or river is the number of cubic feet of water passing a given cross-section of the stream in a second of time. The unit generally employed in the western portions of the United States is the miner's inch. This is an entirely arbitrary standard, varying in different States, and is defined by statute. In California one second-foot of water is equal to about 50 miner's inches, whereas in Colorado it is about 38.4 miner's inches. Again, for large bodies of standing water the unit is the acre-foot. This is 43,560 cubic feet of water; that is, the amount of water required to cover an acre of land to the depth of one foot. Recollecting that a cubic foot of water is taken at 7.5 gallons, any of

these units can be expressed in terms of the other, the element of time being introduced. For instance, 1 second-foot per 24 hours = $60 \times 60 \times 24 = 86400$ cubic feet = 2 acre-feet nearly; 100 miner's inches = 2 second-feet = 4 acre-feet in 24 hours; 115.2 Colorado inches = $86,400 \times 3 = 6$ acre-feet in 24 hours; and so on.

90. The irrigation period extends from about April 15th to August 15th—about 120 days; the number of waterings during this period ranges from 2 to 5, and the period of service from 12 to 24 hours at each watering, according to the climate, soil, and crop. A good rain 5½ inches deep soaks into the earth to a depth of 16 to 18 inches. Three applications amounting to 16½ inches seem to be ordinarily sufficient. Some crops require 24 inches, others as low as 12 inches.

An average depth of 3 inches of water is sufficient to thoroughly water average soils. Sandy soils will require 4 inches of water. For average soils, then, each watering would require about 10,800 cubic feet, and an average of four waterings during the irrigation period would require 43,200 cubic feet, or nearly an acre-foot per acre.

This is probably a minimum, the maxima varying from 1¼ to 2 acre-feet per acre. The above allowance per acre thus determined upon, the total actual service quantity is found by multiplying by the number of acres. But as a rule only from three quarters to four fifths of the total average commanded has to be actually irrigated in any one season.

On the other hand, the losses by evaporation, absorption, etc., in conducting the water to the fields will rarely fall short of 25 per cent. We must therefore provide from 1¼ to 2½ acre-feet per acre in estimating the storage capacity of reservoirs.

HEAD-WORKS.—WEIRS.

91. Having determined upon the quantity of water required, and the highest point to be irrigated, which fixes the elevation of the canal at that point, the source of supply must then be higher than that point by the necessary fall in the canal between the two. This may, however, be determined by questions of convenience and economy in the construction of the head-works and the canal. Points higher than necessary in the hills may be selected for the

head-works on these accounts. This point of diversion having been decided upon from whatever considerations, the question of water-supply has to be considered.

If it has been found that the least annual flow of the stream during the irrigation period is more than sufficient to meet the demand for water. The head-works will simply consist of such constructions as will permit of the diversion of the water from the stream into the canal, and prevent an unnecessary and destructive flow of water thereto during periods of floods. In such cases weirs of small heights are required, and these may be built open so as only to partially obstruct the channels, some of these openings being permanent; or the whole of the openings may be provided with gates sliding in grooves in a series of piers, of wood, iron, or masonry, which generally rest on masonry or timber floors. Safety against underscouring of the floor must be provided, as will hereafter be explained. Such piers are from 3 to 10 feet apart. Timber beams or flashboards are sometimes used instead of gates, being dropped in place or raised one by one. This is simply an economical expedient. In some cases the entire weir is so constructed that it can be lowered in times of flood and raised when the water is low. Such arrangements also prevent silting above the weir, which would obstruct the flow into the head of the canal. If only partial storage is required in order to supply a deficiency of flow, the weir may still be constructed as an open weir, with such an adjustment of the gates as will allow the surplus water to pass, and also prevent silting, especially at that side from which the canal leaves; or it may be in this case built as a solid weir, with only a sufficient number of under-sluices to prevent silting at the head of the canal, or as may be required during the construction of the weir. The surplus flood-water may be allowed to flow over the entire length of the dam, or waste-weirs may be constructed so as to confine this flood-water to special points at one or both ends of the dam; or, if desired, special channels can be provided, passing around the ends of the dam, and emptying into the stream at some point well below the dam.

92. Where reliance has to be placed on water stored during the rainy seasons very high dams will usually be required, and often of great length, in order that the storage capacity may be sufficient.

If a wide valley is selected the dams will be long and low; if narrow, the dams will be short and high.

In this latter case particularly the geological formation of the bed and sides of the valley should be examined into. If the reservoir is to be placed in a synclinal valley, as seen in Fig. 40, there

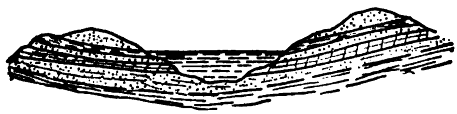


FIG. 40.

will be the most favorable lay of the strata not only to prevent leakage from the reservoir, but it will gain something from seepage-water. If in an anticlinal valley, as seen in Fig. 41, there will certainly be danger of leakage, and it may be so excessive as to entirely destroy the value of the reservoir. Sometimes a combina-

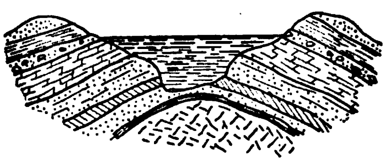


FIG. 41.

tion of these formations may be found which may be better than the last, but neither of them is favorable. A good coating of puddle over the surface may make any of them safe.

93. High dams may be constructed of timber, earth, or masonry, or of combinations of these. It is rarely advisable, however, to let the flood-water flow over their tops or crests, except at specially arranged and constructed waste-weirs. It is generally better to carry this surplus water through specially constructed channels leading around the dam. The fall of water from such great heights produces vibrations and shocks, combined with erosion, which may undermine or cause cracks and greater or less disintegration of the entire dam. Of course with earthen dams any overflow would absolutely destroy the dam. To avoid this the down-stream face of the dam has been given many different sections, polygonal and curved, water-cushions being also provided at the bottom to break the effects of the fall. Benches and steps have also been

provided. While these provisions have been effective in some cases, they have resulted in failure in others.

The theory and practice in the construction of dams will be explained under the head of Dams, including special precautions in the foundation work.

HEAD-WORKS.—REGULATORS.

94. Weirs or dams retard the flow of the stream and raise the water so that it may enter the canal. Regulators are the valves or gates which control the flow of water into the canal, or which prevent its admission entirely. The location should be such as to admit the water with a minimum loss of head.

The best practice is to make the regulator a part of the weir, the canal head being immediately above the weir and perpendicular to it. When the regulator is at a distance from the weir silting is likely to take place in front of the canal head. These regulators may be timber gates sliding in timber, masonry, or iron guiding-walls, or the gates may be iron in iron or masonry guides. The gates themselves may be made of flashboards, each board inserted or removed separately, or the gates may be framed and raised by levers, chains, and windlass, or by screw-gearing. Where the pressure is very great the gates may be placed in series one above the other, and each section raised separately.

Regulators must be so constructed that the proper quantity of water can be admitted to the canal at any stage of the stream. The gates must be opened quickly when necessary, and also closed quickly. Consequently they must not be too large, and in case of large canals with wide regulators there must be several openings formed by pillars or walls and closed by the gates, which are seldom less than 2 nor more than 6 feet wide. The walls and flooring of the regulator should be of timber or masonry to prevent scouring. The guide-pillars should be as narrow as practicable to avoid unnecessary obstruction. These openings are arched over or otherwise covered to form platforms from which to handle the gates. The regulator frames and gates must be carried well above the water surface in the highest rises. They must be strongly constructed, so as to resist the greatest pressure to which they are subjected.

95. Sometimes where the rise would require excessively high and strong regulator-gates these are set well back from the head of the canal, from 200 to 1000 feet, and large escape-weirs are placed in the side of the canal, allowing the flood-water to return to the river.

Long tunnels have been excavated through solid rock opening into an open cut, across which the regulator-gates are placed, and in front of these escape-weirs, the wasting capacity of these being sufficient to carry the greatest discharge that the greatest head can force through the tunnel. These arrangements relieve the regulators from excessive pressure.

VELOCITY OF FLOW IN AND DIMENSIONS OF CANALS.

96. The head-works of a canal are usually placed high up on the supplying stream. In order to command a large area and also secure water free from silt. It is usually possible to reach the irrigation lands with the shortest possible diversion line. But on such lines a large quantity of expensive hillside cutting will be encountered; many ravines, gullies, and much hillside drainage will have to be crossed, entailing difficulties and expense. All of this so-called diversion line means that portion of the canal from the source of supply to the lands to be irrigated, which in this sense brings in no revenue. It should therefore be the object to reduce the length and cost of this portion of the line to minimum. Having reached the highest part of the lands to be irrigated, the line should follow this highest land, skirting along the surrounding foothills. A good topographical map with contours from 5 to 10 feet apart will greatly aid in the determination of the best lines. Such a map will often indicate low divides, by which many miles in length may be saved. The cost of crossing ravines, gullies, low marshy places, or rocky spurs and barriers must be carefully considered, and that method which insures the best work with the least cost of first construction and maintenance should be adopted. It may be less expensive to carry the canal around these obstructions, or it may be better to cross them with aqueduct, flumes or inverted siphons, and to cut or tunnel through the spurs and ridges. To save expense, the highest possible velocity consistent with safety from erosion may be taken, and then proportion the area of cross-

section to the discharge required. A too close adherence to regular grade-lines is often an error. By occasionally inserting a fall it is often possible to obtain a better location and lessen the costs.

97. *Curvature.*—A straight line would be the most economical, and least endanger the banks. There is also less absorption and evaporation. Sharp bends should be avoided. If used, the surfaces of the banks must be paved. Curvature diminishes the delivering capacity of canals, and to keep up the discharge of a canal either the cross-section or slope must be increased in proportion to the sharpness of the curve. In large canals with moderate velocity the radius should be at least from three to five times the depth of the canal. With small cross-sections or increased velocities of current the radius should be increased.

98. Slope and cross-section are quantities interdependent upon each other. The proper discharge required being known, it can be obtained with small area of waterway and high velocity, or large area and small velocity. If the materials forming the sides and bed of the canals will admit, it is well to use a high velocity. But too rapid fall may unduly lower the irrigating portions of the canal. Too little slope requires much excavation. These should be adjusted then to the best advantage. The velocity of flow should be such that it will not erode the banks, nor allow silt to be deposited or permit the growth of weeds.

In a light, sandy soil a surface velocity of from 2.3 to 2.4 feet per second, or a mean velocity of from 1.85 to 1.93 feet, has been found to give satisfactory results. Velocities of from 2 to 3 feet per second will also prevent the deposition of silt and growth of weeds. These are the most favorable velocities.

In ordinary firm soil and sandy loam from 3 to 3½ feet per second is a safe velocity, while in firm gravel, rock, or hard-pan the velocity may be increased to from 5 to 7 feet per second. It has been found that brickwork, rubble, or heavy dry-laid paving will not stand velocities as high as 15 feet per second. For greater velocities the most substantial masonry is required.

The slopes necessary to produce these velocities depend mainly on the cross-sectional area of the channel. Higher grades are required in smaller than in larger canals to produce the same velocity. The velocity being known, the slope can be found by Kutter's formula, eq. (41). In large canals with 60-feet bed and light or

sandy soils 6 inches in a mile produce as high velocities as the material will stand. In firmer soils slopes may be increased to from 12 to 18 inches per mile. Smaller channels will stand from 2 to 5 feet per mile, according to materials and dimensions of channel. Through loose shale a grade of 7.9 feet per mile, producing a velocity of $7\frac{1}{2}$ feet per second, has been used. For a short distance on the Del Norte Canal in Colorado there is a slope of 35 feet per mile through a rock cut. On several miles of this canal the grade is 8 feet per mile, but after it reaches the earth soil in the valley it is reduced to 1.2 feet, nearly.

99. *Form of Cross-sections.*—The most economical channel has vertical sides, with a depth equal to one half the bottom width. This can only be obtained in solid rock.

The best trapezoidal section is one in which the width at the water surface is equal to the sum of the side slopes and double the bottom width. This requires firm, compact material. The exact form will usually have to depend on the material and the topography along the line. The greater the depth the greater will be the velocity and also discharge for.same form of cross-section. In a large canal having a capacity of 2000 second-feet, with a velocity of 2 feet per second, the cross-sectional area should be 1000 square feet. This could be made with a bottom width of about 45 feet and a depth of $22\frac{1}{2}$ feet. Such a proportion would greatly increase the cost of construction. A better relation would be a width of from 100 to 125 feet and a depth of from 8 to 10 feet.

It is as well, where it can be done, to have the channel partly in excavation and partly in embankment. It will, however, be often necessary to have it wholly in excavation or wholly in embankment. Sometimes it will be in excavation on one side and embankment on the other. Proper drains and channels should be provided on the up-hill side to prevent wash of the slopes, especially if an embankment is required on that side ; the drainage water can either be let into the canal at certain points, or carried to the natural water-courses.

Bermes are often left from 2 to 6 feet wide on the slopes where partly in excavation and partly in embankment.

100. In the case of large canals there should always be a road-bed on one or the other side. With a roadway the top width would generally be about 10 feet. Without a roadway, and when its top is

well above the water surface, 4 feet will be sufficient. When the water is well upon the slope the top should be from 6 to 10 feet wide, according as the material may be firm or light. In such banks either a puddle-wall should be built in the interior or the bank should be lined with puddle on the inner slope, or, better still, the entire bank can be made in thin layers and well rolled. Sodding or sowing grass-seed on the bank is beneficial. The water should never be allowed to rise nearer than 1½ to 2 feet from the top of an embankment. In firm materials slopes of 1 to 1, in mixed soils 1½ to 1, in lighter soils 2 to 1, and with the lightest sands slopes of 4 to 1 may be required. In rock excavation the depth to width of channel may be as 1 to 2, the side slopes 1 to 4. In less firm rocks lighter slopes and less proportional depth are required. Sometimes a rubble wall is built on the lower side, the upper side in excavation.

101. *Shrinkage of Earthworks.*—Sand shrinks about 10 per cent; sand and gravel, 8 per cent; earth and loam, 10 to 12 per cent; gravelly clay, 8 to 10 per cent; puddled clay and soil, from 20 to 25 per cent. Rock excavation produces a larger mass by from 25 per cent in case of small fragments to 60 or 70 when in large blocks carelessly piled up.

EXAMPLE OF CANAL ALIGNMENT.

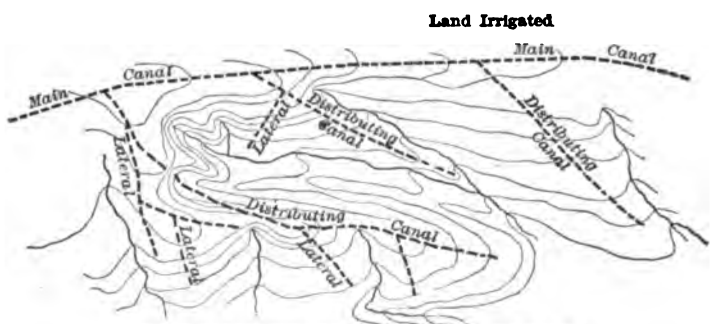
102. *Turlock Canal.*—This canal may be taken as typical of American practice. It is diverted from the Tuolumne River in California, where it emerges from the Sierras between high rocky canyon walls. For the first five miles the canal is built along steep side-hills, crossing ravines, bluffs, and many drainage channels. The difficulties of the ground were so great that only after a careful topographical survey was a feasible route discovered.

Fig. 42 shows location of canal with respect to the river and the high adjacent hills.

The canal leaves the river about 50 feet from the diversion weir, by a tunnel about 560 feet long, 12 feet wide at bottom, and 5 feet high to springing, above which is a semicircular arch of 6 feet radius. Its slope is 24 feet to the mile. It enters an open cut at *B*. The escape *C* and the regulator *D* are at right angles to each other. These have for each six gates with a clear width of 3 feet and a height of 12 feet.

Under a flood head of 16 feet over the sill of the tunnel the discharge is 4000 second-feet, with a velocity of 20 feet per second. The wasting capacity of the escape is 6000 second-feet. Below D the canal

FIG. 42.



MAIN, DISTRIBUTING AND LATERAL CANALS
FIG. 43.

proper begins, having a capacity of 1500 second-feet. It is excavated for the first 6200 feet in slate rock on steep hillside, from D to E. It has a bed width of 20 feet, with depth of 10 feet. The upper

rock slope is ½ to 1. The lower bank is a puddled core 12 feet in top width, faced with an 18-inch rubble wall laid dry, the inner slope being ½ to 1, where it crosses gullies; but where it simply rests on sloping ground, the inner slope is alone faced with rubble, the outer face being of ordinary earth, with its natural slope. The top width in such cases is only 5 feet, which is also the width of the puddle core. This portion of the canal has a slope of 7.92 feet, with a velocity of 7.5 feet per second. At *E* it enters a settling reservoir formed by damming up a ravine. This is formed by an earth dam 20 feet wide on top, 318 feet long on the crest, with slopes of 2 to 1, and a maximum height of 52 feet. It then enters some old hydraulic cuts and washouts; this consumes a distance of some 4120 feet, extending from *E* to *F*, where the water is discharged into Dry Creek. It flows down this creek for 6500 feet on a grade of 12 feet to the mile, from which it is diverted by an earth dam 460 feet long, maximum height 23 feet, with wide slopes of 3 to 1, riprapped to a depth of 3 feet on its upper surface. This dam is provided through the rock at one end with a waste-weir. The waste-gates are arranged to discharge 4000 second-feet. They fall automatically on a concrete floor laid on solid rock.

This brings the line to some point *G*, from which the canal passes through sandy loam for about one mile with a bed width of 30 feet, side slopes 2 to 1, depth 10 feet, and grade of 1½ feet per mile. At this point the canal crosses Dry Creek in a flume 450 feet long, and 62 feet in height, and enters a series of three tunnels, their lengths being 211, 400, and 400 feet, separated by open cuts 250 and 300 feet in length. It then crosses a gulch, a dam about 40 feet high and 20 feet top width being built across below the canal. After crossing this the canal enters a cut 8 feet in maximum depth, with the same cross-section as above given; this runs for a length of 3300 feet. The canal is then enlarged to 35 feet bed and to a depth of 10 feet, with a grade of 1 foot per mile. A mile and a half farther on it crosses another creek in a flume 20 feet wide, 7 feet deep, 360 feet long, and 60 feet high. The flume is supported on a trestle. It has a waste or escape placed in its bottom, the water discharging in two small inclined flumes which lead the water into the creek below. At the end of this flume the main canal is reached, and followed for 11 miles, on which there are two rock cuts each 3000 feet long, respectively 20 and 30 feet bed

width, with depth of water $7\frac{1}{2}$ feet, and grade of 5 feet per mile. The remaining distance has a varying cross-section depending on the soil, but generally 70 feet wide at bottom, $7\frac{1}{2}$ feet deep, with side slopes 2 to 1, and a grade of 1 foot per mile.

103. About 18 miles of the above canal are necessary to reach the lands to be irrigated. At the end of this distance the canal at once begins to do duty watering lands. Below this point the main line is divided into four main branches, each having a bottom width of 30 feet, depth of 5 feet, and grade of 2 feet per mile, their aggregate length being 80 miles; and in addition smaller distributaries having a total length of 180 miles, which lead the water to the several sections of land. The discharge of the branches is so designed as to give a uniform velocity of $2\frac{1}{2}$ feet per second in order to prevent deposition of sediment.

Fig. 43 is designed to show in a rough way the best positions for the main branches, distributaries, small laterals, etc. The main canal is shown as following the watershed line of a district to be irrigated. In this case the land on both sides of the ridge can be watered from the same canal. It is rarely practicable to obtain so favorable a position. The main distributaries are shown as emerging from the main canal and extending along the ridges of the spurs, and from these the laterals branch out so as to reach all portions of the district. The methods of actually placing the water where it is needed are various. It may fill little furrows dammed up at intervals, or furrows may be run parallel to each other and close together, the spaces between these filled with water; or the ground may be divided, checker-board style, into independent squares by furrows run in two directions, and these alternately filled and emptied; or the water can be carried underground in pipes and allowed to soak upwards to the surface; etc., etc.

ART. X.

BUILDING MATERIALS.

104. THE more common and useful building materials are: Timber, stone; metals, such as steel, wrought iron, cast iron, tin, copper, zinc, lead, and some of their alloys; cement, mortar, con-

crete, brick, tiles, terra-cotta, bituminous cement and concrete, and certain artificial stones.

TIMBER.

105. Woods suitable for building purposes are usually called timber, and are almost exclusively confined to trees that grow by the formation of layers of wood over the external surface, and therefore called *exogenous*.

Trees of the palm family do not grow by the formation of successive and perfect layers, and belong to the endogenous class. The timber of this class of trees is too light and flexible for use in most structures, but is sometimes used for piles and light framing in those localities where it is found in abundance.

Timber of the exogenous class is one of the most useful building materials, owing to its lightness, strength, the ease with which it can be cut, framed, and handled, its abundance and cheapness, and its great durability when properly seasoned, protected from the effects of alternate wetness and dryness, and when entirely immersed under water.

106. If a timber tree is cut or sawed off, it will be seen that it is composed of three distinct parts:

1st. On the outside is the bark, of a thickness from one quarter of an inch to one and one half inches or more. This has no value for building purposes, though often useful in other respects; and as it hastens the decay of the rest of the timber after fellng, it should always be removed.

2d. The sap-wood, which lies next to the bark, having a thickness varying from one half inch to as much as three or four inches, is generally indicated by a lighter color than the heart-wood. Although the sap-wood has not as much strength as the heart-wood, and is liable to a more rapid decay when exposed to unfavorable conditions, yet it can be safely used when entirely immersed in water, or when impregnated with certain solutions, or carefully seasoned and painted.

3d. The centre portion surrounded by the sap-wood is called the heart. This is easily distinguished from the sap-wood by its greater heaviness, hardness, strength, and compactness. It also has, as a rule, a darker color—sometimes very dark, or even black.

The successive layers of wood called annual rings are com-

monly formed at the rate of one a year, and serve to indicate the age of the tree. They are much more distinctly marked in some kinds of wood than in others, and these, together with certain more or less defined and glistening rays radiating from the centre, serve to indicate the different kinds of wood.

TIMBER CLASSIFIED.

107. Some authors divide timber trees into two classes:

1st. Pine-wood, including all timber trees of the coniferous species. These are often called in this country soft-wood trees, in which a few leaf-wood trees, poplar, chestnut, etc., are also included.

2d. Leaf-wood, including all trees not found in the first class. These are usually called hard-wood trees in this country.

The above classification is not of any very great value to engineers, as timber can be obtained from either class possessing practically equal strength and durability, and equally suitable for any structures except where very great lengths are required. Preference is, however, given by engineers to one or the other class, even without reference to the relative cost. Abundance and cheapness, however, are usually the controlling considerations.

The soft-wood trees usually contain turpentine, and are characterized by the great height to which they grow, straightness of fibre, freedom from large branches over the greater portion of their length, and the gradual and uniform increase in diameter from the top to the bottom. These qualities, together with their great strength, durability, and the ease with which they can be handled and framed, render them the most valuable building material in many parts of the country; especially in those structures requiring long and straight square or round pieces, and for piles to be used in bridges and trestles. When watercourses are convenient they can be, on account of their lightness, collected in large rafts and floated to great distances at a merely nominal cost.

Embraced in this class is the short-leaved Yellow or Spruce Pine, found somewhat widely distributed in the Northern, Middle, and Southern States. It is strong and durable, and extensively used in those localities where it grows.

The White Pine, commonly called the Northern Pine, is found

mainly in the northern sections of the country. It has considerable strength and durability. It is light, soft, and free from knots. It is used in many structures, such as bridges and trestles, where it grows, and often is brought to the Middle States and used for the same purpose.

The Long-leaf Pine, or Southern Yellow Pine, is found in all of the Southern States, especially in Alabama, Georgia, North Carolina, and Florida. It has little or no sap-wood. The heart is compact and fine-grained. It furnishes the strongest and most durable timber, and is generally preferred to oak. Large quantities of it are shipped to other parts of the country as well as abroad. We find distributed in the same forest a much inferior grade of timber, which cannot be distinguished, except by expert lumbermen, from the first variety until it is cut into or felled. When cut down, however, it shows a very large proportion of sap and correspondingly small proportion of heart timber. This should only be used for submerged structures.

There are several varieties of the Fir-tree. The timber obtained from these decays rapidly on exposure, and also splits and twists.

Hemlock grows only in the Northern States. It is suitable for many submerged structures, but does not stand exposure. It is full of knots, splits on exposure, and cannot for these reasons be framed satisfactorily, and is used practically from considerations of economy.

The cypress-tree is are found in large quantities in the Southern swamps. It is not considered as strong as the Southern pine, but is considered more durable when exposed to unfavorable conditions. It is not used in important structures when the pine can be obtained. The Black Cypress is much sought for use in pile trestles on account of its great durability. Large quantities of shingles and weather-boarding are made from the cypress species.

HARD-WOOD TREES.

108. Of the hard-wood trees there are only a few varieties that have any special value to the engineer for use in important structures. These are Live-oak, White and Post Oaks, and Chestnut-oak and Black Walnut. The other varieties, such as Swamp-oak, Red Oak, Hickory, Chestnut, Ash, Elm, Beech, Sycamore, and

Poplar, although valuable and useful in many ways—such as making furniture, buggies and carriages, spokes, hubs, shafts and bodies, buckets and other wood-ware, fence-rails, etc.,—are either wanting in strength or durability, and sometimes in both, and often only abundant in more or less inaccessible situations. It has been claimed that certain processes have been recently discovered that will make the timber of many of these varieties of trees suitable for building purposes.

Live-oak.—The timber from the live-oaks is heavy and compact; it is stronger and more durable than any other species. It does not exist in large quantities, being confined to a rather narrow belt of country, extending along the Atlantic coast from Virginia southward. These forests are reserved exclusively for ship-building.

White Oak grows in any part of the United States, but is more abundant in the Middle States. It is strong, tough, and durable, provided it has grown in a good, well-drained soil. In moist or swampy soils the tree loses its strength and durability to a great extent. It is well suited for any kind of structure, and is invaluable in ship-building. The timber made from it is apt to split and crack in large pieces, and in thin boards or planks it warps badly in addition, splitting at nail-holes or drawing the nails driven into posts through it. It is harder and more difficult to work than the pines. On account of its great heaviness it is more costly to transport to distant points, especially as it is unsafe to attempt to float it in large rafts on the water unless buoyed up by a liberal intermixture of poplar logs or other light timbers.

Post-oak resembles white oak, but is considered by some as possessing greater strength and durability. It does not usually grow as large in diameter, but is eminently suitable for any structure not requiring a too great section and length. It makes most excellent cross-ties. It grows more or less abundantly in many of the Middle and more Southern States.

Chestnut-oak grows abundantly in some of the Southern States. The timber is strong and durable, but is not considered as good as the other varieties of oak. It is used to a considerable extent for cross-ties on many railroads. Large quantities of this timber are cut down simply to secure the bark, which has great value in tanning processes.

We rarely in this country have any use for foreign timbers.

Some very superior pines and oaks are found in foreign countries. Our own forests, however, furnish nearly all the varieties required for structural purposes. Our importations of timber are confined to a few varieties which are required only for ornamental purposes or for some special properties, such as mahogany, which comes principally from the West Indies and Central America, and lignum-vitæ, which is obtained from the same countries.

There is, however, a great need of better information concerning the qualities and properties of our own timbers. The Government has, to a limited extent, taken hold of this subject in recent years. What has thus far been accomplished will be referred to in another article.

DEFECTS, DURABILITY, AND DECAY OF TIMBER.

109. Many natural defects of timber are developed during the growth of trees, such as "upsets," where the fibres have been crippled or injured; rind-galls, or wounds, which have been partially or entirely concealed by subsequent layers of wood,—these are likely to be local in their effects, and can be commonly separated from the remaining and sound portions of the trunks; the growth of large limbs near the bottom and at short intervals from top to bottom, resulting in what are called knots; the actual separation or a tendency to the separation of the annual rings, called shakes, and also cracks radiating from the centre. Knots, when large or in great numbers, and reaching far inward from the surface, are necessarily objectionable for many purposes of construction.

Shakes and cracks will rarely be seen in a freshly cut tree; but if the fibres have been unduly strained by high winds or other similar causes, they will rapidly develop when the timber is sawn or hewn into square timber or plank. All timber when thus shaped and exposed to sun, wind, and rain will generally have these defects developed to a greater or less degree, often rendering the timber unsightly, and even unfit for purposes requiring strength and durability. Very slight cracks at the exposed ends or on the sides of large pieces of timber cannot usually be avoided. If, however, they extend to any appreciable depth in or length along the pieces, they

constitute a very serious defect, and the timber should be condemned. This splitting and cracking is more common and noticeable in oak and hemlock than in the white and yellow pines.

110. The durability of timber will depend largely upon using timber without the above-mentioned defects, and upon the means taken to prevent the development of cracks in sawed sticks. The most effective means of accomplishing this latter result is natural seasoning, that is, piling the timbers, with open spaces between them in dry and well-ventilated places, protected from high winds and hot sunshine. The too frequent habit of painting green timber cannot be condemned too severely. This is a common cause of so called dry-rot.

Timber kept in confined air, with no ventilation, either without or with moisture, decays by dry-rot, and the wood is at last converted into a fine powder. This condition is often seen in sills and posts of old framed buildings and at the ends of joists imbedded in brick and masonry walls. Some means of ventilation should always be provided.

Wet-rot is the gradual disintegration and decomposition of all organic matter when exposed to air and moisture, that is, alternate wetness and dryness, as seen in timber lying on the ground or as exposed in a structure. It also occurs in all surfaces of contact, as in the joints of timber frames where the air is more or less confined and heat is developed, although the degree of moisture is slight. Moisture is essential in wet-rot. This form of decay is also developed in the tree while standing and growing, as is often seen in hollow trees.

111. Timber-trees in a healthy growth reach their maturity in fifty to one hundred and fifty years. They should not be felled before reaching maturity, as there is a larger proportion of sap-wood, even the heart-wood is not as firm, strong, and compact, and in addition the yield of timber is proportionately small.

The owners of forests are likely to take precautions against the destruction of young and healthy trees, as will also lumbermen and sawmill owners.

After reaching maturity, the centre portion of the tree begins to deteriorate, the wood begins to show signs of becoming brittle, sometimes becomes spongy and soft, losing both strength and durability.

Owners of forests are particularly anxious to dispose of their timber when evident decline has commenced. Arguments are not wanting to prove that sponginess, or, as it is commonly called, dotiness, is only a local defect, and when this portion of the timber has been cut off, the remaining portion is sound and healthy. It plainly indicates a diseased and unsound tree well advanced on the decline.

Except by such plain indications none but the more expert lumbermen can determine the age of a tree.

The time of the year in which timber-trees should be felled is, however, of great importance. It is admitted on all hands that the felling should be done at a time when the sap is not circulating. This may be either at a certain period during the summer, or during the winter when it is certainly not circulating. When regular supplies of timber are required to meet current demands the felling is usually done in the late fall and winter months, as labor is usually cheaper at that time, and in addition it is desirable to have a large number of logs ready for floating, either pell-mell or in rafts down the watercourses in the late winter and spring rises. But when large quantities of special-sized timbers are required to meet a sudden demand, such as the construction of large bridges and trestles, no attention is given to the question of the proper time of the year. The trees are felled, the logs sent to the mills, sawed, shipped, and erected in structures with the sap still oozing out. While this is to be regretted, it cannot well be avoided.

The subsequent shrinking and consequent derangement of the structure, and the early and rapid destruction of the more exposed timber structures are directly and primarily due to the above causes.

112. It has long been considered that the custom of boxing or turpentining the resinous pine-trees of the Southern forests diminished both the strength and durability of the timber derived from them, and it has been the custom to prohibit the use of such timber in important structures. Recent experiments, however, seem to indicate that it has no such injurious effects. It has, however, become so universal, that it is difficult to obtain large quantities of Southern pine that have not been turpentined; and as it is impossible by ordinary methods to distinguish the one from the other, it is useless to put such prohibition in the specifications, as

it only adds to the cost, and does not prevent the use of the bled timber.

NATURAL AND ARTIFICIAL SEASONING.

113. The seasoning of timber has for its object the removal of moisture, and also the removal or alteration of the albuminous substances in it. These substances are fermentable, and cause decay in the wood.

When these have been removed the main causes of decay are also removed; and if the pores are filled with some impervious substance so as to keep moisture from again permeating the timber, there is no reason for decay ever commencing.

Natural seasoning simply consists in exposing the timber to a free circulation of dry air, but sheltered from the heat of the sun and high winds. This method is admittedly the best; but owing to the time required—from two to four years or more—it is only adopted for timber used in the finer kinds of joiner's or cabinet work.

For those portions of buildings requiring careful seasoning, such as the doors, window-frames, flooring-plank, etc., of houses, an artificial or hot-air process is often, if not exclusively, used. In this process the timber is exposed to a current of hot air. The temperature of the air and time required vary with the size and kinds of timber. The time may vary from one week to two or three months. The temperature may be from 200° to 300° Fahr. or more. Immersing timber in water soon after being felled, and allowing it to remain two or three weeks, and subsequently removing it to some dry and well-ventilated place seems to hasten the seasoning process.

When timber has been well seasoned, and kept constantly in a dry and well-ventilated place, it will last for centuries, and has been found in a sound state after a thousand years. It is the custom to paint or finish in oil all of the interior finishings and fittings in houses, and also furniture, not only for appearance' sake, but to exclude moisture. Timber should not be built in contact with lime mortar.

Timber, when entirely and constantly immersed in water, whether fresh or sea water, does not seem to require any seasoning

to prevent rot, and, so far as records show, we may conclude that it will last indefinitely. It is doubtlessly weakened and softened.

In sea-water, however, it is attacked by worms, known as the *Limnoria terebrans* and the *Teredo navalis,* which eat between the rings or completely honeycomb the timber. They are very minute when they first enter the surface, but grow in size afterwards. One of the effective remedies is to creosote the piles or timbers. These worms, however, only attack the timber between the mud-line and low-water surface. Trestles have been completely destroyed in a short time after being constructed.

PRESERVATION OF TIMBER.

114. It is worse than useless to adopt any artificial means of preserving timber, unless it has been first seasoned by natural or artificial methods. When well seasoned, two or more coats of mineral, asphalt, or tar paints, or of hot oils, will greatly increase the durability. On the contrary, these applied to the exterior surfaces of green timber prevent the escape of the moisture from within, resulting in internal rot, which is the more dangerous as it is kept from view.

Other artificial means of preservation are intended to expel the albuminous substances and supply their places by more durable and impervious ones. Solutions of the salts of mercury, zinc, and iron have been tried, but, either on account of the expense or inapplicability of the methods, are rarely used.

Boucherie's process consists in forcing a solution of sulphate of copper through a stick of green timber. The solution, contained in a tank about 40 feet above the top end of the stick, descends through a flexible tube to a cup fixed on the end. The pressure of the column of the fluid forces the liquid through the tissues, and drives the sap out at the other end of the log. When the solution begins to flow out the pores have been filled with it. It is said that the timber is thus protected against wet and dry rot, and also against white ants and sea-worms. This process has been used successfully and satisfactorily in France, and is largely used in preserving cross-ties on the railways.

In this country creosoting has been used to a great extent, and almost exclusively.

Mr. Bethell's process consists in saturating the timber with the liquid creosote—a heavy oil of tar. The timbers are inclosed in an iron tank or cylinder, and the air partially exhausted. Creosote at a temperature of about 120° Fahr. and a pressure of 150 pounds is applied, which is continued for several hours, or even days, until the timber is saturated.

The Seeley process is a modification of the Bethell process. The timber is immersed in creosote, and at a temperature of 212° to 300°, for a time sufficient to expel the moisture. The hot oil is then replaced by cold oil, condensing the vapors in the pores of the timber, into which the oil is forced by atmospheric pressure.

There is a more recent process called "vulcanizing," which, it is claimed, not only renders the timber more durable, but also stronger and stiffer. The timber is heated in closed cylinders from eight to twelve hours at a temperature ranging from 300° to 500° Fahr. while under a pressure of 150 to 200 pounds per square inch. A circulation of superheated and dried compressed air removes the moisture. Wood-vulcanizing is heating wood and timber under great pressure. This process fills the pores with the antiseptic compounds, among which is creosote. The theory in this process is that the albuminous substances are coagulated by the high heating and rendered insoluble, and that the cellulose also decomposes, and that a chemical combination between it and the sap results in a most powerful antiseptic, which becomes solid on cooling. In the Seeley process the antiseptic is forced into the pores. In the vulcanizing it is formed in the timber and kept there.

APPEARANCES AND CHARACTERISTICS OF GOOD TIMBER.

115. The heaviness and darkness in color generally indicate good timber; also the slowness of growth as indicated by the narrowness and closeness of the annual rings, and a larger proportion of heart-timber; the timber of trees grown on sandy, elevated, and well-drained areas, is generally of good quality. Of the same species, that grown in a cold climate, and of different species that grown in a warm climate, is considered best.

ART. XI.

BUILDING-STONES.

116. STONE suitable for building purposes is either natural or artificial.

The most important properties or characteristics of rock for building are the structural and the chemical.

In regard to the structural character, they are divided into the stratified and the unstratified, or those which show more or less distinct layers and those which do not. These properties are of the greatest importance as regards the strength, durability, and economy of structures.

Stratified rocks are often, if not always, found in a series of parallel layers, either horizontal or inclined or curved, and evidently deposited from water. They are more or less easily separated along certain planes or seams between these layers, which may be several feet, or only a fraction of an inch in thickness. When in very thin layers, it is called a laminated structure, and the planes of division may have any inclination between a perpendicular and a parallel to the main layers or strata. The stratified stones, although commonly not as hard, strong or compact as the unstratified, yet are more easily quarried, cut, and dressed into desired shapes; are somewhat lighter, and consequently more easily and cheaply handled and transported; and at the same time possess a high degree of durability and strength when properly laid in structures. They are more porous, as a rule, and are consequently more injuriously affected by atmospheric influences and changes, and by the disintegrating effects of frost and certain acids. They are found in a great variety and combination of colors well suited for ornamental purposes, and withal are more widely distributed, and constitute the most valuable and useful building-stones for general purposes.

The unstratified rocks are hard, compact, strong, and durable; and although not found in distinct layers, yet they generally have seams or planes of division, not well defined perhaps, along which they can be split into large blocks of regular shapes. They are

more difficult to cut and dress to regular surfaces and desired forms, are commonly heavier, and are not so easily disintegrated by exposure. The difficulty and cost of quarrying, working, and transporting prohibit their use in any but the most solid and important structures, or where such things are not considered.

117. In regard to the chemical composition of stones, they are divided into three classes, viz., siliceous, calcareous, and argillaceous, according as the predominant constituent is silica, lime, or clay.

SILICEOUS STONES.

Of the many varieties of siliceous stones, such as granite, syenite, gneiss, sandstone, soapstone, porphyry, trap, basalt, etc., only granite, syenite, gneiss, and sandstone have any great value as building materials. The others may be valuable and useful, but are seldom used in ordinary structures.

While the composition of granite, syenite, and gneiss is practically the same, consisting of quartz, feldspar, mica, and hornblende, yet granite and syenite are unstratified and gneiss is stratified.

Owing to the great cost of quarrying, cutting, and dressing, requiring the skill and services of experienced men, granite, and syenite are only used in large and important structures, such as light-houses, large piers for bridges, sea-walls, public buildings, post-offices, custom-houses, etc. It has a very high degree of hardness, strength, and durability, and can be quarried in blocks of very great size. It does not, however, stand exposure to heat, under which it cracks, splits, and chips off.

Gneiss, being stratified, can generally be split in layers of moderate thickness, and where it abounds it is often used in structures. When capable of being split into thin layers it is valuable for flagstone, door-sills, and other similar purposes.

Sandstone is stratified, and consists of grains of sand cemented together by a compound of silica, alumina, and lime. In the strongest and most durable stones the cementing material is almost pure silica, and those in which alumina is the principal constituent in the cementing material are the weakest and least durable. When

much lime is found in the cementing material it decays rapidly near the sea-coast and in localities where much coal is burned, caused by the presence of muriatic and sulphuric acids. Sharp, well-defined grains with little cementing material indicate a good quality of stone; on the contrary, rounded grains with much cementing material indicate a poor stone. The best sandstone is found in thick strata, and it is often difficult to determine the direction of the stratification in the blocks cut from them. It is found in many colors, is easily cut or sawn into desired shapes, resists the action of heat fairly well, is widely distributed, and is used in buildings of every kind. It is porous, and consequently absorbs much water. Alternate thawing and freezing will cause it to separate along the layers unless it is laid with its beds horizontal and perpendicular to the pressure. It often contains crystals of iron pyrites, which decompose on exposure to air and moisture, disfiguring and disintegrating the blocks.

It is impossible to determine the qualities of sandstone from its appearance, chemical analysis, or even specimen tests of its crushing strength when freshly quarried, owing to the presence of the quarry sap. Exposed faces of large masses known to have been undisturbed for a long period of time will furnish some indication of its durability. If the color is dark and overgrown with mosses, all angle lines sharp and well defined, no cavities or hollow spaces, it may be assumed that the stone will weather well—which is the important consideration, as nearly all varieties of sandstone will have the requisite strength to resist crushing under the usual loads or pressures to which it is subjected. If possible, its durability should be determined by examining structures, buildings, steps, water-tables, etc., built of the same material. Some varieties are soft when first quarried, but harden on exposure.

CALCAREOUS STONES.

118. The calcareous stones are those in which lime is the predominant constituent.

The building-stones are either pure carbonates of lime or they may be mixed with sand, clay, and metallic oxides.

The common or compact limestones are found in large masses, often showing distinct stratification, and capable of being quarried

in slabs and blocks from a few inches to two or more feet in thickness. When thus found they constitute one of the most useful and valuable kinds of building-stones, being easily quarried and requiring but little if any cutting and dressing, and found in many localities in the greatest abundance. It varies in hardness and compactness. It is found in several colors—white, grayish-blue, and sometimes variegated. Some varieties are soft when first quarried, but harden on exposure. Some varieties are more porous than others. Compactness is essential to the durability of limestones. The more porous kind absorb water, and are disintegrated by alternate freezing and thawing, and by exposure to an acid atmosphere.

The stones colored by metallic oxides furnish the many colored and variegated marbles. These are capable of receiving a high polish, and owing to their scarcity and cost are rarely used for any but ornamental purposes. White marble is often used in public buildings and occasionally in private residences, and to a large extent in columns and statues. Among other varieties may be mentioned bird's-eye marble, Lumachella marble, and verd-antique.

All limestones effervesce with acids freely, and when exposed to a high temperature the carbonic acid is driven off, and the lime is left either in lumps or powder, and known as quicklime.

Good compact limestone is preferred by many engineers to sandstone for building purposes, especially in the foundation-walls of houses. It may be stated that sandstone should rarely be used for this latter purpose when limestone can be obtained.

Limestones containing certain proportions of silica or alumina when calcined furnish a large proportion of the hydraulic limes and cements used in building masonry and making concrete.

Gypsum or calcium sulphate, when burned and reduced to powder, is called plaster of Paris. When formed into a paste by mixing it with water it hardens rapidly. It is principally used for the hard finish on walls of houses, and for making models and ornamental figures. It absorbs water freely, and swells and cracks, when exposed to moisture.

ARGILLACEOUS STONES.

119. Argillaceous stones are those in which clay is the predominant constituent.

Roofing-slate is the only variety of this stone of any value in buildings, and owing to its imperviousness to water is principally used for roof-coverings and lining water-tanks. It is hard and compact. The best slate splits into thin layers with smooth surfaces, will admit of driving nails through it without splitting or chipping, has a dark-blue or purple color, and should give a clear ringing sound when struck.

DEFECTS, DURABILITY, AND DECAY OF STONE.

120. *Granite.*—The principal defects in this variety of rock are: 1st. The presence of pyrites and other iron ores, which deface and disintegrate the stone when exposed to the weather. 2d. An excess of feldspar and mica indicates the weaker and more perishable varieties. 3d. In some granites there are often blind seams, which may have any inclination to what may be called the planes of the quarry beds, and which weaken the blocks and also cause disintegration when exposed to moisture, with alternate freezing and thawing.

Granites are liable to disintegrate from the constant expansion and contraction caused by changes in temperature. Their compactness and non-absorbent properties prevent to a great extent if not entirely injury from freezing alone, and as these stones are poor conductors of heat the injurious effects from this cause are confined to the exposed surfaces. Decay, then, is caused more by mechanical than by chemical agencies, such as exposure to high winds or blasts of any kind, driving dust, grit, and rain-drops against the surfaces.

Observations in great conflagrations, such as have occurred in Portland, Boston, and Chicago, have proved beyond question that granite, instead of being a fire-proof material, resists but poorly the effects of heat, ranking certainly below sandstone and probably limestone in resistance to high temperatures, cracking, splitting, and flying off in fragments.

At the burning of a church in England, the body of the

church being built of granite and the tower of sandstone, around which the heat was intense enough to melt the bells in the belfry, the tower was but little injured, although the granite window-jambs and sills were destroyed. Sandstone is often used for the backing to fire-brick lining of furnaces.

Limestone and dolomites, both the marbles and common varieties, while less affected than granite by purely mechanical agencies, are yet more susceptible to the solvent action of gaseous atmospheres. Limestones are not, in this respect, as durable as the dolomites. Compact limestones are but little, if any, more absorbent than granite, and are therefore but little affected by freezing.

Marbles are liable to disintegration on the surfaces, caused by carbonic and sulphuric acids and also by internal disintegration. In the first case the surfaces lose their polish and become roughened, followed by minute rifts and final rapid disintegration. These conditions are sometimes apparent in from fifteen to twenty years. Internal disintegration seems to arise from reactions between sulphuric acid and calcium carbonate, forming a superficial coating of sulphate of lime. When this is removed by mechanical agencies the stone crumbles rapidly, the cohesion of the crystalline granules beneath the coating being destroyed.

Although the contraction and expansion of the constituent minerals of many varieties of stone may cause disintegration near the outer surfaces, the expansion and contraction of a block of stone taken as a whole is very small, and will not be appreciable or noticeable unless blocks are laid close together in a long line.

Chemical analyses, artificial imitations of the effects of freezing and thawing, and determinations of the resistances to crushing have been resorted to in order to determine the strength, durability, and suitableness of the various kinds of stone for building purposes. While the information may be useful and valuable, yet these tests do not prove entirely satisfactory. Experience and observation will alone determine the matter definitely.

It may in general be stated that, with the exception of a few evidently soft varieties of stone, all stones worthy of the name will be capable of bearing safely the loads ordinarily used, and the question of durability is the important one.

PRESERVATION OF STONE.

121. As resistance to disintegration of stone from either mechanical or chemical agencies is of so much importance, much time and thought have been spent on some means of preservation. After proper seasoning or driving out the quarry moisture, all methods have for the main object to exclude air and moisture from the surface and from penetrating the interior of the stone.

Paints, oils and coal-tar, are considered efficient, at least for a time, but are more or less unsightly.

Silicate of potash or soluble glass, in solution, has been applied, which hardens after the evaporation of the water and removal of the potash by the carbonic acid of the air. Or the pores of the stone are filled with a solution of silicate of potash, and then introducing a solution of chloride of calcium or nitrate of lime, the chemical reactions produce silicate of lime, which forms an artificial stone. The chloride of potassium or nitrate of potash being soluble in water can be washed out. Soft soap dissolved in water, followed by a solution of alum ($\frac{3}{4}$ lb. soap to 1 gal. water ; $\frac{1}{2}$ lb. alum per gallon of water), has been used.

Whatever merit may exist in either of these processes, the fact stands that rarely is an attempt of any kind made to protect the natural stone. Much has been said and written upon the disintegration and decay of stone, and examples given where it has been observed, and in some cases isolated blocks have been removed on account of it. This would rather indicate a soft and porous stone when originally used. A few examples have been given of the actual crumbling of structures, and along with these examples of structures built from the same species or varieties of stone which have remained in a good state of preservation, indicating in the former the use of an inferior grade of stone.

APPEARANCES AND CHARACTERISTICS OF GOOD STONE.

The relative hardness of stone can be determined by scratching with other stones or metals, and by the use of the hammer. This property is especially important for paving, building-steps, sills, and all cases where wear and abrasion occur; and in general terms, hardness, fine grain, uniform and compact structure, darkness of color,

great heaviness or specific gravity, and a small capacity of absorbing water are good indications of the strength and durability of stone.

122. A few useful points in regard to the qualities and durability of stones are collected below.

Durability, or the power of resisting atmospheric and other external agencies and influences, is the most important and essential property in any variety of stone.

The destroying substances are usually taken up by the moisture in the air, especially during rains, and by this means conveyed or driven into the stone. Without this vehicle they will not penetrate so deep and frost can have no effect. Hence the number of rainy days in a year, and the positions of the stones in a building with respect to the direction of the prevailing winds and rains, have an important bearing upon the durability of stones. High winds alone, variations in temperature, exclusion of sunshine, all are causes of disintegration.

The destroying substances are sulphur acids, carbonic acid, hydrochloric acid, and nitric acid in the atmosphere of cities. Carbonic acid will destroy any stone composed of carbonates of lime or carbonate of magnesia ; oxygen acts upon stones containing much iron ; and fumes from bleaching-works and factories decompose many varieties.

We therefore conclude that most stones will better stand a pure air than one charged with fumes and smoke ; a dry better than a wet, moist situation ; those not facing the prevailing winds, and those exposed to sun and gentle breezes, than when otherwise situated. The decay is rapid on those facing the winds and rains, soffits of arches and lintels, etc.

Stones should be compact and not porous ; homogeneous in structure, containing no soft patches. If the grains and cementing materials are both durable, the stone will be durable. If the grains are easily decomposed and the cementing material durable, the stone becomes porous and spongy ; if the reverse, the grains separate and the stone crumbles.

A hard stone does not necessarily wear or weather well ; it should also have toughness. Some hard stones are more affected by atmospheric influences than softer varieties.

Running or dripping water wears away the surfaces of stone.

Blocks of stone in marine works are liable to serious injury, not only from the impact of waves, but also from the wearing action of sand, grit, and bowlders driven against them. For these works the heaviest stones should be used, and in large blocks, in order to resist displacement.

A light, strong, durable stone is well suited for the construction of arches; whereas the heavier stones are valuable in retaining-walls, abutments, and piers liable to great pressures from ice, drift, etc.

All stones work better when first quarried, but should be allowed to season before being placed in structures. Some varieties harden on exposure to the air.

ART. XII.

QUARRYING.

123. BUT little can be learned of quarrying except by experience. Few men can predict, except in case of very favorable external conditions, how far a proposed site will develop into a good quarry, as determined by the cost, ease of quarrying, and the ability to secure stone in large quantities and of the proper sizes and shapes. Often, after spending much time and money in stripping and facing up, the work has to be abandoned. A quarryman of experience and good judgment is of inestimable value, both for selecting good quarries and for getting out stone in good shape and at a reasonable cost.

When the rock lies in well-defined seams and of the proper thickness of layers the conditions are favorable, as most rocks will split in planes perpendicular to the strata without any very great labor or cost. In such cases crowbars, picks, wedges, and hammers, with a limited amount of drilling in a direction normal to the layers, or the aid of plugs and feathers, are the only tools required. This is true of many sandstones and limestones, especially the latter. In many sandstones it requires expert and experienced quarrymen to detect the planes of division between the stratifications, as well as the less well-defined seams perpendicular to them. They commonly exist, and much depends upon the finding of them. In

these stones but little blasting should be done, and then only with judgment and care, using a number of small charges rather than a few large ones, as the latter will not only result in much waste, but by springing hidden seams shatter or otherwise render the stone unfit for dimension blocks of large size.

In many varieties of these stones the stratification and seams are not well defined and not easily detected, and it will be necessary to resort to blasting. In such cases it will generally be found advisable to loosen and detach large masses of the stone by a series of blasts, and subsequently subdivide these into blocks with blasting or by a series of small blasts. Where rubble-stone only is required, the most economical and rapid means of getting out large masses of all shapes should be adopted, and blasting with dynamite instead of powder will prove both effective and satisfactory.

When blasting has to be resorted to, holes from one and one-half to two and one-half inches in diameter are drilled into the rock to depths varying from four or five to twenty feet or more.

There are two common methods of hand-drilling. In the one, steel drills of varying lengths, formed with a broad chisel-shaped end, are required. One man holds the drill and turns it slightly after each blow, and two men with heavy hammers strike on the head of the drill alternately. In the other, two men simply lift the drill and let it fall into the hole, turning it slightly after each fall of the drill. The latter is the more efficient method, but more wearing on the drill. A day's work in drilling may be from five to fifteen feet per man.

In quarries doing a large regular business, or where sufficiently large quantities of stone justify it, the drilling is done by steam or compressed air, machine-drilling, as it is called. The diamond-drill is often used in the harder varieties of rock. In any case the principle is the same, the drills being turned and moved forward by blows or the application of pressure.

A certain quantity of water is necessary to keep the end of the drill cool. It also facilitates the removal of the powdered rock, and when supplied under a sufficient head it can be made to remove it from the hole. In ordinary cases it is removed by small iron spoons, or by the broomed end of a small branch of a tree.

124. The line of least resistance is the shortest distance from

the charge to the surface in uniform rock, and this should never be allowed to be in the direction of the bore-hole. The amount of rock loosened is roughly estimated at one cubic yard for each one half to two and one half pounds of powder, and sometimes stated as equal to two times the cube of the line of least resistance. The quantity and kind of blasting material must be determined by experiment in each case.

Enlarging holes near the bottom has been effected by the use of acids, but more commonly by one or more small blasts of powder or dynamite.

The holes—after the explosive material is deposited at the bottom and connected by fuses, or wires if the discharge is to be effected by electrical currents from the surface of the ground, are to be tamped, first placing a foot or two of clay or other material free from sand or grit of any kind, and ramming it well with a wooden rod or stick; above this any heavy material can be rammed until the hole is full, in order to make the line of least resistance as nearly as practicable perpendicular to the bore-hole.

Even in the unstratified rocks, such as the granites, there are seams or planes of division which should be traced out in order to quarry the stone cheaply and effectively. There are no differences in the mode of quarrying stones of this kind from that adopted for other stones. They are harder, larger blasts can be used, and the cost of quarrying is greater. Larger and more regular-shaped blocks can be obtained; and what would usually be called débris or waste in sandstone quarries and also in limestone—the smaller stones and blocks—can be worked into paving-blocks and other useful and valuable purposes, such as macadamizing streets and roads, making most excellent spawls, backing masonry, and making concrete. Both sandstone and limestone are used for these latter purposes, but are not as good.

When practicable, quarries should be opened on hillsides, as high and broad faces both in vertical and horizontal planes can be exposed with a minimum cost in stripping the soil, and the blocks are more readily handled and loaded on cars or barges for transportation.

The cost of quarrying will vary from 50 cents to \$1.00 for rubble-stone, and from \$2.50 to \$4.00 per cubic yard for good-sized dimension-stone.

ART. XIII.

ARTIFICIAL STONES.

125. OF the artificial stones the more common and the more useful for building purposes are Brick and Concrete.

Bricks are divided into three classes, namely, Common Brick, Pressed Brick, and Fire-brick.

COMMON BRICK.

Brick Earths.—The earths used in making the ordinary or common bricks generally consist of alumina and silica, either alone or in combination with other materials, such as lime, magnesia, iron, and certain other substances which are invariably found in small quantities in all earths. It is difficult to determine, either by appearances or even chemical by analysis, the suitableness of earths for brickmaking. The only reliable test is to actually convert the earth into a paste, dry, and burn it. Expert brickmakers can, however, form a reliable opinion as to the probabilities of the earths making good bricks.

Alumina is the principal constituent in every kind of clay. Its plastic qualities are due to this material, and it is also the cause of its shrinking, cracking, and warping under the influence of heat.

Silica, either as sand, mechanically mixed with the clay, or in chemical combination forming silicate of alumina, is found in the clay.

Pure silicate of clay is practically infusible; it is plastic when combined with water, but shrinks and cracks on drying. To prevent this the presence of a certain amount of sand is necessary. An excess of sand, however, renders the brick brittle. Sand also provides the silica necessary for a partial vitrifaction of the materials.

Although pure clay swells when mixed with water, and subsequent shrinkage is prevented by the presence of sand, yet owing to the infusible nature of the sand the burnt mass would not be coherent unless some substance were present to cause the grains of silica to melt and thus cement the particles of the mass together.

The presence of a very small quantity of lime in a finely divided state acts as a flux, as also does oxide of iron. Carbonate of lime or ordinary limestone in lumps renders the clay unfit for making good brick, as the carbonic acid is driven off by heat, and the quicklime, absorbing water from the atmosphere, slakes and disintegrates the bricks. Iron pyrites, the alkalies, common salt, and silicate of lime in excess, acting as a flux, cause the clay to melt and become distorted.

Loam is a mixture of clay and sand. Marl is a mixture of clay and carbonate of lime.

A good earth for making bricks should contain a sufficiency of flux to fuse its constituents at a furnace-heat, but not enough to cause the bricks to run together or become vitrified. Such earths contain from one fifth to one third alumina, one half to three fifths silica, and the remaining portions should be carbonate of lime, carbonate of magnesia, oxide of iron, etc.

Bricks made of such clays are silicates of alumina and lime or other fluxes used. If ordinary earths do not contain these substances in the proper quantities or proportions, they should be added to them.

The color of bricks depends upon the composition of the clay, upon the kind of sand used for moulding, on the state of dryness of the bricks before burning, on the temperature at which they are burnt, and upon the amount of air admitted to the kiln. Pure clay, free from iron, will burn white, but the color of white bricks is generally produced by adding *chalk* to the clay.

Iron in the clay causes it to burn yellow, orange, and red, and when in large quantities a bright red is produced; if raised to an intense heat, a dark blue or purple is produced; a little lime in the presence of a small quantity of iron produces a cream color; magnesia in the presence of iron makes the brick yellow; a clay containing alkalies, and burnt at a high temperature, becomes a bluish green.

To make 1000 bricks of ordinary dimensions there will be required from 1¼ to 3½ cubic yards of earth, measured before excavating.*

It requires a high and long-continued temperature to expel the

* See Notes on Building Construction.

water, but when properly burnt the mass becomes hard, compact, and gritty; and when good materials have been used it will not again combine with water, nor disintegrate on exposure to acid atmospheres. It stands the effects of exposure to high temperatures, as in large fires in the cities; and a wall of burnt clay or brick, if it does not fall over, is regarded as the most perfect barrier against the spread of flames.

126. It is better to dig and pile up the earth so that it may be pulverized by freezing and thawing during the winter. The clay is then mixed with about one half of its volume of water and thoroughly kneaded until it becomes a uniform paste. This is effected by shovels or by heavy wheels attached to levers and carried around by horses, the wheel being confined to a channel or trough, and the clay and water thrown in previously. This mashes the lumps and mixes the materials, which are still further and more thoroughly mixed in the pug-mill. This consists of a vertical cylinder or box, in the middle of which is placed a vertical shaft turning on a pivot in the bottom of the mill. It has a series of iron arms radiating from it, and set in such a manner that while they mix the paste they also force it downwards and out through an orifice at the bottom, where it is pressed into moulds by the moulder. The moulds are usually made of plank, so framed as to form three spaces of the proper dimensions, open at top and bottom. The hollow spaces are a fraction larger than the intended sizes of the bricks, in order to allow a reasonable shrinkage. The bricks after burning vary in dimensions from $8\frac{1}{2}$ to $8\frac{3}{4}$ inches long, 4 to $4\frac{1}{4}$ inches wide, and from $2\frac{1}{4}$ to $2\frac{3}{8}$ inches thick. The moulds are kept moist by dipping them in water, and the moulder throws or sprinkles a little sand on the surfaces before throwing a batch of clay in the moulds. He then throws the paste with sufficient force into the moulds, which rest on a smooth bottom-board, to fill them thoroughly, cuts off the surplus clay with a small, smooth straight-edge. The perfectness in form and smoothness of the surfaces depend much on the care and skill with which this operation is performed. Small boys then carry the moulds containing the clay to the yards, where by a skilful jerk they are lightly deposited on the ground or on boards to be cured or dried in the sun, or they are sometimes deposited on plate-iron floors in a house prepared for the purpose, the iron floors being kept at the proper temperature by steam or

hot air. Usually they are laid in only one course, sometimes in three or four courses or more, the one on the other. They are first placed parallel to each other in each course or layer, and when partially dried they are laid diagonally and a little distance apart in order to afford a better circulation of the air. These open walls of 10 to 12 courses should be covered over with boards or temporary sheds to protect them from rains and intensely hot sunshine.

It will require a week or more of drying before the bricks can be safely piled in the open walls, and they should be left fully as long in the walls to complete the drying, but this depends greatly upon the temperature and moisture in the atmosphere.

127. When properly dried the bricks are built in large masses called *kilns* preparatory to the burning.

Kilns are constructed in small brick-yards by simply placing the bricks in layers, with narrow, open spaces between the individual bricks, crossing each other header-and-stretcher fashion. At the bottom a series of flues or eyes are formed at intervals by roughly arching the space with short overlaps, and when these come together at the top of the flues the subsequent layers are then laid over the entire area, leaving the required open spaces. This is carried to the height of ten or twelve feet. The horizontal section of the kiln is rectangular. The smaller side of this rectangle is such that the flues can be easily fed with fuel from the two longer sides. The longer side or length of the kiln is determined by the number of bricks to be burned, the number ranging from 80,000 to 200,000 bricks. The outer surfaces are inclined or battered inward slightly from the bottom to the top. Entirely over the sides and ends old, broken, or underburnt bricks are built in mud-mortar, or a paste of mud is simply smeared over them in order to confine the heat. In larger establishments the flues and walls of the kiln are built of good bricks and are permanent, the sun-dried bricks being built in the interior, as above described.

A moderate fire is then started in the eyes or flues to still further dry the bricks. The completion of this process is indicated by the smoke issuing from the kiln being no longer black. The intensity of the heat is increased until the eye or arch brick attain a white heat. It is allowed to lower somewhat and again increased in order to prevent vitrifaction. This process is continued until examination or experience determines that the burn-

ing is sufficient. The ends of the flues are closed, and all other openings covered with moist clay, and the kiln remains in this condition until it has cooled sufficiently to be opened. This operation consumes from two to three or more weeks.

Under favorable conditions it will require about one-half ton of soft coal for each 1000 bricks burned.

One great advantage in using the permanent kilns is that all of the bricks are equally burnt and more uniform in color, and less time is required in drying in the sun, as the temperature in the kilns can be better regulated during the kiln-drying, thereby preventing warping and cracking, which cannot be done if placed in an ordinary kiln when too moist; and although the kiln-burning requires more fuel, the time required is less, and the bricks are of a better quality, and there is less waste.

128. Bricks burned in ordinary kilns are always found in several very different conditions. Those on the outside, on top, sides, and ends, are really of no value. Next to these is found a large number of underburnt bricks, called soft, pale, or sammel bricks, only fit for filling in between walls, as they are wanting in strength, hardness, and durability, and will not consequently stand exposure.

The centre of the kiln contains the best-burnt bricks, called body, hard, or red bricks. These possess great strength, hardness, and durability; are suitable for any building purposes, being but little inferior to many of the building-stones, and in some respects are superior to them.

The arch or eye bricks are hard and brittle, are frequently vitrified, warped, and cracked, and are not as good as the body bricks for most purposes.

Pressed bricks are usually made of the same earths as the ordinary bricks, and are made by taking the ordinary raw bricks when nearly dry and subjecting them to a great pressure, usually by machinery. They are more perfectly formed, with sharp and well-defined angles, are heavier and stronger, and shrink but little in burning. Ordinary bricks shrink from one tenth to to one twelfth of their linear dimensions.

129. Fire-bricks are made from fire-clay or refractory clay, as the clays capable of standing a high temperature without melting or becoming soft are called.

Clay suitable for fire-bricks is found in the coal-measures.

underlying the several seams of coal. It is composed of nearly pure hydrated silicate of alumina. It contains from 59 to 96 per cent silica, 2 to 36 per cent alumina, and from 2 to 5 per cent oxide of iron.

A good fire-clay should have a uniform texture, a rather greasy feel, and should be free from any of the alkaline earths.

Stourbridge Fire-bricks.—It is customary to mix burnt ordinary clay with the natural clay instead of sand where the presence of iron requires some substance to prevent its cracking. These fire-clays will resist very high temperatures, but are not sufficiently refractory to resist the temperature of a furnace.

The Dinas fire-bricks are made from a so-called clay, but it really consists almost entirely of silica. It is found in the state of sand. About one per cent of lime is added, and enough water to make it cohere. The bricks are moulded by machinery under pressure, dried, and burnt in a close kiln.

The bricks made from this substance will bear a most intense heat, being the only description that will resist the temperature (4000° to 5000° Fahr.) of a regenerative furnace. They should not be exposed to the action of slags rich in metallic oxides. They expand under heat, are porous, and will not stand rough usage.

The general processes of making fire-brick are the same as in making ordinary brick.

TERRA-COTTA.

130. Terra-cotta is largely used as a substitute for stone in the ornamental parts of structures. To prevent too great shrinkage, different clays are mixed, and a large proportion of ground glass, pottery, and sometimes sand is added. After being properly kneaded and forced into moulds smeared with soft-soap, it is carefully dried, gradually baked in a pottery kiln, and slowly cooled.

Fire-clay is also often used in the manufacture of terra-cotta. This becomes ragged and porous on removing the outer skin, which shows a good texture, color, and surface.

When properly prepared and burnt, terra-cotta is not affected by the atmosphere or any acid fumes. When solid it weighs 122 lbs. per cubic foot, but when hollow it only weighs from 60 to 70 lbs. Its resistance to compression is greater than that of limestone.

It wears well, and is therefore suitable for floors. It can be made in many colors.

Porous terra-cotta is made by mixing clay and some combustible material, such as sawdust, charcoal, cut straw, etc. When burned these combustible materials are consumed, leaving the terra-cotta full of holes. It is light, strong, easily cut, will hold nails, gives a good surface for plastering, and is fire-proof.

Stoneware is the name given to articles made from the plastic clay of the Lias formation. It contains about 76 per cent of silica and 24 of alumina. It is burnt at a high temperature, being vitrified throughout. It is hard, of close grain, and gives a ringing sound, and is well suited for all purposes requiring strength and resistance to destructive atmospheric and chemical influences. It is commonly salt-glazed, but are non-absorbent without it.

Owing to its strength, durability, imperviousness, and resistance to destructive influences it is well suited for drain and sewer pipes. For these purposes salt-glazed ware should alone be used.

TILES.

131. Common tiles are used for paving and roofing. They are made of about the same materials as ordinary bricks, but of purer or stronger clays, and with somewhat more care.

Paving tiles are of many shapes and sizes, and about one inch thick.

Encaustic tiles are those in which the colors are produced by substances mixed in the clay, such as manganese for black, cobalt for blue.

Each tile consists of three layers: the face, which is of very pure clay, of the color required; the body, which is of coarser clay; and the back, to prevent warping, which is formed of a thin layer of clay different from the body.

Roofing-tiles are of many forms and sections, such as plain, corrugated, Venetian, ridge, etc.

Tile roof-coverings are heavy; they are apt to absorb water and keep the roof wet unless glazed, they do not usually fit together closely, and require pointing to make a tight roof.

132. Pipes are made from clay very finely ground, washed,

passed through sieves, tempered, passed through the pug-mill, then forced by machinery through a mould, dried, and baked in a circular kiln. Agricultural drain-pipes are made in lengths of 2 feet and diameters from 1 to 6 inches, often in ◠ shaped sections. Collars are used for the joints; these are 3 or 4 inches long, and with diameters 1 inch greater than the pipes. They are often omitted.

Sewer-pipes should be of imperishable and vitreous materials, strong and tough so as to resist fracture and shocks, hard, tenacious, in texture and uniform thickness, straight and of uniform cross-section, impervious, and glazed both inside and outside; free from cracks or flaws, and should give a clear ringing sound when struck. Elbows and other special forms must be accurately moulded to the proper curve. They are made either from stoneware-clay or fire-clay. The former are stronger and better, and do not require so great a thickness. They should be salt-glazed, and if this can be picked off it indicates an inferior quality.

ART. XIV.

LIME, CEMENT, MORTAR, AND CONCRETE.

133. OF all the building materials none are more useful and important to the engineer, architect, and builder. And as lime, cement, and sand are the essential constituents of mortar and concrete they will be first considered.

If any limestone be calcined at a bright-red heat or higher temperature, the carbonic acid and water will be driven off, and the residue will be substances known as quicklime or caustic lime, hydraulic lime or hydraulic cement, or simply cement, depending on the amount and nature of the impurities in the limestone.

The calcination, at a bright-red heat or some higher temperature, of a pure carbonate of lime, such as marble and chalk, leaves a residue in the form of white lumps or powder, called quicklime. This residue by weight will be about 56 per cent of that of the limestone.

Almost all ordinary limestones contain certain foreign constituents or impurities, principally silica, alumina, magnesia, etc.

If the percentage of these impurities does not exceed 10 per cent the residue is still called quicklime, as the properties of the residues in the two cases are practically the same.

134. The calcining is performed in a kiln, which for ordinary purposes consist simply of a rough masonry structure with thick walls, enclosing a hollow space of varying dimensions, the horizontal section circular and the vertical oval, the diameter at the bottom being about six tenths of the greatest diameter; and in order not to diminish the draught nor the capacity of the kiln the height varies from 10 to 15 feet.

This is the general construction of an intermittent kiln, in which the fuel is all placed at the bottom. The fuel is usually wood. The larger pieces of broken limestone are formed into an arch above the space for the fuel, and above this the kiln is filled with broken stone somewhat above the top; or a permanent open arch of fire-brick can be built, over which the broken limestone is placed. The heating should be gradual at first to prevent a too rapid expulsion of the carbonic acid and water, shivering the lumps and choking off the draught. After a uniform red heat has been reached the temperature should be kept up until the burning is completed, which is indicated by the settling of the mass, and the ease with which an iron rod can be made to penetrate it. The kiln should then be closed to keep the heat confined and complete the burning of the upper layers, when the whole mass is removed and a new charge put in. A cylindrical kiln is sometimes used, the heat being supplied from a furnace. In this case the limestone is fed continuously from the top, and is completely burned by the time it reaches the bottom, whence it is gradually removed. This is a form of the perpetual kiln. Or this kind of kiln may be of the shape of an inverted cone. Such a kiln, having a diameter at the bottom of about 4 feet and at the top 9 feet, and 16 feet high, may contain 25 tons of limestone. It has a grating at the bottom with movable bars, upon which is placed a layer of brushwood, then alternate layers of coal and moistened stone piled to the top, the larger fragments placed in the centre, where the heat is more intense. By removing the alternate bars when the burning has been sufficient the lime is withdrawn, and as the mass settles additional layers of stone and fuel are thrown in at the top. Or a pair of vertical cones may be placed base to

base, or a cylinder on top of a lower vertical cone. This is fed in the same way.

These perpetual kilns are economical in fuel; the lime is not as uniformly calcined, and more skill and experience are required.

The kilns may be constructed of brick, stone, or concrete. It is better to line the kilns with fire-brick, especially if built of concrete or rough masonry. The space between the two is left hollow.

The amount of fuel will be from one fifth to one fourth the weight of the lime produced. A pure, rich or fat lime, i.e., quicklime, requires only heat enough to thoroughly drive off the carbonic acid and moisture. Limes containing clay require a high temperature in order that the silicates and aluminates may be formed, which give the hydraulic properties required.

135. Great care must be taken, however, that the heat is not sufficient to fuse the particles of lime or cement. The Roman cements, which contain a quantity of iron and alumina together equal to the silicic acid, are burnt with little fuel and at a low temperature. Portland cement, in which the iron and alumina are less than one half the silicic acid, is burnt at a very high temperature. There is but little danger of fusing the particles, and the temperature may be raised to nearly that of vitrifaction.

The stones should not be overburnt nor underburnt. So long as any carbonic acid is left in the stone it will remain at a dull-red heat, but when it is all expelled the stone becomes peculiarly bright. If overburnt, a hard, heavy substance is produced, burnt to a clinker, which slakes with difficulty and only after the lapse of a long time, and may cause damage to the masonry. If underburnt, the substance produced is partly a perfect cement and partly free quicklime, which is prevented from slaking by the setting of the cement, and may also cause injury to the work.

Underburnt lime has a core of the carbonate left, and will not slake. Hydraulic lime overburnt may also have a core of carbonate left.

If the limestones contain from 10 to 20 per cent of impurities, the hydraulic-limes are produced. If the impurity is chiefly clay, they are called argillaceous limestones; if silica, silicious limestones.

If between 20 and 60 per cent of impurities exist in the limestones, the hydraulic cements are produced.

If more than 60 per cent of impurities are found, the resulting product is called a calcareous pozzuolana. If 10 per cent or less of lime, it is called pozzuolana.

136. Pure, rich, fat or quick lime is a calcium monoxide, and is produced when a nearly pure limestone is heated sufficiently to drive off the carbonic acid. It is an amorphous, infusible, caustic mass, and has a specific gravity of 2.3. It combines readily with about ⅓ of its weight of water, and forms hydrate of lime. During this process the lime swells to from 2½ to 3½ times its original volume, becoming very hot and boiling violently, forming a soapy, unctuous paste. Owing to its affinity for moisture it should not be exposed to air, as it air slakes, and partly returns to the condition of carbonate of lime.

It should be kept in barrels, and protected from moisture until needed.

When made from impure limestones it is called meagre or poor lime.

The perfectness with which the lumps fall to powder when water is applied is a fair indication of the quality of the quicklime. No mashing the lumps or stirring should be necessary, though the slaking will be somewhat hastenened by so doing.

The paste made from quicklime will not harden at all under water. It hardens slowly in air by the absorption of carbonic acid and the crystallization of the carbonate of lime formed. This is a slow process, but after the lapse of years it attains a considerable degree of hardness. It is at least doubtful whether in large masses the interior will ever harden. A thin crust forms on the outside, which stops the necessary absorption of carbonic acid.

Hydraulic limes slake somewhat slowly when mixed with water, little heat is developed, and the increase in volume rarely exceeds one third the original bulk. A paste made of these will harden slowly under water. It is generally stated that these limes are not manufactured in this country. The silicious limestones produce on calcination the hydraulic limes of Teil, in France. These hydraulic limestones are found extensively in the United States, but owing to the fact that the so-called hydraulic-cement stones are found abundantly distributed also, it has neither been found necessary nor remunerative to manufacture to any great extent the inferior grades.

Hydraulic limes made into a paste set well under water, but rather slowly.

137. *Hydraulic Cements.*—The so-called hydraulic limes pass imperceptibly into the class of hydraulic cements, which do not slake when mixed with water. There is no sensible elevation of temperature nor increase in volume, and the best grades do not shrink in hardening like the fat limes, and can be used without any addition of sand. They set rapidly under water. The rapidity of setting and the hardness attained vary greatly with the proportions of clay and lime, and also the degree and duration of the heat reached in burning.

The same stone may produce a quick-setting or a slow-setting cement, depending on the temperature during the burning. Some of them when burnt at a certain temperature may set rapidly when first mixed into a paste and immersed in water, but are wanting in permanence and stability; whereas sometimes a temperature above or below, or both, would produce a good cement. On the contrary there are stones that will only produce a good cement at a certain maximum temperature, and produce an inferior grade at a lower or higher temperature.

Pure limestones, as well as those containing not more than 22 to 23 per cent of clay, will stand a high temperature without fusing, whereas those containing a much larger proportion of clay will fuse at a temperature not much above a red heat. If those containing from 22 to 23 per cent of clay are burnt at a high temperature the resulting product is what is called a heavy, slow-setting cement; but if calcined at a moderate temperature the product may be a light, quick-setting cement. In the latter case, where the stone contains larger proportion of clay, and is burned at a moderate temperature, the product will be a light, quick-setting cement.

The Roman cement and the hydraulic cements manufactured in the United States are of the class known as light, quick-setting cements.

POZZUOLANA.

138. Natural pozzuolana is of volcanic origin. It is usually found in the form of a more or less coarse-grained powder, of a brown, red, yellow, or gray color.

Trass is also a naturally burnt argillaceous earth. Arènes are natural mixtures of sand and clay which apparently have not been subjected to heat.

All of these are clayey earths containing 80 to 90 per cent of clay, with a little lime and small quantities of magnesia, potash, soda, and oxide of iron.

In consequence of the presence of clay they confer hydraulic properties upon lime, and when powdered in the raw state they may advantageously be added to fat-lime paste. The following makes a good proportion of ingredients: 12 parts Italian pozzuolana, well pulverized; 6 quartzose sand, well washed; 9 rich lime, recently slaked.

Artificial pozzuolana is prepared by burning clay. Powdered bricks or tiles can also be used for the same purpose. The proper proportions of lime to the pozzuolana can only be determined by experiment.

139. It may be stated in general that experience and experiment alone can be relied upon to determine, 1st, whether any particular limestone will produce hydraulic lime or cement at all; 2d, what is the proper temperature at which a single variety or a mixture of several different limestones should be burnt to produce a cement of a certain grade.

These experiments can be made by calcining samples in small crucibles, heated to different temperatures, and subsequently tested, or they may be made on a larger scale. Appearances and chemical analyses will not determine the character of the resulting product from calcination.

ARTIFICIAL CEMENTS.

140. Almost any grade of hydraulic lime or cement may be manufactured by artificial processes. To produce a light, quick-setting cement, the proper proportions of fat lime and clay are mixed, resembling in composition the natural hydraulic limestone, and then calcined at a moderate temperature. The mixture may be made by violently stirring the materials in water, or they may be ground in the dry state and formed into a paste with water. The paste is then moulded into bricks, which are dried and calcined like ordinary lime. Owing to the present facilities for obtaining

and the cheapness of the ordinary natural cements manufactured, hydraulic limes are seldom manufactured.

Artificial cements, known as Portland cements, are, however, manufactured in almost every country. The raw limestone or chalk is crushed and then intimately mixed with the proper proportion of clay, which varies with the composition of the chalk, the mixture containing before burning from 23 to 26 per cent of clay. This mixture is then calcined at high temperature, which reaches the point of incipient vitrifaction. The product after burning should contain from 58 to 63 per cent of lime combined with about 22 per cent of soluble silica, 7 to 12 per cent of alumina, and small percentages of oxide of iron, magnesia, etc.

The two principal processes are known as the English or wet process, in which chalk and clay are used, and the German or dry process, in which limestone and clay are used.

With white chalk (containing no clay) 3 volumes of chalk and 1 volume of alluvial clay or mud from the lower Thames are mixed, and with gray chalk containing some clay the proportions are 4 parts of chalk and 1 of clay.

In the wet process these ingredients, chalk and clay, are mixed in water to the state of a semi-fluid; when thus thoroughly mixed, this liquid is run into reservoirs or tanks, and allowed to settle during several weeks. The water is then drawn off. The solid residue is next removed, sometimes further mixed in a pug-mill, dried on iron plates properly heated over flues of kilns or other sources of heat, burned in intermittent kilns at a very high temperature—but little below the point of vitrifaction, and then ground to a very fine powder.

In the dry process the limestones are crushed by machinery, the clay is roughly burnt, and the two are then mixed in the proper proportions, as explained above, to give the right percentages of lime and clay, and are ground to a fine powder. This powder, slightly moistened, is run through a pug-mill to mix it thoroughly, and then moulded into bricks. These bricks are then dried upon heated plates, burnt as above described, and ground to powder.

Ordinary slag ground and mixed with the proper proportion of lime and properly burned produces a good Portland cement.

The Portland cements are known as the heavy, slow-setting cements when properly burned.

It is more economical and less troublesome to burn and grind an underburnt cement, which no doubt is often resorted to. It will also set more quickly, but will never attain the same degree of ultimate strength. It contains an excess of quicklime, which may slake in the structure and cause damage.

Pure carbonate of magnesia, when burned at a rather red heat, ground to powder, and made into a paste, makes a good hydraulic cement.

141. All limes and cements when not intended to be used at once should be kept in tight barrels, which should also be lined with brown paper in order to exclude moisture, and should in addition be stored in cement sheds or houses, as they will deteriorate if moisture has access to the lime or cement. If, however, they can be procured direct from the manufacturer in no very large quantities at a time, it is not unusual to deliver the lime in open bulk, and the cement in sacks or bags. The difference in cost is a little over that of the barrel. There is always, however, more or less waste through the interstices of the bags or from torn or untied bags; and even when exposed to a slight degree of moisture or a slight shower much damage is often done. Good barrels will waste but little in handling; they will stand considerable exposure to a more or less moist atmosphere, without serious damage; and will not be injured when caught in an ordinary shower, or rain if not long continued, and rapidly dried subsequently. Although there is a seeming saving in the cost, there will practically be none when these things are balanced against each other.

THE THEORY OF THE SETTING OF LIME AND CEMENT PASTES.

142. Neither the exact changes and combinations which take place during the operation of calcination, nor the actual changes and combinations during the process of setting, are known or thoroughly understood. To the engineer, however, these matters are not of much importance, as he is more concerned with results than with processes.

A few general principles will be interesting, and may be useful.

Ordinary quicklime, when slaked and exposed to the air, will absorb carbonic acid and be reconverted into a carbonate of lime. This action, however, rapid at first, constantly decreases, and practi-

cally ceases after forming a thin surface crust scarcely more than half an inch in thickness—the interior of the mass remaining in a pasty or soft condition if in a damp situation, and becoming friable and easily pulverized if in a dry situation. The hardening ceases in the interior in either case. Sand found in the limestone as an impurity has by itself no chemical action with the quicklime; when calcined at the temperature ordinarily reached in the calcination it is merely a mechanical mixture, as when mixed with sand after burning.

If the lime paste is immersed under water no hardening whatever takes place; on the contrary it will dissolve slowly, and if the water is frequently changed it will disappear entirely.

The conclusion is inevitable that under no circumstances should lime paste be used in works under water or even in damp situations, such as the foundation-walls of houses, and it should not be used even in ordinary dry air in large thick masses. When used in thin walls built of porous stones or bricks a certain degree of hardness is ultimately attained.

143. The hydraulic limes and cements owe their setting or hardening properties almost entirely to the presence of clay, which either reduces the slaking action or prevents it entirely, and causes the paste to set under water or in situations and in those portions of the mass from which air is excluded.

These effects are more marked as the proportion of clay is increased, provided it does not exceed about 40 to 50 per cent of the whole mass. If a greater proportion is present the setting properties are injured, and if over 60 per cent of clay is present it will not set under water at all. It then becomes pozzuolana, which can be rendered hydraulic by mixing it with the proper proportion of lime.

Limestone containing less than 10 per cent of clay will produce only quicklime.

Any limestone containing from 10 to 50 per cent of clay will produce hydraulic lime, hydraulic cement, or Portland cement, depending on the proportions of clay and lime and the degree of calcination to which it is subjected; and upon the same conditions depend the various grades of these cements in respect to the heaviness, the slowness or rapidity of setting, and the ultimate strength and hardness attained.

After calcination of limestones containing clay the resulting sub-

stance contains a certain proportion of quicklime, and certain compounds of clay and lime which have the property of becoming hard under water.

The silica in the clay must be soluble and in combination with other substances. The mere presence of sand, flint, etc., in a mechanical mixture with the lime is of no value. This explains the reason why a chemical analysis is not of much value, as it is not easy to determine the state of the silica or sand.

When limestone containing clay is burnt, the carbonic acid and moisture are driven off, and the remaining constituents are formed into new and different compounds, which are usually free quicklime mixed with silicate of lime and aluminate of lime.

If there is a large proportion of clay in the limestone, and the burning is at a moderate temperature, the lime is converted into silicate of lime; the alumina in the clay will not combine with the lime. The result is a quick-setting cement. If the amount of clay is not sufficient to furnish enough silica to convert all of the lime into silicate of lime, there will be a certain proportion of quicklime in the free state; this will slake when water is added. The slaking will be slow, owing to the presence of clay. This is characteristic of the hydraulic limes.

If the burning is at a high temperature, some of the lime is converted into silicate of lime, and the alumina also combines with the lime, forming aluminate of lime. At the same time more silicate of lime is formed, and other combinations occur which produce double silicates of lime and alumina. These compounds have the property of setting under water, and when properly burnt the whole of the lime is converted into either silicate or aluminate. The paste becomes hard and strong. These furnish the Portland cements, heavy and slow-setting.

If underburnt the aluminate is not formed, and the result is a quick-setting cement, but one which will not attain great strength.

If much iron is found in the clay, and the combined amount of alumina and iron is large as compared with the silica, fusion will take place unless burnt at a low temperature. This is the case in Roman cement, which should not weigh more than 75 lbs. per struck bushel, and sets quickly—within 15 minutes after being mixed in a paste—but attains no great ultimate strength. The

weight of this and similar cements should be specified, as if too heavy it indicates an overburnt cement.

If the iron and alumina are comparatively small in proportion to the silica, the mixture can be burnt at a high temperature. These are the conditions in the Portland cements, which are the heavy, slow-setting cements. They should weigh 106 lbs. per struck bushel, and set in from two to five hours.

When the smallest amount of clay necessary to produce a cement, calcining at a low temperature, will not produce the necessary combinations for producing a strong cement, the product will result in either a hydraulic lime, which slakes and then sets, or a mixture of quicklime and quick-setting cement, the latter sets and is then broken up by the slaking of the lime. Limestones, then, containing about 20 to 25 per cent of clay—as usually stated, 22 per cent—should be burnt at a very high temperature in order to produce cements of great strength, such as the Portland cements. If, however, the calcination is carried beyond the point of vitrifaction, the resulting product is inert, and without value as a cement. Stones containing, however, from 23 to 36 per cent of clay should be calcined at a low temperature. A high degree of heat will give a mixture of lime and cement similar to the underburnt slow-setting cement, or it may result in a slow-setting cement. With such stones the point of vitrifaction is soon reached, and a poor product then obtained.

We then find that after burning such stones there remains a mixture of pure lime and silicates, or of pure lime, silicates, and aluminates, which are ready to combine if favorable conditions are offered. If the cement remains in a dry state the particles may lie close together without combining. When placed in water, the water dissolves and distributes the particles of lime, bringing them in contact with the particles of silica and alumina; these combine one by one and form hydrated silicates. A certain portion of water combines with the silicates and aluminates, forming hydrated compounds, which set by crystallization and become solid.

This explains the fact that cements set better and more perfectly under water, and if allowed to dry out too rapidly the setting process is impeded.

This is also true of the quicklimes: the presence of a certain amount of moisture facilitates the absorption of carbonic acid.

Walls built with lime mortar in hot climates should be kept moist for a time, otherwise the mortar becomes granular, is apt to crumble, and will not absorb so readily, if at all, the necessary carbonic acid.

144. What degree of hardness or firmness is to be understood as a "set" is rather ill-defined. Gen. Gillmore defines it as a state of the lime or cement paste in which it will not change its form without fracture, or when it has entirely lost its plasticity, and also when it will support without indentation or depression a $\frac{1}{12}$-inch wire loaded with $\frac{1}{4}$-pound weight and a $\frac{1}{24}$-inch wire loaded with 1 pound, or when it will resist the pressure of the finger or some blunt instrument. These are all vague and uncertain standards of either actual or relative "set." The time necessary will vary with the temperature of the air, that of the water used in mixing, and of the water in which the paste is immersed. And, after all, the time of this initial setting is not reliable indication of the ultimate and progressive increase of hardness and strength. Cement as a rule not only improves by keeping under proper conditions, but the older the cement the less danger of its "blowing," which is caused by the slaking of free lime, as time is given for the lime to slake before using, and the time of setting is also longer than in fresh cements.

The weight and fineness to which cement has been ground are important factors in the value and use of cements. The weight is influenced by the fineness to which it is ground. Hence in specifying the required weight a certain degree of fineness should also be specified. A coarse cement is heavier than one equally well burnt which is more finely ground. The weight is affected also by exposure; the samples should be taken from various parts of the heap. The difference may be from 1 to 1½ pounds. The weight, again, depends on the manner of filling the measure. It should be allowed simply to slide down a board or flow through a sieve, and when filled to overflowing the surplus over the top of the measure should be carefully struck off with a straight-edge.

The heavier elements are slow-setting, but will generally become ultimately harder and stronger than the lighter and quick-setting. But it is necessary to see that the weight is not increased by a large proportion of coarse and unground particles which, being overburnt, will slake slowly and cause injury after being used in the work. The weight test alone would lead to coarse grinding; the sieve test only would lead to furnishing a light, easily-ground cement of

an inferior strength. Both should be required, but by specifying a given weight a definite degree of fineness is secured within limits.

Quick-setting cements, although showing an initial strength and hardness greater than slow-setting, yet the latter will in a few days, or at any rate in a week or two, far exceed, as a rule, the quick-setting in strength and hardness, and ultimately will be greatly superior in these respects.

All particles of cement which can be detected by the gritty feel when the hand is inserted into a barrel of cement are either inert, and therefore reduce the amount of useful and available cement, or they are or may be positively injurious.

Hydraulic cement of the Rosendale type weighs from 60 to 70 pounds per struck bushel, which is equivalent to from 49 to 56 pounds per cubic foot (taking the bushel as equal to 1.244 cubic feet). Portland cement weighs from 100 to 125 pounds per struck bushel, or from 80 to 91 pounds per cubic foot.

As to the requisite fineness, the practice of engineers differs widely, some requiring that when sifted through a No. 80 sieve, that is, one having 6400 meshes or openings per square inch, the residue not passing through should not exceed 12 to 15 per cent of the whole. Others are satisfied if only 10 per cent remains after sifting through a No. 50 sieve or one with 2500 meshes to the square inch. This last result should certainly be required, and is easily attained. For ordinary cements a No. 80, having 6400 meshes, and for best Portland cement a No. 180 sieve having 32400 meshes per square inch may be taken as extreme requirements as to fineness.

One barrel of lime weighs about 150 lbs. to 250 lbs.
" " Rosendale cement weighs about 300 "
" " Portland " " " 400 "

A barrel contains 3 ordinary bushels, or 3¾ cubic feet nearly. The usual dimensions are: length 2′ 4″, middle diameter 1′ 4½″, and end diameter 1′ 3½″.

A trade bushel is some specified weight.

According to the English standards a struck bushel contains 1.28 cubic feet, by the American standard 1.25 cubic feet. A cubic yard is about 21 struck or "striked" bushels. A struck bushel is when the measure is lightly filled and smoothed off with a straight-edge at top.

TABLE XII.

Cements.	Weight per Cu. Ft., lbs.	Weight per Trade Bushel, lbs.	Weight per Striked Bushel, lbs.
Portland..........	74 to $101\frac{1}{2}$	100	95 to 130
Roman...........	60 " $62\frac{1}{2}$	70	77 " 80
Medina...........	61	68	78
Keene's...........	64	75	82
Parian............	60	66	77
Plaster of Paris....	50	..	64
Whiting..........	64	..	82

See Notes on Building Construction.

SAND.

145. Sand is an aggregation of loose incoherent grains of a crystalline structure, derived from the disintegration of rocks and other mineral matter, and is called silicious, argillaceous, or calcareous, according to the character of the rocks from which it is derived.

It is obtained from pits, shores and beds of rivers, sea-shores, or may be made by grinding sandstones. Pit sand has an angular grain and a somewhat rough surface, but often contains clay and organic matter. When washed and screened it furnishes a good sand for general purposes.

River-sand has more or less rounded grains, and may or may not contain clay or other impurities. It is commonly of fine grain, is often white in color, and when clean is suited for plastering.

Sea-sand has also more or less rounded grains. It contains alkaline salts, and attracts and retains moisture.

Dirty and sea sand can be washed either by hand or machinery; and if lumps, pebbles, or vegetable matter such as leaves, grasses, and small twigs and the like, are found, it should be screened.

Clean sand should leave no stain when rubbed between moistened hands. Sharp sand gives a gritty feel and grating sound when rolled in the hand, and may be detected by the eye alone or by the aid of an ordinary magnifying-glass.

If the grains are from $\frac{1}{16}$ to $\frac{1}{8}$ of an inch in diameter, the sand would be called coarse; if from $\frac{1}{16}$ to $\frac{1}{24}$ of an inch, fine sand;

and mixed sand when it is of varying sizes within the above limits.

Burnt clay is sometimes used as a substitute for sand; also the arènes, which, though not burnt, being mixtures of clay and sand, are often used.

Silicious sands derived from the quartzose rocks are the most abundant, and are generally preferred.

A finer sand, which seems to be used in testing cements, will generally pass through a No. 30 sieve containing 900 meshes to the square inch, and will be retained by a No. 40 sieve containing 1600 meshes to the square inch. The grains of this sand would be between $\frac{1}{30}$ and $\frac{1}{40}$ of an inch in diameter.

Sand is used in mortar; also for distributing the pressure on soft materials; as a foundation for broken stone; a cushion and bed for paving-blocks; and as a joint filling. The proper qualities of sand for these purposes will be discussed under those heads.

ART. XV.

MORTAR.

146. MORTAR is composed of a mixture of lime or cement, sand, and water. The proportions of these ingredients vary greatly with the purposes for which the mortar is required, the time of setting desired, and the strength needed.

The uses of mortar are many and varied. It is used in brick and stone masonry; in concrete; in plaster for walls; in stuccoing; in lining cisterns, tanks, and reservoirs; in making artificial stone; in making tiles and pipes, and for many other similar purposes.

Ordinary mortar is composed of quicklime, sand, and water sufficient to form a paste of the proper consistency. This mortar is used solely from considerations of economy. It is absolutely unsuitable for damp situations or for thick walls, or in large masses as an ingredient in concrete; it will remain moist or become friable, and has but little strength under the most favorable circumstances. It should never be used in the foundations of houses, and in no part of any important structure. The large majority of brick houses are, nevertheless, built with lime mortar; but that the practice is

not good in any respect, and often bad even from an economical point of view, cannot be denied. The structure is composed of alternating strong and weak layers. A thin crust is formed on the outside, but the mortar remains soft on the inside, or at any rate in a crumbly condition; consequently the pressure is not uniformly distributed, being concentrated towards the faces, often causing chipping and scaling; headers or bond-stones are sheared off; the slightest shrinking or yielding of any portion of the walls causes cracks, which meander and follow the joints; the mortar absorbs and retains moisture, which on freezing is thrown out, requiring repointing; and the necessary patching and repairing may cost a great deal. Damp cellars and damp walls are both inconvenient and unhealthy. As it is, however, used in great quantities it demands more than a passing notice.

147. *Slaking.*—Ordinarily the quicklime is measured, or the barrels are emptied into spaces surrounded by a circular embankment of sand, or in large boxes made of plank, the depth being only 12 or 15 inches, and water enough to slake it thoroughly is poured or sprinkled over it. It is poured over it in considerable quantities, the amount of water varies from one third to one half of the bulk of the lime. Fresh lime requires more water than that which has been kept for some time. Great heat is evolved, the lime swells, falls to powder, and forms a paste of the hydrate of lime. There will usually be left granules or pieces of core which will not slake. These are removed by screening, the semi-fluid lime being allowed to run from one box to another through a grating. Time is given to thoroughly slake the lime, the surplus water rising and covering it; or it may be covered over with sand: this prevents deterioration by absorption of carbonic acid. It then remains in this condition until required for use. More frequently, after being well slaked and screened, the proper portion of sand is mixed with the paste more or less thoroughly and intimately. This is then heaped up in large piles and covered over with sand.

Lime mortar is improved by being thus kept after mixing with sand. When ready to use the mortar, a certain portion of the mixture is shovelled into a box, and properly tempered with a little water. The quantity of water varies with the condition of the sand as to dryness and moisture. The quantity of sand in lime mortar varies considerably—from 2½ to 3½ of sand to 1 volume of pure slaked

lime in paste. The paste is usually soft and plastic, so that it may be spread rapidly.

One barrel of lime, 230 lbs. by weight, will make about $2\frac{1}{2}$ barrels of paste—equivalent to 0.3 cubic yard of stiff lime paste. One barrel of lime paste and 3 barrels of sand will make about 3 barrels of good lime mortar—equivalent to 0.4 cubic yard; or, 1 barrel of unslaked lime with 6.75 barrels of sand will make about 6.75 barrels of mortar equivalent to 0.95 cu. yd.

Another proportion: 1 cu. ft. lump lime, 3 cu. ft. sand, $7\frac{1}{2}$ gallons water, $2\frac{3}{4}$ cu. ft. mortar.

For ordinary use lime mortars are mixed by hand. Machine-mixed mortars are usually regarded as better. As thorough mixing is essential, each and every grain of sand should be well coated with the paste.

148. *Uses of Sand.*—Sand is essential in lime mortar to prevent the excessive shrinking and cracking that would take place in the setting of lime paste.

It is supposed to increase the resistance to crushing, but diminishes the tenacity. In excess it makes the mortar brittle and friable on drying. It facilitates to some extent the absorption of carbonic acid by the mortar, which should be kept moist for a time. For this reason, among others, a rather coarse sand is preferred for lime mortar. Finally, sand is cheaper than lime; therefore the larger the proportion of sand the cheaper the mortar.

One barrel of lime produces about 0.3 cubic yard of mortar, and $\frac{1}{3}$ of a barrel of lime and 1 barrel of sand produce about 0.32 cubic yard of mortar. Since a barrel contains 3 bushels or $3\frac{3}{4}$ cubic feet, there will be 7.2 barrels to the cubic yard. Then 1

barrel of lime or 0.3 cu. yd. of mortar @ $1.00 =		$1.00
and $\frac{1}{3}$ of a barrel of lime @ $1.00 per barrel =	0.33\frac{1}{3}$	
1 barrel of sand @ $1.00 per cu. yd. = $\frac{1.00}{7.2}$ =	0.13$\frac{2}{3}$	0.47
Amount saved for each 0.3 cu. yd. or 8.1 cu. ft. of mortar =		$0.53

Practically the cost is only one half if shrinkage is not considered, but considerable when it is.

Sand only enters the mortar as a mechanical mixture, and little or no chemical action ever takes place between the sand and the lime. The hardened mortar is simply an inferior artificial sand-

stone, the grains of sand being cemented together by a carbonate of lime far from uniform in strength and hardness.

On many works of even some degree of importance and magnitude it is not unusual to mix lime and cement together—either 1 barrel of lime and 1 barrel of cement, or 1 barrel of lime and 2 barrels of cement. The lime should be thoroughly slaked before mixing with the cement, otherwise the lime may not slake until the mortar has been used and partially set, resulting in "blowing" and serious injury. When properly proportioned it is said not to injure the cement, and at the same time is somewhat economical. All things considered, it is of doubtful value.

149. *Cement Mortar.*—Either hydraulic cement or Portland cement, sand, and water are used in what is commonly known as cement mortar, the only difference being in the proportion of the ingredients used and the ultimate hardness and strength obtained.

As there is no appreciable slaking when water is mixed with cement to form a paste, and as setting and hardening results from the chemical actions which commence almost immediately upon the exposure of cement to the air, or upon the adding of water to the cement powder, no time is required between the mixing and the use of the mortar. On the contrary, positive harm may result if the use is delayed. The paste will immediately begin to stiffen and lose its plasticity, and it becomes necessary to retemper with water; and any disturbance of the paste after the chemical combinations begin, which occur simultaneously throughout the mass, impedes the further setting, if it does not permanently injure the ultimate strength, of the mortar.

It is usually required that cement mortar shall be mixed in small quantities at a time and used without delay. This requirement can be easily complied with, and should be insisted upon. Very few cements set so rapidly when mixed with sand that a reasonably sized batch cannot be used before losing its plasticity.

150. *Proportions of Ingredients.*—The proportions of cement, sand, and water will vary according to the kind and quality of the cement, the proportion of lime and clay, the degree of calcination, the degree of fineness to which it has been ground, the size of the grains of sand, and the purposes for which the mortar is required. The best practice limits the water to a very small quantity; any amount of water beyond that which the necessary chemical com-

binations require impedes the setting, and it is claimed injures the mortar.

In the cement mortar used in the concrete under the Washington Monument Gen. T. L. Casey, U. S. Engineer, used 13⅛ gallons of water to the barrel of cement in dry weather, and no water at all in wet, soaking weather. In England from 5½ to 7⅛ gallons of water per barrel of cement have been used. Some engineers make a very soft mortar—almost a semi-fluid—without measuring the water. Both of the proportions apply to Portland cement.

In practice, where no attention is given the subject, as a rule as much as 50 per cent of water is used, so that about 30 per cent has to be drawn off by evaporation. This reduces the tensile strength. A good working rule for cement mortars as determined by numerous experiments seems to be about 1 part of water to 3 parts of cement by volume, or 1 to 3½ by weight, both as regards mixing, handling, and ultimate results.

The proportion of sand to hydraulic cement rarely exceeds 2 sand to 1 cement for ordinary purposes, often only 1 to 1; and in unimportant structures 3 to 1 is often used.

The proportion of sand to Portland cement is rarely less than 2 to 1, very commonly 3 to 1, and in unimportant works 4 and 5 to 1. It is stated that even 8 sand and 1 Portland cement will give better results than ordinary lime mortar.

151. One barrel of Rosendale cement, as usually packed, will measure in the loose state from 1.25 to 1.40 barrels, weigh about 300 pounds net when finely ground, and make from 3.70 to 3.75 cubic feet. One barrel of dry cement mixed with 0.33 of a barrel of water will make about 0.66 cubic foot of stiff paste. This reduced to cubic feet of course holds the same ratios.

The Western Rosendales are lighter, weighing about 265 pounds net per barrel, and will measure in the loose state about 1.1 barrels.

Portland cement weighs 400 pounds gross and 375 net, and will, measured in the loose state, make about 1.2 barrels. Volume for volume Portland cement dry will make about the same amount of paste as the Rosendales, 100 pounds making a cubic foot of stiff mortar.

As there is neither swelling nor shrinking in the case of cement mortars, the addition of sand simply diminishes the cost of the

mortar in proportion to the increase in the quantity of sand. Sand is not necessary as in lime mortars.

Uses of Sand.—In cement mortar it is not necessary to use sand, as no shrinking or cracking should occur if the cement is good. It retards the setting, diminishes the tensile strength, increases the resistance to crushing, and is principally used as a matter of economy.

As cement mortars harden from internal causes and not like lime mortars from external causes, it is natural to assume that a rather fine sand would be better than a coarse-grained sand; and, moreover, cement will carry a larger proportion of fine sand, and it is therefore economical. In general terms it may be said that the size of the grains of sand does not materially affect the strength of the mortar.

Yet there seems to be a decided objection to the use of exceeding fine sands. Thé principal things to consider are cleanness, sharpness of grain, angular rather than rounded, and rough surfaces.

It is generally assumed that sand enters only as a mechanical mixture.

152. There seems to be great difference of opinion in regard to the effects of freezing on mortars. Some maintain that it seriously retards the setting and positively injures the mortar; others maintain with equal positiveness that even if it somewhat impedes the setting it does not injure it. Alternate freezing and thawing may be injurious, while a continuous frozen state may not. It is undoubtedly wise to suspend the building of masonry during extremely cold weather, but it is not always done. Many add salt in the form of a weak brine and continue the work; others continue without this, to say the least, doubtful practice. One quart of salt to the water required to mix a barrel of cement is ample.

It has been found that from $\frac{1}{8}$ to 2 per cent of coarse sugar dissolved in water and mixed with cement increases the strength.

153. The sand and mortar should be first thoroughly mixed in the dry state until the color of the mixture is uniform, and the water then added; otherwise there will not be an intimate mixture of the two. The practice on this point also differs; water, sand, and cement being often mixed together.

The essential result to be attained is that every grain of sand

should be entirely covered with paste. The mortar is either mixed by hand or by machinery; the latter is regarded as producing the better results. A large worm-screw eight or ten feet long, revolving in a trough but little larger than the screw, works well. The cement and sand being deposited at one end in the proper proportion, is revolved over and over and carried forward until well mixed; water is then allowed to mix with it in the proper proportions, which is turned over with it and carried forward. At the end of the trough it drops into barrows, buckets, or boxes. When mixed by hand the sand and cement are first mixed in a box with shovels, and the water added. The mixing is then completed with hoes and shovels.

The cost of English and German Portland cements vary from \$2.60 to \$3.50 per barrel, American Portland from \$2.30 to \$2.50 per barrel, Rosendales and other American hydraulic cements from \$1.10 to \$1.30 per barrel.

ART. XVI.

CONCRETE.

154. Concrete is one of the most useful, convenient, and economical materials used by engineers.

It is an artificial compound or stone, composed of lime or cement, water, sand, and some hard material, such as broken stone or brick, gravel, shingle, shells, slag, etc. The mortar formed, called the matrix, cements together the hard material called the aggregate.

The strength and other qualities of the concrete depend mainly upon the mortar, but are necessarily influenced to some extent by the hard materials.

As lime mortar is unfit to be used where great strength is required, or when placed in a damp situation, so also concrete made of lime mortar should never be used under like conditions. It is, however, often used for building the walls of houses above ground, and seems to make a good and a cheap wall. An ordinary proportion of ingredients is 1 quicklime, 2 sand, and 5 or 6 of broken stone or brick or gravel.

Concrete of Cement Mortar.—Only the best mortar that can be made of the available materials should be used in concrete, as the qualities of the mortar for this purpose are of greater importance than when it is used in stone or brick masonry. More depends on the character of the mortar, and only the best brands of hydraulic cement and, better still, of Portland cement should be used.

155. *Hard Materials.*—For the *aggregate* of concrete almost any hard material may be used. Where the weight of the concrete is unimportant, the lighter materials, such as broken brick, shells, the lighter and more porous stones, and breeze, may be used. Where weight is desirable, as in the air-chambers and cribs of caissons, breakwaters, and sea-walls, the more ponderous materials, such as the heavier stones, should be used.

All porous stones should be thoroughly wet, especially if a slow-setting cement is used; and all stones kept moist, especially in hot weather.

Angular broken stone is usually preferred to rounded pebbles or shingles, as the mortar takes better hold of the rougher stones, and, overlapping, they make a better bond. It may be stated generally that within a certain limit the smaller the broken stones or gravel the better—from the size of a walnut to that which will pass through a 2-inch ring or mesh. A mixture of broken stone, varying from 2½ inches to 1 inch, is favorable for packing, and if the open spaces can be filled with chips of stone and gravel from the size of a pea to a diameter of half an inch, less mortar will be required and a more solid and compact mass obtained. The stone can be broken by hand or by machinery. The Blake and the Gates crushers are both good machines. There is a difference of opinion and practice as regards screening the broken stone. It is doubtless best to remove by air-blasts, if practicable, the impalpable powder that always is formed when stone is crushed by machinery. Gravel and shingle should generally be screened to remove the larger-sized pebbles, dirt, and vegetable matter, and should be washed if they contain silt or loam. Slag from furnaces, if not too glassy, but somewhat porous, makes an excellent *aggregate.* If it contains lime, it may be injurious.

156. With the weaker cements and a good hard variety of stone, the greater the quantity of stone consistent with having all voids filled the better will be the concrete.

With the stronger cements and softer stones the reverse will give the better result. For economy, as much stone as the mortar will carry should be used. In any case there should be sufficient mortar to thoroughly fill all voids, and a small excess in addition. The amount of mortar can easily be obtained by filling a box containing a cubic yard of well-soaked stones, but not dripping with water, and then pouring a sufficient quantity of water to fill all voids. This volume of water increased somewhat will give the cubic feet of mortar required. Some authorities give as the result of numerous experiments the voids at from 20 to 30 per cent of the whole with mixtures of broken stone and gravel. The voids in broken stone of uniform size may be as much as 50 per cent. If the pieces vary in size the voids may be only 40 per cent or less. The following gives the results of some experiments:

TABLE XIII.

Stone	broken	to $2\frac{1}{2}$	inches,	voids	10	cubic	feet	in	one cubic	yard.
"	"	" 2	"	"	$10\frac{2}{3}$	"	"	"	"	"
"	"	" $1\frac{1}{2}$	"	"	$11\frac{1}{3}$	"	"	"	"	"
Shingle				"	9	"	"	"	"	"
Sand				"	6	"	"	"	"	"

A mixture of different sizes materially reduces the amount of voids, and therefore requires less mortar.

Below are given fair averages of proportions of ingredients, not including water, the amount of which varies so greatly with the opinion of engineers from the almost dry mixture to a very soft and plastic one, that it cannot be fixed by any rule:

TABLE XIV.

Hydraulic cement	1 part.
Sand	2 parts.
Broken stone	3 to 4 parts.
Portland cement	1 part.
Sand	2 to 3 parts.
Broken stone	4 to 7 "
Portland cement	1 part.
Sand	$2\frac{1}{2}$ parts.
Gravel	3 "
Broken stone	5 "

For one cubic yard of concrete, with sand, gravel, and broken stones, the following will be proportions:

Broken stone with 50 per cent of voids......1 cubic yard.
Gravel to fill voids in stones...............$\frac{1}{2}$ " "
Sand " " " " gravel......... .....$\frac{1}{4}$ " "
Cement " " " " sand.................$\frac{1}{8}$ " "

For one cubic yard of broken stone and sand:

Broken stone with 50 per cent of voids..1.00 cubic yard.
Sand to fill voids in stone............... .50 " "
Cement to fill voids in sand............. .25 " "

157. Having fixed upon the proportion of the ingredients according to the kind of cement, the sizes of the broken stone, the strength of the concrete, and the importance of the work, the proper mixing of the concrete is to be decided upon. The rules and the practice governing the mixing vary as widely as the proportion of the ingredients. It may be stated, in general, that if too much time is not consumed in the mixing, a good result can be obtained in any of the many ways practised, if only the mixing is thorough.

For hand-mixing, it can safely be said the best American practice is to first mix the sand and cement dry, and then by addition of the proper amount of water form the paste as described for mixing mortar. Next spread the mortar in a uniform layer, upon which the broken stone is laid evenly and uniformly; or, better, perhaps, first spread a thin layer of mortar, on this a thin layer of stone, then another layer of mortar on this and another layer of stone on top, the stone, of course, properly moistened. This mass is now turned over and over with shovels, first towards the centre, and then out from the centre towards the sides; or it may be heaped up from the sides and ends to an oval-shaped pile. As each shovelful is thrown up, a few cuts into the mass with the shovel almost vertical will not only prevent the stones from rolling down the sides, but will materially aid in the mixing; then the mass is turned outward from the centre, spread out into layers again, and the operation repeated. Two, three, or more turnings may be necessary. It should be turned until every stone is coated with

cement and the entire mass presents the uniform color of the mortar, and the mortar and stones are uniformly distributed.

158. The English method consists in first spreading all the materials in the dry state, and turning them over two or three times to thoroughly incorporate the ingredients. The dry mixture is then sprinkled, not drowned, the water being added gradually through a rose, only sufficient water being used to bring it to the proper consistency. If poured on too freely the cement will be washed away. The moist mass must then be turned over several times, as above described.

159. In machine-mixing either method may be adopted, and both produce good results. Only two of the machines will be described, both of which are common in this country.

One machine has a cubical-box about 4 feet on each edge, rotating on a diagonal axis passing through the box, into which a batch, consisting of $\frac{3}{4}$ barrel of cement, $1\frac{1}{2}$ barrels of sand, $2\frac{1}{4}$ barrels of pebbles, and 3 barrels of broken stone, to which was added in the case described about 10 gallons of water, is thrown. The mixer is then turned eight times, which seemed sufficient to thoroughly mix the batch. The concrete, when emptied from the box, appeared about as moist as moist brown sugar. No water was added in wet weather. Cylinders of various capacities, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$ or 1 cubic yard, turning similarly on a diagonal axis, have been used.

In another machine the sand and cement are allowed to drop from a hopper into a trough with a worm-screw turning in it, the mortar being mixed as described under the head of Mortar. This drops from the other end of this trough into an iron receptacle, slightly inclining downwards from the trough. The receptacle is about two thirds of a cylinder, in the axis of which is placed an iron axle with arms or ribs at intervals, so arranged that on turning the axle the tendency is to turn over and over and move forward any materials in the cylinder. If exact proportions are required, a few cubic feet of mortar could be allowed to drop into the mixer at the upper end, with which could be mixed the proper proportion of stone. This mixture could then be fed into the mixer at the rate proportioned to its capacity. More commonly, however, a barrel of cement, and 2 barrels of sand are fed through hoppers into the worm-screw trough, and the 4 or 5 barrels of stone are collected at the end of the mixer. Both the worm-screw

and the shaft of the mixer are run each at a special and different but uniform speed. As the mortar falls from the trough one man, or two men alternating, shovel the broken stone into the mixer at a uniform rate. The mortar and the stone are then turned over and carried forward, being thoroughly mixed when reaching the lower end of the mixer, from which it drops into barrows or boxes run under it, and is carried to the place where used. This, although not producing a perfectly uniform mixture, gives the results within a reasonable limit of uniformity, and with due care and skill will answer reasonable and practical requirements as regards the proportions of ingredients.

160. In whatever manner concrete may be mixed, it is usually deposited in layers varying in thickness from 6 to 12 inches. As each layer is spread it should be carefully rammed, and even with a small amount of moisture a thin skim of water will appear on the top; this indicates a sufficient degree of compactness. If too much water is present the top will be flooded with a half-inch or more of water mixed with pure cement, which will be forced up with the water, or a more or less semi-fluid paste will form on top. This will generally be accompanied with a sponginess and a springing or wave-like motion of the concrete mass. This indicates that the mortar is too wet; also, that much of the cement has been brought to the surface, thereby reducing the strength of the mortar; and, furthermore, that the stones have been driven in too great proportion to the bottom. This will always occur in soft concretes even when not excessively wet, if the ramming be too heavy and long continued. With mortar of the proper consistency, the author has found that a rammer, not too heavy to be handled by one man, will sufficiently compact the mortar. It is always best, when practicable, to cover a certain area with a second layer before the first has had time to set, as the layers will not unite or bond to each other as well after the under layer sets, especially if the semi-fluid cement has been forced on top, and formed an imperfectly setting and slimy skim, and in addition there is danger of loosening the coherency of the under layer. If a layer has set, it is well to scratch the surface if found smooth, and to spread over it a thin layer of mortar before depositing another layer, which should be at least 12 inches thick, and the ramming should be as light as is consistent with compactness. It is not always in prac-

tice possible to have everything in perfect condition, or to do what may be considered absolutely the best workmanship. But our aim should be to do as near the right thing as circumstances, guided by a good sound judgment, will permit. Frequently on top of a layer of concrete one or even two man-stones are placed, and over and around these the second or an upper layer is placed and rammed. If this is done, the stones should not be closer than from 12 to 18 inches. There is a little economy in this, but otherwise no advantage.

161. Depositing concrete under water always leaves the condition of the mass uncertain. The cement is likely to be separated from the sand, and both to a greater or less extent from the stone; and this is true whether it be poured down through a timber shoot or hollow cylinder, or lowered in specially designed boxes which only open when the bottom is reached. In the lowering water is likely to force its way into and upwards through the concrete, thus washing out the cement; and, in addition, when the doors are opened at the bottom the concrete falling into the water is separated, and again the cement is washed out. That all of the methods are used and structures built on them stand, does not disprove the above statement. The best method known to the author is to fill sacks from two thirds to three fourths full of concrete and lower these into the water, placing them side by side and in layers crossing each other, sending down divers if necessary for this purpose. If this cannot be done deposit them in the best way practicable, and then lower, as the layers are placed, mortar rich in cement in order to fill around, under, and over the bags. And in all cases or by whatever means the concrete is lowered, it should be allowed to take an initial set, losing to some extent its plasticity before being lowered through the water.

162. *Uses of Concrete.*—Concrete has been extensively used for the foundations of structures of all kinds, and to a great extent for the filling or backing in piers and over arches. It can be made easily and readily to conform to the inequalities of foundation-beds, avoiding unnecessary hammering and blasting. It furnishes the most satisfactory means of enlarging the bases of foundations, in order to reduce the unit-pressure on the foundation-beds. It affords a uniform surface upon which to build piers, abutments, walls of houses and other structures, and for lining cisterns, cellars, and reservoirs.

Entire piers and abutments may be built of this material with proper care and using the best cements. It is used in filling the air-chambers of pneumatic caissons and the cribs above. It can be made into slabs for paving. Drain and sewer pipes are made of it. It is used as a foundation for paving-blocks, etc., etc.

ART. XVII.

OTHER ARTIFICIAL STONES OF CEMENT MORTAR..

163. BÉTON AGGLOMÉRÉ, or Coignet-Béton, is an artificial sandstone, in which a large proportion of sand is mixed with Portland cement and lime; no gravel or broken stone is used. It is made by placing the cement with a small proportion—not more than one third of its volume—of water in a mill and mixing until a plastic paste is formed. Sand, lime, and cement in the proportions of 1 cement, 1 hydraulic lime, and 5 to 6 sand are used. The paste first formed is mixed with the perfectly dried sand in a mill until a pasty powder is formed; this is then placed in strong moulds in layers from 2 to 6 inches in thickness, and rammed with an iron rammer until the layers are reduced to about one third of their original thickness.

The upper surface is smoothed off with a trowel and then removed from the mould. Small blocks can be handled in one day; the larger ones require a longer time. If too much water is used, the mixture cannot be properly rammed; if too little, it will not be sufficiently strong. It is used for aqueducts, bridges and piers, sewers, cellars, etc. It is used to a considerable extent in Paris, France.

Ransome's patent stone is made by mixing artificially dried sand with silicate of soda and a small proportion of powdered stone. These are thoroughly mixed in a pug-mill, and then forced into moulds. The blocks are then turned out of the moulds, and a cold solution of chloride of calcium poured over them; they are then immersed in a boiling solution of the same, under pressure, in order to fill the pores with the solution. The chlorine and soda combine to form chloride of sodium, which is washed out, and the silica acting on the calcium forms silicate of lime, which is a strong and durable cement, holding the hard materials together. The

result is a stone as hard as most building-stones. The facility with which these and similar stones can be moulded renders them suitable for many purposes. They are considered by some as good as the ordinary limestones and sandstones for building. Other stones of the kind differ only in the method or process of forcing the calcium solution into the pores of the mass.

ASPHALTUM, ASPHALT MASTICS, AND CONCRETE.

164. Asphaltum is supposed to be a product resulting from the decomposition of vegetable and mineral substances. Bitumen, or mineral pitch, as it is sometimes called, was used in ancient times as a cement, and was obtained from the surface and shores of the Dead Sea. It is found in various degrees of consistency or firmness, and is of a black or brownish-black color; it melts a little below the temperature of boiling water, and burns with a smoky flame. Deposits of asphaltum are found in many parts of the world, especially in tropical regions, and in varying composition. Large and valuable deposits are found in the island of Trinidad, W. I. It there forms a lake of more than 100 acres in area and of unknown depth, and deposits are also found in the adjacent country, and in Cuba, Mexico, and Peru. It is found in the interstices of ancient rocks. In many places it is disseminated through limestones and sandstones. Below a temperature of 50° Fahr. it is solid and brittle; from 50° to 70° it is soft and plastic; from 70° to 90° it has a pasty condition; from 90° to 120° it is glutinous; and above 120° it is liquid. It is a compound of oxygen, hydrogen, and carbon. The larger proportion used in the United States is the Trinidad asphaltum. There it is mixed with vegetable and earthy matter. When liquefied over a slow fire the vegetable matter rises to the surface and is removed; the earthy matter sinks to the bottom, leaving a residue of about 52 per cent of bitumen.

Asphalt is a mixture of pure bitumen and silicious or calcareous substances, and may be either natural or artificial. The natural asphalt is either a sandstone or a limestone combined with bitumen, and is called and known as bituminous sandstones or bituminous limestones. The natural asphalt is ground, mixed with sand, to which more bitumen is added, and then run into moulds. This mixture is known as mastic.

Artificial asphalts are made by mixing natural bitumen with crude petroleum in the proportion of about 100 parts bitumen to from 5 to 20 parts of petroleum. This mixture is known as asphalt cement. Then from 12 to 15 parts of this is mixed while hot with from 60 to 83 parts of sand and from 5 to 30 parts of powdered carbonate of lime. This material is usually used for asphalt pavements in theUnited States.

The prices in New York of crude Trinidad asphaltum were, in 1889, $13.00 per ton; for the refined, $30.00 per ton; hard Cuban, $28.00 per ton; California bituminous rock at the mines, from $2.50 to $10.00 per ton; and the Kentucky, $2.40.

Good mastic should be proof against frost and damp; tough, not brittle, and not inflammable. It should stand a temperature of from 140° to 160° Fahr. without softening, and should not become fluid below 260° Fahr.

There are several kinds of asphalt: the Seyssel, found in the Jura Mountains—this contains from 90 to 92 per cent carbonate of lime and 8 to 10 per cent of bitumen; the Val de Travers, found in Switzerland, containing from 11 to 20 per cent of bitumen; and Trinidad asphalt; also, Limner asphalt, from Hanover.

Coal-tar pitch, the residue obtained by distilling coal-tar, is sometimes used instead of bitumen for mixing with asphalt. It is brittle, becomes softer when heated, and is easily crushed.

Patent asphalt or mastic is water-proof, fire-proof, easily applied, and slightly elastic. It is used as damp-proof courses of walls; for water-proof courses over arches and on flat roofs; for floors of cellars; for footpaths and streets; surface coats for carriage-ways or light traffic, and for filling the joints in pavements of stone, as it prevents the penetration of water. It is slippery in wet weather.

PLASTER AND STUCCO.

165. Plaster is the term commonly given to the covering of mortar on the interior walls and ceilings of houses.

Plastering is usually done in at least three courses, commonly known as the scratch-coat, brown-coat, and hard-finish, respectively.

The first or scratch coat is usually a pure lime mortar, containing a large proportion of sand, and mixed with a certain quantity

of ox-hair obtained from the tanneries. The hair should be long, free from grease or dirt, beaten so as to straighten and separate the hairs, and then dried. The proportions vary, but may be taken at from 1 lb. of hair to 2 cubic feet of the paste, and sometimes only 1 lb. of hair to 3 cubic feet. After being mixed carefully, so as not to break the hair into short bits, the mixture should stand several weeks to "cool," or thoroughly slake the lime. When properly mixed with sufficient hair, a stick or trowel dipped into the mass should lift out a perceptible quantity hanging in streaks from it. When ready, this is applied on the open-work lathing of walls and ceilings; the portion pressed through the openings between the laths swells behind, forming a key, which the hair holds to the surface coat, thus bonding the whole together. This coat is subsequently scratched with a pointed instrument, and when dry enough the second or brown-coat is put on, bonding and keying into the first; this coat does not need so much hair. Upon this the third or hard-finish coat is placed, which is a paste of pure lime carefully prepared, and run through a sieve to get rid of coarse particles. This has no hair in it. It is then mixed with plaster of Paris, in the proportion of three fourths to four fifths lime paste and one fourth to one fifth plaster of Paris. The plaster of Paris causes the mixture to set rapidly. An excess of this causes the coat to crack. It is used for finishing walls and for cornices.

There are many patent plasters used by architects. Plaster of Paris is produced by the gentle calcination of gypsum. The paste of this material sets in a few minutes, and reaches its full strength in a few hours. It expands in volume while setting, which makes it valuable for casts, filling small holes, and sharp angles; and it is added to other mortar in order to make it harden rapidly. It is used for making ornamental finishes for walls and ceilings by the use of suitable forms or moulds. It is very soluble in water, and should not therefore be used except in situations free from moisture and dampness.

There are several so-called artificial marbles, the principal ingredient of which is usually plaster of Paris, mixed with various coloring matters dissolved in glue or isinglass, or with fragments of alabaster or colored cement intermixed. This is used for walls, columns, etc., in order to give a finish resembling marble This is

called Scagliola marble. Another variety is the Marezzo marble. Neither of these is capable of standing exposure to the weather. There are other artificial marbles not using plaster of Paris as a base. These may or may not stand exposure.

STUCCO.

Stucco is the term applied to the mortar coverings on the exterior of walls, and is used to protect the materials of the wall from the usual causes of disintegration and prevent the rain from penetrating the joints in masonry walls; also to give a smooth finish, and to imitate stone. Portland cement and sand mortar are generally used, a clean white sand being preferred. Hydraulic cement is sometimes used. The proportions are usually 1 cement and 3 sand. A rough stucco contains a larger proportion of sand and of coarser grains.

WHITENING AND COLORING.

166. Whitewash is simply pure white lime mixed with water. It gives a clean, fresh, and white appearance to walls, ceilings, outhouses, and fences. For sanitary considerations it should be made of hot lime and applied promptly, as it then adheres better. It will not stand rain for any great length of time, and is easily rubbed off. The addition of 1 lb. of pure tallow to every bushel of lime has some advantage. For outside work, put half a bushel of lime in a water-tight barrel, pour boiling-hot water over it, covering it five or six inches deep, and stir rapidly till well slaked. Dissolve this in water, and add 2 lbs. sulphate of zinc and 1 lb. of common salt. This prevents cracking, and causes the wash to harden. To produce the following colors, add to each bushel of lime 4 to 6 lbs. of ochre for a cream-color; 6 to 8 lbs. amber, 2 lbs. Indian-red, 2 lbs. of lampblack, for fawn-color; 6 to 8 lbs. raw amber and 3 or 4 lbs. lampblack for buff or stone color.

Whiting is pure white chalk ground to powder and mixed with water and size (glue). It will not stand exposure to the weather. Proportions, 6 lbs. whiting to 1 quart of strong glue. The whiting is first covered with cold water for six hours, then mixed with the size and left in a cold place until it turns to a jelly. It can then be diluted with water and applied to the ceilings and walls.

ART. XVIII.

METALS.

167. THE metals are not, with few exceptions, found in the pure metallic state, but are obtained from the sulphides, oxides, or carbonates. These are known by the general term of Ores.

IRON.

The more common iron ore found in the British Isles is clay ironstone, a carbonate found in great abundance. It is very impure, containing clay, pyrites, and sulphur, often producing only from 20 to 50 per cent of iron, but is largely used on account of its abundance and its nearness to coal and limestone. When it is colored black by from 10 to 25 per cent of bituminous and carbonaceous matter it is called Blackband ironstone. It is easier to smelt on account of these impurities. The carbonate of iron when pure and crystalline is called Sparry or Spathose iron ore. The carbonates furnish the main reliance, though the red and brown hematites are also used.

In the United States the following are largely used:

Red iron ore, a peroxide of iron, either pure or mixed. When pure it is called Specular iron ore, or Iron-glance. When nearly pure and found in kidney or globular shaped masses with fibrous structure it is called Red Hematite. When mixed with clay and sand it is called Red Ironstone and Red Ochre. The purer ores produce from 50 to 70 per cent iron. Many of these are valuable for making Bessemer steel.

The Brown iron ore is hydrate of peroxide of iron. When compact and nearly pure it is called Brown Hematite; when earthy and mixed with clay, Yellow Ochre. It yields from 50 to 60 per cent of iron.

Magnetic iron ore is a very pure oxide, containing over 70 per cent of iron.

Native iron, supposed to have fallen from the heavens, contains from 80 to 100 per cent of iron. It is combined with from $\frac{1}{4}$ to $\frac{1}{100}$ part of its weight of nickel. It is very rare.

Such of these ores as contain much manganese are used in the manufacture of Spiegeleisen, a cast iron rich in carbon and manganese.

168. The extraction of iron from its ores is effected, by the process called smelting, in a large vertical furnace of masonry or iron lined with fire-brick. In some cases the iron ores are broken up by stamping or crushing, and washed, generally by machinery, in order to separate the lighter impurities or earthy matters adhering to them. They are sometimes heated to drive off the moisture and carbonic acid, leaving oxide of iron containing earthy and other impurities. The ore is then mixed with a substance called the flux. This is generally an earthy substance, which will combine more readily with the special impurities in the ore. If the "gangue" or impurities of ore are chiefly clay, limestone is added as the flux. If the gangue is chiefly quartz, a clayey ore and limestone are used with it. If the gangue is limestone, then clay or clayey ores are added.

The furnace should be partly filled with fuel, commonly coke. After the furnace is heated up by burning this, ore mixed with flux is thrown in from the top, and then alternate layers of fuel and ore with the flux are thrown in. It will not be necessary to explain the chemical combinations which take place. It will be sufficient to say that strong blasts of air are forced in near the bottom to support the combustion, and aid in the other processes which take place. Either a "cold blast" or "hot blast" may be used,—commonly the latter,—the temperature being from 800° to 1400° Fahr. The intense heat developed causes fusion of the substances. The molten metal sinks to the bottom, and over this is collected a glassy refuse, composed of the lighter and more fusible impurities, also containing a certain quantity of the iron. This is called the slag. The slag is drawn off, run into iron carts or cars, and carried to the waste or dumping grounds. At stated periods the molten metal is also drawn off, and run through trenches made in a previously prepared bed of sand. These trenches consist of one large main trench with secondary and somewhat smaller trenches at right angles to it; from these, on each side, branch out small trenches 3 or 4 inches deep, and 2 to 2½ feet long. These smaller trenches filled, the run from the furnaces is stopped, and the molten iron allowed to cool. The

cooled iron is easily broken into bars of the length of the shortest trenches,—2 to $2\frac{1}{2}$ feet long,—which are called "pigs" or "pig iron." These are piled up for shipment.

169. In this form the iron is sold to the manufacturer, and afterward subjected to various processes, which have for their object either to reduce or entirely remove the foreign substances or impurities, such as carbon, sulphur, phosphorus, silicon, calcium, or magnesium. The only one of these of any value in the iron is the carbon. All of the others are either useless or positively injurious, even in exceeding small proportions, except as hereafter noted. Sulphur, calcium, and magnesium produce "red-short" iron, or that which is brittle at high temperatures. Phosphorus and silicon make it "cold-short," or brittle at low temperatures.

The pig iron, when put through certain processes by the manufacturer, results in the production of three classes of iron, varying with the process and the quantity of carbon retained, and also with the relative proportion of the carbon mechanically mixed or chemically combined with the iron. These products are, respectively, cast iron, wrought iron, and steel. These, although they may not differ materially in the chemical composition of their constituents, and pass from one to the other by imperceptible shades of differences, yet within the limits of their respective classes they differ very widely in their general characteristics and mechanical properties.

PIG IRON.

170. Pig iron is classed under several heads.

Bessemer Pig.—This is made from the purer hematites. It should contain as little sulphur or phosphorus as possible. A small percentage of silicon and manganese is useful. It is required for conversion into steel by the Bessemer process.

Foundry Pig.—This should contain a large proportion of carbon mechanically mixed, and is produced with a high temperature and much fuel. The fracture is mottled and of a gray color, due to the graphite or free carbon.

Forge Pig.—This should only contain the chemically combined carbon. It is not suitable for castings, being mainly useful for conversion into wrought iron. The fracture is of a white color

with little lustre. It is produced under a low temperature and with little fuel.

It may here be stated that the properties of cast iron do not depend wholly upon the absolute quantity of carbon contained, but upon the condition in which the carbon is found, that is, whether mechanically mixed, as free carbon or graphite, or chemically combined.

The varieties containing mainly free carbon are of a dark-gray color, are soft, easily fusible, and run readily into moulds.

Those containing mainly chemically combined carbon are not mottled with black specks, are of a white color, hard and brittle, and do not fuse easily, but become pasty, and will not flow freely in moulds. Calling the first No. 1 and the second No. 6, the intermediate numbers 2, 3, 4, and 5 gradually pass by imperceptible degrees from the one to the other.

No. 1 contains from 3 to 5 per cent of carbon, has a dark-gray color and high metallic lustre, is fusible, and very valuable for foundry-work. No. 6 contains about 5 per cent of carbon—4 per cent in chemical combination; is white in color, hard and brittle, has little lustre, and is only used for the manufacture of inferior grades of wrought iron. Nos. 4 and 5 also are used only in the manufacture of wrought-iron for the same reasons as have been given for No. 6.

CAST IRON.

171. Cast iron is produced by remelting the foundry pig iron Nos. 1, 2, and 3, and running the molten metal into moulds. It is often purified by a second melting in a cupola with a little addition of limestone. There are several varieties of cast iron thus produced.

No. 1 cast iron fuses easily, runs freely, and expands lightly at the point of fusion or solidification, which adapts it for making delicate castings and those having sharp angles. But it is soft and wanting in strength, and is not therefore suitable for large and important members of structures.

No. 2 cast iron contains less free carbon, is lighter in color, more compact, hard without brittleness, strong, and durable, and consequently well adapted for beams, girders, and castings to bear weight.

No. 3 contains still less free carbon, is harder, somewhat more brittle, and is employed in heavy castings.

White cast iron contains little or no free carbon, is very hard and brittle, and only fit for the most ordinary castings, such as weights for window-sashes.

It can be converted into gray iron by melting followed by slow cooling, and gray cast iron can be converted into granular white cast iron by melting and sudden cooling. White cast iron resists the oxidizing or rusting better, and is not so soluble in acids.

Where it is advisable to form a skin or exterior surface which shall be very hard, and resist the destructive effects of exposure, as in beams, girders, and the tires of wheels, this may be effected by running the molten metal into moulds lined with cold iron coated with loam. The outside is chilled suddenly and converted into hard, brittle iron, whereas the remainder of the casting will be of strong and tough gray iron.

Malleable cast iron is applied to a cast iron from which a portion of the carbon has been extracted. This may be done by imbedding the casting in hematite ore and heating to a bright-red heat. The casting may be affected entirely through or only for a short distance from the surface. It is somewhat malleable. It is used for pokers, tongs, buckles, gun-locks, etc.

Toughened cast iron is produced by melting cast iron with one fourth to one seventh of its weight of wrought iron scrap.

172. The engineer is not so much concerned with processes of manufacture and methods of moulding and casting as he is with results. And while he should know the general properties of the materials with which he has to deal, he should also know whether the results are what were required and expected.

It is far better to specify what properties the material should possess, and the degree of perfectness in form, solidity, and freedom from special defects, than to prescribe modes and methods of producing these results. And however instructive and interesting the methods of producing the many and varied castings of all kinds, it is more important to be able to determine their good and bad features.

To examine castings: If the iron is of good quality a blow on the edges with a light hammer should make a slight indentation or impression. If hard and brittle, fragments will be broken off. Air-

bubbles may be formed if the air and gases evolved when the hot metal is poured in the mould cannot escape; and pipes and columns should be cast in vertical positions so as to let air-bubbles rise to the top; moreover, the casting will be more compact.

Flaws and air-holes can generally be detected by the sound when tapped at different parts of its surface.

The exterior surface should be smooth and uniform, angles sharp and well defined.

Cast iron, though strong, hard, and durable, is often injuriously affected by changes of temperature, vibrations, and shocks or blows. It is difficult to detect the hidden defects, and altogether it is an uncertain and treacherous material. For these reasons it is rarely if ever used for members of important structures, especially if exposed to vibrations and shocks under changing temperatures, but is largely used for pipes, columns, beams, girders, etc., which carry steady and permanent loads. Its use for compression-members of iron bridges is almost entirely abandoned; and even in details, where its indestructible qualities combined with its ability to carry heavy weights would render it valuable, its use is almost prohibited.

WROUGHT IRON.

173. While cast iron is rich in carbon, as it contains from 2 to 6 per cent, wrought iron, properly speaking, should be pure iron, and at most should not contain more than 0.15 per cent of carbon.

Wrought iron may be produced direct from the ore, but is commonly obtained from forge pig or the harder varieties of pig iron. The object to be attained is the removal of the carbon, phosphorus, silicon, and other impurities. The conversion is made by the puddling process. The iron is sometimes put through the refining process, which means exposing the iron in a melted state, and subjecting it to currents of air. The oxygen removes a part of the carbon and converts the silicon into silica, which forms with a portion of the iron a fusible slag, which flows away. Commonly it is placed directly in a reverberatory-furnace and puddled. Puddling simply means bringing the melted iron into close contact with the air by stirring with a rake. The molten metal is at the same

time mixed with oxidizing substances, such as hematite ore, sometimes with limestone and common salt. The remainder of carbon and silicon are oxidized and pass off in the slag. As the iron becomes purer, it also becomes less fusible, and stiffens, when it can be rolled into balls, or loups, or blooms. In this condition it is removed, and then "shingled," that is, subjected to blows of a tilt-hammer, which force out the cinder and weld the particles of iron together. The red-hot mass is passed between grooved rollers, which convert it into bars 3 or 4 inches wide, $\frac{3}{4}$ to 1 inch thick, and 10 or 12 feet long. The puddled bars constitute the lowest grades of iron, and have to be improved by piling, reheating, and rolling. The effect of rolling is to produce a fibrous structure; but to produce the best iron the bars have to be cut into short lengths, piled, reheated, and rolled several times. Each piling and rolling improves the quality of the iron. It will not, however, stand too many rollings. The fifth reheating seems to be the limit.

Cold-short iron, due to the presence of phosphorus, is brittle when cold, and will crack if bent double. It can be worked at high temperatures. Red-short iron, due to the presence of sulphur, will crack when bent at a red heat, but has considerable tenacity when cold. It cannot be welded. It is tough when cold, and is used for tin-plate. Copper, arsenic, and other impurities also cause red-shortness.

There are no very simple tests of the quality of wrought iron. If a bar is broken suddenly, it will have a crystalline appearance on the fractured surface; if gradually, the appearance will be fibrous. Good wrought iron should always be fibrous. The breaking-test is therefore no criterion.

By varying the shape and the application of the force, pieces cut off the same bar will present different appearances. Again, if the pieces be too small or too thin they may not show fully the characteristics of the larger bar from which they are taken.

If the bending-test is applied, the angle through which the same quality of iron should bend without cracking will depend upon the thickness and mode of bending.

Therefore, since wrought iron is so extensively used, in many important structures and in very large dimensions, and upon the strength and uniformity of these so much depends, no crude or

imperfect tests should be relied upon. After explaining the subject of stresses and strains and their relations, the proper tests to be made will be given in another article of this work. It will be sufficient here to say that good wrought iron should have a clear gray color, metallic lustre, and fibrous appearance. It should have a high tensile strength, be tough and ductile, and uniform throughout, and when cold should bend through many degrees without cracking. But to determine these qualities requires careful and accurate means of applying the tests and estimating how near the results conform to the requirements of the best engineering practice.

Wrought iron can be rolled into a great variety of shapes, but it is generally better, where complicated forms are required, to build them of plates, and so with the other simpler forms. These matters will be discussed in another article.

STEEL.

174. Steel, as we have seen, is intermediate in chemical composition between cast iron and wrought iron. Cast iron contains from 2 to 6 per cent of carbon, steel from 0.15 to 2 per cent, and wrought iron from zero to 0.15 per cent.

Any such classification, however, is vague and unsatisfactory. A classification based upon tensile strength and ductility has been proposed. For example, any iron giving a tensile strength over 40,000 pounds per square inch shall be called steel, and any falling below this limit is termed wrought iron; and, again, to include under "the general denomination of cast steel all compounds consisting chiefly of iron which have been produced through fusion and are malleable." This does not exclude from the denomination of steel such materials, though not produced by fusion, but which are capable of tempering, such as shear, blister, and puddled steels; nor does it interfere with the distinctions between cast steel produced by different methods, as pot steel, Bessemer steel, or open-hearth steel; and, finally, "iron containing a small percentage of carbon, the alloy having the property of taking a temper," is steel.

The material known as steel depends on its composition, the

methods or processes of production, and also upon certain distinctive qualities and characteristics.

If the iron does not contain more than from 0.15 to 0.45 per cent of carbon, it is called *soft steel;* if from 0.45 to 0.55 per cent, an *intermediate* or *mild steel;* and if from 0.55 to 1.50 per cent, *hard steel.* When the carbon exceeds 1.50 per cent the steel becomes harder, but loses its tenacity and welding properties; and when it reaches 2.0 per cent the metal cannot be drawn out under the hammer without cracking and breaking. It is or has passed into a cast iron.

Within the above limits, the larger the proportion of carbon the harder, stronger, more brittle, and more easily melted will the steel become. The less carbon contained, the tougher, more easily welded and forged will it be, but weaker in tenacity.

175. In general terms, steel is produced either by adding carbon to wrought iron, or by partly removing carbon from pig iron, the product containing the proper proportion of carbon. Steel is made by many processes, of which the following varieties are the most important:

Blister steel is made by the process called cementation, in which bars of the purest wrought iron are imbedded between layers of charcoal-powder, and subjected to a high temperature during a period of one to two weeks, depending on the quality of steel required. It cannot be used for ordinary forging nor for making cutting-tools, but may be used for facing-hammers and other similar purposes. It principal use, however, is for making other varieties of steel.

Shear-steel is made by taking the bars of blister-steel,—which is in fact only steel for a certain depth below the surface, and a material between steel and wrought iron in the central portions,—and breaking them into short lengths; these are then piled into bundles, sprinkled with sand and borax, and while at a welding-heat are rolled or hammered until the composition and texture are found to be uniform. The hammering and heating should be repeated. The first produces *single-shear steel;* the second, *double-shear steel.*

It is used for such tools as large knives, scythes, etc., requiring toughness but no great hardness. It is capable of being welded, is

more malleable and tougher, and will take a higher polish and a finer edge than blister-steel.

Cast steel may be produced by melting fragments of blister-steel in covered fire-clay crucibles, and running the molten metal into iron moulds. Crucible-steel is now largely produced direct from bars of the best wrought iron made from the purer magnetic ores. The bars are cut into lengths and placed in crucibles, together with the required proportion of carbon to produce the desired grade of steel. Spiegeleisen or oxide of manganese is subsequently added.

Cast steel is the strongest, purest, and most uniform steel that is made. It is hard and compact If raised beyond a red heat it becomes brittle. It cannot be welded. It is used for the finest cutlery, and for all steel cutting implements when great hardness is required.

Heath's process consists in adding to the molten metal a small quantity of carburet of manganese. The resulting product has a greater tenacity at high temperatures, and it becomes capable of being welded to other portions of the same material or to wrought iron. Other modifications of this process are Heaton's and Mushet's.

176. *The Bessemer Process.*—In this well-known process pig iron of a dark-gray color, containing a large proportion of carbon, with but small percentage of silicon and manganese and practically no sulphur and phosphorus, is melted in a cupola, or carried direct from the blast-furnace and run into a *converter,* which is a pear-shaped iron vessel lined with fire-brick. While in the converter a strong blast of air is forced through the molten metal for a period of about twenty minutes. The color of the flame indicates to the experienced eye when all of the carbon is removed, or more accurately determined by means of a spectroscope. Then from 5 to 10 per cent of spiegeleisen is added. The air-blast is again started in order to thoroughly incorporate the two metals. The steel is then run into ladles and thence into the moulds. The ingots thus obtained are not as compact as required, but are rendered so by proper hammering. They are then rolled into the desired sizes and shapes for use. This steel is used for the members of roof and bridge trusses, for boiler-plate, rollers for the ends of bridge trusses

to rest upon, called *expansion-rollers;* for rails, tires for wheels of railway-cars, and for some forms of cutlery, hatchets, hammers, etc.

The Thomas-Gilchrist process, known as the *basic method,* is similar to the preceding. The converters are, however, lined with a magnesian limestone or some refractory substance which contains practically no silica. Lime having been added, the air-blast is turned on. In this process not only the silicon and carbon are removed, but also the phosphorus. Ores containing even large proportions of phosphorus can be used in the manufacture of steel.

In the Siemens Process pig iron and ore are fused on the open hearth of a regenerative gas-furnace. The pig iron is first melted and raised to a temperature which will melt steel; rich and pure ore and limestone are added gradually. The chemical reactions convert the silicon into silicic acid, which forms a fusible slag with the lime, and the carbon passes off as carbonic acid. A modification of this process consists in treating the iron ore in a rotary furnace with carbonaceous matter, by which both phosphorus and sulphur are removed, thereby producing a high grade of steel.

The Siemens-Martin Process.—In this process a bath of highly heated pig iron is prepared in a furnace, and three or four times its weight of scrap iron or steel are added and dissolved in the bath, with enough ore to reduce the carbon to about 0.1 per cent. The furnace then contains a fluid malleable iron, to which is added silicious iron, spiegeleisen, or ferro-manganese in such proportions as experiment and experience indicate are necessary to produce a steel of the requisite hardness. The metal is then run or forced into the ingot-moulds.

It is used in ship-building, for rails, tires, boilers, bridges, and roofs.

There are many other processes and corresponding grades of steel produced.

The general term *mild steel* may be taken as applying to all iron containing from 0.2 to 0.5 per cent of carbon, When a larger percentage of carbon is present it is called *hard steel.*

Mild steel is more uniform in texture than wrought iron, and stronger and superior in most respects. It will weld; is used for rails, spades, hammers, etc.; and when made by the Bessemer and Siemens-Martin processes is used for boiler-plates and ship-building.

CHARACTERISTICS OF STEEL.

177. *Hardening Steel.*—If steel when at a red heat be suddenly cooled it becomes very hard, and the more suddenly it is cooled the harder it will be. It may be plunged in water or oil. But it is made very brittle at the same time, and in order to give it the toughness required for most purposes it must be ***tempered.***

Tempering Steel.—If, after hardening, steel be reheated, the hardness diminishes as the heat increases. If then it is desired to have specified toughness, and hardness without brittleness, it is gradually reheated, and, when it reaches a temperature which experiment and experience have shown will produce the desired degree of hardness, it is cooled suddenly. It also changes color as it is heated, owing to the formation of a thin film of oxide, which is indicative of the temperature attained. A simple and good example of the application of these processes is the following:

If a mason desires to temper his tools, he heats the point or cutting-edge to a bright-red heat, and then hardens it by dipping the point into cold water. Having cleaned off the scale thus formed, he allows the point to be reheated by conduction from the main body of the tool, which was not cooled, and when the temperature of the point or cutting-edge has reached the proper point as indicated by the color, he suddenly plunges the entire body into cold water, and moves it about until the heat has been entirely removed.

If the tool after heating is allowed to cool in the air more or less slowly, a softer degree of temper is secured.

In hardening, the metal must not be overheated before plunging into the cooling substance. A low-red heat will in general be sufficient. Cast steel, if overheated, becomes permanently brittle or is otherwise injured.

When the entire mass of any article or bar is to be tempered, it may be either cooled suddenly or allowed to cool in the air; but when only a portion of it is to be tempered by the conduction of heat from another portion of the body the method previously described must be followed.

When toughness and elasticity rather than hardness are desired, oil is used for cooling both in hardening and tempering.

The temperature of the cooling liquid may be regulated to suit the purpose desired to be accomplished. Cooling in oil can be better regulated than in water, as the oil will not cool so rapidly.

Annealing is a term applied when the hardened steel is raised to a red heat and then allowed to cool gradually, by which process the steel regains its original softness.

Case-hardening is applied to a process by which the exterior portions of wrought iron may be converted, to a depth of from $\frac{1}{16}$ to $\frac{3}{8}$ inch, into steel, the iron combining the toughness of the interior portion with the hardness and resistance to wear of the steel coating. Gun-locks, keys, and similar articles requiring toughness combined with a hard surface, are case-hardened.

Steel can be distinguished from wrought iron by placing a drop of dilute nitric acid upon it. If a dark-gray stain is produced it is steel.

178. There are no simple and reliable tests of the useful and important properties of steel, and, as in the case of wrought iron, we must subject it to certain careful and specified tests to determine its suitableness for any proposed purpose. This will be explained in another article.

The fracture of steel does not necessarily indicate its quality. If the steel is low, or if some of the high grades are thoroughly annealed, the fracture is fine and silky, provided it has been produced gradually. In other cases the fractured surface may be partly granular and partly silky, or entirely granular. In any case a sudden fracture may produce a granular appearance.

ART. XIX.

COPPER.

179. THE peculiar red color of copper is well known. The metal is very malleable, and can be hammered or rolled into thin sheets and drawn out into wire.

Copper has less tenacity than wrought iron, but is superior to all other metals in this respect. It is not as ductile and cannot be drawn into as fine wires as wrought iron. It cannot be welded. It may be worked either cold or hot. It oxidizes slowly in air, be-

coming covered with a thin film of the carbonate, called *verdigris.* This protects it from further oxidation. It is corroded by salt water when it is exposed to the air.

It is used for slate nails, water-pipes and gutters, roofs, lightning-rods, wires for various purposes, and especially, in very large quantities, for the conducting wires in electric lighting and transmission of power, from $\frac{1}{16}$ to $\frac{1}{4}$, and even $\frac{1}{2}$ inch in diameter, usually insulated with a rubber or some other insulating cover.

Ores.—The more common ores are the gray and red, copper pyrites, and copper-glance. The metal is obtained from these by roasting, calcining, and refining, and melting them with suitable fluxes. The forms used are sheet copper, designated by the thickness, and in the form of wire, described as the BWG or Birmingham gauge, or the SWG or standard wire-gauge. Sheet copper is designated by the weight per superficial foot.

LEAD.

180. Lead is soft, heavy, malleable, and fusible, and is wanting in elasticity and strength. Sheet lead is either cast or rolled, the latter in thinner sheets than the former. When exposed to air or water a thin film of oxide is formed. This protects it from further oxidation, unless some acid or other substances dissolve the oxide. It is used for pipes, tanks, roofs, for holding iron cramps into masonry, and for distributing the pressure between arch-stones, as cushions when testing stones by crushing, and for distributing heavy weights on their supporting beds. White lead, the basis of paints, is prepared from it.

There is much difference of opinion in regard to the use of lead pipes for carrying water for drinking purposes. It is claimed that only the purest waters, or those containing only organic matter or sewage-polluted waters, act upon and are poisoned by lead. The pure or soft waters seems to require the presence of much air in order to act on the lead and become poisoned, and lead pipes should not be used for carrying rain-water, or for roofs when the water-supply is collected from them.

The hard waters, or those containing carbonate of lime and also phosphate of lime, do not act upon the pipes, sometimes even forming a coating which protects it.

Ores.—Lead is obtained principally from the sulphide called galena, by roasting or smelting in a reverberatory-furnace.

ZINC.

181. Zinc is easily acted upon by moist air. The oxide formed protects it from further oxidation. If the air contains an acid, as in large towns and near the sea-coast, it is destroyed. It is easily fusible. Cast zinc is brittle. It becomes malleable at about 220° Fahr., and can be rolled into sheets which remain malleable. At a red heat it takes fire and blazes up. It should not be used in contact with iron, copper, or lead, as voltaic action is set up, especially when moisture is present, destroying the zinc; nor should it be in contact with lime or water containing lime, or such timber as oak, which contains an acid. It expands and contracts under changes of temperature to a greater extent than any other metal. This must be allowed for in zinc roofs.

Zinc is used for coating iron, which is then called galvanized iron; also for light gutters and pipes, and for roofs and ventilators. The oxide is used as a basis for zinc paints.

Sheet zinc of good quality is tough, bends without cracking, and should have a uniform color.

Ores.—The metal is obtained from the carbonate, the sulphide, and the red oxide. The ore is roasted, mixed with charcoal, and heated in retorts. It is converted into vapor, which is condensed and subsequently fused.

TIN.

182. Tin is very soft and malleable, is of low strength and ductility, resists oxidation better than any of the metals except gold and silver, and fuses at a low temperature.

It combines readily with iron. Tin-plate is prepared by immersing well-charred sheets of iron in melted tin. The iron is coated with an alloy of iron and tin, passing into pure tin at its outer surface, and is used for roof-coverings, rain-pipes, and the common house utensils. Such plates are durable until a hole is made in the covering, when galvanic action sets up between the tin and the iron. The tin is then rapidly eaten away. Block-tin

or doubles has a thicker coating of tin. It is used for the best tin-ware. Pure block-tin is seldom used for building purposes. Crystallized tin-plate is made by heating ordinary tin plate with hydrochloric and nitric acids, which give it a variegated appearance. Tinned copper is also used for kitchen utensils. The metal is obtained from the binoxide and tin-pyrites ores, roasted, washed, mixed with flux, and smelted in a reverberatory-furnace. The liquid is run into a basin, and thence into moulds. The ingots are refined and boiled.

ALLOYS.

183. Alloys are produced by melting two or more metals together. They seem to be something more than mechanical mixtures, often being harder than either of the metals, and possessing different properties.

Speculum metal, for example, formed of two parts of copper with one of tin, is very hard and brittle. Copper and tin are both malleable.

Brass is an alloy composed of copper and zinc. It is tough, fusible, but weaker than copper.

Bronze is an alloy composed of copper and tin. It is harder than copper. The hardness increases with the proportion of tin up to a certain point. Gun and bell metal are also alloys of copper and tin, and are harder, stronger, and more fusible than copper. The proportions of the elements vary.

Pewter is an alloy of tin and lead. It is used for cups, spoons, and other similar purposes.

Solder is a name given to a number of different alloys, and is principally used for making joints between sheets or bars of metals. The solder forms a second alloy with the pieces united. It must be more fusible than the metals to be joined. Hard solders are those that only fuse at a red heat. Soft solders fuse at a low temperature. The hardness and malleableness should be nearly equal to that of the metals to be united.

Hard solder, made of copper and zinc, is used to make joints in iron, copper, brass, etc. Silver solder is composed of silver and copper, and is used for the finer and neater joints in the above-named materials.

Soft solders are composed of tin and lead, and are used for joints in lead, thin sheets of tinned iron, zinc, copper, and other metals. Boxax and several other fluxes are used for hard soldering, and for soft soldering several substances, depending on the metals to be united, commonly resin.

The tensile strength of an alloy is generally greater than that of either of the metals.

If the metals have different specific gravities, they must be stirred while melted to produce a homogeneous compound.

The more infusible of the metals should be melted first, and the others then added.

The specific gravity of an alloy is seldom equal to the average of that of the metal composing it.

ART. XX.

PAINTS AND VARNISHES.

184. *Paints.*—These consist of a *base,* usually a metallic oxide; a *vehicle;* and, lastly, a *solvent.*

Bases: White lead, red lead, zinc white, oxide of iron.

Vehicles: Water, oil, spirits of turpentine.

Solvents: Spirits of turpentine.

In most cases a *drier* is used in order to make the vehicle dry more rapidly; and if the finished color desired is different from that of the base, *coloring pigments* are used. These are commonly called *stainers.*

Driers are red lead, litharge, acetate of lead, sulphate of zinc, binoxide of manganese, etc.

Stainers are ochres, lampblack, umber, sienna, indigo, Prussian blue, yellow ochre, chrome yellow, red lead, carmine, vermilion, Indian-red, Venetian-red, lake, and orange. The green stainers are made by mixing yellow and blue, but these are not as durable as those obtained from copper, arsenic, etc.

White lead is the most useful and common base used. It is permanent, has good body, and is dense. It is especially useful for painting surfaces of wood. It should be pure. There are many impurities and adulterations found in it. Sulphate of baryta

is the more common adulterant. It absorbs but little oil, and can often be detected by the gritty feeling when rubbed between the fingers. By treating the dry substance (after removing the oil) with nitric acid and boiling, the white lead dissolves, the baryta is insoluble, and remains, it can then be washed, dried, and weighed.

White lead can be used as the base of paints in all colors. It is improved by keeping. It is sold either dry in powder or ground into a paste with from 7 to 9 per cent of linseed-oil. Fresh white lead has a yellowish tinge. White lead is a carbonate. When heated on a glass plate it turns yellow. It is unhealthy to those handling it. It blackens on exposure to sulphur acids. It is probably the best of the paints for covering surfaces of wood.

Red lead is an oxide of lead known as minium. When ground by itself in oil or varnish it is durable, and retains its color unless it contains preparations of lead or metallic salts. Brick-dust is often an adulterant. This can be detected by heating the red lead and treating it with nitric acid. The lead will be dissolved, the powdered brick will not. Sesquioxide of iron is also used as an adulterant. Red lead is used for painting iron, as the priming-coat for wood, and as a drier.

Sulphide of antimony is used as a substitute for lead. It is durable and permanent in color, and can be mixed with white lead.

185. Oxide of zinc is the basis of the zinc paints. It is wanting in body and density; it does not combine so freely with oil, and consequently has not the same covering power. It is more difficult to work, and dissolves in hydrochloric acid, but does not blacken when exposed to sulphur acids, nor is it injurious to the workmen. It weathers badly, as carbonic acid dissolves the oxide. It is also acted upon by the acids contained in unseasoned wood. When exposed to sulphurous vapors it should not be mixed with driers containing lead. Sulphate of manganese or zinc should then be used as driers. It is durable in oil and water.

186. Oxide of iron is used as the basis of many important and useful paints, which are free from injurious constituents such as are found in lead paints. They are especially recommended for painting ironwork. If the iron is rusty the rust is absorbed by the paint. Their covering power is great; for equal weights they cover a greater surface than the lead paints.

They must be made from the sesquioxide or red oxide of iron. One pound of the oxide paint mixed with two thirds oxide and one third linseed-oil should cover 21 square yards of sheet iron. There are many varieties of these paints.

Torbay paint contains from 50 to 65 per cent of oxide of iron combined with 35 to 50 per cent of silicious matter; brownish, red, and black are the usual colors. It is especially useful in painting ironwork, is durable, weathering well, and is not injured by fumes from manufactories. There are many other varieties of silicious paints which are considered valuable. They will cover nearly double the surface the same weight of white or red lead paints.

Bituminous paints made from vegetable bitumen, asphalt, and mineral pitch, dissolved in paraffin, petroleum or naphtha, and various oils, are well adapted for painting the inside of pipes, and for ironwork, such as cylinders and screw-piles, under water. The finer varieties can be used for general purposes, especially to resist the action of water and foul vapors.

Tar Paint.—Coal-tar thickened with lime is often used to paint joints between timbers.

The canvas roof over the tubes of the Britannia Bridge was painted with a compound containing 9 gallons of coal-tar, 13 lbs. slaked lime, and 2 or 3 quarts of turpentine or naphtha, and dredged over with sand.

VEHICLES.

187. The oils are generally classed under two heads—Fixed Oils and Volatile Oils.

Fixed oils are obtained by pressure from vegetable substances. They are of a fatty nature, will bear a temperature not exceeding 500° Fahr. without decomposing, and do not evaporate on drying.

Of these drying-oils those which become thick upon exposure to air are alone used as an ingredient in paint, and of these linseed oil is the most commonly used. Nut and poppy oils are sometimes employed.

Volatile or essential oils are usually obtained by distillation. They have an odor characteristic of the plant from which they are obtained. They are colorless at first, but become darker, thicker, and form a variety of resin after exposure to air and light. Of

these the oil or spirits of turpentine is the only variety used for paint. Petroleum-oil and naphtha are used as vehicles or solvents.

Linseed-oil, obtained by compressing flaxseed, is the best of the oils used in paint, putty, etc. It oxidizes and thickens upon exposure to the air. It is superior in body, tenacity, and drying power to the other fixed oils.

Raw Linseed-oil.—The oil first obtained is allowed to settle until it can be drawn off clear. It may be further clarified by the addition of an acid, which is subsequently washed out. The oil should be clear and light in color, almost perfectly transparent, with little odor, and sweet to the taste. Darkness in color and slowness in drying indicate an inferior quality. These defects are diminished and the quality of the oil is improved by age. It should never be used in a less period of time than six months after being produced. The raw oil being thinner and lighter in color is well adapted to fine work.

Both the drying and the color can be improved by adding a certain proportion of white lead and allowing it to settle. The lead can be removed and used.

Raw oil, when placed on a non-absorbent body, will require two, three, or more days to dry. It is used for grinding up colors, and for interior work.

Boiled Linseed-oil, owing to the comparative rapidity with which it dries, is the properly called Drying-oil. Placed on a non-absorbent body it dries in from twelve to twenty-four hours. It can be boiled alone, but usually other substances are added, which cause it to dry more quickly, as red lead and litharge. A little umber gives a darker color. Boiled oil is thicker and darker. It is not suited for grinding colors. It is commonly required for outside work, as it has more body and dries more quickly.

188. *Spirits of Turpentine* is obtained by the distillation of the turpentine obtained by tapping or boxing, usually called bleeding, the pine-trees. The residue remaining is called resin. It is largely obtained from the yellow-pine trees of the Southern States. On exposure to air it oxidizes and becomes resinous. It is used in paints to make them work more smoothly; will not stand exposure to the weather; should dry in twenty-four hours, forming a hard, dry varnish; and is also used as a solvent for resin and other materials in the preparation of varnishes.

The driers are substances containing a large proportion of oxygen, the more common of which have already been mentioned. They should not be used in excess.

PROPORTIONS OF INGREDIENTS.

189. The proportions of the materials used in preparing paints vary greatly. They depend upon the material to be painted, being different for wood and iron. The kind of surface, whether porous or not, the porous requiring more oil, and the degree of exposure to which the paint is to be subjected.

If the surface is subsequently to be varnished, the paint must contain a minimum of oil. Raw oil is used for inside, boiled oil for outside, work. If the work is exposed to the sun, turpentine is necessary to prevent blistering. The proportions also depend upon the quality of the materials used. More oil and turpentine will combine with pure than with impure white lead. And the different coats of paint vary in composition; the first coat on new work requires more oil. Turpentine is necessary to cause adherence to old work.

VARNISH.

190. Varnish is made by dissolving resin in oil, or in turpentine or alcohol. In the one case the oil dries, and in the other the turpentine or alcohol evaporates, leaving in either case a film of resin over the surface, smooth, solid, and transparent. The quality of the varnish is determined by the amount of gloss, and its permanence, durability on exposure to the weather, toughness and hardness of the coating, and rapidity of drying.

Oil varnishes made from the hardest gums, such as amber, gum animé, and copal, dissolved in oil, are the hardest and most durable, but dry more slowly. They are suitable for work exposed to the weather, but require frequent cleaning and polishing. They are used for carriages, japan-work, joining, and house-fittings, and always for outside work.

Turpentine varnishes are made by dissolving the softer gums, such as mastic, dammar, and common resin, in the best turpentine. They dry more rapidly, are lighter in color, but not not so tough and durable. They are less costly. The still softer gums

dissolved in spirits of wine dry more quickly, are harder and more glossy than the turpentine varnishes, but are apt to crack and scale off, and will not stand exposure.

ART. XXI.

STRUCTURES.

DEFINITIONS.

191. ALL fixed structures consist, in the most general sense, of parts of solid materials either resting against each other or connected along certain surfaces called joints, and so arranged as to resist any tendency to alter the forms or arrangements of its parts. In a more restricted sense only such solid materials are included as have already been mentioned in the preceding pages—iron, wood, stone, brick, concrete, etc. In this volume the term will be used in its more general sense, so as to include earthwork and similar constructions.

If the parts of a structure are movable relatively to each other, or relatively to each other and at the same time movable as a whole as regards their positions on the surface of the earth, the structure is called a machine, engine, locomotive, etc. These will only be explained or discussed so far as may be necessary to a clearer understanding of their relations to and effects upon fixed structures.

The parts of a fixed structure should touch or be connected with each other, so that they may act as a unit in transmitting their own weights, as well as any weight or load resting upon them, either directly or indirectly, to the solid materials of the earth, such as rock, clay, sand, gravel, or any combinations of two or more of these materials, the whole of these being so constructed and arranged as to preserve the form, arrangement, and position in which they are put together or otherwise prepared.

192. In this sense a structure may be divided into three principal parts, each of which is composed of smaller parts joined or connected together.

1st. The solid materials of the earth, called the foundation-beds,

which support the total weight or load, consisting of the weights of the substructure and superstructure, as well as loads or weights resting upon them.

2d. The substructures, which may consist of any of the materials used in structures, such as earth, wood, brick, concrete, stone, iron, etc. These rest directly on the foundation-beds, and the parts are so arranged or connected as to transmit their own weights and any weights or loads upon them to the foundation-beds.

3d. Superstructures, which may be defined as structures whose parts are so designed, arranged, and connected as to transmit their own weights and any loads upon them to one or more supports, these supports constituting the substructures, which transmit the weights and loads to the foundation-beds.

The supporting resistances or pressures are (1) the resistances of the solid materials of the earth to being crushed, torn apart, or to any displacement of any kind; (2) the resistances of the parts composing the substructure to crushing, tearing, or displacement of its parts by the weights or loads resting upon them; and (3) the resistances of the parts of the superstructure to crushing, tearing, or displacement of any kind.

Structures may be rendered unfit for the purposes for which they were constructed by alterations in form, bending, twisting, yielding, etc., without actually crushing, tearing apart, or being displaced from their proper positions.

These loads, weights, and resistances, or supporting pressures, are called the external forces, and must balance each other, as the structures, either in respect to the parts or as a whole, are assumed to be fixed.

DESIGNING STRUCTURES.

193. In the designing of any structure it is necessary to know the nature and relations of the forces acting upon the structure as a whole or on any of its parts, and the properties of the materials with which it is to be constructed, in order that the form, dimensions, and arrangements of its parts may be properly determined and adjusted so as to perform safely the duties imposed upon them with the least labor and cost consistent with a due regard to strength, stability, permanence, durability, and suitableness for the purposes in view.

Certain principles, rules, and conditions are necessary to be understood and considered, which are common to all forms of fixed structures, of whatever material constructed.

The conditions of equilibrium of a structure are:

1st. That all of the external forces acting upon a structure as a whole shall balance each other. These consist of the weight of the structure, any load that may be placed upon it, and the supporting pressure or resistances of its foundations.

2d. That the forces exerted on each piece of a structure shall balance each other. These consist of the weight of the piece itself, any other load upon it, and the resistances at the joints between it and other pieces in contact with it.

3d. That the forces exerted on each part of a piece into which it may be divided shall balance each other. These consist of the weight of the part itself and of any external force applied, and the internal force or stress exerted between it and the other parts of the piece.

194. Stability requires the fulfilment of the first two conditions under allowable variations of the load.

195. Strength requires the fulfilment of the third condition within the limits of the prescribed load, which must be such that no injury to the piece shall occur, such as stretching or shortening, twisting, or any other alteration in the form or dimensions of the piece beyond certain prescribed limits.

The load or other external forces produce strains or alterations in the form or dimensions of the piece, which develop internal forces or stresses that resist such strain or alteration. For every kind of strain produced there is developed a corresponding stress—compressive stress with compressive strain, tensile stress with tensile strain, shearing-stress with shearing-strain. Strain and stress vary in the same proportion, and the ratio of stress to strain is constant for a given material within certain limits.

196. If the direction of the external force is parallel to the axis of the piece the strain is either one of compression or extension: the piece will be shortened or lengthened, and if the piece breaks it will do so by crushing or tearing. If the direction of the external force is normal or inclined to the axis of the piece, the strain will be one of distortion, twisting, or bending; and if the piece breaks it will do so by shearing, wrenching, or breaking across,

respectively. In either case, however, the ultimate condition of the piece is one of compression or extension of its particles or fibres, and its internal stress resisting these tendencies is either compressive or tensile, or both, one in one part and the other in the other part. The surface dividing the two is called the neutral surface, which passes through the centres of gravity of the cross-sections of the piece. The ultimate strength is measured by the load that will break the piece in some specified way; the proof-strength, by the greatest load that does not so far alter the form or dimensions of the piece as to impair its strength; and the working strength, by that load which for reasons of safety is the greatest to which the piece should be subjected repeatedly or continuously. The ultimate strength or load divided by the working load gives what is called a factor of safety. This varies between wide limits, from 2 to 20 or more, with the degree of safety required, but is on an average from 3 to 5 for a steady or dead load—such as the weight of the structure, and from 4 to 10 for a live or moving load—such as a rapidly moving train.

197. Coefficients, or Moduli of Strength, are simply quantities which express the intensity of the stress developed when a piece of a given material gives way when strained in any specified manner. There are as many coefficients of strength as there are ways of breaking a solid body. They vary with the nature of the material, and also with the relative direction of the applied force and that of the grain or fibres of the piece.

198. Elasticity is that property of bodies by which they resist any strain or alteration of form, commonly called stiffness, together with the power to recover more or less perfectly its original form. No body is perfectly elastic, and recovery is not perfect. Coefficients of elasticity vary both with the kind of material and the kind of stress, and simply express the ratio of stress to strain. The coefficient of elasticity is usually indicated by the letter E, and if p is the intensity of stress producing a certain strain α, then $E = \frac{p}{\alpha}$ is the coefficient of elasticity. α is the increase in length per unit of length for tension, the decrease per unit of length for compression, and the special alteration in form for shearing, torsion, or bending. If, however, the coefficient of elasticity is to be found by experiment upon a bar of any given

cross-section and length, let A = area of cross-section ; L = original length of the bar; P = total stress or tension uniformly distributed over the area of cross-section; l = total elongation caused by the pull P. Since the intensity of a stress is equal to the total stress divided by the area $= \frac{P}{A}$ and the elongation per unit of length $= \frac{l}{L}$, and as it is assumed that the relation between the intensity of the stress and strain is constant, then $\frac{P}{A} \div \frac{l}{L} = E$, the constant called the coefficient of elasticity; and since $\frac{P}{A} = p$ and $\frac{l}{L} = \alpha$, we have, as above,

$$E = \frac{p}{\alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Or, solving with respect to P, $P = EA\frac{l}{L}$, if in this equation, A = unit of area, and $l = L$; $\therefore \frac{l}{L}$ 1, substituting, we have $P = E$, which has led to the following definition of the coefficient of elasticity, viz., the coefficient of elasticity is that force which, applied to a bar whose area of cross-section is unity (one square inch or one square foot), will produce an elongation equal to the original length of the bar, assuming it to be perfectly elastic up to that limit.

Equation 55 is true only for certain limits of stress, beyond which it is not true, E being assumed constant for the same material and same kind of stress. This limit is called the limit of elasticity, or the elastic limit. This limit is not usually well-defined. All coefficients are determined directly from experiments or deduced from some law devised by experiment.

ART. XXII.

GENERAL PRINCIPLE OF THE BALANCE OR EQUILIBRIUM OF FORCES.

199. It will not be necessary to consider the balance of forces in space, whether inclined or parallel to the co-ordinate planes. And all forces considered in this article, whether inclined or parallel, will be assumed to have their lines of action in the same plane.

Force Defined.—A force is defined as any action between two bodies that causes or tends to cause a change in their relative conditions of rest or motion. If two or more forces so act on a body that no change in its condition of rest or motion is produced, these forces are said to be in equilibrium, or to balance each other.

A force can be represented by a line—the length of the line representing the magnitude of the force to any desired scale, the direction of the line indicating the direction of action of the force, and one end of the line, or some point in its prolongation, being the point of application of the force.

A force is completely known or determined when we know: 1st, its point of application; 2d, its direction of action; and 3d, its magnitude in pounds or tons, or some other unit of force. Forces are usually distributed over a surface of greater or less extent, and if treated as a single force it is supposed to act or be concentrated at a point called the centre of pressure or resistance. The single force is supposed to have the same effect as that caused by the combined action of the distributed force, and is called the RESULTANT. For the purposes of this volume all the forces acting on a body are supposed to be in the same plane. The following are the conditions of equilibrium for any system of forces in the same plane.

200. If any number of forces, acting in the same plane and at the same or different points of a rigid body, are in equilibrium, the algebraic sum of their components in any given direction is equal to zero, as commonly stated.

1st. The algebraic sum of the horizontal components is equal to zero;

2d. The algebraic sum of the vertical components is equal to zero; and

3d. The algebraic sum of the moments of the forces, taken with respect to any axis perpendicular to the plane of the forces, is equal to zero.

The only two cases which we have now to consider will be: 1st. When the lines of action of the several forces are parallel and generally vertical, as the forces are the weights of the structure and the loads upon it. 2d. When the lines of action of the forces are inclined to each other. In this latter case the forces will generally consist of the weights, loads, and reactions of the supports and the stresses developed in the members of the body meeting at one

point. If more than three stresses act at one point the determination of these stresses is impossible unless we can determine one of them independently.

The general principle or condition of the balance or equilibrium of any number of inclined forces acting in the same plane and through the same point, is that if lines be drawn parallel in direction and proportional in magnitude to the forces themselves, these lines will form a closed polygon. If they do not, the forces are not in equilibrium, and the line necessary to close the polygon will represent *the direction and magnitude of the force required to produce equilibrium, if it acts in the direction of the other forces continuously in one direction around the polygon; but if it acts in the opposite direction it will be the resultant of the other forces, and will replace the others, or have the same effect on the body as that of the combined action of the other forces.*

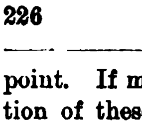


FIG. 44.

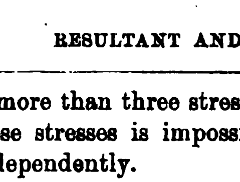


FIG. 45.

If the system of inclined forces, Fig. 44, acting on a body, and represented in magnitude and direction by the lines 1, 2, 3, 4, and 5, and lines be drawn as in Fig. 45, parallel to the direction of the forces in Fig 44, and their lengths proportional to the magnitude of the forces; that is, if forces 1 = 5 tons, 2 = 6 tons, 3 = 6 tons, 4 = 7 tons, 5 = $5\frac{1}{2}$ tons, then Fig. 45, 1 = $\frac{1}{2}$ in., 2 = 0.6 in., 3 = 0.6 in., 4 = 0.7 in., 5 = 0.55 in., the scale used being 10 tons to the inch. As a gap exists between *a* and *b* the system of forces is not in equilibrium, and to produce a balance a force represented by *E* in magnitude and direction must be introduced into the system. If *ab* is 0.4 in. the magnitude of the force must be 4 tons and applied as shown at *E*, Fig. 44, and must act in the direction as shown by the arrow-head from *a* to *b*. If its direction of action is reversed so as to act from *b* to *a*, or in the direction shown by *R*, Fig. 44, it becomes the resultant of the system, and is the single force that could replace all of the others. The balancing force *E* and the resultant *R* are equal in magnitude, but opposite in direction, and in the same line of action. It is immaterial in what order the forces are taken in constructing the polygon Fig. 45. The shape of the polygon is of no moment. The lines may cross each other. The only requirement is that the closing line *ab* shall be drawn

from the end of the last line to the beginning of the first line. Fig. 45 may be called the force polygon.

The important and usual case of this general problem is where there are only three inclined forces, and the force polygon becomes a triangle, commonly a right-angle triangle, the same construction and same remarks apply. If the three forces considered are Nos. 3, 4, and 5, Fig. 44, then the force polygons will be as shown in Figs. 46, 47. These polygons only differ in the direction of action of the force No. 4. Fig. 46 shows the direction for a resultant or replacing force, and Fig. 47 the direction for equilibrium. It is evident that either force may be considered as the resultant or the balancing force, the other forces being called the components. The more usual case is as shown in Figs. 44a and 47a, Fig. 44a showing the three forces in equilibrium at a panel point in a frame or truss, and Fig. 47a the force polygon for a condition of equilibrium. It is evident from Figs. 45, 47, and 47a, as shown in

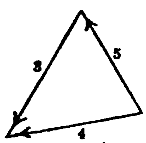


FIG. 46.

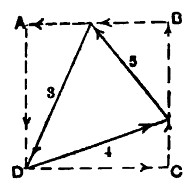


FIG. 47.

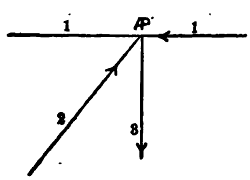


FIG. 44a.

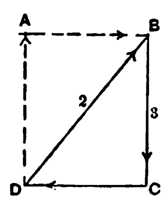


FIG. 47a.

the rectangles $ABCD$ formed by dotted lines enclosing the force or stress polygon, that the first two conditions of equilibrium are fulfilled, viz., algebraic sum of vertical components $= 0$, and algebraic sum of horizontal components $= 0$.

201. The above may be considered as a graphical solution of the problem, as the force polygon is drawn to an exact scale of so many pounds or tons to the inch. The analytical solution is based upon the same principles as above enunciated, and in addition to knowing the magnitude of one of the forces we must know the angles between the direction of the forces. The same relations then exist between the magnitudes of the forces as exist between the lengths

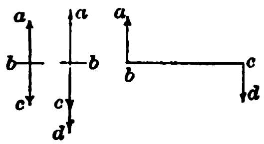
of the sides of the polygon or triangles. Knowing one force, the others can easily be determined by the formulæ for the solution of plane triangles. The only relations that are necessary in most cases are, first, that the sides are proportional to the sines of the opposite angles, or, in case of a right-angle triangle, either side about the right angle is equal to the other side multiplied by the tangent of the angle opposite the first side, or that the hypothenuse is equal to either side about the right angle multiplied by the secant of the included angle.

ANY SYSTEM OF PARALLEL FORCES IN A PLANE.

202. The same conditions of equilibrium are applicable, but the first two conditions are equivalent to only one, viz., that the algebraic sum of the parallel forces shall be equal to zero. The third condition is in the same as the preceding paragraph, viz., that the algebraic sum of the moments of the forces with respect to any axis perpendicular to the plane of the forces shall be equal to zero.

The resultant of a system of parallel forces acting in the same direction is a single force acting in the same direction, and equal in magnitude to that of all the forces added together. If there are two sets of parallel forces acting in opposite directions, the resultant will be either a single force or a couple. 1st. If the resultant of each set acts through a common point, and is of the same magnitude, the system is completely balanced, and the final resultant is zero. 2d. If acting through the same point, and unequal, the final resultant will be a single force equal to the difference between partial resultants and acting in the direction of the greater of the two, and the tendency would be to move the body in the direction of the resultant. 3d. If the partial resultants are equal in magnitude, but do not act through a common point or in the same line of action, *the resultant of the system is a couple, which may be defined as two equal and opposite parallel forces not acting on a body in the same line of action.* The tendency is to turn the body about some axis perpendicular to the plane of the couple. 4th. If the resultants of the two sets of parallel forces are unequal but not acting through the same point, the resultant will be a single force, equal to the difference of the two and acting

parallel to and in the direction of the greater, but on the opposite side of the greater from the smaller of the two.



FIGS. 48, 49, 50.

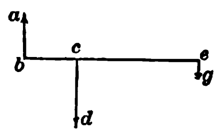


FIG. 51.

The four cases described are illustrated in Figs. 48, 49, 50, 51. In Fig. 48 the partial resultants are equal and in the same line of action. The system is completely balanced.

In Fig. 49 the partial resultants are unequal and in the same line of action; the final resultant is a single force $cd = bd - ab$. In Fig. 50 the partial resultants ab and cd are equal, opposite, but not in the same line of action. The final resultant is a couple. In Fig. 51 the partial resultants are unequal and not in the same line of action; the resultant is a single force (eg) acting beyond the greater and in the same direction, and at a distance from the greater such that $cd : ab : eg :: be : ce : bc$. If equilibrium in Fig. 51 is required, then the direction of the force eg must be changed so as to act upwards instead of downwards. This condition is shown in Fig. 52, and shows that the least number of parallel forces that can balance each other is three, leaving out two equal and directly opposed forces, as shown in Fig. 48. This fact, known as the Balance of Three Parallel Forces, called also the Principle of the Lever, taken together with the principles explained in paragraph 200, and illustrated in Figs. 44a, 46, 47, 47a, is a fundamental principle underlying the equilibrium of bridge and roof trusses. The principle of the lever is as follows: *If three parallel forces in the same plane are in equilibrium, the extreme forces must act in one direction. The intermediate force must act in the opposite direction, and must equal the sum of the extreme forces, and each force must be proportional to distance between the other two.* These conditions are expressed by the following relations and equations:

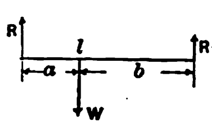


FIG. 52.

$$W = R + R_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

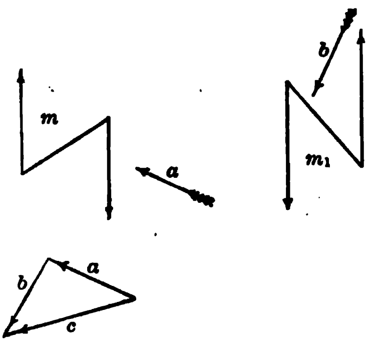
$W : R : R_1 : : l : b : a$, which expresses the last condition; from which we obtain

$$R_1 = \frac{Wa}{l} \text{ and } R = \frac{Wb}{l}; \qquad W = \frac{R_1 l}{a} = \frac{Rl}{b}. \quad . \quad . \quad . \quad . \quad (57)$$

From these equations, when any one of the forces with the distances between it and the other two are known, the other forces can be found.

203. These equations are continually used, and should be thoroughly understood. The proof of the above relations depends upon the theory of couples. Referring to Fig. 50, par 202, the moment of the couple is *ab* (or *cd*) multiplied by *bc*; or, in words, the moment of a force is the product of the force by its perpendicular distance from the force to any axis perpendicular to the plane containing the force. The force will always belong to a couple, and the perpendicular distance called the lever-arm will always be the distance between the two forces of the couple. The equilibrium of couples is in every way similar to that of single forces. When parallel couples have the same moment, and tend to turn the body in the same direction, the resultant moment is the sum of two. If they tend to turn the body in opposite directions and are equal, they balance each other. If unequal, the resultant couple is one whose moment is the difference between the moments of the other two, and tends to tnrn the body in the direction of the greater. If the turning is to the right, in the direction of motion of the hands of a watch, it is called a right-handed couple; if in the opposite direction, a left-handed couple. Applying these principles to the principle of the lever (see Fig. 52), the force W may be supposed to be divided into two forces, one of them $= R$ and the other $= R_1$, both acting at the same point, thus forming two couples—the one whose moment is Ra, and the other whose moment is $R_1 b$; the one right-handed, the other left-handed. For equilibrium, $Ra = R_1 b$, or $R : R_1 : : b : a$, which corresponds to the conditions of equilibrium. The tendency of a couple to turn a body is completely known, when the moment and direction of the couple are known, and the position of the axis is known, the moment of a couple can be represented by a line in magnitude and direction by drawing a line propor-

tional to the magnitude of the moment and perpendicular to the plane of the couple, so that on looking along the line towards the couple it shall appear right-handed. If, then, two such lines be drawn to represent two couples, the third line forming a triangle will represent the resultant couple, or the closing side of any polygon, if there are more than two couples. The direction that is given to the closing line will determine whether it is a resultant couple, or a couple necessary to produce equilibrium, as in the case of inclined forces. If the lines a and b represent the two couples m and m_1 respectively in amount, direction, and position of the axes, and are so drawn that on looking along those lines the couples appear right-handed, then will c, Fig. 53, represent the resultant moment; or, if the direction of c is reversed, it will represent the couple necessary to produce equilibrium.



FIGS. 53.

204. A force polygon can also be formed, if the forces are all parallel, by taking a set of three parallel forces, a and b acting downward and c upward. Now laying off vertically downward a line to represent the force a, and in continuation the line to represent the force b, then turning upward a line to represent the force c, we find a gap from B to A represented by the dotted line. The resultant force is represented by AB acting downward. The force necessary for equilibrium would be the force BA acting upward. The above principles are all that are necessary to determine the stresses upon any framed structure, simple or complicated, if the external forces or loads are known. These in any particular case are given or assumed. As the external forces are generally parallel, it is necessary to have some simple mode of determining what is called the *centre of parallel forces, being the point through which the final resultant passes;* and as the forces are generally weights, they will be vertical, and the centre of parallel forces becomes the centre of gravity of all the loads. The principle is identical in

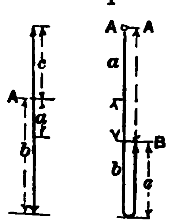


FIG. 54.

either case. We will find the centre of gravity, or more properly the position of the line of action of the resultant of any system of loads or weights, such as those on the wheels of a locomotive and tender, the weights and distances of which, apart, are known. This applies to any other kind of isolated or, as they are called, concentrated loads, and, while establishing the general principle involved, will familiarize us with the nature, amount, and distribution of the loads actually used in determining the stresses on and the dimension of the parts of beams and trusses. The type of locomotive is among the heaviest now in use, and is about the standard required in the usual specifications. Each pair of tender-wheels carries 15,000 lbs., also the front truck. Each of the four pairs of drivers carries 24,000 lbs. Only half of these loads is on each rail and borne by each of the two bridge trusses. The distances apart of the wheels from centre to centre are shown on the diagram Fig. 55. As the position of the axis is arbitrary,

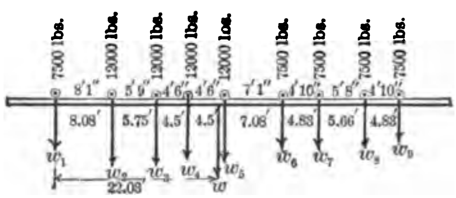


FIG. 55.

we can take it anywhere, assuming it to pass through the centre of w_1, the front wheel of the locomotive. The simple principle is that the moment of the resultant weight must be equal to the sum of the component weights taken with respect to the same axis. The lever-arm of w_1 is 0. Calling W the sum of the loads or the resultant, and its unknown lever-arm x_0, we have

$$Wx_0 = w_1 \times 0 + w_2 \times 8.08 + w_3 \times 13.83 + w_4 \times 18.33 + w_5 \times 22.83 + w_6 \times 29.91 + w_7 \times 34.74 + w_8 \times 40.40 + w_9 \times 45.23.$$

Or, since $w_1 = w_6 = w_7 = w_8 = w_9 = 7500$ lbs., $w_2 = w_3 = w_4 = w_5 = 12{,}000$ lbs., and $W = 85{,}500$, we have $x_0 = 22.03$ ft. to the right of w_1, which places the resultant 0.8 ft. to the left of w_5, and similarly for any other system of weights.

205. The principles of the balance and equilibrium of forces, considered in the abstract, that is, without reference to its relations to or effects upon the material objects to which they may have been applied, have been fully discussed in this article. These simple, elementary, and fundamental principles are essential to an understanding of the theory and practice of engineering construction, and should be thoroughly understood. In the following articles forces will be considered in their relations to the parts of structures considered separately, or to a structure taken as a whole. In the following discussion the materials of which the structures are composed, and the forms in which the materials are aggregated in the pieces considered, do not enter into the discussion; in other words, the strength of the parts or pieces is not considered. The questions of equilibrium and stability alone is considered. (See paragraphs 193 and 194.) Stability of a structure consists in the fulfilment of the first and second conditions of equilibrium of a structure under all variations of load within given limits. A structure which is deficient in stability gives way by the displacement of its pieces from their proper positions.

ART. XXII (A).

GENERAL PRINCIPLES OF THE EQUILIBRIUM AND STABILITY OF FRAMES.

206. In this article the forces, loads, and resistances will be considered as single forces concentrated at points.

As these conditions alone affect the question of stability. The mode of distribution of the intensity of the loads and resistances, which affects the strength and stiffness of a member or piece of a structure, will not be considered at present.

It will only be necessary to know the direction, magnitude, and point of application of the resultant load. The point of application is called the centre of pressure.

Similarly, it will only be necessary to know the resultant of the resistance or stress developed between two pieces of a structure at the joint or surface of contact between them, and treat this resultant as a single force, of which the direction, magnitude, and point

of application are known. The point of application of the resultant is called the centre of resistance for that joint.

A line of resistance is a straight, broken, or curved line passing through the centres of resistance of the several joints in a structure. The direction of this line at any joint does not necessarily coincide with the direction of the resultant resistance at that joint.

207. There are three kinds of joints. 1st. Those in which the two pieces are so connected by glue, cement, bolts, rivets, or other rigid connections, that the relative angular position of the pieces are fixed with respect to each other, and the joint is capable of resisting both a thrust and a pull. In this case the centre of resistance of the joint may be at any point of surface of contact; or there may be no centre of resistance, the resultant being a couple and not a single force. Under the latter condition the two pieces act as one piece, and the resistance is a question of strength, and not of stability.

2d. Those in which the two pieces are joined along surfaces of considerable area as compared with the dimensions of the parts connected, as the joints in masonry or between masonry and the materials of the earth. These joints are capable of resisting a more or less oblique pressure or thrust, but in which the resistance to a pull or tension is small in amount, and not considered in practice. In such joints the centre of resistance may be varied within certain limits from the centre of the joint.

3d. Those in which the pieces are bars, links, ropes, or iron or timber beams or struts, which, while connected at their ends, offer but little resistance to change in the relative angular position of the pieces. The centre of resistance in such joints admits of no deviation from their centre.

208. A Frame, considered from the condition of stability alone, is a structure composed of rods, bars, ropes, or links, connected and supported by joints of the third class, free to change their relative angular positions. The pieces may be all struts, all ties, or there may be both struts and ties in the same frame.

A Tie is a piece or member in a state of tension, and the stress developed at any section that may be conceived as dividing it into two parts is called a pull or tensile stress, and the strain is one of elongation, extension, or stretch. Fig. 56 represents a single bar or tie, acted upon by a force or load P, the direction of which

coincides with the axis of the bar, and acts outward with respect to it. This force is resisted or balanced by an equal and opposite force or resistance at the other end or joint. The points of application of these forces A and B are the respective centres of resistance, and the line AB is the line of resistance. As equilibrium is assumed, P and R must be equal, opposite, and have the same line of action. A bar or rod in this condition is called a *tie*. The equilibrium of a tie is stable, for if the forces P and R are unchanged in direction, while the angular position of the tie is changed, the equal forces P and R constitute a couple, tending to bring the bar back to its original position, as shown by the dotted lines.

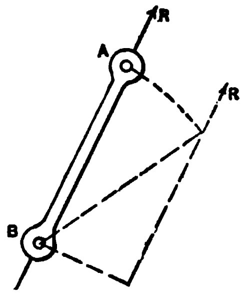


FIG. 56.

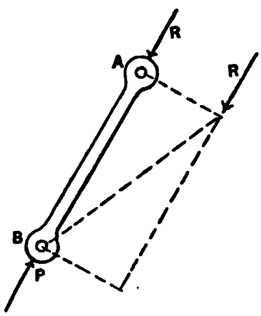


FIG. 57.

Struts.—If the equal forces or resistances P and R are directed inwards or towards each other, the bar or member is called a *strut*. It is in a state of compression, and the stress developed at any ideal section is called a compressive stress, or a thrust, and the strain is one of shortening, or contraction. The equilibrium of a strut is unstable, for if the angular position of the piece is changed the originally directly opposed forces constitute a couple tending to remove it still farther from its original position. A flexible body, such as a rope, chain, or small rods and bars, will not be suitable for a strut. The dotted lines show the direction of action of the couple.

The weights of the pieces themselves are never considered when they are very small.

The balance of a load supported by two parallel forces or reactions has been discussed under the head of the Principle of the Lever.

The balance of a load and two inclined forces has been discussed under the head of a Triangle of Forces. This is also illustrated by the inclined bar or beam, Figs. 58 and 59. Fig. 58 shows the position and direction and relative magnitudes of the load W and the supporting resistances R and R_1. The conditions of equi-

librium are: 1st, that the lines of action of the forces must be in one plane; 2d, that they must intersect in one point; and, 3d, that the three forces must be proportional to sides and diagonal of a

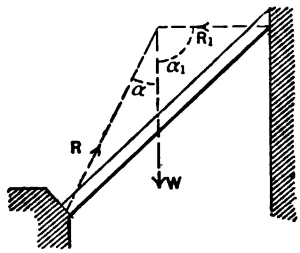


FIG. 58.

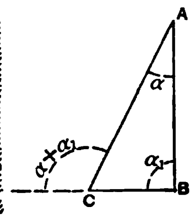


FIG. 59.

parallelogram, or to the sides of a triangle, Fig. 59, respectively parallel to their directions.

$$W : R : R_1 :: AB : AC : CB :: \sin (\alpha + \alpha_1) : \sin \alpha_1 : \sin \alpha.* \quad (58)$$

209. If four parallel forces balance each other, required the relations that must exist between the magnitudes of the forces. Let W be a weight acting vertically downward at D, and supported by three props or struts at A, B, and C, the supporting forces or resistances R_1, R_2, and R_3 acting vertically upwards at these points, respectively. (See Fig. 60.) If lines be drawn from the vertices of a triangle formed by connecting the points of application of the three forces acting upward to the point D, these points being supposed to be in the same plane (either inclined or horizontal) with the point of application of the load W at D, four triangles will be formed, namely, ABC, ADB, ADC, and BDC. And each force will be proportional to that triangle whose vertices are in the lines of action of the other three forces; i.e.,

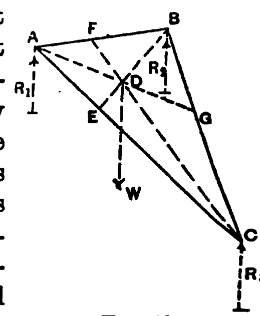


FIG. 60.

$$W : R_1 : R_2 : R_3 :: ABC : BDC : ADC : ADB.$$

In Fig. 60, extending the lines first drawn, AD, BD, and CD, until they intersect the sides BC, AC, and AB at G, E, and F,

* See Rankine's Civil Engineering, p. 35.

respectively, the resultant of R_1 and R_2 will intersect the line AB at some point; and since this resultant must be in equilibrium with the remaining two forces W and R_3 and parallel to them, all three must be in the same plane containing the line FDC, this plane being vertical in this case because the forces are vertical, otherwise it would be a plane parallel to the forces. Hence the resultant of R_1 and R_2 will pass through the point F, and be equal to $R_1 + R_2$ by the principle of the lever. Then also, by the same principle,

$$W : R_3 : R_1 + R_2 :: CF : FD : DC. \quad (\text{A}) \quad . \quad . \quad (59)$$

The same reasoning applied to the resultants of R_2 and R_3 at G, and of R_1 and R_3 at E, enables us to write the following proportions:

$$W : R_1 : R_2 + R_3 :: AG : DG : AD; \quad (\text{B}) \quad . \quad . \quad (60)$$
$$W : R_2 : R_1 + R_3 :: EB : ED : DB. \quad (\text{C}) \quad . \quad . \quad (61)$$

Since triangles having the same bases are to each other as their altitudes, we can write

$$\left.\begin{array}{ll} \triangle ABC : \triangle ADB :: CF : FD; & (\text{A}) \\ \triangle ABC : \triangle ADC :: EB : ED; & (\text{C}) \\ \triangle ABC : \triangle BDC :: AG : DG. & (\text{B}) \end{array}\right\} \quad . \quad . \quad . \quad (61\tfrac{1}{2})$$

These lines, CF, FD, etc., are not the altitudes, but they are proportional to them.

Combining the two sets of equations in their order,

$$\begin{array}{ll} W : R_3 :: \triangle ABC : \triangle ADB; & (\text{A and A}) \\ W : R_2 :: \triangle ABC : \triangle ADC; & (\text{C and C}) \\ W : R_1 :: \triangle ABC : \triangle BDC. & (\text{B and B}) \end{array}$$

Since W and $\triangle ABC$ are common, we have

$$W : R_1 : R_2 : R_3 :: \triangle ABC : \triangle BDC : \triangle ADC : \triangle ADB. \quad (62)$$

Knowing the relative positions of the forces and the distances between them, we can readily find the areas of the triangles, since in each we know the three sides; and also knowing W, we can find the values of R_1, R_2, and R_3.

If the load is supported by three inclined supports, the lines of action of the three supporting forces must intersect that of the

load in one point, and the magnitudes of the three supporting forces are represented by the three edges of a parallelopipedon whose diagonal represents the load. (See Rankine's Civil Engineering, p. 37.)

210. A frame of two bars may consist of two ties, two struts, or a strut and a tie joined or connected at one point, the other end of each fastened or fixed. Fig. 61 represents the frame of two ties

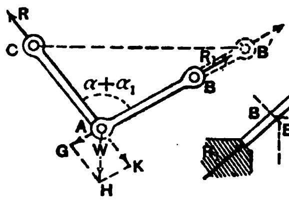


FIG. 61.

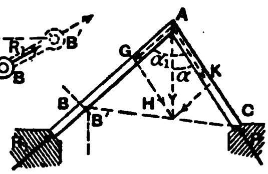


FIG. 62.

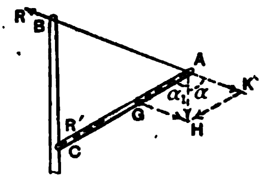


FIG. 63.

jointed at the ends A and fixed at their upper ends B and C, respectively. Fig. 62 is the condition of two struts jointed at their upper extremities A and fixed at their lower extremities B and C, and Fig. 63 is the case of a strut and a tie jointed at one end and fixed at the other. These conditions exist provided the applied forces are acting downward.

If acting upwards, the frames should be revolved through 180° in Figs. 61 and 62 and the pieces should change position in Fig. 63, unless the pieces in each frame are capable of resisting both a tensile and compressive stress. In Fig. 61 the pieces may be composed of bars, links, wires, ropes, or any flexible bodies.

In Fig. 62 both pieces must possess more or less stiffness, such as posts or columns of timber or iron.

In Fig. 63 the piece AC is a strut or column of timber or iron, and the piece AB may be flexible—commonly a rope either of manilla or hemp, and often a wire rope. The forces applied are usually loads acting vertically downward, and in either of the three cases the stresses on each piece are found by decomposing the applied force into components acting in the direction of the axis of the pieces, according to the principles of the parallelogram or triangle of forces, as shown in $AGHK$ or AHK, respectively, and in either case the relations between load and supporting forces are

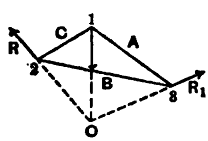
$$W : R_1 : R : : AH : HK : AK : : \sin(\alpha + \alpha_1) : \sin\alpha : \sin\alpha_1. \quad (63)$$

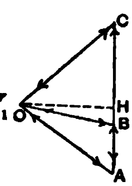
These principles and relations hold true whether the two adjacent pieces of the frame compose the entire structure or are only parts of large frames consisting of many pieces, as in bridge and roof trusses.

A frame of two ties is not only stable in the plane of the frame, but in planes perpendicular to it.

A frame composed of two struts is unstable laterally or in planes perpendicular to the plane of the frame, and must be stayed or held in position. It is important to remember this fact in erecting bridge and roof trusses.

A frame consisting of a strut and a tie is stable laterally when the direction of the load inclines from the line joining B and C, the points of support, and unstable when the direction of the load inclines towards that line. This construction embodies the essential principles used in derricks and cranes, in which the ends B and C are fastened to a strong, upright column of wood or iron. The column is fixed in position at top and bottom, but is capable of revolving on a pivot at the bottom. This column is called the mast, the strut CA the boom, and the rope or cable BA the tie or boom-fall by which the distance between the points A and B can be varied as desired, the boom AC assuming any position from the horizontal to almost a vertical position.





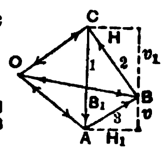


FIG. 64. FIG. 65. FIG. 66.

211. If a triangular frame (Fig. 64) composed of three pieces, A, B, and C, which are connected at the joints 1, 2, and 3, is acted upon by a force w applied at the joint 1 and acting in any direction, and supported at the joints 2 and 3, the reactions or resistances at these points being represented by R and R_1, then the first condition of equilibrium requires that these three forces shall balance each other. They may be inclined to each other, or they may be parallel. The force w is generally a weight or load, consequently acting vertically downward. The reactions R and R_1

may be inclined to w and to each other, or all three forces may be parallel.

If inclined to each other, the conditions of equilibrium require that they shall be in the same plane, that their lines of action shall meet in one point, and that their magnitudes shall be proportional to the lengths of the sides of a triangle parallel to their directions, respectively. Therefore in Fig. 66 draw a line parallel to w, and make its length CA proportional to the magnitude of w, on any desired scale, such as 1 inch to 1 ton, 2 tons, or 3 tons; then from its extremities A and C draw lines respectively parallel to R and R_1, such as BC and AB, intersecting at the point B. CAB will be the triangle of forces. For equilibrium the directions of these forces must be continuous around this triangle; and as CA acts downward, the other two must act in directions shown by the arrowheads. Knowing w and R_1, and the direction of R, we can find R and its direction, or *vice versa*. In other words, if any two of the forces are known and the angle between any two of them, or if one of the forces and the angles between it and the other two be known, the remaining forces and angles can be found.

Having then the triangle of forces CAB, in which CB represents the reaction at the joint 2, and observing that at the joint 2 there are three and only three forces acting, namely, the reaction R, and the stresses in the members C and B, and these being in equilibrium, they must be respectively proportional to the sides of a triangle parallel to their directions. Therefore from C and B, Fig. 66, draw lines parallel to the members C and B, Fig. 64, intersecting at O, Fig. 66. OC will then represent the stress in the piece C and OB the stress in the piece B. In the triangle OBC, BC is known, and all the angles; hence OC and OB can be found. Again, at the joint 3, Fig. 64, the three forces R_1 and the stresses in A and B are in equilibrium, and can be represented by the triangle OAB, Fig. 66, from which OA and OB can be found.

It will be observed that OB is common to the two triangles OCB and OBA. Again, at the joint 1, Fig. 64, it will be seen that the load w and the stresses in A and C are the only forces acting. These then must be represented by the triangle OAC, Fig. 66.

We find, then, that at each of the joints 1, 2, and 3 there is a triangle representing the external force and the two stresses in the members meeting at that joint; and, that all of the lines repre-

senting the stresses OA, OB, and OC meet at a common point O. This constitutes a stress diagram, and in each of the triangles in this diagram a sufficient number of parts are always known to find the remaining parts, whether they be angles or stresses, the lines representing the stresses being to the same scale as that adopted for the first line known and drawn, viz., CA.

Further, in each of the triangles the direction of the action of one of them is known, and for equilibrium the direction of the action of the others must be continuously around that triangle as indicated by the arrow-heads. OB acts to the right considered as a part of the triangle OCB, and to the left as a part of OAB.

These considerations and relations are of the greatest importance, and should be thoroughly understood. They underlie the essential and fundamental principles applicable to all framed structures.

212. The direction of action of the stress lines in the stress diagrams, when transferred to the members of the frame to which they are parallel, determines the kind of stress in those members. Referring now to Fig. 61, we see that when the members are under tension the direction of the forces or stresses is outward from the joint under consideration, as is also shown in the piece AB, Fig. 63. But in Fig. 62 and in the piece AC, Fig. 63, the forces or stresses act inward towards the joint, and indicate a compression.

Applying these principles to the members of the triangular truss, Fig. 64, we find that the stress CO, Fig. 66, acts inwards towards the joint 2, indicating a compressive stress on the piece C, and OB acts outwards from the joint 2, indicating a tensile stress in the piece B. Similarly for the joint 1, where the triangle of stress is COA, Fig. 66: as W acts downward, AO acts to left or upwards on the piece A towards joint 1, and OC acts upwards to the joint 1, indicating a compressive stress in each piece, A and C, meeting at joint 1. Similarly for the joint 3: OB, forming a part of the triangle OBA, acts away from the joint 3, indicating tension on the piece B, as before found, and OA acts towards 3, indicating compression in the piece A. It is to be carefully noted that stress lines in stress diagrams have different directions of action, depending upon the directions of the other lines of action in the same triangle, viz., the line OC, forming a part of the

triangle OCB, which is the stress diagram for the joint 2, acts from C to O, because BC acts from B to C; but the same line OC forming a part of the stress diagram COA acts from O to C, because CA acts from C to A; but it acts inwards towards 2 in the first case, and inwards towards 1 in the second case, thereby indicating, as it should do, compression in the piece A in both cases, and of the same amount. A failure to understand and carry these facts in mind is the main difficulty encountered in studying and applying the graphical method to the determination of stresses and strains in bridge and roof trusses, or framed structures in general.

As all of the triangles in the above case are oblique-angled triangles, the trigonometrical relation that sides of the triangles are proportional to the sines of the opposite angles, is the only one usually required. Sometimes the two sides and the included angle are given, and occasionally the three sides may be given, to find the remaining parts, and the appropriate formulæ must be used. In all cases, however, a sufficient number of parts are given to find the remaining ones, whether forces or angles.

213. Recollecting that one of the conditions of equilibrium of any system of forces in the same plane is that the sum of the horizontal components and the sum of the vertical components, taken separately and with their proper signs, shall be equal to zero. Then decomposing R and R_1 into horizontal components H and H_1, and also into vertical components v and v_1, (see Fig. 66), since H and H_1 are the only horizontal forces acting, they must balance each other, and consequently must be equal, as shown by the dotted lines, Fig. 66; and also the three vertical forces W, v, and v_1 must also balance, or $W - (v + v_1) = 0$. This condition is also shown by the dotted lines v and v_1, and the line CA representing W, in Fig. 66. These conditions must be true whatever may be the directions of R and R_1, Fig. 64. As these approach more and more nearly vertical, the smaller will the values of H and H_1 become, while $v + v_1$ remain the same. When they become vertical both H and H_1 become zero, and $v + v_1$ coincides with CA or W, but acting in the opposite direction. The stress lines CO and AO remain the same, but the stress line OB becomes less and less and eventually OB_1. The diagram of forces CAB becomes the straight line CA acting downwards, and AB and BC upwards on the same line. The whole diagram becomes as

shown in Fig. 65, corresponding lines representing corresponding stresses; and OH is a horizontal component common to each piece in the frame, and is a compression in some members and a tension in others. The reactions R and R_1 become v and v_1, since H and $H_1 = 0$. When the load and reactions are all parallel and vertical, it is only necessary to draw a vertical line to represent the total load, divide this into two segments to represent the reactions, and from the upper and lower ends of the line and the intermediate points of division draw lines parallel to the pieces forming the frame. These lines will all meet in one point, and represent the stresses on the members of the frame. Or, conversely, assuming any point O, and drawing lines to the ends and the point of division of the force diagram, any frame built with its members parallel to these lines will be stable, and the lines will represent the stresses.

The reactions are found when vertical, which is the more common condition, by the application of the principle of the lever.

The principles thus established for a triangular frame are equally true for a polygonal frame of any number of sides or pieces, whatever may be their lengths or relative directions, and when the pieces are indefinitely small the polygon becomes a curved line.

214. Taking a polygonal frame of any number of sides as shown in Fig. 67, and acted upon by any system of forces applied at their

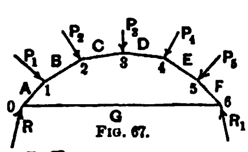


FIG. 67.

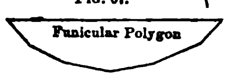


FIG. 69.

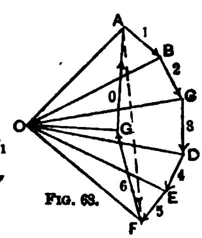


FIG. 68.

several intersections, the diagram of forces and stresses are found as follows. It is immaterial whether the relative direction of the pieces of the frame are given, and the direction and magnitude of the applied forces, to find the kind and magnitude of the stresses; or whether the latter are given, and it is required to find the former. Let the applied forces P_1, P_2, P_3, P_4, and P_5 at the joints 1, 2, 3, 4, and 5 be given, and the reactions R and R_1 at the joints 0

and 6 be found; and let the directions of the pieces A, B, C, D, E, F, and G with respect to each other be given, to find the stresses in magnitude and direction in each of the pieces.

To find the reactions R and R_1: In Fig. 68 draw a series of lines to any scale, such as 1 inch to 3 tons, AB, BC, CD, DE, and EF, respectively parallel to the applied forces. As these lines do not form a closed polygon, the line AF necessary to close the polygon will be the resultant of this system of forces. This line AF can be readily found, since the other sides and angles of the polygon $ABCDEFA$ are known. Having then the direction and magnitude of AF, and assuming the directions of both reactions, or the magnitude and direction of one of them, we can find the lines AG and FG representing the reactions R and R_1 from the triangle AGF. This completely determines the force diagram or polygon $ABCDEFG$.

If, then, lines be drawn from the angles of this polygon parallel to the pieces A, B, C, D, E, F, and G of the frame, they will all meet at one point O, and represent the kind of stress and its magnitude to the same scale used in the force diagram in the pieces to which they are respectively parallel. For each joint in the frame there has been formed a triangle representing the external force at that point and the stresses in the two members meeting at that point. Taking the joint 1, the triangle of stress is OAB, Fig. 68. Since AB acts to the right and downward, BO, representing the stress in the piece B, acts towards O or inwards on the piece B to the joint 1—hence it is compression; and OA must act from O to A or inwards on the piece A to the joint 1, also showing compression. The triangle OBC applies to the joint 2. In this triangle OC is the stress on the piece C, and acts inwards on it towards 2—hence it is compression; and OB, which acted towards the left as forming a part of the triangle OAB, now acts to the right as forming a part of the triangle OBC, and acts inwards towards the joint 2 on the piece B, giving compression, as before. Each triangle must be taken separately, and the direction of one force being given, the directions of the other two are found by going continuously around the triangle similarly for the other joints. For the joint 6 the triangle is OGF, in which FG is the reaction at 6, acting upwards. GO is the stress in the piece G and acting from G to O it acts outwards on the piece G from the

joint 6, giving tension on piece G; OF acts downwards on the piece F towards the joint 6, giving compression on F.

As all of the members of the above frame, Fig. 67, are under compression, except the piece G, which is under tension, flexible members cannot be used, except for the piece G. If this frame should be turned upside down, or revolved through 180° of arc, the kinds of stress would be reversed in each of the members. A, B, C, D, E, and F becoming tension members, can be made of ropes, links, bars, or chains; whereas the piece G being under-compression must be of such form as would give requisite stiffness. This condition is shown in Fig. 69. The same force and stress diagram, Fig. 68, still applies. The directions of the stresses would, however, act outward from the joints, indicating tension, except in the piece G, where it would act inwards towards the joints 0 and 6, indicating compression.

The whole frame in this case is called a funicular polygon. It is also a line of resistance. The broken line 0, 1, 2, 3, 4, 5, and 6, Fig. 67, is sometimes called the "equilibrium polygon." If this polygon is made up of an indefinite number of short lines it becomes a curve, such as the arc of a circle, parabola, or catenary. In this case the lines of stress in Fig. 68 are tangents to the curves, and represent the stresses at the tangent points; and the line represents the cable of a suspension-bridge.

215. The reverse of the above conditions is to assume that the direction and magnitudes of the stresses are known, and to find the directions of the pieces of the frame and the magnitudes and directions of the external forces P_1, P_2, etc., and R and R_1. In Fig. 68, assuming any point O, draw a series of lines OA, OB, OC, OD, OE, OF, and OG, each representing the known stresses to any scale. These represent the stress diagram. Then join the extremities of these lines by AB, BC, CD, DE, EF, FG, and GA. This forms the force polygon, and in each of the triangles thus formed one angle and the two including sides are given to find the other side and angles. If, then, a frame be constructed, the sides of which are respectively parallel to the stress lines and the forces parallel to the sides of the force polygon and proportional in magnitude to them, they will be the forces to be applied in order to produce equilibrium. The above discussion gives all of the condi-

tions and principles connected with the general solution of the problem.

If all of the external forces are vertical and parallel, all horizontal components of the force polygon $ABCDEFGA$ become zero, the force lines become vertical and are projected into the vertical line AF, and the force polygon becomes a straight line. The sum of the applied forces is equal to the line AF, each line following the other and acting downwards, balanced by the two reactions acting upward, thereby closing the polygon. The stress lines radiating from the point O and intersecting the vertical line at the points of division determined the magnitude of each stress taken separately.

216. The more common case occurs under the last conditions; and, in addition, the line or piece G is horizontal.

The following are the trigonometrical relations existing between external forces and the stresses developed in the different members. Having explained the principles by which the force polygon passes

FIG. 70. FIG. 73.

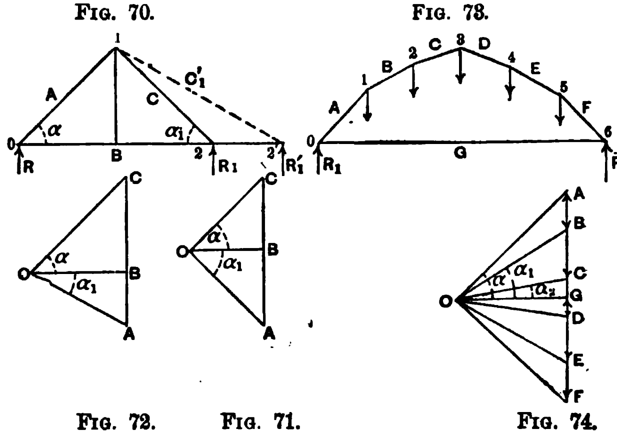


FIG. 72. FIG. 71. FIG. 74.

into a straight line consisting of forces acting downward, and an equal aggregate total amount of force acting upward and balancing the aggregate force acting downward, the usual method of constructing the diagram will be first explained, and then the mathematical relations will be found. The frame and the loads are usually given to find the stresses. Figs. 70 and 71 represent the

frame and the stress diagram, respectively, for a triangular truss with a centre load at the apex, joint 1. Taken at 3000 lbs., the reactions R and R_1 will then be equal, and each equal to 1500 lbs. In Fig. 71 lay off a vertical line, CA, downwards equal to 3000 lbs., then upwards from A to the centre B, AB to represent 1500 lbs., and BC also 1500 lbs. Using a scale of 2000 lbs. to the inch, CA will be $1\frac{1}{2}$ inches, and AB and BC each three-fourths inch. If the stress lines are laid off to the left of CA, AB will be R_1 and BC will be R. If to the right, then AB will be R and BC will be R_1. Then from C, B, and A draw lines parallel to the pieces A, B, and C, respectively. They will meet at a common point O, and represent the stresses to the same scale. In this case OC and OA scales off $1\frac{1}{16}$ inch, equal to $2000 \times 1\frac{1}{16} = 2125$ lbs., which are the stresses in the pieces A and C. Taking the triangle OCA for the joint 1, CA acts downwards, AO acts inwards towards the joint 1 on the piece C, and OC acts inwards on the piece A to the joint 1; hence the stress in each is compression.

Taking the triangle OAB for the joint 2, AB or R_1 acts upward, hence BO acts to the left or outward from the joint 2 on the piece B, indicating tension on B. OA acts to the right and downwards, or inwards towards the joint 2 on C; hence compression, as before found.

The triangle OBC for the joint 0, BC acts upwards, and CO acts from C to O, or inwards on the piece A towards the joint 0; hence compression, as before found, and OB acts from O to B, or outwards from the joint 0 on the piece B; hence tension, as before found. This line OB scales off $\frac{3}{4}$ inch, and $2000 \times \frac{3}{4} =$ 1500 lbs. tension in piece B. In the triangle OCB we have

$$OC = \frac{CB}{\sin \alpha}, \quad OB = \frac{BC}{\text{tang}\, \alpha}; \quad OA = \frac{AB}{\sin \alpha_1}, \quad OB = \frac{BA}{\text{tang}\, \alpha_1}. \quad (64)$$

In the case taken the angles α and α_1 are equal; $OC = OA$ and $OB = BC = BA$. If, however, they are unequal, let the piece C be longer than the piece A, as shown by the dotted line C_1. The load at joint 1 is the same as before. Precisely the same formulæ hold true, as shown in Fig. 72; but AB and BC are not equal; and will vary in proportion to the lengths of the lines or pieces A and C, *inversely*, as $R : R_1 :: C : A :: 0B : B2'$,—which is nothing more than the principle of the lever. If $0B$ is $\frac{1}{3}$ of $02'$, then R

will be $\frac{3}{4}$ of the load at the joint 1, and R_1' will be only $\frac{1}{4}$. Then in Fig. 72, making $CA = 3000$ lbs., $R_1' = AB$ will be 1000 lbs., and $R = BC = 2000$ lbs. It will be noted in this case that the stress line CO is longer than the stress line OA. The stress line OB is increased in length, but its value from either triangle OBA or OBC' is the same. AB is less and BC' is more; but the tangents of the angles α and α_1 vary in the same proportion, consequently the results are the same. If the angles α and α_1 become smaller and smaller, the distance 02 or 02′ between their lower extremities either remaining the same or increasing, the greater will the stress lines become with the same apex load. This is evident when we recollect that $OC = \frac{CB}{\sin\alpha}$, $OA = \frac{AB}{\sin\alpha_1}$, $OB = \frac{AB}{\text{tang}\,\alpha_1}$, and $OB = \frac{BC}{\text{tang}\,\alpha}$ are, respectively, the same as $OC = CB\frac{\text{length } A}{\text{length } B1}$ and $OA = AB\frac{\text{length } C}{\text{length } B1}$, since, from Fig. 70, $B1 = C\sin\alpha_1$, $\therefore \sin\alpha_1 = \frac{B1}{C}$; and $B1 = A\sin\alpha$, $\therefore \sin\alpha = \frac{B1}{A}$. Substituting the values of $\sin\alpha$ and $\sin\alpha_1$, we find the above results. And, similarly, $B1 = 0B \text{ tang}\,\alpha$ and $B1 = B2 \text{ tang}\,\alpha_1$, from which $\text{tang}\,\alpha = \frac{B1}{OB}$ and $\text{tang}\,\alpha_1 = \frac{B1}{B2}$. These substituted for tang α and tang α_1 give, respectively, $OB = AB\frac{OB}{B1}$ and $OA = BC\frac{B2}{B1}$; or, in other words, knowing the load transmitted through any member, the stress is found by multiplying the load or reaction by the length of the piece and dividing by the depth of the truss. Hence the longer the member and the smaller the vertical reach or rise of the member the greater will be the stress.

For the polygonal frame and its stress and force diagram the construction is the same. (See Figs. 73 and 74.) First lay off a vertical line composed of the separate loads at the joints 1, 2, 3, 4, 5, AB, BC, CD, DE, DF. Then returning upwards the two reactions R_1 and R_2, equal to FG and GA respectively, the radiating lines drawn from the ends A and F, and the intermediate points parallel to the several pieces of the frame, Fig. 73, will meet in a common point O.

$$OA = \frac{AG}{\sin\alpha}; \quad OB = \frac{BG}{\sin\alpha_1}; \quad OC = \frac{CG}{\sin\alpha_2}; \quad . \quad . \quad . \quad (65)$$

α, α_1, and α_2 being the angles made by the respective pieces with the horizontal. $AG = R_1$; $BG = R_1 - P_1$; $CG = R_1 - (P_1 + P_2)$, etc. If the truss is symmetrical and symmetrically loaded, the corresponding stress lines in the lower half of the diagram, Fig. 74, will have the same values. The horizontal stress OB will be the same no matter which triangle it may be taken from; hence

$$OG = \frac{AG}{\text{tang } \alpha} = \frac{R_1}{\text{tang } \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

will be sufficient. The same remarks made in the discussion of the triangular frame in regard to the diminution in the values of α, α_1, and α_2 as increasing the stresses, apply also in the polygonal frame, and need not be repeated.

It will be also noted that the system of notation adopted is simple, and assists in locating any stress triangle for its appropriate joint. For example, the triangle for the joint 2 is OBC, the load BC in the force polygon being the one applied to the joint 2 between the pieces B and C. Similarly OEF for the joint 5, EF being the load between the pieces E and F; and for the joint 0 the triangle OAG, AG being the reaction R_1 at 0, between the pieces A and G; and so on. The direction of the action of the stress lines OA, OB, OC, OG, etc., as indicating the kind of stress in each piece of the frame, should be traced out for each triangle, recollecting that its direction of action will change from triangle to triangle when the same line forms a part of different triangles.

In any of the frames the single line 0 2, 0 6, and so on,—improperly called a tie-beam, as it may be flexible, like a rope or chain,—and the compression line 0 6 in the funicular polygon may be omitted; but the walls or abutments at the extreme ends must be able to take up the stress, whether tension or compression, that has been found to exist in this member when inserted.

217. Stability consists in the fulfilment of the first and second conditions of equilibrium of a structure. (See paragraphs 193 and 194.) If a bar be free to change its angular position, then if a tie it is stable; if a strut it is unstable, but may be made stable by fixing its ends. A structure wanting in stability gives way by the displacement of its pieces from their proper position.

Stability should exist not only under one condition of loading,

but under reasonable variations in the loading, and each bar or member should be stable.

The external forces applied to a polygonal frame so as to produce equilibrium considered as an entire system, may be distributed in a manner not consistent with the equilibrium of each bar taken separately. It then becomes necessary to introduce pieces called braces connecting any two joints not contiguous. These supply the forces necessary at the joints to produce the equilibrium of each bar.

These braces may be either struts or ties. They introduce two equal and opposite forces, which cannot therefore alter the resultant of forces applied to that pair of joints in amount nor in position, but only the distribution of the components of that resultant on the two joints so connected. The triangle is the only geometrical figure the relative angular position of whose sides cannot be altered within the limits of strength of its sides. The introduction of braces, then, is simply to subdivide a many-sided polygon into a series of triangles connected together, and hence the triangle is the frame or truss element. A polygonal frame of more than three sides may change its form without altering the lengths of its sides. A rectangle may be flattened out into various parallelograms with the same base and various altitudes.

The polygonal frame Fig. 73 might be changed into a triangle or a four-sided figure; but if in this case braces be introduced joining the joints 0, 2, 2, 4, 4, 6, 2, 6, and 4, 0, no piece could change its relative angular position within its limit of strength; or if 1, 6, 5, 0, and 2, 4 be connected by braces, the stability of the frame in the plane of the frame will be secured; or if in the case of a rectangle diagonal braces be introduced, the frame will be stable.

None of these frames are, however, stable laterally. They may fall or be blown over as a whole unless held in position. The necessity of introducing braces will always be indicated in the stress diagrams by gaps, which must be closed by lines giving the magnitude and direction of the stress in the braces, and by consequence the position and direction of the braces themselves.

218. A simple illustration of the necessity of introducing braces in a polygonal frame is given in Figs. 75 and 76. Fig. 75 represents a frame composed of five pieces, *A*, *B*, *C*, *D*, and *E*, loaded with a single weight at the apex or joint 1, the members being

symmetrically arranged with respect to a vertical line through joint 1. The load being at the centre of the span, the reactions R and R_1 will be equal, and each equal to one half of the total load. The force polygon Fig. 76 is formed by laying off a vertical line EA to represent the total load at 1. AB acts upwards to represent R_1 and $DE = AB$ upwards for R. Then for the joint 0 draw Ee and Dd parallel to the pieces E and D of the frame, for the joint 2 draw Aa and Bb parallel to the pieces A and B, and for the horizontal member C draw an indefinite horizontal line Cc. These radiating lines do not meet in one point. Equilibrium requires a closed triangle or polygon representing the stresses and forces at each joint in order to secure the stability of each bar.

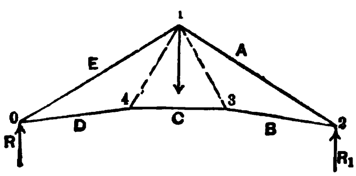


FIG. 75.

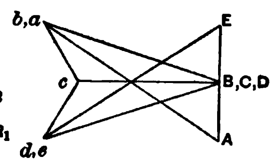


FIG. 76.

The diagram with full lines gives a triangle for the joints 0 and 2, but there are no closed triangles or other polygon for the joints 1, 3, and 4. Stress lines must then be introduced in the diagrams. The choice of the precise directions for these lines is entirely arbitrary, provided the pieces introduced are parallel to these lines and the lengths of the members D, C, B are proportioned so that the braces may join the joint 1 with the joints 3 and 4. As this might introduce a want of symmetry or other objectionable feature, it is usual to fix upon the lengths of the pieces D, C, and B, thereby fixing the position of the joints 3 and 4, and introduce the braces 1, 4 and 1, 3 as shown by the dotted lines in Fig. 75, drawing lines in the stress diagram ac and dc parallel to them and intersecting the horizontal stress line at c.

Then ac represents the stress in 1, 3; dc the stress in 1, 4; and Cc the stress in the piece C. Then for every joint there will be a closed stress triangle or polygon.

Joints.....	0	1	2	3	4
Polygons....	$DEeD$	$EAaceE$	$ABbA$	$BcbB$	$DcdD$

It is well to note that although equilibrium requires the stress polygon to close, it does not require it to be a regular polygon with all salient angles. There may be re-entrant angles, and the stress lines may cross each other, as seen by following the stress lines *EAaceE*, forming a closed polygon.

Although the principles upon which the kinds of stress in each piece are determined have been fully explained, the importance of not only a clear understanding of them, but a facility in making these determinations will justify the repetition, as confusion and error often result in applying this method, called the Graphical Method, of determining stresses from a want of clearness on this subject. See paragraphs 212 and 214.

For the joint 0 the stress angle is *DEeD*. Since *DE* represents the reaction *R* it acts upwards; hence *Ee*, the stress in the piece *E*, acts inwards on it towards the joint 0, and consequently the stress is one of compression.

De, the stress in the piece *D*, then acts to the right or outwards from the joint 0, hence the stress is one of tension; and similarly for the joint 2. The stress polygon for the joint 1 is *EAaceE*; then *EA*, the load at 1, acts downwards; *Aa*, the stress in piece *A*, acts upwards to the left, hence inwards on the piece *A* towards the joint 1, and so compression; *ac*, the stress in 1, 3, acts downwards towards the right or away from joint 1, hence the stress is one of tension; *ce* also acts downwards to the left or outwards on 1, 4 from the joint 1, hence tension; *Ee* acts upwards towards the right or inwards on the piece *E* towards the joint 1, hence compression. *Ee*, as forming a part of the triangle *EeD*, acts towards the left, but as a part of the cross polygon it acts towards the right, and in both cases indicates compression on the piece *E*; and similarly for other stress lines.

219. The preceding discussions, as regards the equilibrium or balance of any system of forces in one plane, the relations between the externally applied forces, and the kind, magnitude, and directions of the stresses developed, embody all of the essential principles required in the solution of all problems connected with forces, stresses, and stability in framed structures. The same force and stress polygons are required, whether it be desired to use the trigonometrical or graphical methods in their determination. In the trigonometrical solutions the diagrams are not drawn to scale; the

relative dimensions are calculated from the known lengths and angles by means of the simple formulæ for the solution of triangles, for which the diagrams give sufficient data, whether the external forces are parallel or inclined. This method for the simpler and less complex frames is the easiest, most desirable, and accurate.

For the more complex frames the graphical methods are far more simple and require much less labor. But absolute dependence must be placed upon the accuracy with which the diagrams are made to a proper scale. An error made in one part will be carried entirely through the diagrams, which consist of a series of lines, triangles, and polygons built one on the other and each dependent upon the other. The method is, however, used to a great extent, especially with complicated frames. With care the results will agree closely with those determined by the more troublesome calculations, and usually near enough for practical purposes. The results should be checked occasionally by exact calculations in order to avoid errors and confusion.

The preceding discussions have been entered into without any reference to the strength and stiffness of the pieces composing the structure.

The subject of stability of block-work structures, such as masonry piers, abutments, and arches, will be discussed under their appropriate heads.

That of fixed and rigid connections of joints, involving the questions of strength and stiffness, will be considered in another article.

Certain more complicated structures, as bridge and roof trusses or frames, will be further discussed in another article.

ART. XXIII.

SHEARS AND BENDING MOMENTS ON GIRDERS AND BEAMS.

220. THE principles discussed and formulæ deduced in this article are also entirely independent of the material and the distribution of the material in the cross-section of the beams, being the same for timber, iron, or stone, and for solid, hollow, or any other form of cross-section, and are dependent entirely upon the lengths of the beams, the manner of supporting or fixing their ends,

and upon the magnitude and distribution of the load, that is, whether loaded at a single point or at several points, or uniformly loaded over a part of its length or over its entire length, or, finally, a combination of these systems of loading.

Bending moments are often combined and associated with direct compression on the same member, as in long struts or columns, and also in the chords of trusses under certain conditions of loading and arrangements of the member. Where practicable it is well to avoid this combination of bending and direct stress.

Beams are horizontal or inclined pieces of any material, such as wood or iron, that are acted upon by forces, generally weights resting on its top or suspended from it. They may be fixed at one end and free at the other, in which case they are called cantilevers; or they may be supported only or fixed at both ends; or they may be continuous over three or more supports. The conditions of strain will be different in each case. Again, they may be loaded at any single or isolated point or at a series of points; or they may be uniformly loaded over the whole or a part of their length. The strength of any given beam varies with each of the above conditions. The only cases, however, that will be considered here are: first, a beam or cantilever fixed at one end and loaded at the other, or uniformly loaded over its entire length; and, second, a beam supported at both ends and loaded with a single weight at the centre, or uniformly loaded over its entire length. But as the greatest effect of the load is what we wish to determine in any case, only two cases of loading are really necessary to consider, for the effect of a uniform load is the same as that of half its amount concentrated at the end, or at the centre of the beam; or, in other words, a beam that will carry safely a given concentrated load at its end, or centre when supported at both ends, will carry safely twice that load when uniformly distributed over its entire length.

221. In either case the load produces a bending action on the beam, and if sufficiently great will break the beam crosswise or transversely to its length. This necessarily requires both the elements of force or weight and a distance or lever-arm, and the product of these two elements is called a moment, and as applied to beams the bending moment, or moment of the external forces. A moment necessarily involves what is called a couple in mechanics, which may be defined as two equal, parallel, and opposite forces act-

ing in the same plane but not in the same straight line. The perpendicular distance between these forces is called the lever-arm of the couple, and the moment of the couple is the product of one of the parallel forces by the perpendicular distance between them. Then, as applied to beams, we have in the first case a weight W acting at the end of the beam, and an equal upward force at the point of support, constituting the couple, the lever-arm of which is l, and the bending-moment Wl. If the load W is uniformly spread over the beam, as represented by the shaded rectangles on top of the beam, the maximum bending action would be the same as if $\frac{1}{2}W$ were placed at the free end, and in this case the bending-moment would be $\frac{Wl}{2}$. In addition to this bending action, which we see would gradually increase from the end to the point of support where it is the greatest possible, it is evident that the load must be transmitted, as it were, from A to B, and consequently it would tend to cut or sever the beam, in each successive position, along certain planes. This is called the shearing force, and for a beam loaded at the end it would be constant at every point and equal to W. But, as will be easily seen, when uniformly loaded the shearing force is nothing at the extreme free end A, but gradually increases as each successive weight is added until it reaches a maximum at B equal to $W = w + w + w + w + w$, etc. In the first case it can be represented by the ordinates of a rectangle, and in the second by the ordinates of a triangle. (See Fig. 77.)

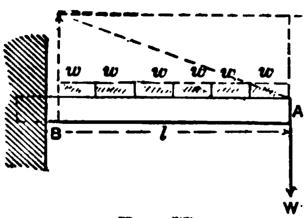


FIG. 77.

222. The beam AB, supported at both ends and loaded with the weight W at the centre, is evidently in the same condition as two beams fixed at the centre C, and pulled upwards at the two points of support A and B by a force equal to $\frac{1}{2}W$, in which case the bending moment at the centre would be $\frac{1}{2}W \times \frac{1}{2}l = \frac{1}{4}Wl$, the force W being conceived to be divided into the two forces $\frac{1}{2}W$, forming the two couples, the one a left and the

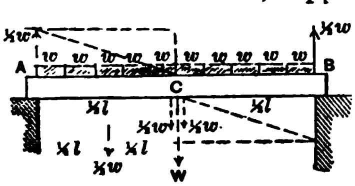


FIG. 78.

other a right handed couple, and the moment of either couple being the bending-moment at the centre C, where it is a maximum. For the uniform load we may, as before, assume $\frac{1}{2}W$ concentrated at the centre, in which case the forces of the couple would be $\frac{1}{2}W$ and the lever-arm $\frac{1}{4}l$, and the bending moment equal to $\frac{1}{2}W \times \frac{1}{4}l = \frac{1}{8}Wl$, W being equal to wl, and $\frac{1}{8}Wl = \frac{1}{8}wl^2$. Or we may proceed as follows: The upward forces at A and B are called the reactions at the points of support. We would then have for the moment of the reactions, with respect to an axis through C, $\frac{1}{2}W \times \frac{1}{2}l = \frac{1}{4}Wl$, a right-handed couple. The weight between A and $C = \frac{1}{2}W$, and being uniformly distributed its resultant would act half-way between A and C, or at a distance $\frac{1}{4}l$ from C; hence its moment with respect to C would be $\frac{1}{2}W \times \frac{1}{4}l = \frac{1}{8}Wl$, a left-handed couple, and consequently the resultant moment at C would be $\frac{1}{4}Wl - \frac{1}{8}Wl = \frac{1}{8}Wl = \frac{1}{8}wl^2$. For the shearing force in the first case we have the reaction at $A =$ the shearing force, which would be constant and equal to $\frac{1}{2}W$ until the centre is reached, where it would suddenly be cancelled and made $= 0$ by the downward weight at C. The same condition would exist between B and C. This, again, could be represented by the ordinates of a rectangle, the length being $= \frac{1}{2}l$, and height to scale $= \frac{1}{2}W$. In the second case, for a uniform load the shearing force at A and B would be equal to the reaction $\frac{1}{2}W$, but would gradually diminish as each of the small loads w are reached, as they act downwards, whereas the reaction is upwards, and would be $= 0$ at the point C. This can be represented by two right-angle triangles, the base of each being $= \frac{1}{2}l$, and the altitudes $= \frac{1}{2}W$ at A and B. The shearing force vanishes at the apex of each triangle C, where $\frac{1}{2}W-(w + w + w + w + w + w + w + w + w + w) = 0$. It will be observed that the value of the bending moment is independent of the form of the beam, it making no difference whether the beam is solid or hollow, round or square, or any other shape, but that it depends on the way in which the beam is loaded or supported, and can be expressed by the general form MWl, in which M will be 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, as seen in the above special cases. (See Figs. 77 and 78.) This bending action must be resisted by the stresses developed in the beam, and these depend entirely upon the dimensions of the beams and upon the form of cross-section, and the material of which it is composed. The idea of the couple is also involved in this resistance, and the bending moment is resisted by,

and must not exceed, the moment of the resistances or stresses developed. If a beam is subjected to the action of both an isolated load and one uniformly distributed, the resultant bending moment at any point is the algebraic sum of the two moments.

Continuous beams and beams fixed at both ends will be discussed later.

223. With the above illustrations and principles understood, we can now find the general formulæ for bending moments and shearing stress. Fig. 79 represents a cantilever beam fixed at one end and loaded with a single weight W and a uniformly distributed weight of w per unit of length; l is the length of the beam and x any distance from the free end. The bending moment at the distance $x =$

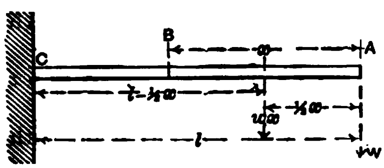


FIG. 79.

$$M = Wx + \frac{wx^2}{2}. \quad \ldots \ldots \quad (67)$$

For the point c, $x = l$; hence max. bending moment =

$$M_0 = Wl + \frac{wl^2}{2}. \quad \ldots \ldots \quad (68)$$

If there is only a uniform load on the beam, $W = 0$ and

$$M = \frac{wx^2}{2}, \quad \ldots \ldots \quad (69)$$

and for maximum bending moment

$$M_0 = \frac{wl^2}{2}. \quad \ldots \ldots \quad (70)$$

If there is only a single load on the beam, $w = 0$ and

$$M = Wx, \quad \ldots \ldots \quad (71)$$

and for maximum bending moment

$$M_0 = Wl. \quad \ldots \ldots \quad (72)$$

For the shear at any point B,

$$S = W + wx. \quad \dots \dots \dots \quad (73)$$

For the shear at any point $C =$ max.,

$$S_{\circ} = W + wl. \quad \dots \dots \dots \quad (74)$$

For the shear at any point with only a uniform load $W = 0$,

$$S = wx; \quad S_{\circ} = wl. \quad \dots \dots \quad (75)$$

For the shear at any point with only a single load $w = 0$,

$$S = W; \quad S_{\circ} = W. \quad \dots \dots \quad (76)$$

$M_{\circ}$ and $S_{\circ}$ stand for max. bending and max. shear, respectively.

The following are the general formulæ for bending moments of beams simply supported at both ends:

In Fig. 80 the beam is loaded with a single weight at the centre and a uniformly distributed weight. In this case the reactions are equal, and each equal to one half the sum of the loads.

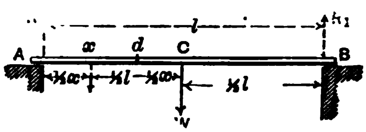


Fig. 80.

$$R = R_1 = \frac{W + wl}{2};$$

$$\text{the bending moment at } d = M = Rx - \frac{wx^2}{2}; \quad . \; . \; . \quad (77)$$

and when $x = \frac{l}{2}$, we have

$$\text{max. bending moment at } c = M_{\circ} = \frac{Rl}{2} - \frac{wl^2}{8}. \quad . \; . \quad (78)$$

Substituting the value of R, we have

$$M = \frac{Wx + wlx}{2} - \frac{wx^2}{2}, \quad \dots \dots \quad (79)$$

and

$$\frac{Rl}{2} - \frac{wl^2}{8} = \frac{Wl}{4} + \frac{wl^2}{4} - \frac{wl^2}{8} = M_{\circ} = W\frac{l}{4} + \frac{1}{8}wl^2. \; . \quad (80)$$

For a single load at the centre $w = 0$,

$$M = \frac{Wx}{2}, \quad \ldots \ldots \ldots \ldots \quad (81)$$

and for maximum bending moment $w = 0$,

$$M_e = \frac{Wl}{4}. \quad \ldots \ldots \ldots \ldots \quad (82)$$

For a uniform load alone $W = 0$,

$$M = \frac{wlx}{2} - \frac{wx^2}{2}, \quad \ldots \ldots \ldots \quad (83)$$

and for maximum bending moment $W = 0$,

$$M_e = \frac{Wl^2}{8}. \quad \ldots \ldots \ldots \ldots \quad (84)$$

For the general value of the shear,

$$S = R - wx. \quad \ldots \ldots \ldots \ldots \quad (85)$$

For the max. value of the shear $x = 0$,

$$S_e = R = \frac{W + wl}{2}. \quad \ldots \ldots \ldots \quad (86)$$

This max. value is found at the point A, where

$$x = 0.$$

The min. shear is at the centre, where

$$x = \frac{l}{2}.\ S_1 = R - \frac{wl + W}{2} = 0. \quad \ldots \quad (87)$$

For convenience, positive bending moments cause convexity downward or produce tension in the lower fibres, and negative moments the reverse. Positive shears tend to cause the left-hand portion to move upward on the right, and negative the reverse.

Had we placed the single load W at any point other than the centre, at d for instance, the principle would remain the same. Instead, however, of the reactions due to this load being $\frac{1}{2}W$, the reaction at A would be more and at B less than $\frac{1}{2}W$; the sum of the two would still be $= W$, that part of the reaction due to the

distributed load remaining the same $= \frac{wl}{2}$, to find the reactions due to W acting at d, from the principle of the lever,

$$r : r_1 :: dB : Ad, \quad \text{and} \quad r : W :: dB : l;$$

hence

$$r = \frac{WdB}{l} \quad \text{and} \quad r_1 = \frac{WAd}{l}.$$

If Ad is $= \frac{1}{3}l$, $dB = \frac{2}{3}l$, and hence $r = \frac{2}{3}W$, $r_1 = \frac{1}{3}W$, and the total reaction at $A = R = \frac{2}{3}W + \frac{wl}{2}$; then the bending moment due to the load W would be greatest at d, that due to the uniform load would be greatest at the centre c, and the greatest bending action due to both loads would be at some point between d and c, its position depending on the relative amounts or value of the two loads (the resultant of the uniform load acting at c). This point is found by the rule for finding the centre of parallel forces or centre of gravity. If the uniform load covers only a part of the length of the beam, its resultant would pass at the middle of the distance occupied by the load, its reactions would be found by the principle of the lever, and the moments at any point would be found as before. Whatever may be the positions of the loads, the reactions due to them must be found; then the moment at any point will be the algebraic sum of the moments of the reaction, and that of any load or loads, with respect to the axis passing through the point and perpendicular to the plane of the forces. If no loads exist between the reaction and the point, the reaction by its distance to the point is the final moment. If loads exist, the sum of the products of each load by its distance from the axis must be deducted from the moment of the reaction. This, of course, applies only to beams supported at both ends. In the cantilever the moment at any point is simply the sum of the moments of all the forces between the point and the free end of the beam.

224. As an infinite number of different conditions of loading can be used, it would be impossible to express by a general formulæ all such conditions; but as the principle is simple and the same for all, one or two illustrations will be sufficient. Instead of finding the expression for the final moment in terms of the loads, and the

distance from each load to the axis, it will be found in terms involving the distances between the centres of gravity of the several loads, as these are always known. Only one term will then contain a variable factor. This method simplifies the numerical work, makes the formulæ applicable to the conditions of loading usually found when a train comes on a bridge for instance, and is at the same time applicable to any other system of loads. Assuming, then, a series of loads with fixed distances between their centres to move on a cantilever beam and occupy the position shown in Fig. 81, and designating the several loads as w, w_1, w_2, w_3, etc., to w_n, as in Fig. 81, the moment at any point c would be the sum of the moments of each load. Calling the distances between the centres of gravity of the respective loads a, b, c, d, etc., the sum of the moments will be

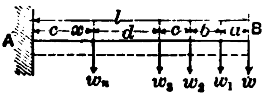


Fig. 81.

$$M = w(a + b + c + d + \ldots + x) + w_1(b + c + d + \ldots + x) + w_2(c + d + \ldots x) + w_3(d + \ldots x) + w_n x$$
$$= w\,a + (w\,a + w_1)\,b + (w + w_1 + w_2)\,b + (w_1 + w_2 + w_3)\,d + \ldots + (w + w_1 + w_2 + w_3 + \ldots + w_n)\,x, = M, \quad (88)$$

in which x is the only unknown quantity. Knowing the distance between w and w_n, and the position of w with respect to the end of the beam, x can be easily found for any position of the loads on the beam. For the max. bending moment at A, x becomes the distance between wheel w_n and the fixed end of the beam, which would be $= l - (a + b + c + d +$ etc.), if w is at the free end of the beam. If w moves to the right, x would increase, or if the loads move to the left x would decrease, by the same amount of distance. Eq. (88) is general, whatever be the distance between loads, or whether they vary or not. But in using the locomotive-wheel concentrations the distances a, b, c, etc., would be constant for the same locomotive. We might have found the position of the resultant of all the loads, and then have found the moment. The position of the resultant is found by dividing the algebraic sum of the moments of each weight by the sum of the weights; the quotient will be the distance of the resultant from the axis of moments, at c in this case. The bending moment on a cantilever beam at any point is

simply due to the loads between that axis or point and the free end of the beam B, and is in no wise affected by the loads between the point or axis and the fixed end of the beam A. The same condition determines the shearing stress at any point, being the sum of the loads between the point and the free end of the beam.

$$S = w + w_1 + w_2 \cdot \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (89)$$

for any point between the loads w_2 and w_3, and for any point to left of w_n,

$$S = w + w_1 + w_2 + w_3 + \ldots w_n . \quad . \quad . \quad . \quad . \quad . \qquad (90)$$

For beams supported at its two ends and loaded with any system of isolated weights, as those of a locomotive called the wheel-concentrations, we have the following: Fig. 82 shows the relation between the forces or loads and the reactions. We find the reactions R and R_1 by the principle of the lever, after finding the position of the resultant by the rule for finding the centre of parallel forces. To find the position of the resultant W, let x_0 be its distance from the centre of gravity of the front load w_1, which we desire to find; then

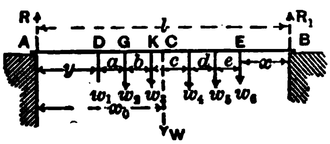


FIG. 82.

$$\begin{aligned}
&(w_1 + w_2 + w_3 + w_4 + w_5 + w_6)x_0 \\
&\quad = w_1 o + w_2 a + w_3(a + b) + w_4(a + b + c) + w_5(a + b + c + d) \\
&\quad + w_6(a + b + c + d + e) \\
&\quad = (w_2 + w_3 + w_4 + w_5 + w_6)a + (w_3 + w_4 + w_5 + w_6)b \\
&\qquad\qquad + (w_4 + w_5 + w_6)c + (w_5 + w_6)d + w_6 e;
\end{aligned}$$

hence

$$DC = x_0 = \frac{(w_2 \ . \ . \ . \ w_6)a}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6}$$
$$+ \frac{(w_3 \ . \ . \ . \ w_6)\ b + (w_4 \ . \ . \ . \ w_6)\ c + (w_5 + w_6)\ d + w_6 e}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6} . \qquad (91)$$

The loads and the distances between their centres of gravity being known, we can easily find x_0, the distance from w_1 to the centre of gravity of the entire system of loads. Then

$$R = \frac{W \times CB}{l} \quad \text{and} \quad R_1 = \frac{W \times AC}{l}.$$

The length of the span $AB = l$, and the distance of w_1 from A being known,

$$AC = AD + x_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

Or we can find the reactions by finding the reactions for each load separately and adding these together. For load w_1, by the principle of the lever,

$$r_1 = \frac{w_1(a + b + c + d + e + x)}{l};$$

for w_2,

$$r_2 = \frac{w_2(b + c + d + e + x)}{l};$$

for w_3,

$$r_3 = \frac{w_3(c + d + e + x)}{l}.$$

and so on for the other loads. Hence

$$\begin{aligned} R &= \frac{w_1(a + b + c + d + e + x) + w_2(b + c + d + e + x)}{l} \\ &+ \frac{w_3(c + d + e + x) + w_4(d + e + x) + w_5(e + x) + w_6 x}{l} \\ &= \frac{w_1 a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + (w_1 + w_2 + w_3 + w_4)d}{l} \\ &+ \frac{(w_1 + w_2 + w_3 + w_4 + w_5)e + (w_1 + w_2 + w_3 + w_4 + w_5 + w_6)x}{l}; \quad (93) \end{aligned}$$

which gives the total reaction at A, Fig. 82, which is also the shear at A, and contains but one unknown or variable quantity, viz., x. If the loads move forward towards A, x increases; if to the rear, x decreases. If another load comes on the beam, w_7 and so on to w_n would be introduced in the formula, and x in each case would represent the distance between the rear weight and B the right-hand support, which will always be known. Having R,

$$R_1 = W - R;$$

and we can now find the bending moment at any number of points.

Moment at $D = R \times y$; at G, $M = R(y+a) - w_1 a$; at K,

$$M = R(y + a + b) - w_1(a + b) - w_2 b;$$

and at C, making $Kc = q$,

$$M_0 = R(y + a + b + q) - w_1(a + b + q) - w_2(b + q) - w_3 q;$$

or

$$M_0 = Ry + (R - w_1)a + (R - w_1 - w_2)b + (R - w_1 - w_2 - w_3)q. \quad (94)$$

R in equation (94) is also the shear at A, where it is a maximum. This continues constant to D. Between D and G it is $R - w_1$; between G and K it is $R - w_1 - w_2$; between K and C it is $R - w_1 - w_2 - w_3$; and at C, the point of maximum bending, it is zero,—the shear being least where the bending is the greatest, and, *vice versa*, the bending moments at A and B being zero and the shear a maximum and equal to R.

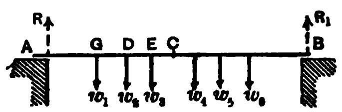


FIG. 83.

225. If there are the same number of equal loads or forces symmetrically situated with respect to the centre of the length of the beam, the expression and calculation for the bending moment can be somewhat simplified. (See Fig. 83.)

$$R = R_1 = w_1 + w_2 + w_3 = w_4 + w_5 + w_6 = 3w,$$

maximum bending moment at the centre.

$$C = M_0 = R \times AC - w_1 \times GC - w_2 DC - w_3 EC.$$

Assuming $w_1 = w_6 = 1000$ lbs., $w_2 = w_5 = 2000$ lbs., $w_3 = w_4 = 3000$ lbs., $l = 20$ feet, $AG = 4'$, $GD = 2'$, $DE = 2'$, $EC = 3'$; then we have

$$M_0 = 6000 \times 11 - 1000 \times 7 - 2000 \times 5 - 3000 \times 3 = 40000 \text{ ft.-lbs.}$$

By the usual method this requires finding the reaction, and the use of five terms. But the result will be the same in this case if we simply add together the product of the loads on one side of the centre by their respective distances from the nearest point of support, viz.:

$$M_0 = 1000 \times 4 + 2000 \times 6 + 3000 \times 8 = 40000 \text{ ft.-lbs.}$$

226. There is one condition of loading of great importance and common occurrence. When a heavy wagon, road engine, or railway locomotive comes on a bridge the weight upon one pair of wheels is supported directly by cross-girders or floor-beams. These with the reactions constitute two equal couples, the force of which is the weight upon one of the wheels and its lever-arm equal to the distance between each wheel and the ends of the beam nearest it, in the case of a single-track bridge, when the wheels are equal distance from the centre line. If the wheels are not symmetrically placed with respect to the centre line, it would be necessary to find the reactions by the principle of the lever, and then the bending moment at any point by the ordinary rules. This case usually occurs in double-track highway and railway bridges, as it rarely will happen that both tracks are loaded at exactly the same time; but to provide for this contingency the bending moment should be found on the assumption of both tracks being loaded at the same time. Each reaction or force of the equal couples will be the weight on a pair of wheels, and the lever-arm of the couple will be the distance from the resultant, of the weight on a pair of wheels, which would act midway between each pair, to the ends of the beam. These three cases are represented in Figs. 84, 85, and 86, and by the corresponding equations.

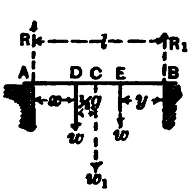


FIG. 84.

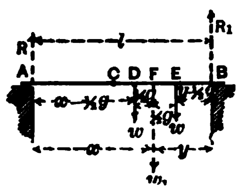


FIG. 85.

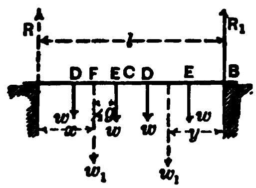


FIG. 86.

In each of the three figures the same letters correspond to the same quantities, points, and distances.

R and R_1 are the reactions, w and w the loads on the wheels, l the length of the spans, C the centre of the spans, D and E the position of the loads, x and y the lever-arms of the couples, DE the distance between a pair of wheels called the gauge $= g$. In Fig. 84 the maximum bending moment occurs at points D and $E = M_0 = wx = wy$. $R = R_1 = w$. The moment increases from A to D and from B to E

gradually from 0 to wx and wy; between D and E it is constant and uniform $= wx = wy$. Taking moments with respect to C, we have

$$R(x + \tfrac{1}{2}g) - w\tfrac{1}{2}g = Rx = wx.$$

If the load was uniformly distributed from A to D and B to E the resultant of the load would be distant from $A = \frac{1}{2}x$ and the load $= w_2x = w$, and the moment would be

$$= \frac{w_2x^2}{2} = \frac{wx}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

or one half what it was in the case of the isolated loads. The shearing force at $A = S = R = w$, and is constant to D, where it is zero, and continues zero between D and E; beyond E it is equal $R = R_1 = w$ to B. For uniform load $S = R = w_2x = w$, gradually decreasing to zero at D.

In Fig. 85 $R = \frac{w_1y}{l}$ and $R_1 = \frac{w_1x}{l}$. The bending moment gradually increases from 0 at A to $R(x - \frac{1}{2}g)$ at D, and at the point F it becomes $R(x - \frac{1}{2}g) + R\frac{1}{2}g - w\frac{1}{2}g = Rx - \frac{1}{2}wg$. From B the moment increases from 0 at B to $R_1(y - \frac{1}{2}g)$ at E, and at the point F it becomes $R_1(y - \frac{1}{2}g) + R_1\frac{1}{2}g - w\frac{1}{2}g = R_1y - w\frac{1}{2}g$; but substituting values of R and R_1, we find that

$$Rx - \tfrac{1}{2}wg = R_1y - \tfrac{1}{2}wg = \frac{w_1xy}{l} - \tfrac{1}{2}wg. \quad . \quad . \quad . \quad (96)$$

The shear at $A = R$, and is constant to D; from D to E it is $= - w + R$, and downward. From B to E it is constant and equal to R_1; from E to F it is $= R_1 - w$, and constant to F. With a single load w_1 not at the centre of the beam the bending moment at that point $= Rx = R_1y = \frac{w_1yx}{l}$.

In Fig. 86 $R = R_1 = w_1$. The moment increases from A to D where it is $R \times AD$; at F it is $Rx - w\frac{1}{2}g$, at E it is $R\,(x+\frac{1}{2}g) - wg$, and at C it is $R \times Ac - w \times DC - w \times EC$, and is the same as at E, as the last two expressions can be reduced to the same value. Similar expressions can be obtained in using R_1. The bending moment is uniform from E to D. The shear is constant from R to D and equal to R, from D to E constant and $= R - w$, at $E = R - 2w$

$= 0$, and is zero between E and D. These expressions obtained in this paragraph are of constant application in floor-beams of bridges, and are also of use for continuous beams.

The weight of the beam or truss itself is not considered in these relations. When necessary it is simply treated as a uniformly distributed load, and the two results are combined for the final moment or shear.

If the load is carried to floor-beams by stringers, as in ordinary bridge trusses where the floor-beams are from 15 to 25 feet apart, the loads w_1 are to be taken as the resultant loads supported by the floor-beams. . This does not change the equations except in the amount of the loads w. This condition will be fully explained in another article.

The floor of a highway bridge is assumed to carry so much weight per square foot. Each floor-beam carries an area of floor equal to the length of the floor-beam multiplied by the distance between them, if placed at equal distances apart; if not, by a distance between the centres of the intervening spaces on each side of it. The same rule applies to the joist or floor-beams of a wharf or warehouse of any kind. In both cases it is assumed to be uniformly distributed over the length of the floor-beam.

POSITIONS OF LOADS FOR MAXIMUM BENDING MOMENTS AND FOR MAXIMUM SHEARS IN BEAMS.

227. In the preceding discussion the positions of the loads have been assumed, and the bending moments and shears determined from the given conditions.

This is all that is needed when the loads are fixed, or, as usually called, dead loads, which include the weights of the beams themselves when of sufficient magnitude to be considered, and also the weights of the walls, or the floors with their loads above or resting upon them. But whenever the loads are shifting, moving, or rolling, which are called live loads, it is important to be able to determine those positions or conditions which will produce maximum effects. In this case, assuming a beam to be horizontal and to be subjected to vertical loads, let w be any single vertical load, and let x be any horizontal co-ordinate measured from any point as an origin. Let x_1 represent the co-ordinate of the point of applica-

tion of any load w. Required to determine the external bending moment M at any section of the beam. The lever-arm of w will evidently be $(x - x_1)$ and moment $M = w(x - x_1)$; hence for any number of forces w_1 the moment will be

$$M = \Sigma w(x - x_1). \qquad (97)$$

The summation sign Σ refers only to x_1, and is to cover that portion of the beam on one side of the section x, as is evident from the manner of forming the equation. If the origin of x is in the section considered,

$$M = -\Sigma w x_1. \qquad (98)$$

From equation (97),

$$\frac{dM}{dx} = \Sigma w = S. \qquad (99)$$

$\Sigma w = S$ is the algebraic sum of all the forces on one side of the section considered. *It is consequently the total force acting in the section tending to move one portion of the beam past the other.* It is therefore called the *shear* in the section. It is a most important quantity in the subject of the resistance of materials. The reaction or supporting forces applied to the beam are to be included both in the Σw and in the moment $\Sigma w(x - x_1)$.

Equation (99) shows that *the shear at any section is equal to the first differential coefficient of the bending moment considered as a function of x.* The sum of all other loads on the opposite side of the section x would give the same numerical shear, but it would evidently have a contrary direction. The analytical condition for a maximum or minimum bending moment in a beam is

$$\frac{dM}{dx} = 0. \qquad (100)$$

From equations (99) and (100) we learn that the greatest or least bending moment in any beam is to be found in that section for which the shear is zero. From this principle the section subjected to the greatest bending moment can be determined by a simple inspection of the loading. As an example of the above, assume a beam resting on two supports bearing two equal weights w, w,

which are kept at a constant distance apart $= a$; $l =$ length of span. Find the position for greatest bending moment on the beam, and the value of the moment. (See Fig. 87.)

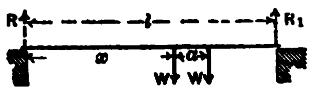


FIG. 87.

The reaction is found by the principle of the lever, x being the unknown distance from R_1 to the nearest load; then the distance of the resultant, $2w$ acting half-way between the two loads, from R_1 will be

$$l - \left(x + \frac{a}{2}\right), \quad \text{and} \quad R = 2w\frac{l - \left(x + \frac{a}{2}\right)}{l}, \quad . \quad . \quad . \quad (101)$$

as R can never be equal to $2w$, (Σw), or the shear, it must be equal to zero at the point of application of one of them. Hence in finding the greatest moment it will be better to take the moment about the point of application of one of them. Take that one nearest to R. The moment will then be

$$M = Rx = 2w\frac{\left\{l - \left(x + \frac{a}{2}\right)\right\}x}{l},$$

or

$$M = 2w\left(x - \frac{x^2}{l} - \frac{ax}{2l}\right). \quad . \quad . \quad . \quad . \quad . \quad (102)$$

$$\frac{dM}{dx} = 2w\left(1 - \frac{2x}{l} - \frac{a}{2l}\right) = 0; \quad \therefore \ x = \frac{l}{2} - \frac{a}{4}.$$

Substituting in equation (102)

$$M_1 = \frac{w}{2}\left(l - a + \frac{a^2}{4l}\right). \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Since $\frac{d^2M}{dx^2} = -\frac{4w}{l}$, it appears that M_1 is a maximum, as it is essentially negative.

If the load is uniformly continuous, and of the intensity w, wdx_1 must be put for w in equations (97), (98), and (99), and the integral sign for Σ. Then

$$M = w\int(x - x_1)dx_1, \quad M = -w\int x_1 dx_1, \quad \frac{dM}{dx} = w\int dx_1. \quad . \quad . \quad (104)$$

But since the dx and dx_1 are perfectly arbitrary, they may be taken equal to each other; hence $\frac{d^2M}{dx^2} = w$, *or the second differential coefficient of the moment considered as a function of x is equal to the intensity of the continuous load.*

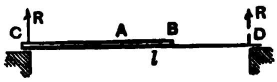


FIG. 88.

228. *A continuous train of any given varying or uniform density advances along a simple beam of span l. It is required to determine what position of loading will give the greatest shear at any specified section.* The load advances continuously from C to any point B. Now find the position of the load for the greatest transverse shear at any section A. If ΣP is the load between A and B, then the shear S' at A gives $S' = R_1 - \Sigma P$. Now if r be that portion of R_1 due to the ΣP and r_1 that due to the load on CA, r must be less than ΣP, and $r + r_1 - \Sigma P = S'$. If there is no load on AB, both r and ΣP become 0, and we have $r_1 = S$, the shear for the head of the train at A. S is evidently greater than S', because ΣP is greater than r; but if any load is taken from AC, the value of r_1 is diminished. *Hence the greatest shear will occur at any section when the load extends from the end of the span to that section.* The load may reach from either end of the span, and the longer or shorter segments may be covered. Both conditions will give a maximum shear, but these greatest shears will act in opposite directions. When the load covers the greater segment, the shear is called a main shear; if it covers the shorter segment, it is called a counter-shear. It will always be found at the head or end of the moving load.

229. When a continuous load of any given varying or constant density traverses a non-continuous beam, required to find what position of the load will cause the greatest bending moment at any specified section. Since every load which can come upon the beam will produce the same kind of bending at any given section, it follows that the greatest possible bending moment at any section will occur when the greatest possible amount of load has been brought upon the beam. *Hence the greatest bending moment at the specified section will exist when the load covers the whole span.* Also, that all sections will suffer their greatest bending moments with the same position of load.

A beam is a cantilever when it projects or hangs over; one end being fixed in some manner, the other free or entirely unsupported.

A beam is said to be continuous if its length is equal to two or more spans; or, if in the case of only one span, its ends are constrained or fixed.

A beam is said to be non-continuous if its ends simply rest or are supported at its ends, and does not suffer any constraint.

230. The preceding principles and formulæ embrace all those that can be satisfactorily discussed or explained without reference to the materials composing the members or pieces acted upon by forces and moments, or which are independent of the form of cross-section adopted. Hence the next article will be devoted to the properties of materials in respect to their capacities to resist stress or strain developed by the application of external forces and moments; in other words, the strength, elasticity, and resistance of materials used in engineering constructions.

ART. XXIV.

THE ELASTICITY AND RESISTANCE OF MATERIALS.

231. STRENGTH consists in the fulfilment of the third condition of equilibrium (see paragraphs 193, 194, and 195); and the greatest internal stress developed in any piece of a structure, by the prescribed greatest load, must be such as the material can bear, not only without immediate breaking, but without such injury to its texture as might render it liable to break under a long-continued, or oft-repeated application of a certain prescribed load.

It may be permanently injured and rendered unfit for its purpose by being stretched, compressed, bent, twisted, or otherwise altered in form or dimensions beyond a certain limit. The stress required to produce this limiting strain of a specific kind is called the proof-stress, or proof-strength. It is also called the limit of elasticity, or the elastic limit. This term will be more specifically defined in another paragraph.

It is therefore necessary that each piece should have such form and dimensions that these may not be altered beyond the given limits. The term *stiffness* is applied when this condition exists.

The terms strength and stiffness have to be necessarily considered together.

232. The strength of the various materials used in engineering can only be determined by experiment on the same kind of materials when subjected to the same kind of stress and strain. Unfortunately, the number of experiments, although seemingly great, have been far from sufficient to fully determine the strength of any given material when placed under the conditions constantly recurring in practice. The large majority of experiments have been made with small test specimens or pieces, and from these have been deduced the relative behavior of large pieces or members of structure made of the same kind of material. The strength is greatly affected by small differences in chemical composition; by the form, shape, and dimensions of parts; by the kind of stress to which they are subjected, by the manner in which the parts are connected; by the manner of supporting and loading the pieces; whether the load is distributed over a greater or less surface or concentrated at single points or a combination of these; whether the loads are supplied suddenly or more or less uniformly and gradually; whether repeatedly applied and removed, or continuously applied for a greater or less period of time; and, finally, by the mode of its manufacture and the temperatures to which it has been subjected during the process of manufacture or subsequently, as well as the actual temperature under strain. While from these considerations and conditions exact and reliable determinations may not have been made, and may never be made, yet the results of the numerous experiments that have been made have enabled engineers and builders to arrive at safe working loads, stresses, and strains. A few of the more important and recent results will be given for those materials which are the more commonly and extensively used. The definitions of the terms usually employed will be first given.

233. Ultimate strength of a solid is the stress required to produce fracture in some specified way, The following shows the direction of the external load, nature and direction of the stress and strain produced, and the character of the fracture when the stress is sufficiently great to cause it:

If the load or externally applied force is in the direction of the longitudinal axis of the piece, then—

1st. The stress is tensile. The piece is in a state of tension. The strain is one of extension or stretching or elongation in the direction of the applied force, and one of contraction of area in planes inclined or perpendicular to it.

The fracture results from tearing asunder the solid body. This may indicate a crystalline, granular, or fibrous substance.

2d. The stress is compressive. The piece is in a state of compression. The strain is one of compression, contracting, or shortening in the direction of the applied force, and one of bulging or enlargement of area in planes inclined or perpendicular to it. The fracture results from crushing, splitting into layers, or shearing along planes approaching an angle of 45° to the direction of the applied force. Some substances will not fracture under any pressure, however great, but will simply bulge or swell laterally and shorten longitudinally.

If the external force is applied transversely or perpendicularly to the longitudinal axis of the piece, then

1st. The stress is shearing; that is, tending to cause a sliding of one part of the body on another part along planes parallel, inclined, or perpendicular to the direction of the applied force. The strain is one of distortion, not in the sense of twisting, but rather of dragging the particles of the body past each other. This may be accompanied by either a contraction or enlargement of the area of cross-section, according as the kind of stress is tensile or compressive.

The fracture results from a cutting and sliding along certain planes.

2d. The stress is torsional. The piece is in a state of torsion or twisting, which necessarily requires the action of a couple. The strain is torsion, and the fracture results from wrenching the parts asunder.

This kind of stress and strain rarely exists in the pieces of fixed structures.

3d. The stress is bending. The piece is in a state of combined tension, compression, and shearing, and sometimes of torsion. The strain is one of shortening some of the parts of the beam and lengthening other parts. The fracture results from tearing asunder some parts of the beam or crushing other parts, or both, and is known as breaking across or transversely.

It is evident from the above definitions that, in whatever manner a solid body be strained, the state of the body is one of tension or compression, or of both.

234. The proof-strength of a solid body is the stress required to produce the greatest strain of a specific kind consistent with safety; that is, with the strength of the piece unimpaired. A stress exceeding the proof-strength of the material, although it may not cause immediate fracture, may result in fracture by long-continued or oft-repeated application of the load.

The proof-load is the load which will produce the proof-stress, and varies with the mode of application and distribution of the load, and also with the manner of connecting or supporting the ends of the piece.

Any load less than the proof-load might be considered as a safc or working load, but usually it is not more than $\frac{1}{2}$ or $\frac{1}{3}$ of the proof-load, in order to secure absolute safety. These fractions are sometimes called factors of safety. They are also sometimes expressed as fractions, such as $\frac{1}{4}$ or $\frac{1}{6}$ of the ultimate or breaking load. More commonly, factors of safety are expressed simply as 2, 3, 5, 6, etc., meaning $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., of the ultimate or breaking load. This will be the notation adopted in this volume.

235. Coefficients, or moduli of strength, are quantities expressing the intensity or unit of stress under which a piece of given material gives way when strained in any given manner. This intensity is generally expressed as so many pounds or tons per square inch of sectional area of the piece; or, if this unit is very small, as so many tons or pounds per square foot of area.

These coefficients are of as many different kinds and values as there are different ways of breaking a piece or member of a structure, and when of the same kind they may vary with the direction of the applied force with respect to the grain or fibre of the body.

Stiffness or rigidity is that property of a body by which it resists any forces tending to change its figure or form. The coefficient of stiffness may be expressed as a quantity, by dividing the intensity of the stress producing any change in the dimensions of a body by the change or strain in that particular dimension considered. Coefficient of stiffness $= \dfrac{\text{intensity of stress}}{\text{strain}}$.

The coefficient of elasticity is the same as the coefficient of

stiffness when the intensity of the stress is such that the stress and strain vary in such a manner that the ratio is sensibly constant for all values of the stress, the elasticity of the body being sensibly perfect. It varies with the kind of material and kind of stress.

This coefficient is sometimes given as the force which, applied to a bar whose cross-section is unity, would produce an elongation equal to the original length of the bar, its elasticity being perfect up to this limit. This is purely a theoretical force.

The coefficient of elasticity simply expressing the ratio of stress to strain, assumes the body to be perfectly elastic, and the ratio to be constant. But no body is perfectly elastic, nor possesses a perfectly constant coefficient of elasticity. Yet, within certain not well-defined limits, the assumptions are sufficiently near the truth to give results of great value.

This limit is called the elastic limit or limit of elasticity, which may be defined as that intensity of stress within which the ratio of stress to strain is constant.

DUCTILITY.—PERMANENT SET.

236. Ductility is that property by which a solid material is enabled to change its form beyond the limit of elasticity before fracture occurs. It is measured by the permanent *set* or stretch, in the case of a tensile stress, which the test-piece possesses after fracture, and also by the decrease in area found in the piece at the place of fracture. Under whatever strain the determination is made, the permanent set is the strain which remains in the piece when the external forces are removed or cease to act. In many cases this so-called permanent set decreases immediately after the removal of the stress, and under small strains it may disappear entirely, although it is commonly claimed that a permanent set is produced under any degree of stress whatever.

In many grades of wrought iron and steel the limit of elasticity can be quite accurately determined. In other materials there seems to be no simple relation between stress and strain for any condition of stress. For such materials there is no definite elastic limit or coefficient of elasticity. Between these are found all grades of material.

ART. XXV.

ELASTICITY AND RESISTANCE AS DETERMINED BY EXPERIMENT.

TIMBER.

237. THE results of experiments made to determine the elasticity and resistance of timber vary so greatly, owing to the sizes of the specimens used, their condition as to seasoning, and manner of making the experiments, that but little reliance can be placed upon them.

In timber especially is it important that the experiments should be made upon pieces as nearly in those conditions which are necessarily used in practice, such as conditions of growth and seasoning, with the usual small defects, and in sizes suitable for columns, posts, and beams. The large majority of tests have been made upon specimens of from $\frac{1}{2}$ of an inch to 2 inches round or square sections. The specimens were usually free from defects, and commonly well seasoned.

Recently the United States Government has established a bureau known as the Forestry Division of the Department of Agriculture, under the direction of Mr. B. E. Fernow, Chief of Division. One object of this bureau is to determine "the interrelation between physical condition, anatomical structure, and mechanical properties" of American timbers. Although up to date experiments have only been made on what is generally known as the Long-leaf Pine of the Southern States, and these only very limited in extent, the results thus far obtained have great value, and will be instructive as showing the proper methods of investigation and the character of information required. All that will be said on this subject is taken from the reports of Mr. B. E. Fernow, entitled "Timber Physics," which he has kindly sent the writer.

238. The following questions indicate the scope of the investigations being carried on:

What are the essential working properties of our various woods, and by what circumstances are they influenced? How does age, rapidity of growth, time of felling, and after treatment change

quality in different timbers? How far is weight a criterion of strength? What macroscopic or microscopic aids can be devised for determining quality from physical examination? What difference is there in wood of different parts of the tree? How far do climatic and soil conditions influence quality? In what respect does tapping for turpentine affect quality of pine timber?

The test-specimens were taken from the logs as shown in sections Figs. 89 and 90. The small sticks were nominally 4 inches square, dressed down to $3\frac{1}{2}$ inches square. The large sticks varied from from 6×12 to 8×16 inches in cross-section. The logs varied from 12 to 18 feet in length. The "green tests" were usually made within two months after sawing. The "dry tests" were made at various subsequent times. One end of each small stick was tested green, and the other end tested after seasoning. The seasoning was hastened in some cases by means of a drying-box. Temperature of inflowing air, 100° F. Precautions were taken to prevent checking of the wood.

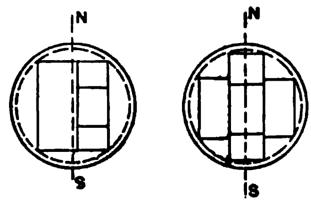


FIG. 89. FIG. 90.

The testing apparatus consisted of a 1,000,000-pound column-testing machine; one 100,000-pound beam-testing machine; one 100,000-pound Universal testing-machine of Riehle's "Harvard" pattern; various steam-engines, planes, lathes, etc.

239. *The Cross-breaking Tests.*—The large beam-testing machine is shown in Fig. 91. The base of this machine consists of two long-leaf pine sticks (*Pinus palustris*), 6 × 18 inches and 24 feet long, with steel plate three fourths of an inch by 18 inches by 20 feet long, all bolted up as one beam. The power is applied by hydraulic pressure upon a plunger below, to the cross-head of which are attached the two side-screws, on which the upper cross-head is moved by sleeve-nuts and spur-gearing. The beam to be tested rests on pivots at the ends, placed on top of the base-beam, and the upper cross-head is moved down by means of the gearing until the central pivot attached to it comes in contact with the beam, or rather with the distribution-blocks placed on the beam at this point. The test then begins, the power originating in a double plunger-pump, operated by hand or by steam power, in another part of the room.

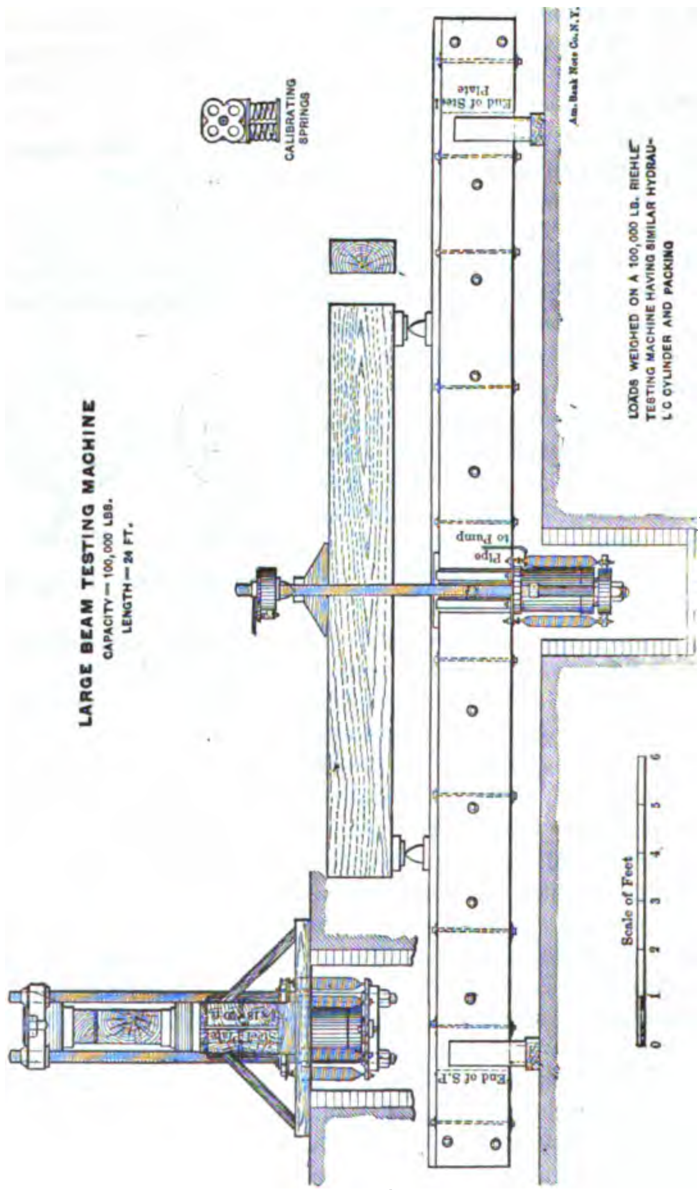

LARGE BEAM TESTING MACHINE
CAPACITY—100,000 LBS.
LENGTH—24 FT.
CALIBRATING SPRINGS
Pipe to Pump
End of Steel Plate
End of S.P.
LOADS WEIGHED ON A 100,000 LB. RIEHLE TESTING MACHINE HAVING SIMILAR HYDRAULIC CYLINDER AND PACKING
Scale of Feet
0 1 2 3 4 5 6
Am. Bank Note Co.,N.Y.

FIG. 91.

In the tests of all beams, both large and small, the load is put on at the same uniform rate, so as to eliminate the time effect, which is very great in timber tests.

The load on the small beams is increased at such a rate as to produce an increase in the deflection of $\frac{1}{8}$ inch per minute without any pause until rupture occurs. This causes rupture in from ten to fifteen minutes' time. The time required for the large beam tests is about the same, the deflection rate being greater when the total deflection is to be greater, which is the case with 4 × 8 inch sticks 12 feet long.

Small Beams.—The small beams, which are nominally 4 inches square and 60 inches long between supports, are tested on the small

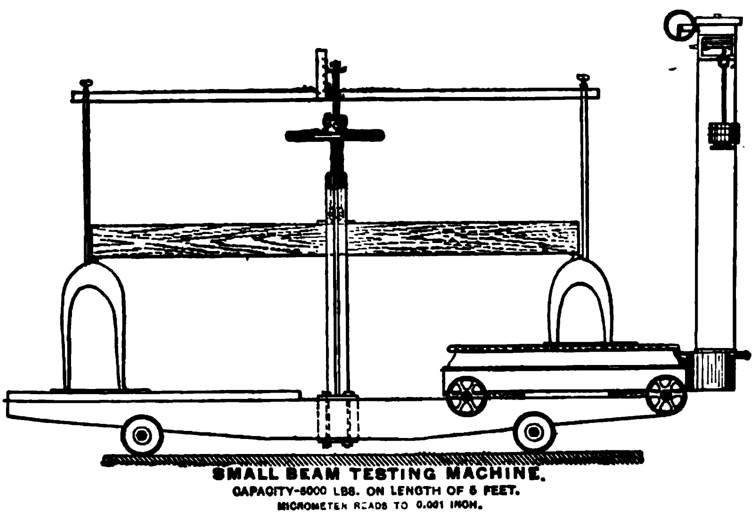


FIG. 92.

machine shown in Fig. 92. The load is put on by the hand-wheel and power-screw, and the weighing-beam kept in balance by putting on overweights and moving the poise. One man moves the power-screw, which has one-fourth-inch pitch, so as to make one revolution every two minutes, and this uniform motion continues until rupture occurs. Another keeps the scales balanced and calls off the

even hundreds of pounds. A third keeps the micrometer-screw in contact with the head of the power-screw, reads it for certain even hundred-pound loads called off, and records the time of each such reading to the nearest minute, the load, and the corresponding reading of the micrometer-screw.

240. *Moisture Tests.*—After rupture the sticks are bored about 20 inches from each end and at about one third the width from either side, in order to get samples for the moisture tests. These are weighed, and then dried at a temperature of 212° F. until they reach a constant weight. If the original weight is twice the dry weight, there were equal quantities of water and woody fibre, and the moisture, if computed on the basis of the original weight, would be 50 per cent of the original; but if computed on the basis of the dry weight, it would be 100 per cent. This latter is the method used.

241. The specific gravity is determined by measuring very carefully one of the end-pieces, usually $4 \times 4 \times 8$ inches, and with its volume and its weight the weight of a cubic foot is calculated. This weight divided by the weight of a cubic foot of distilled water gives the specific gravity.

242. *The Tension Test.*—A piece 16 inches long and $2\frac{1}{2} \times 1\frac{1}{2}$ inches cross-section is cut from one end of the broken beam. Its thickness at the centre is reduced to about $2\frac{1}{2} \times \frac{3}{8}$ inches by cutting out circular segments. This is then tested similar to the test of a bar of iron in the Universal machine. (See Figs. 93 and 94.)

Care is taken to cut the specimen parallel to the grain of the wood, so that failure will occur in a condition of pure tension.

243. *Compression Across the Grain.*—Specimens 4 inches square and 6 inches long are tested in compression across the grain. An arbitrary limit of distortion, namely, 3 per cent of the height, has been chosen as a reasonable maximum in practice. The load, then, on the specimen is called the compressive strength across the grain. This load is indicated by the ringing of an electric bell. The test is then continued until the distortion has reached 15 per cent of the height; both results are recorded. (See Fig. 95.)

244. *The Shearing Tests.*—Since timber fails by shearing or splitting oftener than in any other way, the shearing test is of great importance. The specimen is taken 2 inches square and 8 inches long; rectangular holes are mortised 1 inch from each end, and at

right angles to each other (see Fig. 96). The specimen is then pulled in the Universal machine (see Fig. 93) by means of suitable stirrups and keys, as shown in Fig. 96. The ends are kept from spreading or splitting by putting on small clamps with just enough initial stress in them to hold them in place. After one end shears out two auxiliary hoops or stirrups are used to connect the key, which is sheared out to a pin put through the hole at the centre of the specimen as shown. The other end is then sheared out, and

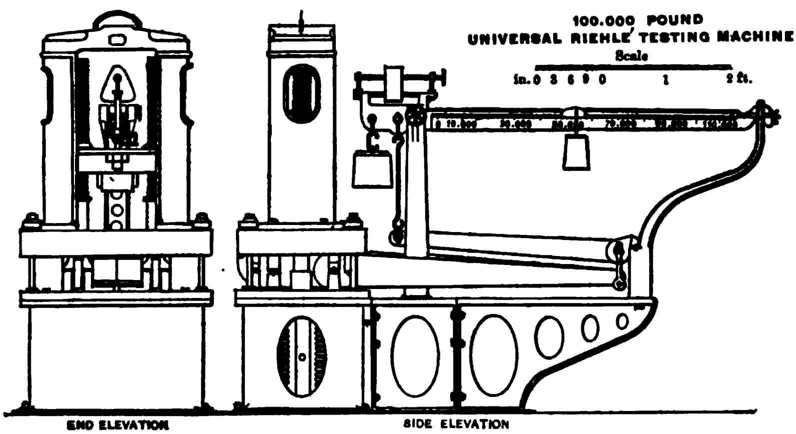


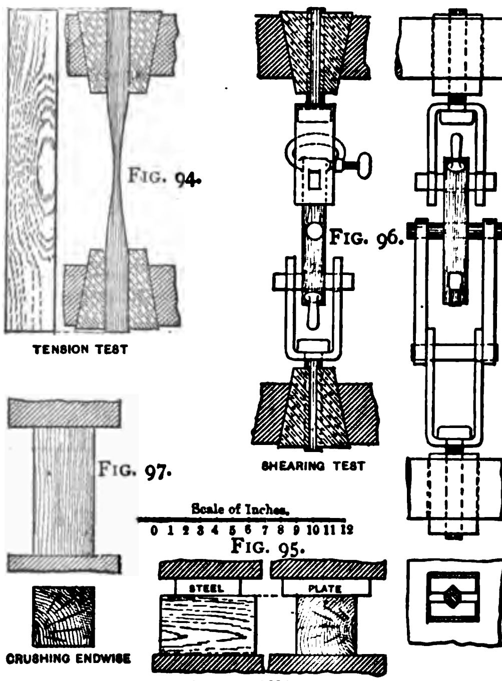
FIG. 93.

two results are obtained at right angles to each other. The details of connections, grips, stirrups, and keys are shown in Fig. 96.

245. *Endwise Compression Tests.*—Most of the tests for compressive strength were made on sticks 4 inches square and 8 inches long. The ends were cut true and square and at right angles to the sides. They are tested in the Universal machine (see Fig. 93). The compression is continued until the stick has been visibly crushed and has passed its maximum load. The crushing usually manifests itself over a plane section by crushing down or bending over all the fibres at this section, which may be either a right or oblique section. The section of failure, however, is seldom at the very end. The slightest source of weakness may determine its position, as for example a very small knot—for knots are a source of weakness in compression as well as in tension.

Some tests were made on columns 40 inches long by 4 inches square on the large beam machine. These usually failed as in case of the short blocks, and not by bending sidewise. (See Fig. 97.)

Compressive Tests of Full-sized Columns.—To make these tests requires a machine of at least 1,000,000 pounds capacity



capable of crushing to failure columns from 12 to 14 inches square and at least 30 feet long. It will receive a column of 36 feet in length or less. The sides or tension members of this machine are made of four long-leaf yellow-pine sticks (*Pinus palustris*), from Georgia, each 8 × 12 inches by 45 feet long. The power is applied by the same hydraulic pump which operates both the large beam machine and the 100,000-pound Universal machine. The loads are

weighed on this latter machine the same as for the beam tests. The plunger in the column machine has just ten times the area of that in the weighing-machine, and hence the loads in the column tests are just ten times those indicated on the weighing beam, whereas in the beam machine they were the same. The tail-block is of cast iron resting in a spherical socket, which is carried on a car, and can be held by struts resting in slots in the timber. This is to make the distance between the face-plates any even number of feet from two to thirty-six. The spherical socket in the tail-block will produce an accurate adjustment of the end-bearings at the beginning of the test, but after the load is on it is thought that this joint will remain rigid. This socket is not intended to serve as a round end-bearing for the column. No tests have been made on this machine up to the present time.

246. *Significance of Results.*—From the cross-breaking tests are obtained the cross-breaking modulus of rupture, the modulus of elasticity, or measure of stiffness, and the elastic resilience, or measure of toughness.

The loads and their corresponding deflections are plotted as rectangular co-ordinates, and the modulus of elasticity and the elastic resilience are obtained from a study of this strain diagram.

The following is an example of the record made for every beam test. This is a record of a test made on a 4 by 8 inch stick of long-leaf pine, 12 feet long, which was placed on supports 140 inches apart.

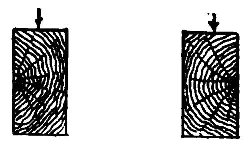
CROSS-BREAKING TEST.

Mark $\begin{cases} 16 \\ 8 \\ 1 \end{cases}$	Strength of extreme fibre, where $f = \dfrac{3\,Wl}{2bh^2} = 10{,}910$ pounds per square inch. . (105)
Length, 140.0 inches.	Modulus of elasticity = 2,070,000 pounds per sq. inch.
Height, 8.04 inches.	Total resilience = 35,440 inch-pounds.
Breadth, 4.02 inches.	Resilience per cub. in. = 7.88 inch-pounds.
	Total elastic resilience = 8650 inch-pounds.
	Elastic resilience, per cubic inch = 1.91 inch-pounds.

The observed data are given in the columns headed "Time," "Load," and "Scale Reading." These results are recorded on this sheet in ink as they are observed. The result in the "Deflection"

TABLE XV.

(Number of annual rings per inch = 14.)

August 27, 1891.	Load.	Deflection.	Scale Reading.	Remarks.
h. *m.*				
1 58	1,000	0.17	11.02	
59	2,000	0.34	11.19	
59	3,000	0.50	11.35	
2 00	4,000	0.66	11.51	
00	5,000	0.82	11.67	
01	6,000	0.98	11.81	
02	7,000	1.13	11.98	
02	8,000	1.27	12.12	Butt end. Top end.
03	9,000	1.46	12.31	
03	10,000	1.65	12.50	FIGS. 98.
04	11,000	1.93	12.78	
05	12,000	2.27	13.12	
07	13,000	2.85	13.70	
09	13,500	3.85	14.70	Maximum load.

column is computed from the scale-reading. It is placed next to the column of "Loads" for convenience in plotting the strain diagram, which is done on the ruled squares at the bottom of each sheet. These plotted results fall in all cases on a true curve, similar to the one shown in Fig. 99. The total area of this curve, *O D E*, properly evaluated by the scales used, represents the total number of foot-pounds or inch-pounds of work done upon the stick before rupture occurred. This is called the *Total Cross-breaking Resilience* of the stick, and when divided by the volume of the stick in cubic inches it gives approximately the total cross-breaking resilience of the stick in inch-pounds per cubic inch of timber.

A better criterion of toughness, or resistance to shock, is some definite portion of this strain-diagram area, as *OPK*, Fig. 99, for example. This amount of resilience or spring can be used over and over again, and is a true measure of the toughness of the timber as a working quality. To locate the point *P*, the following arbitrary rule has been followed:

Draw a tangent to the curve at the origin, as *OA*. Lay off $AC = \frac{1}{2}BA$, and draw *OC*. Draw *mn* parallel to *OC* and tangent to

the curve. Take the point of tangency as the point P, and draw PK. The area OPK is then called the *Relative Elastic Resilience.**

There is no "elastic limit" in timber as there is in rolled metals. In this respect it is like cast iron. The point P is the point where the rate of deflection is 50 per cent more than it is at first, and usually falls on that part of the curve where it begins to change rapidly into a horizontal direction or where the deflection begins to increase rapidly. The areas of these curves are measured

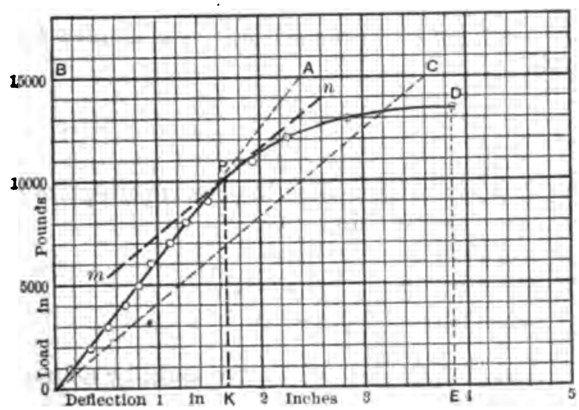


FIG. 99.

with a planimeter and reduced to inch-pounds. Thus, if 1 inch vertically represents 5000 pounds, and 1 inch horizontally represents 1 inch deflection, then 1 square inch represents $5000 \times 1 = 5000$ inch-pounds. If the area OPK is 1.73 square inches, then the corresponding resilience is 8650 inch-pounds. This means that a weight of 100 pounds, falling 86.5 inches, or 1000 pounds falling 8.65 inches, would have strained the beam up to the point P or it would have deflected it 1.66 .inches, and the beam would have been then resisting with a force of 10,000 pounds, since P falls

* This term has been coined to define this particular portion of the resilience which will be used for comparing the relative elasticity or toughness of different timbers.

on the 10,000-pound line. If this result—8650 inch-pounds—be divided by the number of cubic inches in the stick between end-bearings, the result is the true *Relative Resilience in Cross-breaking* in inch-pounds per cubic inch. This result is independent of the dimensions of the test specimen, and is therefore a true measure of the quality of timber which is usually known as toughness. It depends, as toughness in the usual understanding does, on both the strength and the deflection; in fact, it is very nearly the half-product of the strength developed and the deflection produced at this particular point P. It is probably the nearest quantitative measure of the toughness that can be arrived at.

247. *The strength of the extreme fibre* is computed by the ordinary formula

$$f = \frac{3Wl}{2bh^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

where f = stress on extreme fibre in pounds per square inch,
W = load at centre in pounds,
l = length of beam in inches,
b = breadth of beam in inches,
h = height of beam in inches.

At the time of final rupture this formula by no means represents the actual facts. It assumes that the neutral plane remains at the centre of the beam till rupture occurs, which is far from correct. In green timber, where the crushing strength is greatly reduced by the presence of the sap, the crushing resistance is only about one third as much as the resistance to tension, so that the stick invariably begins to fail on the compression side. This causes the neutral plane or plane of no stress to be lowered, and at the time of final rupture this plane may be from one fourth to one sixth the depth from the bottom side of the beam. The value of f computed by this formula from a cross-breaking test, therefore, will always be intermediate between the crushing strength and the strength in tension. Thus the crushing strength of a given stick was found to be 5820 pounds per square inch, while the tensile strength was 15,780 pounds; the cross-breaking strength was found by this test to be 10,900 pounds.

The modulus of elasticity is computed from the formula

$$E = \frac{Wl^3}{48DI} = \frac{Wl^3}{4Dbh^3} = \frac{W}{D} \cdot \frac{l^3}{4bh^3}, \quad . \quad . \quad . \quad (107)$$

where E = modulus of elasticity, W, l, b, and h as in eq. (106),
D = deflection of beam, and
I = moment of inertia of the cross-section $= \frac{1}{12}bh^3$ for rectangular sections.

To find this modulus, a tangent line is drawn to the strain diagram at its origin, as OA, and the co-ordinates of any point on this line used as the W and D from which to compute E.

The modulus is thus seen to vary directly as the load and inversely as the deflection, hence it is a true measure of the stiffness of the material. It is the most constant and reliable property of all kinds of engineering materials,* and is a necessary means of computing all deflections or distortions under loads.

In using the modulus of elasticity of timber for computing deflections, it must be remembered that in this case the time effect is very great (it is nearly zero in metals) and that this factor can only be used to compute the deflection for temporary loads. The deflection of floor or roof timbers, for instance, under constant loads is a very different matter, as it increases with time.

Relation between Strength and Stiffness.—In Fig. 100 is shown the relation found by Professor Bauschinger † between the modulus of elasticity (stiffness) and the cross-breaking strength, from tests on pine, larch, and fir timber. Although the results show a wide range, there is evidently a general relation between these two quantities, as indicated by the straight line drawn through the plotted points. The algebraic expression of the law shown by this line, rendered into pounds per square inch, is, in round numbers,

Cross-breaking strength = 0.0045 modulus of elasticity + 450. (108)

* The wide range of values of the modulus of elasticity of the various metals, found in published records of tests, must be explained by erroneous methods of testing.

† See Pl. II, vol. 16, of Professor Bauschinger's Reports of Tests made at Government Testing Laboratory at Munich.

If it should be found that there is such a law for all kinds of timber, then there may be derived an equation of this form, but with different constants, for each species.

Cross-breaking Strength = 0,0045; Modulus of Elasticity (or Stiffness) + 450.

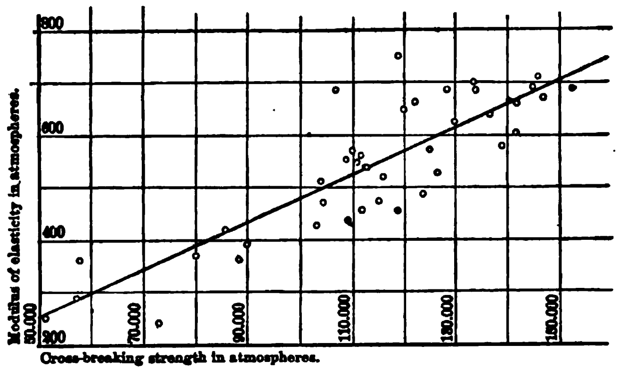


FIG. 100.—Relation between Cross-breaking Strength and Modulus of Elasticity or Stiffness.

Compressive Strength = 13,800; Sp. G. – 900.

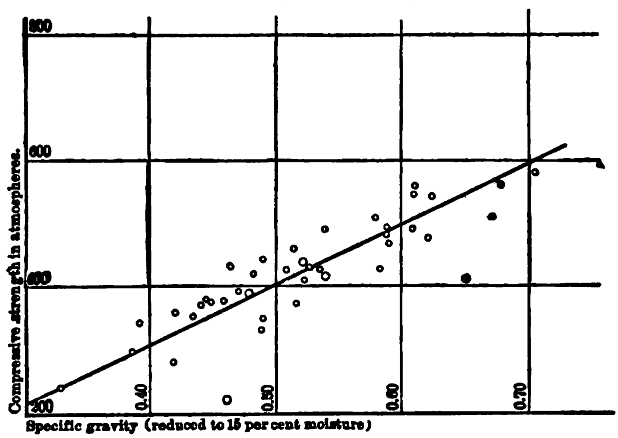


FIG. 101.—Relation between Compressive Strength and Sp. G. or Weight.

Relation between Strength and Weight.—In Fig. 101 is shown the relation between the crushing strength and the specific gravity, when both are reduced to the standard percentage of moisture, which was taken at 15 per cent.

These results are also taken from Professor Bauschinger's pub-

lished records of tests on pine, larch, and fir timbers, and they conclusively show that the greater the weight the greater the strength of the timber. The law here is a well-defined one, so far as these timbers are concerned. When rendered into English units (pounds per sq. in.), the equation of this line is

$$\text{Crushing strength} = 13{,}800 \text{ specific gravity} - 900, \quad . \quad (109)$$

when the timber contains but 15 per cent of moisture. This equation would also vary in its constants for each species of timber.

Relation Between the Compressive Strength and the Percentage of Moisture.—In Fig. 102 are plotted some very careful tests by Professor Bauschinger to show the relation between the percentage of moisture and the crushing strength.

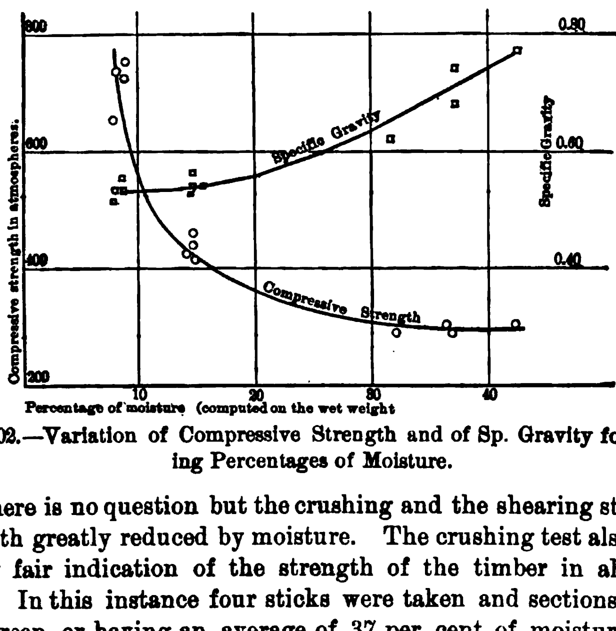


FIG. 102.—Variation of Compressive Strength and of Sp. Gravity for Varying Percentages of Moisture.

There is no question but the crushing and the shearing strength are both greatly reduced by moisture. The crushing test also gives a very fair indication of the strength of the timber in all other ways. In this instance four sticks were taken and sections tested first green, or having an average of 37 per cent of moisture when computed on the wet weight, or 59 per cent of moisture when computed on the dry weight, as is the practice in the tests made by this Department. The sticks were then dried until there was an average of 14.6 per cent moisture on the wet weight, or 17 per cent on the dry weight. The remaining portions of the sticks were further seasoned until there remained but 8.2 per cent moist-

ure computed on the wet weight, or 9 per cent moisture on the dry weight, and then tested. This is a smaller percentage of moisture than outdoor lumber ever reaches, as the ordinary humidity of the external air will usually maintain at least 10 per cent of moisture in all kinds of timber.

When these three groups of results are plotted, and the most probable curve drawn through them, there is seen to be a remarkable increase in the crushing strength when the percentage of moisture falls below fifteen or twenty. The variation in strength above that limit is very small. Professor Bauschinger has published a great many such curves, all showing the same general law. This curve illustrates the necessity for finding the percentage of moisture for every test of strength made.

Professor Bauschinger has published very few tests showing the relations between the cross-breaking strength and the moisture, but Fig. 103 is a reproduction of such results as he has given.

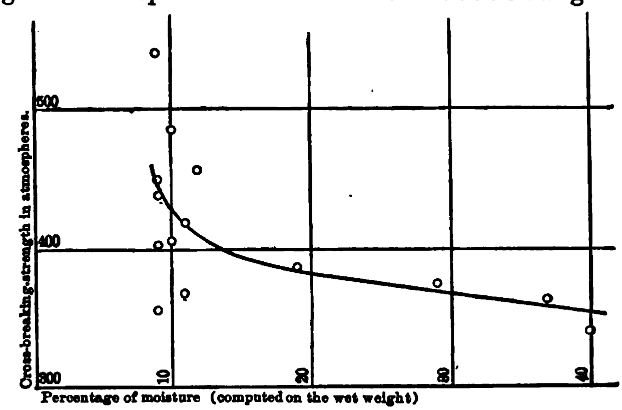


FIG. 103.—Relation between Cross-breaking Strength and Percentage of Moisture.

When the percentage of moisture sinks as low as 10 there appears a wide variation of strength, not satisfactorily explained. There would seem to be a law of dependence, however, but less marked than in the case of compressive strength.

Relation Between Specific Gravity and Moisture.—In Fig. 102 the "specific-gravity" curve shows the relation between the specific gravity and the percentage of moisture. At first the specific gravity diminishes rapidly as the percentage of moisture is reduced, but when this has been reduced to 15 per cent the

specific gravity changes very little for any further reduction in moisture. This shows that the shrinkage is insignificant until the timber becomes nearly dry, when it swells and shrinks almost directly with the percentage of moisture, so that the weight of a unit volume, which is a measure of the specific gravity, remains nearly constant. This curve is also only one of a great many similar ones given by Professor Bauschinger.

248. There is much other interesting and valuable information contained in Mr. Fernow's reports, to which the reader is referred. We will close this subject with the following conclusions:

The data contained in the report refer altogether to the timber of long-leaf pine (*Pinus palustris*) from Alabama. These data refer to over 2000 tests on material furnished by twenty-six trees collected from four different sites by Dr. Charles Mohr.

These tests may be said to represent fairly well the range of strength pertaining to the species. The following table represents the range of value of the various exhibitions of strength as compiled from Professor Johnson's report:

TABLE 16.

CONDENSED TABLE OF MECHANICAL PROPERTIES OF LONG-LEAF PINE.

[Ranges reduced to 15 per cent moisture.]

	Specific Gravity.	Cross-bending tests.		
		Strength, $f=\frac{3Wl}{2bh^2}$.	Modulus of Elasticity.	Relative Elastic Resilience in inches = lbs. per cu. in.
Butt logs....	0.449—1.089	4762—16200	1118800—3117370	0.23—4.69
Middle logs...	0.575—0.859	7640—17128	1186120—2981720	1.34—4.21
Top logs.....	0.484—0.907	4268—15554	842000—2697460	0.09—4.65

	Crushing Endwise.	Crushing across Grain.	Tension.	Shearing.	Modulus of Strength at Elastic Limit.
	Strength, per sq. in.	Strength, per sq. in.	Strength, per sq. in.	Strength (mean), per sq. in.	Per sq. in. $f=\frac{3Wl}{2bh^2}$.
Butt logs....	4781—9850	675—2094	8600—31890	464—1299	4930—13110
Middle logs..	5030—9300	656—1445	6330—29500	539—1230	5540—11790
Top logs....	4587—9100	584—1766	4170—23280	484—1156	2543—11950

here

Professor Johnson has attempted to relate the values of strength to other qualities, especially moisture contents of the test-piece, and compared the various exhibitions of strength with each other, to find if possible their relation. It is to be understood that this discussion refers only to the species in hand, and does not admit of generalization to other timbers.

Some of the deductions for the long-leaf pine may even have to be modified upon further study. We summarize the more important deductions as follows:

(1) With the exception of tensile strength, a reduction of moisture is accompanied by an increase in strength, stiffness, and toughness.

(2) Variation in strength goes generally hand in hand with variation in specific gravity.

(3) The strongest timber is found in a region lying between the pith and the sap at about one third of the radius from the pith in the butt log; in the top log the heart portion seems strongest. The difference in strength in the same log ranges, however, not over 12 per cent of the average, except in crushing across the grain and shearing, where no relation according to radial situation is apparent.

(4) Regarding the variation of strength with the height in the tree, it was found that for the first 20 to 30 feet the values remain constant, then occurs a more or less gradual decrease of strength, which finally, at the height of 70 feet, amounts to 20 to 40 per cent of that of the butt log for the various exhibitions of strength.

(5) In shearing and crushing across and parallel with the grain, practically no difference was found.

(6) Large beams appear 10 to 20 per cent weaker than small pieces.

(7) Compression tests seem to furnish the best average statement of value of wood, and if one test only can be made this is the safest, as was also recognized by Bauschinger.

The investigations into the effect of bleeding the trees for turpentine leave now no doubt of the fact announced in a preliminary circular, that bled timber is in no respect inferior to unbled timber.

This conclusion, to which the mechanical tests lent countenance, is strengthened by the chemical study of Mr. M. Gomberg.

into the distribution of resinous contents throughout the trees bled and unbled. These show what physiological considerations would lead us to anticipate, that the resinous contents of the heart-wood take no part in the flow of resin induced by the "boxing" or "chipping" of the tree, being non-fluid, and also being found present in larger amounts in the heart-wood than in the sap-wood, as well before as after bleeding. The drain appears to be entirely from the sap-wood, and as this does not enter into lumber production, being hardly more than two inches on the radius, it may be left out of consideration.

The result of the tests, to the effect that bled timber is stronger than unbled, which Professor Johnson proposes to explain as a result of the bleeding, does not seem to admit of such reference. It is suspected that the timber from the orchard might have come from a locality the soil conditions of which were apt to produce better quality, the comparison of bled and unbled timber having been made on material from different localities.

From the field report of Mr. Roth it would appear that opinions of practical men are so much at variance as to the effect of bleeding as to be of no special value, and we can claim that the discrimination made against bled timber, be it on account of inferior strength or inferior durability, is due to an unwarranted prejudice.

The physiological considerations and the processes employed in the gathering of turpentine in this and other countries will be found fully discussed in the annual report of the Forestry Division for the year 1892.

Much stress has been laid on the above government tests and methods, as these are the first systematic efforts that have been made in this country to obtain satisfactory knowledge on the strength and other valuable properties of American timbers.

The following tables give the data of other experiments upon American timbers, and are the results upon which we have heretofore and do now base our calculations.

249. For convenience of reference the usual coefficients for timber, stone, cast iron, wrought iron, and steel will be collected in this place. Special reference, however, to each material will be made under their proper heads.

The following are average results taken from many sources. The usual values adopted in practice will be given in their appropriate places.

TABLE XVII.

IN POUNDS PER SQUARE INCH.

	Coefficients of Elasticity.	Limits of Elasticity.
Steel	30,000,000 to 35,000,000	37,000 to 60,000
Cast iron	12,000,000 " 14,000,000	20,000
Wrought iron	28,000,000 " 29,000,000	25,000 to 30,000
Timber	354,000 " 1,079,500	5,000 to 9,000
Stone	5,000,000 " 13,500,000	

TABLE XVIII.

COEFFICIENTS OF STRENGTH.

[☞ *The figures represent thousands.*]

	Resistance to Crushing, lbs. per sq. in.	Resistance to Tearing, lbs. per sq. in.	Resistance to Cross-breaking, lbs. per sq. in.	Resistance to Shearing, lbs. per sq. in.
Steel	76 to 148	65 to 114	115 to 142	53 to 68
Cast iron	80 " 100	16 " 20	42 " 70	16 " 20
Wrought iron	36 " 40	50 " 60	62 " 75	49 " 54
Timber	4 " 14	7 " 15	10 " 17	0.369 " 8.48
Stone	3 " 20		1.062 to 3.6	

The resistance to cross-breaking for steel is given at about 1.66 to 1.85 times that of direct tearing; for wrought iron 1.5 times, and for cast iron 2 to 2½ times, the direct tensile strength.

TABLE XIX.

FOR THE MORE COMMON AND USEFUL TIMBER.

	Resistance to Crushing, lbs. per sq. in.	Resistance to Tearing, lbs. per sq. in.	Resistance to Cross-breaking, lbs. per sq. in.	Resistance to Shearing, lbs. per sq. in.	
				Along the Grain.	Across the Grain.
White pine	9,500	7,000	9,000	482	2,480
Yellow pine	11,500	20,700	15,000	848	5,735
Locust	18,700	29,000	21,000	1,165	7,176
White oak	8,500	13,200	12,000	1,250	4,425
Hemlock	6,280		8,000	369	2,750
Spruce	8,410	11,600	10,000	542	3,255
Black walnut	7,000	9,790	13,500		4,728
White ash	8,150	15,500	16,000		6,280
Live-oak		10,310			8,480

These tables are merely given as samples of the ultimate strength. Different experimenters have arrived at widely different results,

not only on account of the varying qualities of materials used, but the different shape and sizes of pieces tested, the different mode of making the tests, and many other causes. The important values to the practical engineer, and for the purposes of this volume, are those for the working loads. The following are fair average values, subject to certain modifications on account of the length of the pieces, or their position in the structure requiring special values, which will be subsequently explained.

TABLE XX.

SAFE LOADS.

	Resistance to Crushing, lbs. per sq. in.	Resistance to Tearing, lbs. per sq. in.	Resistance to Cross-breaking, lbs. per sq. in. Modulus of Rupture f.	Resistance to Shearing, lbs per sq. in.
White oak..............	500	1,000	1,500	500
Pine....................	500	1,000	1,500	200
Wrought iron...........	8,000	10,000	8,000	7,500
Cast iron...............	16,000	4,000	4,000	
Steel....................	14,000	14,000	14,000	10,000

For the steel used in the bridge at Memphis, recently constructed, the requirements were as follows:

TABLE XXI.

Ultimate strength, maximum for high grade.........	78,500 lbs.
" " minimum " " "	69,000 "
" " maximum for soft................	63,000 "
" " minimum " "	55,000 "
Minimum elastic limit for high grade...............	40,000 "
" elongation in 8 inches....................	18 per cent.
" reduction of area........................	38 " "
" elastic limit for soft......................	30,000 lbs.
" elongation in 8 inches....................	28 per cent.
" reduction of area........................	50 " "
Laboratory tests:	
Maximum strength...............................	70,000 lbs.
" elastic limit............................	40,000 "
" elongation in 8 inches..................	15 per cent.
" reduction of area.......................	18 " "

Full-size bars:

Ultimate strength	62,000 lbs.
Elastic limit	32,000 "
Average stretch	12 per cent.
Minimum "	10 " "

TABLE XXII.

WEIGHT OF TIMBER.

Oak timber weighs from 50 to 75 lbs. per cubic foot.
Pine " " " 30 to 50 " " " "
Cast iron " 450 lbs. per cubic foot.
Wrought iron " 480 " " " "
Steel " 490 " " " "

The expansion of cast iron is 0.0000062 of its length for each degree Fahr., or about 0.0004 for the ordinary range of temperature; and for wrought iron, 0.0000067 to 0.0000075 of its length per degree Fahr.

The following are tables of the comparative values of timber, as determined by Mr. Goff, of the Railway Institute, Sydney, New South Wales:

Timber loses one third of its weight when perfectly dry, as compared with green timber, with the following shrinkage ratios in breadth:

TABLE XXII (A).

English oak	$\frac{1}{12}$	Riga fir	$\frac{1}{32}$
Dantzic "	$\frac{1}{38}$	Elm	$\frac{1}{24}$
Yellow pine	$\frac{1}{38}$	Kauri	$\frac{1}{64}$
Pitch "	$\frac{1}{40}$		

Strength is defined as the property by which it resists fracture; stiffness, the property by which it resists bending or flexure; toughness, the capability of bending to the greatest extent without fracture.

TABLE XXII (B).

Kind of Timber.	Weight per cubic foot.	Strength.	Stiffness.	Toughness.
Standard	45 to 58	100	100	100
Baltic Riga	48 " 45	108	93	125
American Oak	37 " 47	86	114	117
Dantzic	42 " 53	107	117	99
Elm	35 " 46	82	78	86
Pine or fir	29 " 42	80	114	58
Poplar	33	86	66	112
Mahogany	35 to 53	96	93	99
Tamarac	32 " 40	102	80	130
Walnut	50	90	70	110

ART. XXVI.

ELASTICITY AND RESISTANCE, AS DETERMINED BY EXPERIMENT.

STONE.

250. *Fracture.*—A fresh surface should be bright, clean, and sharp, with grains well cemented together. A dull, earthy appearance indicates an inferior grade, and the stone is likely to disintegrate and decay on exposure.

Acid Test.—A few drops of dilute solutions of sulphuric and hydrochloric acid will cause a brisk effervescence if the stone is a carbonate of lime or magnesia. It will also, if fragments of the stone are soaked in the solutions, determine the presence of earthy or mineral constituents easily dissolved, thereby indicating its probable weathering qualities when placed in acid atmospheres.

Test with Small Fragments or, better, Crushed Stone, when Soaked and Stirred in Water.—If the stone is crystalline and the grains well cemented together the water will remain clear, but if it contains earthy matter the water will become turbid or milky.

Absorption.—This is a useful test. First weigh a good-sized lump after removing any loose grains or powder on the surface by gently wiping with the hand or with a cloth. Let the stone be immersed in water for twenty-four hours or more, and after allowing the surface-water to drip off weigh the fragment again. The increase is the weight of water absorbed. The best stones absorb the least percentage of water. This is an important factor in the absorption of acids and gases contained in rain-water or carried into the stone by them. High absorption indicates a stone light in weight, and is necessary for the destructive effects of freezing. It has been attempted to imitate the action of frost by soaking the stone in a concentrated solution of sulphate of soda. The subsequent crystallization of the salt forces off fragments of greater or less size. The difference in weight of the specimen indicates the extent of disintegration. This is not much relied upon.

251. *Crushing.*—Specimens of the stone are usually carefully dressed to exact cubes, 1, 1½, or 2 inches on each edge, commonly 2 inches. The specimens should be dressed, but not polished.

Cushions, from $\frac{1}{8}$ to $\frac{1}{4}$ inch in thickness, of soft pine, lead, pasteboard, plaster of Paris, or any substance that will tend to produce a uniform distribution of the pressure, are placed on top and bottom of the cube, which is then placed in a hydraulic or other power machine. The pressure is gradually increased until the cubes give way. Any indications of gradual yielding, cracking, or shearing are noted, together with the pressure at the time. Some stones give way without any warning or indications of yielding. The hardest variety of stones, such as basalts and primary limestones, give way suddenly. The softer varieties crack or show signs of yielding under from one-half to two-thirds the crushing loads. The crushing resistance is generally less in fresh or green specimens which contain the quarry sap than in seasoned. The larger the cubes the greater will be the unit crushing resistance for the same kind of stone. It would give more accurate, or rather reliable, results if the length of the specimen is about $1\frac{1}{2}$ times the sides of the base rather than a perfect cube. Though useful in many ways, the crushing resistance of small cubes is but an approximate indication of the resistance to crushing per square inch of large blocks. The crushing resistance is usually divided by a safety factor of from 8 to 10 to obtain the safe or working load.

Tearing and Shearing.—But few experiments have been made on the tensile resistance of stones, as this material is seldom used to resist direct tension. Resistance to shearing is an important property.

Cross-breaking.—The transverse strength or resistance to cross-breaking, though important, has been but little experimented upon.

Stone is but little used for beams of any great length, and generally only for lintels over doors and windows.

Weight.—The specific gravity or heaviness of stones of different kinds is a good indication of their valuable qualities, the heaviest being generally the best. As has already been mentioned, the durability or weathering properties of stones are the most important as regards the suitableness of any stone for building purposes. The above tests, so far as they have any bearing upon durability, are important. It is better when practicable to be guided by the actual weathering properties as seen in old structures.

The chemical composition of stone is of little practical value (see paragraphs 116, 117, and 118), though it is well to note the

quantities of silica, alkalies, and lime that it contains. Microscopic examination, determining the way in which the materials are cemented together, is of much more importance. Stones which may be of good quality if quarried in warm weather, may be entirely ruined if quarried in very cold weather. This is especially true of marble and limestone.

252. The following table will be useful when a proper factor of safety is used:

TABLE XXIII.

Kind of Stone.	Moduli of Resistance or Strength. Crushing, in pounds per sq. in.	Tearing, in pounds per sq. in.	Cross-breaking in pounds per sq. in. Length of Beams. In ft. f	In in. $f_1 = 18f$	Coefficients of Elasticity, in pounds per sq. in. E.	Absorption, in per cent.	Weight of, in pounds per cu. ft.
Granite:							
Connecticut	15,780 to 23,190						166.0
New Hampshire	14,480 to 24,000		108	1,854			166.0
Portland	18,700 to 18,500				5,000,000 to 6,400,000		166.5
Richmond	18,875 to 21,250				18,500,000	0.066 to 0.155	164.4
Missouri Red	12,700 to 13,600						
Massachusetts	14,750 to 17,750		50 to 150	940 to 2,740			166.2 to 168.7
Port Deposit	19,750						168.0
Canadian	11,916						167.0
Scottish	10,900						165.0
New Haven, Conn.	7,750 to 9.750						162.5
Patapsco, Md	5,340						168.0
Michigan	18,125						164.4
Staten Island, N. Y.	22,250						178.8
Trap and Basalt	19,700 to 22,250	1,469					178.8 to 189.5
Slates	7,200 to 21,347	9,600 to 12,880	200 to 450	8,600 to 8,100	18,000,000 to 16,000,000	0.066 to 0.155	165.0 to 181.0
Limestones—Marble:							
White Statuary	4,000 to 6,050						
White Italian	12,156 to 21,778	84 to 722	87 to 128	666 to 2,304	2,520,000		168.2
Wisconsin	13,700 to 20.025						175.0
New York	12,950		8 to 155	144 to 2,800			179.7
Vermont	7,612 to 8,670						164.7 to 167.8
Illinois	9,687 to 9,787	84 to 500					156 9 to 160.6
Limestones—Common:							
New York	10,750 to 25,000						168.8 to 171.9

TABLE XXIII—*Continued.*

Kind of Stone.	Moduli of Resistance or Strength. Crushing, in pounds per sq. in.	Tearing, in pounds per sq. in.	Cross-breaking, in pounds per sq. in. Length of Beams. In ft. f	In in. $f_1 = 18f$	Coefficients of Elasticity, in pounds per sq. in. E.	Absorption, in per cent.	Weight of, in pounds per cu. ft.
Limestones—Common:							
Illinois	12,775 to 16,900		32 to 128	576 to 2,340		0.20 to 5.0	158.8 to 162.5
Lime Island, Mich.	15,425 to 25,000						159.4 to 161.2
Marquette, Mich	7,600 to 7,825				1,533,000		146.3
Bardstown, Ky	15,000 to 16,250						166.9
Canton, Mo	5,650 to 9,250						146
Marblehead, Ohio	12,600						
Grafton Magnesian	6,000 7,200 to 10,100				6,000,000 10,000,000 12,000,000		
Sandstones:							
Potsdam (Red), N.Y.	42,804						162.3
Medina, N. Y.	14,812 to 17,250						149.3 to 150.6
Little Falls, N. Y.	9,150 to 9,850						140.6
Belleville, N. J	10,250 to 11,700						141.0
Middletown, Conn.	5,550 to 6,050	100 to 200	20 to 131	360 to 2,360		0.41 to 5.48	148.5
Berea, Ohio	7,250 to 10,250						137.5 to 131.9
Vermillion, Ohio	6,000 to 8,250						135.3
Marquette, Mich.	5,730 to 7,450						135.0 to 158.0
Seneca, Ohio	9,687 to 10,500						149.3
Fond du Lac, Wis.	5,110 to 6,250					Some varieties from 3 to 10.	138.8
Albion, N. Y.	11,350 to 13,500						151.0
Haverstraw, N. Y.	4,350						
Derby and Cheshire, England	2,185 to 3,100						133.0 to 150.0
Edinburgh, Scotland.	11,250 to 12,000						141.3
Pavement:							
Arbroath, Scotland.	7,884	1,261					155.0
Craigleith, Scotland.	5,287	453					153.0
Bluestone:							
Flagging			200	3,600			167.1 to 171.5
Bricks:							
Common	600 to 3,000	150 to 300	10 to 30	180 to 540		2.0 to 25.0	100.0
Paving	9,000 to 15,000					0.15 to 8.00	
Best pressed	14,973		30 to 50	540 to 900			150.0
Hard	12,000						125.0

The preceding tables are taken from various sources, and are designed to show fair averages. The smaller values for resistances to crushing refer in most cases to crushing on edge, the larger values when crushed on natural beds.

The tables show how incomplete are the records in regard to tensile, transverse, and shearing strengths, and also how different are the results of experiment in regard to crushing strength, as indicated by the wide range of resistances per square inch even in the same kinds of stone, resulting from the dimensions of the specimens, modes of experimenting, conditions of surfaces—whether polished or not, the kinds of cushions used, etc. For instance, the Connecticut granite of Millstone Point gives with unpolished surfaces a crushing resistance of 17,750 pounds per square inch, whereas with polished surfaces for the cubes the same stone gives 22,880 pounds. The same stone with steel cushions gives 23,190 pounds, with wood 22,880, and with lead 15,730.

Massillon (Ohio) sandstone gives with leather cushions 3640 pounds, with lead 5500, with wood 6730, and with steel 5660. And similar results are found for many other varieties of stone.

The tables, however, indicate the relations between resistances to crushing, percentage of absorption, and weights per cubic foot, which will be of much assistance in determining the probable value of any given stone for building purposes, especially in regard to wear and durability.

Stones are seldom used where tensile resistance or transverse strains are called into play, and our imperfect knowledge on these kinds of strain will not be likely to cause injury or damage.

And, fortunately, even the weakest of the weak stone have ample crushing resistance to bear safely the heaviest loads that are likely to be placed upon them. From 15 to 20 tons per square foot, equivalent to from 233 to 380 pounds per square inch, are very unusual pressures or loads, and usually not more than from one half to two thirds of these are actually known to exist, it being estimated that in very high winds the pressures on the leeward sides of high towers may reach the values above given.

ART. XXVII.

STRENGTH OF CEMENT, MORTAR, CONCRETE, BÉTON, AND ARTIFICIAL STONES.

253. THE strength of artificial stones, other than brick, terra-cotta, and similar materials, depends to a large extent on the qualities of the cementing materials used. It will be better, then, to consider first the strength of cements and mortars.

Comparatively but a small number of experiments have been made to determine the compressive resistance of mortars, this resistance being inferred from or based on the tensile resistance of the same mortars. Both of these vary with the kind and proportion of sand used, and the time which has elapsed between the mixing of the ingredients and the tests made, as well as upon disposition of the samples meanwhile, whether exposed to the air or immersed in water for a part or the whole of this period of time. It is essential that all of these facts and conditions should be known.

254. The following are some of the experiments made by General Q. A. Gillmore, Mr. Bremermann, Mr. John Grant, and Mr. Henry Reid on the compressive resistance of many brands of cement mortar.

General Gillmore used cubes dressed from the parts of briquettes torn asunder in determining the tensile resistance of the same cements. Both the tensile strength and compressive strength will be here given in advance of explaining the methods used in determining the tensile strength of mortars, in order to avoid repetition.

The mortar was composed of one part of cement and one part of sand mixed dry and tempered with water to the consistency of stiff mason's mortar. The cubes were $1\frac{1}{2}$ inches on each edge, equivalent to $2\frac{1}{4}$ square inches on each face; exposed one day in air and immersed six days in water.

TABLE XXIV.

PORTLAND CEMENTS.

	Resistance to Crushing in lbs. per sq. in.	Resistance to Tearing in lbs. per sq. in.
Stettin German cement.......	1439	216
London "	1140 to 1330	199 to 216
Saylor's American "	1078	184
Teil, France.................	931	158
Ottawa, Canada..............	882	141
Boulogne-sur-Mer, France.....	764	108

ROMAN AND OTHER CEMENTS.

Coplay Cement Co., Pa........	292	38
Cumberland Cement Co., Md..	196	41
Howe's Cave, N. Y...........	170 to 183	28 to 43
Scott's selenitic cement...... } Howe's Cave lime and plaster }	208	52
Parian cement, London.......	205 to 1175	51 to 181

The following tests were made on specimens $2\frac{3}{4}'' \times 4\frac{1}{4}'' \times 9''$ Pressure applied on flat sides $9 \times 4\frac{1}{4} = 38.25$ sq. in. The results are given in pounds per square inch.

PORTLAND CEMENT.

Compressive resistance in lbs. per sq. in.

	Age 3 months.	Age 6 months.	Age 9 months.
Neat............	3795	5388	5984
1 cement, 1 sand..	2491	3478	4561
1 " 2 " ..	2004	2752	3647
1 " 3 " ..	1436	2156	2393
1 " 4 " ..	1331	1797	2208
1 " 5 " ..	959	1540	1678

The following experiments were made by Mr. Bremermann, under the direction of Capt. Eads, during the construction of the St. Louis Bridge. The specimens were 12″ long and 2.5″ in diameter. The cement was the Fall City, Louisville.

TABLE XXV.

Louisville Cement.	Resistance to Crushing, in lbs. per sq. in.			Coefficiency of Elasticity, in lbs. per sq. in.			Limit of Elasticity, in lbs. per sq. in.			Age in Days.
	Max.	Mean.	Min.	Max.	Mean.	Min.	Max.	Mean.	Min.	
1 cement, 0 sand	1889	1320	620	1,500,000	800,000	500,000	1502	800	424	123 to 143
1 " 1 "	783	494	261	1,910,000	607,000	217,333	587	365	191	117 to 141
1 " 2 "	489	213	131	6,633,330	1,285,000	220,450	424	182	96	127 to 135

The coefficient of elasticity of concrete may be taken at about one-fortieth that of mild steel.

TABLE XXVI.

Another set of experiments, made on 2½″ cubes, left for 12 days in water and 6 months in the air, resulted as follows:

				Lbs.
Akron cement.........	1 cement,	0 sand;	ultimate crushing resistance,	2140
" "	1 "	1 "	" " "	733
" "	1 "	2 "	" " "	240
" "	1 "	4 "	" " "	480
Fall City..............	1 "	0 "	" " "	1587
" "	1 "	1 "	" " "	400
" "	1 "	1½ "	" " "	240
Beach & Co., Louisville.	1 "	0 "	" " "	1615
" " "	2 "	1 "	" " "	1280
" " "	1 "	1 "	" " "	560
" " "	1 "	1½ "	" " "	400
" " "	1 "	2 "	" " "	280
Hulme & Co., Louisville	1 "	0 "	" " "	2320
" " "	1 "	1 "	" " "	740
" " "	1 "	1½ "	" " "	600

These tables show the effect of increasing the volume of sand in reducing the resistance to crushing. A 4-inch cube of Ransome's patent siliceous stone gave a resistance of 4200 pounds per square inch.

255. Concrete cubes composed of Akron and Louisville cement in the proportions given in the following table gave the resistance to crushing as shown. The blocks were kept in water twelve days and then exposed to the air for six months. Cubes 6″ on each edge.

TABLE XXVII.

	Resistance to Crushing in lbs. per sq. in.
1 Akron cement, 1 sand, 4 broken limestone........	889 to 1170
1 " " 2 " 4 " "	722 " 1361
1 Louisville " 1 " 4 " "	1194
1 " " 2 " 4 " "	640 to 950
½ Akron and ½ Louisville cement, 1 sand, 4 broken limestone..................................	1170 to 1445
½ Akron and ½ Louisville cement, 2 sand, 4 broken limestone..................................	611 to 1361

2-inch cubes of silicated stone composed of 1 part Portland cement and 3 parts Thames ballast, gauged with water, and placed in silicate bath 11 days, gave crushing resistance of 4257 to 5650 pounds when 12 months old.

Blocks of Sorel stone, made by a French chemist, containing from 12 to 15 parts oxide of magnesium and mixed with sand or powdered marble, after being made into paste, hardens sufficiently to be handled in twenty-four hours to three or four days; and at the age of from one to three years has a resistance to crushing of from 4920 to 21,560 pounds per square inch.

The following table, taken from Mr. J. Grant's experiments, shows the effects of compressing the concrete in layers and not compressing the layers in making the blocks, and also the relative results obtained from specimens exposed to the air and when immersed under water. The resistance is given in tons per square foot, the ton being 2240 pounds.

In these blocks the Portland cement used weighed 110.56 pounds per bushel, and had a tensile strength of 427 pounds per square inch after seven days' immersion in water. The compressed blocks were in layers of 1 inch thick, and compressed by ramming.

It is seen that the blocks containing the larger proportion of cement are the stronger nearly in the proportion to the quantity of cement. The average of those kept in water are the stronger when the concrete was compressed or rammed, and somewhat irregular when not compressed.

In a recent report on the proposed Hudson River Bridge, by a

TABLE XXVIII.

ULTIMATE RESISTANCE IN TONS PER SQUARE FOOT.

One Volume of Portland Cement to Volumes of Ballast of Sand and Gravel.	Compressed.				Not Compressed.	
	Blocks 12″ × 12″ × 12″.		Blocks 6″ × 6″ × 6″.		Blocks 6″ × 6″ × 6″.	
	Kept in Air.	Kept in Water.	Kept in Air.	Kept in Water.	Kept in Air.	Kept in Water.
1	Exceptional 107	170	152	134.5	120	150
2	149	160	172	138	154	144
3	118	115	120	142	96	112
4	103	108	120	112	112	108
5	89	99	98	142	96	94
6	80	91	81.6	74	72.8	68
7	75	80	66	64	56	50
8	61	76	54	54	50	44
9	54	68	48	44.5	40	36
10	49	48	42	42	32	28

board of eminent engineers, the strength of concrete to resist crushing is given as follows: Proportions, 1 cement, 2 sand, 5 broken stone in cubes 1 foot on edge. Hardened in water 45 days, 425 pounds per square inch, or 30.5 tons per square foot. Hardened in water one year, 1,520 pounds per square inch, and in air one year, 1,620 pounds per square inch.

THE DETERMINATION OF TENSILE STRENGTH OF CEMENTS.

256. As has already been mentioned, concretes and mortars are almost exclusively subjected to compressive strain, rarely to tensile strain; yet for reasons of convenience in testing the tensile strain is determined, and from it the compressive resistance is inferred or deduced from the experiments. It is usually taken as equal to from 8 to 10 times the tensile strength of the same mortar at the same age.

The tensile strength of the same cement paste or mortar depends upon the form of the specimen or briquette, the method of gauging the cement, the amount of water used, and the manner of making the test.

The briquettes are made either of neat cement or of cement mixed with any desired proportion of sand.

Briquettes of Neat Cement.—The sample barrels of cement should be bored into with an auger, and the portions taken out by the auger should be mixed, turned over, and allowed to cool. If sacks are used, samples should be taken out of a number of them and treated as above.

From this heap a small quantity is worked into a paste with sufficient water to cause a little moisture to appear when pressed or gently tapped a few times with the trowel. This paste should then be made into cakes 3 or 4 inches in diameter and from $\frac{1}{2}$ to $\frac{3}{4}$ inch in thickness.

If it requires two hours or more to become firm enough to resist the pressure with the finger or a nail, it will be classed as a slow-setting cement, and many briquettes can be made at the same time. If quick-setting, only enough for three or four briquettes should be mixed. The paste then is placed in moulds of brass. These moulds are generally split longitudinally, and the two parts held together by springs, screws, or catches. Sometimes they are hinged at one end.

The mould should be wiped out with a greasy cloth, and placed on slate, marble, or glass plate. Sometimes a few pieces of blotting-paper are first placed on the plate, the moulds resting on the paper. The proper quantity of cement is then mixed with sufficient water to bring it to a good paste. The mixing should continue until the paste presses slick and smooth under the trowel. The mixing should be done on the non-absorbent plate. The moulds are then filled quickly, pressing or gently ramming the paste so as to fill the moulds with a homogeneous mass free from air bubbles or spaces. It is then smoothed off even with the edges of the moulds. The moulds should then be numbered and placed in some damp place, or kept damp by spreading a wet cloth over them. In some cases the moulds can be opened and removed in a few hours, sometimes only after twenty-four hours. This should be carefully done, so as not to leave flaws or mashed edges. Unless the cement is very slow-setting, the briquettes can be placed in water in about twenty-four hours after mixing and moulding. If necessary they must be kept on glass plates in a damp place until they can be placed in water. The water should have a temperature of 60° to 70° Fahr.

257. *Briquettes of Cement and Sand.*—The cement and sand may be proportioned by weight or by volume: 1 cement to 1 sand, 1 to 2 or 1 to 3, etc. These should be mixed dry and then worked into a paste with the proper quantity of water, clean and pure and of the proper temperature. The paste is then pressed or rammed into the mould. It must be recollected that with sand it requires a longer time to become firm enough for removal from the moulds or immersion in water. If done too quickly, the samples will not keep their forms.

An excess of water will give a porous specimen, which also takes longer to harden. From 10 to 20 per cent of water will be sufficient. Experiments have shown that an increase from 19 to 25 per cent of water was accompanied by a loss of from 28 to 40 per cent in tensile strength. With hot and quick-setting cements more water is required than with cool and slow-setting ones. Salt water increases the strength somewhat, but it should not be used in walls of houses. The water should not be dirty or muddy, as it injures the cement. Hot water should not be used unless it is desirable to hasten the setting.

Many forms of briquettes have been adopted—with square heads connected by a square neck, or rounded heads with square necks or connecting prisms. The form recommended by the American Society of Civil Engineers is shown in the following diagrams. The smallest section between the heads is usually 1 square inch, 1 inch on each side. It is sometimes made $1\frac{1}{2}'' \times 1\frac{1}{2}'' = 2\frac{1}{4}$ square inches.

258. When the samples are to be tested they are placed in clips connected with properly arranged levers so as to give a straight pull. The clips are made so that they only touch the specimens at four points. If the pressure is distributed over any appreciable surface on the specimen there will be a want of uniformity and directness in the stress. In the more approved forms of briquettes the changes in dimensions are very gradual from the head to the middle or smallest cross-section.

The dimensions and sections of a common form of briquette are shown in Figs. 104, 108, and of the more approved form as giving the better results are seen in Figs. 105, 109. The clips and the method of taking hold of the specimens are shown in Figs. 106, 108, and 109.

A simple testing apparatus for crushing, shearing, and tearing resistance is given in Fig. 107, and a more delicate apparatus in Fig. 111.

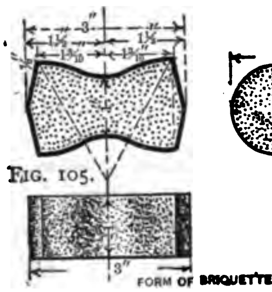


FIG. 105.

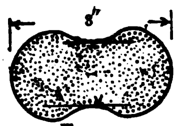


FIG. 104.

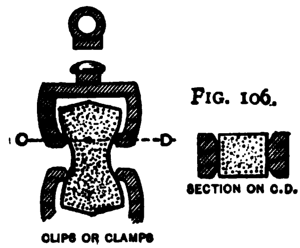


FIG. 106.

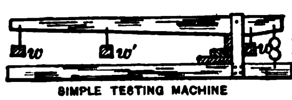
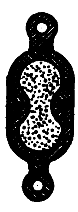
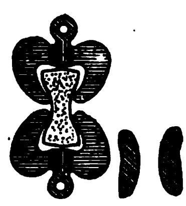


FIG. 107.



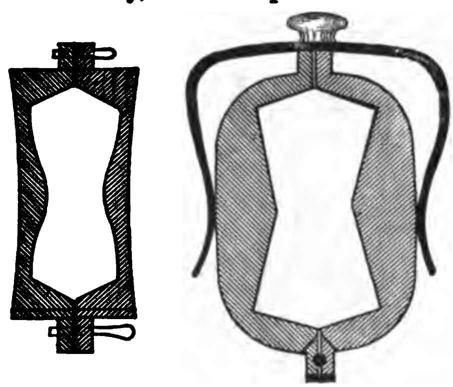


CLIPS OR CLAMPS AND BRIQUETTE IN POSITION.

FIG. 108. FIG. 109.

The usual forms of moulds are shown in Fig. 110. During the testing the weights should be applied uniformly and very gradually, at a rate not exceeding 400 pounds per minute, especially when nearing the breaking-point. There are other forms, methods, and apparatus used. Care and accuracy are essential.

The principles and application of the simple beam machine, Fig. 107, are clear. In this case it can be readily made by any mechanic, but is of course rough, and will give only approximate results. These beams are often carefully mounted on knife-edges set in a pillar mounted on a stand. Arrangements are made by chords and pulleys by which the sliding weight *w'* can be moved gradually and uniformly, and also prevented from slipping when



MOULDS FOR BRIQUETTES
FIG. 110.

rupture of the specimen occurs. This apparatus can be made to register very accurate results.

A more delicate apparatus is shown in Fig. 111, which is known as Michaelis' Double-lever Cement-testing Apparatus, the combined leverage of which is 1 to 50; that of the longer is 1 to 10, and that of the shorter 1 to 5. Each lever has three hardened-steel knife-edges acting upon hardened-steel concave bearings, so that an extremely accurate balance is obtained. The short arm of the upper lever is provided with a movable counterpiece to secure the correct position of the levers, which is indicated by a mark on the upright catch at the top of the column. At the extremity of the long arm is suspended a small brass frame to carry the shot-bucket.

On the lower lever, near the fulcrum, is suspended the upper clamp or clip for holding the briquettes. The lower clip is fixed to the base of the column and adjusted by means of a screw. When ready to make the test, the briquette is removed from the water, dried, and placed into the clamps, which must be accurately

adjusted to the sides of the briquette, and the screw applied until the upper edge of the long lever is opposite the mark on the upright catch. Fine shot are then allowed to roll from the shoot just above into the bucket suspended from the long lever until the briquette breaks, and at this point the supply of shot is cut off. The breaking stress is then exactly fifty times the weight of the shot and the bucket. This weight must be accurately determined.

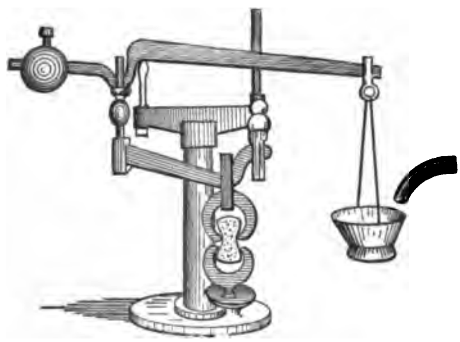


FIG. 111. Michaelis' Cement-testing Apparatus.

The above is taken from "Notes on Building Construction," where will also be found a large number of other machines, with full description of the manner of using them. These machines are capable of testing to 1000 pounds.

259. The following table gives a fair average range of tensile strength of Rosendale and Portland cements in pounds per square inch, in which the weakening effects of the sand are shown in all conditions and ages:

TABLE XXIX. (Byrne).

Composition of the Mortar.		Rosendale.				Portland.			
		Age of Mortar when Tested.				Age of Mortar when Tested.			
Cement.	Sand.	1 Week.	1 Month.	6 Months.	1 Year.	1 Week.	1 Month.	6 Months.	1 Year.
1	0	100	180	275	300	300	400	450	500
1	1	60	100	180	225	175	250	340	375
1	2	25	60	125	170	120	150	245	290
1	3	20	40	80	120	90	110	175	220
1	4	15	25	60	90	75	75	130	170
1	5	10	15	50	80	60	65	110	130
1	6	6	10	45	75	50	35	90	100

260. As to the fineness of sand and its effects on the strength of mortars there is much difference of opinion. The following tables give the results of a few experiments, showing the weakening effects of fine sand. Portland cement was used. The figures represent pounds per square inch.

TABLE XXX.

Age of Mortar.	1 Cement, 1 Sand.		1 Cement, 2 Sand.		1 Cement, 3 Sand.	
	Fine Sand.	Coarse Sand.	Fine Sand.	Coarse Sand.	Fine Sand.	Coarse Sand.
1 week....	85	95	33	68	19	41
1 month...	162	202	81	94	49	74

The following table is designed to show the results of some experiments made to determine the relations between the weight and tensile strength, the specimens being seven days old:

TABLE XXX (A).

Weight per Bushel.	Brand.	Tensile Resistance in lbs. per square inch.	Per cent. passing a No. 50 Sieve, 2,500 Meshes.
101.5 to 108	Alsen's (German)	326 to 340	93 to 91
112 " 120	Burham	252 " 317	84 " 90
121 " 132	Saylor's (American)	260 " 369	78 " 90

261. From these and many other experiments the following conclusions have been drawn by the experimenters. Not that all agree on these: in fact on many points decidedly different opinions are given.

The tensile strength is increased materially at first by raising the temperature of the water from 32° to 70° Fahr., but after an interval of several weeks the effects are not so marked, and may be actually diminished by an increase in the temperature of the water.

The slower the cement is in setting the more its strength increases. Mortar made of equal quantities of sand and cement will have, at the expiration of one year, three fourths the strength of neat cement; 1 cement, 2 sand, one half; 1 cement, 3 sand, one third; 1 cement, 4 sand, one fourth; and 1 cement, 5 sand, one sixth the strength of neat cement.

The cleaner and sharper the sand the greater the strength. The stiffer the cement is gauged, that is, the less the amount of water used, the better.

If immersed in water the mortar will be stronger than out of it. Salt water is as good as fresh water in mixing cement (Portland). Neat Portland cement bricks when six to nine months old are as strong as the best bricks.

Portland cement bricks, 1 part cement and 4 to 5 sand, will be as strong as the best stocks.

The strength of concrete increases with the proportion of cement used, except in case of very weak cements. It is essential that the bricks and stones used should be soaked in water.

A No. 80 sieve should pass from 80 to 90 per cent of the cement, and a No. 50 should pass not less than 90 per cent and often as much as 95 per cent. A No. 80 sieve contains 6400 meshes per square inch; a No. 50, 2500 meshes. Cements are ground much finer than the above.

Without any well-established standards for coarse and fine sands, we may say that a sand would be called coarse that will pass a No. 8 sieve, 64 openings to the square inch, but will not pass a No. 16, 256 meshes per square inch; and a fine sand, one that will pass a No. 16 but will not pass a No. 25, 625 meshes per square inch. Much finer sands are, however, often used.

262. *Adhesion.*—But few experiments have been made to determine the adhesion of mortars to other materials. The following results are taken from various sources:

TABLE XXXI.

	Adhesion to—	Pounds per sq. in.	Age in Days.
Quicklime mortar:			
1 lime, 2 sand	limestone	9 to 15	16
1 " 2 "	"	15	180
1 " 2 "	hard bricks	40	180
1 " 2 "	soft "	18	180
Quick-setting mortar (cement):			
1 cement, 2 sand	hard bricks	28	7
1 " 2 "	" "	59	30
Slow-setting mortar (cement):			
1 cement, 2 sand	hard bricks	15	7
1 " 2 "	" "	30	30

TABLE XXXI (*Continued*).

	Adhesion to—	Pounds per sq. inch.	Age in Days.
Rosendale mortar (cement):			
1 cement, 0 sand (neat)	Croton bricks	30.8	30
1 " 1 "	" "	15.7	30
1 " 2 "	" "	12.8	30
1 " 3 "	" "	6.8	30
1 " 4 "	" "	5.2	30
1 " 0 " (neat)	" "	68.4	320
1 " 1 "	" "	40.0	320
1 " 2 "	" "	24	320
Rosendale mortar:			
neat	cut granite	27.5	30
1 cement, 1 sand	" "	20.8	30
1 " 2 "	" "	12.0	30
1 " 3 "	" "	9.2	30
1 " 4 "	" "	7.9	30
Portland mortar:			
neat	bricks	218	28
1 cement, 1 sand	"	105 to 146	28
1 " 2 "	"	45 to 73	28
1 " 3 "	"	24 to 48	28
1 " 4 "	"	14 to 45	28
neat	sawed limestone	78	30
"	cut granite	97	30
"	Bridgewater bricks	66	30
"	sandstone	49	30
"	bricks	68.8	42
1 cement, 2 sand	"	46.9	42
1 " 2 "	"	569	56
1 " 2 "	"	54	56
1 " 2 "	"	45 to 123	1 yr.
1 cement, 1 sand	"	44 to 62	1 "

The test for adhesion is made by cementing bricks together and separating by a direct pull. With strong cements the pull may either break the bricks themselves or the cement mortar may give way by tearing apart itself. This was the case with some of the above tests, and is consequently no measure of the adhesion. Some cements which have high tensile strength give low values for adhesion, and *vice versa.* The adhesion of mortars to bricks or stones varies greatly with the kind of brick or stone, especially with

their porosity. It also varies with the kind of cement, kind and quantity of sand, the age of the mortar, and the condition of the material with regard to cleanness and wetness or dryness.

263. The resistance to shearing of mortar in joints of masonry and parallel to the joints has been found as follows:

TABLE XXXII.

Portland cement	mortar,	neat					78.9	pounds,	when	49	days	old.
"	"	"	1 cement,	1	sand,		155	"	"	52	"	"
"	"	"	1	"	2	"	106.6	"	"	52	"	"
"	"	"	1	"	3	"	72.5	"	"	42	"	"
Hydraulic lime		"	1	"	3	"	76.8	"	"	90	"	"
Quicklime		"	1	"	3	"	7.1	"	"	90	"	"

SHEARS IN CUBES OF CEMENT MORTARS DRIED IN AIR.

Portland cement (Bonn): neat, 369.7; 1 to 1, 369.7; 1 to 2, 284.4; 1 to 3, 142.2 lbs. per sq. in .... 50 days old.

Portland cement (Perlmoos): neat, 256.0; 1 to 1, 405.8; 1 to 2, 383.9; 1 to 3, 362.6; 1 to 4, 320 lbs. per square inch .... 60 days old.

The shearing resistance of mortars in cubes seems to be greater than the tensile strength at the same age and under the same conditions by about 20 per cent.

Quick-setting cements seem to give greater adhesive strength than slow-setting cements. The adhesive and shearing resistance of mortars in those masonry structures which are subjected to lateral pressures are more important considerations than they seem to have been credited with; and in fact so little is known in regard to these matters that but little if any value is given to them in providing stability against sliding or overturning of masonry structures. Friction of masonry on masonry and sufficient weight are alone considered.

264. The crushing resistance of mortar as determined with cubical blocks furnishes no idea of its strength when in thin layers, as in the joints of masonry structures.

It is an undoubted fact that the strength of mortars against crushing increases, as the thickness compared with the base or bed area decreases, per unit of area of base, but no experiments seem to

increase?

have been made to determine the law of this decrease. Altogether, it seems that the determinations of tensile strength alone of mortars furnish but a vague and unreliable standard by which to judge of the value and suitableness of a given mortar for building purposes.

265. The true principle in mixing mortars is to use as much sand as possible, without unduly reducing the strength of the mortar, from considerations of economy. So long as the mortar is weaker than the stone or bricks used, the strength of the wall will increase as the strength of the mortar increases, until the two are nearly equal. When they are equal in strength, the fracture would follow a straight line rather than along the joints. This can be brought about in brickwork and in walls built of ordinary stone, if not with the hardest stones. It is, however, not only unnecessary but wasteful to make the mortar stronger than the bricks or stones to be united. But little if any attention seems to be given to these considerations in fixing the proportions of cement to sand in preparing mortars, exactly the same proportions being frequently used both in large thick masses of concrete supporting masonry structures and in the thin masonry joints of piers.

266. *Concrete.*—The tensile strength of concrete is of no great importance, as concrete is seldom, and should never be, put under a tensile stress. When used in foundations it may be called upon to act as a beam in case of undermining or unequal settling of the foundation-bed, and in large thick masses may save a structure from destruction, at least for a time. It then acts, however, as a beam. The following tables give some average results of experiments on the transverse resistance of cement and concrete beams. The prisms were all broken by the application of a centre weight, and from this the value of f, the modulus of rupture, was then calculated from the usual formulæ. If the weight of the concrete prism is W_1, and is taken into consideration, then

$$f = \frac{3}{2}\frac{(W + \frac{1}{2}W_1)l}{bd^2}, \quad \ldots \ldots \quad (110)$$

W being the applied centre weight, and b, h, l all in inches. If the weight of the beam is not considered, or is to be included in W, the centre weight, the value of f is found from the usual formula,

$$f = \frac{3}{2}\frac{Wl}{bd^2}. \quad \ldots \ldots \ldots \quad (111)$$

TABLE XXXIII.

SECTION OF PRISMS 2 INCHES SQUARE. CLEAR LENGTH BETWEEN SUPPORTS 4 INCHES.

Kind of Cement.	Composition of Mortar.	Pressure White Setting per sq. in.	Weight at Centre W, in lbs.	Modulus of Rupture f, in lbs.	Age of Mortar.
James River......	2.6 vols. water...... 4 vols. dry cement...	0.0	281.5	211	59 days.
" "	1.4 vols. water...... 4 " dry cement.	32.0	497.5	373	" "
Hoffman's Rosendale.	Pure cement and water (thin).......	0.0	646.0	485	320 "
Hoffman's Rosendale.	Pure cement and water (thin).......	32.0	692.5	519	" "
Delafield and Baxter, Rosendale.	Pure cement and water (thin)	0.0	613.0	459	" "
Delafield and Baxter, Rosendale.	Pure cement and water (stiff)	32 0	871.5	654	" "

These samples were kept in a damp place for twenty-four hours, the remaining time in salt water.

Kind of Cement.	Pure Cement.	1 vol. cement, 1 vol. sand.	1 vol. cement, 2 vols. sand.	Age of Cement.
	(f in pounds per square inch.)			Days.
English Portland........	1152	945	713	320
Cumberland, Md.........	716	690	419	"
Newark and Rosendale...	631	420	375	"
Shepherdstown, Md.....	560	464	338	"
Akron, N. Y............	573	489	453	"
Lawrence, Hoffman brand	656	684	—	1 year.
Round Top, Md.........	—	630	—	1 "

These samples were also kept in salt water after the first twenty-four hours. Samples 2 inches square; clear length, 4 inches.

TABLE XXXIV.

Concrete prisms 6 × 6 inches × 2 feet; clear distance between supports 12 inches, the ends overhanging. In this case, considered as simply supported at ends,

$$f = \frac{3}{2}\frac{Wl}{bd^2} = \frac{3}{2}\frac{W \times 12}{6 \times 36} = \frac{W}{12}. \quad . \quad . \quad . \quad . \quad (112)$$

All of the prisms were composed of 1 vol. Portland cement, 2 vols. sand, 5 vols. small broken stone. All prisms were one month old.

Temperature, Fahr. Of Air.	Temperature, Fahr. Of Concrete when Mixed.	Exposure of Samples after Mixing.	Centre Weight W in lbs.	Modulus of Rupture f, in lbs.
18°	40°	In river.	525	44
18	40	Exposed outside.	775	60
18	40	" indoors.	1125	94
18	98	In river.	175	15
18	98	Exposed outside.	325	27
18	98	" indoors.	750	63
24	40	" outside.	1800	150
24	97	" "	800	67
32	40	" "	1475	123
32	98	" "	700	58

267. The following is taken from the *American Architect and Building News* of September 2, 1893: "The extended and extending use of concrete in the form of arches, beams, and slabs in bridges, floors, pavements, etc., render any recent experiments on the strength of concrete very important. In the following experiments the slabs, or, more properly, beams, were made with good Portland cement (having a tensile strength of 665 pounds per square inch when seven days old) and clinker obtained from furnaces which burned ash-pit refuse. The clinker was crushed and passed through a screen with three-quarter-inch meshes, and thoroughly washed with clean water. This was done to prevent the concrete from swelling and 'blowing' after setting. The proportions were: (1) 1 part cement to 4½ clinker; (2) 1 part cement to 6 clinker; (3) 1 part cement, 6 parts of the coarse clinker, and 2 parts of clinker ground to the fineness of coarse sand. The con-

crete was well rammed into greased wooden moulds. From each of the above mixtures three slabs or beams were made of the following dimensions, respectively: 21 × 18 × 4 inches; 30 × 18 × 6 inches; and 39 × 18 × 9 inches. They were kept dry until tested. The clear length of span for the above beams were 12, 18, and 27 inches, respectively. Using the formula for modulus of rupture, $f = \frac{3}{2}\frac{wl}{bd^2}$, f taken in hundredweights (cwt.) of 112 lbs., it was found that for concrete (1), proportions 1 to $4\frac{1}{2}$, $f = 1.9$ cwt. $= 202.8$ pounds, age 15 days, and 2.8 cwt. $= 313.6$ pounds, age 21 days; for concrete (2), 1 to 6, $f = 1.2$ cwt. $= 134.4$ lbs. in 14 days, and only 1.1 cwt. $= 123.2$ pounds in 21 days; for concrete (3), 1, 6, and 2 parts, or 1 to 8, $f = 0.3$ cwt. $= 33.6$ pounds in 14 days, and 0.4 cwt. $=$ 44.8 pounds in 21 days. These tests were not entirely consistent, some of the beams giving two times the strength of others of the same series. The beams yielded suddenly, with no signs of yielding before the total collapse. The age of the concrete was not sufficient to give satisfactory results. These experiments confirm the belief of the unsuitableness of concrete, when used alone, for resisting transverse stresses. These beams were simply supported at the ends. If they had been fixed, the probability is that the lower surfaces would have cracked before the total collapse, and the strength might have been increased three or four times that actually found.

A beam composed of equal volumes of Portland cement and coke breeze, tested in 1891, yielded a value of $f = 5.9$ cwt. $= 660.8$ pounds when only seven days old; and another beam, composed of 1 part Portland cement and 4 parts clean breeze, gave a value for $f = 4.1$ cwt. $= 459.2$ pounds when 43 days old.

268. *Accelerated Tests for Soundness of Cements.*—It is generally conceded that neither chemical analysis nor the usual tests for setting and tensile strength of cements are of much value in determining the durability of a cement. The cold-water test requires a longer time to give reliable results than is usually available. Therefore, heat tests of various kinds, such as subjecting the mortar to the heat of a flame or kiln, or to hot water and steam, have been recommended and tried. A series of such tests have been made by Mr. Fred. P. Spaulding. (See *Engineering News*, Aug. 24, 1893.)

The durability or soundness of hydraulic cement is more important than its tensile strength when a few days old. Most cements

will prove strong enough for the usual purposes of construction, but their strength is of small moment if in a few weeks or months they will crack and disintegrate. Unsoundness is commonly due to an excess of the lime, or to an imperfect combination of the lime with the silica and alumina. An over-limed cement rapidly disintegrates or swells in sea-water, owing to the dissolution of the lime by the salts therein, or to the formation of crystalline compounds with the sulphates in the water, causing expansion. This is usually called "blowing" of the cement. This may occur in air, but more commonly in water, especially sea-water; it may occur in a few hours after mixing, and it may not show itself for months after mixing. Disintegration may arise from the use of dirty water, or dirty sand, exposure of green mortar to severe frosts, or from many other causes. It is proposed to determine the question of soundness by hastening those changes and combinations which bring about the setting and hardening of cement mortars. There are other materials than lime which cause disintegration, such as aluminate and ferrate of lime, sulphate of lime or gypsum, and also magnesia. The dry-heat test, such as subjecting the mortar to the intense heat of a flame would seem to act injuriously, whereas much valuable indication of the soundness of a cement may be obtained from the moist-heat tests. The presence of moisture seems essential to proper hardening and setting. Whether the temperature should be above or below the boiling point of water—that is, in water under 212° Fahr., or as steam at 15 atmospheres of pressure, 390° Fahr.—is not settled. Moderate temperatures would seem to be advisable, as corresponding more nearly to the normal conditions accompanying the setting of cement. (For an interesting and full discussion of this subject, see the *Engineering News*, Aug. 24, 1893.)

A cement which resists well a hot-water test of 180° is pretty certain to be free from uncombined lime. No cement should be used in sea-water which cannot resist the ordinary hot-water test. In this test M. Dival advocates the immersion of a pot of neat cement in water at 80° C., 177° Fahr., and asserts that if the cement disintegrates it is unsound. Mr. Henry Faija recommends a vapor-bath at about 100° Fahr., until the cement sets hard, and then the immersion of the pot of cement in water at from 112° to 117° Fahr. for twelve or fifteen hours. He claims

that good cements will fail in water at 117° Fahr. Some Portland cements have been boiled (at 212° Fahr.) for two weeks without signs of failure. Probably most of the failures of cement in sea-water are due to want of care in making the mortar or concrete and in placing it in the structure. A concrete made of a comparatively poor cement will withstand the disintegrating action of sea-water (not including the action of the force of waves) if care is taken to make it impervious to the water—that is, by covering it with a skin of cement mortar or rich concrete.

269. Mr. R. Feret, Director of the Laboratory of the Ponts et Chaussées at Boulogne (see *Engineering Record*, Sept. 2, 1893), attributes the failure of mortar in sea-water to the want of sufficient cement and inferior grades of sand in the mortar, and not entirely to the use of bad cement. He claims that it is not reliable to trust to the ordinary determinations of the volume of voids in sand as the measure of the weight of cement-paste to fill them; that with sands having the same weights per unit of volume when measured under identically the same conditions, and consequently having the same volume of interstices, the mortars obtained by mixing the same quantities of cement with equal volumes of these sands are far from possessing the same qualities; and recommends making a number of samples with varying proportions of cement to determine that proportion best suited, with the materials used, for the purpose intended. The following table shows the volumes and weights of coarse and fine sand necessary to be mixed with a given weight of cement to produce the same strength:

TABLE XXXV.

PROPORTIONS OF SAND AND CEMENTS IN MORTAR FOR SEA-WORKS.

Resistance to compression after five months in sea-water per square inch.	....	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
Weight of cement to be mixed with unit weight of sand to attain corresponding strengths.	Coarse sand..	0.09	0.15	0.20	0.25	0.29	0.35	0.41	0.50	0.62	0.82
	Fine sand....	0.21	0.34	0.45	0.54	0.63	0.73	0.86	1.04	1.26	1.58
Volume of cements to be mixed with 1 volume of sand to attain same strengths.	Coarse sand..	0.10	0.16	0.22	0.27	0.33	0.39	0.46	0.56	0.69	0.92
	Fine sand....	0.20	0.34	0.44	0.53	0.62	0.72	0.84	1.02	1.24	1.56

The volumes were deduced from the weights by assuming the weights of the materials per bushel as follows: Cement 120 pounds, coarse sand 134 pounds, and fine sand 118 pounds. It is to be observed that it requires about twice as much cement with the fine sand as it does with the coarse sand to obtain the same strength of mortar. Therefore the determination of the proper proportions of sand and cement by analogy from those used in other places should be avoided.

TABLE XXXVI.

Proportionate volumes of sand and cement...	1 cement, 6 sand	1 cement, 8 sand
Volume of mortar resulting from 1 volume sand..................................	0.940 to 1.030	0.970 to 1.180
Weight of cement in 1 cubic yard of mortar, pounds..............................	410 to 450	710 to 870
Absolute volume of solid matters (cement and sand) contained in 1 volume of mortar...	0.570 to 0.787	0.565 to 0.728
Resistance to compression after immersion for a year in sea-water in pounds per square inch..........	425 to 1,490	1,060 to 3,700
Proportions of weight of sand and cement, pounds..............................	1 cement	8 sand
Weight of cement contained in 1 cubic yard of mortar.................	670 to 850	
Absolute volume of solid matter (cement and sand) contained in 1 volume of mortar...	0.580 to 0.734	
Volume of spaces remaining in 1 volume of mortar after drying (porosity)...........	0.030 to 0.190	
Resistance to compression after exposure for nine months in air, followed by immersion for three months in sea-water, in pounds per square inch.................	1,140 to 4,400	

TABLE XXXVII.

Weight of water wetting a weight 100 of sand........	0.0	0.5	1	2	3	5	10
Weight of 1 bushel of sand, in pounds...................	116.8	105.0	99.2	97.2	96.9	96.9	101.4
Weight of dry sand in 1 bushel of wet sand, in pounds.....	116.8	104.5	98.2	95.3	94.1	92.3	92.2

Sands, and also broken stone, when the grains are as 1 to $\frac{1}{10}$ in size, may have the same volume of voids, but in the latter case may have 10 times the grain surface to coat and cement.

Since the foregoing matter on cement and concrete was written, and in fact printed, the author found the following interesting article in *Engineering* (an English magazine) of July 13, 1894:

The tensile strength of coarsely ground cement, gauged neat, is much greater than that of the same cement finely ground, but the tensile strength of a mixture of the former with sand is much less than that of a similar mixture with the latter. Messrs. Dycherhoff found that cement which would leave a residue of 10 per cent of coarse particles on a No. 50 sieve (2500 meshes per square inch) has a tensile strength, when gauged neat, nearly 42 per cent greater than that of the same cement from which the coarse particles had been removed by a No. 180 sieve (32,400 meshes per square inch), but that a mixture of 1 part of the fine sifted cement to 3 parts of sand had 41 per cent greater tensile strength than a similar mixture made with the unsifted cement, and that it was increased to 64 per cent in case of a mixture of 1 cement to 5 sand, the samples in each case being twenty-five weeks old.

Adding 20 per cent of the coarse particles to fine sifted cement decreased the tensile strength by 47 per cent, with samples seven days old.

Coarse particles not passing a No. 180 sieve are inert, and are to be regarded as an adulterant. It is easy to grind German cement so as to pass a No. 180 sieve without leaving a residue.

Cement as now generally sold in England contains an average of from 35 to 40 per cent of inert material, i.e., this amount is stopped by a No. 180 sieve.

If a concrete is specified to consist of 1 part ordinary cement to 8 parts of sand, broken stone, etc., a not uncommon proportion, the actual proportion of active cement would be 1 to 13 or 14. Such concrete contains slightly over 7 per cent of cementing material.

Coarse particles of cement, when cleaned by washing off the fine cement powder adhering to them, are devoid of all adhesive or cohesive strength.

A clear mesh of 0.003 in. by 0.003 in. is sufficient.

The weight per bushel has nothing to do with the tensile

strength. It is often specified that a cement shall have so much weight, say 112 pounds per bushel, and shall leave a residue not exceeding 10 per cent on a No. 50 sieve. The cement fulfilling these conditions, if more finely ground, say to pass a No. 180 sieve, and thereby increasing its value 35 to 40 per cent, would fall considerably short of the specified weight.

Standard sand is such that will pass a No. 20 sieve and be stopped by a No. 30 sieve. The coarse particles in cement should count only as so much sand.

The following conclusions are drawn by the writer of the article:

(1) That the strength of a mixture of cement and sand is the most reliable of the present tests.

(2) That the tensile strength of neat cement may be omitted altogether as a test of quality.

(3) That the weight per bushel is misleading, and should be omitted altogether.

(4) That color is not of sufficient importance to be considered as a test.

(5) That extreme fineness of grinding is so absolutely essential, that a sieve of not less than from 175 to 180 meshes to the lineal inch should be used for testing purposes.

ART. XXVIII.

STRENGTH OF CAST IRON.

270. Cast-iron has but little tensile strength, but has a high degree of compressive strength. It is hard and brittle, very deficient in toughness and elasticity, and gives way without warning, especially when subjected to shocks or changes of temperatures. It is easily melted and cast into various shapes. It does not rust or deteriorate as rapidly as wrought iron on exposure. In salt water, however, it softens and is weakened. It can be cut and turned, but is not malleable, and is not capable of being welded.

It is not, therefore, suitable for large structural members of bridge-trusses liable to shocks under changing temperature; and when used in buildings where not liable to shocks, if heated dur-

ing fires and suddenly cooled by pouring water upon the beams or columns, it cracks and gives way.

Otherwise it is peculiarly adapted to columns, struts, chairs, shoes, bed-plates, etc., where it has to bear only steady compressive loads or stresses, and for pipes, grates, railings, stoves, and ornamental purposes generally.

Its use in important bridge constructions has been practically prohibited either by law or custom, even in those situations where it would seem eminently suitable, such as the bed-plates on masonry piers for supporting the heavy concentrated loads of the ends of long trusses, especially as these are more exposed to rust and deterioration, and cannot be so easily reached for painting, etc.

TABLE XXXVIII.

The following results of experiments on the strength and stiffness of cast iron are taken from various sources:

271. Resistance to crushing of cast iron with test-pieces ¾ inch diameter and 1½ inches high, with different makes of iron:

No. 1 cast iron,	from 25.2 to 39.6	tons	per	square	inch.
" 2 "	" 30.6 " 45.5	"	"	"	"
" 3 "	" 34.3 " 46.8	"	"	"	"
Other results,	" 19.8 " 62.5	"	"	"	"

From these and other experiments the resistance to crushing of good cast iron can be safely taken at from 36 to 46 tons, equivalent to from 80,640 to 103,040 pounds per square inch.

272. The coefficient of elasticity for compression has been found by numerous experiments to vary from 6,896,600 to 18,750,000, or an average of 12,823,330 pounds per square inch.

There is no clearly defined elastic limit. It is so imperfectly elastic that it takes a permanent set, even with very small loads. As this set is very small at first, and it may disappear after an interval of rest, an appreciably permanent set may not occur until the applied load is from one quarter to one third of its ultimate resistance. The elastic limit is therefore taken at from 20,000 to 27,000 pounds per square inch.

The elastic limit is determined by observing at what load the ratio of stress to strain ceases to be constant. This load is very

difficult to determine with accuracy. The coefficient of elasticity is then calculated by assuming that the ratio will continue constant. If under a compression of 2000 pounds per square inch of cross-sectional area a bar of cast iron is found to be shortened $\frac{1}{6412}$ of its length, then under 4000 pounds it would be shortened $\frac{1}{3206}$ of its length, and under 80,000 pounds it would be shortened by $\frac{1}{160.3}$ of its length. And since the ratio of stress to strain is assumed to be constant and equal to the coefficient of elasticity, then coefficient of elasticity

$$E = \frac{\text{stress per sq. inch}}{\text{strain}} = \text{constant} = \frac{2000}{1/6412} = \frac{4000}{1/3206}$$

$$= \frac{80000}{1/160.3} = 12{,}824{,}000 \text{ lbs. per square inch.}$$

It can then be readily understood why different experimenters arrive at different values for the coefficient of elasticity: each one determines the extent of the shortening at some particular stress, when the set is appreciable and permanent, by the means of measurement adopted or available. One experimenter determines that a set of $\frac{1}{3206}$ of the length is obtained at 3000 pounds per square inch, and another at 5000 pounds, either from errors in measurements or from a different period of time of continuous application of the load; the smaller load remaining for a longer time on the test-piece may have caused a set equal to that of the greater load removed in a less time. The first gives the coefficient of elasticity,

$$E = \frac{3000}{1/3206} = 9{,}618{,}000 \text{ lbs.};$$

the second,

$$E = \frac{5000}{1/3206} = 16{,}030{,}000 \text{ lbs.}$$

Especially is this true with cast iron which takes a permanent set at a very low unit pressure.

The elastic limit is lower for a continued stress than for one simply applied and removed in a short time.

273. Resistance to tearing of cast iron is found by pulling a

specimen asunder. From experiments made on cylinders $\frac{3}{4}$ inch in diameter and $1\frac{1}{2}$ inches long it has been found that the tensile resistance varies as below:

TABLE XXXIX.

No. 1	cast	iron	from	5.7	to	7.2	tons	per	square	inch.
" 2	"	"	"	5.9	"	7.9	"	"	"	"
" 3 3	"	"	"	6.5	"	6.9	"	"	"	"

These vary from 12,768 to 17,696, with an average of 15,234 pounds per square inch.

The best qualities of warm-blast charcoal iron taken from second and third fusions vary from 22,888 to 42,884 pounds per square inch.

Cast iron is but little used to resist direct tensile stress. Fair average values seem to be from 15,000 to 20,000 pounds per square inch. The best charcoal iron gives a coefficient of elasticity under tension of from 3,454,000 to 50,000,000 pounds per square inch. Ordinary cast iron gives from 9,549,120 pounds to 13,603,520 pounds, and an average of 11,576,320 pounds per square inch. These are calculated as for the coefficient of compressive elasticity already explained, except that the strain is one of elongation instead of shortening. If a weight of one ton, 2240 pounds per square inch, produces an elongation $\frac{1}{6073}$ of the length of the bar, then coefficient of elasticity $E = 2240 \div \frac{1}{6073} = 13{,}603{,}520$ pounds per square inch. The same remarks apply to the coefficients under tensile stress as were made for those under compressive stress. The elastic limit for tensile stress varies from 2000 to 6000 pounds per square inch.

From three to four successive fusions of cast iron seem to increase its tensile strength and also its compressive strength.

No. 3, hot blast, according to Sir Wm. Fairbairn, had its compressive strength increased up to the fourteenth smelting.

The surface of a casting is harder and stronger than the interior portion. The effect of this is to give a small casting a proportionately greater strength than a larger one. Cast iron becomes weaker when heated over 120° Fahr., and is unsafe at or below 32° Fahr. If raised to a red heat it will fall to pieces when struck.

274. Under a compressive stress cast iron gives way by portions, shearing off along planes making angles from 46° to 62.5° with the normal sections of the specimens. The characteristic fracture of cast iron is granular and crystalline, with little stretch or reduction of area. It gives way suddenly under shocks and changes of temperature. It is a treacherous and unreliable material in tension, as any brittle material must be.

275. *Shearing Resistance.*—The shearing resistance has been found to vary from 17,000 to 20,000 pounds per square inch. It is usually taken as equal to the tensile resistance of cast iron. Coefficient of elasticity for shearing from 6,000,000 to 7,000,000 pounds per square inch.

276. *Flexure of Solid Cast-iron Beams.*—The transverse strength of solid cast-iron beams has been found usually by experiments on the cross-breaking either of the girders or beams themselves when of small dimensions, or from specimens cast from the same mass of molten metal. The test-bars have usually the dimensions of 24 inches long and cross-sectional area of 2 × 2 inches or 2-inch diameter, equivalent to 4 or 3.1416 square inches respectively, or 3 feet 6 inches long, with a clear span of 3 feet, and with transverse dimensions of 1 inch wide and 2 inches deep. These are loaded at the centre points between two supports, the weights being gradually increased until fracture occurs.

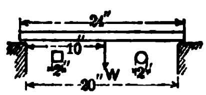


FIG. 112.

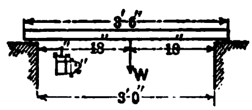


FIG. 113.

Fig. 112 represents the first case and Fig. 113 the second. If W is the centre load in pounds, l the clear length between the supports A and B in inches, f modulus of rupture—that is, the greatest stress in those fibres which yield first, whether compression or tension; b the breadth or horizontal width of the bar or beam in inches, and d the depth in inches, or in case of bars of circular cross-section d is the diameter, then the following expressions represent the relations between these several quantities:

For rectangular sections $\frac{1}{4}Wl = \frac{1}{6}fbd^2$; $\therefore f = \frac{3}{2}\frac{Wl}{bd^2}$.

" square " $\frac{1}{4}Wl = \frac{1}{6}fbd^2$; and $b = d$; $\therefore f = \frac{3}{2}\frac{Wl}{d^3}$.

" circular " $\frac{1}{4}Wl = \frac{f\pi r^3}{4}$; $\therefore f = \frac{Wl}{\pi r^3} = \frac{8\,Wl}{\pi d^3}$.

In the above expressions l, b, and d are in inches. If l is taken in feet, b and d remaining in inches, then the equations become, respectively,

$$f = \frac{18\,Wl}{bd^2};\quad f = \frac{18\,Wl}{d^3};\quad f = \frac{96\,Wl}{\pi d^3}.$$

The methods of deducing these expressions will be explained in a subsequent article. They are introduced here to show the method of finding the modulus of rupture $= f$.

If, then, the clear span is 20 inches for the square and round bars, and the centre load is 11,000 pounds when fracture occurs, we have the values for f as follows:

$$f = \frac{3}{2}\frac{11000 \times 20}{(2)^3} = 41{,}250 \text{ pounds per square inch.}$$

$$f = \frac{8wl}{\pi d^3} = \frac{wl}{\pi r^3} = \frac{11000 \times 20}{3.1416 \times 1} = 70{,}028 \text{ pounds per square inch.}$$

In this manner a number of values of the moduli of rupture have been determined with different varieties of cast iron, both for solid square and round bars. These values range from 40,000 to 60,000 pounds for square bars, and from 50,000 to 72,000 pounds for round bars; from which it has been concluded that the ultimate coefficient of resistance to cross-breaking is about twice that of the tensile strength for square bars, and about two and a quarter times the same for bars with circular cross-sections.

The above experiments were made with rather high grades of cast iron, and may not be true for the poorer varieties.

A good quality of iron in the form and dimensions given in Fig. 113 should just break under a load at centre of about 3050 pounds. See next paragraph. Substituting in

$$f = \frac{3}{2}\frac{Wl}{bd^2}, \ f = \frac{3}{2}\frac{3050 \times 36}{1 \times 4} = 41175,$$

the modulus of rupture practically the same as before found.

If, on the other hand, the modulus of rupture or ultimate fibre strain under a transverse load is given, then the ultimate or breaking load for beams of rectangular cross-section can be found by finding W from the equations,

$$W = \frac{2}{3}\cdot\frac{fbd^2}{l}, \text{ when } l, b, \text{ and } d \text{ are inches,}$$

or

$$W = \frac{1}{18}\frac{fbd^2}{l}, \text{ when } l \text{ is in feet, } b \text{ and } d \text{ in inches.}$$

A fair average value of f for cast-iron beams may be taken $= 40,000$ pounds per sq. in. Assuming clear length of bar equal to 36 inches $= 3$ feet, $b = 1$ inch, $d = 2$ inches, $f = 40,000$ pounds, then from equation above

$$W = \frac{2}{3}\frac{40000 \times 1 \times 4}{36} = \frac{1}{18}\frac{fbd^2}{3} = 2963 \text{ lbs. centre breaking load.}$$

277. Many practical works, such as Trautwine's Engineering Pocket-book, while not recognizing the theoretical formulæ for centre breaking load, but admitting that the ultimate transverse strengths of any given beams vary directly with the breadth and the square of the depth, and inversely as the length, or as $\frac{bd^2}{l}$, contain tables of quiescent breaking loads of beams, or moduli of rupture for beams of various materials, determined by actually breaking bars 1 inch wide, 1 inch deep, and 1 foot long, between supports; and for any other rectangular beam they simply multiply

the expression $\frac{bd^2}{l}$ by this constant in order to find the centre breaking load. Mr. Trautwine gives for good castings the modulus at 2300 pounds; hence $W = \frac{bd^2}{l} \times 2300 = \frac{1 \times 4}{3} \times 2300 = 3067$ pounds, as compared with 2963 pounds by the regular formulæ with l in inches.

The two values of the moduli of rupture, as l is in inches or in feet, should be in the ratio of $1 : \frac{1}{18} :: 40000 : 2222$, instead of 2300 pounds. A failure to understand these relations and conditions often leads to errors and confusion on the part of young engineers.

278. The deflection of cast-iron beams should also be accurately determined. This subject will be further discussed in another article. The measured deflections in the above-mentioned tests varied from 0.15 to 0.26 of an inch, according to the load. The questions of safe or working loads and factors of safety will be discussed later. The following may be taken as fair averages :

	Factor of Safety.
For beams and girders and pillars........	From 3 to 6 dead load.
Pillars and girders under live loads.......	" 8 to 10 live "
Crane-posts and parts of machinery liable to shocks and vibrations............	" 8 to 10 " "
Water-tanks	4

ART. XXIX.

STRENGTH OF WROUGHT IRON.

279. WROUGHT IRON is characterized by a high tensile resistance and a relatively low resistance to compression. But it has the properties of toughness and ductility, which combined with its tensile resistance render it one of the most valuable building materials. It therefore, under ordinary conditions, gives way gradually instead of suddenly. It is practically infusible, is readily forged by hammering or rolling into a great variety of patterns or

shapes, is malleable cold or hot, and possesses the valuable property of being weldable at high temperatures.

Many grades of the iron have a more or less well-defined elastic limit, which is about one half its ultimate strength. Wrought iron also has a high coefficient of elasticity. In all of the above properties and characteristics, except compressive resistance, it is superior to cast iron, and in many of them distinct from it.

It, however, rusts more readily and more rapidly than cast iron, requires greater precautions for its preservation, and better stands exposure to salt water.

It is liable to the defects of hot and cold shortness, caused by the presence of sulphur, or phosphorus and silicon, respectively.

Both the strength and ductility of wrought iron depend upon the quality of the material, and the care and method used in its manufacture. There is always a small percentage of carbon present, and its strength increases as the proportion of carbon increases, until it passes into steel.

It is used extensively for all members under tension, in all kinds of structures, especially in bridge and roof trusses, such as straps, bolts, main and counter diagonals, bottom-chord bars, stringers, and floor-beams, and now has practically supplanted cast iron for chords, struts, and columns of trusses which are under compression. It should also be used for beams and girders when liable to vibrations and shocks. Although large thick pieces may sometimes have flaws or other defects, yet it can be manufactured with a more homogeneous and uniform texture, and is far more reliable in all respects, than cast iron.

280. Although the following remarks apply in many respects to cast iron, they are not so necessary to be considered in connection with cast iron, owing to its limited use for structural purposes.

We may, however, say, that the ultimate resistance of any given material will be influenced by its mode of manufacture, dimensions of normal cross-sections, form of cross-section, the actual and relative dimensions of test-pieces, also shape of test-pieces, and such conditions, if the materials admit of them, as hardening, tempering, annealing, etc. All of these circumstances should be noted and considered in giving results of experiments. It would be beyond

the legitimate scope of this volume to give tables of experiments made upon the many different forms and conditions of the test-pieces, and it will only be necessary to give the usual average values used in practice. Owing to the impracticability of maintaining a uniform distribution of the stress over the entire area of cross-section in ductile materials, the greatest intensity of the stress may be found either towards interior portions of the bar or near the surface, depending upon the manner of applying the external forces. For instance, in testing eye-bars the tendency is to develop a greater intensity of stress in the interior portions of any cross-section; on the contrary, if the grip is applied to the surface of the piece, the greatest intensity is found at or near the surface. In the first condition, the intensity on the surface being less than the mean, and the strain or stretch being measured on the surface, and therefore less than that due to the mean stress, it follows that the mean stress divided by measured strain gives a too large coefficient of elasticity; whereas in case of the surface grip the surface stretch or strain which is measured is greater than that due to the mean stress, hence mean stress divided by measured strain gives a too small coefficient. The true coefficient would be somewhere between the two. As the difference between the interior and surface stress is greater in large bars, experiments on these will usually give greater coefficients of elasticity than the smaller pieces.

281. From a number of experiments made on bars of rectangular cross-section varying from $2'' \times 1''$ to $4'' \times 1\frac{3}{8}''$, and from $24'\ 9\frac{1}{2}''$ long to $35'\ 0''$ long, with a tensile stress of 20,000 pounds per square inch, causing an elongation varying from 0.1948 inch to 0.2692 inch, the calculated coefficients of elasticity under these conditions varied from 30,554,000 to 33,600,000 pounds per square inch. Taking the bar $35.0 \times 4'' \times 1\frac{3}{8}''$, with an elongation of 0.2692 inch in the entire length, then $\frac{0.2692}{35 \times 12}$ is the elongation expressed as a fraction of the entire length in inches, and under a unit stress of 20,000 pounds. The coefficient of elasticity $E = \frac{\text{stress}}{\text{strain}} = 20{,}000 \div \frac{0.2692}{420} = 31{,}203{,}232$ pounds, and similarly for round iron from $2\frac{3}{8}$ to $2\frac{1}{8}$ inches in diameter, and about $11'\ 9''$ long. The value of elongation varied from 0.0940 to 0.1008 inch, under 20,000

pounds stress per square inch; the value of E varied from 29,380,000 to 27,777,777 pounds. Take a bar 11′ 9″ = 11.75′ × 2⅛″, with a stretch of 0.1008 inch in entire length. Then, in fractions of length, $\frac{0.1008}{11.75 \times 12}$ = stretch; and $E = 20{,}000 \div \frac{0.1008}{141} = 27{,}976{,}190$ pounds. These results are particularly valuable, as they were made on full-sized bars. The recovery of each of the many bars experimented upon was perfect with 20,000 pounds. A permanent set was obtained with about 25,000 pounds in some cases. This would then be the elastic limit. The stretch per foot of length in case of the rectangular bars was less than that of the round bars, i.e., $\frac{0.2692}{420} = 0.0064$ is less than $\frac{0.1008}{141} = 0.0071$; consequently the rectangular bars gave a higher coefficient of elasticity. If the coefficients had been taken at some other intensity of stress the values might have been different. A number of other experiments on smaller test-pieces of different grades of iron give coefficients of elasticity varying from 9,000,000 to 34,000,000 pounds in round numbers, and limits of elasticity varying from 24,000 to about 27,000 pounds per square inch. To illustrate the effect of passing the elastic limit: if the total stretch in a bar 10 feet long at the elastic limit of 24,000 pounds is 0.100 inch, then $E = \frac{24000 \times 120}{0.1} = 28{,}800{,}000$. If, then, the stretch is 0.12 at an intensity of stress beyond the elastic limit of 27,000 pounds, then $E = \frac{27000 \times 120}{0.12} = 27{,}000{,}000$, and for 37,000 pounds per square inch and a stretch of 1.095 inches, then $E = \frac{37000 \times 120}{1.095} = 4{,}100{,}000$ pounds, showing the great and irregular variations in the stretch and coefficients of elasticity when the intensity of the stress is greater than the elastic limit. The conclusions drawn by good authorities from many experiments is that for good wrought iron the coefficient of elasticity will be found to be between 25,000,000 and 30,000,000 pounds per square inch.

282. Reversing the above operations, with these coefficients of elasticity and an intensity of stress of 10,000 pounds per square inch the elongations of good iron, under the usual working loads, will be $l = \frac{10000}{25000000}$ to $\frac{10000}{30000000}$, or $\frac{1}{2500}$ to $\frac{1}{3000}$ of its length; and

we may, therefore, say that the coefficient of elasticity is a measure of the stiffness of any material.

A fair result from experiments gives the ultimate tensile resistance of bar iron from 50,000 to 60,000 pounds per square inch, and for plates along the fibre about 15 per cent less, or from 44,500 to 51,000 pounds, and across the grain about 10 per cent less than along the grain, or from about 40,000 to 46,000 pounds per square inch.

TABLE XL.

RECTANGULAR BARS.

No.	Kind of Iron.	Size of Bar in inches.	Stress in pounds per square inch. At First Stretch.	Ultimate in lbs. per sq. inch of Original Section.	Elastic Limit.	Original Length in inches.	In inches. Per Cent of— Final Elongation.	Final Contraction.
1	Single refined..	3 × 1		52,470	29,000	80	18.0	31
2	Double " ..	3 × 1		53,550	31,000	80	16.0	27.7
3	Single " ..	5 × 1¼		50,410	27,330	80	16.6	24.1
4	Double " ..	5 × 1¼		50,920	27,170	80	19.0	25.7
5	Single " ..	3 × 1		48,700	28,330	80	13.1	27.1
6	Double " ..	3 × 1		51,370	29,170	80	22.2	35.6
7	Single " ..	5 × 1¼		49,240	24,830	80	16.0	18.1
8	Double " ..	5 × 1¼		51,010	27,170	80	19.7	29.5

TABLE XLI.

CIRCULAR CROSS-SECTION.

1		*0.97	29,678	54,888		10	23.1	38.2
2		0.97	28,011	55,288		9½	24.3	36.5
3		0.97	29,345	55,355		9	21.5	31.1
4		0.97	29,345	55,622		8½	22.0	31.2
5		0.97	30,840	54,890		7½	25.0	39.9
6		0.97	30,412	55,488		7	25.8	38.6
7		0.97	28,562	51,800		6½	22.1	40.0
8		0.97	30,600	55,418		6	22.3	34.7
9		0.97	29,475	55,333		5½	25.4	39.3
10		0.97	29,278	55,887		5	21.2	32.2
11		0.97	29,705	55,532		4	25.7	37.4
12		0.97	31,817	55,482		3½	26.7	36.6
13		0.97	31,123	56,190		3	27.0	38.3
14		0.97	33,428	56,428		2	27.0	36.2
15		0.97	42,249	57,096		1½	26.0	34.0
16		0.97	34,288	58,933		1	37.0	34.3
17		0.97	57,565	59,388		½	30.0	37.9

* Original diameter in inches.

Other things being the same, bars of the smaller cross-sections will have a greater ultimate tensile resistance, as was shown in regard to the coefficient of elasticity, and in addition a local transverse strain or contraction takes place at the point of fracture. This only manifests itself shortly before rupture. This contraction will also be greater in small cross-sections, and also a greater ultimate resistance and greater final contraction will result in a greater final stretch with the same length of piece.

283. The elastic limit varies between one half and two thirds the ultimate resistance. The ultimate resistance is usually referred to the original area, and not to the contracted area at the place of fracture. The foregoing tables are interesting and instructive as showing the relations between the dimensions of the cross-section, the character of the iron, the elastic limit, ultimate tensile resistance of original section per square inch, percentages of final elongation, and final contraction.

In the rectangular bars the larger ones show, as a rule, the lowest elastic limit and ultimate resistance. The double-refined iron gives principally, the highest results for both.

284. As examples of the effect of annealing at different temperatures, the following are given:

Tensile resistance of iron wire 0.19 inch in diameter was found to be 83,380 pounds per square inch.

After annealing by heating to redness and cooling in dry ashes 58,101 pounds, and heating to redness and quenching in water 53,578 pounds, per square inch.

For boiler-plate iron various specimens having a tensile resistance of from about 48,000 to 77,000 pounds per square inch before annealing, were reduced, by annealing at temperatures varying from 1037° Fahr. to bright welding-heat and cooling, from 57,137 to 56,678, 53,185 to 46,212, 48,407 to 39,333, and 76,986 to 50,074, pounds per square inch.

From which it is seen that the tensile resistance is materially decreased by annealing. But annealing renders the iron more ductile; consequently a hard, stiff iron can be rendered suitable for purposes requiring resistance to shocks and sudden applications of loads, which unannealed would be entirely unsuitable.

Annealing or softening means raising a hardened steel to a red heat and then allowing it to cool gradually.

Hardening means heating to red heat and cooling suddenly. Tempering means reheating after hardening; as the heat increases the hardness diminishes.

Experiments indicate that at a temperature below 500° Fahr. the tensile resistance of wrought iron is not essentially decreased, but at a temperature of 1000° it may lose more than one half of its resistance. This is of great importance in the construction of boilers.

With respect to the effects of low temperatures on wrought iron it seems to be established that it is not diminished in strength under steady strain, but that at low temperatures its resistance to shocks is much affected, depending upon mode of manufacture, chemical composition, etc.

285. Iron wire is the strongest form for resisting tensile stress, and the ultimate tensile resistance increases as the diameter of the wire decreases. An increase of iron wire from 0.122 to 0.124 inch in diameter decreases the tensile resistance from 94,871 to 86,776 pounds per square inch.

Other experiments give a tensile strength for diameters from 0.134 to 0.029 inch of from 92,890 to 113,546 pounds per square inch.

286. It has been found that if wrought iron be subjected to a stress greater than the elastic limit, and then allowed an interval of rest, both the elastic limit and ultimate resistance may be increased. The gain in some cases was from about 3 per cent for the coarse iron to about 18 per cent for the soft iron, the interval of rest varying from 1 day to 8 days or more.

RESISTANCE TO CRUSHING OF WROUGHT IRON.

287. The crushing resistances of materials depends even in a greater degree than the tensile resistances upon mode of manufacture, size, form, and dimensions of pieces both actual and relative, quality and composition of materials. Not only do the elements of strength, toughness, and ductility modify its capability of offering resistance to compressive stresses, but questions of nature of cross-section and distribution of actual metal areas with respect to certain axial lines, and relations between actual lengths and

dimensions of normal cross-sections, are matters of the greatest importance, as the relations between stress and strain are not in many cases direct and simple, but direct stress is combined with bending stress whenever the length exceeds a certain multiple of the smallest dimensions of normal cross-section. These considerations naturally lead to the consideration of two distinct conditions: 1st, the resistance to pure compression which only takes place in what are called short columns; and, 2d, the resistance to combined compression and bending, which takes place in long columns.

The second condition is not considered in this article, and only the resistance to pure compression on short blocks or columns. These may vary from those in which the height is only a small portion of the area of the base (as in thin mortar beds, in joints of masonry: of such resistances we know practically nothing), to those in which the height may be equal to twice or three times the least dimensions of the base or normal cross-section.

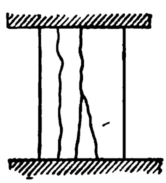


FIG. 114.

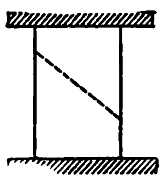


FIG. 115.

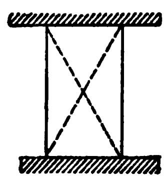


FIG. 116.

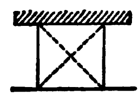


FIG. 117.

288. Crushing of short blocks may take place in many different ways. The phenomenon is not as simple as in tearing asunder, and gives rise to the following cases:

Crushing by splitting (Fig. 114) into a number of more or less parallel layers, which are also nearly parallel to the direction of the compressive force. This mode of fracture is characteristic of hard and homogeneous substances of a glassy texture, such as vitrified bricks.

Crushing by shearing or sliding of portions of the block along certain planes. The shearing or sliding may take place along a single plane, as in Fig. 115, sometimes along two planes, as in Figs. 116 and 117, dividing the blocks into pyramids, the upper and

lower ones approaching each other and forcing those on the sides outward. The greatest intensity of the shearing stress is on planes making angles of 45° with the direction of the crushing force; but the friction on the end surfaces and upon the surfaces of sliding materially reduce these angles, which have been found for cast iron to vary from 42° to 32°. These modes of crushing are characteristic of granular substances, such as cast iron and most kinds of stone and brick.

Those substances which give way by splitting and by shearing have a higher resistance to crushing than tearing.

Crushing by bending or crippling may occur as in Fig. 114, with a buckling or bending of the layers into which the block is split. This takes place in timber, plates of wrought iron, and in bars of considerable length. Short bars or blocks give way by swelling or bulging. This is characteristic of ductile and tough substances, which really never fail by crushing or shearing. An increase of compressive force simply produces an increased area of resistance by swelling or bulging laterally.

289. Crushing by cross-breaking takes place when the length is great as compared with the diameter or least dimension. Under an excessive crushing force the columns yield laterally, and are broken across similarly to beams under the action of transverse loads.

Since ductile wrought iron does not give way usually otherwise than by shortening in its length and increasing in the area of normal cross-sections when applied to short blocks or columns, it is necessary to define the meaning of the failure of wrought iron under compressive stress. A short block of wrought iron is considered to have failed when its length has been shortened by from 5 to 10 per cent.

290. From a large number of experiments on short blocks or columns it has been found that the ultimate resistance to compression may be taken at about 60,000 pounds per square inch. This must not be confounded with the compressive resistance in case of long columns.

The elastic limit is found to vary from 0.4 to 0.5 of the crushing resistance, or from 24,000 to 30,000 pounds per square inch.

The average of a large number of experiments have led to the

conclusion that the coefficient of elasticity for wrought iron in compression is about 28,000,000 pounds per square inch. This may be found as follows: If at the intensity of stress of 26,200 pounds, about the elastic limit, the shortening or rather compressive strain is 0.113 inch in a bar 10 ft. or 120 inches long, then

$$E = \frac{\text{stress}}{\text{strain}} = 26200 \div \frac{0.1133}{120} = 28{,}000{,}000 \text{ pounds.}$$

And under a working load of 5000 pounds per square inch a piece of wrought iron would be shortened by

$$\frac{5000}{28000000} = \frac{1}{5600} \text{ of its length.}$$

291. Resistance of wrought iron to shearing will depend to a great extent upon the thickness of the test-piece. In thin pieces the shearing is simultaneous throughout the section, whereas in thick pieces those portions near the jaws of the shears begin to separate before those at a distance from it, and failure extends gradually or in detail. Usually shearing in detail will give the least ultimate resistance to shearing per unit of area of the entire section. From experiments the shearing resistance of wrought iron may be safely taken at about 0.8 of its tensile resistance, or say from 50,000 to 60,000 × 0.8, equal to from 40,000 to 48,000 pounds per square inch; and the coefficient of elasticity for shearing may be taken for soft wrought iron at 8,571,000 pounds, and for iron bars at 9,523,000.

The shearing resistance of good rivet-iron has been found about equal to the tensile resistance of the same.

292. *Test by Falling Weight or Impact.*—When iron is to be subjected to severe and sudden blows or shocks, it is tested for strength by a weight falling from a height. If a ton-weight is allowed to fall 30 feet, with an apparatus so arranged that the blow can act in the direction of the length of a bolt,—for instance, so as to produce a tensile strain,—it has been found that this will tear a 3-inch bolt asunder in six or seven blows. It may be required that it shall break with a fibrous and not at all crystalline fracture, and with a contraction of 40 per cent in area. Iron rails are also thus tested.

FLEXURE OF SOLID WROUGHT-IRON BEAMS.

293. The moduli of rupture for solid wrought-iron beams are found as in timber and cast-iron beams by supporting them at two points and loading them at the centre. The following example illustrates the nature of the experiments and the calculations for the moduli of rupture and of elasticity: With a centre load of $W = 15,885$ pounds, clear length of 25 inches $= l$, and $b = d = 2$ inches, then

$$f = \frac{3}{2}\frac{Wl}{bd^2} = \frac{3}{2}\frac{15885 \times 25}{8} = 74,461 \text{ pounds};$$

and with a least load of 13,338,

$$f = \frac{3}{2}\frac{Wl}{bd^2} = 62,522 \text{ pounds}.$$

The mean of these results gave about 68,000 pounds for the modulus of rupture; the tensile strength of the same iron was 45,000 pounds. The conclusion reached is that the modulus of rupture is about 1.5 times the tensile resistance: $45,000 \times 1.5 = 67,500$ pounds.

294. From the same set of experiments before loading to rupture the coefficient of elasticity was determined at the elastic limit with a load of 6625 lbs. Hence $f = 31,000$ nearly—about the elastic limit. The deflection was found to be 0.082 inch $= v$; deflection is the strain for bending; then the coefficient of elasticity is

$$E = \frac{Wl^3}{48vI}; \quad I \text{ for rectangular beams is } \tfrac{1}{12}bd^3;$$

$$\therefore\ E = \frac{Wl^3}{4vbd^3} \doteq \frac{6625 \times (25)^3}{4 \times 0.082 \times 2 \times (2)^3} = 19,725,000 \text{ pounds}.$$

A mean of a large number of experiments was found to be $E = 19,049,000$ pounds per square inch.

ART. XXX.

STRENGTH OF STEEL.

295. PERHAPS more thought, labor, and experimenting has been given to the production of uniform and reliable grades of steel than to that of any other building material. Many difficulties have been encountered and overcome in the main, and while much is yet to be determined and many improvements may be desired, manufacturers can now produce the necessary grades required for all structural purposes with a reasonable degree of uniformity. But it is still difficult to answer the question often asked, What is steel?

It is first a form of iron containing from 0.12 per cent of carbon for very soft steel to 1.5 per cent for very hard steel; and, generally speaking, all other impurities should be removed as far as practicable. The effects of such as necessarily remain have already been alluded to.

After this it may be said that steel is a material resulting from certain manufacturing processes by which many properties are acquired either not possessed at all by other materials or at least to only a limited extent, such as annealing, hardening, and tempering; it has a higher tensile and compressive resistance, and a higher coefficient and limit of elasticity, even in the mild grades, and decidedly so in case of the hard steels, than wrought iron. It is weldable, ductile, and altogether a reliable material. It is more easily oxidized than wrought iron, and far more so than cast iron.

296. *Tests for Steel.*—The appearance of the fractured surface will depend upon the circumstances, in the same steel, of the fracture. If rupture occurs slowly it will be silky fibrous, if sudden granular, in appearance.

A better test is to heat the steel and judge of its tenacity, resistance to crushing, by bending on a small radius and through a large angle, flattening out of angle-bars, and by its welding powers.

297. *Admiralty Tests for Steels.*—Plates for ship-building, bars, angles, ties, or tie-bars, made by the Bessemer or Siemens-Martin process, strips cut lengthwise, or for plates either lengthwise or crosswise, or pieces from the bar in round bars, shall have a tensile resistance of not less than 26 tons = 58,240 pounds, and not more than 30 tons = 67,200 pounds, per square inch of section, with an elongation of 20 per cent in 8 inches.

298. *Tempering Tests.*—Similar strips, $1\frac{1}{2}$ inches wide, heated uniformly to a low cherry-red heat, and cooled in water at 82° Fahr., must stand bending in a press to a curve of which the inner radius is $1\frac{1}{2}$ times the thickness of the steel tested, the strips all to be cut in a planing-machine, and to have the sharp edges removed. The ductility of every bar is to be ascertained by the application of one or both of these tests to the shearings, or by bending them cold with the hammer. The testing pieces must be of same width for at least 8 inches in length.

299. *Percussive Tests for Round Bars.*—A specimen bar of 2 inches in diameter may be taken from every charge, or from every 50 bars. This must stand without injury a weight of 15 cwt., = 1680 pounds, falling 30 feet, or 20 cwt. falling $22\frac{1}{2}$ feet, at least for one blow; and the following facts must be noted:

(1) The number of blows to break the bar; (2) the character of the fracture; (3) the reduction in diameter after each blow; (4) the reduction in area at place of fracture; (5) the elongation in 8 inches and in the inch containing the fracture.

Welding Tests.—Sample pieces to be welded together and bent in the way of the weld when cold.

Tests by Repeated and Falling Blows.—A length of 6 feet will be cut from the sample rails, placed in position on solid supports, clear interval 3′ 6″, the ends of the piece projecting equal distances beyond the supports. Then a weight of one ton (2240 pounds) will be allowed to fall freely upon the rail at its centre from a height 12 feet 6 inches. The rail must stand two such blows without sign of fracture, and the permanent set caused by the first blow must not exceed 2 inches. A similar piece supported in the same way must bear at its centre 18 tons, = 40,320 pounds, without deflecting more than $\frac{3}{16}$ inch. These requirements were for rails weighing 79 pounds per yard.

300. Steel for bridges and roofs should have a high elastic limit, which will permit of high working stresses. Good steel for such purpose can be obtained having a tensile strength of 35 tons = 78,400 pounds, and an elastic limit of 20 tons = 44,800 pounds, per square inch, and an elongation of 20 per cent in 8 inches of length. Such steel would endure a working stress of 8 tons = 17,920 pounds per square inch.

The following are some of the requirements from the India Office for large steel bridges:

Steel bars and plates must weld perfectly, and not crack or crumble at all when hammered at a welding heat; strips 1 inch wide and 8 inches long to have a tensile strength not less than 28 tons = 62,720 pounds, nor more than 31 tons = 69,440 pounds per square inch, an elongation of not less than 20 per cent, and a limit of elasticity of 15 tons = 33,600 pounds per square inch. The same tempering as for the Admiralty tests, except that the radius of the curve to which the steel is bent is 3 inches instead of 1½ inches.

Buckle-plates of roadway to bear a concentrated load of 12 tons at centre without permanent set, and 24 tons at centre without fracture.

Rivets.—Tensile strength 26 to 28 tons per square inch, in test-pieces of 10 diameters; elongation not less than 25 per cent. A piece of bar heated to cherry-red, quenched in water at 82° Fahr., to bear being doubled quite close without injury. A piece heated to full red or orange, dropped into a hole in a cast-iron block, so that 1½ to 2 diameters project, to bear having the end hammered out to a thin edge all around without showing signs of cracking. All of the above tests are under the requirements of English engineers.

301. For the steel used in the construction of the bridge across the Mississippi at Memphis, completed May, 1892, the requirements were as given in Table XXI, page 295.

Another instance of the requirements for steel for use in bridge construction is the following:

A specimen about ⅝ inch diameter should be taken from every melt. This bar should bend double when cold 180° around its own diameter without cracking; should have an elastic limit

of at least 50,000 pounds, and an ultimate strength of at least 80,000 pounds; should elongate 12 per cent before breaking, and show a reduction of 20 per cent at the point of fracture. The percentage of carbon was fixed at 0.35. A difference in the strength of large and small sized bars corresponding to that which exists in iron bars was found in the steel. The finished bars measured 6 × $1\frac{1}{4}$ inches to $1\frac{5}{8}$ inches, and were found to have an elastic limit of 37,000 pounds and ultimate resistances of 66,000 to 73,000 pounds per square inch. The modulus of elasticity below the elastic limit was exceedingly uniform.

Sample sizes used in counters and laterals approximated closely in their strength and elastic limit to the test samples.

302. *Steel Cable-wire for the East River Suspension Bridge.*—The steel must be hardened and tempered; and it must be galvanized. The size of the wire shall be No. 8 full Birmingham gauge. Each wire must have a breaking strength of not less than 3400 pounds. This corresponds in wire weighing 14 feet to the pound to a rate of 160,000 pounds per square inch of solid section. The elastic limit must be not less than 0.47 of the breaking strength = 1600 pounds. Within this limit of elasticity it must stretch at a uniform rate corresponding to a modulus of elasticity of not less than 27,000,000 nor exceed 29,000,000 pounds.

Mode of Testing.—1st. One ring in every forty (40) will be tested as follows: A piece of wire 60 feet long will be cut off from either end of the ring, and it will be placed in a vertical testing-machine. An initial strain of 400 pounds is now applied, which should take out every crook and bend. A vernier gauge capable of being read to $\frac{1}{10000}$ of one foot is so attached as to indicate the stretch of 50 feet of the wire. Successive increments of 400 pounds strain are then applied and the vernier read each time, until a strain of 1600 pounds is reached. It is required that the amount of stretch for each of these increments shall be the same, and that the total stretch between the initial and terminal strains shall not be less than $\frac{62}{1000}$ of one foot, equal to $\frac{124}{100000}$ of the 50 feet, and on reducing the strain to 1200 pounds there shall be a permanent elongation not exceeding $\frac{1}{100000}$ of its length. The same wire shall then be subjected to a breaking strain. The minimum re-

quired is 3400 pounds per wire or 160,000 pounds per square inch. The minimum stretch when broken shall have been 2 per cent in 50 feet, and the diameter of the wire at the point of fracture shall not exceed $\frac{15}{100}$ of one inch.

2d. Every ring shall be subjected to a bending test by cutting off from each ring a piece of wire one foot long, and coiling it closely and continuously around a rod one-half inch in diameter, when, if it breaks, it will be rejected.

3d. All the wire must be straight wire; that is to say, when a ring is unrolled upon the floor the wire behind must lie perfectly straight and neutral, without any tendency to spring back in the coiled form, as is usually the case. This straight condition must not be produced by straightening machines, as they only injure the strength and elasticity of the wire.

General Specifications for Steel Suspenders, Connecting-rods, Stirrups, and Pins.—All steel must be of uniform quality known as mild steel, and give these results: Ultimate tensile strength 75,000 pounds per square inch of full section; elongation not less than 15 per cent in one foot of length; a reduction of area of not less than 25 per cent at the point of fracture; limit of elasticity not less than 45,000 pounds per square inch; and a modulus of elasticity between 26,000,000 and 30,000,000 pounds per square inch.

For Other Steel Work.—All of the steel used in this work must be of a mild, uniform, elastic, and ductile quality, suitable for bridge members. Siemens-Martin or open-hearth steel, or Bessemer steel under the Hay process, will be preferred. Two specimens direct from the rolls, each 1 inch square and 24 inches long, are required. All of the steel must be capable of sustaining a tensile strain in every full-sized member, round or flat bar, of not less than 70,000 pounds per square inch of cross-section. It must have an elastic limit in all shapes of not less than 40,000 pounds per square inch, and a modulus of elasticity of not less than 26,000,000 nor more than 30,000,000 pounds per square inch. An ultimate elongation of 10 per cent of the full length of uniform sections, and 15 per cent in one foot of length, inclusive of the fractured section, is also required. The area of the reduced section at the point of fracture must not exceed 80 per cent of the original section.

Small specimens of one foot in length, of even section of one square inch or less, should reach a tensile strength of 75,000 pounds per square inch, with a modulus and limit of elasticity and reduction of area before mentioned, and an ultimate stretch of 15 per cent.

All round or flat bars, or flat pieces cut from the web of any shaped bars, must be capable of being bent cold for 180° to a curve whose diameter is no greater than the thickness of the bar, and that without cracking. The rivets must be made of very ductile steel. The rods from which the rivets are made must have a tensile strength of not less than 70,000 pounds per square inch, and elongate at least 20 per cent in a length of one foot, and a reduction in area at point of fracture of 30 per cent. If the minimum is reached in any one of these requirements, the others must be exceeded by at least 10 per cent. The rod must be capable of being bent cold under a hammer 180°, and the inner surfaces brought into contact without producing any fracture.

Cold straightening must be avoided; but when resorted to, the pieces must be annealed afterwards, and of every piece, any portion of which for any cause is reheated, the whole must be annealed and very slowly cooled; and all pieces in which, from tests or otherwise, a want of uniformity is suspected must be annealed if required.

All rivet-holes must be drilled, unless some system of punching and reaming be authorized whereby all of the compressed section around the punched hole will be cut away. The spacing must be accurately done, as no gauging or drifting will be allowed.

The above extracts from specifications give more succinctly and clearly all of the actual requirements for a structural steel, and it is consequently unnecessary and would be unprofitable to introduce a large mass of detailed experiments, as the above embodies the deductions from them by able and skilled engineers, so far as they are applicable to steel as a building material.

EXTRACTS FROM SPECIFICATIONS FOR WROUGHT IRON.

303. As wrought iron and steel are now exclusively used in the more important iron structures, especially bridge construction, a

few of the more useful requirements are introduced here for the sake of comparison with similar ones for steel.

The iron subjected to tensile stress shall be tough, ductile, and of uniform quality, capable of sustaining not less than 50,000 pounds per square inch of sectional area when tested in large and long lengths, and have an elastic limit of not less than 26,000 pounds per square inch. If the bars are larger than 4.5 square inches, a reduction of 1000 pounds will be allowed for each additional square inch of section down to a minimum tensile resistance of 46,000 pounds per square inch. Test-pieces taken from bars, and having a uniform section of 1 square inch for a length of 10 inches, must give the following results:

From bars of 4.5 square inches or under, an ultimate resistance of 52,000 pounds per square inch, with a stretch of 18 per cent in 8 inches. From bars over 4.5 square inches in area of cross-section, a reduction of 500 pounds will be allowed per square inch for each additional square inch of section down to a minimum of 50,000 pounds per square inch. Specimens from angles, beams, channels, or plates must show an ultimate resistance of 50,000 pounds per square inch, with 15 per cent elongation in 8 inches.

All iron for tension members, whether bars, angles, or plate, must permit of being bent cold, without cracking, on a curve of which the diameter is not greater than twice the thickness of the bar, plate, or angle. Any of the above classes of iron, when nicked and broken, must exhibit a fibrous structure, almost entirely free from crystalline specks. Or,

Round bars up to $1\frac{1}{2}$ inches diameter must bend double, when cold, until the inner sides are in contact, without showing signs of fracture. Square bars must bend cold through 180° around a cylinder having a diameter equal to two-thirds the length of the side, without showing signs of fracture. Flats must bend cold through 180°, around a cylinder having a diameter equal to the length of the shortest side, without sign of fracture. Elastic limit not less than 26,000 pounds per square inch, and the elongation of the bar not less than 15 per cent in 12 diameters. The reduction of area at breaking-point not less than 25 per cent of the original section.

304. Probably the largest eye-bar ever broken in a machine is the following: A steel bar $10 \times 2\frac{5}{8}$ inches and 50 feet from centre to centre of pin-holes was broken on the 1200-ton hydraulic test-

ing-machine of the Phœnix Iron Works, Pa. The elongation in a length of 47 feet was 9 feet 9 inches, or 20.47 per cent, and the reduction of area was 50.4 per cent. Elastic limit 33,250 pounds per square inch. Ultimate tensile strength of the bar, 1,626,322 pounds, or 61,720 pounds per square inch. The fracture was silky, half cup and half angular, not at all crystalline. The originally circular pin-hole was made oblong, the elongation being $2\frac{5}{16}$ inches. The pin diameter was 9 inches.

COMPRESSIVE RESISTANCE OF STEEL.

305. The results of experiments on the resistance of small specimens of steel gave from 175,000 to 193,000 pounds per square inch of original section. This is the ultimate resistance.

Taking the ultimate resistance of normal untempered steel at from 100,100 to 112,400 pounds per square inch, when heated to a light cherry-red and plunged in oil at 82° Fahr. its ultimate resistance will be from 173,200 to 199,200 pounds per square inch. Heated as before, and plunged in water at 79° Fahr., with final temper drawn on heated plate, ultimate resistance 325,400 to 340,800 pounds per square inch. Heated as before and plunged in water at 79° Fahr., and tested at maximum hardness, ultimate resistance 275,640 to 400,000 pounds per square inch.

The coefficient of elasticity for compression varies between 28,000,000 and 35,000,000 pounds per square inch, the elastic limit from 36,000 to 60,000 pounds per square inch.

The shearing resistance of steel is about three quarters the tensile resistance.

The modulus of rupture f for steel is about 1.66 times the ultimate tensile resistance for square Bessemer-steel bars, and about 1.85 times the tensile resistance for square crucible-steel bars.

The elastic limit and ultimate resistance to tension, in mild-steel plates, increase as the thickness of the plates decrease.

The process of annealing decreases both the elastic limit and ultimate tensile resistance, but increases considerably both the final stretch and contraction; in other words, increases the ductility of the material.

306. In determining the tensile strength of iron and steel bars the hold or grip can be made by the use of sleeves and screw-threads

at the ends of the bar; by simple enlargements at the heads for the clips; by special formed ends with wedge-shaped bearings; or by holes and pins, as shown in Fig. 118, which represents a bar ready for the test. In Fig. 119 the general form after rupture is shown; *ab* and *cd* show the total stretch or elongation, *AB* the original section, and *A'B'* the contracted area at the point of fracture. The

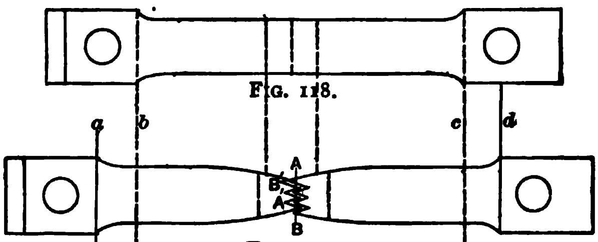


FIG. 118.

FIG. 119.

elongation in any given length is indicated by the dotted lines at the centre of the length of the bar.

The chief hindrance to the common and almost universal use of steel for structural purposes has been due to its untrustworthiness and its inability to withstand shearing and punching without serious injury. Experiments, however, show that soft and mild steels, with ultimate tensile resistance of from 55,000 to 70,000 pounds per square inch, in fact any steel under 80,000 pounds tensile resistance, is not more injured by punching and shearing than wrought iron is. Planing and reaming for shearing and punching, respectively, restore the original condition.

The effects of punching and shearing are: (1) reduction of ductility; (2) elevation of tensile strength at the elastic limit; (3) reduction of ultimate tensile strength. Annealing sheared or punched steels restores ductility to a great extent, and reduces the exaggerated elastic limit for thicknesses up to one-half inch.

ART. XXXI.

EFFECTS OF FORM OF CROSS-SECTION ON THE STRENGTH AND STIFFNESS OF MATERIALS USED IN ENGINEERING.

307. HAVING discussed in Art. XXI the equilibrium and balance of forces in general, in Art. XXII the equilibrium and stability of frames, in Art. XXIII the general subject of shears and bending moments, and in Articles XXIV to XXX, inclusive, the capacity of the different materials to resist the action of forces and moments and the methods of determining the intensity of these resistances, we will in this article consider the effects of the form and cross-section in increasing the strength and stiffness in order to secure maximum resistances with the least weight, and, consequently, in general, the least cost and convenience in the manufacture, handling, and erection of structures. It is well, however, to remember that a saving in weight of pieces or structures does not always correspond with a saving in cost, as so much labor and time may be expended in producing the desired forms and making the connections that the cost will be increased. Especially is this true in departing from the use of standard forms and weights in order to conform with the results of over-nice calculations or with some fanciful idea possessing no real advantage. Such things may be necessary under some circumstances, but generally only where architectural or ornamental considerations are necessary or desired rather than convenience or utility. Strength, economy, and suitableness for the purpose in view are the important matters to be considered in this place.

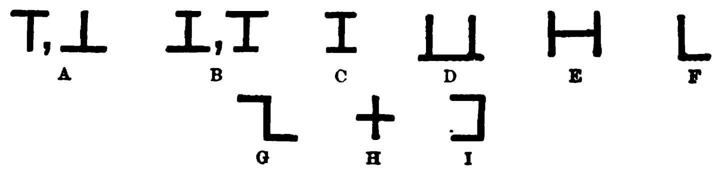
308. *Timber.*—Except for ornamental purposes, timber is mainly used, whether for struts, ties, beams, planks, deals, and battens, in the square, rectangular, or circular cross-sections, and in solid sections.

Stone.—Stone is also used mainly in solid sections, and commonly in square, rectangular, and circular sections. Parts of certain structures, such as ring-stones of arches and the ends of piers, have some of their surfaces elliptical, and of other curved forms. In

both of the above materials economy and strength require such forms.

Iron.—Cast iron, wrought iron, and steel can be cast, rolled, hammered, or built into a great variety of forms adapted to any purposes, giving the greatest strength with the least material, and admitting of convenient and strong connections.

The usual forms are known by some letter of the alphabet which they most resemble. The forms are: the solid or hollow round; square, rectangular; the tee or inverted tee (A); the double tee (B); the eye-beam, having two equal flanges (C); the trough (D); the H (E); the angle (F); the zee (G); the cross (H); the channel (I); and flat bars and plates of any desired thickness.



Cast iron can be moulded in any of the above forms, but owing to the limited use of cast iron for structural purposes the forms more commonly used are the hollow circle, the single or double tee, and the trough forms.

Wrought iron and steel can be rolled or hammered into many of the above forms. The rolled forms are more commonly confined to the solid, round, square, rectangular forms; the eye-beams, the angles, the zees and channels; while the tees, the H's, and the crosses are usually built of plates and angles, as shown in Figs. 121.

a is the simple built beam, composed of one vertical plate and four angles, riveted, and may or may not have top and bottom plates; *b* is the ordinary box beam, composed of two channels and usually only a top plate, and the bottom may have a plate or simple lattice-bars; *c* is the zee column, composed of four zees and plate or lattice-bar; *d* is the Phœnix column, composed of four, six, or eight circular segments with flanges; and *e* is the cross, composed of three plates and four angles. In each form the parts are riveted with single or double rows of rivets. It is evident that a great variety of forms can be built.

309. *Tension.*—To resist simple tensile stress any of the above

forms are or can be made suitable by proper connections. But as it is important to have a uniform distribution of the stress, it is best to collect the necessary material into as compact a mass as possible; hence the solid round or square sections are used for small stresses, and the rectangular for the larger. If, however, a tension member is to be under transverse stress, as in the bottom chords of some bridges and the tie-beams of roof-trusses, some of the forms in the preceding paragraph must be used.

310. *Compression.*—For members under compressive strain, if short columns, any of the above-mentioned forms may be used, as these give way by simple compression. But for long columns, which yield by direct compression and bending combined, the solid forms, round, square, or rectangular, are too flexible without the use of unnecessary material; consequently, if made of cast iron, the hollow cylinder is almost exclusively used, as giving the maximum strength and stiffness with the minimum amount of material. If made of wrought iron or steel, one of the forms a, b, c, d, or e (see Figs. 121) is always used. The form b is frequently if not commonly made of plates and angles, as shown in the form g.

311. *Long Columns.*—Where the ratio of the length to the least dimension of cross-section is great, a member under a crushing load does not give way by pure compression, but by bending sideways, crushing on one side and tearing apart on the opposite side. The formulæ which have been adopted for practical use have been deduced partly by theoretical and partly by empirical investigations, and are commonly called Gordon's formulæ. The coefficient of strength f is supposed to be composed of two intensities or coefficients: first, the intensity due to the uniform distribution of the load over the cross-section, $= p'$; and, second, an intensity which arises from lateral bending, and takes place in the direction of its least side or diameter. Let (in Fig. 120) h = least side or diameter and b = the greater, l = length of the strut, and v = greatest deflection from the original straight position. The greatest moment of flexure $= Pv$. The greatest stress p'' produced by that moment is directly proportional to the moment, and inversely as the breadth and the square of the thickness, or $p'' : \frac{Pv}{bh^2}$. But the greatest deflection consistent with safety is directly as the square of the length and

inversely as the thickness, or $v : \frac{l^2}{h}$; also, $bh^2 = bh \times h =$ the area $S \times h$. Substituting, $p'' : \frac{Pl^2}{Sh^2}$; and since the stress due to the direct pressure $p' = \frac{P}{S}$, $\therefore p'' : \frac{Pl^2}{Sh^2} : p'\frac{l^2}{h^2}$; that is, the additional stress due to bending is to the stress due to direct pressure as the square of the length is to the square of the least diameter. Multiplied by some constant a, determined by experiment, there then results

$$f = p' + p'' = \frac{P}{S} + \frac{P}{S}\frac{l^2}{h^2}a = \frac{P}{S}\left(1 + a\frac{l^2}{h^2}\right); \quad . \quad . \quad (113)$$

or the crushing load

$$P = \frac{fS}{1 + a\frac{l^2}{h^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

Eq. (114) is found by experiment to apply only to columns with fixed ends or with flat capitals and bases. If a column is rounded at both ends, that is, having more or less freedom of motion, its strength is the same as that with both ends fixed and double its length. Hence for rounded or hinged ends

$$P = \frac{fS}{1 + 4a\frac{l^2}{h^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

And for pillars with one fixed and one round end a mean between the two is taken, or

$$P = \frac{fS}{1 + 2a\frac{l^2}{h^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

In equations (114), (115), and (116) $S =$ area of cross-section, f is the coefficient of resistance to crushing of the material, a is a constant determined by experiment, l is the length, and h is the least side of the column if rectangular, or diameter if circular. All dimensions to be in the same unit.

The following values of f and a will be sufficient in this place; special values will be given in another place:

TABLE XLII.

		Breaking Load, lbs. per sq. in.	Proof Load, lbs.	Working Load, lbs.
For cast iron,	$f=$	80,000	$\frac{1}{3}\times 80{,}000 = 26{,}700$	13,300 to 20,000
" wrought iron,	$f=$	36,000	$\frac{1}{2}\times 36{,}000 = 18{,}000$	6,000 to 9,000
" stone,	$f=$	5,000 to 20,000		2,000 to 6,000
" brick,	$f=$	1,000 to 8,000		500 to 1,000
" timber,	$f=$	1,000 to 5,000		500 to 1,000

For wrought-iron rectangular columns $a = \frac{1}{3000}$, for cast-iron hollow cylinders $a = \frac{1}{400}$, for timber $a = \frac{1}{250}$, and for stone or brick $a = \frac{1}{800}$. The above formulæ are simple, easy of application, and approximately accurate. Other formulæ are expressed in terms of the moment of inertia or square of the radius of gyration, which will be fully explained when discussing beams. f is still equal to $p' + p''$, and $p' = \frac{P}{S}$; but $p'' = \frac{MPvy}{I}$, as shown in Fig. 120, for all formulæ. The greatest moment of flexure of the column is proportional to Pv, and may be expressed by MPv, M being a constant depending upon the form, and the bearings of the ends of the column. By the common theory of flexure $MPv = \frac{p''I}{y}$, y being the distance of the extreme fibre from the neutral axis, and I the moment of inertia of the cross-section of the column (see Art. XXXII). Then

FIG. 120.

$$p'' = \frac{MPvy}{I}; \quad \therefore f = p' + p'' = \frac{P}{S} + \frac{MPvy}{I} = \frac{P}{S}\left(1 + \frac{MvyS}{I}\right);$$

$$\therefore P = \frac{fS}{1 + \frac{MvSy}{I}},$$

which is the most general form of the value of the ultimate crushing load, corresponding to the unit strain f. It can be shown that $v = a_1 . \frac{l^2}{y}$, and $I = S\rho^2$, S being area of cross-section and ρ^2 the square of the radius of gyration. Substituting, and making $a_1 = a_1 M$, we have

$$P = \frac{fS}{1 + a\frac{l^2}{\rho^2}}.^* \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

* See Burr, Elasticity and Resistance of Materials.

The integration by which the value of v is obtained, being taken between limits, causes everything to disappear which depends upon the condition of the ends of the column; hence eq. (117) applies to all columns, whether the ends are rounded or fixed. Assuming the latter condition, and as the column must be bent symmetrically, there will be at least two points of contraflexure, B and C. Then $BC = \frac{1}{2}AD$, and the centre bending moment at E must be equal to that at A or D. Hence the free or round end column BC must possess the same resistance as the fixed or flat end column AD. Hence $l = 2BC = 2l_1$, and eq. (117) becomes

$$P = \frac{fS}{1 + 4a\frac{l_1^2}{\rho^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

for free or round end columns. The flat or fixed end column AD has also the same resistance as the column AC, with one end fixed and one end free or round. Hence $l = \frac{4}{3}AC = \frac{4}{3}l_2$, and eq. (117) becomes

$$P = \frac{fS}{1 + 1.8a\frac{l^2}{\rho^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

for columns or struts with one end fixed and the other rounded.

Columns are often of larger cross-section at and near the centre of their lengths than at or near the ends. Such columns are called "swelled columns." Having the same form and area of cross-section, they will sustain a greater load if solid, but for built columns of shaped iron it is found that they will not, the reasons for which it is not necessary to state.

Struts or columns with pin ends are often taken to be in the condition of columns with both ends round or free, but it is found that such columns approximate more nearly those with one end fixed and one end round, so far as ultimate strength is concerned. Which will be used depends solely upon the degree of safety that may be required. Eqs. (114) to (119) give the two sets of three equations each which are applicable to the resistances of columns.

struts, posts, or pillars. The following are generally used, and are the common forms in which they usually occur in books:

$$P = \frac{fS}{1 + a\frac{l^2}{\rho^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

for flat or fixed end columns.

$$P = \frac{fS}{1 + 4a\frac{l^2}{\rho^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

for both ends, free or round.

$$P = \frac{fS}{1 + 1.8a\frac{l^2}{\rho^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

for one end fixed and the other round. Also for pin ends, as the strut members in truss bridges.

In these formulæ P is the breaking load in pounds, f is the coefficient of resistance to crushing in pounds per square inch, S is the metal area in square inches, a is a constant determined by experiment, l is the length of the column in inches or feet, and ρ is the radius of gyration of the cross-section in inches or feet. l and ρ must be in the same unit. When the bearings at both ends are on pins, which is usually the case in large truss bridges for members under compression, P is often found from equation (118) above for both ends rounded; but it seems that the resistance of such columns actually approach more nearly the conditions in equation (119) as indicated above. These formulæ are applicable to straight columns—that is, for those of the same diameter throughout. Theoretically, swelled columns or those of a greater diameter at the middle of their length should be stronger, but for wrought iron they are really not so strong. The following values for f and a are taken from Mr. Burr's work:

TABLE XLIII.

Flat	ends,	square columns,	$f = 39000$;	$a = \frac{1}{35000}$;	. .	(117)
Pin	"	" "	$f = 39000$;	$1.8a = \frac{1}{17000}$;	. .	(118)
Flat	"	" Phœnix,	$f = 42000$;	$a = \frac{1}{50000}$;	. .	(117)
Pin	"	" "	$f = 42000$;	$1.8a = \frac{1}{22700}$;	. .	(118)
Round	"	" "	$f = 42000$;	$4a = \frac{1}{12500}$;	. .	(119)

Flat ends, American Bridge Co. column,

$$f = 36000; \quad a = \frac{1}{18000}; \quad . \quad . \quad (117)$$

Round ends, American Bridge Co. column,

$$f = 36000; \quad 4a = \frac{1}{11000}; \quad . \quad . \quad (118)$$

Pin ends, American Bridge Co. column,

$$f = 36000; \; 1.8a = \frac{1}{31000}. \quad . \quad . \quad (119)$$

The American Bridge Co. column is composed of an I beam and plates riveted to flanges: 

For steel columns, flat ends: For mild steel, $f = 67{,}000$ lbs., $a = \frac{1}{22400}$; for strong steel, $f = 114{,}000$ lbs., $a = \frac{1}{14400}$.

"The formula for square columns may be used for the common-chord section composed of two channels and plates, with the axis of the pin passing through the centre of gravity of the cross-section. By putting 36,000 for 39,000 the same formula may be used for compression members composed of two channels with zigzag bracing."

312. If we substitute for equation (117), $P = \dfrac{fS}{1 + a\dfrac{l^2}{\rho^2}}$, the following, $P = \dfrac{fS}{1 + a\dfrac{l^2}{d^2}}$, in which d is the least dimension of the column, it much simplifies the solution of the problem, as we do not have to find the moment of inertia or the radius of gyration. In this case the experimental constants are as follows:

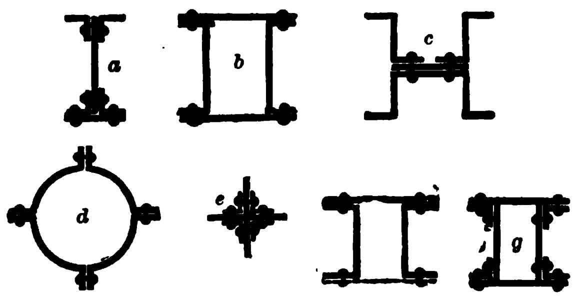
TABLE XLIV.

Flat ends, square columns,	$f = 38500$; $a = \frac{1}{5850}$;
One pin end, square "	$f = 38500$; $a = \frac{1}{3000}$;
Two pin ends, " "	$f = 37500$; $a = \frac{1}{1900}$;
Flat ends, Phœnix "	$f = 42500$; $a = \frac{1}{4500}$;
One pin end, " "	$f = 40000$; $a = \frac{1}{2250}$;
Two pin ends, " "	$f = 36600$; $a = \frac{1}{1500}$;
Flat ends, American Br. Co. column,	$f = 36500$; $a = \frac{1}{3750}$;
One pin end, " " " "	$f = 36500$; $a = \frac{1}{2250}$;
Two pin ends, " " " "	$f = 36500$; $a = \frac{1}{1750}$.

From experiments made by Mr. C. Shaler Smith, the following constants are recommended for wooden columns, using the same formulæ:

$$f = 5000 \quad \text{and} \quad a = \tfrac{1}{250}.$$

For the column f, as shown in Figs. 121, called the common column, which is composed of two channels connected on one



Figs. 121.

side by a solid covering-plate and on the other by lattice-bars,

For flat ends, $f = 36500$; $a = \tfrac{1}{2700}$;
" one pin end, $f = 36500$; $a = \tfrac{1}{1600}$;
" two pin ends, $f = 36500$; $a = \tfrac{1}{1200}$.

Steel.—For mild steel, $f = 67{,}000$ lbs., $a = \tfrac{1}{22400}$; and for strong steel, $f = 114{,}000$ lbs., $a = \tfrac{1}{11400}$.

313. As in all matters pertaining to formulæ used in engineering construction, simplicity is a most important end to be obtained, many efforts have been made to simplify the formulæ applicable to long columns; and recently new formulæ have been introduced, and in many cases adopted by practical engineers. The following are fair samples:

Mr. Theodore Cooper has recommended the following formulæ for the safe or working loads, using Cooper's Extra A type of locomotive, namely: Front truck, 16,000 pounds; 4 pairs of drivers, 30,000 pounds per pair; and load on each pair of tender-wheels, 18,000 pounds. Then if p = the working load per square inch of

metal area, $\rho =$ the radius of gyration and $l =$ the length, both in the same unit,

For top chords $p = 8000 - 30\frac{l}{\rho}$ for live-load stresses; . . (120)

" " " $p = 16000 - 60\frac{l}{\rho}$ " dead-load " ; . . (121)

For all posts $p = 7000 - 40\frac{l}{\rho}$ " live-load " ; . . (122)

" " " $p = 14000 - 80\frac{l}{\rho}$ " dead-load " ; . . (123)

" " " $p = 10000 - 60\frac{l}{\rho}$ " wind stresses;

For laterals, $p = 9000 - 50\frac{l}{\rho}$ " assumed initial stress.

$\frac{l}{\rho}$ is generally less than 100, and always less than 150.

In a recent work by Mr. Johnson he recommends as safe working loads (using a factor of safety = 4):

For wrought iron, pin ends, $p = 8500 - 0.17\left(\frac{l}{\rho}\right)^2$; . . . (124)

" " " flat " $p = 8500 - 0.11\left(\frac{l}{\rho}\right)^2$; . . . (125)

For mild steel, pin " $p = 10500 - 0.24\left(\frac{l}{\rho}\right)^2$; . . . (126)

" " " flat " $p = 10500 - 0.16\left(\frac{l}{\rho}\right)^2$. . . . (127)

Breaking loads per square inch.

Cast iron, $p = 60000 - \frac{25}{4}\left(\frac{l}{\rho}\right)^2$, round ends; . . . . . (128)

" " $p = 60000 - \frac{9}{4}\left(\frac{l}{\rho}\right)^2$, flat ends. . . . . (129)

For white pine, $\frac{l}{d} \lessgtr 60$; $p = 2500 - 0.6\left(\frac{l}{d}\right)^2$; . . . . . (130)

" yellow " $\frac{l}{d} \lessgtr 60$; $p = 4000 - 0.8\left(\frac{l}{d}\right)^2$; . . . . . (131)

" white oak, $\frac{l}{d} \lessgtr 60$; $p = 3500 - 0.8\left(\frac{l}{d}\right)^2$; . . . . . (132)

in which d is the diameter or least side; p is considered as not exceeding the elastic limit.

ART. XXXII.

MOMENT OF RESISTANCE TO BENDING OF BEAMS AND GIRDERS.

314. Although in beams of all forms of cross-section the resistance to both shearing and bending is necessarily developed in all parts of the cross-section, it is usual in all forms, except those of the solid round, square, or rectangular cross-sections, to assume that the resistance to bending is confined to the top and bottom flanges and the shearing resistance to the web, whether of solid plates or of open work; this being practically true when the material is so distributed that the maximum strength is secured with the least material. Iron and steel beams are universally cast, rolled, or built into these latter forms.

The general moment of resistance which will be discussed in this article applies theoretically to any form of cross-section and to any material, from which special expressions will be deduced applicable only to special forms.

Beams are horizontal or inclined members or pieces, and usually acted upon by vertical forces or loads.

315. The effects of the load is to bend the beam and to cut or shear it along certain planes. The shearing is constant, as in case of a single isolated load, over the entire length of the beam or over a part of it. Where several isolated loads exist, it is constant over that section between the loads, but varies from section to section. When uniformly loaded the shearing force varies uniformly, being a maximum in the cantilever beam at the point of support, where also the bending moment is the greatest; but in a beam supported

at both ends it is a maximum at the points of support, where the bending moment is nothing, and gradually decreasing to nothing at some intermediate point where the bending moment is a maximum. These effects will be considered separately.

316. The effect of the bending action is to compress or crush the fibres above the neutral axis and to stretch or tear the fibres below in the case of a beam supported at both ends, and the reverse in the case of a beam fixed at one end, the upper fibres in this case being stretched and the lower fibres compressed, as seen in Figs.

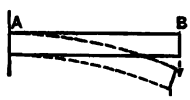


FIG. 122.

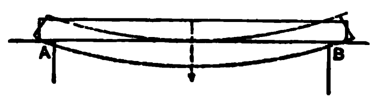


FIG. 123.

122 and 123. It is evident that in passing, in the same beam, from a condition of compression to that of extension there must be a surface or layer of fibres which is neither compressed nor extended, or a surface of no bending strain. This is called the neutral surface. The length of this layer of fibres is not changed, and it can be shown that this surface contains the centres of gravity of the cross-sections of the beam. The condition of these resistances to crushing and to tearing must be one of equilibrium; that is, the total thrust and the total tension at any cross-section must be equal to each other, as these are the only forces acting in the direction of the length of the beam, and act in opposite directions. The neutral surface passing, as it does, through the centre of gravity of a cross-section, will vary in position with the form of the cross-section of the beam, sometimes nearer the top and sometimes nearer the bottom of the beam. As the strength of any beam depends upon the total resistance of the fibres to being lengthened and shortened, upon the number of fibres or layers of fibres, and the position of these fibres with respect to the neutral surface, the elements that enter into the algebraic expression for this resistance will be:

1st. The coefficients of elasticity of the material to resist compression and tension. These two coefficients are generally assumed to be the same, which is not exactly true, though practically so. They vary both with the kind of material and kind of stress, and are constant in any given beam. Either is equal to the

ratio of stress to the strain produced. Stress is an internal force which resists an alteration in the form or linear dimensions of a body; this alteration is called strain. In symbols, $E = \frac{\rho}{l}$ = a constant, in which E is the coefficient of elasticity, ρ is the intensity of the stress, and l is the increase or decrease in length, per unit of length, in the line of action of the force. 2d. Some function of the length, breadth, and thickness of the beam, expressed generally by some power of l, b, and d, respectively. 3d. By some coefficient depending on the form of cross-section.

317. The general value of the moment of resistance of any beam is found as follows:

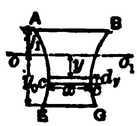


FIG. 124.

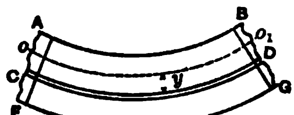


FIG. 125.

Let Fig. 124 be a cross-section of any beam, and Fig. 125 a longitudinal section of a beam, originally straight, but which under the action of the external bending moment has assumed the curved form represented in Fig. 125, concave upward and convex downward; CD, any layer perpendicular originally to the direction of the loads, but now curved as in Fig. 125. The layers at and near the upper surface of the beam are compressed or shortened; those at and near the lower surface EG have been stretched or lengthened. There must be some intermediate layer, such as oo_1, which is neither shortened nor lengthened; this is called the neutral layer or surface. This layer is, however, compressed or extended in a pair of inclined directions making an angle of 90° with each other or 45° with the neutral surface, and is not therefore a layer or surface of no strain, as often stated. If the external forces or loads are sufficiently great to break the beam crosswise or transversely, they must do so by crushing the material of the beam above the neutral surface and tearing it asunder below. The resistance to breaking or bending is the internal stresses of compression and extension developed by the external bending moment, and by the laws

of equilibrium the moments of these stresses must balance the external moment.

In finding these moments of resistance it is determined by experiment that the longitudinal stresses vary uniformly, and that their intensities are proportional to their distances from the neutral surface. If p is the intensity of the stress along any layer CD, Figs. 124 and 125, and y its distance from the neutral surface, then $\frac{p}{y}$ is a constant, $\frac{p}{y} = a$; the area of the layer is $= xdy$; the total stress on the layer is $pxdy = axydy$. The total stress on the entire cross-section, Fig. 124, will be $= a\int xydy$, and these are the only stresses or forces in the direction of the length of the beam, as the loads are all parallel and vertical. Then $\int xydy = 0$ by the conditions of equilibrium, and the resultant of these stresses must pass through the centre of gravity or centre of parallel forces. and being in the neutral surface, the neutral axis, oo_1, Fig. 124, which is the intersection of the surface of the cross-section with the neutral surface, must pass through the centre of gravity of any cross-section; hence by finding the centre of gravity of any cross-section, we can locate the position of the neutral axis. As p varies with y, it will have its greatest value when y has its greatest value. If, then, f_0 be the greatest tensile strain and f_1 be the greatest crushing strain that the beam can sustain, y_0 the distance from the neutral axis to the extreme outside or lowest fibre and y_1 from the neutral axis to the extreme upper fibre or surface, then $\frac{p}{y}$ can be made $= \frac{f_0}{y_0}$ or $\frac{f_1}{y_1}$; but as the smaller measures the ultimate strength of the beam, and as in some materials the resistance to crushing is greater than the resistance to tearing, we should make $\frac{p}{y}$ equal to the smaller of the two ratios, or $\frac{f}{y'}$. We have then $\frac{p}{y} = \frac{f}{y'}$. $p = \frac{fy}{y'}$, $\therefore$ total pressure on the layer $= pxdy = \frac{f}{y'}yxdy$, and its moment $= pxydy = \frac{f}{y'}xy^2dy$; and for the total moment of resistance of the cross-section,

$$M_0 = \frac{f}{y'}\int xy^2dy. \quad \ldots \ldots \quad (133)$$

The quantity under the integral sign is called the moment of inertia of the cross-section. Making, then, $\int xy^2dy = I$, which is the symbol for the moment of inertia, we have

$$M_0 = \frac{fI}{y'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

The moment of inertia, I, for present purposes simply means the sum of all the elementary areas, xdy, into which the cross-section can be conceived to be divided, multiplied by the square of the distances from their respective centres of gravity to the neutral axis of the section.

In equations (133) and (134) f is called the coefficient or modulus of rupture of the material. It is stated by some authorities that the value of the modulus of rupture is intermediate between the resistance to direct crushing and tearing. Later authorities, however, give the value of the modulus of rupture as equal to or greater than the direct tensile resistance; or, taking the usual materials, the relative values are as follows:

	Steel.	Wrought Iron.	Cast Iron.	Timber.
Direct tensile resistance	(a) 75000	(b) 50000	(c) 20000	(d) 10000
	$a \times 1.66 =$	$b \times 1.5 =$	$c \times 2$ (or $2\frac{1}{4}$) $=$	
Modulus of rupture f..	125000	75000	40000 to 45000	10000 to 17000

But doubtless the more common practice would not make f greater than the direct resistance to tearing in the above table for breaking loads, and not over one sixth to one fifth for working loads, or $\begin{cases} 12500 \\ 15000 \end{cases}$, $\begin{cases} 8333 \\ 10000 \end{cases}$, $\begin{cases} 3333 \\ 5000 \end{cases}$, and $\begin{cases} 1666 \\ 1000 \end{cases}$, respectively, for steel, wrought iron, cast iron, and timber.

318. For sections of similar figures I, the moment of inertia are to each other as their breadths and the cubes of the depths, or $I = n'bd^3$, and y' as the depths. $\therefore y' = m'd$;

$$\therefore M_0 = \frac{fn'bd^3}{m'd} = nfbd^2, \quad \text{when } n = \frac{n'}{m'}. \quad . \quad (135)$$

The moment of resistance of beams of the same material, then, varies as the breadth b and the square of the depth d. By doubling the depth we increase the strength four times and double the breadth only two times. Hence the deeper the beam the greater

the economy of material for a given area of cross-section. The ratio of $\frac{b}{d}$ should not be smaller than about one sixth or one eighth, otherwise the beam would be wanting in lateral stiffness.

For some of the more simple forms of section we can readily find the values of I and y', equation (134). For the solid rectangular beam

$$I = \int_{-y_0}^{+y_1} xy^2 dy, \quad x = b, \quad y' = y_1 = y_0 = \tfrac{1}{2}d = r;$$

hence

$$I = \tfrac{1}{12}bd^3; \quad M_0 = \frac{f\frac{1}{12}bd^3}{\frac{1}{2}d} = \tfrac{1}{6}fbd^2. \quad . \quad . \quad . \quad . \quad (136)$$

For the circle

$$I = \int_{-r}^{+r} xy^2 dy = \frac{\pi d^4}{64}, \quad r = \tfrac{1}{2}d, \quad M_0 = \frac{2f\pi d^3}{64} = \frac{f\pi r^3}{4}, \quad . \quad (137)$$

applicable to round logs; for beams of square section $b = d$. Then in equation (130)

$$M_0 = \tfrac{1}{6}fd^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

319. For solid rectangular beams, Fig. 126, the moment of resistance can be found directly. Assuming a beam resting on two points of support A and B, and to be bent so that the original vertical section cd assumes the position c_1d_1, the upper wedge cgc_1 the crushed volume, and dgd_1 the torn volume, the ordinates of these triangles, or volumes per unit of length, would represent the uniformly varying stresses, their resultants C and T would be equal and in opposite directions, and would pass through the centres of gravity of the triangles, which would be at points on the line joining g and the centres of the bases cc_1 and dd_1, and at two thirds of their lengths from g. Their lever-arms would then be equal to two thirds of $\frac{1}{2}d = \frac{1}{3}d$, each, as the neutral axis would be at the centre of the depth of the beam; the area over which the stresses are distributed would be $b \times \frac{1}{2}d$. Since $C = T$, they form a couple whose lever-arm is $\frac{2}{3}d$; the moment of this couple is then $M_0 = T \times \frac{2}{3}d$. Since f is the stress at the extreme upper or lower surface of the beam, the mean intensity of either

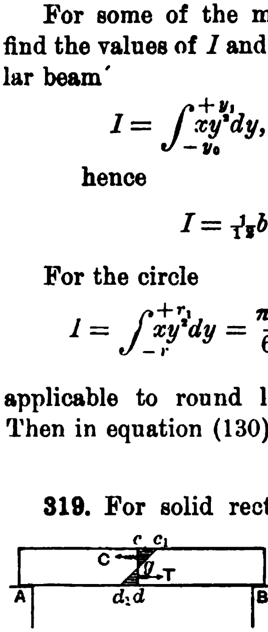


FIG. 126.

stress $= \frac{1}{2}f$; and as $T = C = \frac{1}{2}f \times \frac{1}{2}bd = \frac{1}{4}fbd$, then $M_e = \frac{1}{4}fbd \times \frac{2}{3}d = \frac{1}{6}fbd^2$, the same as equation (136).

320. As was seen in Art. XXIII, the bending moment depended on the length of the beam, upon the load and the way in which it was distributed, and upon the way in which the beam was supported. The general expression for the bending moment $= mWl$, in which the coefficient m depends upon the manner of loading and supporting the beam, $W =$ the total load, and $l =$ length of the beam. Then the equation expressing the equilibrium between the bending moment and the moment of resistance to bending will be

$$M_e = mWl = \frac{fI}{y'} = nfbd^2. \quad . \quad . \quad . \quad . \quad (138\tfrac{1}{2})$$

For a rectangular cross-section we have seen that $n = \frac{1}{6}$, for a square section $n = \frac{1}{6}$ and $b = d$, for a solid circular section $n = \frac{\pi}{32}$ and $b = d$, and for an elliptical section $n = \frac{\pi}{32}$, b the conjugate or shorter axis, and d the longer or transverse axis. For the hollow forms, such as the hollow rectangle, square, circle and ellipse, channels, flanged beams, etc., it is better to use the equation

$$mWl = \frac{fI}{y} = M_e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

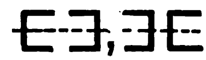
and find I and y for each case, and substitute in equation (139). Mr. Rankine, however, gives tables for the value of n for all forms in common use.

ART. XXXIII.

MOMENTS OF INERTIA.

321. As the meaning and applications of the moment of inertia are but little understood or appreciated by young engineers, the writer will enter into some detailed determination of the value I, the moment of inertia for some of the more common and useful forms of cross-section, before applying the foregoing principles and

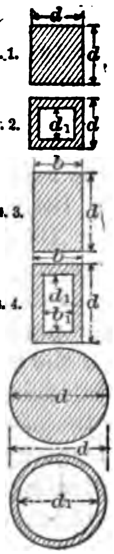
relations to the determination of the actual dimensions of beams or the loads that any given beam will carry safely.

For the hollow square, rectangle, and circle subtract the value of I for a square or rectangle of the dimensions of the hollow portion from the value of I for the entire cross-section. For flanged beams, if the flanges are of equal length and thickness, find the moment of inertia for a solid rectangle whose dimensions are the total depth of the beam and the breadth of the flanges, and subtract the value of I for the two supposed rectangles between the flanges and on either side of the web. This is also true for any of the following forms in which the neutral surface is at the centre of the depth of the beam,  , and in fact for any form of cross-section in which the solid and hollow portions together can be conceived to form a large rectangle, the separate rectangles composing it being symmetrically arranged with respect to the neutral surface, such as  , these forming double channels latticed, generally of wrought iron. For such forms as the  with only one flange, or the  with unequal flanges, the formulæ are somewhat complicated, and two methods are used: one consists in finding the exact position of the centre of gravity of the cross-section through which the neutral surface passes; or we may consider the cross-section as made up of a series of rectangles, and then determine the moment of inertia of each rectangle with respect to the neutral axis passing through its centre of gravity, to which is to be added algebraically the sums of the separate areas of the rectangles multiplied by the squares of their distances from one of the co-ordinate axes Ox or Oy. All of these cases will be readily understood by the following diagrams and values of I obtained.

322. The moment of inertia, so far as it appears in the formulæ for the strength of beams, is simply the sum of the products of the infinitely small areas into which a surface may be divided, or rather conceived to be divided, multiplied by the squares of their distance from the neutral axis, this sum being equivalent to the entire area of the section multiplied by the square of its radius of gyration, so called. In other words, if we find the moment of inertia for any cross-section, the square of the radius of gyration will be that

quantity divided by the area, or, in symbols, $\rho^2 = \frac{I}{a}$, in which ρ is the radius of gyration, I the moment of inertia, and a the area of the cross-section. From which relation and the values of I already determined for solid cross-sections we can easily calculate the values of I, ρ^2, etc., given below.

FIG. 1. FIG. 2. FIG. 3. FIG. 4.



FIGS. 127-132.

TABLE XLV.

For (1), solid square,

$$I = \frac{1}{12}d^4, \quad a = d^2, \quad \rho^2 = \frac{1}{12}d^2;$$

For (2), hollow square,

$$I = \frac{1}{12}(d^4 - d_1^4), \quad a = d^2 - d_1^2, \quad \rho^2 = \frac{d^2 + d_1^2}{12};$$

For (3), solid rectangle,

$$I = \frac{1}{12}bd^3, \quad a = bd, \quad \rho^2 = \frac{1}{12}d^2;$$

For (4), hollow rectangle,

$$I = \frac{1}{12}(bd^3 - b_1d_1^3), \quad a = bd - b_1d_1, \quad \rho^2 = \frac{bd^3 - b_1d_1^3}{12(bd - b_1d_1)};$$

For (5), solid circle,

$$I = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}, \quad a = \frac{\pi d^2}{4}, \quad \rho^2 = \frac{1}{16}d^2 = \frac{1}{4}r^2;$$

For (6), hollow circle,

$$I = \frac{\pi d^4}{64} - \frac{\pi d_1^4}{64}, \quad a = \frac{\pi d^2}{4} - \frac{\pi d_1^2}{4}, \quad \rho^2 = \frac{d^2 + d_1^2}{16}.$$

For the following cross-sections,

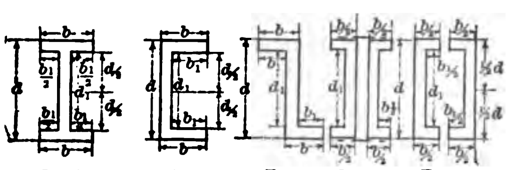


FIG. 7 FIG. 8. FIG. 9. FIG. 10. FIG. 11.

FIGS. 133.

$$I = \frac{1}{12}(bd^3 - b_1d_1^3), \quad a = bd - b_1d_1, \quad \rho^2 = \frac{bd^3 - b_1d_1^3}{12(bd - b_1d_1)}.$$

In the last figures (133, 7–11) it will be observed that b is the outside breadth of the flanges, d is the total depth of the section, b_1 is the total breadth of the hollow portion, i.e., $b_1 = \frac{b_1}{2} + \frac{b_1}{2} =$ $b -$ the web thickness, and d_1 is the depth between inner sides of the flanges.

323. For the following forms, where there is no top flange and the web is either single or double, and when double the two webs together have the same thickness as the single web, we proceed as follows: The web may be at any point on the flange at the middle as in Fig. 134, or at the ends as in Fig. 135. In such figures it is best to find the centre of gravity of the section which locates the position of the neutral axis.

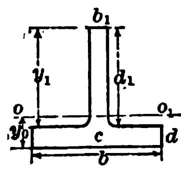


FIG. 134.

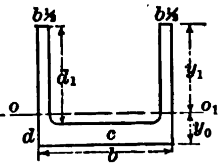


FIG. 135.

For practical purposes this could be found by cutting an exact copy of the cross-section out of cardboard, tin, or other material, and balancing it on a knife-edge, by which we can locate the position of the neutral axis; or we can determine the position of the centre of gravity, as in case of the centre of parallel forces in Art. XXI. Dividing the cross-section into a number of rectangles, the areas may be taken to represent both the volumes and weights, as the unit weight would appear in all of the terms, and can be omitted. Likewise, the length being unity, the areas are equivalent to the volumes; and as the sum of the moments of the parts is equal to the moment of the sum taken with respect to any axis perpendicular to the section, assuming the axis to pass at the lowest point or edge of the flange at c, we have two rectangles whose areas are bd and b_1d_1, their respective centres of gravity from $c = \frac{1}{2}d$ for bd, and $d + \frac{1}{2}d_1$ for b_1d_1. The sum of their moments is then $= b_1d_1(\frac{1}{2}d_1 + d) + bd \times \frac{1}{2}d$, the entire area $= bd + b_1d_1$, and its unknown lever-arm is y_0. As the neutral axis oo_1 must pass through the centre of gravity of the entire section, its moment is $(bd + b_1d_1)y_0$, and as the moment of the sum is equal to the sum of the moments of the parts, $y_0(bd + b_1d_1) = b_1d_1(\frac{1}{2}d_1 + d) + bd \times \frac{1}{2}d$; hence

$$y_0 = \frac{\frac{bd^2}{2} + b_1d_1(d + \frac{1}{2}d_1)}{bd + b_1d_1}; \quad . \quad . \quad . \quad . \quad . \quad (140)$$

$$y_0 + y_1 = d + d_1; \quad \therefore\ y_1 = d + d_1 - y_0 = \frac{b_1 d_1^2}{2} + bd\left(d_1 + \frac{d}{2}\right). \quad (141)$$

These values of y_0 and y_1 give the distances of the centre of gravity of the entire section, and also of the neutral axis oo_1, from the bottom and top of the beam, respectively. Now, having the position of the neutral axis, the moment of inertia of the sections Figs. 134 and 135 is then equal to the algebraic sum of the areas or rectangles into which tho surface may be divided, multiplied by the squares of the distances from the neutral axis to their respective centres of gravity, increased by the sum of the moments of inertia of the several rectangles with respect to a neutral axis passing through their own centres of gravity, respectively. This rule is general, and applicable to all figures that can be formed by symmetrically arranged rectangles, the neutral axis of each rectangle being parallel to the neutral axis of the entire section.

As the neutral axis oo_1 in Figs. 134 and 135 will generally pass a little above the upper surface of the flange, there will be at least three areas to consider: 1st, that portion of the web above the neutral axis; 2d, that portion between oo_1 and the flange; 3d, the flange itself.

But having found the value of y_0, it will be simpler to suppose that the thickness of the flange is increased from d to y_0, and then deduct for the two small rectangles on either side of the web and between oo_1 and the flange. Referring, then, to Figs. 134 and 135, we have areas b_1y_1, by_0, and $(b - b_1)(y_0 - d)$. Their respective distances from the neutral axis to their centres of gravity squared will be $(\frac{1}{2}y_1)^2$, $(\frac{1}{2}y_0)^2$, and $[\frac{1}{2}(y_0 - d)]^2$. The sum of these quantities multiplied two and two $= \frac{1}{4}b_1y_1^3 + \frac{1}{4}by_0^3 + \frac{1}{4}(b - b_1)(y_0 - d)^3$, to which must be added the sum of the moments of inertia of each rectangle with respect to an axis parallel to oo_1 through its own centre of gravity. These can be written directly from Fig. 129, paragraph 322, and for the three rectangles are $\frac{1}{12}b_1y_1^3$, $\frac{1}{12}by_0^3$, and $-\frac{1}{12}(b - b_1)(y_0 - d)^3$; hence

$$I = \tfrac{1}{4}b_1y_1^3 + \tfrac{1}{12}b_1y_1^3 + \tfrac{1}{4}by_0^3 + \tfrac{1}{12}by_0^3$$
$$- [\tfrac{1}{4}(b - b_1)(y_0 - d)^3 + \tfrac{1}{12}(b - b_1)(y_0 - d)^3];$$

hence

$$I = \tfrac{1}{3}[b_1y_1^3 + by_0^3 - (b - b_1)(y_0 - d)^3]. \quad . \quad . \quad (142)$$

By definition, $I = A\rho^2$, A being the area and $\rho^2 =$ the square

of the radius of gyration. The radius of gyration ρ of a body about a given axis is a length the square of which is the mean of all of the distances squared of the centres of gravity of the indefinitely small portions of the body from the same axis,—in this case the rectangles and neutral axis.

$$\rho^2 = \frac{I}{A}; \quad A = bd + b_1d_1.$$

$$\rho^2 = \frac{b_1y_1^3 + by_0^3 - (b - b_1)(y_0 - d)^3}{3(bd + b_1d_1)}. \quad . \quad . \quad (143)$$

If y_0 and y_1 in terms of the dimensions of the beam as found in equations (140) and (141) are substituted in equations (142) and (143), I, ρ, and A can be easily determined.

324. For the double-flange form, **I**, with unequal flanges, we find y_0, y_1, I, A, and ρ^2 by an exactly similar process.

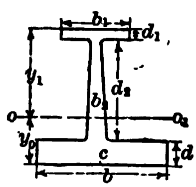


FIG. 136.

In Fig. 136, d = thickness of lower flange,
d_1 = thickness of upper flange,
d_2 = height or depth of web,
b_1 = breadth of upper flange,
b = breadth of lower flange,
b_2 = thickness of web.

Then

$$(bd + b_2d_2 + b_1d_1)y_0 = bd \times \tfrac{1}{2}d + b_2d_2(\tfrac{1}{2}d_2 + d) + b_1d_1(\tfrac{1}{2}d_1 + d_2 + d).$$

$$y_0 = \frac{\frac{bd^2}{2} + \frac{b_2d_2^2}{2} + \frac{b_1d_1^2}{2} + b_2d_2d + b_1d_1(d_2 + d)}{bd + b_2d_2 + b_1d_1}. \quad . \quad . \quad (144)$$

Assuming that the thickness of flanges are increased to y_0 and y_1, respectively, and deducting for the areas on either side of the web and between oo_1 and the flanges, we have areas by_0, b_1y_1 and $-(b - b_2)(y_0 - d)$, and $-(b_1 - b_2)(y_1 - d_1)$; and the squares of their respective distances between their centres of gravity and neutral axis oo_1 are $\frac{1}{4}y_0^2$, $\frac{1}{4}y_1^2$, $\frac{1}{4}(y_0 - d)^2$, $\frac{1}{4}(y_1 - d_1)^2$. The products of these added $= \frac{1}{4}by_0^3 + \frac{1}{4}b_1y_1^3 - [\frac{1}{4}(b - b_2)(y_0 - d)^3 + \frac{1}{4}(b_1 - b_2)(y_1 - d_1)^3]$; and the moments of inertia of the several rectangles with respect to their own neutral axes parallel to oo_1 are

$\frac{1}{12}by_0^3$, $\frac{1}{12}b_1y_1^3$, $-\frac{1}{12}(b-b_2)(y_0-d)^3$, and $-\frac{1}{12}(b_1-b_2)(y_1-d_1)^3$.
Hence

$$I = \tfrac{1}{4}by_0^3 + \tfrac{1}{12}by_0^3 + \tfrac{1}{4}b_1y_1^3 + \tfrac{1}{12}b_1y_1^3 - [\tfrac{1}{4}(b-b_2)(y_0-d)^3$$
$$+ \tfrac{1}{12}(b-b_2)(y_0-d)^3 + \tfrac{1}{4}(b_1-b_2)(y_1-d_1)^3 + \tfrac{1}{12}(b_1-b_2)(y_1-d_1)^3]$$

$$= \frac{by_0^3 + b_1y_1^3 - (b-b_2)(y_0-d)^3 - (b_1-b_2)(y_1-d_1)^3}{3}. \qquad (145)$$

$$\rho^2 = \frac{I}{A};\ A = bd + b_2d_2 + b_1d_1. \quad . \quad . \quad . \quad (146)$$

Any of the preceding forms can be used either as columns or beams; so likewise the following forms, which are built up of plates, channels, and angle irons riveted together; but the hollow circle and the Phœnix column or beam are rarely used for beams. The following forms of cross-sections are used for the strut members of iron trestles and iron bridges:

For columns the more common forms are: 1st. The Phœnix columns with circular cross-section, composed of four or more circular segments, with external flanges, riveted together through the flanges. These form a closed column, which is objected to by many on account of the supposed deterioration on the inside from rust, as the interior cannot be painted after being once closed; but, on the contrary, the advocates of this column or beam claim that the objection is more fanciful than real. This form is largely used by the Phœnix Bridge Co., and often adopted by engineers as being the most economical. Another objection to this form of column is that it is used generally in connection with cast-iron bases and capitals and cast-iron sockets between the different sections in long columns and beams; and, in addition, the connections for the diagonal and horizontal braces between the columns both in horizontal and vertical planes are difficult, and seem somewhat complicated by the number of pins, loops, hangers, etc., required. But there are many bridges, and innumerable iron trestles or viaducts, in which the columns and other strut members are of this construction. Examples of these have been given in previous paragraphs. By reference to the above cross-section we see that it may be made up of a hollow circular cross-section, and a series of rectangles, the number of which may

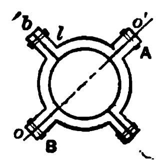


FIG. 187.

be four, six, or eight. In paragraph 322 we see that the moment of inertia I of the hollow circle is $I = \frac{\pi(r_2^4 - r_1^4)}{4}$; and for the rectangles formed by the flanges the moment of inertia would be the area bl multiplied by the square of its distance from any axis passing through the centre of gravity of the entire column increased by the moment of inertia of the rectangle with respect to an axis through its own centre of gravity. It can easily be shown that the position and direction of the diameter of the column assumed for the neutral axis is immaterial; we will therefore assume it as through the centre of gravity of the two opposite flanges, A and B. The sum of the moments of inertia of all the flanges will be $2bl\left(r + \frac{l}{2}\right)^2 + 4 \times \frac{bl^3}{12}$. The last term, being very small as compared with the moment of inertia of the whole section, may be neglected; hence the moment of inertia of the entire section will be $I = \frac{\pi(r^4 - r_1^4)}{4} + 2bl\left(r + \frac{l}{2}\right)^2$; r = exterior and r_1 interior radius, b = breadth and l = length of rectangles. Area of cross-section $= \pi(r_2^2 - r_1^2) + 4bl$; radius of gyration squared

$$\rho^2 = \frac{\frac{\pi(r^4 - r_1^4)}{4} + 2bl\left(r + \frac{l}{2}\right)^2}{\pi(r^2 - r_1^2) + 4bl} = \frac{\pi(r^4 - r_1^4) + 8bl\left(r + \frac{l}{2}\right)^2}{4\pi(r^2 - r_1^2) + 16bl}. \quad (147)$$

2d. If the column is a box composed of two channels and cover-plates, as in Fig. 138, proceed as follows: Let b = breadth and t_1 = thickness of the plates; t = the thickness of the web of the channels, and d = the channel depth; t_2 = the thickness of the flanges of the channel and b_1 = the breadth of the flange projection. In this cross-section we have two axes of symmetry, as the neutral axes oo_1 and o_2o_3. Find first the moment of inertia with respect to oo_1 for the covering-plates,

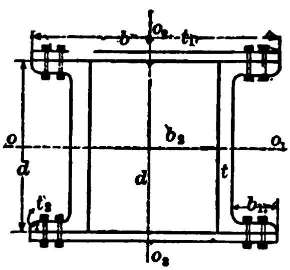


FIG. 138.

$$\left(\frac{bt_1^3}{12} + \frac{bt_1(d + t_1)^2}{4}\right) \times 2 = \frac{bt_1^3}{6} + bt_1\frac{(d + t_1)^2}{2},$$

these being rectangles distant from and symmetrically situated with respect to oo_1. For the moment of inertia of the channels

$$\left[\frac{(b_1+t)d^3}{12} - \frac{b_1(d-2t_2)^3}{12}\right] \times 2 = \frac{(b_1+t)d^3 - b_1(d-2t_2)^3}{6}$$

for both channels. Hence

$$I = \frac{bt_1^3}{6} + bt_1\frac{(d+t_1)^2}{2} + \frac{(b_1+t)d^3 - b_1(d-2t_2)^3}{6} \qquad (148)$$

about the axis o_2o_2. For the two covering-plates about an axis through their own centres of gravity $\frac{t_1b^3}{12} \times 2 = \frac{t_1b^3}{6}$, and for the two channels we may construct the diagram Fig. 139, in which we have two rectangles, each t_2 in thickness and $2b_1 + 2t + b_2$ in breadth, and a rectangle $b_2 + 2t$ in breadth, in height $d - 2t_2$, from which must be subtracted the rectangle of b_2 thickness and d length, the axis o_2o_2 passing through their respective centres of gravity, and we write at once the algebraic expression for the moment of inertia of the whole area of the two channels,

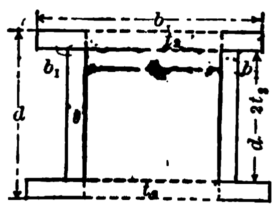


FIG. 139.

$$\frac{2t_2(2b_1+2t+b_2)^3}{12} + \frac{(d-2t_2)(b_2+2t)^3}{12} - \frac{db_2^3}{12};$$

and for the entire cross-section,

$$I_1 = \frac{t_1b^3}{6} + \frac{2t_2(2b_1+2t+b_2)^3}{12} + \frac{(d-2t_2)(b_2+2t)^3}{12} - \frac{db_2^3}{12}. \quad (149)$$

The area of the entire section $A = 2bt_1 + 4(b_1+t)t_2 + 2t(d-2t_2)$, and

$$\rho^2 = \frac{I}{A} \quad \text{or} \quad \frac{I_1}{A},$$

In the above equations (148) and (149) if the covering-plates are omitted, and the channels are connected by thin strips of iron riveted to them at intervals, owing to areas of these being very small

all terms in the above equations involving bt_1 can be omitted. Such are called the latticed columns; these are the usual forms of columns. Eq. (148) then becomes with respect to oo_1

$$I = \frac{(b_1 + t)d^3 - b_1(d - 2t_2)^3}{6},$$

and from Eq. (149)

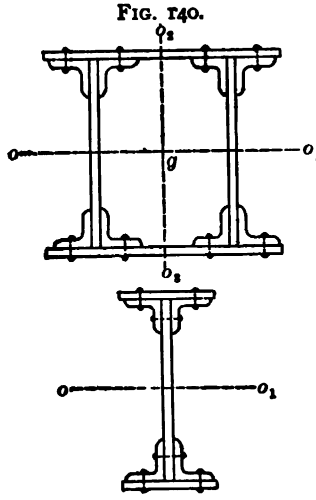
$$I_1 = \frac{2t_2(2b_1 + 2t + b_2)^3 + (d - 2t)(b_2 + 2t)^3 - db_2^3}{12},$$

with respect to o_2o_2.

$$A = 4(b_1 + t)t_2 + 2(d - 2t_2)t; \quad \rho^2 = \frac{I}{A} \text{ or } \frac{I_1}{A}. \quad . \quad (150)$$

3d. If the box columns are composed of two rectangular cover-plates and two web-plates, and these connected by angle-irons four or eight in number, or a built flange beam of two rectangular plates and one web-plate connected by four angle-irons as in Figs. 140 and 141, respectively, having as in the above case two neutral axes at right angles to each other as in oo_1 and o_2o_2, the moments of inertia I and I_1 can be easily found as outlined above by recollecting that the moment of inertia of any cross-section composed of any number of symmetrically arranged rectangles with respect to the neutral of the entire section is found by first adding together the areas of each rectangle multiplied by the square of the distance between its centre of gravity and the neutral axis oo_1 or o_2o_2, and to this sum the moments of inertia with respect to a neutral axis passing through its own centre of gravity and parallel to the neutral axis of the entire cross-section. It will be observed that Figs. 140 and 141 are built up of rectangular plates, and angles the two legs of which may be cosidered as forming two rectangles.

FIG. 140.



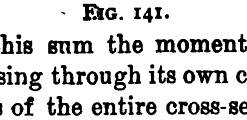


FIG. 141.

If the web in Fig. 134 is moved to the end of the flange, or Fig. 135 be cut by a plane between the two webs, the resulting parts would form the angle-irons, and equations (140), (141), (142), (143) can be easily adapted to the above suppositions. The uses and practical value of the moment of inertia has already been seen in discussing long columns, and will be further explained in the practical application of the foregoing formulæ and principles to the determination of the strength and dimensions of beams.

ART. XXXIV.

SHEARING FORCES.

325. THE subject of shearing forces has been alluded to in connection with that of bending moments, which it generally accompanies, and in that connection it was seen that at any section of a beam it was equal to the algebraic sum of the vertical forces or loads acting on that part of the beam, on either side of the section. And further, that for beams fixed at one end and loaded with a single weight at the free end, or at any point, the shearing force is constant and equal to the load from the point of application to the fixed end, and when uniformly loaded either over a part or the whole of its length, that the shearing force gradually increases from the outer to the inner end of the load, attaining its maximum value at the inner end of the load, and increasing in the same direction as the bending moment, its maximum value being equal to the sum of all of the loads between the free end and the last load (inclusive) preceding the given section, which may be at the fixed end.

If the beam is supported at both ends, it was also seen that the shearing force had its maximum value at the points of support where the bending moment is zero, and decreased gradually towards some intermediate point (in case of a uniformly distributed load) or by successive subtractions of the isolated loads between the points of support and that point where the bending moment was the greatest, at which point the shearing force was zero; and that at any section the shear is equal to the first differential coefficient of the

bending moment $\left(\frac{dM}{dx}\right)$, considered as a function x_1, the co-ordinate x being horizontal and measured from any point in the length of the beam, the loads being normal to the axis of the beam. The above conditions as to loading and supporting beams are alone considered in this article. The shear having been found by the above principles and rules, we will now find—

326. *The distribution of the internal stress which resists the shearing force at any section.*

Without undertaking to discuss the theories or deduce the equations applicable to internal resistances to shearing, a few of the principles and practical results will be explained as regards the intensities, directions, and distribution of this stress.

The First Principle.—Planes of equal shear or tangential stress.—" If the stresses on a given pair of planes be tangential to those planes, and parallel to a third plane which is perpendicular to the pair of planes, those stresses must be of equal intensity."

Let Fig. 142 represent the cross-section of a right prism of any length, say unity, so that the area $ABCD$ is equivalent to the volume, and the lines AB, CD, AD, and BC are equivalent to the areas of the bounding surfaces. Then if p_1 is the intensity of the shear or tangential stress on AB and CD, and planes parallel to them; and p_2 the intensity of the shear on planes AD, CB, and planes parallel to them (see Fig. 142),—the total shears are $p_1 \times AB$, and $p_2 \times AD$. These with the equal forces on the areas CD and BC form couples whose lever-arms are EF and HG, respectively. Hence the moments of these forces are $+p_1AB \times EF$ and $-p_2AD \times HG$, right and left handed, respectively. Hence $p_1AB \times EF = p_2AD \times HG$. But the products $AB \times EF = AD \times GH$ are both equal to the volume of the prism.

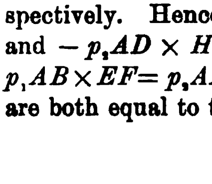


FIG. 142.

$$\therefore p_1 = p_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

Eq. 151 shows that a shear upon a given plane cannot exist alone as a single stress, but must be combined with a shear of equal intensity on another plane. The tendency of such a pair of shearing stresses on the prism tends to distort it, lengthening one diag-

onal and shortening the otner, as seen in Fig. 142. (See Rankine's Applied Mechanics, page 88.)

Again, we have seen that the coefficient of elasticity simply expresses the ratio between stress and strain.

The characteristic strain caused by a pull or tensile force is an increase in that linear dimension of a body in the direction of the external force, or $E = \frac{p}{l}$, p being the intensity of the force, l the increase per unit of length, and E the coefficient of elasticity for tension.

The characteristic strain for a compressive force is a decrease in that linear dimension of the body in the direction of the external force. Then $E_1 = \frac{p_1}{l_1}$, the terms having the corresponding meaning as above. It is usual to assume

$$E = E_1. \qquad (152)$$

327. The characteristic strain of a shearing force is distortion, as seen in Fig. 143. Let $ABCD$ represent one face of a prism, another face being fixed along AD. If a shear acts in the face BC perpendicular to $ABCD$, all layers of the prism parallel to the plane of the shearing force BC will tend to slide over each other, the faces AB and DC taking the position AE and DF. The distortion or strain per unit of length will be measured by the angle ϕ between the old and new position of the faces. If ϕ is small, we can use indifferently ϕ_1, sin ϕ, or tang ϕ.

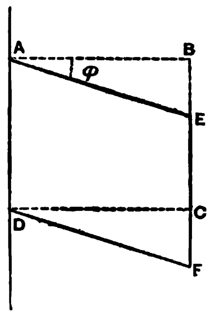


FIG. 143.

$$\therefore E_2 = \frac{p_2}{\phi}. \qquad (153)$$

p_2 = the intensity of shear, ϕ = the strain, and E_2 = the coefficient of elasticity for shearing. As has been stated, there are certain limits of stress within which eqs. (151), (152), (153) are true, but not true beyond. This limit is called the limit of elasticity.

328. If a body or a beam be subjected to tension or compression, as when acted upon by external forces orloads, all of its oblique sections, such as aa_1, bb_1, etc., Fig. 144, will have a tendency to slide

over each other, caused by the action of the tangential components of the direct stresses. (See Fig. 144.) If the direct stress is tensile, the tendency of the tangential or shearing stresses is to reduce the areas of the original or normal cross-sections, thereby reducing correspondingly its power of resistance to the external forces acting on BC and AD. If the direct stress is compressive (the external forces would act inwards towards the body), the shears will be in the opposite direction to those in the case of direct tension, and the tendency of such shears will be to increase the area of the cross-sections, thereby increasing its capacity for resistance to the external forces. These changes in the dimensions of the cross-sections of a body are called "lateral strains," which decrease the resistance of a body to tearing and increase its resistance to crushing. The effect of the shearing stresses is to cut or shear the beam along certain oblique cross-sections. The beam must then have sufficient area to bear this strain as well as that arising from the direct pulls or thrusts. But this strain in a beam is always greatest where the bending moment is least, and becomes zero or nothing where the bending moment is greatest; and for this reason in beams of ordinary length it is not necessary to add any material to the beam above that usually required for resistance to bending. The subject of the value and magnitude of shearing strains is of the greatest importance in trussed bridges, but is of little moment for ordinary beams, as the thickness of the web will always from necessity be sufficient to resist the shearing stresses in flanged beams, and in solid or hollow beams there will generally be a considerable excess of material. The resistance of either wrought or cast iron to shearing may be taken as practically the same or a little less than their resistance to tearing, say from 1 to $\frac{3}{4}$, and the same for steel; or for wrought iron $\frac{3}{4}$ of 50,000 lbs. = 37,500 lbs., for cast iron $\frac{3}{4}$ of 20,000 = 15,000 lbs., and for steel $\frac{3}{4}$ of 80,000 = 60,000 lbs., per square inch.

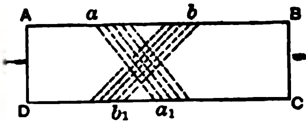


FIG. 144.

The shearing resistance especially of timber varies considerably, not only in different timbers, but also in the same timbers according as it acts along the grain or fibre or across it. Special applications will be made in another article to beams of different material. (See Burr's Resistance of Materials.)

329. We have seen that the intensities of the shears on a pair of planes at right angles to each other, and to the plane parallel to which the stresses act, must be equal to each other. If, therefore, we wish to find the intensity of the vertical shearing stress at a given point in a vertical section of a beam, it is only necessary to find the horizontal shearing stress at the same point. That the latter

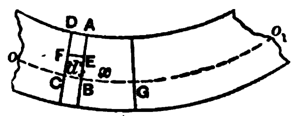


FIG. 145.

stress exists can readily be seen by placing a series of planks the one on the top of the other, forming an unbolted built beam. When loaded these layers slide on each other. In Fig. 145 let AEB be a a vertical section and E a point in that section, and let DFC be another plane near and parallel to AEB. As in a beam, the bending moments are different at these two sections. If supported at both ends, it will be greater at AEB; therefore the direct stress (compressive in Fig. 145 on AE) is greater than on the corresponding portion of the section DF. This excess is a horizontal force acting on the small prism $AEFD$, and as equilibrium is assumed the resistance to sliding or shearing along the plane FE must be equal and opposite to this horizontal excess. It is clear that the shearing stress is zero at the upper and lower surfaces of the beam, since the entire direct stress on each cross-section is zero; and it is also clear that the shearing stress in the vertical layer between the sections AEB and DFC is greatest at CB in the neutral surface oo_1, at which point the direct stress changes from a compressive to a tensile force, for at that surface the horizontal force is a maximum and develops a maximum resistance to shearing.

To Express these Relations and Results in Symbols.—In Fig. 145 let $x =$ distance of the section AEB from the centre of the beam G, y vertical or perpendicular to x, and z horizontal and normal to the plane x, y. The limiting value of x is the length of the beam, of y its depth, and of z the varying breadth of the beam. The beam is supposed to be of uniform cross-section (see Fig. 146). As we have seen in Art. XXXII, Eq. (134) the general expression for the moment of resistance to bending is

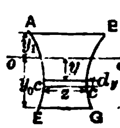


FIG. 146.

$$M = \frac{pI}{y}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (154)$$

in which p is the intensity of the direct horizontal stress at any point E distant y from the neutral axis. The elementary area at $E = zdy$; stress (compression in this case) on this area $= pzdy$. The total stress on

$$EA = \int_y^{y_1} pzdy,$$

y_1 being the distance from B to A, and y from B to E. From equation (154)

$$p = \frac{M}{I}y.$$

Hence total direct stress on

$$AE = \frac{M}{I}\int_y^{y_1} yzdy. \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

Since the horizontal excess of the direct stress on AE over that on DF is the force that develops and is equal and opposed to the resistance to sliding or shearing required, and as this excess arises from the difference between the bending moments at the sections AEB and DFC, M in eq. (154) is the difference of the moments of flexure or bending; and as these sections are distant from each other by dx, and if S is the amount of shearing force at the vertical layer, then $M = Sdx$, and eq. (155) becomes

$$= \frac{Sdx}{I}\int_y^{y_1} yzdy; \quad . \quad . \quad . \quad . \quad . \quad . \quad (156)$$

and this divided by the area of the plane $FE = zdx$. The intensity of the shearing stress

$$q = \frac{S}{Iz}\int_y^{y_1} yzdy; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$

and for its maximum value $z = z_1$ at the neutral surface oo; and the intensity

$$q_0 = \frac{S}{Iz_1}\int_y^{y_1} yzdy. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (158)$$

The same results are obtained for that portion of the beam below the neutral surface where the direct stress is tensile for beams supported at both ends. The complete integral for the

whole section = 0, as y is measured from the neutral axis which passes through the centre of gravity of the section. If A is the total area of the cross-section of the beam $= \int z dy$, the mean intensity of the shearing stress $q_1 = \frac{S}{A}$; and if the maximum intensity is q_0, we have $\frac{\text{max. intensity}}{q_1} = \frac{q_0}{S/A} = \frac{q_0 A}{S}$ as the ratio in which the maximum intensity exceeds the mean—a ratio depending entirely on the form of the cross-section. Substituting the value of q_0, we have

$$\frac{q_0 A}{S} = \frac{A}{Iz_1} \int_0^{y_1} yz dy. \quad . \quad . \quad . \quad . \quad . \quad . \quad (159)$$

For a solid rectangular beam breadth b and depth d,

$$A = bd, \quad I = \tfrac{1}{12} bd^3, \quad z_1 = b, \quad z = b, \quad y_1 = \tfrac{1}{2} d;$$

then the ratio

$$\frac{q_0 A}{S} = \frac{12 b^2 d}{b^2 d^3} \int_0^{\frac{1}{2}d} y dy = \frac{12 b^2 d^3}{8 b^2 d^3} = \frac{3}{2}. \quad . \quad . \quad . \quad (160)$$

For an ellipse the ratio is $\frac{4}{3}$, and similarly it can be found for other forms of cross-section. (See Rankine's Applied Mechanics, pages 338, 339, 340.)

Mr. Burr, on page 121, Resistance of Materials, gives the same ratio as equation (160). It means that the greatest intensity of the shear is one and a half times as great as the mean intensity, is found at the neutral axis, and is expressed by

$$T_2 = \frac{3}{2} \frac{F}{2d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (161)$$

On pages 204 and 206 the following principles and conditions are enunciated:

"At the neutral surface there are two planes on which the stress is wholly normal, and these planes make angles of 45° with the neutral surface, or 90° with each other (i.e., they are principal planes):

$$P = \pm T_2 = \pm \frac{Fd^2}{2I}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (162)$$

F being the weight at the end of the beam, I the moment of inertia of the cross-section, and d the half depth of the beam.

"Hence, from equation (162), each of these normal or principal stresses equals in intensity that of the longitudinal or transverse shear at the neutral surface; also, one of these principal stresses is a tension and the other a compression.

"Hence at the exterior surfaces of the beam the planes of greatest shear make angles of 45° with the axis of the beam, and the intensity of the shear is half that of the direct stress at the same place:

$$T = \pm \frac{Fxd}{2I} = \pm \frac{N_1}{2}; \quad . \quad . \quad . \quad . \quad . \quad (163)$$

at the neutral surface,

$$T = \pm \frac{Fd^2}{2I} = T_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad (164)$$

and the planes of greatest shear are the transverse and longitudinal planes, and the greatest shear itself is consequently the transverse or longitudinal shear."

TO DETERMINE THE THICKNESS OF THE WEB.

330. Referring to equations (161) and (162) and the deductions from them, we can use the following approximate method of determining the thickness of the web-plate in flanged beams (see Burr's Resistance of Materials). The common practice in flanged beams is to consider that the flanges resist the entire bending moment and that the web resists the entire shearing force. This is practically true, and the assumption involves no serious error.

It is further generally assumed that the intensity of the shear is uniform throughout the area of the transverse section of the web. Equations (160) and (161) show that this assumption will make the shear too large at the top and bottom portions of the section and only two thirds of its proper value at the centre or neutral axis.

If, then, we suppose that the web is composed of short columns with fixed ends and rectangular cross-sections, whose axes make an angle of 45° with the neutral surface, one assumption is an error on the side of danger, as it makes the shear only two thirds of its

actual value; while the other, in making the shear at the top and bottom surfaces equal to the mean intensity instead of zero, is on the side of safety. This latter influence largely exceeds that of the former, and the resultant error is on the side of safety.

If, then, in rolled beams, $d =$ total depth and $t_1 =$ thickness of one flange, and $l =$ length of these elementary columns,

$$l = (d - 2t_1) \text{ sec. } 45^\circ = 1.414(d - 2t_1);$$

and in built beams, if $d =$ the depth between centres of rivet-holes, $l = 1.414d$. Then if $S =$ total shear at any section, $A =$ area of section, $s =$ mean intensity $= \dfrac{S}{A}$, s will be the intensity of compression on the small columns; and if t is the thickness of the web or least side of column, Gordon's formulæ,

$$p = \frac{f}{1 + a\frac{l^2}{d^2}}, \quad \text{becomes} \quad s = \frac{f}{1 + \frac{l^2}{3000t^2}};$$

$$\therefore t = l\sqrt{\frac{s}{3000(f - s)}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (165)$$

Or, making $f = \frac{1}{6}$ of 40,000 for a safe value,

$$t = 0.0183l\sqrt{\frac{s}{8000 - s}}. \quad . \quad . \quad . \quad . \quad . \quad (166)$$

$a = \frac{1}{3000}$ applies only to wrought iron. If the depth of the beam is constant, it is only necessary to find t where s is the greatest, i.e., at the points of support.

The above formulæ give a greater thickness than is actually used, but they show the necessity of stiffness near the ends of the beams. The web should never be less than $\frac{1}{4}$ in. in thickness for any beam. The resultant error would lead to an increase in the value of t; and in addition there exists an equal tension at right angles to the greatest compression in the material of the web, as seen in equation (162), which gives support to the elementary columns throughout their entire lengths.

Three quarters of the thickness given by the formulæ would be ample.

331. The practical deductions from all of the foregoing principles and formulæ are that in solid rectangular beams there is always a sufficiency and more of material to resist the shearing forces when the resistance to the maximum bending moment is provided for; that where, as in timber beams, it is convenient and economical to use solid beams, in iron beams, whether cast as in cast iron, rolled as in wrought iron and steel, or built as in plate-girders of plates and angle-irons, the conditions of strength are satisfied and economy of material is practised by making the flanges of sufficient dimensions to resist the bending moments, and only using enough material in the web-plates toresist the shearing alone, such as the Phœnix, box, inverted **T**, or double **I** sections; and further, that where the spans are long the solid web can be dispensed with and a system of struts and ties forming an open web system between the flanges or chords can be used, which resist the shearing action by direct stresses of compression or tension in the direction of their longitudinal axes.

Although many actual conditions are unknown and many assumptions made, the combined results of theoretical investigations and experiments enable us to build safely, and with a fair degree of economy in both material and workmanship.

332. Short columns also give way under a direct crushing force by sliding or shearing along oblique surfaces which make angles approaching 45° with the direction of the crushing force. Theoretically, the greatest intensity of shearing stress is on planes making angles of 45°, and the deviation of these planes from that angle shows that resistance to shearing is not purely a cohesive force, but consists partly of a force analogous to friction. Crushing by shearing or sliding is characteristic of granular substances, such as cast iron, stone, and brick, and occurs in those substances whose resistance to crushing is much greater than that to tearing.

SHEARING STRESS ON RIVETS AND BOLTS.

333. Rivets are short pieces or bolts of wrought iron or steel which are used to connect thin plates together so as to enable them to resist a direct compressive or tensile strain, the stress being transmitted from one plate to another through the rivets, which are thereby subjected to a shearing force in a plane normal to their

longitudinal axis. In order that the shearing force may be uniformly distributed over the area of the transverse section of the rivet it is evident that it must fit the rivet-hole so perfectly that the friction at its surfaces must be at least equal to the intensity of the shearing stress. For this reason rivets must be inserted when intensely heated, and a head formed under repeated and hard blows or by a heavy pressure, so that the rivets not only fit perfectly the holes, but at the same time draw the plates close together. The heads should be formed at right angles to the axis of the rivets, and should bear squarely and fully against the plates. If rivets are loose, the shearing stress is not uniformly distributed over their areas, will be nothing at certain parts of their external surfaces, and the most intense shearing stress will be much greater than the mean stress.

It is possible only approximately to determine the distribution of stress in a riveted joint, hence the amount of stress carried by a rivet, cover-plate, or main-plates cannot be found exactly. The following figures, 147–151, show the ordinary riveted joints.

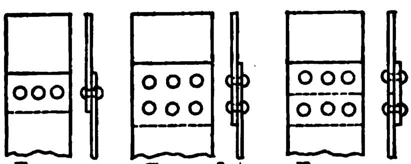


FIG. 147. FIG. 148. FIG. 149.

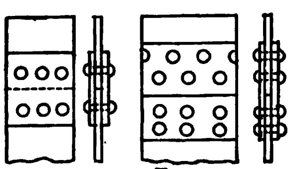


FIG. 150. FIG. 151.

These are commonly used in members under a tensile strain. Fig. 147 is a single-riveted "lap-joint." Fig. 148 is a double-riveted "lap-joint." Fig. 149 is a single-riveted "butt-joint" with cover-plate. Fig. 150 is a single-riveted "butt-joint" with two cover-plates. Fig. 151 is a double-riveted "butt-joint" with two cover-plates; the upper half shows zigzag riveting and lower half chain riveting. In all of these joints the main-plates are single. Two or three

thicknesses are often used: in such cases the overlap must be longer, as well as the cover-plates if used, so as to increase the number or diameter of the rivets in proportion to the increased stress to be transmitted. When two or more rows of rivets are used, the outside rows have to bear a larger portion of the stress, and may have to sustain it all, in consequence of the stretching of the material at the joint. It is seen in Figs. 147 and 148 that the stresses on the plates form a couple, whose lever-arm is usually equal to one half the sum of the thicknesses of the plate. Hence, calling T the mean intensity of the tension on one plate, t its thickness, and T_1 the mean intensity on the other plate and t_1 its thickness, also p the pitch of the rivets and d their diameter, the area of metal between rivet-holes $= t(p - d)$, tension on this area $= Tt(p - d)$, its lever-arm $= \frac{t + t_1}{2}$, and its moment

$$= M = Tt(p - d)\left(\frac{t + t_1}{2}\right) = T_1t_1(p - d)\left(\frac{t + t_1}{2}\right). \quad . \quad (167)$$

The resultant stress can be made to pass through the centre of the joint. In this case the lever-arms are $\frac{t}{2}$ and $\frac{t_1}{2}$, respectively, in which case the moment

$$= M = Tt^2\left(\frac{p - d}{2}\right) = T_1t_1^2\left(\frac{p - d}{2}\right). \quad . \quad . \quad . \quad (168)$$

The bending moment on the plate is $M_0 = \frac{fI}{r}$. $f =$ modulus of rupture, $r = \frac{1}{2}t$. M in eqs. (167), (168) $= M_0$. $\therefore\ f = \frac{Mt}{2I} = \frac{Mt_1}{2I} =$ the greatest intensity of tensile strain due to bending, or ultimate greatest tension in the plates is $T + f$ or $T_1 + f$. The moment of inertia $I = \frac{1}{12}(p - d)t^3$ or $\frac{1}{12}(p - d)t_1^3$. Substituting the values of I and M from equation (167), we have $f = 6T$, and M from equation (168), we have $f = 3T$. These values of the tensile bending stress are only true within the elastic limit, and within that limit the greatest intensity of tension in the plates may reach four to six times the direct tension T.

The rivets themselves are liable to a great bending action,

especially in the single lap and butt joint with a single cover-plate (Figs. 147–9). An approximate equation for this action, and the resistance, can be readily found. The general value $M = \frac{fI}{y}$ gives for a circular cross-section (see par. 322) $I = \frac{\pi d^4}{64}$, $y = \frac{1}{2}d$; hence

$$M = \frac{f\pi d^3}{32}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (169)$$

in which f is the modulus of rupture or greatest intensity of tension or compression on the rivet due to the bending action, and d equals diameter of rivet. If there are n rows of rivets on each side of the joint, then

$$M = \frac{nf\pi d^3}{32}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (170)$$

Substituting for M its value in equation (167), and solving with respect to f, we have

$$Tt(p-d)\frac{(t+t_1)}{2} = \frac{nf\pi d^3}{32}, \; f = 16Tt(p-d)\frac{(t+t_1)}{n\pi d^3}. \quad (171)$$

If $t = t_1$, then

$$f = 32Tt^2\left(\frac{p-d}{n\pi d^3}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (172)$$

These formulæ are merely given as showing the general line of theoretical investigation, and can be of but little practical value. The actual bending moment after a slight deflection of the rivet is much less than M in equation (170). After a distortion of the joint by the bending of the plates and rivets a strain of direct tension is developed.

There is also a very large intensity of pressure exerted between the rivet and the walls of the hole. This pressure is not distributed uniformly over the entire surface of contact, but has its greatest intensity on those portions of the surface adjacent to the ends of that diameter parallel to the direction of the stress on the plates. This greatest intensity may equal the crushing resistance of the material over a large part of the surfaces of contact. The exact distribution of this pressure cannot be determined. The bending

of the rivet tends to increase the intensity of the pressure near the surface of contact between the plates, and this combined with bad fitting of the rivet in the holes causes the intensity of the pressure to vary, both along the axis and on the circumference of the rivet. But assuming the pressure to be distributed over an area found by multiplying the diameter of the rivet by the thickness of the plate equal dt, and f a mean resistance to the pressure on the surface, the total bearing resistance R of one rivet gives

$$R = fdt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (173)$$

This equation will be frequently used in another article.

EFFECTS OF PUNCHING.

334. Punching rivet-holes in both wrought iron and steel plates is injurious. The tensile resistance of the remaining material will be diminished. This injurious effect can be removed by reaming the hole, increasing the diameter by $\frac{1}{8}$ inch. The loss of tensile resistance in wrought iron caused by punching averages from 10 to 15 per cent of its original resistance. Mr. Burr (on page 328, Resistance of Materials) quotes the following conclusions of Mr. Hill:

That the order of strength for holes made in steel plates is: 1st, the reamed hole; 2d, the drilled; 3d, the punched and annealed; and, lastly, the cold-punched. The injurious effect of punching can be entirely removed by enlarging the hole with either drill or reamer, the extent of this increase of diameter of hole varying with thickness of plate and grade of steel. Restoring strength and ductility by annealing is not recommended for bridge work, and in addition punching and reaming is more economical than punching and annealing. And on page 313 it is stated that both the elastic limit and ultimate resistance in mild-steel plates increase as the thickness decreases, and that the process of annealing decreases both elastic limit and resistance, but increases the ductility of the material.

335. Returning to the subject of rivets and riveted joints:
Let t = thickness of plates;
d = diameter of rivets;

p = pitch of rivets (i.e., distance centre to centre in one row);
T = mean intensity of tension in plates between rivets;
T_1 = mean intensity of tension in main-plates;
f = mean intensity of pressure on area $= dt$;
S = mean intensity of shear in rivets;
n = number of rivets in one main-plate;
r = number of rows in one main-plate;
h = distance from outside rivet centre and edge of cover-plate in direction of tensile stress.

Let T, T_1, f, and S be pounds per square inch. The thickness t depends upon the stress to be carried after allowing for loss of metal in holes, and the effects of punching.

Then, as readily seen from Fig. 152, we have the following relations:

$$Tt(p - d) = tpT_1; \quad \therefore \frac{T}{T_1} = \frac{p}{p - d},$$

or

$$\frac{T_1}{T} = \frac{p - d}{p} = 1 - \frac{d}{p}. \quad (173)$$

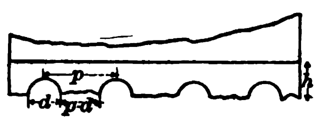


FIG. 152.

and for equal strength of joint in every direction the following must be fulfilled:

$$\frac{n}{r} T_1pt = \frac{n}{r} Tt(p - d) = nfdt = \frac{\pi d^2}{4} nS. \quad . \quad . \quad (174)$$

$nfdt$ is the bearing pressure of the rivets on the plate; $\frac{\pi d^2}{4} nS$ is the shearing resistance of all n rivets; $\frac{n}{r} T_1pt$ is the total tensile stress on the entire plate, $\frac{n}{r}$ being the number of rivets in one row, or the number of divisions p. From equation (174) we have

$$T_1tp = Tt(p - d) = rfdt = \frac{\pi d^2}{4} rS (= 0.7854rd^2S). \quad (175)$$

It may not be possible to make all of these expressions equal in any joint, but T, T_1, f, S, should not exceed perfectly safe working loads.

The diameter d is generally expressed in terms of the thickness of the plates t. From equation (175) we can find either d or p when other terms are known.

$$\left.\begin{array}{ll} d \text{ varies from 1.5 to 3 times } t; & p \text{ varies from 2 to } 2.75d; \\ \text{For } T = 45{,}000 \text{ to } 50{,}000 \text{ lbs.}, & T_1 = 45{,}000 \text{ to } 40{,}000 \text{ lbs.}; \\ \text{“ } f = 55{,}000 \text{ to } 60{,}000 \text{ “} & f = 55{,}000 \text{ to } 50{,}000 \text{ “} \\ \text{“ } S = 40{,}000 \text{ to } 45{,}000 \text{ “} & S = 0.8T \text{ for plates } \tfrac{1}{4} \text{ to } \tfrac{5}{8} \text{ in.} \end{array}\right\} \quad (176)$$

f varies from 1.33 to $1.4T$.

All of these results are for single-riveted wrought-iron lap-joints.

FOR DOUBLE-RIVETED LAP-JOINTS.

f can be taken = 1.1 to $1.25T$; p = 3.25 to $4.0d$; the smaller values for thick plates and the larger for thin ones.

For 1-in. plates, $T = 30{,}000$ to $35{,}000$ lbs. per sq. in.,
“ $\frac{1}{4}$ “ “ $T = 50{,}000$ to $55{,}000$ “ “ “ “

and proportionately for intermediate thicknesses.

The total resistance of a single-riveted lap-joint, when the plates are not over $\frac{1}{2}$ in. thick, is from 44 to 58 per cent of that of the solid plate, and for the double-riveted lap-joint about 60 per cent; with thick plates, as 1 in., from 33 to 36 per cent. Some experiments have shown that $\frac{3}{8}$-in. plates were stronger than those $\frac{7}{16}$ to $\frac{1}{2}$ in. thick. The above apply also to butt-joints in wrought iron with single cover-plate. The cover-plate is of the same thickness as the main plates. It will be safe to make $h = \frac{1}{2}d$.

In chain-riveting the distance between centre line of rows of rivets may be taken equal to the pitch in a single-riveted joint, or about = $2.5d$; and for zigzag riveting at $\frac{3}{4}$ of its value for chain riveting.

336. For steel lap-joints the relations are similar in their general relations to those of wrought iron. With $T = 65{,}000$ to $75{,}000$ lbs. per sq. in., f may be taken = $1.2T$. For thin plates $\frac{1}{4}$ to $\frac{3}{8}$ in. thick, $T = 70{,}000$ lbs. and $f = 1.2T$, we find $p = 2.25d$ for single-riveted lap-joints.. For very thick plates $T = 50000$ to 55000 lbs. For double-riveted steel lap-joints, with same values of f and T,

$p = 3.5d$. In the above the number of rows $r = 2$. For $r = 3$, $f = T$ for thin plates and $0.9T$ for thick plates.

$$\left.\begin{array}{l} \text{For thin plates, } \tfrac{1}{4} \text{ to } \tfrac{5}{8} \text{ in., } p = 4d; \\ \text{" thick " } \tfrac{7}{8} \text{ " 1 " } p = 3.7d. \end{array}\right\} \quad . \quad . \quad (177)$$

For ordinary plates in single and double riveting, with $f = 1.2T$ and $S = 0.75T$, equation (175) gives

$$d = 2t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (178)$$

For thick plates in treble and quadruple riveting, $f = 0.9T$ and $S = 0.75T$,

$$d = 1.6t, \text{ nearly.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (179)$$

The rivet pitch for steel plates vary, then. from 1.6 to 2 t for thick and thin plates, respectively, and maximum diameter of $1\tfrac{3}{16}$ to $1\tfrac{1}{8}$ in.

Strength of joints for single-riveted lap.........55 to 64 per cent.
" " " " double-riveted " ....65 " 75 " "
" " " " treble or quadruple lap....70 " 80 per cent,

of the resistance of solid plates. Distance between rows of rivets and the overlap h same as for wrought iron. All rivets have been assumed to be made of steel in the preceding remarks. For steel plate with iron rivets we may take $S_1 = 0.9S$, $f = T$ for thin plates or $0.8T$ for very thick plates. These values inserted in the preceding formulæ for steel joints and rivets will give corresponding values for p and d.

WROUGHT-IRON BUTT-JOINTS WITH DOUBLE COVER-PLATES.

337. In these joints the rivets are in double shear, and there is no bending action on the main plates; but the cover-plates are subjected to a greater flexure than the plates of lap-joints. These should then be made thick enough to resist this bending. Each cover-plate should be from three-quarters to seven-eighths that of the main

plates, or their combined thickness from 1.5 to 1.75 that of the main plates. Equations (174) and (175) then become

$$\frac{n}{r}tpT_1 = \frac{n}{r}Tt(p - d) = nfdt = 1.5708nd^2S;$$

$$\therefore\ tpT_1 = Tt(p - d) = rfdt \quad = 1.5708rd^2S. \quad . \quad . \qquad (180)$$

$f = 1.25$ for thin plates to $1.5T$, or a mean value $1.4T$, as no bending action exists in main plates. This value is the same for single or double riveting. For single-riveting equation (180) gives

$$p = 2.5d \text{ (nearly)}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (181)$$

In double-riveting,

$$r = 2; \quad p = 4.0d \text{ (nearly)}.. \quad . \quad . \quad . \quad . \qquad (182)$$

For punched 1-in. plates, $T = 40,000$ lbs. per sq. in.;
" drilled ¼ " " $T = 55,000$ " " " "

$S = 0.75T$. For thin plates,

$$d = 1.3t; \quad f = 1.5T;. \quad . \quad . \quad . \quad . \quad . \qquad (183)$$

For thick plates,

$$d = 1.1t; \quad f = 1.25T, \quad . \quad . \quad . \quad . \quad . \qquad (184)$$

The smaller rivets for ¼-in. plate = ⅜ diam. The larger rivets for 1-in. plate = 1⅛ diameter.

Steel Butt-joints with Double Cover-plates.—Taking $T = 70,000$ to 75,000 lbs. per square inch for thin plates and 55,000 to 60,000 lbs. for thick plates, and $f = 1.25$, $r = 2$, equation (180) gives for double-riveted joints with two cover-plates.

$$p = 3.5d, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (185)$$

and for single-riveted with two covers

$$p = 2.5d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (186)$$

$S = 0.7T$, and $f = 1.25T$; $d = 1.14t$, or better, for thin plates $d = 1\frac{1}{4}t$, and for thick plates

$$d = 1\tfrac{1}{8}t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (187)$$

The single-riveted joint should give 60 to 65 per cent, and the double-riveted 65 to 75 per cent of the resistance of the solid plate.

All steel plates are drilled or annealed if punched.

There seems to be some advantage in favor of the chain-riveting over the zigzag.

Rivets in drilled holes have less resistance to shearing than in punched holes, the edges in the former being sharper than in the latter.

Machine-riveting is to be preferred to hand-riveting: the head is more quickly formed before the rivet has time to cool, the hole is better filled, and there is consequently less danger of loose rivets.

ART. XXXV.

DEFLECTION OF BEAMS.

338. If a beam is straight when unloaded it becomes curved when loaded, and the greatest displacement of any point in the beam under the load is called the deflection. The ultimate deflection is that which takes place immediately before breaking. The proof-deflection is that which takes place under the greatest load that does not impair the strength of the beam. Until the load exceeds the proof-load the deflection is very nearly proportional to the load, but beyond the proof and to the ultimate breaking load the deflection increases in a greater ratio, and irregularly, so that it does not admit of exact determination. In Fig. 153, which shows

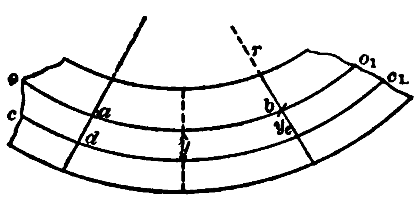


FIG. 153.

a part of a beam bent under the load, that portion of the neutral surface between any two radii is neither lengthened nor shortened,

but that portion of a surface cc_1, between the same radii, in a beam supported at both ends, is lengthened by an amount a; and as y is the distance between the two surfaces, and if r is the radius of curvature of the neutral surface, then $r + y$ is the radius of the surface cc_1. If we call the length of the neutral surface ab unity, the length of $de = 1 + a$; hence $1 : 1 + a :: r : r + y$; $\therefore r = \frac{y}{a}$ or $\frac{1}{r} = \frac{\alpha}{y}$. If p is the intensity of the direct stress, E the coefficient of elasticity, and α the strain, then $E = \frac{p}{\alpha}$, and $\alpha = \frac{p}{E}$; hence $\frac{1}{r} = \frac{p}{Ey}$. But $M = \frac{pI}{y}$ (see eq. (139));

$$\therefore \frac{1}{r} = \frac{p}{Ey} = \frac{M}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

The reciprocal of r, the radius of curvature $\left(= \frac{1}{r}\right)$, is the curvature. When the quantity $\frac{p}{y} = \frac{M}{I}$ varies at different points of the beam the curvature $\frac{1}{r}$ varies also. For the point of maximum bending and $p = f_1$, equation (188) becomes

$$\frac{1}{r_0} = \frac{f}{Ey_0} = \frac{M_0}{EI_0} \text{ and } \frac{1}{r} = \frac{M}{EI} = \frac{M_0}{EI_0}\frac{MI_0}{M_0 I} = \frac{f}{Ey_0}\frac{MI_0}{IM_0}. \quad (189)$$

The latter part of equation (189) results from multiplying $\frac{M}{EI}$ by $\frac{M_0 I_0}{M_0 I_0}$ and changing the position of the terms. If Fig. 154 represents a portion of a beam supported at both ends and symmetrically loaded, C the centre of the span, and Fig. 155 a beam fixed at one end, C being the fixed end. Let these beams be so fixed or supported that at the point C the neutral surface shall be horizontal. Take the tangent at C to the neutral surface AGC for the axis of x. The length of the cantilever beam (Fig. 155) $= l =$ one half the

span of the beam (Fig. 154) $= CB$. Let $CH = x$, C being the origin of co-ordinates in both figures; and let $GH = v$ be the ordinate of any point G in the curve AGC, and $CD = AB = v_1$ be the

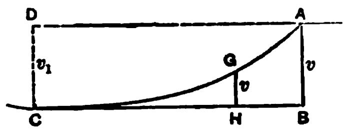


FIG. 154.

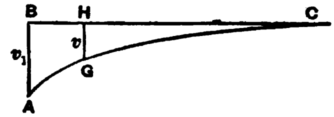


FIG. 155.

greatest ordinate; this is known as the deflection. The inclination of the beam at any point G can be found from the expression

$$\text{tang}\, i = \frac{dv}{dx}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (190)$$

in which i is the angle of inclination; and the curvature, which is the rate of variation of the inclination in a given length of beam, is expressed by $\frac{1}{r} = \frac{di}{ds}$, in which ds is the elementary arc. But $ds^2 = dx^2 + dv^2$, or

$$ds = dx\sqrt{1 + \frac{dv^2}{dx^2}}; \quad \therefore \frac{1}{r} = \frac{di}{dx\sqrt{1 + \frac{dv^2}{dx^2}}}. \quad . \quad . \qquad (191)$$

Practically the curvature is very slight. We can take the arc for its tangent, the slope $\frac{dv}{dx}$, and ds can be taken as sensibly equal to dx; hence slope

$$i = \frac{dv}{dx}; \quad \text{and} \quad \frac{1}{r} = \frac{di}{dx} = \frac{d^2v}{dx^2}, \text{ the curvature.} \qquad (192)$$

When the curvature at each point is given by equation (188) we have, from equations (192) the ordinate

$$v = \int_0^x idx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (193)$$

and the slope

$$i = \int_0^x \frac{dx}{r}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (194)$$

and for the greatest slope i_1 at A, and the greatest ordinate or deflection, $v_1 = AB$. The above expressions must be integrated between the limits $x = 0$ and $x = l$.

Substituting value of $\frac{1}{r}$ from equation (189) in equation (192)

$$i = \int_0^x \frac{dx}{r} = \frac{f}{Ey_0} \int_0^x \frac{MI_0}{IM_0} dx, \quad . \quad . \quad . \quad (195)$$

and value of i, equation (195), in equation (193),

$$v = \int_0^x idx = \frac{f}{Ey_0} \int_0^x \int_0^x \frac{MI_0}{IM_0} dx^2. \quad . \quad . \quad (196)$$

In equations (195) and (196) $\frac{MI_0}{IM_0}$ is a ratio depending on the bending moments and the moments of inertia, the former depending on the manner of loading and supporting the beam and the latter upon the varying form of cross-section. Then we can place the integral in equation (195), $\int \frac{MI_0}{M_0 I} dx = m''l$; and in equation (196) $\int\int \frac{MI_0}{M_0 I} dx^2 = n''l^2$. Then the equations themselves become, respectively,

$$i_1 = \frac{m''fl}{Ey_0}, \quad \text{and} \quad v_1 = \frac{n''fl^2}{Ey_0}. \quad . \quad . \quad . \quad . \quad (197)$$

339. For beams of similar cross-section loaded and supported in the same manner, y_0 simply varies as the depth of the beam and l as the length; hence for such beam the greatest slope i under the proof-load varies directly as the length and inversely as the depth, and the greatest deflection under the proof-load is directly as the square of the length and inversely as the depth, the material of which the beam is made being the same, so that the modulus of rupture f and the coefficient of elasticity E remain the same.

In ordinary cases the beams have uniform and equal cross-sec-

tions, so that the moments of inertia at all sections are equal, i.e., $I = I_0$; $\therefore \frac{I_0}{I} = 1$. In such cases the factors m'' and n'' vary simply as $\frac{M}{M_0}$, that is, as the ratio between M the bending moment at any point of the beam and M_0 the maximum bending moment; which for beams fixed at one end and loaded at the other, or uniformly loaded, Fig. 155, is at the fixed end C; and for beams supported at both ends and loaded at the centre, or uniformly loaded over its entire length, is at the centre of the span C, Fig. 154. With origin at C, and $BC = l$ in both figures 154 and 155, we have, for Fig. 155, $M_0 = Wl$; $M = W(l - x)$;

$$\therefore \frac{M}{M_0} = 1 - \frac{x}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

for a beam fixed at one end and loaded at the other. For the same beam uniformly loaded, with w for unit of length,

$$M_0 = \frac{wl^2}{2}; \quad M = \frac{w(l - x)^2}{2};$$

$$\therefore \frac{M}{M_0} = \frac{(l - x)^2}{l^2} = \left(1 - \frac{x}{l}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

If the beam is supported at both ends and loaded in the centre, the half-span $= l$, and x, measured from C (Fig. 154):

$$M_0 = \frac{W \times 2l}{4} = \frac{Wl}{2}, \quad \text{and} \quad M = \frac{W(l - x)}{2}.$$

Hence

$$\frac{M}{M_0} = 1 - \frac{x}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

And when uniformly loaded

$$M_0 = \frac{wl^2}{2}, \quad \text{and} \quad M = wl(l - x) - \frac{w(l - x)^2}{2} = \frac{wl^2}{2} - \frac{wx^2}{2}.$$

Hence

$$\frac{M}{M_0} = \left(1 - \frac{x^2}{l^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

These ratios a, b, c, and d, substituted in eqs. (195) and (196), give:

For beams fixed at one end and loaded at the other, substituting (a),

$$i_1 = \frac{f}{Ey_0}\int \frac{MI_0}{M_0 I}dx = \frac{f}{Ey_0}\int_{x=0}^{x=l}\left(1 - \frac{x}{l}\right)dx$$

$$= x - \frac{x^2}{2l} = \tfrac{1}{2}l = m''l; \quad \therefore\ m'' = \tfrac{1}{2}.$$

$$v_1 = \frac{f}{Ey_0}\int_{x=0}^{x=l}\int_{x=0}^{x=l}\frac{MI_0}{M_0 I}dx^2 = \frac{f}{Ey_0}\int_{x=0}^{x=l}\left(x - \frac{x^2}{2l}\right)dx$$

$$= \frac{x^2}{2} - \frac{x^3}{6l} = \tfrac{1}{3}l^2 = n''l^2; \quad \therefore\ n'' = \tfrac{1}{3}.$$

In the above the constant factor $\frac{f}{Ey_0}$ is not carried through the deductions, as it does not affect the value of m'' or n'', and similarly for the following relations:

For beams fixed at one end and uniformly loaded, substituting (b) in equations (195) and (196):

$$i_1 = i\frac{f}{Ey_0}\int \frac{MI_0}{M_0 I}dx = \int_{x=0}^{x=l}\left(1 - \frac{x}{l}\right)^2 dx$$

$$= x - \frac{x^2}{l} + \frac{x^3}{3l^2} = \tfrac{1}{3}l = m''l; \quad \therefore\ m'' = \tfrac{1}{3}.$$

$$v_1 = \frac{f}{Ey_0}\int_{x=0}^{x=l}\int_{x=0}^{x=l}\frac{M}{M_0 I}dx^2 = \int_{x=0}^{x=l}\left(x - \frac{x^2}{l} + \frac{x}{3l^2}\right)dx$$

$$= \frac{l^2}{2} - \frac{l^2}{3} + \frac{l^2}{12} = \tfrac{1}{4}l^2 = n''l^2; \quad \therefore\ n = \tfrac{1}{4}.$$

For beams supported at both ends and loaded in the centre, substitute equation (c) in equation (195).

Substituting (c),

$$i_1 = \frac{f}{Ey_0}\int_{x=0}^{x=l}\frac{MI_0}{M_0I}dx = \int_{x=0}^{x=l}\left(1 - \frac{x}{l}\right)dx$$

$$= x - \frac{x^2}{2l} = \tfrac{1}{2}l = m''l; \quad \therefore\ m'' = \tfrac{1}{2}.$$

$$v_1 = \frac{f}{Ey_0}\int\int\frac{MI_0}{M_0I}dx^2 = \int_{x=0}^{x=l}\left(x - \frac{x^2}{2l}\right)dx$$

$$= \tfrac{1}{3}l^2 = n''l^2; \quad \therefore\ n'' = \tfrac{1}{3}.$$

And when uniformly loaded substitute equation (d) in equation (196):

$$i_1 = \frac{f}{Ey_0}\int\frac{MI_0}{M_0I}dx = \int_{x=0}^{x=l}\left(1 - \frac{x^2}{l^2}\right)dx$$

$$= x - \frac{x^3}{3l^2} = \tfrac{2}{3}l = m''l; \quad \therefore\ m'' = \tfrac{2}{3}.$$

$$v_1 = \frac{f}{Ey_0}\int\int\frac{MI_0}{M_0I}dx^2 = \int_{x=0}^{x=l}\left(x - \frac{x^3}{3l^2}\right)dx$$

$$= \frac{x^2}{2} - \frac{x^4}{12l^2} = \tfrac{5}{12}l^2 = n''l^2; \quad \therefore\ n'' = \tfrac{5}{12}.$$

Collecting the above results, we have the values of m'' and n'' in eqs. (197), $i_1 = \dfrac{m''fl}{Ey_0}$, and $v_1 = \dfrac{n''fl^2}{Ey_0}$, as follows:

For beams fixed at one end and loaded at the other, $m''=\frac{1}{2}$; $n''=\frac{1}{3}$.
For beams fixed at one end, uniformly loaded... $m''=\frac{1}{3}$; $n''=\frac{1}{4}$.
For beams supported at both ends and loaded at centre.................................. $m''=\frac{1}{2}$; $n''=\frac{1}{3}$.
For beams supported at both ends and uniformly loaded.................................. $m''=\frac{2}{3}$; $n''=\frac{5}{12}$.

It is to be recollected that in the first two $l =$ the entire span, in the last two $l =$ half the entire span. (For an extended discussion of this subject see Rankine's Applied Mechanics, pages 322 to 326 inclusive).

In the majority of treatises the general moment M is expressed as $M = EI\frac{d^2v}{dx^2} = EI\frac{d^2w}{dx^2} = EI\frac{d^2y}{dx^2}$; v, w, and y being the deflection according to the notation used by different writers. From eq. (188), $\frac{1}{r} = \frac{M}{EI}$; from eq. (192), $\frac{1}{r} = \frac{d^2v}{dx^2}$. Hence

$$\frac{M}{EI} = \frac{d^2v}{dx^2}. \quad \therefore M = EI\frac{d^2v}{dx^2}. \quad . \quad . \quad . \quad . \quad (198)$$

Eq. (198) is commonly used, M being the external bending moment. The common theory of flexure is completely expressed by this equation, and is assumed to be true for pure bending, whatever may be the number or manner of application of the external forces or leads. The expression $EI\frac{d^2v}{dx^2}$, is the moment of internal resistance to bending. Pure bending occurs when the external forces are normal to and cut the axis of the beam; and when v is a function of x only, x being the horizontal axis of the beam, v is the ordinate of the curve and is the strain for bending, E the coefficient of elasticity, and I the moment of inertia.

340. The common theory of flexure is also applicable to originally curved beams within certain limits. Calling the radius of the curvature ρ, then $M = \frac{EI}{\rho}$. If M' and M_1 are two moments which will produce the curvatures whose radii are ρ' and ρ_1, respectively,

$$M' = \frac{EI}{\rho'} \quad \text{and} \quad M_1 = \frac{EI}{\rho_1}; \quad \therefore M_1 - M' = EI\left(\frac{1}{\rho_1} - \frac{1}{\rho'}\right).$$

Then a bending moment $M = M_1 - M'$, applied to a curved beam with a radius of curvature ρ', will cause a change in curvature expressed by

$$\left(\frac{1}{\rho_1} - \frac{1}{\rho'}\right) = \frac{d^2w_1}{dx^2} - \frac{d^2w'}{dx^2}, \quad w = w_1 - w', \quad \text{and} \quad d^2w = d^2w_1 - d^2w';$$

and hence

$$M = EI\frac{d^2w}{dx^2}; \quad w = v. \quad . \quad . \quad . \quad . \quad . \quad (199)$$

The writer prefers forms of expression determined by Rankine's method, as possessing more adaptability to the determination of many questions connected with the deflection of beams. But as they seem to be different from those more commonly met with, he will deduce these more common formulæ, and show that the two are equivalent, only differing in form. The position of the origin of co-ordinates will bo changed, and the first results obtained from a combined single and uniform load.

341. With the above general remarks, we will take the case of a beam fixed at one end, originally straight, and loaded with a single weight at the end and a uniform load over its entire length. $W =$ single load, $w =$ uniform load per unit of length, $wl =$ total uniformload $=$ length of beam, and x is measured from the free end towards the point of support. The value of the bending moments are found as in Art. XXIII. Then we have, from Fig. 156 and equations (67) and (198),

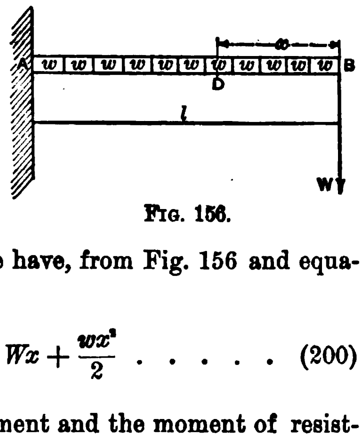


FIG. 156.

$$M = EI\frac{d^2v}{dx^2} = Wx + \frac{wx^2}{2} \quad . \quad . \quad . \quad . \quad . \quad (200)$$

for the relation of the bending moment and the moment of resistance at the point D, x distance from B.

Integrating equation (200) between the limits x and l, recollecting that for $x = l$, $\frac{dv}{dx} = 0$, as the curve of the beam is horizontal at A,

$$EI\frac{dv}{dx} = \frac{W}{2}(x^2 - l^2) + \frac{w}{6}(x^3 - l^3);$$

hence

$$v = \frac{1}{EI}\left[\frac{W}{2}\left(\frac{x^3}{3} - xl^2\right) + \frac{w}{6}\left(\frac{x^4}{4} - l^3x\right)\right], \quad . \quad . \quad (201)$$

for $x = l$, $v = v_1$, and

$$v_1 = -\frac{1}{EI}\left(\frac{Wl^3}{3} + \frac{wl^4}{8}\right). \quad \ldots \ldots \ldots \quad (202)$$

The greatest bending moment is at A, where $x = l$; hence equation (200) gives

$$M_0 = Wl + \frac{wl^2}{2}. \quad \ldots \ldots \ldots \quad (203)$$

For the same beam, loaded only with a single weight W at B, make $w = 0$ in equation (202);

$$\therefore\ v_1 = -\frac{1}{EI}\frac{Wl^3}{3}, \quad \ldots \ldots \ldots \quad (204)$$

and $M_0 = Wl$. For a uniformly distributed load alone $W = 0$;

$$\therefore\ v_1 = -\frac{1}{EI}\frac{wl^4}{8}; \quad \ldots \ldots \ldots \quad (205)$$

$M_0 = \frac{1}{2}wl^2$. The general expression for the shear is the first differential coefficient of the bending moment

$$= S = \frac{dM}{dx} = EI\frac{d^3v}{dx^3} = W + wx,$$

and for maximum shear $x = l$;

$$\therefore\ S_0 = W + wl. \quad \ldots \ldots \ldots \quad (206)$$

342. In the case of a beam supported at both ends and loaded

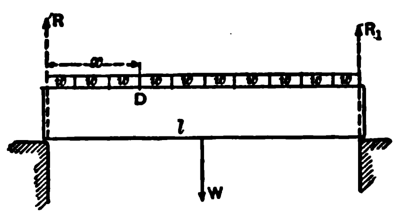


Fig. 157.

with a single weight W at the centre of its length, and a uniformly distributed load of w per unit of length, as shown in Fig. 157, we

have, by the preceding principles and from equations (77) and (198),

$$M = EI\frac{d^2v}{dx^2} = Rx - \frac{wx^2}{2}; \quad . \quad . \quad . \quad . \quad . \quad (207)$$

x being now measured from the point of support A. $R = R_1$ the reactions, $l = AB$ the length of span, and D the point of bending, x distance from A. The origin for both x and v is at A.

Integrating equation (207) between x and $x = \frac{l}{2}$, remembering that $\frac{dv}{dx} = 0$ when $x = \frac{l}{2}$ as the tangent of the angle at C, the centre of the span is zero, as the curve of the neutral surface is horizontal at that point. There results

$$EI = \frac{1}{v}\left[\frac{R}{2}\left(\frac{x^3}{3} - \frac{xl^2}{4}\right) - \frac{w}{6}\left(\frac{x^4}{4} - \frac{x^3}{8}\right)\right]. \quad . \quad . \quad (208)$$

The greatest deflection v_1 occurs when $x = \frac{l}{2}$; and since

$$R = R_1 = \frac{W + wl}{2},$$

substituting in equation (208), making $v = v_1$,

$$v_1 = \frac{1}{EI}\left(-\frac{Wl^3}{48} - \frac{5}{8 \times 48}\frac{wl^4}{1}\right) = -\frac{l^3}{48EI}\left(W + \frac{5}{8}wl\right), \quad (209)$$

and M becomes $M_0 = \frac{1}{4}Wl + \frac{1}{8}wl^2$ for maximum bending at centre. For a single centre load W we have $w = 0$, and equation (209) becomes

$$v_1 = -\frac{Wl^3}{48EI} \quad \text{and} \quad M_0 = \tfrac{1}{4}Wl. \quad . \quad . \quad . \quad (210)$$

For a uniform load w, $W = 0$, $M_0 = \frac{1}{8}wl^2$;

$$v_1 = -\frac{5}{8}\cdot\frac{1}{48}\frac{wl^4}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

For a single centre load equation (209) gives $v_1 = -\frac{Wl^3}{48EI}$, in

which l and all other dimensions involved in I are to be expressed in inches. If we desire l to be in feet,

$$l^3 = 1728 \quad \text{and} \quad v_1 = -\frac{36\,Wl^3}{EI}. \quad . \quad . \quad . \quad (212)$$

For cast iron E may be taken $= 12{,}000{,}000$ lbs.; hence when l is in feet the centre deflection in inches

$$= v_1 = \frac{3\,Wl^3}{1000000I}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (213)$$

in which W is the centre load or, as seen from equation (211), is equal to five eighths the total load $= \frac{5}{8}wl$ for a uniform load.

For wrought-iron rolled beams, making $E = 22{,}000{,}000$, and for built beams 5,500,000, substituting, equation (212) becomes

$$v_1 = \frac{36\,Wl^3}{22000000I} \quad \text{and} \quad \frac{36\,Wl^3}{5500000I}. \quad . \quad . \quad . \quad (214)$$

(See Burr, page 563.) It must be recollected that W is the centre load increased by $\frac{5}{8}$ of the weight of the beam, which must also be included if W is made $\frac{5}{8}$ of the total uniform load. If the above values are substituted in equations (209) and (211), all dimensions must be in inches in (212), (213), and (214); l alone is in feet, v_1 and all other dimensions in inches.

343. We can now compare the values of v_1 as obtained by the two methods. Referring to equation (139), Art. XXXII,

$$M_0 = mWl = \frac{fI}{y} = \frac{fI}{y_0}; \quad \therefore f = \frac{mWly_0}{I}. \quad . \quad . \quad (215)$$

Then from equation (197)

$$v_1 = \frac{n''fl^2}{Ey_0} = \frac{n''mWl^3y_0}{EIy_0} = \frac{n''mWl^3}{EI} \quad . \quad . \quad . \quad (216)$$

For a beam fixed at one end and loaded at the other, $n'' = \frac{1}{3}$ $m = 1$;

$$v_1 = \frac{n''fl^2}{Ey_0} = \frac{n''mWl^3}{EI} = \frac{1}{3}\frac{Wl^3}{EI} \quad \text{[same as eq. (204)].} \quad (217)$$

When uniformly loaded, $n'' = \frac{1}{4}$, $m = \frac{1}{2}$, $W = wl$, then

$$v_1 = \frac{n''fl^2}{Ey_0} = \frac{n''m\,Wl^3}{EI} = \frac{1}{8}\frac{wl^4}{EI} \quad \text{[see eq. (205)].} \quad . \quad . \quad (218)$$

For value of small m see Art. XXIII, pars. 221 and 222, it depending on the method of loading and supporting the beam.

To make the comparisons in the case of beams supported at both ends, it must be recollected that in equation (197) l was taken as only one half of the span (see Fig. 154), whereas in equations (209) and (211) (see Fig. 157) l was taken as the length of the entire span. Then l in the first case is equal to $\frac{1}{2}l$; in the second, substituting, in equation (216) for $l = \frac{1}{2}l$,

$$v_1 = \frac{n''fl^2}{Ey_0} = \frac{n''m\,Wl^3}{EI} \quad \text{becomes} \quad v_1 = \frac{n''m\,Wl^3}{4EI}. \qquad (219)$$

Then for a beam with a single weight at the centre we have $n'' = \frac{1}{3}$, $m = \frac{1}{4}$, and

$$v_1 = \frac{1}{48}\frac{Wl^3}{EI} \quad \text{[see eq. (210)];} \quad . \quad . \quad . \quad . \quad (220)$$

and when uniformly loaded, $n'' = \frac{5}{12}$, $m = \frac{1}{8}$, and $W = wl$. Hence

$$v_1 = \frac{n''m\,Wl^3}{4EI} = \frac{5}{8}\cdot\frac{1}{48}\frac{wl^4}{EI} \quad \text{[see eq. (211)].} \quad . \quad . \quad (221)$$

344. Either of the above sets of equations enable us to find the greatest deflection of any given beam supported and loaded in either of the ways above mentioned.

In the case of a series of isolated loads alone or combined with a uniform load over the whole or any part of the span, the expression for the bending moment can be found as explained in Art. XXIII, and equated to $M = EI\frac{d^2v}{dx^2}$; and integrated as already fully explained, we can find v or its maximum value v_1 for the deflection.

In most cases the actual loading can be reduced to an equivalent single load concentrated at the centre of the span, or to an equivalent uniform load, which would bring it under some of the

above forms; and, as seen above, a uniformly distributed load produces only five eighths of the deflection caused by the same load concentrated at the centre of the span: in other words, for beams supported at both ends it is only necessary to make W, the single load at the centre, equal to five eighths of the total uniform load, really reducing all cases under the equation for a single load at the centre, or equations (197) or (209).

345. An important extension of the above principles is to determine the proper depth to be given to a beam, so that under a given load the deflection of the beam shall not exceed a certain value, or, in other words, the ratio of the depth to the length of a beam so that the ratio of the greatest deflection to the length of the beam shall not exceed a certain fraction, say from $\frac{1}{200}$ to $\frac{1}{1200}$. For this purpose we will use for a beam fixed at one end, eq. (216),

$$v_1 \times \frac{n''m\,Wl^3}{EI} = \frac{n'''\,Wl^3}{EI}; \quad . \quad . \quad . \quad . \quad . \qquad (222)$$

and for a beam supported at both ends, equation (219),

$$v_1 = \frac{n''m\,Wl^3}{4EI} = \frac{n'''\,Wl^3}{4EI}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (223)$$

$n''' = n''m$ for convenience. From equations (134) and (135)

$$M_0 = \frac{fI}{y_0}; \quad I = n'bd^3 \text{ and } y_0 = m'd;$$

and making $\frac{n'}{m'}, = n$, we have

$$M_0 = m\,Wl = \frac{fn'bd^3}{m'd} = nfbd^2,$$

this being the general relation between the bending moment and the moment of resistance to bending, from which

$$f = \frac{m\,Wl}{nbd^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (224)$$

Then, substituting in eq. (222), $n''' = n''m$, and for I, its value $v_1 = \dfrac{n''m\,Wl^3}{En'bd^3}$, in which $n' = m'n$;

$$\therefore\ v = \frac{n''m\,Wl^3}{Em'nbd^3} = \frac{n''}{E}\cdot\frac{m\,Wl}{nbd^2}\cdot\frac{l^2}{m'd}; \quad . \quad . \quad . \quad (224\tfrac{1}{2})$$

substituting f for $\dfrac{m\,Wl}{nbd^2}$ from eq. (224), $v_1 = \dfrac{n''fl^2}{Em'd}$, in which f is the modulus of rupture and must be taken as f_0 or $f_1 - f_0$ if the material has a smaller resistance to tearing than to crushing, or f_1 when the reverse. Then

$$v_1 = \frac{n''f_0(\text{or } f_1)l^2}{Em'd}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

f_0 and f_1 being the proof-strain or working strain, according as W is the proof-load or working load. Hence

$$\frac{v_1}{l} = \frac{n''f_0(\text{or } f_1)}{Em'}\cdot\frac{l}{d} \quad \text{or} \quad \frac{d}{l} = \frac{n''f_0(\text{or } f_1)}{Em'}\frac{l}{v_1}, \quad . \quad . \quad (226)$$

which gives the value of the ratio of the depth of a beam to its length so as not to exceed a certain fraction expressed by $\dfrac{l}{v}$, n'' $= \frac{1}{3}$ for a single load at the end, and $n'' = \frac{1}{4}$ for a uniform load.

For a beam supported at both ends we find, by the same substitutions in eq. (223), $v_1 = \dfrac{n''\,Wl^3}{4EI}$ becomes

$$v_1 = \frac{n''f_0(\text{or } f_1)}{4Em'}\frac{l^2}{d} \quad \text{or} \quad \frac{d}{l} = \frac{n''f_0(\text{or } f_1)}{4Em'}\frac{l}{v_1}, \quad . \quad . \quad (227)$$

$n'' = \frac{1}{3}$ for a single load, and $n'' = \frac{5}{18}$ for a uniform load.

In both equations (226) and (227) m' depends upon the position of the neutral axis with respect to the bottom and top surfaces of the beam, or, in other words, upon the form of the beam, values of which will be given when we come to apply the formulæ. These equations are applicable for any beam loaded and supported as

described, whatever may be its figure of cross-section, in which f_0 or f_1 is the proof or working stress per square inch on that side of the beam which will give way first on the compressed side, where the resistance to crushing per square inch is less than its resistance to tearing as in the case of timber or wrought iron, and on the extended side as in cast iron, the neutral axis being at or near the centre of the depth of the beam, or where the beam is so proportioned that the strength is the same above and below the neutral axis. In the first case there are other simpler methods of determining the ratio of depth to length of span.

The advantage in the above equations arise from our ability to determine the proper depth of the beam directly from the working values of f_0 and f_1 which are always prescribed in the specifications, and the loads must be so regulated as not to exceed these values.

346. For beams of equal strength above and below the neutral axis, as in properly proportioned flanged beams, equations (226) and (227) can be put under still better forms. In such beams we should have

$$f_0+f_1 : f_0 : f_1 :: y_0+y_1 (\text{or } d) : y_0 : y_1; \quad \therefore \frac{d}{y_0} = \frac{f_0}{f_0+f_1} = m'.$$

Equation (226) then becomes

$$\frac{d}{l} = \frac{n''(f_0+f_1)}{E}\frac{l}{v'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

$n'' = \frac{1}{3}$ for a single load at the end, $n'' = \frac{1}{4}$ when uniformly loaded, and eq. (227) becomes

$$\frac{d}{l} = \frac{n''(f_0+f_1)}{4E}\frac{l}{v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

$n'' = \frac{1}{3}$ for a single weight at the centre, and $n = \frac{5}{12}$ for a uniform load.

347. Taking the value of v_1 from the first part of eq. ($224\frac{1}{2}$), $v_1 = \frac{n''m}{Em'n}\frac{Wl^3}{bd^3}$, we see that in general the deflection varies as

$\frac{Wl^3}{bd^3}$, or, in words, directly as the load and cube of the length and inversely as the breadth and cube of the depth. Such purely practical works as Trautwine use simply the following:

$$\text{Deflection under working load} = \frac{Wl^3}{bd^3} \times a;$$

a being a constant to be determined by experiment, and depending on the kind of material,—that is, for E in the above equation,—the manner of loading and supporting the beam, and on the form of cross-section involved in $\frac{n''m}{m'n}$. For beams of pine timber Mr. Trautwine gives the constant $a = 0.00032$, for oak $= 0.00023$, and for cast iron $= 0.000025$. Such expressions save the trouble of understanding the principles, and are easy and simple of application provided you have always at hand a table of the constants. But the chances are that at the very time you want to solve the problem approximately, as it must be at best, you have left your book of tables in the office, and, ignorant of any of the principles involved, you are perfectly helpless. For these reasons the writer has discussed this subject in great detail, although he has endeavored not to introduce any difficult or purely theoretical analytical work. The most elementary differentiation or integration has been introduced, though he has endeavored to make the principles and their relations to each other clear, and to put the results in forms that can be easily remembered and applied.

ART. XXXVI.

CONTINUOUS BEAMS.

348. In the writings of many authors but little attention has been given to the determination of stresses and strains in continuous beams. The subject is an important one, both in connection with the solution of theoretical and practical problems in the design and construction of draw or swing bridges, and also of cantilever and other truss bridges.

The special application, however, in this article will be made

to the cases of continuous beams of some solid cross-section. This general discussion will follow the line as developed in Rankine's Applied Mechanics, pages 332 to 337, inclusive.

Beams are considered as continuous when resting on more than two points of support, whether the extreme ends are fixed or constrained, or free. Also in the case of a beam of a single span with fixed ends.

When a beam simply rests on two points of support and is loaded in any manner, the beam bends under the action of this load, becoming convex downwards. The greatest angle of slope which is measured by the tangent of the angle which a tangent line at any point of the curve assumed by the neutral axis makes with the horizontal at that point $= \frac{dv}{dx}$. This is a maximum at the points of support, and becomes zero at some intermediate point where the bending moment is the greatest, the tangent being horizontal at this point; and the deflection is the greatest, as at this point a tangent to the curved neutral surface is horizontal.

349. If a beam, then, is so acted upon by forces at the points of support as to reduce the slope to zero over the points of support, that is, to keep its neutral surface horizontal at those points, the effect will be to diminish the curvature, slope, and deflection at intermediate points. This can be accomplished by firmly fixing the ends of the beams in masonry walls, or by making the beam

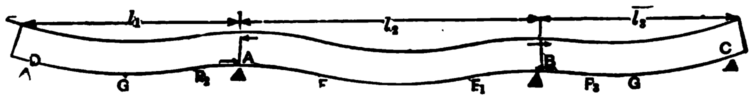


FIG. 158.

a part of a continuous beam over several points of support, or by simply allowing the beam to project beyond each of the two supports, the projecting ends being loaded or counterbalanced by weights, which is often done in case of small drawbridges; or the projecting arms can be anchored down to masses of masonry, or connected with ordinary trussed spans or beams of sufficient weight, the other ends resting on a pier or other point of support,—any of these methods constituting a cantilever truss or beam. Fig. 158 illustrates a continuous beam over two points of support, the ex-

treme ends resting on other supports, thus dividing the beam into three spans, which may or may not be of equal lengths. The end spans are generally of equal length, and the middle span of a different length. Let l_1, l_2, and l_3, be the lengths of the three spans continuous over the two points A and B, and resting on the extreme points D and C; $DA = l_1$; $AB = l_2$; $BC = l_3$. Calling $\frac{1}{2}l_2 = l$, and referring to Art. XXXV, equation (188),

$$\frac{1}{r} = \frac{p}{Ey} = \frac{M}{EI};$$

from equation (192)

$$i = \frac{dv}{dx} \quad \text{and} \quad \frac{1}{r} = \frac{di}{dx} = \frac{d^2v}{dx^2};$$

and from equations (193) and (194), respectively,

$$v = \int_0^x idx, \quad \text{and the slope } i = \int_0^x \frac{dx}{r}.$$

Since from

$$\frac{1}{r} = \frac{di}{dx}, \quad i = \int \frac{dx}{r},$$

we have for the deflection, substituting the above value of $\frac{1}{r} = \frac{M}{EI}$

$$v = \int\int \frac{dx^2}{r} = \frac{1}{E}\int\int \frac{M}{I}dx^2, \quad . \quad . \quad . \quad . \quad (230)$$

and

$$i = \int \frac{dx}{r} = \frac{1}{E}\int \frac{M}{I}dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

These being general values, if now we take the section at which the neutral curve is horizontal, then $\frac{M}{I} = \frac{M_0}{I_0}$ and constant, the

actual value of M_0, however, depending on the mode of distributing the load and supporting the beam, which can be represented in equation (230) by n'' and in equation (231) by m''. Hence, integrating between the limits $x = 0$ and $x = l$, and representing maximum values by v_1 and i_1,

$$\text{Equation (230) becomes } v_1 = \frac{n'' M_0 l^2}{EI_0}; \quad . \quad . \quad . \quad (232)$$

$$\text{Equation (231) " } \quad i_1 = \frac{m'' M_0 l}{EI_0}; \quad . \quad . \quad . \quad (233)$$

after integrating between the limits $x = 0$ and $x = l\ (= \frac{1}{2} l_1)$.

Equations (232) and (233) are the general expressions for the maximum slope and deflection. We have already found and frequently used the values of n'', and in the same Article XXXV we found m''; in fact m'' is the constant resulting from the first integration of equation (231) after substituting the limits $x = 0$ and $x = l$, and n'' is the constant in the second integration after substituting the same limits. Hence from equations (195), (196), and (197), Art. XXXV, we have

	m''	n''
For a beam fixed at one end and loaded at the other........	$\frac{1}{2}$	$\frac{1}{3}$
" " " " " " " " " uniformly..........	$\frac{1}{3}$	$\frac{1}{4}$
" " " supported at both ends, single centre load.......	$\frac{1}{2}$	$\frac{1}{3}$
" " " " " " " loaded uniformly.......	$\frac{2}{3}$	$\frac{5}{12}$

Referring to Fig. 158, either of the end spans BC or AD are said to be fixed at one end A and B, and supported at D and C, respectively. The middle span AB is said to be fixed at both ends, assuming that the neutral surface is horizontal over the points of support A and B. In Fig. 158 this condition can only be fulfilled by assuming that, on vertical sections of the beam over each point A and B, we have a couple the moments of which are equal and opposite, that is, one right-handed and the other left-handed; the forces of these couples being a uniformly varying resistance to crushing on that part of the vertical cross-section, at each point, below the neutral axis, and a corresponding uniformly varying resistance to tearing on that part above the neutral axis, and the magnitude of the moments of these couples sufficient to maintain the neutral surface horizontal at A and B. These couples are

indicated by the arrows, Fig. 158. We then desire to find these moments and the effect of these moments upon the strength, deflection, and curvature of the other portions of the beam. For the present we need not consider the spans DA and BC on either side of the span AB now under consideration. As, however, they may be loaded, or whatever may be their length and other dimensions, they simply combine to develop the resistances over the points A and B; and what we now desire to know is the moment of these resistances without regard to the manner in which they may be developed.

350. It has been shown in Art. XXIII, par. 226, that if a beam is supported at both ends, and loaded with two equal weights placed at equal distances from the centre of the beam, there would be formed two equal and opposite couples near the ends of the beam, that the maximum moment would occur at each weight, and that portion of the beam (not considering the weight of the beam itself, or allowing an equivalent weight in the loads) between the weights would be under a uniform bending moment or moment of flexure.

If, then, under the actual conditions of loading, we find from equation (233) the value i_1 of the maximum slope, assuming that the beam AB is simply supported at the ends, and then the value of the uniform moment of flexure which would produce an equal but opposite slope i_1 at the ends of the beam, or, in other words, neutralize the tendency of the actual loads, so far as the slope at the ends is concerned, this uniform moment would be equal to the required moment of resistances at the ends, the result being that the neutral surface over the points of support would be horizontal, and the beam would assume the form shown in Fig. 158, that is, convex upward at and near the points A and B, and convex downward at intermediate points, but less in extent than when simply supported at the ends. It is clear that this bending moment over the points of support and near the centre must act in different directions, and being assumed to be in the same longitudinal and vertical plane, that the resultant moment is equal to their difference; or calling M_0 the maximum bending moment at the centre when the beam is simply supported at both ends, and M_0' the resisting moment at A and B, then $M_0 - M_0'$ is the resultant at any point, moments tending to produce convexity upward being negative and downward positive. Hence if $M_0 > M_0'$, these points of

the beam will be convex downward, $M_0 < M_0'$ for points convex upward; and if $M_0 = M_0'$, the resultant moment is zero, and also the curvature $\frac{1}{r} = 0$. These are points at which the neutral surface passes from convexity downward to convexity upward, hence points of no bending, but at which only a shearing force acts. These points are called points of contrary flexure or contra-flexure, for which radius of curvature will be infinite.

351. To find the uniform moment of flexure at the points of support which would produce a slope equal to i_1, but in a contrary direction, assume, equation (231), $i_1 = \frac{1}{E}\int \frac{M}{I}dx$. Since the moment of flexure is uniform, and also the cross-section of the beam $\frac{M}{I} = \frac{M_0'}{I_0}$, see equation (233),

$$i_1 = \frac{1}{E}\int_{x=0}^{x=l} \frac{M}{I}dx = \frac{m''M_0'l}{EI};$$

and since $\frac{M}{I}$ is constant,

$$i_1 = \frac{m_0'}{EI_0}\int_{x=0}^{x=l} dx = \frac{M_0'l}{EI_0}.$$

$$\therefore \frac{m''M_0'l}{EI_0} = \frac{M_0'l}{EI_0}, \quad \text{that is,} \quad m'' = 1. \quad . \quad . \quad (234)$$

From equation (230) [see equation (232)],

$$\left.\begin{aligned} v_1 &= \frac{1}{E}\int_{x=0}^{x=l}\int \frac{M}{I}dx^2 = \frac{n''M_0'l^2}{EI_0}; \\ &\text{but since } \frac{M}{I} \text{ is constant,} \\ v_1 &= \frac{M_0'}{EI_0}\int_{x=0}^{x=l}\int dx^2 = \frac{M_0'}{EI_0}\cdot\frac{l^2}{2}. \end{aligned}\right\} \quad . \quad . \quad (235)$$

Hence $\frac{n''M_0'l^2}{EI_0} = \frac{1}{2}\frac{M_0'l^2}{EI_0}$, from equation (235);

$$\therefore n'' = \tfrac{1}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

And equation (233) gives $M_0 = \frac{EI_0 i_1}{m'' l}$; and eq. (234), $M_0' = \frac{EI_0 i_1}{l}$; hence

$$M_0' = m'' M_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (237)$$

If M_1 is the actual moment of flexure at the centre of the beam after its ends have been fixed as shown in Fig. 158, then

$$M_1 = M_0 - M_0' = M_0 - m'' M_0 = M_0(1 - m'').$$

In the table, page 401, we find for a beam fixed at both ends and loaded in the centre $m'' = \frac{1}{2}$; and when uniformly loaded, $m'' = \frac{2}{3}$. Hence $M_1 = \frac{1}{2} M_0$, or $M_1 = \frac{1}{3} M_0$.

When the beam is fixed at both ends, if we take $M_1 = \frac{1}{2} M_0$, then the bending moment at A and B is $M_0' = M_0 - M_1 = \frac{1}{2} M_0$; or if with the uniform load $M_1 = \frac{1}{3} M_0$, then the bending moment at A and B is $M_0' = M_0 - M_1 = \frac{2}{3} M_0$.

From this discussion it is seen that the bending moment at the centre M_1 must either be equal to or less than the bending moment M_0' at the points of support, but never greater; that is, $M_1 = M_0'$, but never greater.

For beams of uniform cross-section $I = I_0$, as assumed above. In other words, the bending moments in continuous beams, or those fixed at both ends with uniform cross-section, over the points of support A and B will always be either greater or at least equal to the bending moment at the centre of the span, but never less.

352. From equation (237),

$$M_0' = m'' M_0 = m'' m W' l_2 = \frac{fI}{y} = nfbd^2;$$

hence the ultimate proof or working load (according to the value of f), when the ends are fixed, is

$$W' = \frac{nfbd^2}{m'' m l_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

and when the beam is supported at both ends simply,

$$M_0 = mWl_2 = nfbd^2; \quad \therefore W = \frac{nfbd^2}{ml_2}; \quad . \quad . \quad (239)$$

hence

$$W' = \frac{W}{m''}, \quad \text{or} \quad 1 : m'' : : W' : W. \quad . \quad . \quad . \quad . \quad (240)$$

From which we see that the strength of the beam is increased in the ratio of $m'' : 1$.

353. To find the deflection, subtract that due to the uniform moment for beams with fixed ends from that due to the moment of flexure if the beam had simply been supported; that is, $v_1 - v_0'$, equations (232) and (235) respectively; and $n'' = \frac{1}{2}$ [see equation (236)]. Hence

$$v_1 - v_0' = \frac{n'' M_0 l^2}{EI_0} - \frac{M_0' l^2}{2EI_0};$$

or, since $M_0' = m'' M_0$,

$$M_0 = \frac{M_0'}{m''};$$

$$v_1 - v_0' = \frac{n'' M_0' l^2}{m'' EI_0} - \frac{M_0' l^2}{2EI_0} = \left(\frac{n''}{m''} - \frac{1}{2}\right)\frac{M_0' l^2}{EI_0} = v_2. \qquad (241)$$

Hence the deflection is diminished in the ratio

$$v_1 : v_1 - v_0' :: \frac{n''}{m''} : \left(\frac{n''}{m''} - \frac{1}{2}\right), \quad \text{or} \quad n'' : \left(n'' - \frac{m''}{2}\right), \qquad (242)$$

$M_0' = \dfrac{fI_0}{y_0}$, and equation (241) becomes

$$v_2 = \left(\frac{n''}{m''} - \frac{1}{2}\right)\frac{fl^2}{Ey_0}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (243)$$

When simply supported [see equation (197)], $v_1 = \dfrac{n'' f l^2}{Ey_0}$. Hence $v_2 : v_1 :: \left(\dfrac{n''}{m''} - \dfrac{1}{2}\right) : n''$ [same as in equation (242)]. This expresses what Mr. Rankine calls the diminutive in the proof deflection, f being the proof strain; but it holds good for any value f_0 or f_1 less than the proof strains.

The position of the points of contraflexure are found by placing

$$M_0' = M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (244)$$

M_0' being the uniform bending moment, or the moment of resistance at the fixed ends, A and B, and M being the moment at any point of the beam under the actual loads, if the beam is simply supported at both ends.

The above equations are general for any condition of loading the beam, the beam being of uniform cross-section. For any particular case it is only necessary to give proper values to m'', n'', and m, and for beams supported at both ends make $2l = l_2$, the beams being fixed at both ends.

BEAM CONTINUOUS OVER TWO POINTS OF SUPPORT.

354. For middle span, uniform cross-section, single load at centre (see Fig. 158), $m = \frac{1}{4}$; $m'' = \frac{1}{2}$; $n'' = \frac{1}{3}$; $M_0' = m''M_0$.

$$\therefore M_0' = \tfrac{1}{2}M_0 = \tfrac{1}{2}mWl_2 = \tfrac{1}{8}Wl_2 = \tfrac{1}{4}Wl = \frac{fI}{y} = nfbd^2. \quad (245)$$

Since M_0 is the maximum bending moment at centre of beam, we wish to find the point where the bending moment $M = \frac{1}{2}M_0 = \frac{1}{2}Wx$, x being measured from one of the points of support A or B; hence $x = \frac{1}{2}l = \frac{1}{4}l_2$; $M = \frac{1}{4}Wl = M_0'$, which is the condition for the points of contraflexure; that is, these points are midway between the centre of the span G and the points of support A and B, as shown in Fig. 158 at F and F_1 on middle span. At these points the resultant bending moment is zero, and as the shearing force is the first differential coefficient of the bending moment we have $M = -M_0' + \frac{W}{2}x$; hence $\frac{dM}{dx} = S = \frac{W}{2}$, which is the value of the shearing force at any point between the end and centre.

For the maximum deflection eq. (243) gives

$$2l = l_2,\ v_2 = \left(\frac{n''}{m''} - \frac{1}{2}\right)\frac{fl^2}{Ey_0} = \left(\frac{\frac{1}{3}}{\frac{1}{2}} - \frac{1}{2}\right)\frac{fl^2}{Ey_0} = \frac{1}{6}\frac{fl^2}{Ey_0} = \frac{1}{24}\frac{fl_2^2}{Ey_0}. \quad (246)$$

And since the resultant bending moment at the centre of the beam is $M_1 = M_0 - \frac{1}{2}M_0 = \frac{1}{2}M_0 = \frac{1}{2}mWl_2 = \frac{1}{8}Wl_2 = \frac{fI}{y_0}$, we have $f = \frac{y_0 Wl_2}{8I}$. Substituting in equation (246), we have

$$v_2 = \frac{1}{192}\frac{Wl_2^3}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

In the above case the bending moments at the centre and at each

of the points of support are equal, but of opposite signs and each $= M_1 = \frac{1}{8} Wl_2$; whereas if simply supported the bending moment at the ends is zero and at the centre $M_0 = \frac{1}{4} Wl_2$.

The points of contraflexure are distant from either end

$$= \tfrac{1}{4}l_2 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (248)$$

355. For The same beam uniformly loaded, $m = \frac{1}{8}$; $m'' = \frac{2}{3}$; $n'' = \frac{5}{12}$; $2l = l_2$. Then

$$M_0{}^1 = m''M_0 = \tfrac{2}{3}M_0 = \tfrac{2}{3}m\,Wl_2 = \tfrac{1}{12}Wl_2 = \tfrac{1}{6}Wl = \frac{fI}{y} = nfbd^2. \quad (249)$$

$$M_1 = M_0 - M_0' = \tfrac{1}{3}M_0 = \tfrac{1}{2}M_0' = \tfrac{1}{24}Wl_2 = \tfrac{1}{24}wl_2^2. \quad . \qquad (250)$$

This is the bending moment at the centre of the span and for the bending at the points A and B.

$$M_0' = 2M' = \tfrac{1}{12}Wl_2 = \tfrac{1}{12}wl_2^2. \quad . \quad . \quad . \quad . \qquad (251)$$

In equations (250) and (251), $W = wl_2$, w being the intensity of the uniform load or the load per unit of length.

For the maximum deflection at the centre of the span

$$v_2 = \left(\frac{n''}{m''} - \frac{1}{2}\right)\frac{fl^2}{Ey_0} = \left(\frac{\frac{5}{12}}{\frac{2}{3}} - \frac{1}{2}\right)\frac{fl^2}{Ey_0} = \frac{1}{8}\frac{fl^2}{Ey_0} = \frac{1}{32}\frac{fl_2^2}{Ey_0}. \quad (252)$$

$$M = M_0 - \tfrac{1}{3}M_0 = \tfrac{2}{3}M_0 = \tfrac{2}{3}m\,Wl_2 = \tfrac{1}{12}Wl_2 = \frac{fI}{y_0};\ f = \frac{Wy_0l_2}{12I}.$$

Hence

$$v_2 = \frac{Wl_2^3}{384EI} = \frac{wl_2^4}{384EI}. \quad . \quad . \quad . \quad . \quad . \qquad (253)$$

In Art. XXIV, page 188, eq. (64), (Burr),

$$EIv = \frac{px^2}{12}\left(l_2x - \frac{x_2}{2} - \frac{l_2^2}{2}\right).$$

The value of v in this equation is the general expression for the

deflection of the middle span as deduced by Mr. Burr, in which $p = w$, and for maximum deflection v_1, $x = \frac{l_2}{2}$. Hence

$$EIv_1 = \frac{wl_2^3}{48}\left(\frac{l_2^3}{2} - \frac{l_2^3}{8} - \frac{l_2^3}{2}\right) = \frac{1}{8} \cdot \frac{1}{48} wl_2^4 = \frac{1}{384} wl_2^4,$$

or

$$(v_1) = v_2 = \frac{wl_2^4}{384EI},$$

the same as eq. (253) above.

356. To find now the points of contraflexure, we have $M_0' = \frac{2}{3}M_0$; and since we desire to find a point on the beam where the general bending moment is

$$M = M_0' = \frac{2}{3}M_0 = \frac{wl_2x}{2} - \frac{wx^2}{2} = \frac{1}{12}wl_2^2, \quad . \quad . \quad (254)$$

x being measured from the point of support A to the right. Hence $l_2x - x^2 = \frac{1}{6}l_2^2$; or $x^2 - l_2x = -\frac{1}{6}l_2^2$. Solving with respect to x, we have

$$x = \frac{l_2}{2} \pm \sqrt{\frac{l_2^2}{4} - \frac{l_2^2}{6}} = l_2\left(\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{6}}\right)$$

$$= l_2\left(0.5 \pm \frac{1}{3.464}\right) = l_2(0.5 \pm 0.289).$$

$$\therefore\ x = 0.789l_2 \quad \text{and} \quad x = 0.211l_2. \quad . \quad . \quad . \quad (255)$$

Which simply means that there are two points of contraflexure at 0.211 of the span from A, and the other at 0.789 of the span from A, or 0.211 from the point of support B; or the two points are $0.5 - 0.211 = 0.289$ of the span from the centre. This same result is found by a different method in Burr's Elasticity and Resistance of Materials (see page 188).

CONTINUOUS BEAMS. FIXED AT ONE END AND SUPPORTED AT THE OTHER.

357. The above solution was based on the application of a moment of resistance on a vertical section of the beam at each of

the points of support A and B capable of maintaining the neutral curve at those points horizontal, without reference to the means used to develop that moment. And, in fact, in the preceding discussion the ends of the beam in Fig. 158 were simply conceived to be held by being fixed in masonry, or in some other manner. In this case the weight of the spans DA and CB, or the load upon them, must be sufficiently great to develop the required moment of resistance. These spans are then said to have one fixed end, and to be simply supported at the other end D or C. It is evident that as the bending moment or its equivalent moment of resistance must remain the same over the points of support A and B as for the span AB, and the bending moment at the points D and C must be zero, as the beams are merely supported at those points, these end spans DA and CB are sensibly in the same condition as those portions of the span AB extending from A to F, and from B to F; that is, similar to a span fixed at one end and supported at the other, and so loaded that the neutral curve should be horizontal at the points A and B. A general solution of this problem can be found in Burr's work.

The following solution will be, however, more in the same direction, and in fact forming a part or continuation of the problem just solved for a beam fixed at both ends and forming with these end spans a continuous girder. As we have already found the value of the moment M_0' necessary to hold the beam horizontal at the points A and B, we have already found one of the quantities to be considered, and it now only remains to find the necessary weight or load, including the weight of the span DA or CB necessary to develop this moment. This load may be considered either as a concentrated load acting at a single point or uniformly distributed over the span DA. Taking the case of a single weight W' acting at any point x_1 distance from D (see Fig. 159), the span $AD = l_1$, the beam being assumed as fixed at A and supported at D. If the beam was simply supported at both ends the reactions could be found at once by the principle of the lever, but in this case one of the reactions will be greater and the other will be less by the same amount than would be found by the principle of the lever.

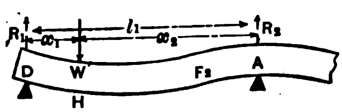


FIG. 159.

This transference of a portion of one reaction to the other can only be effected by a couple, the equal forces of which are the differences between the reactions in the two cases, and the lever-arm is the distance between A and D, or the length of the span.

The conditions of equilibrium require an equal and opposite couple to be applied to the beam, and the bending action of this couple will be of an opposite kind to that caused when the beam is simply supported at the ends. We first desire, then, to find the reactions R_1 and R_2 at the points A and D. Let x_1 be the distance of the load from D and x_2 its distance from A. Then the bending moment at any point between D and H, measuring x from A, will be $R_1(l_1 - x)$, and between H and A will be $R_1(l_1 - x) - W'(x_2 - x)$. Hence

$$EI\frac{d^2v}{dx^2} = R_1(l_1 - x) \quad \text{and} \quad EI\frac{d^2v}{dx^2} = R_1(l - x) - W'(x_2 - x). \quad (256)$$

Integrating, we have

$$EI\frac{dv}{dx} = R_1 l_1 x - R_1\frac{x^2}{2} + C, \quad . \quad . \quad . \quad . \quad . \quad (257)$$

and

$$EI\frac{dv}{dx} = R_1 l_1 x - R_1\frac{x^2}{2} - W'x_2 x + \frac{W'x^2}{2} + C_1. \quad . \quad . \quad (258)$$

In equation (258) $x = 0$; $\frac{dv}{dx} = 0$ also; then $C_1 = 0$.

For $x = x_2$, $\frac{dv}{dx}$ has the same value in the two equations (257) and (258), as the beam is horizontal at H, and the slope $\frac{dv}{dx}$ is the same on both sides of H; hence

$$C = - W'x_2^2 + \frac{W'x^2}{2} = - \frac{W'x_2^2}{2}.$$

Equation (257) becomes $EI\frac{dv}{dx} = R_1 l_1 x - R_1\frac{x^2}{2} - \frac{W'x_2^2}{2}$;

and

equation (258) becomes $EI\frac{dv}{dx} = R_1 l_1 x - R_1\frac{x^2}{2} - W'x_2 x + W'\frac{x^2}{2}$.

Integrating,

$$EIv = R_1 l_1 \frac{x^2}{2} - R_1 \frac{x^3}{6} - \frac{W' x_2^2 x}{2} + C_2, \quad \dots \quad (259)$$

and

$$EIv = R_1 l_1 \frac{x^2}{2} - R_1 \frac{x^3}{6} - \frac{W' x_2 x^2}{2} + \frac{W' x^3}{6} + C_3. \quad . \quad (260)$$

In equation (260) $x = 0$, $v = 0$ also; then $C_3 = 0$.

$x = x_2$, then in the two equations $v = v$; hence $C_2 = \frac{W' x_2^3}{6}$.

Substituting in equation (259) for C_2, then, when $x = l_1$, v in equation (259) is zero, and there results

$$R_1 \frac{l_1^3}{2} - R_1 \frac{l_1^3}{6} - \frac{W' x_2^2 l_1}{2} + \frac{W' x_2^3}{6}.$$

Hence

$$R_1 = \frac{9 W' x_2^2 l_1 - 3 W' x_2^3}{6 l_1^3} = \frac{3 W' x_2^2 l_1 - W' x_2^3}{2 l_1^3}. \quad . \quad (261)$$

This gives the reaction at D, the supported or extreme end of the span DA, and the reaction due to the load on the same span at A will be

$$R_2 = W' - \frac{3 W' x_2^2 l_1 - W' x_2^3}{2 l_1^3} = \frac{2 W' l_1^3 - 3 W' x_2^2 l_1 - W' x_2^3}{2 l_1^3}. \quad (262)$$

These equations are general, and give the reactions in terms of the length of the span and the distance of the point of application of the load from the fixed end of the beam. Having now found the reactions due to the load on the span DA, and knowing that $W' = R_1 + R_2$, we can find the point of contraflexure by placing the second member of equation (256) equal to zero, or, what is the same thing, the algebraic sum of the moments on either side of this point must be zero. This moment is

$$R_1(l - x) = W'\{l - (x_1 + x)\} = W'(x_2 - x);$$

$$\therefore\ x = \frac{W' x_2 - R_1 l_1}{W' - R_1}. \quad \dots\dots \quad (263)$$

If the origin is taken at D instead of A, then the moments with respect to F_2 distant x from D are $R_1 x = W'(x - x_1)$;

$$\therefore\ x = \frac{W' x_1}{W' - R_1}. \quad \dots\dots \quad (264)$$

This last is the same form as equation (49), page 182, Burr's work.

The points of greatest deflection can be found by placing the second member of equation (257) equal to zero, after substituting the values of R_1 and C, and solving with respect to x, since at these points $\frac{dv}{dx}$ is equal to zero. Equation (263) shows that there is but one point of contraflexure.

It is plain that the portion of the span DA between D and F_2 is simply in the condition of a beam supported at both ends, since the bending moments at those points are zero, and the point of maximum positive bending, that is, causing convexity downward, is at the point of application of the resultant W'. The maximum negative moment is M_0' at the point A. The same solution applies to the span $BC = l_2$ by substituting l_2 for l_1 in the above equations. These spans are, however, generally equal, $l_1 = l_2$.

358. In beams simply supported at the two ends the shearing force at the points of support were equal to the reactions and diminished towards the intermediate point where the bending moment was a maximum. In continuous beams the reactions must not be confounded with the shearing forces. In Fig. 158 the reaction is equal to the shear at D, the supported end; this shear reduces to zero at the point of application of the resultant load W'; it then increases to the point F_2 of contraflexure, where it is equal to its value at D, and at the fixed point A, or rather immediately to the left of that point, it is equal to R_2, which is the reaction due to the weight or load upon the span DA. Similarly the shear at the point A from the load on the span AB is equal to the reaction R_1' due to that load, immediately to the right of the point of support. But the full reaction at A is $R_1 + R_1'$. The shear, then, in either span independently considered follows the same laws as for beams supported at both ends, after the proper reactions are determined for that particular span. But the reactions at each point of support are the sum of the shears, or reactions arising from the two spans adjacent to the point of support.

359. If the span DA, Figs. 158 and 159, is uniformly loaded, the solution is somewhat simplified from the fact that it is not necessary to deduce but one set of equations for the bending moment on one side of H and another set for the other.

With the origin at A, the fixed end of the span, we have the general relation for the bending moment at any point

$$EI\frac{d^2v}{dx^2} = R_1(l_1 - x) - w\frac{(l - x)^2}{2}. \quad . \quad . \quad . \quad (265)$$

Integrating twice and bearing in mind that for $x = 0$, $\frac{dv}{dx} = 0$, and also $v = 0$, then

$$EI\frac{dv}{dx} = R_1l_1x - \frac{R_1x^2}{2} - \frac{wl_1^2x}{2} + \frac{wl_1x^2}{2} - \frac{wx^3}{6}; \quad (266)$$

$$EIv = R_1l_1\frac{x^2}{2} - R_1\frac{x^3}{6} - \frac{wl_1^2x^2}{4} + \frac{wl_1x^3}{6} - \frac{wx^4}{24}. \quad (267)$$

For $x = l_1$, $v = 0$, substituting in equation (267),

$$R_1\frac{l_1^3}{2} - \frac{R_1l_1^3}{6} - \frac{6wl_1^4}{24} + \frac{4wl_1^4}{24} - \frac{wl_1^4}{24} = 0;$$

$$\therefore R_1 = \tfrac{3}{8}wl_1,$$

which is the reaction at D. Hence for reaction at A,

$$R_2 = wl_1 - \tfrac{3}{8}wl_1 = \tfrac{5}{8}wl_1.$$

It will be convenient to substitute for R^1 its value in the above equations. Equation (266) then becomes

$$EI\frac{dv}{dx} = \frac{3}{8}wl_1^2x - \frac{3}{16}wl_1x^2 - \frac{wl_1^2x}{2} + \frac{wl_1x^2}{2} - \frac{wx^3}{6}$$

$$= -\frac{1}{8}wl_1^2x + \frac{5}{16}wl_1x^2 - \frac{wx^3}{6};$$

$$\therefore \frac{dv}{dx} = -\frac{wx}{48EI}(6l_1^2 - 15l_1x + 8x^2); \quad . \quad . \quad . \quad (268)$$

$$v = \frac{wx^2}{48EI}(3l_1^2 - 5l_1x + 2x^2). \quad . \quad . \quad . \quad . \quad (269)$$

To find the point of maximum deflection make $\frac{dv}{dx} = 0$ and solve equation (268) with respect to x. We find

$$x = \frac{15}{16}l_1 - \frac{l_1\sqrt{33}}{16} = \frac{l_1(15 - \sqrt{33}}{16} = 0.5785l_1.$$

This value substituted in eq. (269) will give the maximum deflection

$$v = v_1 = 0.0054\frac{wl_1^4}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (270)$$

Then find the maximum bending moment from eq. (265) by forming the first differential coefficient of

$$M = R_1(l_1 - x) - w\frac{(l - x)^2}{2};$$

$$\therefore S = \frac{dM}{dx} = -R_1 + wl_1 - wx = 0; \quad \therefore wx = -\frac{3}{8}wl_1 + wl_1;$$

$x = \frac{5}{8}l_1$, distance of maximum bending from A, or from

$$D = l_1 - \tfrac{5}{8}l_1 = \tfrac{3}{8}l_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (271)$$

that is, half way between D and F_2, Fig. 159, as might have been anticipated, as that portion of the beam from D to F_2 is equivalent to a beam supported at both ends and uniformly loaded. This is also the point where the shear is zero, as seen by substituting in the general value of S for the shear the proper values of R_1 and x.

360. It only remains now to compare the values thus obtained with the same quantities for beams fixed at one end and free at the other, or supported at both ends, of the same length, and loaded with the same weight distributed in the same way, so as to determine in what way and to what extent a continuous beam is stronger or stiffer than in other cases.

These conditions are shown in the following diagrams. For convenience of illustration all spans are supposed to be equal in length; and loads, whether single or distributed, to be the same in amount and distribution. The left-hand span is assumed as fixed at one end and supported at the other, the middle fixed at both ends, and the right-hand span fixed at one end and free at the other.

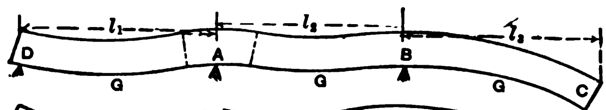
Let $l = l_1 = l_2 = l_3$; $W = W'$, the load concentrated at the middle of the spans, and $wl = wl_1 = wl_2 = wl_3$, the uniformly distributed loads over the spans. See Fig. 160. The corresponding spans for comparison are shown in Fig. 161.

Let DA be a span fixed at one end and supported at the other, and D_1A_1 a span supported at both ends.

Assuming first a single load $W = W'$ at the centre,

$$\text{Reaction and shear on } DA \text{ at } D = R_1 = S_1 = \tfrac{5}{16}W;$$
$$\text{" " " " } D_1A_1 \text{ " } D = R_1' = S_1' = \tfrac{1}{2}W;$$
$$\text{Shear " } DA \text{ " } A = R_2 = S_2 = \tfrac{11}{16}W;$$
$$\text{Reaction and shear on } D_1A_1 \text{ " } A = R_2' = S_2' = \tfrac{1}{2}W.$$

FIG. 160.



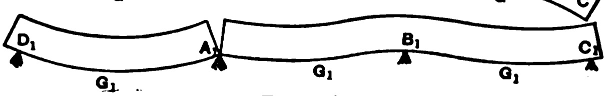


FIG. 161.

The reaction on DA at $A = S_2 + S_3 = R_2 + R_3$, in which S_3 and R_3 are the shear or reaction from the span AB.

$$\text{Max. bending mom. on } DA \text{ at } A = M_0' = R_1 l - \frac{wl}{2} = \tfrac{3}{16}Wl;$$
$$\text{" " " " } D_1A_1 \text{ " } A = M' = 0;$$
$$\text{" " " " } DA \text{ " } G = M_2 = R_1\frac{l}{2} = \frac{5}{32}Wl = 0.159\,Wl;$$
$$\text{" " " " } D_1A_1 \text{ " } G_1 = M_4 = R_1\frac{l}{2} = \frac{1}{4}Wl = 0.25\,Wl.$$

Max. deflection of AD at $G = x = 0.554l$ from A

$$= v_1 = 0.009\frac{Wl^3}{EI};$$

Max. deflection of A_1D_1 at $G_1 = x = 0.5l$ from A

$$= v_1 = \frac{1}{48}\frac{Wl^3}{EI} = 0.021\frac{Wl^3}{EI}.$$

From the above

$$\frac{5}{32}W'l = \frac{fI}{y} = \frac{1}{4}Wl;$$

hence

$$W' = \frac{32}{20}W = 1.6\,W;$$

that is, the same beam fixed at one end and supported at the other will carry 1.6 times the load at its centre that it will carry when simply supported at both ends.

The maximum deflection is reduced in the ratio of 0.021 : 0.009, or $2.3 : 1 = \frac{1}{2.3}$ of the deflection when simply supported. More material is required at A to resist the shearing in the ratio of $\frac{11}{16}$ to $\frac{1}{2} = 1.38$ as much, and also material to resist the bending moment at A, $\frac{3}{16} Wl = \frac{fI}{y} = nfbd^2$. For a solid rectangular beam

$$bd^2 = \frac{9}{8}\frac{Wl}{f}.$$

361. For the same beams uniformly loaded:

Reaction and shear on DA at $D = R_1 = S_1 = \frac{3}{8}wl$;

" " " " D_1A_1 at $D = R_1' = S_1' = \frac{1}{2}wl$;

Shear on DA at $A = R_2 = S_2 = \frac{5}{8}wl$;

Reaction and shear on D_1A_1 at $A = R_2' = S_2' = \frac{1}{2}wl$.

Reaction at $A = R_2 + R_2 = S_2 + S_2$;

Max. bending moment on DA at $A = M_0' = R_1 l - \frac{1}{2}wl^2 = \frac{1}{8}wl^2$;

" " " " D_1A_1 at $A = M' = 0$;

max. bending moment on DA at G

$$= M_2 = R_1 \times \tfrac{3}{8}l - \tfrac{3}{8}wl \times \tfrac{3}{16}l = \tfrac{9}{128}wl^2;$$

" " " " D_1A_1 at $G_1 = M_0 = \frac{1}{8}wl^2$;

Max. deflection of AD at G ($x \doteq 0.5785l$ from A,) $v_1 = 0.0054\frac{wl^4}{EI}$;

" " " A_1D_1 at G_1 ($x = 0.5l$ from A,) $v_1 = \frac{5}{8} \cdot \frac{1}{48}\frac{wl^4}{EI}$.

By fixing one end the breaking load is increased in the ratio $\frac{9}{128}W' = \frac{1}{8}W$; $\therefore W' = 1.78W$. The deflection is reduced in the ratio 0.013 to 0.0054, or is only $\frac{1}{2.4}$ that when supported simply. Bending moment on DA at A has to be provided for.

$$\frac{1}{8}wl^2 = \frac{fI}{y} = nfbd^2.$$

For solid rectangular sections, therefore $bd^2 = \frac{3}{4}\frac{wl^2}{f}$.

362. Comparing now the spans AB fixed at both ends and D_1A_1 supported at both ends, first with single weight at centre of spans:

$$\text{Shear on } AB \text{ at } A \text{ and } B = \tfrac{1}{2}W;$$

$$\text{Reactions and shear on } D_1A_1 \text{ at } D_1 \text{ and } A_1 = \tfrac{1}{2}W;$$

$$\text{Reaction at } A \text{ and } B = R_2 + R_3 = (\tfrac{11}{16} + \tfrac{1}{2})W = \tfrac{19}{16}W;$$

$$\text{Max. bending moment at } A \text{ and } B = M_1' = \tfrac{1}{2}W \times \tfrac{1}{4}l = \tfrac{1}{8}Wl;$$

$$\text{" \quad " \quad " \quad " } D_1 \text{ and } A_1 = M' = 0;$$

$$\text{" \quad " \quad on } AB \text{ and } G = M_1 = \tfrac{1}{8}Wl;$$

$$\text{" \quad " \quad " } D_1A_1 \text{ " } G_1 = M_0 = \tfrac{1}{4}Wl;$$

$$\text{Max. deflection at centre of span } AB = v_1 = \frac{1}{192}\frac{Wl^3}{EI};$$

$$\text{" \quad " \quad " \quad " \quad " \quad " } D_1A_1 = v_1 = \frac{1}{48}\frac{Wl^3}{EI}.$$

Then

$$\tfrac{1}{8}W'l = \frac{fI}{y} = \frac{1}{4}Wl; \quad \therefore W' = 2W,$$

or a beam with fixed ends will carry twice as much as for a beam supported at both ends.

The deflection is reduced in the ratio of $\frac{1}{192}$ to $\frac{1}{48}$, or it is only $\frac{1}{4}$ of that caused when supported at both ends.

363. The spans AB and D_1A_1 uniformly loaded:

$$\text{Shears on } AB \text{ at } A \text{ and } B = \tfrac{1}{2}wl;$$

$$\text{Reaction and shears on } D_1A_1 \text{ at } D_1 \text{ and } A_1 = \tfrac{1}{2}wl;$$

$$\text{Reactions at } A \text{ and } B = (\tfrac{5}{8} + \tfrac{1}{2})wl = \tfrac{9}{8}wl;$$

$$\text{Max. bending moment on } AB \text{ at } A \text{ and } B = M_0' = \tfrac{1}{12}wl^2;$$

$$\text{" \quad " \quad " \quad " } D_1A_1 \text{ at } D_1 \text{ and } A_1 = M' = 0;$$

$$\text{" \quad " \quad " \quad " } AB \text{ at centre } G = M_1 = \tfrac{1}{24}wl^2;$$

$$\text{" \quad " \quad " \quad " } D_1A_1 \text{ at centre } G_1 = M_0 = \tfrac{1}{8}wl^2;$$

$$\text{Max. deflection on } AB \text{ at centre } G = v_1 = \tfrac{1}{384}wl^4;$$

$$\text{" \quad " \quad " } D_1A_1 \text{ at centre } G_1 = v_1 = \tfrac{5}{8}\cdot\tfrac{1}{48}wl^4.$$

Then

$$\frac{1}{24}(w'l)l = \frac{fI}{y} = \frac{1}{8}(wl)l; \quad \therefore w'l = \frac{3wl}{2},$$

or the beam with fixed ends will carry three times the load when supported only. At the points A and B the moment is twice as as great as at the centre, and

$$\frac{1}{12} wl^2 = \frac{fI}{y_0} nfbd^2.$$

For solid rectangular beams $bd^2 = \dfrac{wl^2}{2f}$.

The deflection is reduced in the ratio $\frac{1}{384} : \frac{5}{384}$, or $\frac{1}{5}$ of that for beams only supported.

364. The last case that will be taken is that of the spans BC fixed at one end and free at the other and B_1C_1 fixed at one end and supported at the other. For this span the same moments, reactions, shears, and deflections will be taken as for the span AD. Then we have for a single centre load:

Shear at B_1, $R_2 = S_2 = \frac{11}{16} W$;
Reaction and shear at $B, R_2' = S_2' = W$;
" " " " C_1, $R_1 = S_1 = \frac{5}{16} W$;
" " " " $C, = 0$;
Max. bending moment on B_1C_1 at $B_1 = \frac{3}{16} Wl$;
" " " " BC at $B = \frac{1}{2} Wl$;
" " " " C and $C_1 = 0$;
" " " " B_1C_1 at $G = \frac{5}{32} Wl$;
" " " " BC at $G_1 = 0$;
Max. deflection on B_1C_1 at G, $v_1 = 0.009 \dfrac{Wl^3}{EI}$;
" " " BC at G_1, $v_1 = \dfrac{1}{3} \dfrac{W(\frac{1}{2}l)^3}{EI} = \dfrac{1}{24} \dfrac{Wl^3}{EI}$.

Then

$$\frac{3}{16} W'l = \frac{fI}{y} = \frac{1}{2} Wl; \quad \therefore W' = \frac{16}{6} W = 2.66 W.$$

The deflection is reduced in the ratio 0.042 : 0.009, or is only $\dfrac{1}{4.67}$ as much as for the beam with one free end. The above, however, is equivalent, neglecting the weight of the beam itself, to a beam whose length is only $\frac{1}{2}l$, or that of the beam fixed at one end and supported at the other.

365. For the beams uniformly loaded:

Reaction and shear on B_1C_1 at C_1, $R_1 = S_1 = \frac{3}{8}wl$;

" " " " BC at C, $R_1' = S_1' = 0$;

Shear on B_1C_1 at B_1, $R_2 = S_2 = \frac{5}{8}wl$;

Reaction and shear on BC at B, $R_2' = S_2' = wl$;

Max. bending moment on B_1C_1 at B_1, $M_0' = \frac{1}{8}wl^2$;

" " " " BC at B, $M_0 = \frac{1}{2}wl^2$;

" " " " B_1C_1 at G_1, $M_1 = \frac{9}{128}wl^2$;

" " " " BC at G, $M_2 = \frac{1}{8}wl^2$;

$$\text{Max. deflection of } B_1C_1 = v_1 = 0.0054\frac{wl^4}{4};$$

$$\text{" " " } BC = v_1 = \frac{1}{8}\cdot\frac{wl^4}{EI} = 0.125\frac{wl^4}{4}.$$

Comparing bending moments at the centre of the spans,

$$\tfrac{9}{128}w' = \tfrac{1}{8}w; \quad \therefore w' = 1.78w.$$

But comparing maximum bending moments at B_1 and B,

$$\tfrac{1}{8}w' = \tfrac{1}{2}w; \quad \therefore w' = 4w;$$

that is, the beam fixed at one end and supported at the other will carry four times the load that the same beam will carry with one end free.

The deflection is reduced in the ratio of 0.125 to 0.0054, or only $\frac{1}{23.1}$ as much as with the free end.

Other similar comparisons can be readily made by any variations desired in the loading and supporting or fixing the ends.

The writer is aware of the fact that some authors lay but little stress upon the subject of continuous beams, and often, but very slightly at best, give only a passing allusion to it, or simply give a few results as deduced by others. The writer thinks it a subject of some importance, as developing interesting relations and conditions and also of considerable practical value; he has consequently discussed the subject with some detail.

In Art. XXXV the expression for slope and deflection were found to be $i = \frac{f}{Ey_0}\int \frac{MI_0}{IM_0}dx$ and $v = \frac{f}{Ey_0}\int^l\int^l \frac{MI_0}{IM_0}dx^2$, and for uniform cross-sections $I = I_0$; hence, $\frac{MI_0}{IM_0} = \frac{M}{M_0}$, a constant for any given beam supported and loaded in any specified manner. Then

$$\int \frac{MI_0}{IM_0} = \frac{M}{M_0}\int dx = m''l;$$

also,

$$\int\int \frac{MI_0}{IM_0}dx^2 = \frac{M}{M_0}\int\int dx^2 = n''l^2;$$

from which conditions the values of the constants m'' and n'' were found as given and used in the preceding paragraphs, and in Art. XXXV. If, however, the cross-sections are not uniform, either varying in breadth or depth, or in any other manner, the above values of m'' and n'' would not apply; and if both varied, the problem would be more difficult and complicated.

ART. XXXVII.

BEAMS OF UNIFORM STRENGTH.

366. As both the bending moments and shearing stresses vary from point to point or from maximum to minimum, it is evident that it is not necessary to provide the same amount of material at all points to resist the action of these external forces and moments. In open-work structures, such as cantilevers or brackets to support balconies or platforms, and in open-work frames or trusses, as trussed beams and bridge trusses, as also in solid-built girders, these principles are taken advantage of, both to save material and for ornamental purposes. The laws according to which these changes in dimensions can be made will be discussed in this article. So far as resistance to bending alone is concerned, both the breadth and depth can be diminished from their greatest values at the point of maximum bending moment to nothing at the point of zero bending; but since in beams supported at both ends the shearing

stress increases from the point of maximum bending to the point of zero bending, it becomes necessary to provide material to to resist the shears. For solid timber beams this will vary from one third to one half the maximum cross-section of the beams. Disregarding, however, the necessary material to resist shearing, the laws of change can be easily found by comparing the general values of the maximum bending moments with general bending moment, at any other point. Either the breadth b or the depth d is generally constant.

1. For a beam fixed at one end and loaded with a single weight at the other,

$$M_0 = mWl = nfbd^2, \text{ and } M = mWx = nfb_1d_1^2;$$

hence

$$\frac{mWx}{mWl} = \frac{nfb_1d_1^2}{nfbd^2} = \frac{x}{l} = \frac{b_1d_1^2}{bd^2}.$$

For constant breadth, $b = b_1$;

$$\therefore d_1^2 = \frac{d^2}{l}x. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (272)$$

Eq. 272 is that of a parabola; hence the depth varies as the ordinate of a parabola. (See Fig. 162.)

If the depth d is constant, $d_1^2 = d^2$;

$$\therefore b_1 = \frac{b}{l}x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273).$$

which is the equation of a straight line; that is, the breadths are ordinates of a triangle. (See Fig. 163.)

If the beam is uniformly loaded, we would have

$$\frac{mwx^2}{mwl^2} = \frac{nfb_1d_1^2}{nfbd^2}; \quad \therefore \frac{x^2}{l^2} = \frac{b_1d_1^2}{bd^2}.$$

If the breadth is constant, $d_1 = \frac{d}{l}x$, and the depth varies as the ordinate of a triangle. (See Fig. 164.) If the depth is constant, then $b_1 = \frac{b}{l^2}x^2$, which is also the equation of a parabola. (See Fig. 165.)

For beams supported at both ends and loaded with a single weight W at the centre each reaction will be $\frac{W}{2}$ and $M_0 = \frac{1}{4}Wl$, and $M = \frac{1}{2}Wx$; $\therefore \frac{2Wx}{Wl} = \frac{b_1 d_1^2}{bd^2}$. If breadth is constant, $d_1^2 = \frac{2d^2}{l}x$, and the depth would vary as the ordinates of two parabolas

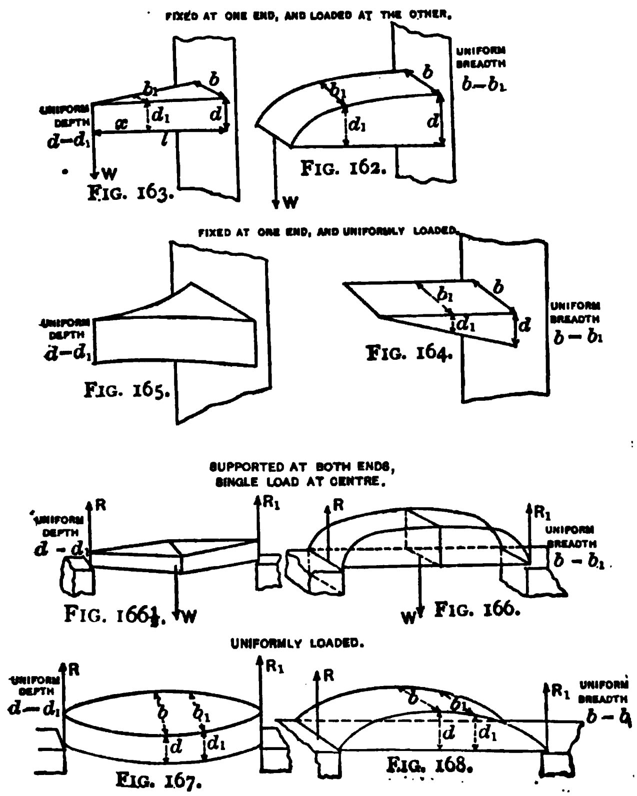


FIG. 163. FIG. 162. FIG. 165. FIG. 164. FIG. 166½. FIG. 166. FIG. 167. FIG. 168.

meeting at the centre with vertices at the points of support (see Fig. 166); and if depth is constant, then $b_1 = \frac{2b}{l}x$, or as the ordinates of two triangles with a common base equal to greatest breadth of

beam and vertices at the points of support. (See Fig. 166½.) When uniformly loaded,

$$M_0 = \tfrac{1}{8}wl^2; \ M = \frac{wl}{2}x - \frac{wx^2}{2} = nfb_1d_1^2 \text{ and } \tfrac{1}{8}wl^2 = nfbd^2.$$

If the depths are constant, $nfd^2 \doteq nfd_1^2 = \dfrac{wl^2}{8b}$.

$$\therefore \frac{wl}{2}x - \frac{wx^2}{2} = \frac{b_1wl^2}{8b}; \ \therefore b_1wl^2 = 4wblx - 4wbx^2;$$

$$\therefore b_1 = \frac{4bx}{l} - \frac{4bx^2}{l^2}.$$

Make $b_1 = b - y$, and $x = \dfrac{l}{2} - x_1$;

$$x_1^2 = \frac{l^2}{4b}y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

Equation (274) is that of a parabola vertex at the centre of the beam, the length of the beam being a double ordinate. (See Fig. 167.) If the breadth is constant, $b = b_1$, and $nfb_1 = nfb = \dfrac{wl^2}{8d^2}$;

$$\therefore \frac{wlx}{2} - \frac{wx^2}{2} = \frac{wl^2d_1^2}{8d^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (275)$$

$\therefore 4wd^2lx - 4d^2wx^2 = wl^2d_1^2$, the equation of an ellipse, with origin at one of the points of support. Making $x = \dfrac{l}{2} - x_1$, and substituting, we have $l^2d_1^2 + 4d^2x_1^2 = d^2l^2$; or, if $l = 2c$, there results $c^2d_1^2 + d^2x_1^2 = c^2d^2$, which is the equation of an ellipse with the origin at the centre. (See Fig. 168.)

So far as timber beams are concerned the above relations are not of much value, except that in cantilever beams they show that the amount of material can be reduced towards the free end, or that the beam can be formed or cut in scrolls or other ornamental shapes, but there is no economy of material resulting. But in iron beams considerable material may be saved, and at the same time the beams can be formed into ornamental shapes of either solid or open work, as in the brackets supporting balconies or projecting platforms. Sufficient area of material must be left at the ends to resist the shearing force—from one-sixth to one-half of the largest area.

PART II.

ART. XXXVIII.

CONSTRUCTION.

367. IN the preceding pages included in Part I all of the essential principles necessary in the construction of engineering works have been elaborated and fully discussed. In this part of the volume the application of those principles to the design and construction of engineering works will be made.

Whatever the nature of the structure, of whatsoever material constructed—timber, stone, brick, concrete, iron, or steel—and for whatever purpose constructed, they all require good and safe foundation-beds and foundations; the foundation-beds consisting of the natural materials of the earth, such as rock, gravel, sand, clay, or silt, and the foundations consisting of those structures resting directly on the foundation-beds and reaching to the surface of the ground or water, whether of timber, stone, brick, concrete, or iron.

A discussion of these subjects necessarily requires a discussion of the means necessary to reach safe foundation-beds, and to render the construction of the foundation both convenient and safe. These means will be discussed under their appropriate heads.

FOUNDATION-BEDS AND FOUNDATIONS.

368. Having determined the weight, magnitude, importance and design of any given structure, it is then necessary to determine the kind of foundation-bed, the kind of foundation, and the means of reaching the one and of constructing the other.

369. *Foundation-bed.*—The proper character of the foundation-bed to be adopted depends, first, upon its capacity to bear

safely the load of the structure (this is usually reduced to some intensity or weight per unit of area—either the square foot or the square inch); second, upon the permanence or stability of the material (this, when its bearing-power is sufficient, is commonly dependent upon our ability to protect it from the injurious effects of alternately freezing and thawing, and from the presence of water, especially running water); and, third, the comparative cost required to reach a material of sufficient bearing resistance in itself and that required to make a material of insufficient bearing-power in itself capable of carrying the load safely by artificial means, such as spreading the base so as to cover a large area, thereby reducing the unit pressure within safe limits, or compacting and consolidating the material by driving piles, or other similar means.

370. *Bearing-power of Materials.*—Any of the kinds of rock, such as granite, limestone, sandstone, and slate, can be safely relied upon to carry the weight of any structure likely to be built. The weakest form of limestone, namely, chalk, has an ultimate resistance to crushing of 0.5 ton per square inch or 72 tons per square foot; the weakest sandstones, 0.83 ton per square inch or 120 tons per square foot; the ordinary building-stones have a crushing resistance of from 150 to 500 tons per square foot; and granites and trap-rocks range as high as 700 to 800 tons per square foot: whereas the greatest pressure that is borne at the base of St. Paul's Cathedral does not exceed 14 tons per square foot, and in St. Peter's, Rome, it is only about 15½ tons per square foot. These are unusually great pressures; whereas the above resistances have been determined by crushing small cubes of from 2 to 1½ inches on each edge. These resistances are small as compared with those of the same stone in large masses. It is usually stated that the load should not exceed from one tenth to one eighth the resistance of the stone, this limit will rarely be approached. Loose and decayed portions of the stone at the surface should always be removed. When this is done frost alone will destroy its permanency or stability.

371. With respect to all other earthy materials there is even at the present time a want of reliable knowledge of their bearing powers and of the safe loads to place upon them. This is due primarily to the want of thorough, sufficient, and systematic experiments to determine the bearing resistances for loads, whether applied only

once and for a short period of time, or continuously applied over a long period.

It is well known that in some materials a structure will settle gradually and continuously for months or even years after completion. This settling may not exceed a small fraction of an inch per month, or it may amount to several inches during the same time. This may result in simply compacting a more or less porous soil, thereby squeezing out any contained moisture. In such cases the material will be in a short time capable of resisting permanently and without further settlement the load resting upon it, or it may result in the bulging up of the material surrounding the structure, in which case it may continue to settle during a long period, and even at an increasing rate due to the diminution in the adhesiveness and compactness of the mass, thereby increasing the injurious effects arising from the presence of moisture accompanied by freezing and thawing. The settlement in the one case insures the ultimate stability of the structure, and in the other leads inevitably to its destruction.

372. Some varieties of earthy material resemble both in their bearing power and permanency ordinary rock. These materials are not affected by the presence of water, and do not settle under the heaviest loads used in practice. These materials are called conglomerates, cemented gravel, or hardpan. They are commonly composed of pebbles and river-jacks cemented together by siliceous, calcareous, or argillaceous materials usually containing iron.

And finally there are soft, alluvial, and silty materials which are practically incapable of bearing any considerable loads directly, and must be compacted by driving piles over the space to be occupied by the structures, or must have the superincumbent loads distributed over large areas by platforms of brush, logs, planks, sand, gravel, or broken stone.

Between these extremes of rock and soft alluvial materials an almost infinite variety of materials and combinations of materials is found.

For practical purposes, however, all such materials may be classified or grouped under four heads. The materials composing each of these groups must be prepared and loaded differently, according to the cost, magnitude, and importance of the structure to be placed upon them. This requires the exercise of good judgment.

373. *Classification of Materials for Foundation-beds.*—1st. Those having ample bearing resistance and whose stability is not affected by the presence of water, but may be more or less impaired by alternately freezing and thawing, such as rock, cemented gravel or stones, and other similar materials which can be safely loaded with from 100 to 1000 pounds per square inch or from 14,400 to 144,000 pounds per square foot. The intensity of pressure arising from the heaviest structures of the present day does not exceed from 200 to 300 pounds per square inch or from 28,800 to 43,200 pounds per square foot. We may therefore conclude that any but the softest materials of this class will carry safely the heaviest loads required at the present day, and that all of them will carry safely the ordinary heavy structures.

2d. Those earthy materials whose stability is not affected by the presence of water, that is, standing water. Such materials may be compressible to a limited extent even under reasonable loads, and may under excessive loads give way by displacement or bulging up around the structure. The presence of water in such materials may to some extent impair the stability by acting as a lubricant and thereby reducing the friction between the grains or separate pieces. In this class may be included gravel, coarse sand, river-jacks, and mixtures of these materials. Such materials certainly when dry will carry safely 42 pounds to the square inch or 6000 pounds (2.7) tons to the square foot. As a matter of precaution and in the absence of accurate data it would be better to make the load only one half of the above amounts if standing water is present, and not to build on it at all if exposed to the scouring action of running water or even liable to be thus exposed.

3d. Those earthy materials affected by the presence of standing water, and which may be compressible to a limited extent even when dry, or at most only containing a small amount of moisture, will certainly give way by displacement and bulging in the presence of water. In this class may be included fine sand, clay, and mixed earths, such as *loam, a mechanical mixture of sand and clay, and marl, a mixture of carbonate of lime and clay.* Unless these materials are found in a dry state or can be drained, and water can be subsequently kept from them either by surface or subsoil drains, it is not safe to build on them at all without resorting to special and usually unnecessary and expensive means of holding or confining the

material. When dry they can be kept dry. A layer of sand 10 or 15 feet thick can be safely loaded with 28 pounds per square inch or 4000 pounds (1.8 tons) per square foot. A layer of pure clay of the above thickness, either alone or mixed with gravel, can be safely loaded with 24 pounds per square inch or 3500 (1.6 tons) pounds per square foot. A mixture such as loam or marl should not be loaded with more than 20 pounds per square inch or at most 3000 (1.3 tons) pounds per square foot.

Some authorities state that any kind of sand when mixed with a small amount of clay will form, when saturated with water, a quicksand, which is absolutely unstable, and will run or flow like water; others, that it must be in a state of impalpable powder, so that either alone or mixed with clay it will seemingly float or be suspended in the water. This latter condition would seem to be more consonant with reason, and certainly with the writer's experience. Any finely divided earthy substance will form a mixture with water which will flow freely when unrestrained, and yet may contain only a small per cent of fine sand. This is usually designated as quicksand, and will be so considered in this volume. This material cannot be drained. It is the most troublesome material with which the engineer has to deal.

A homogeneous, compact clay is but little affected by running water—that is, there is little danger of scouring out; but it forms mud when mixed with water, and consequently has little bearing-power or stability when in this condition. Sand, coarse or fine, and gravel, on the other hand, may not be affected by standing water, but readily scours out when exposed to running water.

4th. Earthy materials such as clay and fine sand, mixed with water, and the soft alluvial or marshy soils, can only be depended upon to carry very small loads directly, and require special treatment when required to sustain heavy and important structures, which treatment will be described later.

METHODS OF DETERMINING THE BEARING-POWER OF MATERIALS.

374. For the materials of the first class it is usual to actually crush small cubes, say 2 inches on the edge, having 4 square inches of surface on each face and containing 8 cubic inches.

For the materials of the second and third classes the usual

practice is to excavate small pits or shafts of different depths below the surface, say 2, 4, 6, and 8 feet. In these pits a stick of timber, which should not be smaller in cross-sectional dimensions than 12 × 12 inches, is placed in a vertical position. Weights are then placed gradually and uniformly on its top, and increased until a decided settling takes place; this may or may not be accompanied by a perceptible bulging up of the surrounding material. Careful observations should be made by the use of a level, or by an index or string carefully stretched beforehand and adjusted so as to touch the top of the stick or some point or line marked on its face or edge, to determine the first or initial settling; and also at intervals to determine the rate of settling, and finally the total amount of settlement under a predetermined load. If the settling ceases after a time, the stick and load should be left in position, and daily observations should be made for at least a month in order to determine any settling that may arise from the continued action of the load. Similar observations should be made with smaller loads than the predetermined one, if any appreciable and continued settling occurs under it. The points to be noted are: (1) the least load that produces appreciable settling, (2) the extent and rate of settling under this load, (3) the effect of the continued application of the load. Similar observations should be made with each increment of 100 pounds, more or less.

For the softer varieties of these materials the base of the column may be increased to 2, 3, or 4 square feet of bearing surface; and for marshy soils it may be still further increased to 6 or 8 or more square feet.

In order to determine whether settling arises from simply compacting the material, or from displacement and bulging upward, a series of stakes should be driven surrounding the pit, the tops of which are sawed off to the same elevation. Any change in these elevations would indicate both the fact and the extent of bulging.

If piles have been driven either into the firmer soils, such as gravel, clay, or sand, which is often done, or into the quick and softer alluvial soils, a platform can be constructed resting on 1, 2, or 4 piles, and loaded until settlement takes place; similar observations to those above mentioned being made.

All such experiments should be made at that season of the year most unfavorable for bearing resistance of the material. If at this

season the surface of the ground is frozen, the frozen material should be removed for a considerable area around the pit.

If a soft stratum underlies a firmer one, the test should be made at that depth in the firmer material required for the bottom of the foundation-walls, noting both the total thickness of the firm stratum and also the thickness remaining after the necessary excavation. The pit should also be excavated entirely through the firmer stratum, so that the bearing power of the underlying soft stratum may be independently determined. This latter load would indicate the minimum bearing resistance, that on the firmer soil the maximum, the firmer soil serving to distribute the pressure over a larger area of the softer soil by virtue of its transverse strength, which would vary with the thickness of the stratum and the toughness of the material. Some soils with little cohesive and adhesive strength, being brittle, will carry a load up to a certain amount and then break along the outside lines of the structure under even a small settlement. Such materials cannot be relied upon to act as a beam in distributing the pressure, and the bearing resistance of the softer stratum will be a measure of the safe weight of the structure. In such cases the determination of the safe load between the maximum and minimum loads, as determined by the experiments above mentioned, is difficult owing to our inability to determine the transverse strength of a layer of any kind of earth. Good judgment based upon the data obtained by experiment is alone to be relied upon.

So long as the settlement increases uniformly and in the same proportion as the increase in the load, it may be taken as indicating that the material is acting as a beam; but when the settlement increases at a more rapid rate than the increase of the load, and in an irregular manner, it will be safe to conclude that we have exceeded the limits of safety. By deductions from experiments made in this manner Gen. Wm. Sooy Smith concluded that the firm clay found underlaid by a softer material, in the city of Chicago, would carry safely a load of 4000 pounds per square foot. This result was confirmed by Mr. Rankine's formula for the bearing power of soft materials, viz.,

$$w = w'x \left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (275\tfrac{1}{2})$$

The theory upon which this formula is based will be explained in another article. Assuming the angle of repose of wet clay to be $15° = \phi$, the depth of foundation-bed 13 feet $= x$ below the surface, and the weight of a cubic foot of the material itself = 120 pounds $= w'$, hence $W = 120 \times 13 \times 2.9 = 4524$ pounds per square foot of foundation-bed. In these experiments a tank was placed on top of a timber $12 \times 11\frac{3}{4}$ inches; into this tank water was allowed to flow gradually and without shock, and the tests were made with weights varying from the weight of the stick and tank, 1210 pounds, to 9000 pounds. Up to 4800 pounds settlement varied nearly as the load, the total at this load being $5\frac{1}{16}$ inches, which is rather in excess of the permissible settlement of about 3 inches. At 9000 pounds the settlement amounted to $12\frac{3}{8}$ inches in 40 hours.

The firm stratum above depressed only $\frac{7}{8}$ inch under 7000 pounds in 63 hours, which was the longest time test.

TABLE XLVI.

CONCLUSIONS.

Safe loads per square foot for alluvial soils......	$\frac{1}{2}$	to	1	ton, or	1120	to	2240	lbs.
Ordinary earth..........	1	to	$1\frac{1}{2}$	tons, or	2240	to	3360	"
Dry sand, clay, and gravel and sand............	$1\frac{1}{2}$	to	$2\frac{1}{2}$	" or	3360	to	5600	"
Moist clay and moist sand	1	to	2	" or	2240	to	4000	"
Layer of clay on quicksand	1	to	2	" or	2240	to	4000	"
Rock..................	10	to	15	" or	22,400	to	33,600	"

It might be said that the extent of settlement of a perfectly homogeneous substance, loaded with an equal weight on each square foot of bearing surface, would be immaterial, whether it amounted to an inch or to several feet, as under such circumstances no injury would occur to the structure above. And there are instances of structures settling 6 inches, 1, $1\frac{1}{2}$, and even 4 feet, partly during construction and partly afterwards. The structures still stand, usually, however, with more or less extended cracks. Such settlements are rarely uniform under all parts of the structure, owing to a want of uniformity in material or load, or in both.

In addition, any considerable degree of settling, although it does not result in serious damage, may impair its usefulness or produce great inconvenience, such as depressing floors below the street or sidewalk levels, throwing bridge spans out of level, and piers out of plumb. It is necessary, therefore, to determine upon some maximum settling, say from 3 to 5 inches, beforehand, and so proportion the loads that this will not be exceeded; and for this allowance must be made in designing and erecting the structures, so that ultimately all portions may reach their proper position.

To determine these questions in advance, it will be necessary to ascertain the thickness and character of each layer from the surface to the underlying bed-rock, when this is not more than 100 feet below the surface.

THE DETERMINATION OF THE CHARACTER OF THE STRATA.

375. The thickness and nature of the strata of earthy substances can be ascertained, first, by sinking shafts or pits; and, second, by sounding or boring. The first in some respects is the best, but it is slow and expensive. The second is the more rapid and economical, but may or may not be satisfactory. The method by sinking shafts will be described in another article.

Where great depths are not necessary, an iron rod can be driven into the soil; this is likely to be very deceptive, and may be entirely misleading. A large auger from 2 to 3 inches in diameter can be screwed from 10 to 15 feet, or even 20 feet, into the ground; by this method samples of the materials may be brought to the surface, but it is usually necessary to use a great deal of water, which renders the exact nature and condition of the materials uncertain. The third method is the same as has been used in sinking glazed earthenware pipes to obtain a supply of water. Very great depths can be reached by this method. Successive pipes are placed one on the other as they sink into the soil. The sinking is effected by buckets having a flap-valve at the bottom opening upwards, the sides of the buckets extending a little below the valve-seat so as to form a cutting edge. A bucket is lifted up a few feet and allowed to drop down the pipe, the material is cut up and

forced through an opening at the bottom into the bucket, and at intervals of time the bucket is lifted up and contents emptied. Either by its own weight or by weights added on top the pipe sinks gradually.

This is the same principle as that upon which large cylinders are sunk to great depths for the foundations of many large structures, the main difference being in the mode of lifting out the material. This is more satisfactory than either of the two other methods described. It also requires the use of much water. But by resorting to the method now to be described entirely satisfactory results can be reached.

The fourth method consists in first forcing into the soil an iron pipe about 1½ inches in diameter; a smaller pipe is inserted into this, which has a chisel-shaped bottom section about one foot long; in this section are two small openings near the lower edge. Water is forced into this smaller pipe (¾ inch in diameter) by means of a force-pump; this, issuing through the small openings with great pressure, loosens the material, which with the water rises up between the two pipes and overflows at the top; as this is going on the pipe sinks. The larger pipe may or may not be forced down with the smaller one. At intervals of a few feet in depth the smaller pipe can be taken out, the bottom section removed, and a small open brass section substituted, which is then forced about a foot into the material below the bottom of the hole, and when lifted out it brings a sample of the material just as it exists in the strata. Rapid progress can be made by this method, and entirely satisfactory information obtained. The importance of boring and sounding cannot be overestimated in determining the proper depth for the foundation-bed, and they should be made in sufficient numbers, at the site of every important structure, to furnish full and reliable information. Safety and ultimate economy both imperatively demand it.

376. Having determined the character, thickness, depth, and the safe load for the stratum upon which the structure is to be built, the most economical and expeditious means of reaching this stratum must be decided upon. This will depend upon the magnitude and importance of the structure, the purposes for which the structure is to be built, and whether the structure is to be

built on dry land, in soft and marshy soils, or in water of a greater or less depth.

377. *The proper means of reaching the foundation-beds, or, as it is usually called, constructing the foundation.*—In this connection the magnitude and importance of a structure simply resolve themselves into a question of weight and safe loads per unit of area of bearing surface, and need not be further considered.

The purposes for which structures are built are:

(1) Houses of any description, and particularly what are now called high buildings, fifteen to twenty or more stories in height. These are usually built on dry land, and may rest on rock, or on firm or soft earth; but often now, from the great weights involved, may require special means of insuring stability, such as broad bases of concrete, or timber grillages or platforms, piles driven into or through the ordinary earthy soils to rock or other hard and compact materials; and, finally, sinking shafts, cylinders, or even caissons to the hard underlying material.

(2) Retaining-walls, piers, and abutments of bridges, arches, and viaducts on land, marshes, or water, which require the same methods of reaching foundation-beds as described above for high buildings.

(3) Structures in water, usually piers and abutments of high bridges, viaducts, and arches. These require special methods, as in such cases it is commonly desirable or necessary to reach rock, or at least some firm material, at depths ranging from fifty to one hundred feet or more below the beds of the streams or rivers, in order to get below any possibly injurious action of the currents, which, even when opposed by comparatively small obstructions, cause a scouring out of the materials to great and uncertain depths.

(4) Foundations for earthen or masonry dams, for canals, locks, and the embankments or walls of storage reservoirs. Greater care in many respects is required for the foundation-beds of these structures than for any of those already mentioned.

(5) Foundations for highways and streets. These, while not requiring such great depths, are of the greatest importance and require special methods of construction.

So far as principles and practice are concerned, all classes and

kinds of structures will find their counterparts in the one or the other of the above-described structures.

The construction of the foundations for each of the above types of structures will be described. The same methods will be found in some conditions to apply to each of the first four classes, and will be described only once, under the head of Deep and Difficult Foundations, as the methods and principles of construction will be the same; the only difference being in the dimensions of the designs used and the depths to which they have to be sunk.

ORDINARY HOUSE-WALLS.

378. For houses three or four stories in height the walls are usually made from one brick to a brick and a half in thickness at the top, that is, from 8 to 12 inches, and for the basement story about $2\frac{1}{2}$ bricks or 20 inches. The footing courses increase this $1\frac{1}{2}$ to 2 times, or from 30 to 40 inches in thickness, which gives from $2\frac{1}{2}$ to $3\frac{1}{2}$ square feet of base or bearing area per linear foot of wall. This at 3000 pounds per square foot of foundation-bed would only carry 7500 pounds to 10,500 pounds of weight.

This weight includes the weight of the wall itself, which for brick walls will be about 115 pounds per cubic foot, and for masonry walls about 150 pounds per cubic foot; the weights of the floors and the loads assumed to be carried by them; and finally the weight of the roof itself. The last two items are very variable and must be assumed or calculated in each particular case. In addition the weight of snow on a roof must be considered. Assuming a depth of $2\frac{1}{2}$ feet, there should be allowed 16 pounds per square foot, and an additional allowance arising from the wind pressure, assumed at from 30 to 50 pounds per square foot.

If the sum of these exceed the allowable pressure of 3000 pounds per square foot of base as above given, additional footing courses must be added. It is better, however, when the necessary spread of base exceeds that provided by doubling the thickness of the basement wall, to secure additional spread by means of timber platforms, or beams of timber or iron, or layers of concrete; and where a very great spread is required, timber or iron beams imbedded in concrete are used. These latter means are rarely resorted to except in very high buildings. Knowing the weight to be carried, the

spread of base must be such that the pressure on the soil shall not exceed the usual limits of from 3000 to 4000 pounds per square foot.

The top of walls of houses from 80 to 100 feet in height are made from 12 to 16 inches thick at top and from 24 to 32 inches at bottom, and a proportionate increase for higher walls up to heights of 160 or 170 feet. It is evident that for such structures the weights are very great and require a very great spread of base. For such structures it has been considered advisable, if not necessary, to drive piles to great depths into the earthy materials, or even through these to the underlying hardpan or bed-rock, which is in many cases from 50 to 60 feet below the surface. In a recent foundation construction for the Chicago Stock Exchange such piles were used under those parts of the structure where there was no danger of damaging adjacent structures by the vibrations and shocks caused by the fall of the hammer. Where such was the case, wells 5 feet in diameter were sunk to hard clay 55 feet below the surface. These wells were lined with poling-boards about 3 feet long, supported on the inside by steel hoops. When hard bottom was reached, the well was spread out by a cone-shaped excavation with a bottom diameter of 7½ feet. The entire well or shaft was then filled with concrete. These means were resorted to, as it was found that the overlying material of soft clay settled $\frac{1}{16}$ of an inch per month under a pressure of from 1½ to 2 tons per square foot of surface, and was likely to continue settling for years.

The Chicago building ordinances limit the load on piles driven to hard material or rock to 25 tons per pile, and on concrete to 8000 pounds per square foot.

379. The city of New Orleans is built on an alluvial soil on which only from 1000 to 1500 pounds per square foot is allowed. If the weight of the structure is so great that with a spread of base of ten bricks, about 80 inches in thickness, the pressure cannot be kept within the above limits, piles are driven, 4 feet centres, and at from 25 to 40 feet penetration into the material they are safely loaded with from 15 to 25 tons. When the base of 80 inches is sufficient, the brickwork or masonry is commenced on the bottom of the trench excavated for the purpose. The material is a sandy clay, saturated with water at the depth of 3 or 4 feet below the surface.

380. For the Manhattan Life Building, New York, small caissons and cylinders of iron were sunk through the softer material, layers of sand and quicksand, to the rock 50 feet below the level of Broadway. The pneumatic method was adopted in this case in order to prevent inflow of the quicksand, which would occur if open shafts or cylinders were used, this inflow endangering the stability or safety of adjacent structures.

381. For light masonry houses it is only necessary usually to excavate a trench into the earthy material from 2 to 4 feet deep, upon the bottom of which the brick or stone masonry is commenced, giving the necessary spread of base by means of the projecting or footing courses, or by the use of a layer of concrete from 1 to 2 feet in thickness. A good bed of mortar should always be first spread over the bottom of the trench in order to insure a true and full bearing of the bottom course of brick or stone. The bottom of the trench should be horizontal.

382. For the higher and heavier buildings some of the above-described special means of securing stability must be employed. But owing to the enormous loads and weights arising from the older methods of construction, with solid masonry main and partition walls, and heavy floors necessary to carry great and varying loads both fixed and moving, together with the rapidly-increasing effects of wind pressure, it has become necessary, in order to keep the pressure per square foot within the prescribed limits, to resort to various methods and designs to reduce the superincumbent weights; and further, as with increasing heights and dimensions of structures both the damage to property and danger to life are greatly increased in case of conflagration, much attention has been given by engineers, architects, and builders to the production of proper materials and designs to reduce both weight and damage from fire.

This has led to the introduction of what is called by architects *Skeleton Construction,* in which all external and internal loads and strains are transmitted from the top of the building to the foundation by a skeleton or framework of metal, consisting of iron or steel columns and beams properly connected and braced, and all other parts of the structure composed of as light materials and construction as is consistent with strength and stiffness. And for fire-proof construction all parts which carry weights, floors, stairs, elevator enclosure, and their contents, are made of incombustible mate-

rials, and all metallic structural members are protected against the effects of fire by coverings of an incombustible and slow heat-conducting material, such as brick, hollow tiles, or burnt clay, porous terra cotta, and two layers of plastering on metal lath. In such structures columns and beams take the place of solid-masonry main and partition walls in supporting and transmitting loads and weights. The design and construction of such buildings are to a great extent governed by building ordinances of the large cities and vary in many respects. To these the reader must be referred for further information. That such structures have not come up to the full expectations or hopes of builders and owners might have been expected, as many of the so-called fire-proof structures have been as entirely and absolutely destroyed by fire for all practical purposes as those for which no special claim has been made in this respect. Much good has been accomplished and much progress made, although much yet may remain to be discovered and understood. The writer has introduced these remarks here as no further discussion of this mode of construction will be given in this book.

RETAINING-WALLS, PIERS, ABUTMENTS, PEDESTALS.

383. Structures of this class may have foundations prepared in any of the ways mentioned in the preceding paragraph for those of high buildings. The bottom courses are spread out by footing-courses in order to keep the pressure per square foot within the proper limits. The walls are usually more massive, and cover areas of much greater widths as compared with their lengths; and although the weights per square foot may be the same as in house walls, much larger aggregate weights are concentrated in one place, and greater loads per square foot on some parts of the foundation-beds may exist in case of unequal or irregular settlement, or may result from the pressure of high winds. In the case of retaining-walls and abutments the pressure of the earth upon the walls makes the resultant pressure act near the front or rear of the wall, thereby making the pressure per square foot of base greater at some points than at others, and the direction of the pressure oblique to the base. Abutments are often constructed on or near the sloping banks of streams and ravines, and may cause a yielding of the

underlying material by lateral displacements under relatively small loads. All of these conditions and considerations require special preparation and arrangement to resist these tendencies. The pressures per square foot are nevertheless as great as those already mentioned, and often greater.

384. *For Piers and Abutments, and Pedestals for Bridges.*—(1) The pressure on the foundation-bed consists of the weights of the masonry, which is usually built of stone, commonly the best varieties of sandstone, limestone, or granite. Bricks are rarely used for such structures, except in those sections of the country where it is difficult to secure suitable stone.

The best quality of granite and limestone ashlar masonry will weigh from 150 to 165 pounds per cubic foot.

Second-class abutment masonry and good hammer-dressed rubble masonry weighs from 150 to 155 pounds per cubic foot. Similar grades of sandstone masonry will weigh, respectively, about 145 to 155 pounds per cubic foot.

Concrete weighs from 120 to 135 pounds per cubic foot, according to quality of materials used.

Brick masonry weighs from 115 to 125 pounds per cubic foot, according to quality and workmanship.

(2) Superstructure for highway bridges may weigh more or less than that for railways, and for present purposes may be taken as equal to it and equal to five times the length of span in pounds per linear foot, increased by 700 pounds per linear foot for the weight of floor system. For highway bridges the rolling or live load should be taken at from 80 to 100 pounds per square foot of floor. For railway bridges the live load may be taken as varying from 3000 to 4000 pounds per linear foot of span for spans over 100 feet. It increases as the length of the span decreases, reaching as high as 6000 pounds per linear foot for short spans from 12 to 15 feet in length. The dead and live loads as above given will be close enough for approximate calculations.

W = 5l + 700 = weight per ft in lbs of superstructure.

Assuming a masonry pier 100 feet high; top dimensions 12 × 35 feet, bottom 20.33 × 43.33 feet; average, 16.16 × 39.16 feet; built of limestone weighing 160 pounds per cubic foot, including mortar, and carrying one end of both a 520 and a 480 feet span; the pier resting on a bed of concrete 20 feet thick. Then

		Pounds.
Weight of masonry in pier..........	$16.16 \times 39.16 \times 100 \times 160 =$	10,441,622
" of concrete.................	$20.33 \times 43.33 \times 20 \times 1$[illegible] $=$	2,378,430
" on one half of 520′ span.....	$\frac{520}{2} \times 520 \times 5 + \frac{520}{2} \times 700 =$	858,000
Live load on " " " " "	$3000 \times \frac{520}{2} \quad =$	780,000
Weight on " " " 480′ "	$\frac{480}{2} \times 480 \times 5 + \frac{480}{2} \times 700 =$	744,000
Live load on " " " " "	$3000 \times \frac{480}{2} \quad =$	720,000
Total weight on foundation-bed..........................		15,922,052

Allowing for the bearing resistance of the bed 5000 pounds, there would be required $\frac{15922052}{5000} = 3185$ square feet of base. The calculated area is $20.33 \times 43.33 = 880.9$ square feet. Increasing these dimensions by 25.67 feet, we have a base of $46 \times 69 = 3174$ square feet. It is, however, generally assumed that the admissible spread of a concrete mass should not exceed its depth—in this case only 20 feet. It would therefore be necessary to provide additional base to the masonry pier of 5.67 or 6 feet all around by footing or offset courses. This would require 6 footing-courses with offsets of 1 foot on each, or 8 courses with 9-inch offsets.

The base of the masonry would then be 26.33×49.33 feet; the top dimensions of the concrete about 30×53, and the bottom 46×69. This increase could be obtained by a series of steps or a uniform slope. The bottom of the masonry is $26.33 \times 49.33 = 1198.86$ square feet. The load above this is 13,543,622 pounds, and the pressure on the concrete would be $\frac{13,543,622}{1198.86} = 11,300$ pounds per square foot. Concrete made of ordinary cement when six months old can safely be trusted with 100 pounds pressure per square inch or 14,400 pounds per square foot, and Portland-cement concretes with 200 pounds per square inch, equivalent to 28,800 pounds per square foot, when six months old: when one month old a safe load would be 100 pounds per square inch or 14,400 per square foot. These pressures allow a factor of safety of from 8 to 10. This example should impress the importance of using only the best Portland cements in such cases, and also of

using not only good sand, but in proportions of not exceeding 2 sand to 1 cement.

385. It must be remembered that although the example given would be considered a work of great magnitude, yet it has been more than once exceeded in the dimensions of piers and concrete base, as well as in length of spans. The great pressures on the concrete also show the importance of spreading the base of masonry by the use of a number of footing courses. It is usually desirable not to have any footing courses above the ground- or water-surface. In such cases it is important to avoid bringing the concrete too near the surface, so as to allow ample depth below for the footing courses of the masonry. It is more than probable that very much greater loads are placed upon concrete than those above given, to which there may be no objection when the progress of the work is slow; and fortunately structures of the magnitude above contemplated require from two to three years in construction, when the safe load on ordinary concrete may be taken at 150 pounds per square inch or 21,600 pounds per square foot, and for Portland-cement concrete 250 pounds per square inch or 36,000 pounds per square foot.

386. Piers on land are rarely ever of as great dimensions, nor do they usually carry such long spans. When they do exist, however, it would usually be good practice to carry the excavation to rock, unless its depth below the surface is very great. In such cases concrete could be dispensed with entirely, and the masonry commenced on the rock after removing all disintegrated and loose portions, levelling the surface, or cutting it into a series of steps, or simply blasting a series of holes, producing a series of ridges and depressions, so as to prevent the possibility of sliding occurring. Such hollows should be filled with concrete, in order to bring the entire surface to the same level or, if much slope originally existed, to a series of horizontal surfaces, upon which to commence the masonry. Rock beds are usually composed of a series of ridges and hollows, the latter filled with earthy material. All such material, as well as other loose portions of rock that may be found, should be entirely removed and the hollows filled with concrete. A rock bed should be thoroughly washed so as to remove all loam or clay; otherwise no bond can exist between the masonry

and the rock, which is a matter of great importance. Cement mortar will set neither in contact with clay or silt, nor will it adhere when a skim of these materials is on the rock.

387. A granite pier 200 feet high will exert a pressure of about 46,200 pounds per square foot at its base. This pressure on any good rock would not be excessive, but it is usual to commence masonry on rock by a spread of at least one fourth to one half more than its bottom thickness, the latter in this case about doubling the area and reducing the pressure to 23,100 pounds per square foot.

In smaller piers the practice is frequently to use a thin layer of concrete, not exceeding from 2 to 4 feet, and imbedding in it timber or iron beams. This is supposed to bind the concrete together, and form a strong beam by which the pressure can be distributed over considerable areas. The concrete no doubt adds greatly to the stiffness of the beams. The writer doubts the wisdom of using this combination as often is done, especially with timber beams, owing to its greater compressibility and deflection without fracture, as compared with the brittleness and unyielding character of the concrete, thereby probably preventing a permanently uniform distribution of the load. The usual method of construction is to lay a bed of concrete about one foot in thickness, on this a series of timbers from 2 to 2½ feet centres, filling the spaces between them with concrete, and on this a flooring of solid timbers 12 × 12 inches in cross-section,—sometimes only a 3-inch plank flooring,—on which the masonry is commenced; or two courses of timber are laid crossing each other, forming a series of cells or pockets from 1 to 1½ feet square, which are filled with concrete, and over these the solid flooring may or may not be placed. A crib or grillage constructed of many courses of timber similarly placed on each other allows a spread of base proportional to the number of courses or to its height.

388. Assuming a close layer of solid timbers under a wall or pier, by a simple application of the formulæ for deflection and strength of beams the proper projecting length of beam and corresponding spread of base can be calculated. Let *ABCD* represent a wall or pier 20 feet high, 20 feet square, assumed, for simplicity of the calculation of its weight, as a rectangular parallelopipedon. The flooring would require 20 sticks of timber 12 × 12 inches in

cross-section, and it is required to find the length in order that the greatest deflection shall not exceed $\frac{1}{480}$ of its unsupported length.

Referring to Fig. 169, the weight of the wall, assuming granite or limestone at 165 pounds per cubic foot, will be $165 \times 20 \times 20 \times 20 = 1,320,000$ pounds, on each of the sticks EF $\frac{1320000}{20} = 66,000$ pounds, and on the portion AD $\frac{66000}{20} = 3300$ pounds per square foot. In order to determine the projecting length of the timbers it is necessary to make some assumption in regard to the distribution of the weight of the masonry over the timber EF, and also as to the position of the fixed end of the beam. It is usual to consider the weight distributed uniformly over the entire length EF, or, what is the same thing, to assume an upward pressure arising from the resistance of the soil, equal to this weight and uniformly distributed over its length, as indicated by the arrows, Fig. 169.

The beam may be considered as fixed at the centre G, at the outer edge A, or at H, the middle point between A and G, where the resultant of the half-weight on the half-beam EG acts.

In either case the total half-load is the same, and is $P = 33,000$ pounds $= pl = wl$, l being $= EG$ and $p =$ upward pressure per unit of length EG.

Having assumed the depth of the beams as 12 inches, and that the deflection must not exceed $\frac{1}{480}$ of its unsupported length, which call y, we can at once pass to the length $y = EA$. Assuming A as the fixed end, by means of eq. (226),

$$\frac{d}{l} = \frac{n''f_o(\text{or } f_t)}{Em'} \frac{l}{v_1};$$

for a single load at the end $n'' = \frac{1}{3}$, and for a uniform load $n'' = \frac{1}{4}$ (see table, page 401); for a solid rectangular beam, or any beam whose neutral axis is at the centre of the depth, $m = m' = \frac{1}{2}$; $\frac{l}{v_1} = \frac{y}{v_1} = 480$; $E = 1,000,000$ for timber, approximately; and $f_t = 1000$ pounds per square inch. Substituting,

$$\frac{d}{l} = \frac{\frac{1}{4} \times 1000 \times 480}{\frac{1}{2} \times 1,000,000} = \frac{1}{4.2} \text{ (nearly)};$$

which simply means that $EA = l = y = 4.2d$, d being $12'' = 1'$, and $EA = 4.2$ feet. As the projecting length is $4.2' = 50.4$ inches, the allowable maximum deflection is $\frac{50.4}{480} = 0.105$ inch. Under the conditions given, the formula for maximum deflection, eq. (197), will be

$$v_1 = \frac{n''fl^2}{Ey_0} = \frac{\frac{1}{4} \times 1000 \times (50.4)^2}{1000000 \times 6} = 0.105 \text{ inch},$$

$$y_0 = \tfrac{1}{2}d = 6 \text{ inches}.$$

If, now, we find what upward pressure on the projecting beam will produce this deflection, $m\,Wl = nfbd^2$ (see eqs. (136) and (138)); $m = \frac{1}{2}$; $l = 50.4$ inches; $n = \frac{1}{6}$; $b = 12$; $d = 12$; $f = 1000$. Then $W = 11{,}429$ pounds $= wl = wy$; hence $w = \frac{11429}{4.2} = 2721$ pounds per foot of length or per square foot of bearing surface. Then substituting, in the common formula for deflection, eq. (205),

$$v_1 = \frac{1}{8}\frac{Wl^3}{EI} = \frac{1}{8}\frac{wl^4}{EI} = \frac{1}{8}\frac{11429 \times (50.4)^3}{1000000 \times \frac{1}{12}(12)^4} = 0.107 \text{ inch},$$

practically the same as before found, thus checking all of the preceding results, notably that $\frac{n''fl^2}{Ey_0} = \frac{1}{8}\frac{wl^4}{EI}$. $I = \frac{1}{12}bd^3 = \frac{1}{12}d^4$.

It should be noted that the upward pressure on the projecting beam should not exceed the safe bearing resistance of the soil. We

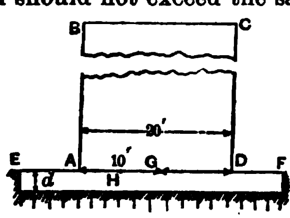


FIG. 169.

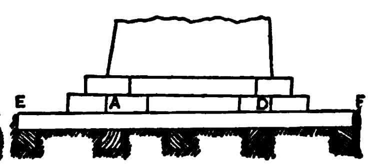


FIG. 170.

have seen in the case above that this amounts to 2721 pounds per square foot. If the soil is not capable of exerting this pressure without undue settlement, it then becomes necessary to increase the projecting length; and if the deflection given is not to be exceeded, the depth of the beam must also be increased.

In such cases it is better to use two or more courses of beams crossing each other, each projecting by an amount to be calculated precisely as given above. The foregoing fully illustrates the principles and methods.

We may desire to know, without considering the weight of the structure, the allowable projection that, with a working maximum fibre strain, can be given without exceeding the safe resistance of the soil—say 3000 pounds per square foot or per linear foot of a 12 × 12 inch beam. Then $mwl^2 = nfbd^2$; $w = \frac{3000}{12} = 250$ pounds, $m = \frac{1}{2}$ the pressure per linear inch; $n = \frac{1}{6}$; $f = 1000$; $b = 12$; $d = 12$; consequently $l = 48$ inches $= 4$ feet. With the weights above given the projection should not be less than 4 feet, and may be as much greater as the safe limit of deflection will allow. But nothing specially is gained by a greater projection, as the full value of the resistance of the foundation-bed would not be brought into action.

Any desired degree of spread can be secured by increasing the number of the courses of beams. As these beams are either placed close together or the spaces between them are filled with mortar or concrete, they are stiffened by the lateral support thus given, in consequence of which the projecting length could be considerably greater than found above for a single isolated beam—possibly as much as two to two and a half times.

In the preceding example the total length of the beam is 28 feet between E and F. Allowing a safe bearing resistance of 3000 pounds per square foot, the total bearing resistance will be $28 \times 3000 = 84,000$ pounds, which is equivalent to a weight of $\frac{84000}{20} = 4200$ pounds per square foot on the area of the masonry base In other words, the height of the masonry could be increased to $20 \times 1.273 = 25.46$ feet, without overloading the foundation-bed, since $165 \times 20 \times 1.273 = 4200$ pounds.

In the next place, assuming the beam EG, Fig. 169, as fixed at G, which will then be the centre of moments, the load on each half stick between A and G is 33,000 pounds, or per foot of length 3300 pounds. The total upward pressure balancing this must also be 33,000 pounds, but distributed over a length $EG = EA + 10 = y + 10$, giving a load per unit of length equal to $\frac{33,000}{y + 10}$. The difference between the moments of these forces with respect to G

must equal the moment of resistance of the beam at G; hence $\frac{1}{2} \times 33{,}000\ (y + 10) - 33{,}000 \times \frac{1}{2}(10) = nfbd^2 = 288{,}000$ inch-pounds $= 24{,}000$ foot-pounds. Hence $16{,}500y = 24{,}000$; $\therefore y = 1.46$ feet $= EA$.

This method of construction is now very common for high buildings, and is of special value when two or more courses of iron beams or rails are used, resting on and imbedded in concrete. It is evident that as the loads rest directly on a definite area—usually square in form—that at least two courses or layers are required in order to secure a symmetrical distribution of load over the foundation-bed.

If, then, a column with a base a^2 in area, length on each side a, be supported on a layer of beams of any form of cross-section having for the projecting length y, and W the total weight, uniformly distributed over the area a^2, W also representing the total reaction or resistance developed in the material upon which the beams rest, which may or may not be the full bearing resistance of the material; and M the maximum bending moment under the assumptions made. Then if the iron pedestal or base-plate of the column is sufficiently strong and rigid, we may consider that the projecting beams are fixed at the outer edges of the base-plate. The beams may then be considered as in the condition of cantilevers acted upon by a uniformly distributed upward pressure per unit of length equal to W divided by the total length $= \frac{W}{2y + a}$, that is, for all of the beams together; and since the projecting length is y, the total load will be $\frac{Wy}{2y + a}$; and for a uniformly distributed load the total maximum moment at the edge of the base-plate is

$$M = \frac{Wy}{2y + a} \times \frac{y}{2} = \frac{Wy^2}{2(2y + a)}. \quad . \quad . \quad . \quad (275\tfrac{1}{2})$$

As both M and y are yet unknown, it will be simpler to take y and a in feet, W in pounds, and consequently M will be in foot-pounds.

If now we determine upon any form of cross-section of beam, its moment of inertia can be found, and the value of y_1, the distance of its extreme upper or lower fibre from the neutral axis

(assumed in the preceding example to be one half the depth). Then the moment of resistance, with a fibre strain f, either ultimate or working strain, will be $M_0 = \frac{fI}{y_1}$ in inch-pounds or $M_0 = \frac{fI}{12y_1}$ in foot-pounds. (See eq. (134).)

Theoretically, $\frac{M'}{Mo} = n$, the number of beams in one layer. The number of beams of any given width of flange will always be known by the numerical value of a, when the spacing centre to centre of beams is known or assumed. We then have

$$n = \frac{Wy^2}{2(2y+a)} \div \frac{fI}{12y_1}. \quad . \quad . \quad . \quad . \quad . \quad (275\ A)$$

The problem can be solved numerically either by finding M and M'_0 separately, substituting the proper values in the above equations, and then dividing M' by M_0 in order to find the number of beams required, or by assuming the number of beams to be used, and substituting in eq. (275A).

In the first case we must assume either M or y. In the second case we find y directly. In either case we can find I by assuming the other unknown quantities.

The value of the moment of inertia can be calculated by rules and formulæ already given, or they can be obtained from the published books of the manufacturers of standard beams; that of f may be taken as high as 20,000 pounds per square inch. As the beams are usually well bedded in concrete, the strength of any beam or all together are greatly increased over that for the same beam or beams not supported laterally. Some comparisons of the relative strength will be given in a subsequent paragraph.

Having in the preceding paragraph taken moments with respect to the outer edges of the base-plate, considering this as the fixed end of a cantilever beam, it may in some cases be considered better and safer to take the centre of the beam as the fixed point. In this case, calling l the entire length of one beam, the uniformed upward pressure will be $\frac{W}{l}$ per linear foot, and its moment with respect to the centre will $\frac{1}{2}l\frac{W}{l} \times \frac{1}{4}l = \frac{1}{8}Wl$. The downward press-

ure over one half the distance a is $\frac{W}{a} \times \frac{1}{2}a = \frac{1}{2}W$, its moment with respect to the centre is $\frac{1}{2}W \times \frac{1}{4}a = \frac{1}{8}Wa$, and the resulting bending moment is

$$M = \tfrac{1}{8}W(l - a), \quad . \quad . \quad . \quad . \quad . \quad . \quad (275\tfrac{3}{4})$$

l being the same as $2y + a$.

Having thus determined the projecting length of the upper layer, it is only necessary to increase the value of W by the weight of the layer of beams and concreting filling; change a^2 to the area covered by the first layer, which will now be a rectangle equal al; and, using l for a in the preceding equations, the length of the second or lower layer will be, as before, $2y + a$. Make these changes, and find any one of the unknowns desired, as before.

These are the general equations applicable to number of layers of beams, dimensions, and form of sections. The general construction is well illustrated in Figs. 181½, paragraph 399¾.

In such cases it is required to know or compute some or all of the following quantities:

(1) The projecting length of beam or beams admissible with any given dimensions of cross-section, and maximum limit of fibre stress, or deflection; (2) the proper dimensions of cross-section with a given projecting length; (3) the bending moments under any conditions of loading, and (4) the moment of resistance of one or any number of beams.

The general formulæ can be applied to the examples already worked out, the moment of inertia I being $\frac{1}{12}d^4$, y_1 in the value of the moment of resistance being $\frac{1}{2}d$, and $f = 1000$ pounds. To apply the general equations 275½ and 275¾ to beams of any form of cross-section it is only necessary to find the value of I from the principles and formulæ fully discussed in Art. XXXIII. For determining moments of inertia, the appropriate values of y_1 are found in the same article. The value of f varies with the material, and may usually be taken at 15,000 pounds for iron and 20,000 pounds for steel. These substituted in $M_0 = \frac{fI}{y_1}$ give the moment of resistance for one beam, and equated to M of equations 275½ and 275¾ and solved will give the projecting length y of the beams.

In Fig. 170 is shown a common construction. The base *AD* is first enlarged by two offset courses of masonry, each 1 foot, and then by projecting timbers, as before. If a further spread of base is required, several courses of timber could be used and arranged as indicated in Fig. 171 (*a*). There is, however, an unnecessary waste

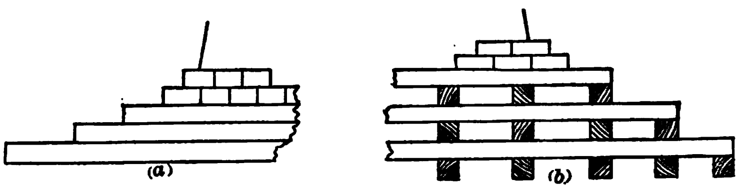


FIG. 171.

of timber in this case. A better construction is shown in Fig. 171 (*b*). This constitutes a crib or grillage, the open spaces of which are filled with gravel, broken stone, or concrete, according to the importance and weight of the superstructure. Such combinations introduce uncertainties as to the exact distribution of the loads, but within small limits for extreme fibre strains the principles above established and the results obtained will provide ample security against undue deflection.

389. If the soil is so soft that the several methods of spreading the base above described are not capable of maintaining the unit pressure within the proper limits, piles may be then driven through the soft stratum to an underlying harder stratum, if any such is to be found within a distance of 50 to 100 feet below the surface. If not, the piles can be driven to these depths in the soft soil. The loads they are capable of bearing will depend upon which of these conditions exist. The piles are then cut off at a convenient depth below the surface and capped with timber, usually 12 × 12 inches in cross-section, upon which a solid floor of timber is placed. Concrete may be placed and rammed around and under the timber, or the timber may be dispensed with and a good layer of concrete may be placed around and over the piles.

These two cases are shown in Figs. 172, 173, and 174.

In Fig. 172 is shown part of a plan for both cases. In Fig. 173 is shown vertical section on *AB* with timber platform, and in Fig. 174 vertical section on *CD*. Fig. 175 shows vertical section on *CD*

when no timber is used, two or three feet of concrete alone being placed over the piles.

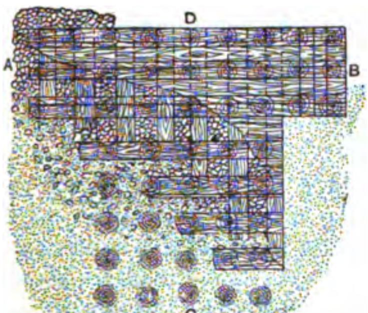
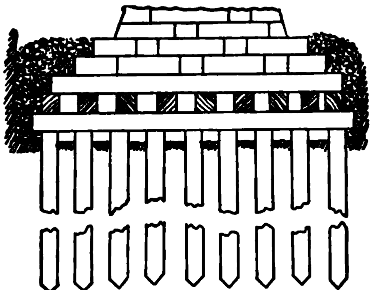
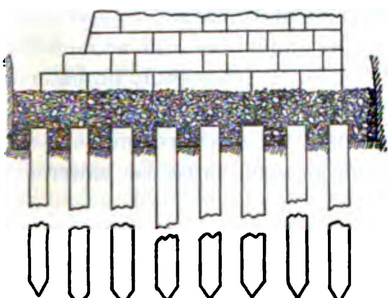


FIG. 172. PLAN



SECTION ON A.B.
FIG. 173.



SECTION ON C.D.
WITHOUT TIMBER PLATFORM
FIG. 175.

SECTION ON C.D.
FIG. 174.

390. There is some difference of opinion amongst engineers as to which of these plans is the better. The writer has used both methods, both on land and under water. Where the timber platform was used on land, even if all the timber was supposed to be below the plane of constant moisture, he has always piled the concrete around and over all timber as shown in Fig. 174, as the moisture line may be lowered either permanently or temporarily by natural causes, or artificial means such as drainage-works, sinking wells, and the like, in which case the timber would rot. When imbedded in concrete its life will certainly be prolonged, and it is believed to be effectual against decay equally with submersion in water. Concrete without timber is unquestionably better, except from one consideration, which seems to have some importance,

namely, the concrete resting partly on the unyielding pile and partly on the natural and more yielding earthy material between and surrounding the piles, there is danger of the solid concrete immediately over the pile separating from the remaining portion, and thereby subjecting a short column of concrete over the pile to an excessive crushing pressure; but as this column of concrete is confined and cannot possibly be displaced, this objection seems more fanciful than real.

In addition, where concrete is used the entire weight is not thrown on the piles, the surrounding material bearing a part of the load, which is not the case where timber caps and platforms are used without concrete. This additional resistance, though not usually counted on, as the piles are intended to carry the entire load, nevertheless exists, and is a factor of safety. All things considered, the possible rotting of the timber both of platform and tops of piles, better -- the better the writer considers concrete alone over and around the piles practice, and has latterly adopted it. It is probably immaterial which plan is used when under-water foundations are constructed. But even there timber may not rot, while it does become very soft and its resistance to transverse compression must be materially reduced.

391. In building piers or abutments on or near the banks of streams one of the above plans should be adopted, even when the soil is firm enough to bear safely the load. The piles in this case are not needed to carry the load, but are necessary to provide against a scouring out of the material from under the pier or abutment; and if this takes place, as is often the case—notably along the Ohio River, where in many places the banks have been washed out, in the memory of men now living, for several hundred feet back from the former shore-line—the piles prevent the collapse of the pier until riprap can be deposited under it and around the piles. As has been stated, a safe load for piles is from 15 to 25 tons per pile, depending on the nature of the earth and the depths to which they are driven in it. The theory of the bearing-power of piles will be discussed under the head of Piles.

392. Even where the heavy walls of houses and piers for land structures cannot be so supported by any of the above methods without undue settling, the piles must be driven to the underlying rock. In this case the piles cannot sink vertically, and can carry

loads even approaching the crushing resistance of the timber; yet the soil may be so soft around them that the whole pier may lean or careen sideways. This may be prevented by the liberal use of gravel or broken stone thrown around near their tops in excavations made for the purpose. But cases may arise where driving piles may be unadvisable, either endangering the stability of adjacent structures or from other causes. In such cases shafts can be sunk through the earthy materials to the rock. These shafts may be timber or iron or masonry lined, and are commonly called *wells*. Their cross-sections may be rectangular, square, or circular. Their sides or diameters may be from only three or four feet to ten, fifteen, or twenty feet. If timber-lined, they would rarely be over five or six feet in diameter, or on the sides. As the sinking of iron or masonry wells will be considered under the head of Foundations in Water, timber-lined shafts will be alone considered in this place.

393. The operation of sinking shafts is the same with those of rectangular and circular sections, consisting in either case of strong frames set at vertical intervals of from 3 to 6 feet, according to the character of the earthy material, in a pit or shaft excavated in it. Behind or outside of these frames short planks or poling-boards are placed or driven. These frames and boards support the sides of the shaft, and prevent caving or bulging of the material. There are three classes of material, each of which requires different methods of construction and precautions: (1) Firm soil, which has sufficient adhesiveness and frictional stability to stand for a time with a vertical face, allowing the excavation to be made first and the lining inserted afterwards. This presents few difficulties. (2) Material which will bulge or cave, not admitting of an excavation with vertical faces, in which the lining must keep pace with the excavation. (3) Where the material is a quicksand and flows almost as freely as water, and at the same time exerts a greater pressure on the lining than water. This case presents the greatest difficulties, and frequently baffles the skill of the best engineer, requiring special methods and precautions.

394. In the first case a large frame is placed over the site enclosing the area of the shaft. This frame has projecting pieces which are set in trenches excavated for the purpose. This frame is shown in Fig. 176. The cross-sections of these pieces vary from

6 × 6 inches to 12 × 12 inches, depending on the size and depth of the shaft, as the frame is intended to hold the frames below from slipping down. It is aided, however, by side friction of the lower frames. Having set this frame in position the excavation is commenced in the enclosed area and carried down to a depth of 6 or 8 feet if the material will admit of it. A square or circular frame is

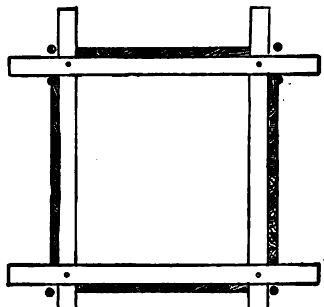


FIG. 176.

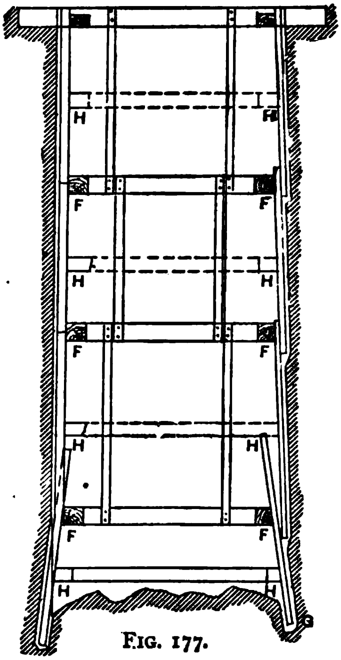


FIG. 177.

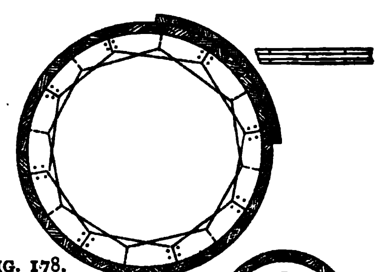


FIG. 178.

FIG. 179.

then placed at the bottom of the excavation somewhat smaller than the excavation. Planks from 2 to 3 inches in thickness are then driven or placed behind both frames and resting against them. The lower frame need not be usually composed of timbers over from 4 to 6 inches square in cross-section. The outer faces of these frames should be set accurately in the same vertical planes. This should be done throughout the entire depth of the shaft. The excavation should then be carried down 5 or 6 feet more, another

frame set at the bottom, and other poling-boards placed or driven outside of this frame and the one above. This is indicated in Fig. 177, which shows an interior view or projection when one half of the shaft is conceived to be removed along a vertical plane. The poling-boards can simply be placed by inclining them somewhat and inserting them behind the frames, and then bringing them to a vertical position. In this case their lengths must be such that they will only reach from centre to centre of the pieces forming the frames, as two sets have to bear upon the same frame excepting the top frame; or the pieces can be driven behind any two frames, and their lower ends kept two or three inches from the lower frame of the two so as to allow another set of boards to be driven between them and the lower frame. Both of these conditions are shown in Fig. 177. The frames can be suspended the one from the other by battens or planks.

In Fig. 178 is shown a timber frame for a cylindrical shaft exhibiting in part both methods of using the poling-boards; and Fig. 179 shows an iron or steel hoop made of angle-iron, to be used instead of the timber frame.

395. In soft materials similar frames and poling-boards are used. The difference is only one of procedure, as the excavation cannot be made first. The sheeting *A*, Fig. 177, is driven ahead of the excavation and behind the frame *F* to a point *G*. The material is excavated to this point. In very soft and flowing materials after excavating about two feet below frame *F* the sheeting will be in danger of being pressed inward. It then becomes necessary to insert a frame *H* to hold the sheeting. This may remain permanently or not. These frames are shown by dotted lines. The excavation is proceeded with and the sheeting driven ahead at the same time. Having reached a depth of three or four feet a permanent and regular frame is put in place. The work proceeds in this manner, always keeping the sheeting ahead of the excavation. This method is shown near bottom of Fig. 177.

With care there are no very great difficulties to be overcome in sinking shafts through soft soils. The frames and the sheeting-planks should be driven closer together. This is not necessary in the firmer soils, two or three planks on a side being sufficient to prevent caving in of the material.

In many soils, whether clayey or gravelly, a certain amount of

water will be forced out of the surrounding soil and finding an avenue of escape, will collect in the bottom of the shaft. In such cases a sump or small pit must be excavated in advance of the bottom of the shaft, into which the water may flow; this is then pumped or lifted out in buckets. It would be well to keep at all times a bucket in the sump, as it will prevent the water mixing with the soil and forming mud, increasing the expense, delaying the work, and not infrequently endangering its success. When much water flows in, constant pumping may be necessary.

396. In the third case, where quicksand is encountered, it becomes a matter of great difficulty to carry forward the excavation. The pressure is much greater than that of water. It is difficult to keep the frames and sheeting in line and position, and not infrequently every effort to hold back the material fails, and the shaft is crushed or broken in by the great pressure of the flowing material aided by the caving in of the overlying strata; if the shaft is not entirely destroyed, it will fill up to a greater or less height above the bottom; great labor and expense will be required to remove the material, and often it will be impracticable, necessitating the abandonment of the work. No enterprise of this character should be undertaken without borings to determine the character of the different strata to be excavated. If, then, it is known, as it should be, that such a layer of quicksand is to be met with, the shaft should be commenced two to four feet larger in each dimension than actually required for the purposes of the work, and the sides of the lining should be strongly braced all the way down. Before reaching the layer of quicksand a double-wall lining should be constructed with a height of 5 to 6 feet, having a cutting edge at bottom; this should be strongly braced between the walls, and filled with clay or sand. Having set this in place beneath the lining the excavation can be commenced, only cutting, however, trenches immediately under the cutting edge, leaving a core of material in the middle of the excavation, if practicable; this can be shored by proper bearing against the lining above, or a platform a little smaller than the shaft may be held down with braces. As the excavation proceeds the double-wall lining will gradually settle. If the layer of quicksand is not over 10 or 15 feet in thickness, the double-wall lining can be made of sufficient height to sink entirely through, reaching below its bottom and extending above its upper

surface. The upper surface of the core can be gradually removed to facilitate the sinking of the lining. If the quicksand is very fluid, it may begin to rise between the wall and the earth core; but having only a narrow passageway, the lining may sink gradually as the material rises on the inside. If it flows in rapidly, the lining may sink proportionately rapidly. If necessary, long iron bars may be used to keep a passageway open for the quicksand to flow through. The sinking may be facilitated by constructing a platform on top of the movable lining, adding weights, and prying

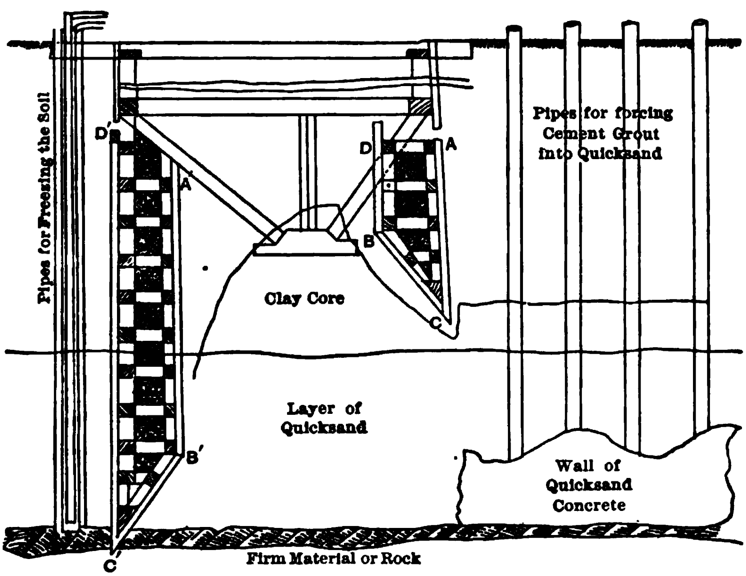


FIG. 180.

down with levers. Care should be taken to fill in between the top of the movable lining and the bottom of the fixed lining above with props, bracing, and sheeting; additional sand or clay, or even broken stone and gravel, being used to fill the vacant space, and by increasing the weight increasing the rate of sinking. By these means a considerable depth of quicksand may be passed through safely. At any rate, a sudden inflow of a large volume of the material will be prevented; the walls are not likely to be crushed in; and in all probability the work will be safely carried through

the most troublesome material with which the engineer has to contend. If by this method failure ensues, the engineer will have the satisfaction of knowing that the same result has been met with by many other good engineers before. This method is shown in Fig. 180. The drawing illustrates clearly the above description. On the right the movable lining *ABCD* is shown just before entering the quicksand and on the left as having passed through it, and the upper sections built up to the lining above, *A'B'C''D'*.

397. By a recently patented process, a series of pipes can be sunk into the layer of quicksand, and through each alternate one a cement grout is forced under pressure. This, seeking an escape by the line of least resistance will make an exit by the adjoining pipe, which opens into the air above; but in so doing the pressure closes a valve at the bottom of the pipe, and results in a diffusion of the grout in the surrounding quicksand which forms with it an artificial stone, and by gradually raising the pipes a wall of stone is formed in the layer of quicksand. This is indicated by the pipes on the right of the shaft, which are shown as partly lifted, leaving a solid wall below. This is Harris' patent process.

Another and somewhat similar method is to force cement powder through pipes into the quicksand by means of compressed air. The effect is similar to the above.

397*a*. The Poetsch-Sooysmith Freezing process is to sink a series of pipes 10 inches in diameter through the earth to the rock; these are sunk in a circle around the proposed shaft. Inside of these 8-inch pipes closed at the bottom are placed, and inside of these pipes are inserted smaller pipes open at the bottom. Each set of smaller pipes are connected in a series. A freezing mixture is then allowed to flow downwards through one set of the smaller pipes and return upwards through the other. This freezing mixture flows from a tank above, giving sufficient head of pressure to cause the fluid to flow with the desired velocity through the pipes. The effect of this is to freeze the earth into a solid wall. This is also a patented process. By either of these means the excavation can be carried on either through the frozen material or within the frozen walls. The shaft is usually lined as shown in Fig. 177. Both of these processes have been sufficiently tested to be accepted as effective, and when the importance of the work justifies the expense necessarily incurred, can be recommended. The arrangement

of the pipes in the Poetsch-Sooysmith process is shown on the left of the shaft, Fig. 180. The principle involved is the same as that employed in the usual refrigerating-machines. This process is also recommended for constructing the foundations of piers, but has not been used on such a large scale, so far as the author is informed.

In whatever manner the shaft has been sunk, after reaching the rock or other hard material the shaft is filled with concrete, rubble, or brick masonry. Under large piers several such shafts are sunk, their tops are connected by masonry arches or strong iron beams, and upon these the masonry of the pier or abutment is commenced.

398. Mr. Rankine's theory of earth-pressure, which will be discussed under the head of Retaining-walls, seems to the writer to be the most plausible, simple, and easy of application of the many theories advanced; and as it is universally admitted that that none of them can be relied upon to give accurate results under all conditions, the writer has determined, after careful study, to adopt Mr. Rankine's theory.

Mr. Rankine's theory is based upon the supposition that in all earthy materials the intensity of the pressure at a point below the surface of the ground is proportional to the specific gravity of the material, the slope of the ground surface, the natural slope of the material or its angle of repose, and the depth of the point below the surface of the ground; and that the direction of the resultant pressure upon any surface upon which it presses is parallel to the surface of the ground.

All of the above assumptions correspond with the well-determined conditions of fluid or water pressure, except the direction of the pressure, which in a fluid is always normal to the surface pressed. When this surface is vertical, all conditions are similar in the two cases. When a material flows freely unsupported, as is the case with quicksand, soft swampy silt, and soft clay mud, it seems clear that its condition of stability is similar in every respect to that of a fluid, and that without sensible error we may deal with these materials as with a fluid having its own special specific gravity. In all but the softest varieties, however, there does exist a sensible degree of frictional stability, as evidenced by our being able to make excavations, and by the fact that the sides

will stand at least for a considerable time with some slope. The formulæ for fluid pressure must be modified by the introduction of some term which depends upon the frictional resistance of the material, usually represented by the letter ϕ. Assuming, then, that the upper surface of this material is horizontal, the surface slope, usually represented by the letter θ, disappears; that is, $\theta = 0$. In a perfect fluid the pressure in all directions around a point in the interior of the mass must be equal, or movement of its particles must occur. In a material, however, with a sensible angle of repose this equality does not exist. The vertical intensity of pressure may be greater or less than the horizontal intensity, depending on the value of ϕ. If, then, p represents the intensity of the vertical pressure at any point and h the intensity of the horizontal pressure at the same point, $h = p \times n\varphi$. Mr. Rankine finds $n\phi$ for a horizontal ground surface to be $n\phi = \frac{1 - \sin\phi}{1 + \sin\phi}$ or $\frac{1 + \sin\phi}{1 - \sin\phi}$.

There is therefore a maximum and a minimum value for the intensity of the horizontal pressure: the one is the least consistent with stability, and the other is the greatest. The maximum horizontal intensity is, then, $h' = p\frac{1 + \sin\phi}{1 - \sin\phi}$, and the greatest vertical pressure consistent with this value of h' is

$$p' = h'\frac{1 + \sin\phi}{1 - \sin\phi} = p\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2. \quad . \quad . \quad . \quad (276)$$

Let $ABCD$ be the volume of a masonry wall, its length being unity (one foot) in a direction perpendicular to the plane of the paper (see Fig. 181). Both the horizontal and vertical intensities (of the *conjugate pressures:* see Retaining-walls) being zero at the surface AD, and assumed to vary uniformly and proportionally to any depth, say at the point E. It is only necessary to use the intensities, that is, the weights of columns with a base of unity in area (one square foot usually), and not the total weights involved. Let $AE = x$, the depth excavated into the soft material of the trench $GHLK$; $AB = y$, the height of

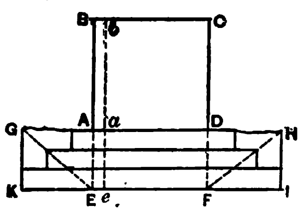


FIG. 181.

the proposed structure; $BE = x + y$, the total height of structure $ABCD$, and that of its foundation $ADFE$, required, by the nature of the soil, to secure stability; w = weight of a cubic foot of the soil; and w' = weight of a cubic foot of the masonry. Then equation (276), since $p' = w'(x+y)$ and $p = wx$, becomes

$$w'(x + y) = wx\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2. \quad . \quad . \quad . \quad . \quad (277)$$

The first member represents the weight of the volume, $BbeE$, having for its base unity (1 square foot), and the second represents the weight of the volume excavated, $AaeE$, also with an area of base of unity, multiplied by a factor depending upon the angle of repose of the material. We have already seen an application of this equation to determine the bearing power of a soft clay in paragraph 374, the result agreeing fairly well with that obtained by experiment.

399. Instead of dealing with intensities, we will assume that the wall in Fig. 181 is 50 feet long, its thickness AD is 6 feet; hence total weight of excavated material $AEFD = 50 \times 6 \times wx = 50 \times 6 \times 110 \times 10 = 330{,}000$ pounds. The total weight of the wall = area $BEFC \times 50 \times w'(x + y) \lesseqgtr 330{,}000 \times \left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2$. If the material is so soft that the angle of repose $\phi = GEK = 10°$, then $\sin \phi = 0.1736$ and $\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2 = 2.0164$; hence total weight of structure $BEFC$ must not exceed $330{,}000 \times 2.0164 = 665{,}412$ pounds. As the area of the base is $50 \times 6 = 300$ square feet, the weight per square foot of base = 2218 pounds. It would hardly be safe practice to load this material so heavily. Since the angle of repose $GEK = \phi = 10°$, $KE = GK \text{ cotan } \phi = 10 \times 5.67 = 56.7$ feet, which would make the total width $KL = 6 + 2 \times 56.7 = 119.4$ feet. Mr. Rankine suggests this spread of base. While this might not be excessive for an earthen embankment of considerable height, say 20 feet, as seems to be contemplated by him, such an embankment would require at any rate $14 + 60 = 74$ feet, and the additional spread of $119.4 - 74 = 45.4$ feet or 22.7 on each side could be secured by throwing over the space large quantities of broken stone, or by building cribs or layers of logs over the required

width, or simply flattening the slopes of the embankment so as to cover the desired width. This is indicated by the dotted lines *GEFH* in Fig. 181.

Practically, this is done by simply piling the ordinary earth along the line of the road, and continuing this until settlement ceases. But a material having an angle of repose of 10° should be capable of bearing at least 1000 pounds per square foot. This would require a little more than doubling the original area of base, increasing the area from 50×6 to about $56 \times 12 = 672$ square feet, or even a greater spread might be necessary. In such material, however, it would seem advisable to drive piles to carry important structures, such as house-walls, masonry piers, and abutments; yet there are many instances of trenches being excavated and filled with sand, gravel, broken stone, and concrete, upon which heavy and important structures have been built.

399½. A modification of the Poetsch-Sooysmith Freezing Process has lately been introduced. The pressure in the circulating pipes being greater than the pressure from without, may give rise to leaks. To avoid this, instead of using a cold brine to remove the heat from the soil, and thereby freeze it for a certain distance around each pipe, the new method consists in introducing an ammoniacal liquid directly into the pipes. This is evaporated in the space between the inner and outer circulating pipes, and the vapor is drawn off, recondensed, and used again. In this manner the pressure inside the pipes can be maintained at any desired degree.

This method has not yet been sufficiently tested to compare its merits with the ordinary method; and to what extent the high pressure of the cold brine is disadvantageous has not been clearly demonstrated.

399¾. *Foundations for Machinery.*—In preparing foundations for machinery it is necessary to provide against the injurious effects of vibrations, either to the foundation itself or to those of neighboring walls. Even very slight vibrations, if long continued, will inevitably loosen or cause the disintegration of masonry.

Concrete is probably the best material for the foundation proper after the foundation-bed is prepared.

Foundation-beds.—A solid rock bed transmits readily and freely vibrations communicated to it; so also does firm, compact earth,

such as clay and hard-pan. Sand and gravel, soft or loose earth, and silt, timber, mineral wool, hair-felt, or asphaltic concrete transmit vibrations feebly.

When founding on rock, a layer of 12 or more inches of asphaltic concrete should be laid on the rock; or a 6-inch layer of hair-felt or mineral wool may be laid in a trench cut in the rock; or a solid course of 12-inch timber—creosoted or vulcanized yellow pine—answers well. Finally, a layer of sand two or more feet in depth may be used. On any of these beds is placed a layer of concrete, on which large stone slabs or a timber floor may be placed.

Where sand is used the drainage of the pit must be thorough; otherwise it will be kept filled with water.

Asphaltic concrete softens and runs at 130° of temperature, and should therefore be surrounded with some non-conducting material, such as porous terra-cotta, mineral wool, or sand.

On firm earths an excavation should be made at least 2 feet below the intended base. Upon this lay the foundation proper and fill around with sand. Or the foundation may rest on a crib of timbers, filling around with sand.

On sand it is only necessary to keep the foundation 2 to 5 feet away from any other foundations. It is well to follow this rule with any machinery foundation.

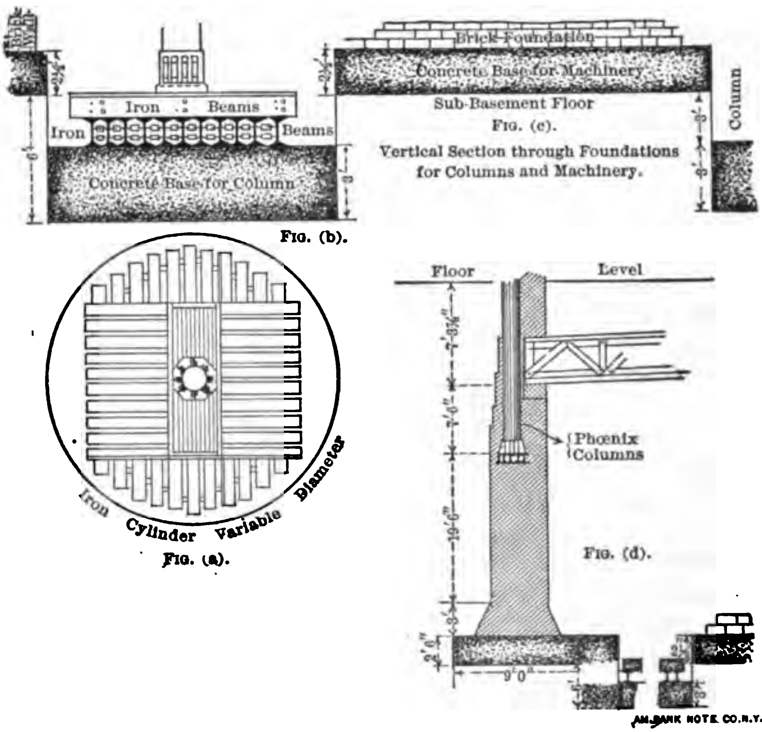
Changing the speed of an engine will sometimes greatly reduce the effects of vibrations. The knocking of pumps when working irregularly causes trouble. In such cases the pipes should be either kept entirely free from the foundation, or the vibrations deadened by a packing of felt, rubber, or leather.

Foundations for the Main Power-station, Broadway Cable Railway.—In order to prevent the communication of vibrations caused by the motion of the powerful machinery used to the walls and the floors above the engine-room, it was necessary to construct the supports of the building and those of the machinery on entirely different foundations.

The building is nine stories high above the street-level, and fronts 125 and 200 feet. The power machinery is placed in the basement, and below this, at a depth of 42 feet below the street-level, is a sub-basement.

The main walls of the building rest on a layer of concrete. These walls are of brick, and at a height of 25 feet they enclose a

series of 29 Phœnix columns resting on a grillage of steel I beams, as shown in Figs. 181½ (*a* and *b*). For the support of the interior walls of the building there are 45 Phœnix columns 38 feet 2 inches high, including base. These are located at proper intervals, and in rows. These interior columns rest upon a double course of steel I beams, forming a grillage enclosed in steel cylinders 6 feet high, with diameters varying from 4 to 12 feet. These cylinders are



FIGS. 181½.

sunk below the sub-basement floor, and are about one-half filled with concrete, upon which the grillage rests. Between the cylinders a bed of concrete about 2½ feet thick is laid, which forms the sub-basement floor. Upon this latter bed of concrete brick piers are built, which constitute the foundation for the machinery.

These column and machinery foundations are shown in vertical section in Figs. (*b*) and (*c*), and a plan of the grillage foundation for the columns is shown in Fig. (*a*). As seen, this construction separates entirely the foundations of the machinery from the deeper foundations of the columns which support the floors above. Consequently the vibrations due to the working of the machinery do not reach the foundations of the columns, as both earth and concrete act as cushions and resist the transmission of the vibrations.

To prevent any interference with or obstruction to the safe and successful construction of the foundations at such a great depth below the normal water surface a series of 30 wells, of 2-inch tubes, in two rows were sunk to a depth of 10 to 12 feet below the sub-basement floor, or from 5 to 6 feet below the foundation-bed of the columns. The two rows were 6 feet apart and the wells in each row 12 feet apart. The water was removed from these by a couple of Smith & Vaile pumps. Beams were extended from the brick walls over the space occupied by the foundations of the column wherever necessary to secure a support between the brick piers.

The allowable projection for the I beams has been fully discussed in paragraph 388. The general equations (275½) and (275¾) can be applied to the beams in Figs. 181½, in the manner already fully explained. Another method is to excavate a pit 10 to 12 feet deep, in which a solid floor of concrete is laid, and the sides of the pit built up with masonry walls. On the concrete is laid a thick sheet of felt and then one of lead; on this a block of brick masonry (not in contact with the side walls); and on this was placed the bed-plate of the engines and dynamos, and anchored down.

Deep Borings.—There is a boring at Paruschowitz, West Silesia, which has at this date reached the depth of 6568 feet. After carefully ascertaining the temperatures at various depths, it is the intention to continue it, if possible, to a depth of 8200 feet. The boring is done by means of the Mannesmann tubes. These consist of a series of tubes in lengths, each tube having an exterior diameter equal to the interior diameter of the one above it. Each length of tube is provided with a diamond-cutting edge. When a hole is bored equal, or nearly so, to the length of the larger tube, then the next smaller is connected on, which also has a diamond-cutting edge, and another section drilled out. There is provided a special device for cutting off the core at the bottom. In case of the boring above

mentioned, the top or largest diameter of the hole is 11.8 inches, and at th depth thus far reached the bottom diameter is $2\frac{3}{4}$ inches.

PILE-DRIVING AND FOUNDATIONS ON PILES.

400. Piles are either round or square sticks of timber of any length desired from 10 to 100 feet. They are usually smaller at one end than at the other. They can be obtained from 12 to 24 inches in diameter at the large end, and from 9 to 12 or more at the small end. They are usually driven small end downwards, sometimes the reverse.

The top should be sawed square to the axis of the pile. The small end may or may not be trimmed to a point. In soft materials it is best not to sharpen the piles. In hard, firm soils they are usually pointed, and often shod with iron.

401. The machine for driving piles is called a pile-driver, and consists essentially of two upright guides or leads, often of great height, erected on a platform, or on a barge when used in water. These guides serve to hold the pile vertical while being driven, and also hold and guide the hammer used in driving. This is a heavy block of iron weighing anywhere from 800 to 4000 pounds; average weight from 2000 to 3000 pounds. All else connected with a driver are simply means of holding the leads in place, of holding the piles in the leads, of lifting and allowing the hammer to fall freely on the head of the piles, and of preventing unnecessary bruising and splitting the piles. Piles can be driven either vertically or inclined to the vertical; for most purposes they are driven vertically.

402. Short piles, not over 15 feet in length, are often driven simply to compact a loose, soft soil, thereby increasing its bearing-power. This is mainly used for house-walls. In this case the piles are driven in a trench in rows of one, two, or more; the trench is then filled with sand, broken stone, or concrete, over and around the piles, upon which the walls are built. Short piles have been driven and then withdrawn, and the holes thus made filled with sand. This plan should certainly be adopted unless there is positive assurance that the piles will be kept constantly wet. Small drivers, and light hammers commonly lifted by hand-power, are used for this purpose.

403. Long piles are driven not so much to compact the soil as to afford direct support to the load. In this case the soil is not commonly supposed to carry any portion of the load directly, but to support the pile by direct-bearing resistance at its lower end and by friction on its sides; these assumptions in either case being on the safe side, as usually the load will be supported partly by the pile and partly by the soil.

Piles may be driven into any earthy material. In hard and firm materials it is not necessary, so far as support is concerned, except under very heavy loads, but is necessary to prevent the collapse of the structure by the scouring out of the material, and the depths to which they are driven is mainly regulated by the depth to which scouring may take place.

Or it may be necessary to provide a waterway to carry the excessive floods on many rivers, the water extending over a considerable area on each side of the stream. Embankments in this case would be destroyed, and as there is always more or less danger of scour occurring, structures called pile-trestles are commonly built.

404. When piles are driven into a firm material it may be safely said that the piles will carry any reasonable loads. A penetration of 20 to 30 feet is generally ample, and about as much hammering as the piles can stand will be required to reach these depths.

Firm materials include compact sand, gravel, and clay. In some sands and soft clays piles can be driven to depths of 40 to 50 feet. Piles thus driven will carry safely from 20 to 30 tons per pile. If driven through to rock, hardpan, or other very hard material, the pile becomes a timber column, and will carry safely from 50 to 70 tons. Such excessive loads are seldom necessary and never desirable, as the piles may be defective and injured in driving to an unknown degree. It is much better to increase the number of piles, driving them $2\frac{1}{2}$ foot centres, when necessary. This is about as close together as piles can or should be driven.

405. Assuming the load on the foundation as given in paragraph 384, $W = 15,922,000$ pounds, and a base of 46×69 feet, we could have 29 rows of 19 piles each, making 551 piles, and only 28,888 pounds per pile, or something less than 13 tons; or, if driven 4 feet centres, 18 rows of 14 piles each, making 252 piles, and giving a load per pile of about 60,000 pounds, or less than 30 tons per pile.

Piles should rarely be over 4 feet centres or less than 2 to 2½ feet. With either of these the loads are within safe limits.

The same conditions hold with piles driven through very soft materials to rock, if precautions are taken by excavating to a certain depth below their tops and filling the trench with gravel, broken stone, or concrete to prevent lateral movement. In any of the above cases there is little difficulty in securing ample support for the piles. The great danger arises from excessive hammering on the piles, by which they are crippled, broken, or split below the ground, rendering them unfit for any purpose, in order to comply with some arbitrary rule that the penetration at the last blow of a hammer weighing 2500 pounds and falling 30 feet shall not exceed an eighth or a quarter of an inch. No such rule should be followed; as above stated, it may result in absolute injury to the pile.

406. Piles have been driven for many important purposes into soft, silty, and marshy soils, and penetrating to 60, 80, or even 100 feet, without reaching firm soil of any kind. Such piles carry safely from 10 to 25 tons when driven from 38 to 60 feet in the ground, the penetration at the last blow varying from 3 to 18 inches. Such facts have puzzled both theoretical and practical men; and those who have worked out formulæ for the bearing power of piles admit that when piles are driven in such materials, with light fall of hammer and several inches or even feet of penetration, it cannot be determined by such formulæ. Although possessing little practical value, the theoretical and empirical lines along which such formulæ have been evolved will be briefly described.

407. In any case of pile-driving the work done is expressed as a quantity by the product of the weight W of the hammer in pounds by the fall h in feet $= Wh$ foot-pounds of energy. This energy is expended in some manner during the penetration or set s of the pile; and if the mean resistance or bearing power is called P, then, considered purely as a question of dynamics, $P = \frac{Wh}{s}$; or, if h is in feet and s in inches, $P = \frac{12\,Wh}{s}$.

In this relation all questions concerning the elasticity and compressibility of hammer, pile, and soil; of relative weights of hammer

and pile; of frictional resistances and air resistances—have been eliminated. Some of these are of very little moment—such as the frictional resistance of air and in the guides of the driver, elasticity of hammer and pile. So far as the relative weights of the hammer and pile are concerned, it may simply be said that the hammer should always be as heavy as the pile; generally it is very much heavier.

With these causes of loss eliminated, the energy of the blow is absorbed in some or all of the following ways:

(1) In brooming the pile at head or point or at some intermediate point, or in all at the same time.

Brooming of the head is an enormous source of loss of energy, both directly and in cushioning the blow. Some brooming always occurs. This can be remedied by cutting the broomed part off, thereby presenting a firm, solid head to receive the blow. Consequently this need not enter into consideration, so far as a formula is concerned. Brooming of the pile at the point does not diminish the energy of the blow, but dissipates it without any useful result. No formula does or can provide against it.

(2) Bouncing simply means that all the energy of the blow cannot be utilized in work. It occurs while the pile refuses to penetrate, as when it reaches rock; or from the hammer being too light or its striking velocity too great, or both, to set the pile in motion before it reacts elastically with more force than the hammer is exerting to drive it down. This bouncing means so much absolutely wasted energy, because the energy escapes from the pile before it begins to move. The remedy is to diminish the fall. A slight bounce at the end of every blow should occur.

(3) In overcoming the inertia of the pile and the static grip of the earth upon it. This is not considered in the formula.

(4) In causing the pile to penetrate against the earth's resistance.

408. Therefore the only important point to be considered is the external resistance to motion of the pile, as the other questions are too insignificant to consider; are either self-compensating, or are not capable of being provided for in a formula; or, finally, can be eliminated, as in brooming by cutting the part off or in bouncing by reducing the fall or increasing the weight of the

hammer. These things granted, the general theory upon which Mr. A. M. Wellington's formula is deduced is briefly discussed below. Reasoning from the general laws of friction and the known nature of earth, there must be, at the moment of impact, a very large excess of external resistance OD, Fig. 182, due (1) to the excess in the coefficient of static friction, or of friction at low velocities over that at relatively high velocities; (2) to the setting of the earth around and into the irregular surface of the pile between the blows. This excessive friction at the

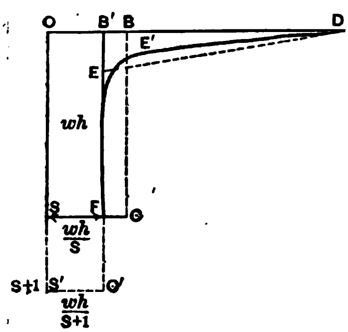


FIG. 182.

moment of impact decreases rapidly to some point E, where it will have a much reduced value $S'C'$, which will remain sensibly constant during the remainder of the set. Then the area of the irregular figure $ODEFSO$ represents the foot-pounds of energy expended $= Wh$, The nearest measure of the future bearing power of the pile will be the comparatively uniform frictional resistance to penetration SF in Fig. 182, after the excessive initial resistance has been overcome, since this excess is due in large part to the suddenness of the blow on the pile, and a smaller continuous pressure of force may overcome it; and, in addition, tremors, seepage of water, and yielding of the surrounding soil may overcome it. The problem, then, is to construct a rectangle

$$OS'C'B' = ODEFSO = OSFB' + DB'E = Wh.$$

To make this construction, the assumption is made (based entirely on observation of the behavior of piles in driving and on a study of the general laws of friction) that the decreasing excess resistance outlined by the line ED, and whose value in foot-pounds is expressed by the irregular area $B'ED$, is confined within the first inch of penetration, i.e., $B'E = 1$; and the initial excess $B'D = 3B'O = 3SF$. Then the triangle

$$B'FD = \tfrac{1}{2}B'D \times B'E = \tfrac{1}{2}(OB' \times 3) \times 1 = 1\tfrac{1}{2}OB'.$$

The irregular area $B'E'ED = \frac{2}{3}$ of the triangle $B'ED = \frac{2}{3}$ of $1\frac{1}{2}OB' = OB' \times 1$. Then, to determine the value of

$$OB' = SF = S'C',$$

we must add to the rectangle $OB'FS$ an area $= OB' \times 1$, which is done by making $SS' = 1$ and $S'C' = OB' = R$, the assumed maximum bearing power of the pile; whence $Wh = R \times (S + 1)$.

$$\therefore\ R = \frac{Wh}{s + 1}.$$

If h is in feet and S in inches,

$$R = \frac{12\,Wh}{s + 1}; \qquad (278)$$

and, assuming a factor of safety of 6,

$$R = \frac{2\,Wh}{s + 1}. \qquad (279)$$

This is commonly known as the *Engineering News* formula for the safe bearing power of piles.

Assuming weight of hammer 2500 pounds, fall 30 feet, penetration 3 inches, $R = \frac{2 \times 2500 \times 30}{3 + 1} = 37{,}500$ pounds is the value of the bearing power. The author of this formula claims that it will always give safe loads. He does not claim absolute accuracy, and recommends that the results be checked by actual experiment, when large numbers of piles are to be driven for important purposes, and where failure would be likely to cause great loss of property, or even of life. The writer has studied conditions and requirements in pile-driving, and in addition built many structures on piles driven in a great variety of soils, and while believing that no formula based simply upon the relations between weight and fall of hammer and the average penetration in the last few blows can be implicitly relied on under all conditions, yet he does not hesitate to commend the above as probably the most rational, simple in application, and, under usual conditions, as safe a guide as any other. There are many other formulæ used, but this alone is given as

developing approximately the actual mode of penetration of a pile when struck with a hammer.

409. There is one point that seems to be lost sight of in measuring the bearing power of piles, when actual bearing resistance and friction of the soil on the pile surface are alone considered, and that is the lateral displacement of the soil as the pile penetrates, either from a blow or steady pressure or load. Assuming a pile 9 inches in diameter at the bottom and 15 inches at the surface of the ground, the area at lower point is $\pi d^2 \div 4 = 3.1416 \times (9)^2 \div 4 = 63.6$ square inches, at top $= 3.1416 \times (15)^2 \div 4 = 176.72$ square inches, and increasing regularly between these limits. When a pile settles one foot, the soil must be pressed outwards for the whole length of the pile in order to make room for the continually increasing area. This suggests a reason for the observed fact that piles often settle under a load for a short distance and then stop. On the theory of the great reduction in frictional resistance after once moving, it would seem that there would be a great tendency to continue until some firm soil was reached but for this continual displacement and consequent check to the reduction in the frictional resistance by additional pressure developed.

410. The only other view that will be taken of this subject disregards entirely the consideration of the weight of hammer, fall, and penetration of the pile, and the manner in which the pile has been driven. It simply considers the condition of a stick of timber or shaft of iron imbedded to any depth in the material, whatever its nature, and whether driven, sunk by hand, or by means of the water-jet, and after such an interval of time that all disturbance has subsided, the material having closed in around the piles, and all conditions normal and permanent, until the pile actually moves or settles under its load, aided by tremors, seepage of water along its surface, or other similar causes.

It is well known that, regardless of all the sources of loss of energy discussed in the preceding paragraphs, the disturbed condition of the surrounding soil prevents the actual penetration during the process of driving from being a measure of the supporting power of the pile. Hence it is contemplated that the value of the penetration s in eq. (279) shall be that found after the pile has had an interval of rest anywhere from 1 to 24 hours in order that the new and permanent conditions be attained, and all broomed portions cut off, so that blows may be delivered on a firm and solid surface. These conditions to a great extent render the application

of such formulæ difficult, except on a few test piles of a group, as too much time would be lost. As it is, 8 to 20 piles driven in a day is good average work.

The pile being in place and under permanent conditions, regardlessly of how it got there, must have an inherent bearing power measured solely by the resistance or bearing power of the material at the lower end of the pile and the frictional resistance of the material on its exposed surface. If, then, P = bearing power of the pile, R = the bearing resistance of the soil per square foot, f = the frictional resistance per square foot between earth and timber, S = the exposed surface of the pile in square feet, and A the area of cross-section of the pile at its lower end, then

$$P = RA + Fs. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (280)$$

As f varies with the pressure, it would be greater as the depth of the pile in the soil increases. Under any circumstances f must be determined entirely by experiment, and deduced by a large number of experiments on a large number of piles in a great variety of material; and the bearing resistances of the soil must be determined by a similar series of experiments in different soils and at different depths below the surface of the ground. Unfortunately our knowledge of either of these forces is meagre and unreliable. The common practice is to allow from 1000 to 5000 pounds, according as the material is soft and silty or firm and compact, as gravel, sand, and clay or mixed earths. The remaining portion of the resistance must then be ascribed to friction. In the absence of more reliable data the author suggests the following values:

TABLE XLVII.

In very soft silt or liquid mud	$R = 0$;	$f = 150$ lbs. per sq. ft.
In ordinary clay or earth (dry)	$R = 3000$;	$f = 300$ " " "
In " " " " (wet)	$R = 2000$;	$f = 150$ " " "
In compact hard clay	$R = 5000$;	$f = 300$ " " "
In sand or sand and gravel	$R = 5000$;	$f = 500$ " " "

A small pile 12 inches top, 11 inches average, and 10 inches at bottom, top area 113.1 square inches, bottom 78.54 square inches. Such a pile driven in the soil presents when 30 feet long about 86.5 square feet of surface over its entire length. Applying, then, eq. (280), $P = RA + fS$.

		Pounds per Pile.
In soft silt	$P = 0 \times A + 150 \times 86.5$	12,900
In dry earth	$P = 3000 \times \frac{78.5}{144} + 300 \times 86.5$	27,600
In wet earth	$P = 2000 \times \frac{78.8}{144} + 150 \times 86.5$	14,000
In hard clay	$P = 5000 \times \frac{78.8}{144} + 300 \times 86.5$	29,600
In sand and gravel	$P = 5000 \times \frac{78.5}{144} + 500 \times 86.5$	48,250

When driven from 60 to 70 feet in the ground the softest of the above materials will carry the heavy loads mentioned in paragraphs 384 and 405.

411. Rankine's theory of earth-pressure can be easily applied to the bearing power of piles. Let

w = weight of one cubic foot of the material;
A = area of cross-section of pile at the bottom;
x = depth of pile in the soil;
S = area of exterior surface of pile;
f = coefficient of friction of the earth on the pile surface.

Then, as in paragraph 398, eq. (277), the bearing power of the earth against the foot of the pile $= Awx\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2$.

The horizontal intensity of pressure against the pile at any depth $x = wx\frac{1 - \sin\phi}{1 + \sin\phi}$ for minimum, and for maximum intensity $= wx\frac{1 + \sin\phi}{1 - \sin\phi}$; and if f is the coefficient of friction, then the intensity of friction $= fwx\frac{1 - \sin\phi}{1 + \sin\phi}$ or $fwx\frac{1 + \sin\phi}{1 - \sin\phi}$; and for mean or average friction over entire lengths of the pile

$$\frac{Sfwx}{2}\frac{1 - \sin\phi}{1 + \sin\phi} \quad \text{or} \quad \frac{Sfwx}{2}\frac{1 + \sin\phi}{1 - \sin\phi}.$$

Then for the total bearing power of the pile

$$P = Awx\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2 + \frac{Sfwx}{2}\left(\frac{1 - \sin\phi}{1 + \sin\phi}\right) \text{ minimum; } \quad (281)$$

$$P' = Awx\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2 + \frac{Sfwx}{2}\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) \text{ maximum. } \quad (282)$$

The only uncertain terms in these equations are the values of ϕ and f. Values for ϕ have been given in tables.

The writer is unable to find any values of f for timber on earth. For masonry on dry clay $\phi = 27°, f = 0.51$; for masonry on moist clay, $\phi = 14°$ to $18\frac{1}{4}°, f = 0.33$; for wood on wood (dry), $\phi = 14°$ to $26\frac{1}{2}°, f = 0.25$ to 0.5.

By actual experiment on piles in liquid mud the frictional resistance was estimated at 144 pounds per square foot of surface on a pile 30 feet in the material; then at a depth of 15 feet the weight of the material per unit of area = 1500 pounds, would give approximately an intensity of horizontal pressure = 1440 pounds. Hence in this case $f = \frac{144}{1440} = 0.1$. This value will be used in the following example: Let $A = \frac{78.5}{144} = 0.545$, as in preceding examples; $w = 110$ pounds; $x = 30$ feet; $S = 86.5$; $f = 0.1$; $\phi = 15°$; hence $\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 = 2.89$; $\frac{1+\sin\phi}{1-\sin\phi} = 1.7$; $\frac{1-\sin\phi}{1+\sin\phi} = 0.59$.

Substituting in eqs. (281) and (282), above, $P = 0.545 \times 110 \times 30 \times 2.89 + 110 \times 0.1 \times 15 \times 86.5 \times 0.59 = 13{,}620$ as the minimum bearing resistance or bearing power, $P' = 0.545 \times 110 \times 30 \times 2.89 + 110 \times 0.1 \times 15 \times 86.5 \times 1.7 = 29{,}461$ as the maximum bearing resistance. This would correspond with wet clay as found in the preceding paragraph. For other materials and depths sunk it is necessary simply to substitute proper values for x, ϕ, w, f, S, and A.

If proper values of ϕ, S, and f in equations above are determined by experiment, it would seem that these formulæ would produce better and more reliable results than the more common formulæ would.

In the absence of good precedents and accurate data the writer recommends the determination of the bearing power of piles by actual experiment in any particular case requiring the use of piles for heavy loads and important structures.

412. *Water-jet.*—If a small pipe be fastened to a pile, its lower end near the lower extremity of the pile, and its upper end connected by a hose to a force-pump, the pile can be lowered very rapidly through almost any material, excepting rock, by forcing water through the pipe. This, issuing with great pressure at the point of the pile, loosens the material, and the pile will often sink under its own weight; if not, weights can be placed on it, or it may be struck with light blows of a hammer. Great depths have been

reached by this process. It is especially advantageous in compact sand and gravel, and much to be preferred to many blows of a heavy hammer in such materials.

FOUNDATIONS IN WATER.

413. For foundations under water, either on the natural beds of the lake or stream, or at great depths below the beds, the preparation and construction will be similar in every respect to the various methods already explained for foundation-beds and foundations on land, and no further description will be needed.

The important problem in this case is the selection and employment of the best means of reaching the foundation-beds, and protecting them from the scouring action of currents.

This naturally divides the subject into the following cases: (1) Where the water is shallow and has no current; (2) where the water is shallow and there is a current; (3) where the water is from 15 to 20 feet deep and has no current, (4) where there is a current, and when in cases (3) and (4) the structure is of no very great magnitude; and (5) where it is necessary, owing to the magnitude of the structure, to reach great depths below the water surface, varying, we may say, from 30 to 200 feet. This case will be discussed under the head of Deep Foundations.

414. In case (1) a simple earthen dam can be constructed around the space to be occupied by the structure. The thickness of this dam need not be over from 3 to 5 feet at the top. and its height above water from 2 to 3 feet. The side slopes should not be less than from 2 to 1 to 3 to 1; they, however, will assume an angle of repose depending upon the material used. These considerations will determine the linear dimensions of the dam. If any depth of excavation is to be made in the bed, widths for similar slopes must be provided and the dimensions of the dam correspondingly increased. The water is then pumped out, the bed excavated and levelled so as to be perpendicular to the direction of the pressure, and upon this timber, concrete, or masonry may be commenced, or, if necessary, piles can be driven.

415. If, as in case (2), there is a current, earth dams alone will not answer. The space may, however, be closed by driving a few long piles, to which are fastened horizontal pieces of timber called *wales;* outside or between these plank or sheet-piles are driven into

the soil, and spiked to the wales, or they may be driven between two wale-pieces. The guide-piles need not be over 8 or 10 inches in diameter, and driven 6 or 8 feet apart; the wales are 6 × 4 or 6 × 6 inches, and the sheet-piles 2 to 3 inches thick. Earth should then be piled up around and against the sheeting—gravelly or sandy clay will be best,—and the exposed surface should be riprapped or paved with stone. Such a dam will usually require bracing on the inside to resist the water pressure, but if it is made large enough earth can be piled against the sheeting on the inside, this is the better plan. It is always bad practice to make such dams too small. The water is then pumped out, and the bed prepared as before described. The sheeting is sometimes made 6 or 8 inches thick, tongued and grooved. This arrangement will make a strong dam. Sometimes two or three layers of plank are driven, breaking joints. These cases are represented in Fig. 183. The drawing

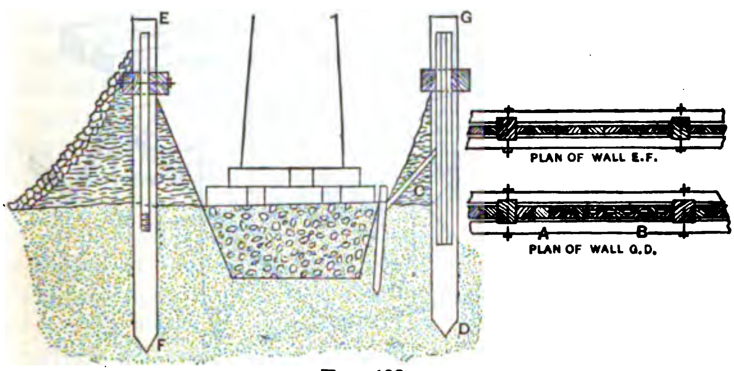


FIG. 183.

shows a vertical section through the coffer-dam. The left wall is shown with earthen embankment on both sides, the outer slope covered with broken stone. On the right a single wall with tongued and grooved sheeting. Plans of left and right walls are shown on the right. Instead of the tongue and groove proper, as shown at *A*, both pieces may be grooved and planks 3 to 5 inches wide and 2 inches thick are driven into the grooves of the adjacent pieces, as shown at *B*, in the plan. This latter arrangement will generally prove more convenient and satisfactory, as the perfect contact of the sheeting plank will not be necessary. At *C* in Fig. 183 shores or braces are shown resting against the main piles at

their upper extremities, and against posts driven into the soil at their lower extremities.

for

416. This construction can be made in somewhat deeper water, but the main piles must be at least 12 inches in diameter, driven well into the bottom and placed close together, as the braces cannot be placed until the water is pumped out, they must carry for a while the entire water pressure; and, in addition, ample room must be left on the inside between the timber wall and the excavation in order that the braces may be well supported. The general problem of fluid pressure will be discussed under the head of Reservoir Walls. Its practical application will be given in this place. Fig. 184

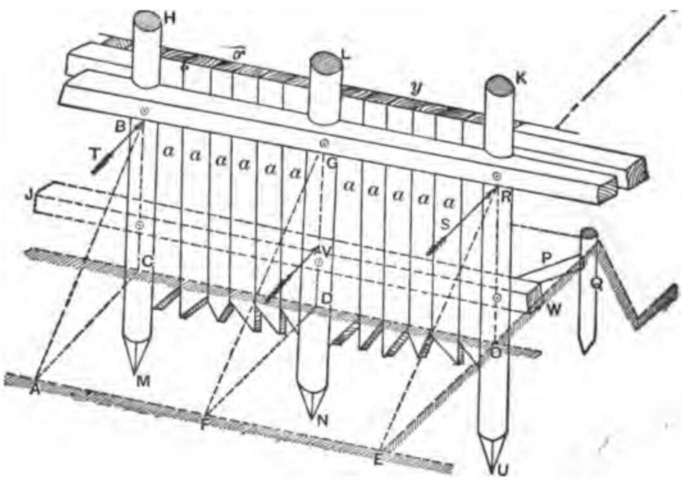


FIG. 184.

represents an oblique view of a part of one side of a single-wall coffer-dam. The depth of the water is taken at 10 feet. *HM, LN*, and *KU* are three of the main piles; $BC = GD = RO = 10$ feet, the depth of the water; *BGRST* the water surface; *CDOEFA* the bed of the river; *a . . . a*, the sheet-piles; *P, P*, the braces; *Q, Q*, the posts to support braces; *I*, the excavation on the interior. The drawing is not made to exact scale, as it is intended to show conspicuously certain parts and details. It is required to find the diameter of the main piles when 4 feet apart in the clear or 5 feet centres, and also the thickness of the sheet-piles, these being 8 inches wide, the pressure acting on 10 feet of their lengths. The extreme maximum fibre strain is not to exceed 1000 pounds, and

the dimensions of the braces P, which are 7 feet long, must be such that the maximum fibre strain on the main pile shall not exceed the limit above given; the braces assumed to bear safely 500 pounds per square inch of their cross-sections. In order that the sheeting may transmit the pressure to the main piles, a wale-piece should be placed against the piles at $\frac{1}{3}$ of the depth of the water from the bed of the stream, that is, $\frac{1}{3} \times 10$ feet, as shown at IVW, in addition to the one near the top, usually at or a little above the water surface. The pressure on any main pile will be equal to the pressure on an area equal to the depth of the water multiplied by a distance xy, extending between the points half way between the pile under consideration, L, and the adjacent piles H and K on either side; and as the main piles are all equally distant, this distance will be equal to the distance between adjacent piles $H, L = L, K = 5$ feet centres. The area supported by each pile $= 5 \times 10 = 50$ square feet. As in a fluid, the pressure is zero at the surface and simply increases with the depth, and at any point below the surface the intensity is equal in all directions and acts normal to the surface pressed. If, then, in Fig. 184, $BC = RO$ be taken to represent the intensity of the vertical pressure at C and D, then, laying off horizontal lines CA and OE equal to BC, they will represent the intensity of the horizontal pressure. If, then, the lines AB and ER be drawn, the horizontal ordinates of the triangles BAC and REO will represent the intensities at any given depth, and the weight of the triangular prism $ABC\ ROE$ will represent the total pressure on the side of a portion of the dam, one half of this will be the pressure supported by each main pile. This pressure is uniformly varying and distributed over the vertical surface supported by each pile; but the pressure is concentrated by the construction at the wale-piece IVW, Fig. 184, or V in the figure. This being placed at one third of the depth from the bottom is the point of action of the resultant pressure of the water, which acting through the centre of gravity of the prism of pressure, this point g is one third the depth from the bottom. The effect then is the same in either case. Hence the main pile LN is in the condition of a beam fixed at one end D and free at the other, and acted upon by uniformly varying load, or its equivalent resultant load applied at the centre of gravity.

Then, from equation (139), $M_e = \frac{fI}{y}$, in which (from table 45),

$I = \frac{\pi r^4}{4}$, $y = r$, and the radius $= \frac{1}{2}d$, the diameter of the pile.

Then $M_e = \frac{f\pi r^3}{4} = mWl$, the bending moment, from which

$$r^3 = \frac{4mWl}{f\pi} \quad \text{and} \quad r = \sqrt[3]{\frac{4mWl}{f\pi}}. \quad . \quad . \quad . \quad (283)$$

The pressure $W =$ area $ROE \times GR \times w'$; $w' =$ weight of a cubic foot of water $= 62\frac{1}{2}$ pounds; area $ROE = \frac{1}{2}RO \times EO = \frac{1}{2} \times 10 \times 10 = 50$; and $GR = 5$. $\therefore W = 50 \times 5 \times 62\frac{1}{2} = 15{,}625$ pounds; and as the load is concentrated at V, $m = \frac{1}{8}$, $f = 1000$, $\pi = 3.1416$, and $l = 10$ feet $= 120$ inches. Substituting, we find $r = 9.3$ inches and $d = 18.6$ inches. This diameter of main pile would be required without braces. If, then, we assume the braces P to be 6×6 inches $= 36$ square inches, and allow 300 pounds per square inch, the horizontal component of the pressure on the braces will be $36 \times 300 = 10{,}800$ pounds. Then in eq. (283), $W = 15{,}625 - 10{,}800 = 4825$ pounds, and $r = 6.2$, and the diameter $= 12.4$ inches. If square piles are used, then $mWl = nf\,bd^2$, $n = 6$, $d = 12$, and other values as above; then $b = 8$, or if $d = 10\frac{1}{2}$, then $b = 10\frac{1}{2}$ inches. With a pile 12 inches in diameter, since the braces cannot be inserted until the water is pumped out, the piles will have to carry the entire pressure for a time. Then, from eq. (283), the fibre strain $f = \frac{4mWl}{r^3\pi}$, $r = 6$. Hence the fibre strain would be $f = 3700$ pounds. Good timber would stand this pressure if brought to bear very gradually and not left unsupported too long. The deflection under this load would be $v_1 = \frac{1}{3}\frac{Wl^3}{EI}$, in which

$W = 15{,}625$; $l = \frac{1}{3} \times 10$ feet $= 40$ inches; $E = 1{,}500{,}000$; $I = \frac{\pi r4}{4}$ $= \frac{3.1416 \times (6)^4}{4} = 1017.88$. $\therefore v_1 = 0.22$ inch.

The sheeting plank being supported at the surface of the water, and also at the wale-piece 6.67 feet below, will only have a clear length of 6.67 feet. The pressure, then, on a plank 12 inches wide will be $\frac{1}{2}(6.67 \times 6.67) \times 1 \times 62\frac{1}{2} = 1391$ pounds. The resultant acting $\frac{1}{3} \times 6.67$ from the lower waling-piece, the reaction at this point will be $\frac{2}{3} \times 1391 = 928$ pounds. Its moment $= 928 \times 26.64$

$= 24{,}722$ pounds $= \frac{1}{6} \times 1000 \times 12 \times d^2$. $\therefore\ d = 3.5$ inches for the thickness. With the above pressure at the centre of its length, the deflection $v_1 = \frac{1}{48}\ \frac{Wl^3}{EI} = 0.0004$ inch, which could not hurt anything. The main piles should be driven well into clay, or sand and gravel—better through the latter into clay, if practicable. The sheet-piles need not be driven over $1\frac{1}{2}$ to 2 feet into the bed; these are not supposed to add any resistance to overturning bodily of the sides. The uniform pressure is treated as a single force.

417. If, however, the main piles are omitted, the sheet-piles should be well driven as stated above for the main piles, or at least every other one, and wale-pieces bolted to them at the water surface. In this case the sheet-piles would have a free length of ten feet, and would be in the condition of a beam fixed at one end and loaded at the distance of 3.33 feet $= 40$ inches from the bed. The load on each piece, if ten inches broad, would be $\frac{10}{12} \times 10 \times 5 \times 62\frac{1}{2}$ $= 2604$ pounds, moment of bending $M_0 = 2604 \times 40 = 104{,}160$ foot-pounds $= \frac{1}{6} \times 1000 \times 10 \times d^2$. $\therefore d = 7.9$, or, say, 8 inches. With grooves $2\frac{1}{2} \times 3$ inches, into which planks $2\frac{1}{4} \times 2\frac{3}{4}$ can be driven.

418. In Fig. 185 is shown a vertical cross-section of a double-wall coffer-dam, in which clay puddle is deposited between the

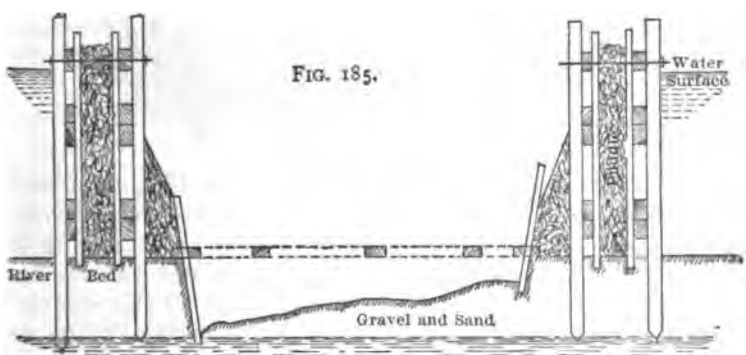


FIG. 185.

walls in order to prevent leaks, and, as commonly stated, to give stability to the walls and prevent overturning by the pressure of the water.

Mr. Rankine states that for any depth of water under ten feet the thickness of the walls of the dam should be equal to the depth,

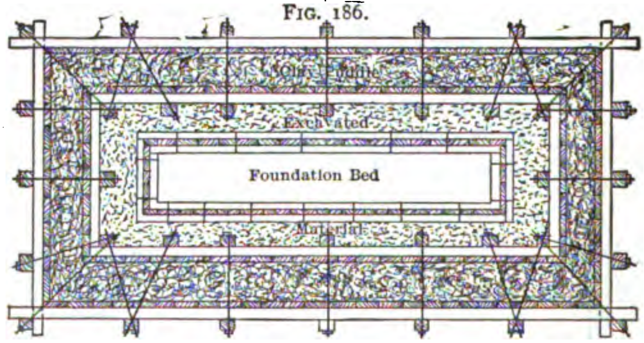
and for greater depths than ten feet add one foot for each three feet over ten feet. This is to give stability to the walls. From two to five feet of clay puddle is necessary to prevent leakage.

This statement has been repeated in books published ever since, and no doubt taught in schools. No special reasons have been given for this assumption. The writer is not aware of any theory setting forth the principles by which the proper thickness can be determined.

Having given the only rule known to the author for the proper thickness, he proposes to discuss this question from what seems to him the proper standpoint. The experience derived from the construction of many coffer-dams under his supervision fully confirms the principles to be stated.

Fig. 186 shows a plan and Fig. 185 a vertical section of a double-wall coffer-dam as usually constructed.

FIG. 186.



419. This represents a coffer-dam which was designed to carry a pressure due to a depth of water of 15 feet, and an excavation through about 6 or 7 feet of gravel and sand to an underlying bed of hard clay, upon which a combined timber and concrete base about 3 feet thick was formed. An excavation for this was made in the hard clay. This was done although the surface of the clay was practically level, as the structure could thus be better bonded into it. The height of the pier was 97.32 feet, built of sandstone; top dimensions, 9×23 feet; bottom of neat work, 42.2×16.50 feet. The great increase in length was due to the addition of circular ends below high-water level.

Four offset courses increased these dimensions to 49.0×23.0 feet. The concrete base gave an additional increase of 5 feet, making

the extreme bottom dimensions 54.0 × 28.0 feet. Profiting by the almost complete failure of another dam, which could be attributed to either one of three following causes: (1) Short piles not driven deep enough into the bed of the stream; (2) piles of too small diameter; (3) making the enclosed area too small, requiring the excavation for the masonry to be carried too close to the inner wall of the dam. The failure was due to causes (1) and (3). The writer, with these experiences and the lateness of the season, the probable rises in the river, and the importance of completing the foundations as rapidly as possible, determined to make no mistake if it could be avoided. Interior bracing was impracticable, owing to the great dimensions required. The methods of constructing an inner crib, open top and bottom, and sinking this by weights and excavation in the interior, thereby holding the sides of the dam in place, are fully explained by the author in a special volume on Foundations.

Sawed white-oak piles 12 × 12 inches at top, 10 × 10 inches at bottom, were ordered. These were 25 feet long. They were driven four centres in each row. The intention was to have the rows 7 feet 6 inches centre to centre, 8 feet 6 inches out to out, which would give 5 feet in thickness of puddle. Owing to bad alignment of the piles the puddle filling varied from 2½ to 5 feet in thickness. A margin of at least 10 feet was provided between the bottom dimensions of masonry and the inner lines of the sides of the dam, making the clear area of the dam 74.0 × 48.0 feet. The water was pumped out under a head not exceeding 10 feet. There was no excessive leak at any point of the dam, no evidence of yielding. The usual bulging of the sheet-piles, which were only 2 to 2½ inches thick, was observable. This the writer attributes mainly to the swelling of the clay when it is thrown into the water, and the subsequent ramming.

The main piles were driven through the gravel and from 1 to 2 feet in the clay, using the ordinary pile-driver. The sheet-piling was driven by mauls a depth of 1½ to 2 feet in the gravel and sand.

After pumping out the water, both for economy and for supporting the dam on the inside, it was determined to shovel as much of the material as practicable from the centre portion to the sides, banking it against the walls of the dam. The extra weight, the finer material, and the water present caused the material to slip or flow back. In order to support this, two rows of 12 × 12 inch timber were laid so as to enclose only the actual area of the base of the

pier; these were placed about 3 inches apart, and small bolts put through them at intervals. In this open space pieces of plank about 7 feet long were inserted and driven into the underlying material. The excavation was again commenced, and the material thrown behind the sheeting. As this advanced men passed around, driving the sheeting a few inches at a time, always keeping it a little below the excavation. This arrangement prevented the material from flowing back, and held it in place against the sides of the dam, it also reduced the excavated material to a minimum, and, further, avoided the expense of lifting it out in buckets and carrying it to the dumping pools on barges. (See also example under head of Tunnelling in Open Cut.)

420. Experience in this and other dams convinced the writer (1) that a width of from one and a half to two times the depth to be excavated should be left between the masonry and the inner walls of the dam; (2) that the stability of a double-wall coffer-dam does not depend upon the thickness of the sides; and (3) that the thickness of the puddle-filling need not be over that required to make the dam water-tight. This will depend upon the material used. With good compact clay, mixed with a little gravel or sand, a thickness of 2 feet is ample; with ordinary loose or porous earths, from 4 to 5 feet in thickness will be required.

In the writer's experience, coffer-dams have only given way by the pressure forcing the lower ends of the inside row of piles inwards, thereby letting the puddle sink downwards and spread laterally, increasing the moment of its pressure on the inner wall, as the upper end is usually tied to the outer row near its top, this continuing to press the lower ends inwards until failure results. This will not occur if the preceding precautions have been taken.

When these precautions have been observed the strength of the dam is simply limited by the strength of the main piles, as the effects of the overturning tendency are resolved into a bending action on the main piles of the inner row. The outer row is simply acted upon by the difference between the outward pressure of the clay puddle and the inward pressure of the water outside, and practically the outer wall simply acts as a fixed support to which the inner wall is tied near the top.

The resultant effect is that, the puddle acting to press the outside wall outwards and the inside wall inwards, the stability is the same as a single-wall dam acted upon by the pressure due to the depth of the water, increased by the conversion of the inner wall

from the condition of a beam fixed at one end and free at the other to that of a beam supported or partially fixed at both ends, the tie-rod at the top bringing about this condition. If these views are correct, it is only necessary to apply the principles discussed in paragraph 416 and Fig. 184 to the condition of stability for the inner wall of the dam, acted upon by the water pressure alone. As the depth of water increases the number of wale-pieces should be increased, so that the unsupported length of the sheeting plank shall never exceed 5 or 7 feet. Hence the thickness of the sheeting plank need not be increased as the depth of water increases.

421. Apply these principles to the dam, Figs. 185, 186, assuming a depth of 10 feet of water $= AB$, and that the centre of pressure is $3\frac{1}{3}$ feet from the bottom B, at which point the middle wale-piece is placed. This would leave the unsupported length of the upper section of sheeting about 10 feet. But the pressure is not great near the surface, and if considered necessary another wale could be placed; and since the main piles are 4 feet centres, the pressure borne by each pile $= \frac{1}{2}(10 \times 10) \times 4 \times 62\frac{1}{2} = 12{,}500$ pounds, and the reaction at the bottom is $\frac{2}{3} \times 12{,}500 = 8334$ pounds. $l = 3\frac{1}{3}$ feet $= 40$ inches; then the moment $M_0 = 8334 \times 40 = \frac{1}{6} \times 1200 \times 144 \times b$. $\therefore b = 11.5$ inches. That is, the cross-section of the pile should be 12×11.5 inches, which would be very close to its dimensions at the bed of the river. The allowable fibre strain for good oak is taken at 1200 pounds. This dam was subjected to several rises in the river, producing a pressure due to a head of water from 12 to 15 feet, but only after it was well supported on the interior by the earth piled against it. In any depth of water over 12 to 15 feet the writer would recommend some other method of construction unless the dam is strongly braced by timbers from side to side. When thus braced the writer has used single-wall dams in water 40 feet deep. This will be explained under the head of Deep Foundations.

422. The above theory of the stability of coffer-dams is in no manner affected by the fact that often three rows of main and sheet piles have been driven where the water was very deep. The inner wall will fail if the excavation on the interior is made too close to it, regardlessly of the thickness of the puddle or the number of timber walls. The tops of the inner rows of piles are more firmly held in place, as there are two fixed walls instead of one to which they are anchored. But the top reaction of the pressure is relatively small, and decreases rapidly as the lower extremities of the piles are

pressed inwards. When the inner wall fails, it is true that the dam as a whole may not fail; but this is due to the fact that the middle or second wall is supported on the interior by a sloping embankment of earth, whose base is the original clear width between the inner wall and the masonry increased by the width between the inner and middle walls. Equal stability would have been secured had only two walls been built, leaving the inner of the three out entirely, only increasing the area of the inclosed space; the only advantage with three walls consists in a feeling of security that if, from defects in material and construction, the inner wall should fail during the pumping out of the water, the entire work would not collapse, as the construction enables us to hold a mass of material in position and readiness to form a sloping bank against the middle wall, which would otherwise have been originally the inner wall. If a coffer-dam of sufficiently large dimensions has been constructed, there seems to be little excuse for failure after the water has been pumped out, as braces or an embankment against the interior wall can, and should, be rapidly placed in position. The great danger to coffer-dams arises from the difficulty, if not impracticability, of bracing the sides and ends until the water is pumped out.

A strong system of bracing, framed and floating, and sinking as the water lowers, can be stopped and wedged against the sides at any desired point, or simply allowed to sink and be in the position to support the sides if necessary; or the inner or third wall, even if constructed but poorly, will hold the material to give support to the middle wall. This inner wall, if it stands, can do no harm; if it fails, it has served at least a good purpose, and can be removed without impairing the strength or stability of the dam.

The above theory simply resolves itself into the following: Whatever may be the number of timber walls and puddle filling between them, the unbalanced inward pressure is due solely to the water pressure on the outside, which is simply transmitted to the inside wall and entirely supported by it. The outer walls, whether one, two, or more, simply act as fixed supports to which the inner wall is tied by iron rods near their tops. So long as the piles of the inner walls are not undermined, or their support is not too much diminished by excavation on the interior of the dam, any tendency to overturn bodily is converted into a transverse strain on the main (sheeting-piles are not considered) piles, which are in the condition of beams supported, or, rather, restrained more or less rigidly, at both ends, and loaded with a uniformly varying load

equivalent to the uniformly varying pressure of water. In order to derive the full benefit from tieing the inner wall to the outer ones the iron tie-rods should be placed at points one third of the depth of water from the bottom, and also near the top of the piles.

423. Frequently the bed of a river is composed of a firm material, or rock. This can be levelled by dredging in case of earth, or by blasting in case of rock; if the bed is of softer materials, piles can be driven and cut off level either at or near the bed of the river. In any of these cases it is not uncommon to construct floating a platform of four or more courses of timber well bolted together. On this a single-wall coffer-dam, composed of vertical posts framed into caps and sills, is constructed; long pieces are placed over the top, projecting one or two feet, and iron bolts are used to connect these securely with the bottom platform. The sides are covered with one or two courses of plank, according to their height; the inner course is usually placed diagonally, the outer horizontally. The entire outside is well calked. One or two courses of masonry are built on the inside to give it ballast. It is then floated over the site prepared.

The masonry is built until it nearly reaches its bed. It is then located in position and sunk on its bed by letting in water. If it sits level and true, the masonry is continued; if not, the water is pumped out, allowing it to float, its position is readjusted, and it is sunk again. (See Fig. 187.)

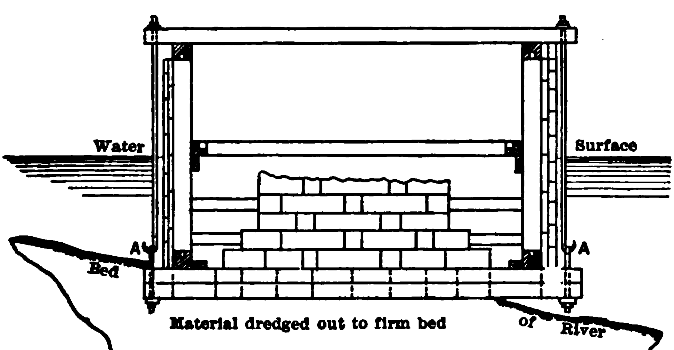


FIG. 187.

Instead of using timber for the platform and sides, the entire caisson has been made with thick concrete bottom and sides en-

closing an open space; or the timber bottom can be used, upon which are constructed relatively thin walls of cement concrete, or of brick and stone laid in cement mortar. These are floated to their proper positions, and sunk by building up the walls, or filling the interior with masonry on concrete. Timber caissons
to
are doubtless to be preferred. Fig. 187 shows a timber caisson sinking on a firm bed levelled by dredging, the lower braces removed, and the sides supported by blocking against the masonry; the upper braces in place, and to be removed and blocking substituted when the masonry reaches them. In soft materials the caisson should rest on piles cut off at a level.

424. Cribs are often built and sunk by weighting the pockets with broken stone, on a bed prepared as in the last paragraph. In case of a rock bed, if much out of level, instead of blasting, broken stone is deposited and well levelled off by divers. Either the crib or open caisson can be sunk resting on the broken stone.

425. Iron cylinders can be sunk around a cluster of piles, or, omitting the piles, they may be sunk either by dredging the material from the interior and loading the cylinder until it sinks to the proper depth; or the cylinders may be closed at the top and made air-tight at all joints, and by connecting an air-lock with the upper section compressed air may be used in sinking the cylinder. In either case the cylinders are usually filled with concrete. In the first case, which will be further explained under Wells for Foundations, the concrete has to be deposited under water, which is always objectionable, and should be avoided if possible. In the second, the water is kept out by the compressed air, and the concrete is deposited in the dry. This method will be explained under the head of Pneumatic Caissons or Cylinders.

DEEP AND DIFFICULT FOUNDATIONS.

426. Under this head will be discussed foundations by (1) well-sinking, (2) open cribs, (3) pneumatic caissons, (4) and iron or timber screw-piles.

The methods of well-sinking and open cribs are similar. The design and construction of the foundation structures differ. The first has been used to a great extent and on a large scale by English engineers; the second by American engineers.

The methods of sinking are the same in both cases, and will

only be described once. The designs and construction will be discussed separately.

427. Well-sinking is the favorite method of constructing foundations in India, and to a greater or less extent in England. The wells or cylinders may be made of iron plates or castings riveted or bolted together in sections, which are built up as the cylinder sinks. This is lined usually with brick or stone masonry, which not only gives strength and stiffness, but also furnishes the necessary weight to sink them against the direct and frictional resistances. The iron casing may extend from bottom to top of the completed structure, or it may only extend a portion of the height from the bottom, leaving the exterior surface of the masonry exposed for the remaining portion of the height.

Occasionally two concentric cylinders of iron are used, the annular space between the two being filled with concrete. These, however, resemble more closely the open crib construction.

The diameters of such cylinders vary from 6 to 40 feet or more. The masonry lining varies from 1 to 5 feet in thickness. The depths sunk vary from 3 feet to 70 feet or more. In all cases a bottom section is made of iron, either solid or hollow, having a wedge-shaped cross-section in order to form a cutting edge. The width of the base of the wedge is about equal to the thickness of the masonry wall resting upon it. Its height varies from 1 to 9 or 10 feet.

The construction and method of sinking is the same for large and small cylinders, whether sunk in dry land or through water into the bed of the river. This being the case the following drawings and description will answer for all types, the only difference being in the dimensions of cylinders and thickness of masonry lining. The example taken shows the largest cylinders of which the writer has any knowledge, and is typical of all such constructions.

Fig. 188 shows a vertical longitudinal section and part elevation of the cylinders for the rectangular piers, and Fig. 189 the same for a round or pivot pier. The total depth sunk was 48 feet below the bed of the river—about 65 below mean high water. They were filled with concrete to a height of 65 feet from the lower end, on which was placed a floor of brickwork several feet in thickness, giving a total height of 70 feet, bottom to top of pier. The wedge-shaped shoe or bottom section was 9 feet in height; the outer face vertical, the inner face on a slope such that at the top its thickness was 6½ feet; the extreme outer diameter is 43 feet; the clear enclosed space 30 feet

in diameter. Upon this were placed two concentric cylinders of plate iron one-half inch in thickness to the height of 15 feet. The total height of iron is 24 feet. The shoe was formed of a specially rolled iron ring inserted between the plates of the cylinders, and the whole bolted and riveted together. This height of ironwork was intended to reach from the bed of the river above the water surface. Concrete was then placed in the hollow shoe after floating the cylinder to its proper site, and under about 200 tons of concrete the cylinder rested firmly in the bed of the stream.

The annular space between the iron cylinders was reduced to 5 feet, as shown in Fig. 189. On the top of the concrete a brick

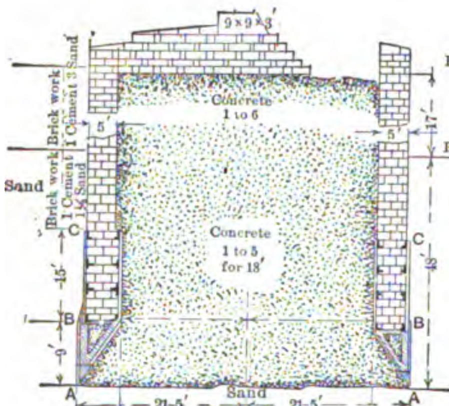


FIG. 189.
VERTICAL SECTION THROUGH COMPLETED PIER FOR PIVOT PIER. ONE LARGE CYLINDER

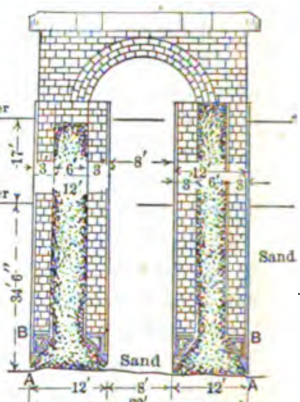


FIG. 188.
VERTICAL SECTION FOR COMPLETED PIER. TWO CYLINDERS CONNECTED BY BRICK ARCH

wall was built, filling the annular space. This wall was carried well above the top of the iron casing and the water surface. The excavation in the interior was then commenced. In Fig. 189, *AABB* is the shoe or cutting edge; *BBCC* the iron plate cylinders, and enclosed in these is the brick wall. The material was removed from the interior by means of the clam-shell bucket or dredge.

428. These buckets open as they descend, penetrate into the material, and close on it when lifted, and bring it to the surface. It is either dumped around the pier or on barges, and carried to some place of deposit. The effect of this is to reduce the resistance of the material on the inside surface, and the weight of

the iron and masonry causes the cylinder to sink, the only resistance being that due to friction on the exterior surface. As the cylinder sinks the iron and brick work are built up. In this case the iron casing only extended to a height of 24 feet from the bottom, the brickwork alone being carried up. It is important to keep the masonry wall well above the water surface, as the entire structure may either sink gradually or rapidly several feet at a time—sometimes from 7 to 10 feet. In addition, great weight is required to cause the sinking. In this case, when the excavation and sinking commenced, the weight on the cutting edge was 6 tons per linear foot around the bottom. The material sunk through was sand, in which the resistance from friction may vary from 300 to 600 or more pounds per square foot of exterior surface.

When bowlders were encountered the water-jet was used to undermine them, which either carried them downwards or forced them either outside or inside the cutting edge. This flow of a water-jet under great pressure had the effect of reducing the friction on the sides. Having reached the proper depth, concrete was lowered through the water in the interior and deposited on the material at the bottom and close around the cutting edge. When 18 feet in thickness of concrete had been thus deposited, it was found practicable to pump the water out of the remaining portion of the enclosed space, and the concrete then could be placed in the dry. The first concrete used should be strong and rich in cement, to prevent leakage under the shoe. In this case it was 1 to 5 concrete of Portland cement. The English engineers use shingle or ballast, which is a mixture of gravel and sand. This proportion would correspond about with 1 Portland cement, 2 sand, and 3 broken stone, or something approaching these quantities. The upper portion of the concrete laid in the dry was about 1, 2, and 4 parts of the above materials, called 1 to 6 concrete. The brick masonry in the side walls was built with 1 cement to 1½ sand. The brick flooring over the concrete and walls was laid in mortar 1 cement to 3 sand. A granite coping and thick pivot-stone was placed on top. The drawbridge on this pier was 287 feet from end to end, consisting of two arms, respectively 140 and 87 feet.

The cylinders for the other piers were from 12 to 14 feet outside diameter and 6 to 8 feet clear interior diameter, with brick walls 3 feet in thickness. These were sunk and filled with concrete as before described. Two or three cylinders are used to a pier. Brick

arches were built from cylinder to cylinder, upon which the remaining portion of the structure was erected.

Under wing abutments cylinders were sunk, upon which both face-wall and wings rest, the number under each depending on the dimensions of the structure. This method is used very commonly in India.

Cast-iron cylinders are sometimes used instead of plate-iron. The frictional resistance is reduced by having the iron cylinders extend at least up to the bed of the river.

Instead of springing arches from cylinder to cylinder, strong iron beams are sometimes used, supported on top of the cylinders, upon which the structure above rests.

429. Open cribs are used by American engineers. These differ mainly from the *wells* already described in having double walls of timber or iron enclosing the open interior space. The two walls forming the sides are spaced from 4 to 10 feet apart, and well braced to each other. This space is usually filled with concrete, and furnishes the weight necessary to sink the crib. The lower section is built to a cutting edge or shoe, generally 8 to 12 feet in height. If timber is used, this shoe may be built of solid timbers, or it may be hollow, and is commonly hollow when built of iron. Whether timber or iron is used is purely a matter of convenience and expense, as either will answer every purpose. A sufficient height of caisson is built floating, and carried to the proper site. The building of the walls, filling the proper space with concrete, and sinking proceed until the crib rests on the bottom. It should then be carried well up above the water surface. The excavation is then commenced, concrete added, walls built up,—all going on at the same time as the crib sinks.

When the proper depth or bed is reached the interior is filled with concrete. Usually single cribs are made large enough to carry the structure above instead of using two or more cylinders. The horizontal sections correspond, then, with the shape of the pier, consisting of a rectangle with triangular or curved end sections: if for pivot piers, usually circular if constructed of iron, and octagonal if of wood. A typical iron crib, used in the Hawkesbury Bridge, and designed by Anderson & Barr, contractors, is shown in the accompanying drawings.

Fig. 190 is a cross vertical section; Fig. 191 shows part horizontal sections at top of the wedge-shaped lower section or cutting edge along *AB* and *CD*, and part horizontal projection. Fig. 192 is part

HAWKESBURY BRIDGE—DETAILS OF FOUNDATION.

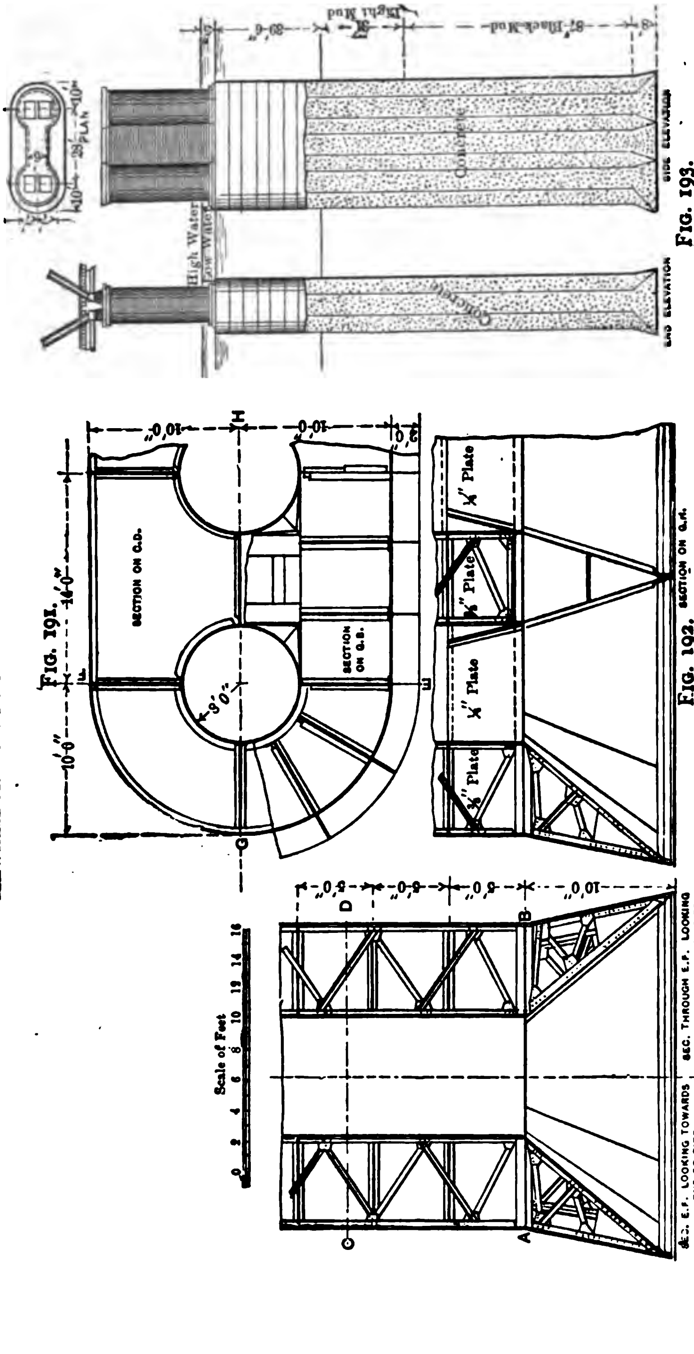


FIG. 193.

FIG. 191.

FIG. 192.

FIG. 190.

vertical and longitudinal section along *GH* of Fig. 191. Fig. 193 shows a general section and elevation of the completed foundation and masonry pier, also plan of the same.

The total height from bottom to top of one of the piers is 197 feet, divided as follows: Top of pier above low water, 42 feet; from low water to river-bed, 47 feet; below bed of river, 108 feet. There were six such piers, the total depths sunk below low water ranging from 94 to 155 feet; height of masonry, 42 feet in all cases.

The walls were composed of $\frac{3}{8}$-inch iron plates, riveted together and stiffened by angle-irons. The double walls enclosed three tubes or cylinders, each 8 feet in diameter. These extended to *AB*, Fig. 190, about 20 feet above the cutting edge, from which plane they were enlarged into a bell or funnel shaped mouth to the bottom, forming with the outside and partition walls strongly built shoes. The horizontal sections were rectangular with rounded ends. Bottom dimensions, 52 × 24 feet, these decreasing to *AB* (Fig. 190), where they were 48 × 20, and continued with these dimensions to the top. They were built up in sections of five feet as the dredging and sinking progressed. The tubes were connected with the side and partition walls by strong iron braces.

The entire open space around the tubes was filled with concrete to provide necessary weight. The materials were excavated and removed through the tubes or dredging-wells by means of the Anderson Automatic Dredge. Each bucketful of material had to be lifted out of the tops of the tubes. Subsequently these tubes were filled with concrete. The foundations were sunk through layers of mud and sand, resting finally on a bed of compact gravel.

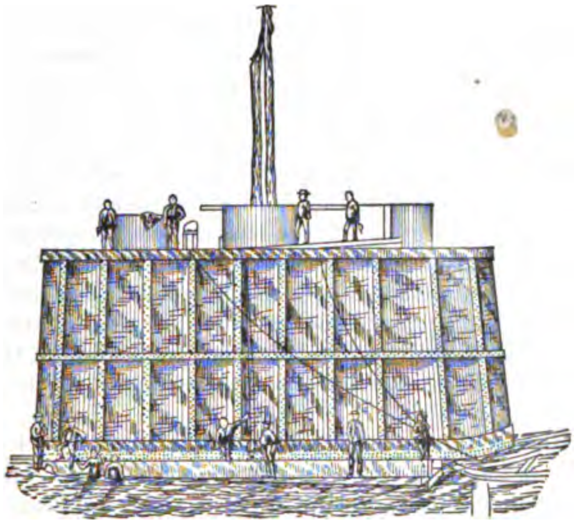
In Figs. 194 and 195 are shown the caisson in process of sinking, also excavators at work, and shore chains for maintaining the caisson in a vertical position.

For a section of the general construction of a timber crib see Fig. 204, combined Open Crib and Pneumatic Caisson.

429*a*. The most recent structure of interest and importance is the foundation for the pivot pier of the 520-feet swing-span of the Interstate Bridge being constructed by the Omaha Bridge and Terminal Railway Company, of which only the pier and swing-span are now completed. This span is the longest of its kind in the world, being 15 feet longer than the swing-span of the Thames River Bridge at New London, Conn. (See Figs. 195a and 195b.)

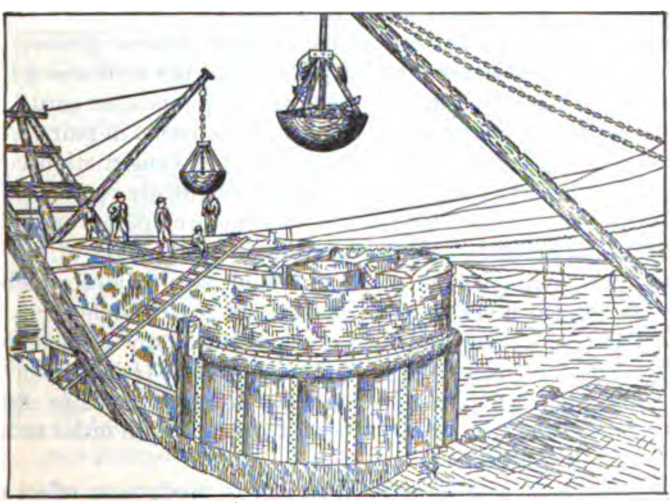
The foundation is constructed by the open-crib process. The

OPEN CRIB HAWKESBURY BRIDGE, AUSTRALIA.



Crib completed and sunk to a great depth.

FIG. 194.



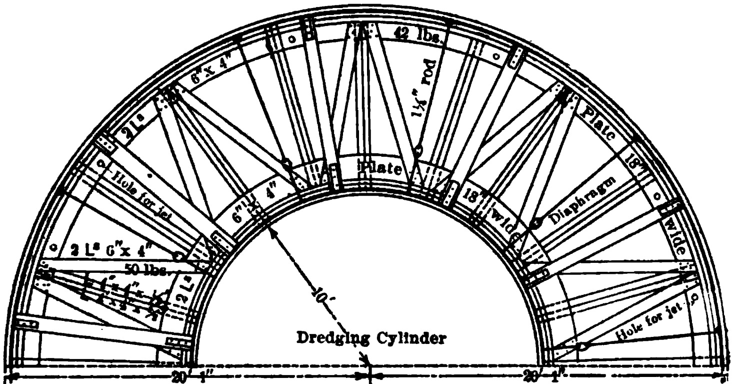
Crib in Process of Construction.

FIG. 195.

crib is constructed of steel plates, angles, and rods. It consists of two concentric cylindrical shells; the outer casing is 40 feet in diameter, while the inner shell enclosing the dredging cylinder is 20 feet in diameter, having an annular space between the two cylinders of 10 feet in width. At a point 10 feet vertically above the extreme lower or cutting edge of the outside casing the inner lining begins to batter outward on a regular slope of about 1 to 1, joining and forming with the outer lining the cutting edge. The lower section or chamber is therefore a frustum of a cone 10 feet high, lower base 40 feet and upper base 20 feet in diameter. The dredging-tube continues of this latter diameter all the way to the top. The shells are made of $\frac{1}{4}$-inch steel plates, butt-jointed, and connected by splice-plates 7 inches wide and single-riveted. The cutting edge is strengthened and stiffened by two bands, the one on the outside 18 inches wide and the one on the inside 6 inches wide, both being 1 inch thick.

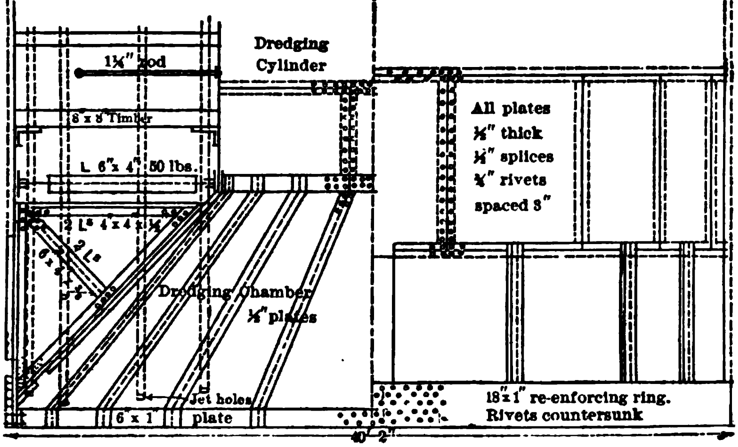
Plate diaphragms built of $\frac{1}{2}$-inch web-plates and $4 \times 4 \times \frac{1}{2}$ inch angle-irons in pairs are riveted to the shells and set on radial lines across the annular space. There are 20 of these diaphragms, which extend to a height of 10 feet above the cutting edge, dividing the bottom section of the crib into pockets, and at the same time strengthening and stiffening the sides of the lower 10-feet section of the crib. Immediately above these and riveted to the two iron casings is placed a horizontal ring-shaped lattice girder. The flanges of this girder are built of plates 18 inches wide and $\frac{1}{2}$ thick, and two 6×4 inch angle-irons weighing 42 pounds per yard. The transverse lateral braces are 6×4 inch angle-irons, in pairs, weighing 50 pounds per yard. The above-described construction constitutes the crib proper, which has a total height above the cutting edge of 16 feet. Above this the two concentric cylinders are built to an additional height of 100 feet, making a total height of 116 feet. These upper cylinders are made of $\frac{3}{8}$-inch steel plates, which are braced across the annular space by timbers 8×8 inches in cross-section, resting on and bolted to brackets which are riveted to the shells. Tension braces are also used, made of $1\frac{1}{4}$-inch iron held between $4 \times 4 \times \frac{3}{8}$ inch angle-irons riveted to the shells or casings. These tension rods are provided with turnbuckles in order to bring all parts to a firm bearing.

In order to facilitate the sinking, and to produce an effect similar to that caused by the escape of air under the cutting edge, which was found to have been so advantageous in sinking the cylinders



HORIZONTAL SECTIONS AND PLAN.

FIG. 195(a).



HALF VERTICAL SECTION. HALF ELEVATION.

FIG. 195(b).

PLAN AND SECTION OF OPEN CRIB MADE OF STEEL, FOR PIVOT PIER, OMAHA, NEB.

of the Hawarden foundations, which was fully explained in previous descriptions, twenty jet-pipes were built in the concrete filling in the annular space between the shells; these opened at or near the cutting edge, where the diameter was reduced to 1 inch. At vertical intervals of 10 feet branch pipes were led from the main 3-inch pipes through the outer shell. These branch pipes were ¾ inch in diameter. This latter is a somewhat novel application of the water-jet for reducing frictional resistance on the exterior surface of cylinders and cribs. After launching, and removing the false bottom used, the caisson drew 6½ feet of water. This represented the weight of the lower section. Concreting between the cylinders, building up the sides, and dredging were then carried on together.

From the cutting edge to the lattice girder Portland cement 1 and sand 2½ were used for the concrete filling. Above these Louisville cement 1, sand 1½, were used. Large stones were bedded in this concrete.

The top 3 feet were filled with Portland cement concrete. A masonry shell was then started on the concrete, and continued as the cylinder proper sank below the water. Within the masonry shell the inner-lining cylinder was built up, which acted as a coffer-dam and protected the masonry backing. When the crib finally rested on its proper foundation-bed, the dredging-tube was filled with Portland cement concrete, deposited under water, up to a point about 50 feet below the top of the pier. The water was then pumped out. Above this Louisville cement concrete was used to within 5 feet of the coping; on which was placed Portland cement concrete.

The pier consisted of a facing of sandstone masonry filled in with concrete backing. The body of the pier is 38 feet in diameter; two courses of coping with projections gave a diameter of 40 feet on top.

To a depth of 50 feet below low water the material passed through was fine river sand mixed with clay. A number of heavy logs were encountered, the removal of which greatly interfered with and retarded the sinking. Below this depth the material was clean coarse sand and occasional bowlders. This material continued to a depth of about 6 feet above the rock, where large bowlders were encountered. At this level the sinking was stopped and filling the dredge-chamber commenced, the structure resting on this layer of bowlders at a depth of 115.9 feet below low water, the depth to solid

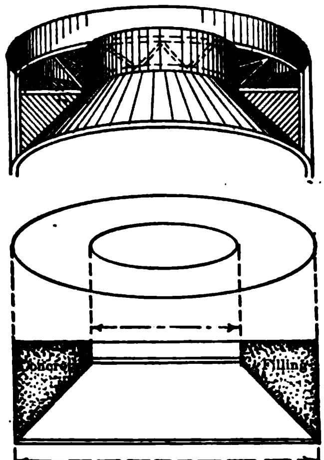
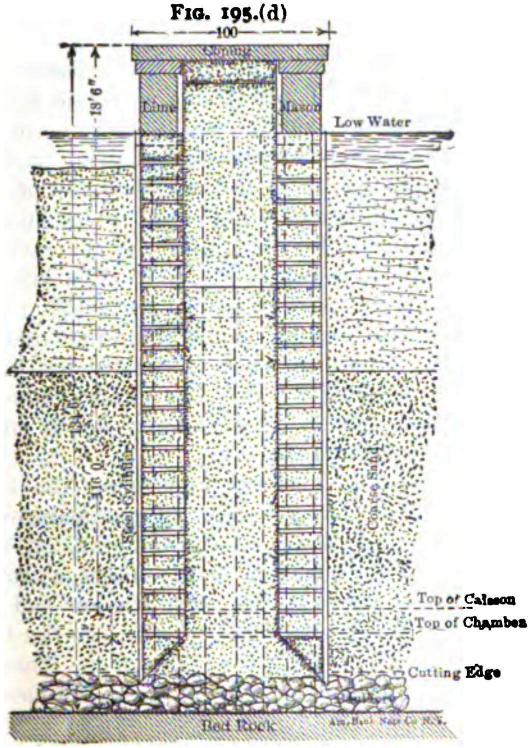


FIG. 195. (c)

FIG. 195.(d)



rock being about 120 feet below low water. The rate of sinking was from 1½ to 2 feet in 24 hours. The height of the masonry pier was only 18½ feet. There was required 5792 cubic yards of concrete, and 376 cubic yards of stone masonry.

The time occupied in the sinking covered a period of about 3½ months, including delays and interruptions from logs, floods, and the severity of the winter. The final completion required nearly a month more.

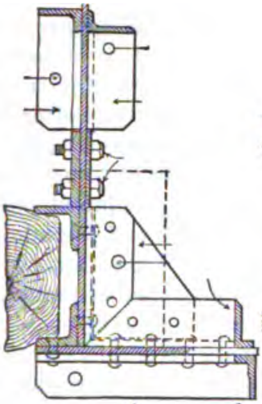
The water-jets facilitated both the sinking and keeping the caisson level and in position.

In Fig. 195(*a*) is shown half plan and horizontal projection of crib, and in Fig. 195(*b*) half elevation and half vertical section. In Fig. 195(*c*) is shown a perspective view of top and bottom of the cylinder, cutting edge, and inside dredging-cylinder, and in Fig. 195(*d*) a vertical section through completed cylinder drawn to a much smaller scale.

THE TOWER BRIDGE, LONDON, ENGLAND.

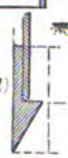
429*b*. There are many novel and interesting features in the design and construction of the Tower Bridge, across the River Thames. The location necessitated either an open span or one so high above the water as not to obstruct navigation; but owing to the very low banks on both sides of the river the necessary cost of approaches practically precluded the use of a high-level bridge, independently of its great inconvenience for the traffic. A general design and elevation of the bridge is shown in Fig. 377. The bridge consists of three spans—two shore spans each 270 feet long, and a channel span 200 feet long; above which, and at the proper height to give sufficient clearance, a fixed footway is constructed between the main towers. The main river towers and smaller shore towers are made of steel columns covered with masonry, and in addition masonry abutments and anchorages.

Foundation-beds.—The material underlying the bed of the river is the well-known London clay. Experiments were made to determine the bearing power of the material. A trial cylinder, sunk into the clay, began to settle under a weight of 6½ tons per square foot. Allowing for skin friction and buoyancy, estimated on the usual data, the actual pressure on the foundation-bed does not exceed 3000 pounds per square foot. These were not taken into consideration, and the dimensions of the piers and cais-

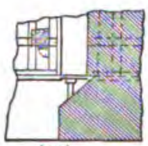


(i)

(k)



(l)



(m)

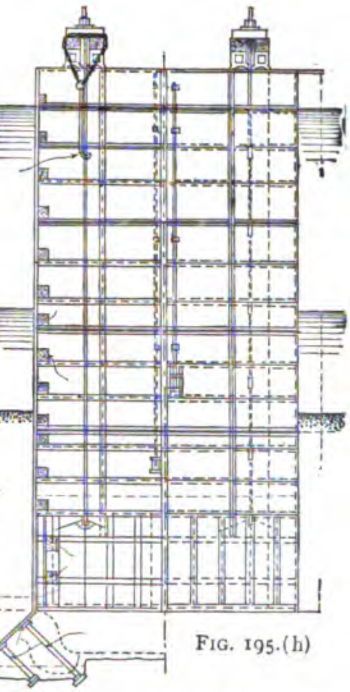


FIG. 195.(h)

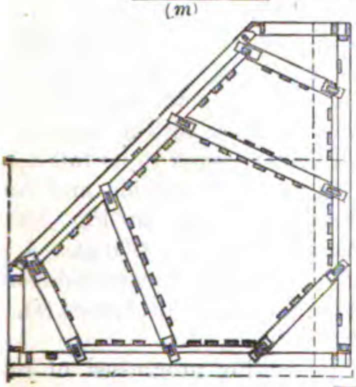


FIG. 195. (e)

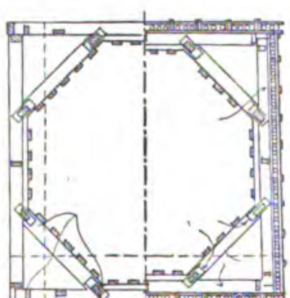


FIG. 195. (f)

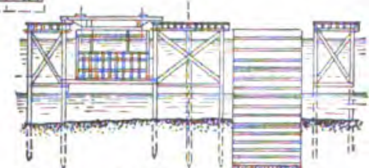


FIG. 195. (g)

sons were made such that the entire weight of structure and load should not be more than 4 tons per square foot of base. The Memphis Bridge has a total of 5 tons per square foot on a compact clay resembling the London clay, and about 1⅜ tons after deducting friction and buoyancy. As the middle span was a bascule span, operated by powerful machinery, it was important to preclude possibility of any settlement. This construction required both long and thick piers for the river towers, independent of the dimensions required to reduce the unit pressure on the base. Owing to the necessity of a number of openings or hollow chambers, required for the free movement of the counterpoise arms, and the proper working of machinery, gearing, etc., the main river piers were somewhat irregular in section, the general length of the pier proper being about 185 feet and width 70 feet, of the caisson proper 90 feet wide and 195 feet long, and extreme bottom dimensions 100 feet wide and 204 feet long. Sections and plans are shown in Fig. 195 (*a*), (*b*), and (*c*). There is no record of a single caisson of as great dimensions as those given above; the caissons of the East River Bridge were very large, the largest having bottom dimensions of 172 × 102 feet. A single caisson could have been used, but, for whatever reason, it was determined in this case to use a number of small steel caissons or cribs, and to sink them by dredging the material from the inside. Twelve small caissons were used; four of these 28 × 28 feet square were used on each longitudinal face, and at each end two of a triangular section. These were sunk in juxtaposition around the outline of the piers, enclosing a central space. Plans of the square and triangular caissons are shown in Fig. 195 (*e*) and (*f*); also a cross-section through the structure, showing one caisson sunk to its full depth; and another in process of sinking, with the outside and inside staging used in sinking the caissons, is shown in Fig. 195(*g*). The positions of these caissons are shown in Fig. 195(*g*). A part elevation and vertical section of one of the square caissons is shown in Fig. 195(*h*). Also, details are shown in figures (*i*), (*k*), (*l*), and (*m*).

The caissons were built up, as the sinking progressed, of plates and angle-irons. The lowest section, 19 feet in height, constituted the caisson proper, and was to be left permanently at the bottom of the structure. The remaining portion above was simply intended for a temporary coffer-dam, and was subsequently removed; other-

wise the general construction is the same as in any iron and steel caisson or crib.

The small caissons were spaced with clear intervals between them of 2½ feet. This is indicated in Fig. 195 (*e*) and (*f*). On the adjacent faces of any two caissons angle-irons were riveted, as shown. These formed grooves or channels into which piles were subsequently driven, thereby making a tight joint between two adjacent caissons, in order to permit of the removal of the adjacent faces so as to secure a monolithic structure throughout. The details of these angle-iron grooves and closing piles are shown in

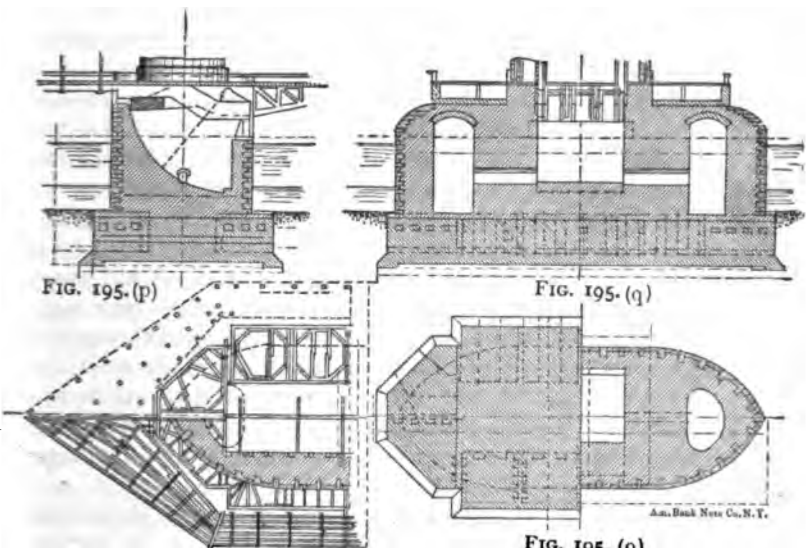


FIG. 195. (p) FIG. 195. (q) FIG. 195. (o) FIG. 195. (n)

Fig. 195(*i*). This portion of the construction will be again referred to under Sinking and Excavating. In order to control and regulate the sinking of the several caissons, a system of stagings were erected around, and also a central staging in the space to be enclosed by caissons. This staging is shown in plan in Fig. 195(*n*), and in section in Fig. 195(*g*). The lower sections of the caissons were built on platforms placed a little distance above low water. Two pairs of trussed girders were then placed on top of the staging, and above the section of the caisson. Rods made of links about 8 feet long, and joined with pins, were used to suspend the caisson from the girders. These rods were 2¼ inches in diameter. The

upper length of each suspending rod was 14 feet long, and had screw-threads cut for a length of 9 feet from their upper ends, which, working through nuts, enabled the sinking to be regulated and controlled. As the caissons were lowered, and it became necessary to add another section on top, the rods were caught by clips at some point below the upper threaded length, a new section built, an additional link added to the rod, and the lowering continued as before. The rods, connections, etc., are shown in Fig. 195(*h*). The material excavated was principally clay. This was removed as far as practicable by means of Priestman grabs, generally to a depth of 5 or 6 feet in the centre of the caisson. Divers were then sent down, who removed the material from around and under the cutting edge, shovelling it towards the middle, where it could be reached by the grabs. Four divers in the square caissons and six in the triangular were worked in double shifts nine hours per day. To remove this material required from two to three hours a day with the dredges. Weights were added to aid in sinking the caisson after it had reached a depth of from 4 to 5 feet in the clay. Seventy-five tons of cast-iron blocks forced the caisson from 1 to 1½ feet into the solid clay. The water was then pumped out, but owing to some error in fixing the height of the staging, which prevented the construction of the third section of the caisson until the cutting edge had reached a depth of 16 feet below the bed of the river, the caisson filled at high tide, and had to be pumped out. After this depth was reached the water was pumped out, and kept out until the final depth of 19 feet below the bed of the river, or 53 feet below mean high water, was reached.

As shown in Fig. 195 (*h*), (*p*), and (*q*), when this depth was reached an undercut was made to increase the bearing surface, having a base of 5 feet on the outside and 2½ feet on the sides next to the adjacent caissons, and to a depth of 7 feet below the cutting edge of the caisson.

The shape and dimension of this undercut are shown in the several sections. This undercutting was done in sections of 3 or 4 feet square, which were sheeted and braced as shown on the outer surface. Each section was filled with concrete when completed to a point six inches above the cutting edge. When this was completed entirely around, the lowering rods were removed, and the caisson rested on this toe of concrete. At first the rate of sinking was only 8 inches per day. This increased during the tide

work to 16 inches, and when the water was permanently excluded the rate of progress was as much as 3¾ feet per day.

Concreting.—After sealing up around and under the shoe the rivets holding the side facing the enclosed central portion to the ends of the caisson were cut, and to prevent the adhesion of the mortar to that side boards were placed against it. In addition boxes were placed against this side, and the concreting was commenced and carried up to within a few inches of the fifth or top frame of the permanent casing or caisson. The boards and boxes were built up as the concreting progressed. The purpose of the boxes was to leave recesses or dovetails, so that when the inner side of the caisson was removed and the enclosed central space filled with concrete it would tooth or bond into the concrete already placed in the caissons proper.

The lower section or permanent portion of the caisson was 19 feet high. When the concrete reached the third frame of this portion, boards and boxes were placed against those sides of the caisson facing the adjoining one, and for the same purposes, as it was intended to remove three sides of the caissons only, leaving the outside shell of iron, it was important that the many distinct columns of concrete should be toothed or bonded into each other. Thus each of the twelve caissons was sunk and filled with concrete to the top of the permanent caisson, giving as many isolated columns of concrete around the outer face of the structure. These columns were separated by the iron sides of the caisson and 2½ feet of clear and open space between them. Before these adjacent sides could be removed it was necessary to make a water-tight connection between the caissons. It was to accomplish this that the grooves were formed by angle-irons on the outside edges of the caisson. Before driving piles in these grooves the caissons were well tied together in order to prevent the wedging action of the piles from spreading them apart, and they were properly braced as shown in Fig. 195(n), in order to prevent any distortion. Piles were then driven in the grooves, as shown in section in Fig. 195(i). This, then, gave water-tight compartments of about 28 × 2½ feet between the caissons. The water was pumped out and the material dredged out by hand, two men working at a time, the material being lifted out in buckets. When the proper depth was reached concrete was placed in these spaces up to the top of the third frame of the permanent caisson. The iron sides of the caisson were then removed, and concrete filling connecting and

bonding the columns was completed. By this means a solid wall of concrete about 28 feet thick was completed, and to a height of 19 feet above the cutting edge and 7 feet below. On top of the concrete and from the level of the bed of the river the masonry was carried up. First a 2-foot course of brickwork was laid entirely over the surface for a base for the granite and brickwork of the pier above. The form of this masonry is shown in Figs. 195 (*o*) and (*q*). It consisted of a facing of rough-picked Cornish granite, in courses of 2 to 2½ feet rise, backed with brick masonry.

All concrete was composed of 1 part Portland cement to 6 parts Thames ballast, spread and rammed in layers of about 18 inches thickness. The mortar for the granite facing was 1½ to 1, and from 1½ to 2½ to 1 for brickwork, using Portland cement. After these walls were completed well above high water the remaining iron on the outer and inner faces of the original caissons, or rather coffer-dams, was removed. There was then a solid enclosing wall of concrete surmounted by granite and brick walls, enclosing a large space, which thus far had remained full of water. Braces were placed across this space from wall to wall at about the water-level. The enclosed space was then pumped out. The excavation was then carried on between concrete walls enclosed in the permanent caissons, and at intervals of about 10 feet below the bed of the river additional braces were inserted and the iron sides of the caissons removed. When the excavation was completed the enclosed space was filled with concrete and other masonry, completing the entire pier to the top. The time occupied in completing the two piers was about four years, the cost of the two $555,600, the total masonry of all kinds 50,960 cubic yards, and the cost per cubic yard, including everything, $10.90.

How far local conditions and circumstances may have controlled the selection of a number of small caissons instead of one large one, and the great length of time taken in completing these two piers—the total depth to which the caissons were sunk being 53 feet below low water and only 19 feet into the compact clay—we do not know, and do not intend to make any invidious comparisons by what is said below. But it will be instructive to call attention to the difference in methods that would probably have been adopted and to give a few examples of the time taken in sinking caissons and completing some large structures in this country.

A single large caisson would probably have been used, built of

timber and concrete or of iron and concrete. For a depth of only 40 feet below low water the method of sinking, whether by dredging in an open crib or by compressed air, would not be a matter of much moment, other than from considerations of economy; but probably the pneumatic process would have been adopted.

The aggregate area of the bottom of the tower caissons is about 33,000 square feet, and depth sunk below low-water surface 40.6 feet.

The aggregate area of the bottom of the five caissons for the Memphis Bridge is about 67,000 square feet, and average depth sunk below low water 70 feet. These were sunk in two years and five months through 40 to 50 feet of sand and clay.

The large caisson of the Washington Bridge, New York, 104.8 × 54.4 feet, was sunk 40.6 feet in six months through sand, gravel, and rock, and 10,400 cubic yards of masonry completed in nine months.

In both of the above cases the pneumatic caisson was used.

The large timber crib of the Poughkeepsie Bridge, 100 × 60 feet and 104 feet high, was sunk by open dredging through 53 feet of water and 82 feet of mud and mixed sand and clay in about three months, and the caisson sunk onto this crib and the masonry completed to a point 149 feet above the cutting edge of the crib, ready for the steel towers, in about three months more. (See *Eng. News*, Jan. 18, 1894.)

Greater depths have been reached by this open-crib process than by any other. The great difficulty lies in the danger of meeting obstacles, such as wrecks of old ships, barges, logs, etc., which retard the progress and may result in entirely stopping the work. The more serious objection is the necessity of depositing the concrete under water. Whether this is done through iron or timber tubes, or in specially designed buckets which only open when the bottom is reached, the cement will inevitably be separated to a greater or less extent from the sand and broken stone. The resulting product is at best uncertain. The only remedy known to the author is to allow the cement to take an initial set before lowering it through the water. It has, however, been used to a great extent when the depth below the water surface is over 100 feet, as this is considered the limit under the pneumatic process, next to be described.

The splaying or spreading out at the bottom has always been considered necessary in sinking such cribs or caissons. Mr. Ander-

son, as a result of his experience in this kind of work, states that it is not only unnecessary, but that the crib can be sunk in better position with vertical sides and ends from top to bottom.

PNEUMATIC CAISSONS.

430. If a roof is constructed closing the interior space of the cylinders or the dredging-tubes of the open cribs just described, they would be converted into pneumatic caissons, and could be used as such. But commonly pneumatic caissons are constructed in an entirely different manner. As the name indicates, air is an essential accompaniment, which, when its tension is increased, can be used to exclude water from a box or cylinder having only one end open, and immersed with the open end downward. If, then, a cylinder or a strong timber or iron box is constructed with air-tight sides and top, and sunk by weights to the bed of a stream, and a pipe is built through the top, reaching to the surface and connected with an air-compressor, air can be forced into the interior of the cylinder or box; if its tension is proportioned to the depth of water above the lower or cutting edge, the water will be forced out of the interior space, and a pressure due to it will be exerted on the under side of the roof or deck, tending to lift the box. If, also, a large shaft 4 or 5 feet in diameter is built through the deck, and extended to the surface, having two air-tight doors which can close the shaft and prevent the escape of the compressed air, both doors opening downwards or against the pressure,—only one of which is or need be closed at the same time,—we have a structure called a pneumatic caisson, which is similar in its operation and construction to a diving-bell on a large scale. Other pipes and shafts are also built through the deck for the purpose of removing the material from the interior or for passing material from without into the box. All spaces between the pipes or shafts and the deck of the caisson must be made air-tight, and must have valves, doors, or caps by which the shafts and pipes themselves can be closed. That portion of the large or main shaft closed by the doors is called an air-lock. This may be at the top, bottom, or any intermediate portion of the shaft. It may be a section of the shaft itself, or may be a specially designed section of larger or smaller diameter, and fitted to or connected with the main shaft. Both the position and design of the air-lock vary in the practice of different engineers. The writer prefers the air-lock to

be a simple section of the main shaft, and to keep it always at the top.

431. The necessary air-pressure varies with the depth of water, or rather the depth of the cutting edge of the caisson below the water surface. A pressure of one atmosphere or 15 pounds—more accurately, 14.7 pounds—to the square inch will support a column of water *in vacuo* 34 feet in height, or about 1 pound pressure for every $2\frac{1}{4}$ feet = 27 inches. If, then, we wish to support a column of water 68 feet in height but not *in vacuo*, it would be necessary to provide a pressure at bottom of 45 pounds,—15 pounds to balance the air-pressure on top and 30 pounds to balance the weight of the column of water.

432. Only using the double walls, omitting the tubes and covering the enclosed space between the walls with an iron roof composed of strong eye-beams placed transversely at intervals of 4 or 5 feet, to the under side of which a full layer of $\frac{1}{2}$-inch plate iron is riveted, Figs. 190–194 will represent the construction of a typical iron caisson. The roof would be usually built at the top of the lower wedge-shaped section, along *AB*, Fig. 190, the height above the bottom being usually from 7 to 9 feet. The iron walls above this roof may or may not be used. The writer believes it is always advisable to use them. Practice on this point differs. The construction of iron caissons will not be further discussed.

433. The following drawings, Figs. 196, 197, show the construction of two typical timber caissons, with air-locks, pipes, shafts, cribs, and coffer-dams used in connection with them.

The vacant enclosed space *A* is the working chamber; *BB* are the walls of the working chamber; open or solid timber work resting on *BB* is called the roof or deck of the caisson, as shown in *CCDD*. The timber and concrete work above the roof is not an essential part of the caisson. The masonry can be commenced directly on top of the roof or deck; this requires the sinking of the caisson to be regulated by the rapidity with which the masonry can be built, which may interfere with the progress of the work. In case of accident the entire masonry work might be submerged, causing both trouble and expense in removing and keeping the water out so as to resume the work. The piers of two of the most important structures in this country, namely, the East River suspension bridge and the St. Louis steel-arch bridge over the Mississippi River, were constructed in this manner.

434. The cribs, as shown in the drawings above the roof, are

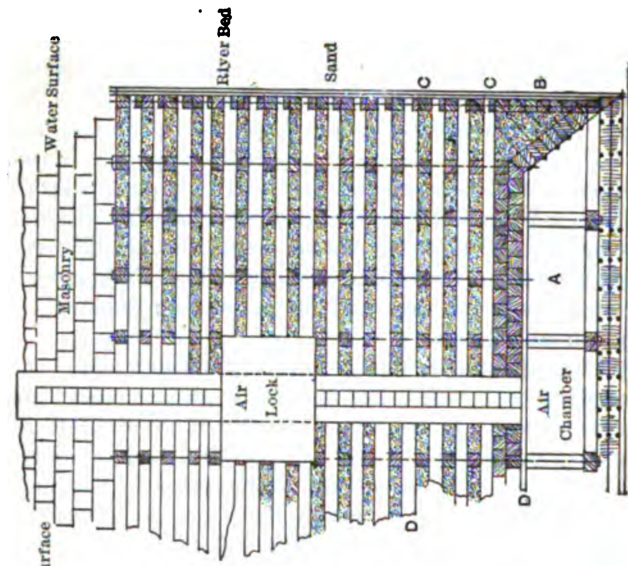


FIG. 196.
VERTICAL SECTION, THROUGH CAISSON, CRIB AND MASONRY.

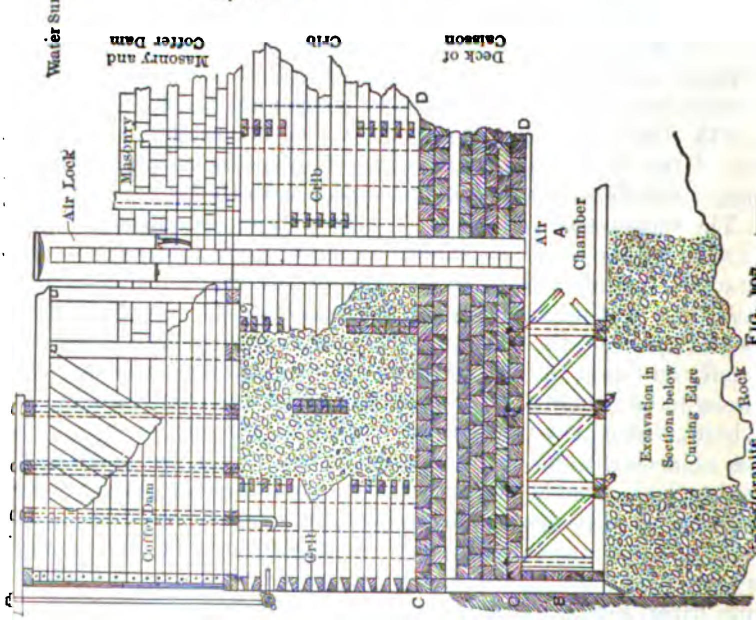


FIG. 197.
VERTICAL SECTION, THROUGH CAISSON, CRIB AND COFFER DAM SHOWING MASONRY IN COFFER DAM

built of timber, and the spaces between are filled with concrete. Whether the cribs shall be constructed with solid outside walls, and solid partition walls built as shown in the drawings, the partitions being carried up solid for five or more courses, then shifted in position, alternating in number 2 and 3 or 3 and 4 according to the size of the crib as shown in Figs. 197–199, by which the concrete is formed into one practically solid and homogeneous mass; or whether the crib shall be open-work timbers simply crossing each other in courses at right angles, thereby dividing the concrete into a series of small columns but poorly connected by horizontal layers of 12 inches in thickness, in addition to the necessarily great difficulty of filling properly under and around so many timbers, and the further danger of the concrete being exposed to moving water before time has elapsed for the setting of the mortar, as seen in Figs. 196, 198—is simply a difference of opinion and practice. Large and important structures have been built on both designs. And whether the walls of the working-chamber shall be constructed as in the drawings, the same remarks apply.

In either case the timber and concrete cribs are more economical, can be built more rapidly, and hence their tops can be kept more certainly above the water than when the masonry is constructed direct on the caisson.

The caissons and cribs shown in Figs. 196–198 are from the designs of Mr. Geo. S. Morison, one of the most prominent engineers and bridge-builders in this country, and have, consequently, the sanction of a high authority. They have been used in many important bridges. The one illustrated was used in the Cairo Bridge across the Ohio River, near its junction with the Mississippi River. Fig. 196 is part longitudinal and vertical section. The drawing shows about two thirds of the full length. The bulk of the timber used is 12 × 12 inches in cross-section. The sloping walls of the working-chamber are formed of 17 × 17 inch timbers. All parts of the structure are thoroughly bolted with screw and drift bolts, and well braced. These caissons were shod with iron plates $\frac{5}{8}$ inch in thickness and 36 inches high, to protect the cutting edge.

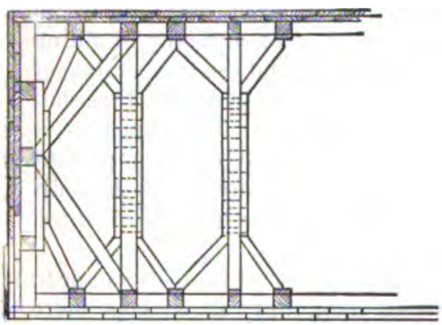
The greatest immersion in water was 90.27 feet, and penetration in sand 86.42 feet. The masonry was started 10 feet below the bed of the river, and on top of the crib.

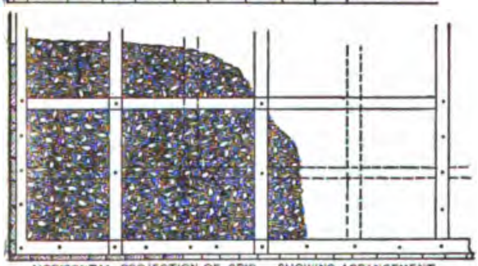
The two designs are shown side by side. Only a part of each is shown, about two thirds, in order to show air-locks, shafts, and pipes

in place. The omitted portion is identical in construction with the parts shown.

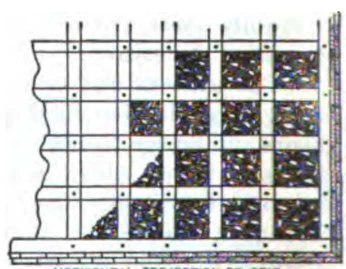
435. The caissons and cribs shown in Figs. 197–199 are from the designs of the author, and have been used in many important foundations, notably those of the Susquehanna River Bridge, B. & O. Ry., at Havre de Grace. Fig. 197 shows part longitudinal and vertical section and part elevation through coffer-dam, crib, and

HORIZONTAL SECTION OF COFFER-DAM.
FIG. 200.





HORIZONTAL PROJECTION OF CRIB. SHOWING ARRANGEMENT OF BRACES AND CONCRETE IN PLACE. FROM FIG. 197.
FIG. 199.



HORIZONTAL PROJECTION OF CRIB, BRACES AND CONCRETE, FROM FIG. 197.
FIG. 198.

caisson; Fig. 199 shows horizontal projection of crib; and Fig. 200 horizontal projection of coffer-dam, showing one set of the system of cross-bracing adopted.

All timbers, excepting the planks 3 × 12 inches, are 12 × 12 inches in cross-section for crib and coffer-dam. All timbers in caisson were also 12 × 12 inches, except the outside vertical pieces, which were 12 × 14 inches in cross-section, and the lining plank on the inside of the air or working-chamber. In some of the larger caissons the cross-braces were 16 × 16 inches square. The whole was thoroughly bolted with screw and drift bolts. The drift-bolts

in the solid timber roof were 1 inch square or round and 22 inches long, spaced 5 feet intervals over each course.

In the largest caisson the deck was composed of eight courses of timber, alternating in direction longitudinally, transversely, and diagonally. Each course was bedded on a layer of cement mortar, and spaced from $\frac{1}{4}$ to $\frac{1}{2}$ inch intervals, which was filled with grout before laying the course above it. The joints between the planks forming the interior lining of the roof and walls of the working chamber were thoroughly calked with oakum. Oakum was also wrapped around the ends of all bolts reaching into the interior against which the nut and washer were pressed hard. The joints on the outside of the caisson were calked, but not so compactly. The walls of the crib were also calked sufficiently to make them water-tight. This was done that the concrete in the crib might always be laid in the dry. Outside vertical plank was spiked to the crib, mainly to hold the oakum in place; it also reduced somewhat the frictional resistance of the material below the bed of the river.

436. It was originally intended that the top of the crib should only reach to the bed of the stream. Owing, however, to the great inclination of the bed-rock, and the time and expense required to blast it to a level or horizontal surface, it was decided to stop sinking the caisson when it rested on or near the higher points of the rock; therefore the top of the cribs reached from 5 to 13 feet above the bed, and were from 5 to 35 feet below the water surface. The masonry was commenced a little before the top of the crib sank below the water surface. No coffer-dam was used on the first crib, the masonry being built up as the caisson sank. The risks and delays caused by this method determined the use of coffer-dams on all other caissons.

437. The coffer-dams were built with caps, sills, and vertical posts framed together, and resting on top of the crib. These frames were sheeted with two courses of 3-inch plank, the first placed diagonally, the outer course horizontally, as shown in section and elevation in Fig. 197, and in plan Fig. 200. Crosspieces were placed over the tops and held down to the cribs by long iron rods, having hooks at the lower ends catching hold of eye-bolts fastened to the cribs. When these bolts were unhooked the sides and ends of the dam could be pulled apart and removed. This was done after the caissons rested on rock, and the masonry was built well above the water. Fig. 200 shows plan of coffer-dam and system of interior bracing adopted.

A longitudinal truss (see Fig. 197) was used to stiffen the roof. The design of the cutting edge provided broad surfaces for bearing on the material, thereby supporting the caisson when desired, and enabling the sinking to be regulated. The truss could be used for the same purpose. At the same time the men had easy access to the cutting edge. The design of the walls was very strong and well connected, and supported by the deck of the caisson. No iron shoe or plates were used to protect the cutting edge, experience amply proving that they are not necessary. It may, however, be wise to use them.

438. Air-locks are used for the passage of men and material, either from the outside to inside of the caisson, or the reverse, without the withdrawal or escape of more than one lockful of compressed air for each such passage. They may be single or double. Double air-locks were used in the Cairo Bridge and others

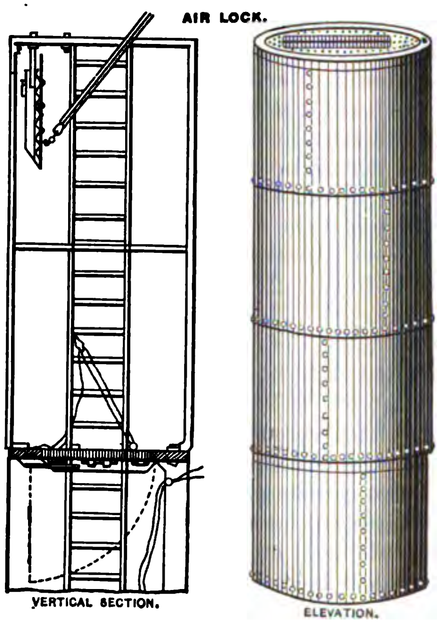


FIG. 201.

constructed by Mr. Morison, and Fig. 201 shows the single air-lock used by the writer in other caissons described. A description of the working of the single air-lock will render the understanding of the other easy by a simple examination of the drawing.

439. Assuming that the air-chamber and shaft contain contain compressed air, and that the lower door is closed and held against its bearings by the compressed air, the upper door of the lock is in this case open. The men enter the air-lock from above and close the upper door, opening at the same time a valve placed in the lower door, or the one of a pair in the double lock, and the compressed air rushes into the lock; its pressure now holds the upper door shut. The air will continue to flow into the lock until the pressure is the same as in the air-chamber. The lower door now opens readily, as the pressure is the same on both sides. The men then descend into the air-chamber. In coming out they pass into the air-lock through the lower and open door, closing this behind them; then, opening a valve in the upper door, the compressed air rapidly escapes from the air-lock, and continues to do so until it is reduced to the atmospheric pressure on the outside when the upper door opens and the men pass out.

In the double lock the doors open sideways or horizontally instead of vertically, as above described for the single lock. The principle is the same: one of each pair of doors is shut when the other is open. Two independent locks are thus formed, each opening at one end into the shaft above and that below by independent doors—these shafts not being directly connected, the two overlapping, as it were. In these caissons the lock is placed in the cribwork, and only 8 feet above the deck of the caisson (see Fig. 196).

In the other case (see Fig. 197) the lock is placed at the top of the shaft, and always above the water surface. The writer believes that this is the safer plan of the two, the men having a better chance of escape in case of an accident.

In either case the lock is usually made of $\frac{1}{4}$-inch iron plates riveted together, and stiffened by angle-irons. The dimensions of the lock vary in the practice of different engineers. This double lock was $9' \times 7' \times 6'$; the single lock was 15 feet high and 4 feet in diameter.

440. The air-pipe is usually 4 inches in diameter, and closed at the lower end by an automatic valve. The discharge-pipes may be from 3 to 4 inches, and the water-pipe for working the sand or mud pump and other purposes is about 6 inches, in diameter.

441. The operation of sinking the caisson is simple. The caisson is partly built on land, then launched, and completed while floating. It is towed to its proper site, and the building of the crib is commenced. The pockets of the crib are filled with concrete as it

sinks. When resting firmly on the bed of the stream air is forced into the air-chamber, and the men descend and commence excavating the material. This, if gravel and sand or finely divided silt, clay, or ordinary earth, is piled around the bottom of the discharge-pipes, the valves are opened, and the material is blown up and out of the pipes with great rapidity and violence. The only skill here is to avoid having the pipes choked up.

For stiff and compact clays or silts it is first necessary to cut the material up with a water-jet, when they can likewise be blown out. Or it may be better, for these materials, to use the mud or sand pump. This is shown in Fig. 202*f*. Water is forced down through a pipe and into the pump. It then passes with great pressure through the annular space around the mouth of the suction-pipe. This creates a partial vacuum, and the material is sucked up through a hose connected to the bottom of the suction-pipe, and passes on upwards with the water.

442. In whatever manner the material is moved from the air-chamber and under the cutting edge the resistance is reduced, and if the outside friction is not too great the caisson will settle gradually. It is, however, more commonly necessary, after removing the material, to reduce the air-pressure by allowing the air to escape through pipes. When sufficiently reduced the caisson will sink from a few inches to several feet. When it stops, or when sunk as far as desirable, the air-pressure is raised and the sinking ceases. Men then descend and commence excavating again. The same operations are repeated.

A very slight reduction of the pressure sets up a dense fog. The loss of air is the principal objection to the blowing-out process of removing the material.

Large bowlders are either simply carried down with the caisson or must be brought out through the main lock and shaft.

443. When a sufficient depth is reached, or a good firm bed is found, the bed is levelled off and the air-chamber filled with concrete. This concrete is usually passed down through a supply-pipe about 18 inches in diameter. The entire length of this shaft is converted into an air-lock by attaching a door at top and bottom. With the lower door closed and properly supported the concrete is thrown into the shaft. When from one-half to one cubic yard has been thrown in, the upper door is closed, the air-pressure equalized, the lower door opened, and the concrete falls on a platform in the working-chamber, whence it is shovelled or carried in barrows, deposited, and rammed.

444. The assumed limit of depth for this process is 100 feet below the water surface. For greater depths the open-crib process has been of late commonly adopted.

445. When a rock bed is reached the rock can be levelled off, or cut into steps if sloping. The first is slow and expensive. The second was adopted by the writer at the Susquehanna and Schuylkill bridges, B. & O. Ry.

The difference of level of the rock within the limits of the caisson varied from 4 to 21 feet. The method adopted in the latter case was to sink the caisson about 8 feet at the highest point, leaving the rock 13 feet below the cutting edge at the other end of the caisson, and varying more or less irregularly between the two. Pits were then excavated to the rock, in sizes about 8 or 10 feet square, and the bottom levelled or well roughened; these were then filled up to and under the cutting edge with concrete (see Fig. 197, below caisson), after which the working-chamber was filled. In some cases, after building a wall of concrete around the cutting edge, to save expense the remaining portion of the caisson has been filled with sand.

446. The depths reached below water surface in some of the largest and most recent bridge structures, together with the sizes of the cribs or caisson, and the materials upon which the structure finally rested, are given in the table below.

TABLE XLVIII.

	Depth Sunk below Water Surface in feet.	Material and Dimensions of Caissons in feet.	Character of Foundation-bed.
Hawkesbury, open crib.	155	Iron.... 52 × 24	Compact gravel.
Poughkeepsie, open crib.	135	Timber, 60 × 100	Compact gravel.
Memphis, caisson.......	108	Timber, 40 × 22 to 92 × 47	Clay.
Cairo, caisson..........	94	Timber, 60 × 26 to 70 × 30	Sand.
Susquehanna, caisson...	94	Timber, 63½ × 26 to 78 × 42½	Rock (solid).

These are the greatest depths reached. Some few caissons of larger dimensions have been used, and many smaller ones.

447. The height of the caisson proper is rarely over 18 feet;

this consists of a working-chamber from 7 to 9 feet pitch, and the thickness of roof or deck, from 1 to 3 feet if of iron, if of timber 8 or 10 feet. The crib on top, whether constructed of iron or timber, may be of any height desired, as determined by the point below the water surface at which the masonry is to be commenced.

448. The effect upon the men working under great air-pressure is not well understood. Some men suffer from pains in the ears; all are liable to pains in the limbs—of a temporary nature, usually passing off in a day or two; a few are stricken with some form of paralysis, either temporary or permanent; and a rather small percentage die. These effects, except the pains in the ear, never occur while under the pressure, but generally in a few minutes after coming out.

SUBSTRUCTURE AND FOUNDATIONS OF THE MISSISSIPPI RIVER BRIDGE, MEMPHIS, TENN.

As seen in Fig. 202(*g*), showing a section of the river and underlying strata, there are four piers and two abutments or anchorage piers. Piers 1 and 4 are close to the banks and may be called shore piers, while 2 and 3 are river piers. High-water surface is about 35 feet above that at low-water. Soundings gave the depth of the bed of the river below low water, at the site of Pier 2, 36 feet, and at the site of Pier 3, 21.6 feet. A layer of sand 47 and 52 feet, respectively, overlaid the clay stratum. This clay stratum, known as the LaGrange formation, is 150 feet thick and perfectly water-tight. One boring was made into this clay to the depth of about 47 feet. Under the clay lies a water-bearing sand or gravel stratum, the outcrop of which is perhaps fifty miles east of Memphis, and is at a somewhat greater distance on the west. The kinds of materials indicated by the borings are shown in section Fig. 202(*g*). The bore-holes are shown by full black lines.

It was evident that the clay must be reached, as it would be unsafe to build on the sand. Owing to the compact nature of the clay it was not deemed necessary to sink the caissons to any great depth into it, but it was determined to lighten as much as possible the weights of the caissons, cribs, and masonry, and to give a sufficient spread of base to keep the pressure in safe limits, even on a compressible clay. In order to do this it was determined to use a high grade of masonry, to reduce the dimensions of the piers to a minimum, and to build the lower portions of the

piers hollow, which, though objectionable in a cold climate, was deemed admissible in that latitude. The construction of the cais-

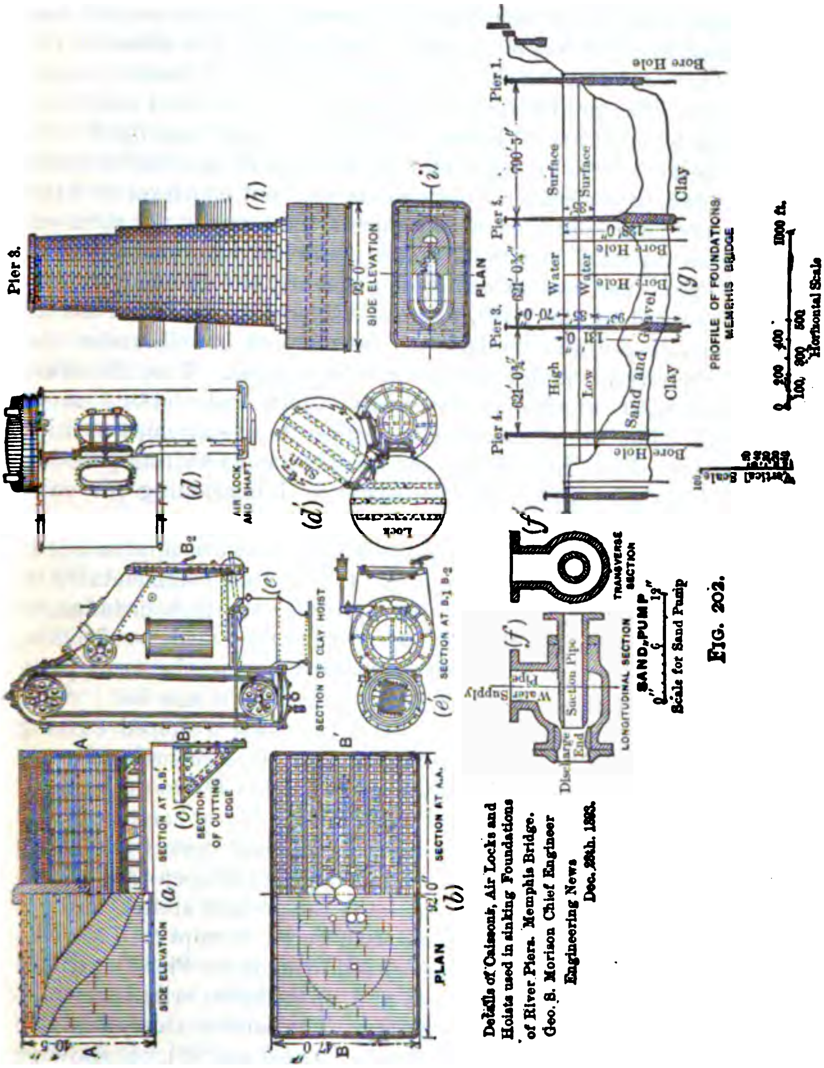


FIG. 202.

Details of Caissons, Air Locks and Hoists used in sinking Foundations of River Piers. Memphis Bridge. Geo. S. Morison Chief Engineer. Engineering News Dec. 29th. 1893.

son was such that its weight should not exceed that of the displaced sand bulk for bulk, and that, after deducting 400 pounds per square foot for friction on the exposed surfaces of caisson and crib, the

weight placed on top of the latter should not produce a pressure on the foundation-bed of clay exceeding two tons per square foot. The base required to thus limit the pressure for the caisson was calculated to be 92 feet long and 47 feet wide. The sides of the caisson below the bed of the river were built without batter, that is, vertically. The total heights of caisson proper, crib and solid-timber work about it are 59.4 feet for Pier 2 and 39.6 feet for Pier 3. These heights were adopted so that the top of the timber-work and the base of the masonry piers would be about on a level with the bed of the river. But, as often happens, matters did not turn out exactly as expected, and good material was found under Pier 2 at a less depth than was expected, consequently the caisson projects above the bed of the river, whereas at Pier 3 it was found necessary to go deeper, and the base of the masonry is well under the bed of the river. Such things cannot be avoided. They illustrate the necessarily uncertain data upon which calculations must be based. The same remark also applies to the attempt at nice adjustment of weights, friction on the sides, and resulting pressure on the foundation-bed referred to above in dimensioning the caisson. All of the elements entering are uncertain, and how near to two tons per square foot of bed the actual pressure approaches is unknown, and can never be known. It is at least commendable in the Chief Engineer, Geo. S. Morison, as it is the first instance, in the writer's knowledge, that any attempt has been made to provide, as far as possible, for a carefully estimated balance between pressures and resistances, however uncertain the result may be.

In the construction of the caissons the usual V-shaped cutting edge was used which has characterized all of Mr. Morison's caissons, as also, the open-work crib of timber to heights above the shoe of 44 feet for Pier 2 and 29.2 feet for Pier 3. To a height of 17.3 feet above the iron shoe the V-shaped walls and open work above were filled with concrete. Above this height the open-timber crib was only partially filled with concrete; the pockets around and adjacent to the sides and ends of the crib were left empty. This construction, together with 15.4 feet of solid timber on Pier 2 and 10.4 feet on Pier 3 above the concrete and open-timber crib, is a somewhat new departure in caisson design. The construction of caisson and crib for Pier 2 is shown in Figs. 202 (*a*) and (*b*), (*a*) showing part elevation and vertical section and (*b*) the top and horizontal section, (*c*) showing iron shoe. The construction as shown is designed

to reduce the weight in order to carry out the idea of making the weight of the caisson equal to that of the displaced sand, and at the same time reducing the cost by the amount of concrete saved, and the difference in cost between solid timber and timber and concrete, if any.

The vertical side walls are bound together by 54 2-inch rods, the lower lengths of which pass through the timbers, the nuts being screwed against the under side of the shoulder of the cutting edge and against washers on the top. In the upper part of the work these rods are placed immediately inside the timbers.

There are also 24 2-inch rods similarly placed in the cross-walls and connected with 1½-inch rods extending through the concrete. In addition, 112 1½-inch rods extend from the roof of the caisson to the top of the concrete, near the timber intersections; the inside walls of the V-shaped portion are tied to the outside walls with 96 1½-inch rods. The sides of each cross-wall are tied together by 36 1-inch bolts, and the timbers of the roof are tied together by 390 1-inch bolts. The successive courses of timber in the outer walls are driftbolted with 34-inch bolts, spaced 5 feet apart. The timbers of the inclined walls of the working-chamber are fastened with drift-bolts 30 inches long, spaced 3 feet apart; and the cross-walls are driftbolted as in the outer walls. The timbers of the solid filling above the concrete are held with 34 × 1 inch drift-bolts, spaced 8 feet apart in every stick. Alternating the position of these bolts in consecutive courses, there are continuous lines of drift-bolts, spaced 4 feet apart. The outside planking is fastened with two 7 × ½ inch boat-spikes per square foot of surface, and all other planking by two 7 × ⅜ inch boat-spikes per square foot.

The corners of the caissons are rounded and plated with ⅜-inch iron.

The largest caisson, for Pier 2, contains 1,548,000 feet B. M. of timber and 424,000 pounds of iron; for Pier 3, 1,078,000 feet B. M. of timber and 340,000 pounds of iron—equivalent to 273½ and 313½ pounds, respectively, of iron per 1000 feet B. M. of timber.

Each caisson was provided with four 24-inch supply-shafts for the removal of material and passing the concrete into the working-chamber. There was also one 36-inch shaft with double air-lock at the bottom, as shown in Fig. 202 (*a*) and (*b*), such as has been commonly used by Mr. Morison; and in addition one 6-foot shaft with special air-lock at the bottom, and fitted with an elevator cage for the use of the men. This is shown in elevation in Fig.

202 (*d*) and in plan in (*d'*). There were, of course, the usual pipes for air, water, and the removal of sand. A special device called the "clay-hoist" was used. This is shown in vertical section, Fig. 202 (*e*) and in plan in (*e'*). It consists of an air-lock at the top of the shaft, behind which is placed a cylinder and piston. The speed of the piston is multiplied by two sets of sheaves, so that the stroke of the piston will lift a bucket from the bottom of the caisson to the air-lock on top. The air-lock is provided with two doors, one of which, opening into the shaft below, is closed by a lever with a balance-weight on the outside, and the other, opening into the open air, is worked by an attendant outside. The only power used is the air-pressure of the caissons. The bucket carried 6½ cubic feet, and 12 buckets have been carried out by a single hoist in an hour. Four hoists were provided, but only two were used at a time. The sand-pump, known as Monson's pump, is shown in vertical and horizontal section in Figs. 202 (*f*) and (*f'*).

The engine working the passenger-hoist is placed on top of the shaft, and is also driven by the compressed air. This shaft is 6 feet above the 6-foot air-lock, and 4 feet in diameter below. These three cylinders are side by side, the shells are connected by cast-iron door-frames carrying doors; while a fourth door, opening outward, is placed at the bottom of the 4-foot shaft. In working, the door between the two shafts was always kept closed, and the door at the bottom of the bottom shaft was always left open.

The only power used to raise either men or material was the air of the caisson. While this may not be as economical as the direct use of steam, it was convenient, and aided in the ventilation of the caissons.

The launching-ways were built on a pile foundation. The piles were driven to a depth of about 25 feet into the ground. These were cut off and capped with 12-inch timbers running parallel to the river, and on these were placed the ways proper, which were driftbolted to the caps and inclined at 1 in 4 for a length of 48 feet and 1 in 3.43 for the remainder. The launching-shoes or carriers, which were simply transverse timbers, were placed on these ways, the cutting edge was set up on the carriers, and the caisson built up to a height of 17.8 feet above the lower edge of the iron. The caisson was fitted with a false bottom. It was not intended that this bottom should be water-tight, but only to prevent a too rapid sinking of the lower edge after launching. The caissons were then launched in the usual manner.

The mattresses used to protect the bed at the sites of the piers from scour were constructed on barges. The upper edge of the mattress was fastened to barges anchored above. As the weaving progressed the weaving barges dropped down-stream, so that when completed the entire mat, 400 feet long and 240 feet wide, was floating on the water. The upper half of the mat was loaded with stone from barges floating over it after it had been weighted sufficiently to sink it well below the surface, it being still held by the mooring-lines. When the floating barges had passed over about one half the length of the mat the mooring-lines were cast off and the upper end sank to the bottom. The first mat sunk was found to be about 120 feet farther down-stream than was intended. Two mats were used at Pier 2, each containing 1000 cords of brush and poles, 900 tons of riprap, and 10,000 pounds of wire.

Twelve box-anchors, made of timber cribs filled with broken stone, were dropped in proper positions well above the site of the pier. These cribs were 10-feet cubes inside of timbers. To these two barges, six anchor-cribs to each, were fastened with 1¼-inch steel wire. The caisson was fastened to the barges with two wire ropes by means of a double-drum winding-engine, by which the caisson could be pulled into position. In addition, anchors, one on either side, were placed about 500 feet from the caisson, and fastened to it with 4½-inch manilla rope. The caisson was then hauled in position and the concrete filling commenced, as was also the timber-work above. As the caisson sank additional lines were run from it to the barges, until the total strength of these lines was about equal to that of those connecting the barges and the anchors. When the caisson grounded in about 36 feet of water the air-pressure was put on, and fourteen days were spent in cutting through the two thicknesses of mat under the caisson.

The building and concreting were continued as the caisson sank into the bed, the sand being removed through the sand-pumps. The masonry had been commenced when owing to a rise in the river it was deemed advisable to suspend the work. The caisson was blocked up under the shoe, and abandoned until low water of the next season.

The launching, placing, and sinking of Pier 3 were conducted in the same manner as for Pier 2, but owing to locating on a rising river, the caisson was carried 50 feet west of its true position before reaching the bed of the river in 44 feet of water. By means, how-

ever, of a quantity wire rope and a large number of anchors the caisson was brought back to its true position and sunk without further trouble. After reaching the clay, borings and wells sunk below the caisson showed that the clay was not sufficiently firm to build upon, and it became necessary to sink the caisson into the clay. Work was then stopped on this caisson for a more favorable stage of water. Subsequently the caisson was sunk 18 feet into the clay, the additional height required being given in the masonry of the pier. Borings were made and a well sunk below caisson 3 on resuming work; the material proving satisfactory, further sinking was stopped at a somewhat higher elevation than was originally intended.

The actual maximum depths at which the men worked below the water surface was 108 feet. At the depths of about 100 feet the men worked only two hours per day, in shifts of 40 minutes each.

The concrete was made of crushed limestone, sand, and cement. The sand was dredged from the bed of the river. The cement was from Louisville, Ky. In special portions of the work German Portland cement was used. The concrete was mixed in a machine mixer. All of the ingredients, including water, were mixed together. The concrete above the working-chamber was gauged, 1 barrel cement, $7\frac{1}{2}$ cubic feet of sand, and $13\frac{3}{4}$ cubic feet of crushed stone. In the V-shaped walls the crushed stone was reduced about one half. In the working-chamber the same character of concrete was used as above described, except that in parts difficult of access the quantity of crushed stone was reduced, and that the lowest 2 feet of concrete was made with Portland cement. Portland cement mortar, 1 cement, 3 sand, was used under the cutting edge, cross-walls, and roofs. The central portion of concrete was made of Louisville cement.

In Figs. 202 (*h*) and (*i*) an elevation and horizontal sections of the masonry piers are shown. The hollow portion near the bottom of the piers is 10 × 20 feet. In the main portion of the pier and in the curved ends a semicircle of 5-feet radius is left hollow, the dividing walls between the two at each end being 3.5 feet thick. The construction is shown in Fig. 202(*i*), which is part plan and part horizontal section.

Tests made on the clay upon which the piers are founded, the clay column entirely unsupported on the sides, showed a resistance to compression of from 13,400 pounds to 19,300 pounds per square foot. The actual weights of piers were, for Pier 2, 10,410 pounds,

and for Pier 3, 9934 pounds, per square foot. After deducting buoyancy, friction, etc., the actual pressures on the foundation-bed are 3339 and 3503 pounds per square inch, respectively.

The following table shows cost of Piers 2 and 3:

TABLE XLVIIIA.

	Pier 2.			Pier 3.		
	Material.	Labor.	Total.	Material.	Labor.	Total.
Launching-ways	$1,792	$1,824	$3,617	$1,885	$1,788	$3,673
Caisson	45,850	24,168	70,018	33,117	17,976	51,094
Concrete above chamber	10,528	3,847	14,376	7,393	2,448	9,842
" in chamber	3,814	1,310	5,125	6,383	2,174	8,558
			$93,136			$73,167
Mattresses	12,362	5,634	17,996	4,543	2,127	6,671
Anchoring caissons	4,619	2,087	6,707	7,557	3,778	11,336
Sinking "	17,215	50,826	68,042	19,432	59,534	78,967
Lighting pier	260	2,398	2,659	55	178	235
Insurance	558	....	558	395	....	395
Protecting foundations	....	....	....	562	101	663
Total, placing, sinking, etc.			$95,957			$98,267
Total for foundations			$189,094			$171,435
Masonry, 4197 cu. yds.			108,669			124,442
Aggregate cost			$297,763			$295,877

THE FORTH BRIDGE.

Length between high-water mark on the two shores, 1900 yards; total length, 2700 yards. Beginning at the Queensferry side, the bridge is constructed for a distance of 1780 feet on 9 piers of granite masonry; average span 160 feet, height of piers 130 feet.

The cantilever tower is at base 103 feet in length and 52 feet in thickness. This supports the shore end of one of the cantilever arms and that of one of the girders.

Three granite piers formed by a group of four columns each give two spans of 1700 feet each. These piers were constructed by sinking iron and steel caissons 70 feet in diameter either to rock or to bowlder clay, the depths sunk varying from 70 to 90 feet. The

caissons were filled to low-water mark with concrete. From this to 18 feet above high-water mark the piers are of masonry. These cylindrical masses of concrete and masonry decrease from a bottom diameter of 70 feet to a top diameter of 50 feet. Joining the centres of the four cylinders, in the centre pier of the three, the oblong formed would measure 270 × 120 feet, while for the two outside piers the oblong is 155 × 120 feet.

These piers carry the two long spans, 1700 feet each. Extending to the right and left of these are the cantilever arms 675 feet in length each, their free ends resting upon the cantilever towers as above described. There are 2 spans 1700 feet in length, and 2 spans 675 each, these together constituting the cantilever spans. The clear height above high water is 150 feet, and the top of the cantilever is 350 feet above high water.

There were also 15 spans 168 feet in length and 5 of 25 feet in length, or a total of 1½ miles. There are some 45,000 tons of steel in the superstructure, and 120,000 cubic yards of masonry in the piers. The contract cost was about $8,000,000.

In locating the piers fine pianoforte steel wire was used. This wire was suspended from supports at the two ends of an accurately measured base-line 1700 feet long, allowing a sag of 24 feet at the centre, and marks fixed to indicate this distance on the wire. The wire was coiled on a roller.

The span was measured by pulling the wire so as to have the same sag at the same temperature as when stretched along the base-line. This measurement was made repeatedly, and as often tested on the base to determine whether it had stretched; but all of the observations were in exact accord.

The reliability of this method for measuring long distances has been especially noted by the writer in his work on Foundations.

COMBINED OPEN-CRIB AND PNEUMATIC CAISSON.

449. In order to utilize the benefits of the pneumatic process, both for depths in which coffer-dams are used as well as for depths beyond the limit of the pneumatic process for which open cribs and dredging have been used, and further to dispense with either a timber or iron roof separating the concrete in the air-chamber from the concrete or masonry above, which is certainly desirable, the writer designed a combined crib and caisson, and patented the same. The general construction is that of a double-wall open crib with air-

tight roof—one or more, according to the depth desired to be reached. The roof or roofs are removable. If in the limit of depth admissible by the pneumatic process only one roof is necessary, two are desirable. The weight required to sink the structure is placed between the two walls, as in the open-crib process, and not on the roof, as in the pneumatic process. The sinking is effected by the latter process. When resting on its final bed the air-chamber is filled with concrete, or to the necessary height, to prevent leakage under the cutting edge. The roof is then removed and the concreting done in the open air. This results in a homogeneous concrete or masonry mass from bottom to top, not broken or separated by timber or iron.

If greater depths than 100 feet have to be reached, the pneumatic process can be used to that depth, and the open crib and dredging process below it. Economy, rapidity, and certainty are secured, and at any depth piles can be introduced and driven in order to secure greater support and stability. Fig. 204 shows a vertical section through the structure, the roof partly removed, and the concrete carried up above it.

450. Coffer-dams have heretofore, as a rule, been resorted to in depths of water and excavations below the bed of the river when the aggregate depth did not exceed 25 to 30 feet. Even in depths not exceeding 15 to 20 feet coffer-dams are usually very troublesome and expensive. Do what we may, in some cases it is impossible to pump them out and keep them clear of water without a number of pumps and corresponding boiler and engine capacity. They are often broken through or undermined, causing great delay and expense. At these depths it has not been deemed desirable to employ the pneumatic caisson and the pneumatic process, (1) because the plant is assumed to be unnecessarily expensive, (2) it was not considered good practice to have a timber deck or roof reaching so near the water surface, as it might be supposed or feared that at very low stages of water the timber would be exposed to conditions of alternate wetness and dryness, and in consequence to decay; (3) the pneumatic process was regarded as in some degree mysterious, only understood by a few specialists, who had to be well paid for the use of their plant, skill, and experience.

The first objection will often prove to be far from the truth, owing to the necessary and additional costs beyond even liberal estimates and allowances for unexpected contingencies.

The third has, or should have, no longer any consideration.

The second, however, has at this time the same force that it always had and always will have. This can, however, be entirely removed by constructing all coffer-dams in a similar way as that for caissons, but providing for the removal of the roof as soon as the sealing up around the cutting edge has been perfected, and all of the advantages accruing from the use of a coffer-dam and caisson will be secured: (1) Leaks, undermining, and destruction of the sides of the structure are avoided; (2) the cost of construction will not be materially increased; (3) the larger portion of the work can be performed in the open air, and all of it free from the injurious effects of the presence of water; and (4) the foundation-bed can be examined, levelled, and washed off—which are important considerations. The writer does not hesitate to recommend the combined crib and caisson as an efficient substitute for the ordinary coffer-dam.

451. Recently an attempt was made to apply compressed air to an annular working-chamber around an open enclosed space. Excavation was made in this space, and the structure sunk 26 to 30 feet below the water surface to rock. The bottom of the annular space was cleaned off, and a layer of concrete, or rather a wall of concrete, laid in it entirely around the enclosed space. Several attempts were made to pump the water out of the enclosed space, but they resulted in failure and practical abandonment of the plan. It became necessary to fill the interior up to a considerable height or to the water surface with concrete deposited under water, notwithstanding its many objectionable features. A general design of such a coffer-dam is shown in Fig. 205. Fig. 205 simply shows a double-wall crib or coffer-dam with the lower portion of the space between the two walls roofed over in order to form a narrow air-chamber, in which *AA* is the air-chamber, *BB* the enclosed space to be pumped out, *CC* the wall of concrete. When it was attempted to pump the water out of *BB* there was a great rush of air into this space, as on that side the air-pressure was unbalanced. This of itself would greatly aid the inflow of water from the outside. As might have been expected, the dam could not be pumped out. It seems that it was intended to removed the roof over the air-chamber and to fill over the entire space with a single solid mass of concrete. This, however, required the removal of the weight on the roof required to sink it, which would have probably resulted in the lifting of the coffer-dam. In the combined crib and caisson, Fig. 204, the space between the double walls is to be filled during the sinking

with broken stone or gravel, or, better, with concrete, which would remain permanently in place. The removable roof is to be placed over the entire closed space corresponding to space *BB* in Fig. 205, and over which no weight is ever placed, or at least it is not required. If this plan had been adopted it would not have been necessary to deposit the large bulk of the concrete under water, and there would have been a solid unbroken mass of masonry or concrete from bottom to top.

451*a*. The following describes an interesting application of compressed air in laying the foundation of a new and large lock

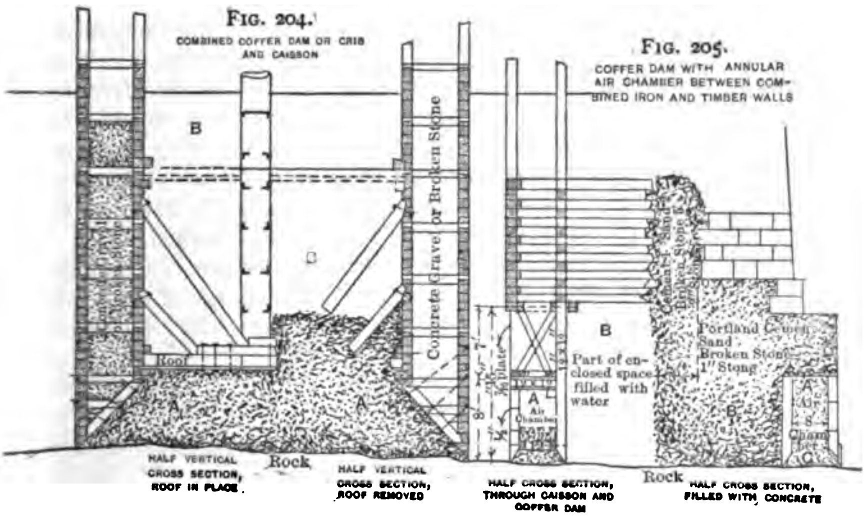


FIG. 204. COMBINED COFFER DAM OR CRIB AND CAISSON

FIG. 205. COFFER DAM WITH ANNULAR AIR CHAMBER BETWEEN COMBINED IRON AND TIMBER WALLS

for the entrance into the Amsterdam and North Sea Canal. the construction of which is described under the head of Canals.

The foundation of the new lock is made of concrete. The concrete was carefully made and laid in place, the bed was protected with sheet-piling, and the enclosed space pumped nearly dry before laying the concrete; but it was found that strong jets of water were forced through the bore-holes used in determining the character of the underlying strata, these holes evidently having entered some water-bearing strata below the clay bed on which the foundation was placed.

The contractors, however, ignoring these jets of water, attempted to lay and ram the concrete over the jets, with the usual result that the jets were not choked down, the cement was washed out, and

the concrete rendered valueless. As it was necessary to stop the flow of these streams, especially against fresh concrete, large concrete domes were built over the holes. These varied in diameter according to the areas more or less softened by the water. Some of them were 30 feet in diameter. An ordinary cylindrical air-lock, with the necessary doors, pipes, valves, etc., was built into the top of the dome. The concrete was well bonded into the clay to prevent leakage. A pipe was then driven outside the dome and into the water-bearing stratum in order to allow the escape of the water above the concrete bed of the foundation. Compressed air was then forced into the dome, which checked the flow through the original bore-holes, and finally stopped it entirely, the flow, however, finding and discharging through the iron pipe outside. The concrete was then laid in the air-chamber without interference from the flow of water, and after hardening it was capable of resisting the pressure of the water after discontinuing the air-pressure. The water that flowed up through the outside pipe was discharged over the bed of concrete, and was entirely harmless in its effect. All uncertain and soft places were thus treated.

This somewhat novel plan was suggested by the contractors, MM. Mortier and Thonvard. The engineers in charge of the work were MM. Van Meren and Bekaar. (See *American Architect*, March 17 1894.)

SCREW-PILES.

452. Screw-piles, whether of timber or iron, are simply ordinary piles to the bottom of which a screw-disk, consisting of a single turn of the spiral, similar to the bottom turn of an auger, is fastened by bolts or pins; and instead of driving them into the ground, they are forced into it by turning them with levers or other machinery suitable for the purpose. The power applied may be that of man, horse, or steam. The screw-disks vary in diameter from 1 foot to 4 or even 6 feet. Considerable power is required, as the resistances developed are very great. This resistance can be reduced by applying the water-jet to the under, upper, or both faces of the disk. Great depths have been reached by this process. Screw-pile piers have been used for bridges, lighthouses, and other similar purposes. They may be solid or hollow. The diameters of the shaft or pile vary from 4 inches to 12 inches, according as they are solid or hollow and as the material is iron or timber. A light, strong pier can be thus constructed.

The foregoing remarks on foundations are but a brief and imperfect summary of the principles, theories, and practice applicable to the construction of the many and varying kinds of foundations. For a more thorough and complete discussion of this subject the reader is referred to "A Practical Treatise on Foundations," by the author.

ART. XXXIX.

ORDINARY EARTHWORK.

453. In this article only the construction of ordinary earthwork, such as that required in railways, highways, and other similar purposes, will be considered.

The special materials and modes of construction required for the embankments of canals, earthen dams for storage-reservoirs, and similar works will be discussed under these special heads.

Foundations for earthworks over swamps were explained in the preceding article, and in this article foundations on firm materials, requiring for their stability only simple drainage, will be contemplated.

454. The embankments are assumed to be made of the earthy materials found along or adjacent to the line. No material is supposed to be excluded except a pure or nearly pure clay when saturated with water. But little attention, as a rule, is given to this material; nothing but practical difficulties in handling it seems to have any weight in the construction of embankments, and when met with in excavations has to be dealt with as best we can. It may be stated, however, that very wet clay should not be used in forming embankments, as it will require a long time to make a firm and satisfactory embankment, if it ever will.

If a selection is to be made, shivers of rock, broken stone, or gravel and coarse sand afford the best materials with which to construct embankments, as they do not retain water and have considerable frictional resistances. Mixtures of clay and sand or gravel furnish also good materials. These compose the ordinary earths.

Embankments made of these materials have stability, and readily yield to good drainage. Fine sand, while there is no objection to its use on the above accounts, forms, however, a disagreeable

material to travel over, on account of the clouds of dust that rise in dry weather, the effect of which is also to injure the rolling machinery, as its particles easily find their way between the sliding or rolling surfaces of engines and cars.

The silt or slush of swamps is often dredged from canals along the site of the road and deposited on the line for the embankments. This is bad practice, both on account of the material being unfit for the purpose and on account of the excavation alongside of the embankment destroying to a great extent the only and necessary support for the material. This method is, however, frequently adopted. If a firmer material cannot be conveniently and economically secured and used in the construction of the embankment, it will doubtless prove economical ultimately to first cross the swamp on a temporary and cheap trestle, and filling under and around this at a subsequent period.

455. Rules for locating the line and establishing the grades in order to promote economy in construction, were fully discussed in Part I of this volume; and the construction alone, consisting of the excavations or cuts and the embankments or fills, need now be explained.

456. Excavations are either in open cut or in tunnels. The latter will be explained under the head of Tunnels. In either case it is the rule to employ the excavated material in making the adjacent embankments.

457. *Open Cuts.*—There is no general rule as to the exact method of carrying on the excavation and disposing of the material excavated. The operation in each case can only be determined by experience and requirements of the contract, length and depth of the cut, character of the material, and length of haul, etc.

Throwing with the shovel is limited to a distance of 12 feet horizontally, or about 6 feet vertically.

The economical length of haul with drag scrapers is about 150 feet, wheeled scrapers 500 feet, wheelbarrows not over 200 feet, dump-carts about 500 or 600 feet. When the haul is to exceed 600 feet, dump-cars drawn by horses, on a track of timber faced with strap-iron or a light iron rail, can be used. And, finally, where large quantities of material are to be excavated and hauled to a distance of 1000 to 5000 or more feet, a well-laid track of light iron rail, over which a train of dump-cars is hauled by a light locomotive, should be built. Often, in this case, light timber trestles are constructed,

having a height somewhat less or greater than the height of the proposed embankment. The material is hauled out on this trestle and dumped under and around it. The top of the trestle, if it projects above the embankment, is simply cut off after the work is completed. The remaining portion is left in the earth.

458. Shovelling can only be used when the embankment is to be formed from ditches alongside, or when, in very shallow cuts, the material is simply wasted, that is, thrown out of and alongside the excavation.

Barrows are used under similar circumstances where the quantity of material needed requires excavating borrow-pits alongside the embankment and extending from 50 to 100 feet on either side of the centre line, or out of cuts of greater depth than about 6 feet. The bulk of the material being wasted, that portion near the two ends can be wheeled onto the adjacent fill for a limited distance, say 100 or 150 feet.

Scrapers are used for the same purposes and under the same conditions. Much higher embankments can be formed with them, as horses can drag them up very steep inclines and descend readily from considerable heights.

For any of the foregoing methods the material is commonly loosened by means of ploughs drawn by two or four horses or oxen. (Spading is only resorted to when barrows are used, as a rule.)

459. When the magnitude of the work justifies the use of carts or cars, drawn by horses or engines, the cut is usually entered at each end on or about the grade surface of the road. It is advisable then to open out at once the full width of the intended road-bed, providing as wide an entrance as practicable to enable carts or cars to easily pass each other, since they should be returning to and departing from the face of the cut continuously. In most materials the prism or gullet cut out will remain for a time with vertical face and sides. When otherwise, the necessary slopes must be excavated as the work progresses. In no material, except solid rock, should the vertical faces remain for any length of time; they should be sloped down well up to the front or head of the excavation, since by caving in they might loosen the soil on either side of the proper slopes, requiring the immediate or ultimate removal of more material than necessary, or, what is worse, obstructing or wrecking trains after the completion of the road. The material is usually excavated by undermining at the face, and by the aid of bars, or

even of blasts with powder, detaching large blocks of the material, which commonly break to pieces in falling.

Where the material is suitable, and large deep excavations are to be made, the steam-shovel or excavator is used, which scoops up large bucketfuls of the material. These can be emptied direct into cars or carts and hauled away to the place of deposit. As the work advances the shovels are moved forward on either a regular track or on movable rails.

460. Many earthy materials are advantageously excavated by blasting. This has to be resorted to in all solid-rock cuts. These are usually taken out to the full width at once, including the slopes, which are but slight. Carts or horse-cars are used mainly in rock cuts. It is usual to use high explosives and large charges, in order to shatter and break up the rock as much as possible.

461. All cuts should be well drained to prevent the bottom getting soft, necessitating the construction of corduroy or plank roads at a considerable cost, or otherwise killing up the horses, and even then only getting a small part of their actual power in useful work. Drains should be excavated and kept open at the foot of either slope or along the centre line, and in addition ditches should be opened on the surface of the ground on the uphill side to prevent surface water finding its way into the cut. Too little attention is paid to this question of drainage during the construction of roads, and often after their completion. Without it work cannot be done economically or expeditiously, and no road-bed can be kept in good order without permanent drainage.

Earth excavation is usually classified under the heads Earth, Hardpan, Loose Rock, or Solid Rock. For each of these materials a specific price is agreed upon. The standard prices under ordinary circumstances are 16 to 20, 35, 40, and 80 cents per cubic yard, in the order mentioned. A special allowance is made of from $\frac{1}{2}$ to 1 cent per cubic yard for a haul exceeding from 300 to 500 feet.

462. Whether the material from the excavation is to be wasted near the ends of a cut or along its sides, is simply a question of economy, or rapidity of construction, or an unavoidable excess of excavation over that of the embankment within a reasonable distance. As a rule, the material is better suited for the formation of good banks, as it can be obtained in a drier condition, and contains a much smaller percentage of surface mould, which is objectionable as a material for embankments; this composes a large percentage of the total quantity when obtained from shallow borrow-pits along the line.

When wasted alongside of the cut, the material should be placed at a distance from the upper edge of the slopes equal to one or one and a half times the depth of the cut, in order not to weight the adjacent material, thereby causing the slopes to cave in.

463. The side slopes of a cut are given in order to provide stability, as the material will slide down until the natural slope is secured. The slopes vary with the material, but are commonly taken at 1 to 1; that is, one foot base for each foot depth. This corresponds with an angle of repose of 45°. It is somewhat steeper than most materials are found to have, as seen in the following table, which gives both the slope and the angle of repose, or corresponding angle of slope.

TABLE XLIX.

Material.	Angle with Horizon.	Slope, or Ratio of Horizontal to Vertical, in feet.	
Solid rock	75° 58′	$\frac{1}{4}$: 1	Angle of repose (ϕ).
Compact and hard earth or hardpan, and loose rock	63° 26′ to 53° 8′	$\frac{1}{2}$: 1 to $\frac{3}{4}$: 1	" " " "
Dry clay	45°	1 : 1	" " " "
Gravel	40°	$1\frac{1}{5}$: 1	" " " "
Shingle	39°	$1\frac{1}{4}$: 1	" " " "
Sand, dry	38°	$1\frac{1}{4}$: 1	" " " "
" wet	22°	$2\frac{1}{2}$: 1	" " " "
Vegetable mould	28°	$1\frac{9}{10}$: 1	" " " "
Wet clay	16°	$3\frac{1}{2}$: 1	" " " "

No slope in solid rock is necessary, but it is better to provide from $\frac{1}{4}$ to $\frac{3}{4}$ to 1, to avoid danger of loose pieces falling down on the track. In those materials classed as hard earths, while they may stand for a time at a steep slope, or even vertically, yet a slope of from $\frac{1}{2}$ to $\frac{3}{4}$, or even 1, to 1 should be usually given. The other cases give average values for angle of repose and ratio of base to rise in slopes.

When the material requires a flatter slope than 1 to 1 it can be cut into a series of short slopes with benches or horizontal surfaces joining them, each slope having its own drains; this saves material, and will be effective in deep cuts, if of a uniform material. Often it is left to assume its own slope, simply removing the ma-

terial as it falls or flows. Again, the caving in may arise from alternate layers of sand and gravel and clay, the sand and gravel flowing under the action of seepage water, and undermining the slopes. In such cases a foot-wall of dry rubble or a stronger wall laid in mortar can be used if the layer is at the bottom, or the entire slope may be paved with a layer of rubble-stone. By these means the water is drained off, while the material is held in place. Sodding or sowing grass-seed will ultimately effect the same purpose. Surface drains on the uphill side will often keep away much of the seepage water, and much can be accomplished by broken stone, or tile-drains imbedded on or under the slope.

Good drains should be provided and kept open a little distance—one or two feet—in front of the foot of the slope. They may be open drains, or filled with broken stone, or tiles covered with gravel may be used. These will prevent the road-bed from being saturated with water. The firmness and stability of the track require complete drainage in some of the ways above described. Safety and economy make the same demand.

464. It is difficult to properly apportion the number of laborers and vehicles required to execute the work economically and expeditiously. In hard clay it may require two picks to one shovel, and a couple of carts will carry to a reasonable distance all the material that ten or twelve men can handle. In loose materials one pick can keep several shovels busy, and several carts are required. Again, all of these relations are altered when a broad face can be exposed, and the material excavated in large blocks by undermining, blasting, or by the use of the steam-shovel. The only rule is to regulate, in each case with its varying conditions, the number of men and the number and kind of vehicles required, so that all will be kept busy. Observation and experience alone can be relied upon to determine such matters.

465. *Drilling and Blasting in Rock.*—Drilling is done either by two men lifting a long iron rod, with a steel section or made entirely of steel, and then letting it drop into the hole, turning it a little at each drop. In very hard rock, with holes from 1¼ to 2 inches in diameter, two men will drill in a day from 7 to 8 feet, and in softer rocks from 10 to 15 feet. The other method consists in using a series of drills of different lengths, depending on the depth of the hole. One man holds the drill, and two keep up a series of blows in quick succession, the man turning the drill slightly at each blow. It is necessary to keep the bottom of the hole well moistened.

When much rock excavation is required the holes are drilled by machinery. The diameters of the holes vary from $\frac{3}{4}$ inch to 6 inches, and from 30 to 100 feet can be drilled in a day.

The quantity of blasting material required to loosen a given proportion of rock depends upon the character of the rock, the kind of explosive, and largely upon a judicious selection of the direction of the hole with respect to the lay of the strata. It is stated that it bears a certain proportion to the line of least resistance cubed. This line is intended to be the shortest distance from the charge to an exposed surface of the rock. Mr. Byrne gives the following expression: $E = CL^3$, in which E is the quantity of explosive in pounds, $L =$ the line of least resistance, and C a constant having the following values: for blasting-powder $C = 0.032$, for blasting cotton $C = 0.005$, and for nitroglycerine or dynamite $C = 0.003$. Practically about three fourths of a pound of powder will loosen one cubic yard of solid rock. A pound of powder occupies about 30 cubic inches. It is fired usually by a fuse burning at the rate of two feet per minute. Either a fuse or a current of electricity is used to explode dynamite.

Although it is not desirable and not as effective to produce a too great shattering and scattering of the broken rock, little attention is paid to this point in ordinary blasting operations. But in blasting near buildings or in the streets of cities special precautions must be taken to avoid projecting the fragments of rock to great distances. This can be done by properly regulating the charge, and covering over and around the holes with brush and logs. A raft of logs chained together, or a matting of ropes weighted with logs around the edges, will prove effective for this purpose.

466. The stability of earthwork depends upon the adhesion and friction between its particles. The adhesive force is destroyed by exposure to air and moisture, and cannot be relied upon to give permanent stability, though of great advantage in maintaining the sides of an excavation vertical or with a steep slope. Friction can then be the only means of giving stability. The material will cave or slide until the natural slope is attained, unless excavated in the beginning to it. These slopes have been given in Table XLIX.

467. *Embankments.*—If embankments are made from side ditches or adjacent borrow-pits, the material is either shovelled or carried in wheelbarrows or scrapers from the pit on to the fill. In such cases no special method is followed: the material is simply piled in any convenient shape, and subsequently dressed off on top

and sides to the proper slope. It is difficult to avoid this when wheelbarrows are used, as it would require a prohibitive frequency in moving the track, consisting of planks resting on light timber supports. In the flat prairie sections of the country much of the work is done in this way. It does not conduce to the formation of the best embankment for early use, as it is only piled up loosely and readily admits water, which in some materials remains for a long time in the fill, causing a want of firmness and stability in the embankment until, by a natural and slow process, it is ultimately compacted. For this reason a liberal addition to the height is given to allow for the settlement that will occur. This settlement or shrinkage varies greatly with the material and its condition at the time of making the fill.

468. It is estimated that any material, except rock, will, when placed in a bank, occupy less space than in the pit from which it is taken. This shrinkage has the following approximate values for the different materials:

TABLE L.

Gravel	8 per cent.
Gravel and sand	9 " "
Clay and ordinary earths	10 " "
Loam and light sandy soils	12 " "
Loose vegetable soil	15 " "
Puddled clay	25 " "

Rock makes a larger volume when broken up, and does not shrink or settle into less than its original bulk. The increase is often as much as 50 per cent.

On this basis an excavation of 1000 cubic yards would make only about 900 cubic yards of embankment if the material is clay or ordinary earth, or it would require 1100 cubic yards in excavation to make 1000 cubic yards in embankment, and a similar proportion for other materials. A rock excavation of 1000 cubic yards will make about 1500 or more cubic yards in embankment. It must, however, be recollected that this excess is consumed in making the embankments from 8 to 12 per cent higher than required ultimately, and, further, that whereas the slopes in excavation are 1 horizontal to 1 vertical, they usually have in embankments $1\frac{1}{2}$ to 1, or even 2 to 1; so that the rule given, of making the excavation and embankments equal or nearly so, holds in practice.

469. In scraper-work, which is largely used, whether hauling material from the borrow-pits or regular excavations, there is too much carelessness in the manner of making fills, these being long, narrow ridges, not wide enough either at base or at top. The material is compacted by the constant tramping of horses and men and scrapers; the desired width and slopes are then made by piling on the sides dirt in a loose state, which settles and slides away from the compacted core, giving an unsatisfactory if not dangerous road-bed, besides adding to the subsequent cost of surfacing and levelling. The same remarks apply to cart and train work. The remedy is simple, and should be enforced. In all such cases the embankments should be commenced with the proper widths of base and maintained with them to the top. The successive layers should present *a concave, and not convex,* upper surface at all times, so that the entire mass may have the same degree, approximately, of settling and compacting as the work progresses. This is especially important in high embankments. Constructed in this manner, little extra height is necessary for shrinkage. When earth is carted from excavations it is usually the practice to make the embankment with nearly its full height in one thick layer, all of the material being dumped at the end of the bank. This does not alter the force of the above remarks as to maintaining the fill its full width. No material should be dumped on the sides in order to widen a too narrow bank. If anything, the width should be a little greater than ultimately required, so that the slopes may be formed by a little trimming off rather than filling up. As to what percentage of shrinkage should be allowed for in these cases is a matter of judgment, controlled by the time taken to make the bank and the manner in which it has been made. Probably one half of the allowances above given would be ample.

470. In all cases it is necessary to have ditches on one or both sides of the banks at a distance, called the berm, of from 3 to 6 feet from the foot of the slopes.

This berm is regulated by the slopes of the banks and the depths of the ditches. The prolongation of the side slopes of the banks should pass under the bottom of the ditch; otherwise the weight would cause caving and filling up of the ditch. The ditches are required to drain and keep dry the base of the bank. They are graded from opposite direction to the usual ditches or streams crossing the line of road. These are carried through the embankments by open trestles, open or closed box culverts of tim-

ber or masonry, and when of large size masonry arches are constructed where the heights of the banks will admit of their use.

471. Embankments on swampy material which can be drained present no special difficulties; but in other cases a broad base must be formed by brush, logs, or broken stone; or the material can be simply piled on until appreciable settling ceases. These conditions and methods have been fully discussed in the preceding article on foundations.

472. Earthwork is often carried on steep hillsides, and is commonly composed partly of excavation and partly of embankment. In such cases the excavated material is simply cast or wheeled from one side to the other.

Should the material extend long distances down the side of the hill, if the natural slope of the ground is very steep it may be necessary to cut it in a series of steps. These steps make an angle with the slope of the embankment varying from 90° at the bottom to zero, or a level, at the top. Such precautions are seldom taken to hold the material. When such methods are necessary, it will commonly be found advisable and economical to build a wall to hold the embankment. This may be dry rubble, or built with mortar. It may only be necessary to build a low foot-wall, or it may be advisable, if material for the bank is scarce, to build a wall extending to the grade of the road. If this is constructed for a highway, a parapet wall or an iron or timber fence or railing should be built to prevent vehicles from falling over the wall.

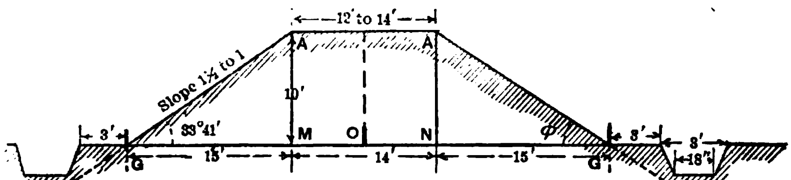
In Fig. 206 is shown an ordinary embankment with side ditches; in Fig. 207 an excavation with side ditches, surface drains, and broken-stone drains in the side slopes. Fig. 208 shows a side-hill cut, with its embankment portion supported by a foot-wall as shown in full lines, or by a high wall as shown by dotted lines. Either may be used.

In Fig. 206 the top and bottom widths are given for a single-track railway. This allows 6 or 7 feet from the centre of the track to the edge of the slopes at *AA*, usually 7 feet. For a double track, since tracks are rarely less than 14 feet centres, to allow good clearance between the cars in passing, the top width should be about 26 to 28 feet.

The bottom width should be increased by the same amount.

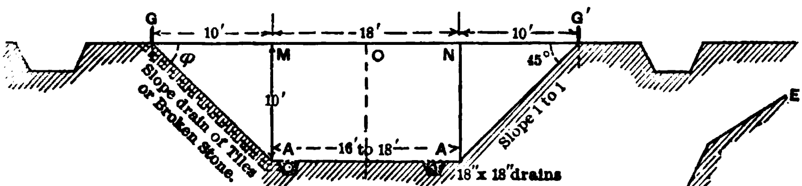
In Fig. 207 the road-bed *AA* is rarely less than 18 feet, except in a rock cut, where it may be reduced to 16 feet. These widths are required to allow for side ditches. With 18 feet for road-bed,

and a depth of 10 feet, with side slopes 1 to 1, the top width is 38 feet for single track. For double track the bottom width is from 30 to 32 feet, and the top increased by the same amount.



Embankment on Level Ground

FIG. 206.



Excavation on Level Ground

FIG. 207.

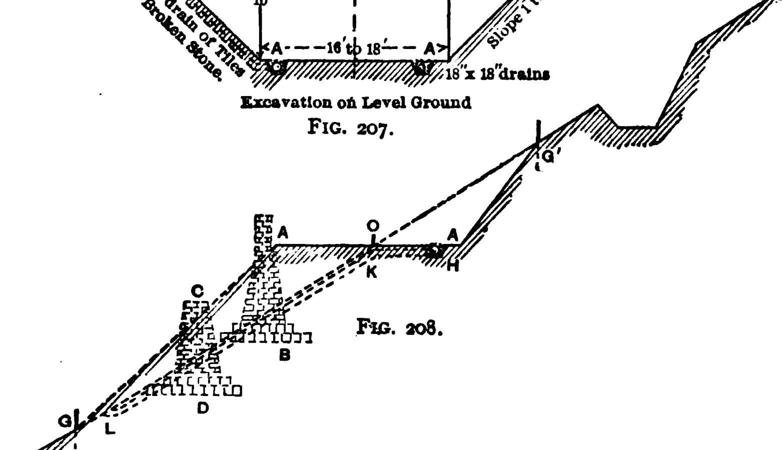


FIG. 208.

AA, Road-bed.	*AG*, Slope of bank 1½ to 1.
AB, Retaining-wall for fill.	*AG'*, Slope of cut 1 to 1.
CD, Foot-wall.	*HKL*, Tile drains.
EF, Slope of ground.	

EXCAVATION AND EMBANKMENT ON SIDE HILL.

In Fig. 208 it is as well to give the same widths as in Fig. 207. If practicable, the line should be laid so that the track may rest on the excavated bed, and not partly on the loose embankment.

The side slopes, berms, and ditches are the same for single and double tracks. These are also the same for railways and highways.

The top widths, or rather the road-bed in embankments or excavations, may vary for highways from the above dimensions to 30 or 50 feet.

473. *Haul.*—Mention has been made several times about extra haul. This simply means that when the excavation is carried to a certain specified distance, say from 300 to 500 feet, the contract price for the material—if earth, from 16 to 20 cents per cubic yard—is all that the contractor receives. If, however, any of the material is carried to a greater distance, he receives an extra compensation of from $\frac{3}{4}$ to 1 cent per cubic yard for each 100 feet or fraction thereof above the free-haul distance. It is then important to have it understood in what manner this amount is to be calculated. A failure to do this often leads to trouble.

There are two methods of estimating the extra haul. *First.* For the actual distance hauled of each cubic yard over, say, 500 feet is paid the extra one cent. As this would be a practically impossible computation, the following approximation is agreed upon: In Fig. 208*a* AEB represents the surface of the ground, DEF the grade-line. E is called the grade-point. The positions of the two lines CD and GH are ascertained by trial, or from approximate formulæ which are given in all field-books for engineers, such that while the distance DG is equal to the free haul, the number of cubic yards in the excavated volume CDE shall be equal to the filled volume EGH. For this much material no extra allowance is given. Then, if the excavated volume $ACDK$ makes the filled volume $GLBH$,

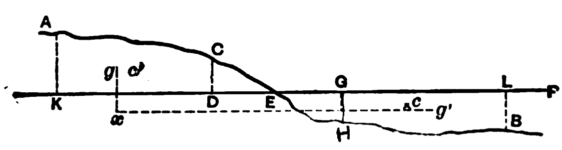


FIG. 208*a*.

the proper allowance for shrinkage being made in both cases, find the centres of gravity g and g' of these volumes and the horizontal distance xg' between them, and the extra haul on the entire volume $ACDK$ will be $xg' - DG$. If the free haul DG is 500 feet and $xg' = 1100$ feet, then the extra haul on the volume $ACDK = 5000$ cubic yards would be $1100 - 500 = 600$ feet; and at one cent per 100 feet, or a total of six cents per yard, the whole mass would cost $5000 \times .06 = \$300$.

The second method is commonly known as the average haul. Find the centres of gravity of the entire masses of excavation *ACEDK* and *LGEHB*, say *c* and *c'*. If this distance is under the limit of free haul, then no extra allowance for any portion of the excavation is made. If, however, it is over the limit, say 800 feet, then the entire number of yards will be paid extra for a distance 300 feet, or three cents on each yard. The first method is probably the better of the two. The second method is carried to the extent sometimes of averaging against one cut in which there is some extra haul other cuts in which all of the material is hauled a less distance than the limit of free haul. This is unjust to the contractor, and no estimate of haul should be made on such cuts. But the company is entitled to the deduction of the free-haul distance for every yard carried over it, as clearly indicated in the first method described.

GENERAL DUTIES OF RESIDENT ENGINEERS.

474. Resident engineers, on assuming the charge of an assigned portion of any line of road, should as soon as practicable retrace the line, replacing missing stakes, checking and correcting small errors in alignment, and running in the curves with more accuracy than is usually practicable by the locating engineers. This may require a slight change in the P. C. and P. T. of curves, or some change in the degree of curvature, not exceeding the maximum allowed. He should also run the levels, thereby detecting any errors in the work or in the notes given him. The lines of neighboring engineers should be carefully connected by proper adjustment of errors, if any are found. He should establish accurate bench-marks at all streams requiring bridges, culverts, or trestles, and should carry out strictly and faithfully all instructions from his superiors. He may suggest material changes in alignment and grades, whereby they may reduce expenses and make improvements. He should then proceed to stake out the work, or set slope-stakes, by which the workmen are to be guided, marking on the stakes in red chalk the cuts and fills. The + and − signs are sometimes used, but it is better to use F. 10 feet or C. 15 feet to indicate a fill or a cut, and its depth at that cross-section.

The records thus obtained are also necessary in calculating the areas of the cross-sections and the prismoids or volumes of earth between them. Duplicate copies of such records should be made. All

transit-points marking the P. C.'s and the P. T.'s should be referenced, as these will be either cut out or filled over during the progress of the work. The referencing is done by selecting two points at some distance from the centre line, and, in positions that are not likely to be disturbed in any way, driving large hubs flush with the ground, in the tops of which tacks are placed. Setting his transit over these points in succession, he lines in two other points on lines intersecting at the transit-point to be referenced. These lines should make as nearly 90° with each other as practicable. By means of these points new points can be located on the road-bed in the same vertical line with the original points of curve. Other points can be referenced in the same way. He also lays out abutments, culverts, and trestles where required. His other and general duties involve the inspection of all works on his residency, and he sees that the work is performed properly and in accordance with the specifications and contract. Necessary points and levels are given as the work progresses, and, finally, as parts of the work are completed he sets grade-stakes. These are stakes driven in the tops of the embankments and the bottoms of excavations at distances from the centre line equal to the half width of the road-bed, and driven until their tops are even with the grade-line at those points, or in cuts at the height of a few inches above the proper grades, or rather subgrades. Upon these subgrades the ballast and ties are placed, the top of which are the grades proper. These are from 10 to 12 inches above the subgrades. The profile grade generally means the subgrade. In rock cuts the bottom is made somewhat lower in order to allow for a filling of broken stone or gravel, in order to keep the ties from resting directly upon a solid and rigid surface. Until the railway can be fenced in care must be taken that the fences are not left down, by which property owners may be greatly damaged.

The reasonable rights of property owners should be carefully protected, and even the unreasonable claims and requests should be granted where no special expense or hindrance to the work accrues; and under all circumstances engineers should be courteous and considerate.

CROSS-SECTIONING AND CALCULATION OF VOLUMES.

475. *Cross-section Work or Setting Slope-stakes.*—It is not the author's intention to enter exhaustively into the methods and

details of staking out work and calculating the quantities of earthwork. A few only of the general principles and formulæ will be given. There are three principal conditions:

(1) When the surface of the ground is horizontal or level across the line.

(2) When the surface of the ground is inclined to the horizon across the line. In this case the work may be all in excavation, all in embankment, or partly in both.

(3) When there is no uniform surface or slope of the ground, but it is broken by a series of depressions and rises, forming an irregular surface. The general lay of the ground may be horizontal or inclined.

In any case, so far as the positions of the slope-stakes are concerned, the entire problem resolves itself into finding points on the ground at such a horizontal distance from the centre line of the road that, commencing a fill or a cut at these points and maintaining its side surfaces at the proper slopes, the top or bottom of these slopes, where they intersect the road-bed, will be at a distance equal to one half the width of the road from the centre line—in other words, the points GG' in Figs. 206–208.

Where the ground is level no difficulty arises. In Fig. 206 it is only necessary to lay off one half width of road-bed OM or ON on either side of the centre stake O, and the fill at the centre being the same as at $N = AN$, and the side slopes being $1\frac{1}{2}$ to 1, $MG = NG' = 1\frac{1}{2} \times AN$; in the case taken $NA = 10$ feet, $NG' = 15$ feet, and $ON = \frac{1}{2} \times 14 = 7$. It is only necessary to measure on each side of O distances of $7 + 15 = 22$ feet. Stakes may or may not be placed at the points M and N. Fig. 207 being in excavation, with side slopes of 1 to 1, $MG = NG' = AN = 10$ feet, and the distances on either side of $O = 9 + 10 = 19$ feet. Otherwise there is no difference in the two operations.

476. If, however, the ground has an inclination across the line, it is evident that the distances from the centre to the side stakes would be unequal, as seen in Figs. 209 and 210. The slope of the ground being uniform, the horizontal distances, or half widths, of the road-beds, $ON = OM$, remain constant for the same road, the variable quantities being $P'G'$ and PG in the figures, or the horizontal distances from N and M to G' and G, respectively, which are the bases of the side slopes. It is required to find these quantities. These vary with the slope or angle, $AGP = A'G'P'$, and height of the fill or the depth of the cut measured vertically from

the points G and G' to the road-bed AA', equal to the vertical distances AP and $A'P'$. It is evident from the drawings that the road-bed will be too narrow if the slope-stakes are set nearer the centre stake than G and G', and too wide if placed beyond G and G', as indicated by the dotted lines parallel to AG and $A'G'$.

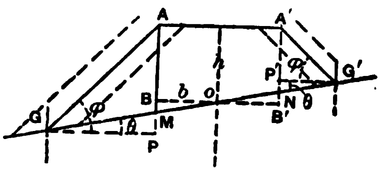


FIG. 209.

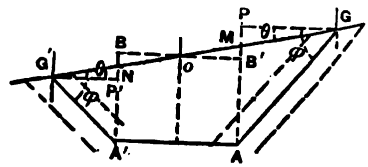


FIG. 210.

It is therefore important to determine accurately on the ground the positions of G and G'.

There are two principal methods: (1) Measure the slope of the ground. This can be done either by means of a clinometer, or by determining with a level, or two graduated straight-edge bars, the vertical rise in any given number of feet measured horizontally. The first gives the angle of slope direct, the second the ratio of the base to the rise. The rise divided by the base is the tangent, or the base divided by the rise is the cotangent, of the angle of slope. Then

$$PGM = P'G'N = \theta$$

is the angle of the slope of the ground.

$$\text{Tang } \theta = \frac{PM}{PG} = \frac{P'N}{P'G'}; \quad \text{cotang } \theta = \frac{PG}{PM} = \frac{P'G'}{P'N}.$$

$$AGP = A'G'P' = \phi$$

Then from the triangles AGP and $AG'P$, we have

$$PG = AP \text{ cotang } \phi \quad \text{and} \quad P'G' = A'P' \text{ cotang } \phi.. \quad (284)$$

$$AP = AB + BM + MP = h + b \text{ tang } \theta + GP \text{ tang } \theta;$$

$$A'P' = A'B' - NP' - NB' = h - b \text{ tang } \theta - G'P' \text{ tang } \theta;$$

in which h is the fill or cut at the centre stake, and b is the half

width of the road-bed. Substituting in equations 284 these values, there results

$$PG = AP \text{ cotang } \phi = (h + b \text{ tang } \theta + GP \text{ tang } \theta) \text{ cotang } \phi;$$

$$PG(1 - \text{tang } \theta \text{ cotang } \phi) = (h + b \text{ tang } \theta) \text{ cotang } \phi;$$

$$\therefore\ PG = \frac{(h + b \text{ tang } \theta) \text{ cotang } \phi}{1 - \text{tang } \theta \text{ cotang } \phi}. \quad . \quad . \quad . \quad (284a)$$

If the slope of the ground is r to 1, then tang $\theta = \frac{1}{r}$, and if the side slope of the cut is s to 1, then cotang $\phi = s$, and equation (284a) becomes

$$PG = \frac{\left(h + \frac{b}{r}\right)s}{1 - \frac{s}{r}} = \frac{(rh + b)s}{r - s} = \frac{rs}{r - s}\left(h + \frac{b}{r}\right). \quad . \quad (285)$$

To which if we add b, the half width of the road-bed, we find the horizontal distance from the centre O to the point G

$$= b_1 = b + \frac{(rh + b)s}{r - s}.$$

The factor $h + \frac{b}{r}$ is the fill AM, Fig. 209, or the cut AM, Fig. 210. Equation (285) is the value of PG, Fig. 209, and also of PG, Fig. 210; that is, the longer distance in either cut or fill.

Using the second of equations (284), $P'G' = A'P'$ cotang ϕ, and substituting the value of $A'P'$ above given, we find, by a similar process,

$$P'G' = \frac{rs}{r + s}\left(h - \frac{b}{r}\right). \quad . \quad . \quad . \quad . \quad . \quad . (286)$$

This applies to $P'G'$ of Fig. 209, or $P'G'$ of Fig. 210. The factor $h - \frac{b}{r}$ is the fill or cut at the point N, in either section.

477. Assuming the depth of the centre fill, Fig. 209, $h = 10$ feet, the side slopes $1\frac{1}{2}$ to 1, or $s = 1\frac{1}{2}$, the slope of the ground

GG_1 as 5 to 1, or $r = 5$, and $b = 7$ feet; then, from equations (285), (286),

$$PG = \frac{5 \times 1\frac{1}{2}}{5 - 1\frac{1}{2}}\left(10 + \frac{7}{5}\right) = 24.43 \text{ feet, and } b_1 = 31.43 \text{ feet;}$$

$$P'G' = \frac{5 \times 1\frac{1}{2}}{5 + 1\frac{1}{2}}\left(10 - \frac{7}{5}\right) = 9.9 \text{ feet, and } b_1' = 16.9 \text{ feet.}$$

The fill at $A = 10 + \frac{7}{5} = \frac{57}{5} = 11.4$ feet, and at

$$A' = 10 - \frac{7}{5} = \frac{43}{5} = 8.6 \text{ feet.}$$

In Fig. 110, with $b = 9$, the side slopes are 1 to 1, or $s = 1$. With everything else as above,

$$PG = \frac{5 \times 1}{5 - 1}\left(10 + \frac{9}{5}\right) = 14.75 \text{ feet, and } b_1 = 23.75 \text{ feet;}$$

$$P'G' = \frac{5 \times 1}{5 + 1}\left(10 - \frac{9}{5}\right) = 6.8 \text{ feet, and } b_1' = 15.2 \text{ feet.}$$

478. When the ground surface intersects the road-bed, as in Fig. 211, the principle involved is the same as the one just employed. The resulting formulæ are somewhat different. This case is represented in Fig. 211. The distance

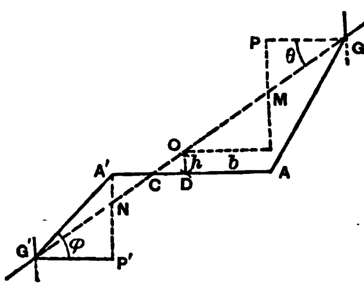


FIG. 211.

$$PG = \frac{rs}{r - s}\left(h + \frac{b}{r}\right),$$

the same as equation (285). But for the distance $P'G'$ on the embankment side

$$G'P' = A'P' \times \text{cotang } \phi;$$

$$A'P' = A'N + NP' = A'C \text{ tang } \theta + G'P' \text{ tang } \theta;$$

$$\therefore\ G'P' = (A'C \text{ tang } \theta + G'P' \text{ tang } \theta) \text{ cotang } \phi.$$

$$G'P' = \frac{A'C \text{ tang } \theta \text{ cotang } \phi}{1 - \text{tang } \theta \text{ cotang } \phi}, \quad A'C = b - CD = b - h \text{ cotang } \theta;$$

$$G'P' = \frac{b \text{ tang } \theta \text{ cotang } \phi - h \text{ tang } \theta \text{ cotang } \phi \text{ cotang } \theta}{1 - \text{tang } \theta \text{ cotang } \phi}.$$

$\text{tang } \theta = \frac{1}{r}$; $\text{cotang } \theta = r$; $\text{cotang } \phi = s$.

$$G'P' = \frac{\frac{bs}{r} - hs}{1 - \frac{s}{r}} = \frac{\frac{bsr}{r} - hsr}{r - s} = \frac{rs}{r - s}\left(\frac{b}{r} - h\right). \quad . \quad (287)$$

$\frac{b}{r} - h$ is the depth of the fill $A'N$, and the distance CD of the grade or zero point from the centre is $CD = rh$. These equations are applied as in the preceding examples. A stake should be placed at C, and marked zero, in order to guide the workmen.

479. The above discussions show clearly the principles involved. But this method is rarely employed except on very steep hillsides, and then only when the slope of the ground is practically uniform across the line.

(2) The usual method is to find the points G and G' by trial, using the level instrument and the tape. In case of an embankment, Fig. 209, with side slopes of $1\frac{1}{2}$ to 1, the engineer estimates by the eye the rate of slope of the ground. He calculates that the distance PG is a certain number of feet. If the centre fill is 10 feet, the fill at $M = AM$ must be $10 + \frac{7}{4.5} = 11.5$ feet, 7 feet being the half width of the road-bed assuming that the slope is 4.5 to 1. If the ground is level outwards from M he would only have to measure $11.5 \times 1\frac{1}{2} = 16.2$ feet, but in this distance the ground has fallen 3.6 feet $\left(\frac{16.2}{4.5}\right)$. He should therefore measure out an additional distance of $3.6 \times 1\frac{1}{2} = 5.4$, or a total of 21.6; but as the ground would again fall over a foot in 5.4 feet, he adds about 2 feet to the above, and tries a distance of 23.6 feet. He then finds with the level and rod how much this point is below the grade-line. He finds it to be 16.1 feet; $16.1 \times 1\frac{1}{2} = 24.1$ feet. This is a little greater than 23.6 feet. If the ground was level outwards from this point it would only be necessary to place the stake at the distance of 24.1 feet; but as the ground is falling it is necessary to go a little farther, and he tries the distance of 24.5 feet, and finding the fill or vertical distance from the grade-line to be 16.3 feet, then $16.3 \times 1\frac{1}{2} = 24.45$ feet. Therefore this is the proper point for the slope-stake at G, Fig. 209. Similarly for the stake G' on the right at the point N, 7 feet from the centre line, the fill is found by the level to be $10 - 1.5 = 8.5$ feet. If the ground is level outward

from this point we should measure out $8.5 \times 1\frac{1}{2} = 12.8$ feet, but in this distance the ground will have risen about 2.8 feet, and the fill at this point would be $8.5 - 2.8 = 5.7$ feet. We should therefore come into a distance of $5.7 \times 1\frac{1}{2} = 8.5$ feet, but in so doing the fill would increase, and require a somewhat greater distance. Then for a trial point take 10 feet out, and finding the fill at this point to be 6.0 feet, which would correspond to $6.0 \times 1\frac{1}{2} = 9.0$ feet, move, then, to a distance of 9.8, and we would in all probability find the fill to be 6.5 feet. The stake G_1 should be then placed at this latter distance, since $6.5 \times 1\frac{1}{2} = 9.75$ feet.

These results correspond with those obtained in paragraph 477 from the general formulæ, as the estimation of the slope of ground was taken to be about the same in order to show the agreement between the two methods. In practice the rapidity of the work will depend upon the accuracy with which the slope of the ground is estimated.

When the work is in excavation the trial method just described is similarly applied, the difference consisting only in the greater width of road-bed and in the slopes being 1 to 1 instead of $1\frac{1}{2}$ to 1, the determined cut at any point is multiplied by 1 instead of $1\frac{1}{2}$; or, in other words, the depth of the cut is equal to the horizontal distance required from the points M and N, Fig. 210.

The principle is still the same if ground surface intersects the road-bed, as in Fig. 211. The point C is determined by finding the point on the ground which has the same elevation as the surface of the road-bed.

A little practice will enable the engineer to estimate with considerable accuracy the slope of the ground, and except on very rough or irregular ground the slope-stake can be readily placed at the second trial. Extreme accuracy is not required. The fill or cut to the nearest tenth of a foot, and the corresponding distance to two or three tenths of a foot, are considered good results. The notes should, however, have the proper distances recorded in them. The writer's practice has been, when a point was determined within two or three tenths of a foot of the proper distance, to place the stake at the proper distance, as the rise or fall of the ground is usually inappreciable in small distance of this amount. This would be proper if the first trial placed the stake in a foot or two of its proper position, if for this distance the ground is practically level.

It is needless to say that all such cross-sections should be placed

at right angles to the centre line, whether on a tangent or on a curve, the slope-stakes and the centre one being in the same straight line. The direction is determined by the eye, and with a little care can be accurately aligned.

480. The form of cross-section notes by the trial method, paragraph 479, is as follows. For embankment:

	Left.	Fill.		Right.	Fill.
Station	16.3	11.5	*F*	8.5	6.5
50	31.5	7	10	7	9.8
	or				or
	24.5				16.8

The centre column gives the fill at the centre stake; the fraction on either side and adjacent to the centre column gives the fill at the point *M*, Fig. 209, $= 11.5$ feet, placed in the numerator, and the half width of the road-bed, $= 7$ feet, in the denominator. The outside fraction on the left gives the fill 16.3 feet at the point *G*, and the horizontal distance from the centre stake $= 24.5 + 7 = 31.5$, or sometimes only the distance $PG = 24.5$ feet is given in the denominator. On the right the outside fraction is a fill of 6.5 feet at G', and either the horizontal distance from the centre *O* or the distance from $N = P'G'$. For an excavation the letter *C* is used for *F*, or the letters *C* and *F* are omitted and the signs $+$ and $-$ used.

These notes are kept in a special field-book called the cross-section book. Field-notes should be kept as neatly as practicable, but a duplicate copy neatly written should always be kept in the office. The field-book should not be defaced by the necessary calculations required in determining the position of the slope-stakes. This figuring should be done on an off piece of paper, which can be thrown away, as only the final results are required. In many cases the necessary calculations can be made mentally.

481. Such cross-sections are usually only required at every regular station, the stations being 100 feet apart, as it will be found that the ground for a distance of 100 feet will have a slope practically uniform. If, however, such is not the case, cross-sections must be taken at 80, 60, 50, or any number of feet apart between which the longitudinal slope of the ground can be considered as uniform. Frequently on very rough ground they have to be 10 or even only 5 feet apart. All such matters are necessarily left to the judgment of the resident engineer.

Cross-sections must also be taken on both sides of all ditches or

streams requiring culverts, arches, or trestles, and the distances apart of such sections carefully noted.

482. Frequently a line is located where, instead of having a slope across the line falling or rising in one direction, it may rise or fall away on both sides of the line, the centre cut or fill being either greater or less than those at the slope-stakes on both sides. The principle of setting the slope-stakes is not, however, altered.

Often the ground is irregular across the line, as indicated in Fig. 212 for an excavation, or, if turned upside down, for an embankment. The positions of the slope-stakes are still determined by trial identically as already described. But evidently, as the object of cross-section work is not only to guide the workmen, but also to enable the engineer to calculate the area of the cross-section, it becomes necessary to take a number of intermediate cuts or fills at every material change in the slope of the ground.

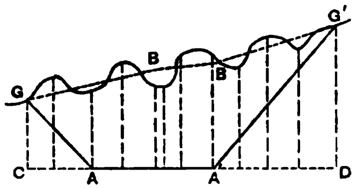


FIG. 212.

At each of the points indicated in the sketch by dotted lines the depth of the cutting or height of filling must be determined and recorded in the notes, with its distance from the centre line in the form of a fraction, as already indicated.

CALCULATION OF AREAS.

483. In Fig. 213 the area required is $GAA'G'G$. This is made up of a trapezoid, $MAA'NM$, and two triangles, MAG and $NA'G'$, and this is true in nearly all cases. The figure can be divided into a series of trapezoids and triangles, the algebraic sum of which will give the required area. The algebraic sum is specified, as it often simplifies the calculation to embrace outside triangles, such as AGC and $A'G'D$, and then to deduct their areas. Upon such combinations of simple geometrical figures all formulæ are based. There are for all figures a number of such combinations, some one of which reduces the labor of calculation to a minimum. It will only be necessary here

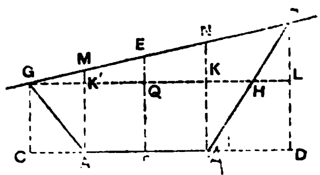


FIG. 213.

to indicate the processes, and not to undertake to give examples of all possible conditions and forms.

In Fig. 213 it is evident that we can add together $MAA'N + GMA + G'NA'$, or $EFCG + EFDG' - (AGC + A'G'D)$ or $GAA'H + G'GH$. In either case sufficient data is known to determine any of these partial areas. As we know the verticals EF, GC, and $G'D$, and the horizontal distances CA, AF, $A'F$, and $A'D$ which give direct the data for calculating any of the above areas except $GAA'H$ and $GG'H$. For these we require to know GH and GL; $GH = CA' + KH = CA' + CA$, and $G'L = G'D - GC$. Then by applying the simple rules for determining the areas of triangles and trapezoids the required area is readily determined. This can be expressed in general formulæ, but they are seldom used, though much labor may be saved by using them.

In Fig. 212 it is only necessary to add together all of the small trapezoids into which the figure is divided and then subtract the outside end triangles AGC and $A'G'D$. Or, for approximate purpose, a thread can be stretched from G, G', respectively to a centre or some intermediate point, in such manner as to equalize the irregularities of the surface, giving and taking, thereby forming regular figures, as indicated by the dotted lines GB, $G'B'$, and BB'.

484. Having found the area of a cross-section by any of the above methods, it is often desirable to find the proper depth of a cross-section on level ground having an equal area with the actual section given. It is evident from Fig. 213 that the level cross-section $GAA'HG$ is composed of the rectangle $AK'KA'$ and the two equal triangles AGK' and KHA'. The rectangle is equal $AA' \times FO = wh$, and each of the triangles is equal $\frac{1}{2}K'G \times K'A = \frac{1}{2}s \cdot K'A \times K'A = \frac{1}{2}sh^2$, in which s is the slope ratio; i.e.,

$$s = \frac{GK'}{K'A} = 1 \text{ to } 1 \quad \text{or} \quad 1\tfrac{1}{2} \text{ to } 1,$$

or any other ratio. Hence for the two triangles the sum of their areas $= sh^2$, and for the entire area $GAA'H = A = wh + sh^2$. Hence

$$h^2 + \frac{w}{s}h = \frac{A}{s} \quad \text{and} \quad h = -\frac{w}{2s} \pm \sqrt{\frac{A}{s} + \frac{w^2}{4s^2}}, \quad . \quad (288)$$

which is the depth of the equivalent level section. If, then, the actual area has been found for any of the previous cross-sections

given, substitute it for A; giving w, the width of the road-bed, its proper value, usually 14 feet for embankment, and $s = 1\frac{1}{2}$, or for excavation $w = 18$ feet, $s = 1$, and we find the depth which must be used with the common earthwork tables. If other widths and other slopes have been used in calculating the tables, these values must be given to w and s.

Sometimes the depth of the equivalent level section is obtained by simply averaging the centre and the two side depths, i.e., adding their sum and dividing by three, or adding the centre and left, the centre and right, cut, or fill, dividing each by two, then adding these quotients and dividing by two for the equivalent level section. Such methods save labor, and give approximately correct results, but are hardly justifiable for the final estimates. The above equation, (288), should be used.

CALCULATION OF VOLUMES.

485. Earthwork is usually estimated in cubic yards. The dimensions are given in feet, which give the volume in cubic feet, which divided by 27 gives the quantity in cubic yards.

The two more common methods of calculating volumes are (1) by the prismoidal formula and (2) by averaging end areas.

The second is the simplest, but gives results in excess of the actual quantities. The first gives accurate results, but requires more labor, and, except in special cases, is rarely used. A modification, however, of the prismoidal formula is often used.

485. *Prismoidal Formula.*—By means of this formula the content or volume of a prismoid can be exactly calculated. If $V =$ volume in cubic yards, $l =$ length of the prismoid in feet, A and A' the areas of the end sections, and $M =$ the area midway between the end sections, all three of which are parallel, then

$$V = \frac{l}{6 \times 27}(A + 4M + A'). \quad . \quad . \quad . \quad . \quad (289)$$

Then in Fig. 214 $A =$ area $BFEDC$; $A' = OPQRS$, in which the dimensions are known from the cross-section notes; $M =$ area $GHIKLN$, in which the dimensions are to be calculated from the corresponding dimensions of the sections A and A' by taking the mean of any two, that is, the centre fill, right fill, and left fill of $M =$ the corresponding fills of A and A' added together and divided by two; also the corresponding distances, added and divided

by two; and from these mean dimensions the area of M is to be calculated for each prismoid, the length of which is the distance between the two end sections in feet, usually 100 feet, but may be

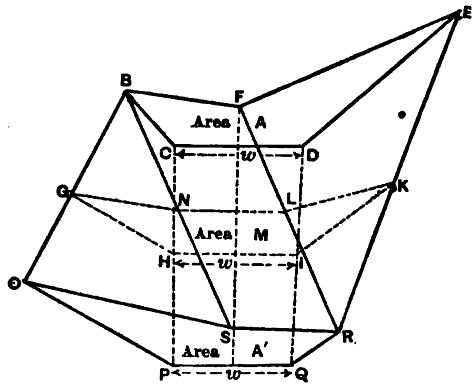


FIG. 214.

any distance greater or less, as governed by the requirements for the distances between cross-sections in paragraph 481. It is not permissible, though often done, to make $M = \frac{A + A'}{2}$. Sometimes it is made a mean proportional, i.e., $M = \sqrt{A \times A'}$. The usual application of this formula is laborious, and many efforts have been made to substitute some modification.

The more common is to assume the ground to be level transversely, as discussed in paragraph 484. Then $A = wh + sh^2$, $A' = wh_1 + sh_1^2$; and using mean dimensions,

$$M = w\frac{h + h_1}{2} + s\left(\frac{h + h_1}{2}\right)^2 = w\frac{h + h_1}{2} + s\frac{h^2 + 2hh_1 + h_1^2}{4}.$$

Substituting these values in equation (289) and reducing,

$$V = \frac{l}{6 \times 27}[2sh^2 + (3w + 2sh_1)h + (3w + 2sh_1)h_1], \quad (290)$$

in which it will be observed that only the dimensions of end sections appear. From this and similar forms of the Prismoidal Formula earthwork tables have been worked out, corresponding to the usual widths of road-bed w, length of prismoids l, and ratio of side slopes s, assuming values of h_1, from 0.1 to any number of feet, while

the horizontal lines have the prefix h, with values from zero (0) to any or an equal number of feet. For any vertical column h_1 is fixed and h variable. The values of V can be then taken from any column corresponding to the given values of h and h_1. It would, however, be an interminable job if the value of V had to be calculated by substituting values for h and h_1, and it will be found only necessary to calculate the values of V for $h =$ a constant, and $h_1 = 1, 2, 3, 4, 5$, etc., and then compute the differences from the expression

$$\varDelta = \frac{4sl}{6 \times 27}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (291)$$

This results from the equation (290) being of the second degree with respect to h, and the second differences of the values of V will be constant and equal to twice the coefficient of h^2. And also the first term of the series of differences in the value of V, i.e., between $h = 0$ and $h = 1$, is expressed by the sum of the coefficients of h^2 and h,

$$\varDelta' = \frac{l}{6 \times 27}[3w + 2s(1 + h_1)]. \quad . \quad . \quad . \quad (292)$$

If, then, we make $h = 0$ in equation (290), it becomes

$$V = \frac{l}{6 \times 27}[3w + 2sh_1)h_1]. \quad . \quad . \quad . \quad . \quad . \quad (293)$$

486. It is only necessary, then, to give successive values to h_1, $h_1 = 1, 2, 3$, etc., and we obtain the first values of V, placing these in a horizontal line opposite $h = 0$, as shown in the following table:

TABLE LI.

Excavation in cubic yards. Width of road-bed $= w = 18$ feet, $l = 100$ feet; side slopes $1\frac{1}{2}$ to 1, $s = 1\frac{1}{2}$.

h	$h_1 = 1$	$h_1 = 2$	$h_1 = 3$	$h_1 = 4$	$h_1 = 5$	$h_1 = 6$	$h_1 = 7$
	Cu. yds.	Cu. yds.	Cu. yds.	Cu. yds.	Cu. yds.	Cu. yds.	Cu. yds.
0	35.25	74.08	116.66	161.73	213.0	266.66	
1	72.29	112.97				312.96	
2	113.03	155.56				362.96	
3	157.47	201.85				416.66	
4	205.61	251.84				474.06	
5	257.45	305.53				535.16	
6	312.99	362.92				599.96	

Having found the values of V when $h = 0$, next find the first difference from eq. (292), $\Delta' = \frac{l}{6 \times 27}[3w + 2s(1 + h_1)]$, when $h_1 = 1$, $\Delta' = 37.04$. The constant difference is found from eq. (291), $\Delta = \frac{4sl}{6 \times 27} = 3.70$, for all values of h and h_1. Add to $35.25 + 37.04 = 72.29$, placing this in column $h_1 = 1$ opposite $h = 1$; $72.29 + 37.04 + 3.70 = 113.03$; $113.03 + 37.04 + 2 \times 3.70 = 157.47$; $157.47 + 37.04 + 3 \times 3.70 = 205.61$; and so on. Then find Δ' when $h_1 = 2$: $\Delta' = 38.89$; $74.08 + 38.89 = 112.97$; $112.97 + 38.89 + 3.70 = 155.56$; $155.56 + 38.89 + 2 \times 3.70 = 201.85$; and so on. The accuracy of the series should be checked by substituting in eq. (290) the values of h and h_1 at intervals; for example, if $h = 6$ and $h_1 = 2$, V should be 362.92 cubic yards. Then for $h_1 = 3$, $\Delta' = 40.74$; for $h_1 = 4$, $\Delta' = 42.60$; for $h_1 = 5$, $\Delta' = 44.44$; and for $h_1 = 6$, $\Delta' = 46.30$. The other columns are run out by similar successive additions; only the first, second, and sixth columns are completed. These tables are thus made for any values of h and h_1. To use the table, if we desired to know the number of cubic yards in a prismoid 100 feet long, with the above widths of road-bed and slopes, when one end cross-section has a depth of cutting $= h = 4$, and the other end $h_1 = 6$ feet, we run down the vertical column $h = 4$, then horizontally to $h_1 = 6$, and we find $V = 474.06$ cubic yards. It must be remembered that this table is only applicable to level cross-sections. But for any cross-sections of the usual forms and roughness of ground we can find the actual areas of cross-section, and substitute these for A in eq. (288), and thereby find the depths h and h_1 of the equivalent level cross-sections, and then with these values of h and h_1 find the value of V from the tables.

These tables can be made out for any values of w, l, and s. The usual values for embankments are $w = 14$ feet, $l = 100$ feet, $s = 1\frac{1}{2}$; for excavation, $w = 18$, $l = 100$, and $s = 1$. The foregoing tables are made out for h and h_1, varying by 1 foot at a time. They can be made varying with 0.5 foot or 0.1 foot, according to the accuracy required. Intermediate values less than 1 foot can be interpolated for approximate results.

487. The tables give correct results in ordinary ground, even with undulating surfaces having ridges and hollows which are parallel to the line of road, and in cases where the surface of the prismoid is plane, whether inclined or not, provided it does not inter-

sect the road-bed within the limits of the prismoid. But they fail on undulating ground, where the ridges or hollows run obliquely to the line of the road, even when the sections seem to be regular. The use of the equivalent depths is accurate when the mid section of the prismoid is its actual area.

488. For the method of average end area it is only necessary to use

$$V = \frac{l}{2 \times 27}(A + A'). \quad . \quad . \quad . \quad . \quad . \quad (294)$$

This gives only approximate results, but gives amounts in excess of the true quantities. It is, however, very often used, on account of the saving in labor. It is required to be used by statute in calculating the cubic yards of earthwork in the State of New York, so far as public works are concerned.

489. When the conditions of the surface of the ground are such that eqs. (290)–(294) are not applicable, then resort must be had to the prismoidal formula, eq. (289), in which the end areas are calculated from exact dimensions, and the middle area from the dimensions obtained by averaging the end dimensions, as already explained. For many very irregular surfaces and for very detailed information on these subjects the reader is referred to "Field Engineering," by Searles.

490. Resident engineers are required to send in to the main office regular monthly estimates of work done during each month. It is not necessary that these should be made out with extreme accuracy. But no material differences should be found between the aggregate of the monthly estimates and the final estimates which are usually calculated either by the prismoidal or some other agreed-upon formula.

ART. XL.

QUARRYING AND STONE-CUTTING.

491. QUARRYING is purely an art. But little can be learned otherwise than by experience; and this knowledge has to be adapted to the varying kinds of stone, to the varying conditions of the same general kinds, the manner in which the stratification, both as to thickness and lay of strata, is found in the quarries, and to the purposes for which the stone is required.

In the preceding article the question of simply loosening stone for excavations was discussed, giving the ordinary methods of drilling holes, the quantity of explosive required to the volume of rock loosened, etc. In that case economy in loosening and subsequent handling was really the only important factor considered. This has application also for many other purposes, such as obtaining rough rubble-stone for masonry or paving, broken stone for concrete, for ballast for macadam road coverings, and the like; that is, for all purposes where the shattering of the stone not only does no harm, but accomplishes the very purpose desired. In such cases neither the quantity nor the kind of explosive is material, these being governed simply by what experience has shown will be most effective and give the best results.

492. When, however, it is required to carry on quarrying operations with a view of obtaining stones of definite shapes and sizes, it becomes important to consider not only the proper methods, but also the quality and quantity of the explosive. It is evident that high explosives in large quantities could not be adopted. Even if the stones were not shattered or broken to pieces, injurious and hidden cracks or seams might be developed. Therefore, when practicable, it is wise to use no explosive.

Many quarries can be economically worked by means of bars, wedges and hammers, and plugs and feathers. Especially is this true of limestones in relatively thin layers, and also of many sandstones, which can be loosened along vertical and horizontal seams. When large horizontal and vertical faces have been exposed by proper stripping of the soil and loose disintegrated portions of the rock, blocks of large size and well shaped can be quarried by means of long rows of plugs and feathers set in holes a few inches deep, and drilled along straight lines at short intervals. In such cases the waste stone is small in quantity.

Where blasting is economical and necessary, several small charges, rather than one or two large ones, will be found effective, as by this means immense blocks can be detached in almost any variety of rock. These blocks can be then cut into smaller blocks, either by the above tools alone, or aided by very small blasts, resulting in but little waste. Owing to many causes it will usually be found that it is very difficult to obtain quantities of large and dimension stone suitable for face and backing stones without also a fair proportion of smaller and ill-shapen stones. The result is that the quarrying will be very expensive unless the smaller stones can

be worked up in rubble masonry, concrete, macadam, Belgian paving-blocks, etc. For these latter purposes, however, only the hard and tough varieties, such as trap, granite, and some varieties of limestones, are at all suitable. For other or soft varieties the bulk of the small stones will have to be wasted.

493. The granites and the best varieties of limestones and sandstones are alone suitable for important structures. There is not much risk in selecting granite quarries. Some varieties are harder, stronger, and better than others, but all will probably be good enough for ordinary purposes. Not so with limestones and sandstones, especially the latter. A hard, tough limestone, however, can usually be distinguished from the softer and weaker varieties. But all signs, indications, and even chemical or mechanical tests fail in giving absolute assurance of the suitableness of sandstones for any specific purpose, unless they have been actually exposed in quarries, bowlders, or structures for a greater or less time. Soundness and durability are, after all, the important properties required, as there is but little danger of using a stone deficient in strength.

Granite is rarely used except in the construction of the most important works, such as light-houses, public buildings, high massive piers of the largest bridges, or, occasionally, where its accessibility, ease of quarrying, and nearness to the site of a structure may render its use either necessary or possibly economical.

For less important purposes sandstones or limestones are universally used. They are abundantly and widely distributed, are relatively easy to quarry, cut, and dress, and have ample strength and durability for all ordinary purposes.

494. *Explosives.*—The principal explosives are nitro-glycerine, dynamite, and powder.

Nitro-glycerine is seldom used in the liquid state in ordinary blasting or quarrying. It readily explodes by percussion; unless confined in cans or in solid rock, is wasted by leakage, and finding its way into crevices or hollows, may be exploded accidentally. It requires but little tamping, is unaffected by immersion, and, being heavier than water, it can be used under water to great advantage. This liquid is produced by mixing glycerine with nitric and sulphuric acids, and is always exploded by means of sharp percussion, produced by means of a cap and fuse. The cap is a small copper cylinder containing fulminate of mercury and some inert substance. Nitro-glycerine develops instantaneously a great force;

at least three or four times as much gas and twice as much heat is developed, pound for pound, as with gunpowder. It costs from 50 to 60 cents per quart.

Dynamite is a mixture of nitro-glycerine and some granular absorbent. The absorbent acts as a cushion to prevent easy explosion by percussion, and is therefore less dangerous to handle. It loses none of its properties or characteristics by being absorbed, is only rendered a little more difficult to explode. If the absorbent is inert, the mixture must contain at least 50 per cent of nitro-glycerine, otherwise it will be too difficult to explode. But if the absorbent contains explosive substances, the percentage of nitro-glycerine may be reduced. A dynamite containing 75 per cent of nitro-glycerine has about six times the explosive force of an equal weight of gunpowder. It is only effective in very hard and solid rock. In soft rock or clay gunpowder is more effective. The lower grades of dynamite are more suitable for sand, loose rock, or earth. For general blasting in open cuts, tunnelling, and mining, a dynamite containing 40 per cent of nitro-glycerine is used; for quarrying, 35 per cent; for blowing out stumps, trees, and piles, 30 per cent; for sand and earth, 15 per cent.

Gunpowder.—For blasting only the cross-grained powder is used. The absolute force developed by gunpowder is not definitely known. It has been determined as high as 200,000 pounds per square inch. Experiment alone can determine the quantity required to produce a given effect.

Dynamite costs per pound from 15 cents to 50 cents, from the lowest to the highest grades.

Powder is sold in 25-pound kegs for from $2 to $2.50 per keg, exclusive of freight. (See Baker on Masonry Construction.)

495. *Stone-cutting.*—It is rarely required of the engineer to do more than specify the sizes and shapes of stones, and for ordinary purposes the forms required are few and simple. It will therefore only be necessary to give a brief description of the methods adopted.

496. The stonecutter's tools consist of a variety of hammers with blunt, chisel, and pointed shapes, sometimes special hammers known as crandalls, or bush-hammers, mallets, and various-sized chisels, and points. With these instruments the stones are cut and dressed to any degree of nicety required.

In large stone yards there are many machines used, such as saws, cutters, planers, grinders, and polishers. The action of these machines are made to resemble as nearly as possible that employed in

dressing stone by hand. The grinders and polishers consist essentially of large iron plates revolving in a horizontal plane, upon which the stone is placed and held in position. By means of water sand is worked in under the stone, which is gradually worn by abrasion to the required degree of smoothness. For ordinary structural purposes it is not desirable to have the surfaces too smooth, as in masonry the stones are usually well bedded in mortar, which will fill all of the little irregularities, and at the same time bond into or adhere better to the stone. Gen. William Sooy Smith states that a column of polished stones simply resting on each other, or having only the thinnest skin of a cement wash between the stones, is at least four times as strong under compression as the same stones unpolished and bedded in mortar would be. A column of limestone 9 feet long and of a uniform cross-section of 1 foot square, composed of several stones, stood a compressive force of 800,000 pounds, or 357 tons.

497. The sides and beds of stones should be dressed to practically plane surfaces, and all angles right angles. These surfaces are not polished, but are about as smooth as can be attained with the point and chisel. Many varieties of sandstone when first quarried can be rough-pointed with an ordinary pick. Harder stones are rough-pointed with a tool called a heavy or blunt point. If a smoother surface is required, the rough-pointed stone is dressed over with the fine point or with the crandall, which is composed of a series of fine points held in a frame with a handle attached. It is simply a means of rapid fine-pointing. In this case the variations from a plane surface do not exceed from $\frac{1}{16}$ to $\frac{1}{8}$ of an inch. If the finish is made with the chisel or patent hammer, the surface is composed of a series of flat chisel marks nearly in the same plane, instead of a series of points and depressions. If when the stone has been rough-pointed or fine-pointed the roughnesses are hammered down by a bush-hammer, a practically plane surface, though not polished, is obtained. This is rarely required.

The face of a stone may be left just as it comes from the quarry, i.e., quarry-faced. If the projections over 3 or 6 inches are knocked off, it is called rock-faced. For certain purposes, such as the up-stream end of a pier, the faces may be rough-pointed or fine-pointed, crandalled, or bushed, but never left with projections over $\frac{1}{2}$ to 1 inch. The down-stream end and likewise the sides may be, in fact usually are, rock-faced. Sometimes the down-stream end is cut to correspond with the other end. For lighthouses and

similar structures all exposed faces are pointed off, or even bushed. Often around the edges on the face a chisel-draught from $1\frac{1}{2}$ to 2 inches wide is made, leaving the remaining portion of the face rough-pointed or rock-faced; or simply the edges are pitched off to a straight line. It is not usual, nor is it advisable, to fine-point the sides of a stone to distances from the face of more than 1 to 2 feet; the rough tails bond well behind the adjacent stones in the same course; especially is this advantageous in headers. All that is necessary is for the dressed surface to clear the width of the bed of the adjacent stretchers. The lower or bottom bed should be true to a straight-edge applied in every direction. It should be out of wind or warp. A very slight concavity towards the centre of the bed is not objectionable if the stone is to be bedded on a somewhat soft mortar; otherwise the pressure will be concentrated on the edges, thereby causing chipping, or even unequal settling of the structure. Stone-cutters are not likely to cut the beds convex, as such stones roll and are difficult to bed; in addition they leave thick and unsightly joints on the face of the work. Except for certain very special and important works, such as lighthouses, the stones are not required to be dressed on all sides, nor to be of exact and definite sizes and shapes. On the contrary, variations in both of these respects are of advantage to the strength of the work. The main requirement for the stones of large, massive masonry is that a sufficient number shall be of the same thickness to complete each course, with due regard to proper lengths and widths to form a good bond both longitudinally and transversely, and that the thickness shall not be less than the specified inferior limit, which varies with the character of the masonry required. In certain special cases every stone has to be cut to a definite size and shape in order to fit accurately in its place, and often so formed as to break joints both in horizontal and vertical directions, the stones being indented or dovetailed into each other, as in lighthouses, or cut to plane surfaces inclined to each other, having one or more curved surfaces, as in the ring-stones of arches. For most purposes it is not desirable that the blocks of stone should be excessively large, as it is difficult to bed well and thoroughly very long and wide stones, and if not well bedded they will split under great pressures. A few long, wide stones are useful in changing the direction of the bond. Contractors are not likely to furnish many very large stones; therefore it is rarely necessary to specify a maximum limit of dimensions.

498. Owing to the fact that the face-stones of large walls and piers are not required to be exact rectangular parallelopipedons, it becomes necessary to specify to what extent they may vary from them ; as, that the width at either end shall not be less than 12, 18, or 20 inches; that the average width shall not be less than from one to two times the depth of the stone; that both side joints shall be dressed true and at right angles to the base, and practically at right angles to the face, of the stone for a distance of 12 to 18 inches or more; and that bottom beds shall be practically dressed to a plane surface, and the top bed to a width not less than from one to two times the depth, leaving the tail beyond this limit to fall away somewhat irregularly, but in no case extending above the plane of the dressed portion. It is also necessary to specify the degree of roughness or smoothness to which the face must be reduced; also whether a chisel-draught or a simple pitch-line is required to be cut around the edges of the face.

For ordinary building purposes the rough block is cut into one of the five following forms: *a, b, c, d, e* (Fig. 215). *a* is a prism, full dressed on beds and joints; *b* is a common trapezoidal form,

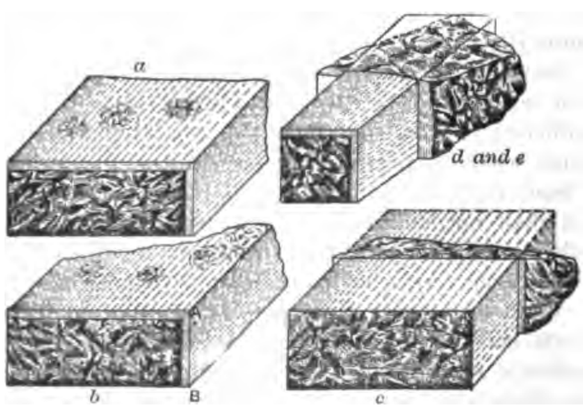


Fig. 215.

c is a stone simply coming within somewhat liberal specifications (the rough tail is contrasted with the perfect form). These are all called stretchers; are rarely less than 3 feet or more than 6 feet in length on the face. *d* and *e* are good forms for headers, their lengths perpendicular to the face varying from 3 to 6 feet.

499. *Dressing the Stones.*—The stone-cutter examines the rough block in order to determine whether the block will work to

better advantage as a header, a stretcher, or a corner-stone which is a header to one face and a stretcher to the other. As the bottom bed, or that on which the stone is to rest in the structure, is to be cut with accuracy and practically to a plane surface for its full width and length, also as the top bed and sides are usually gauged from it, this bed is usually dressed first. The stone is then turned with that face intended for the bottom on top. All rough projections are hammered off, and approximately straight lines are pitched off around its edges; then a chisel-draught is cut on the two longer edges. These draughts are brought to the same plane as nearly as practicable by the use of straight-edges, and the enclosed rough portion dressed down to the plane of the draughts. The entire bed can then be pointed down to a surface true to the straight-edge when applied in any direction—crosswise, lengthwise, and diagonally.

Lines are then marked on this dressed face parallel and perpendicular to the face of the stone, enclosing as large a rectangle as the stone will admit of being worked to, as determined by the other dimensions of the block.

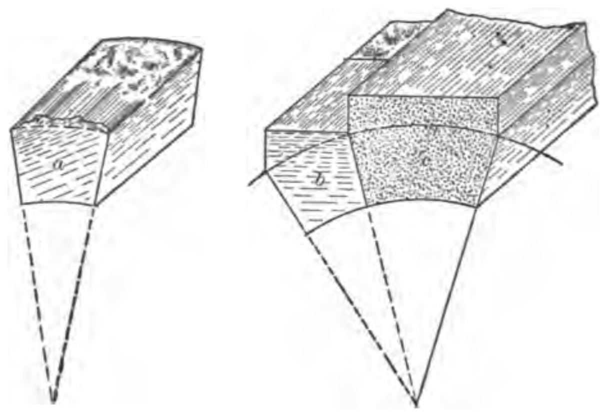
The face and sides are pitched off to these lines. A chisel-draught is then cut down *AB* and along *AC* (see *b*, Fig. 215), and the entire side cut to the plane of these draughts; the same is done on the other side. The stone is turned over bottom bed down, and the top bed dressed down to a plane surface parallel to the bed. It is important that the dressed portion of the top bed should be exactly parallel to the bottom bed in order that the same thickness of stone shall be maintained to this extent.

Small and shallow depressions of the top bed below its general plane surface are not material defects, as they must necessarily be filled with mortar before setting the next course above. A very slight narrowing towards the tails of the stones is permissible, as it facilitates filling the side joints with mortar, which is difficult to do with only half-inch joints if the sides of adjacent stones are exactly parallel all the way back.

If the beds and sides are to be inclined to each other, the proper slope is maintained by using a bevelled straight-edge instead of the ordinary steel square.

All that is necessary for the engineer to do is to tell the foreman what he requires. He will attend to the question of proper procedure if the first stones not coming up to the requirements are condemned.

Arch-stones have two plane surfaces inclined slightly to each other; these are called the beds. The upper surface, left rough as a rule, is called the back or extrados, and the lower surface, the intrados, which is cut to the curve of the arch, usually circular or elliptical. This surface and the beds must be cut as true as possible to the straight-edge or to the curved templet. Where the widths of the various longitudinal courses are all the same, one set of bevelled and curved patterns or templets will answer for every stone. While such a requirement is not necessary, it will usually be found practical to reduce all courses to two or at most three different widths, requiring a like number of templets.



FIGS. 216.

The drawing (*a*), Fig. 216, shows an ordinary voussoir or arch-stone; (*b*) one of the end stones or face-stones; and (*c*) the top or keystone. These may be cut with the square top, as shown by full lines; or the extrados may be cut to the curve of the arch, as shown by the dotted lines. It is evidently necessary for the stone-cutter to know the widths of the voussoirs on both soffit and extrados lines of each stone in order to give the proper inclination to the side surfaces, and also to have a curved templet cut to the exact curve of the soffit. Knowing these, there is little difference in the methods of dressing the stones from those required in the ordinary square blocks. The chisel-draughts are cut to correspond to the plane and curved surfaces, and the stones are dressed down to these

More complicated forms require special templets to guide the workmen. Voussoirs are cut and dressed to practically smooth

surfaces on all sides but the top or extrados, and they should only require the thinnest practicable joints for mortar between them.

500. The large backing or filling stones are selected from those that would not cut into good face-stones, as the requirements of exact thicknesses and shapes are not usually as rigid as for the face-stones. They should have good beds, but the sides are not required to be cut exactly to vertical surfaces, and both bed and side joints in masonry may be thicker than allowed for the face-stones. Specifications on these matters vary between wide limits.

It is to the contractors' interest to use as large stones as consistent with economy in quarrying, cutting, and handling. It is also to their interest to utilize as far as practicable the smaller and ill-shapen stones. The cost of stone masonry will depend largely upon the requirements of the specifications, as well as upon the character of the stone, the nearness and accessibility of quarries, and means of transportation.

The beds of all stones should correspond with the stratification of the stone, as they should be laid on their natural beds, which is commonly the same as the quarry-beds, that is, horizontal or nearly so.

ART. XLI.

MASONRY.

501. MASONRY is classified according to (1) the kind of material used, as stone, brick, or mixed masonry; (2) the dimensions and shapes of the stones; and (3) the manner in which the work is performed, especially in regard to the thickness of the mortar-joints and the extent of the bond required, and also with the uniformity or want of uniformity in the thicknesses of the stones in a given course or vertical distance on the face of the wall. For ordinary massive masonry the appearance on the face, whether rough or pointed, with or without chisel-draughts, is immaterial from an engineering standpoint.

502. The following are terms in common use:

Facing is composed of those stones which are exposed to view; the *Backing* or *Filling*, those which are behind the facing in retaining-walls, abutments, and arches, and between the faces in piers which are exposed to view on all sides.

Headers are stones whose ends are seen on the face of the wall with lengths perpendicular to the face of the walls, and all stones parallel to them.

Stretchers are those whose lengths are seen on the face of the walls, and all stones parallel to them.

Quoins are the corner-stones. These are headers on one face and stretchers on the other.

A Course is one complete horizontal layer of stones, including the facing and backing or filling. Strictly speaking, all of the stones in the same course are of the same thicknesses. It is, however, applied to the thickest unbroken stones, between which two or more layers of thinner stones may be placed.

Joints are the spaces between the stones, usually filled with mortar. They are called bed-joints when horizontal, and side-joints when vertical or inclined to the horizontal.

Bond means the overlapping of the stones, by which no continuous vertical or inclined joints exist, and sometimes the continuity of the horizontal joints are broken.

Batter means the inclination of the face of a wall to the vertical plane.

String-courses are courses of wide stone which project a few inches beyond and outside of the face of the wall.

Coping: One or more courses of well-dressed stones laid with thin side joints, which are intended to protect the rougher masonry below, and to distribute a heavy concentrated load over a larger area. One course, or both courses, projects beyond the face of the wall from 6 to 9 inches.

Raising-stones are large stones made from hard and strong varieties of stone, which are placed on the top of the coping, and serve as a rest for the end posts of bridge-spans, distributing the pressure over a large area of the masonry.

Cramps are bars of iron bent at the ends through a right angle, and inserted in small holes and trenches cut in adjacent stones to hold them together.

Dowels are used for the same purpose. They are, however. straight bars of iron or stone, and are entered into holes on the adjacent sides of two stones in the same course. Bolts are sometimes placed in holes drilled through one stone into the masonry below. Around and over cramps, dowels, and bolts, melted lead or sulphur, or cement grout, is poured. These are used mainly for fastening coping-stones to each other and to the masonry below, or

fastening any stones together that may be liable to be forced out of their positions from any cause whatever.

CLASSIFICATION OF MASONRY.

503. *Rubble Masonry.*—This class of masonry covers a wide range of construction, from the commonest kind of dry stone work to a class of work composed of large stones laid in cement; that is, from walls made of river-jacks or round bowlders to roughly dressed or squared stones, laid with or without mortar, and any combination of these stones. The strength and stability of walls without mortar are both small and uncertain, as there is no adhesion between the stones and no unification of the mass. Each stone is dependent for permanency of position solely on the friction on its surfaces, excessive pressure at any point causing a local displacement of that portion of the wall. If either lime or cement mortar is used to fill the interstices between the stones, a good strong wall can be constructed. The rounded stones are rarely, if ever, used with mortar, except for the interior filling between the face-walls. We may define rubble as masonry of unsquared stones, bedded in mortar or not, according to the strength and stability required. For nearly every important structural purpose mortar is or should be used.

504. Uncoursed rubble is where no regular horizontal courses are used. The joints both in horizontal and vertical planes are broken, and irregularly so. But little attention may be given to the horizontal joints, but special attention should be given to breaking by proper bond all side joints. Whereas the general thickness of the mortar-joints may not be over 1 inch, in many parts of the bed and sides they may be two, three, or more inches in thickness. Such spaces should be carefully filled with small stones or spawls, well bedded in mortar. In this class of work only sharp angles and the rougher projections on sides and beds are hammered off. (See (*a*), Fig. 217.)

505. Coursed Rubble only differs from the uncoursed in having at certain vertical intervals continuous horizontal joints. The thickness of any one course is regulated by that of the corner-stones and an occasional intermediate or bond stone, the spaces between these being filled with uncoursed rubble, the top surfaces of which are levelled by knocking off projections above the corner stones, or levelling up with small pieces and mortar when below, and

upon this course another similar one is laid. No hammering on such stones should be allowed after they are bedded. (See (*b*), Fig. 217.) Masonry of these two classes is the most difficult to build properly. It requires care to properly break the side joints, as well as to properly fill the interstices, which is often done by piling in small stones and smearing a dab of mortar over the top. This should never be allowed. The spaces should be filled either entirely or partly with mortar, and the small stones should be pressed or forced into the mortar. This is the only way of securing sound, solid work. It is stated that the strength of such masonry is about four tenths that of the stone with which it is built; it is probably better to say that it is measured by the strength of the mortar used in it. A cubic yard of rubble masonry requires about $1\frac{1}{5}$ cubic yards of stone and from $\frac{1}{4}$ to $\frac{2}{5}$ of a cubic yard of mortar.

With the proportions of 1 cement and 2 sand, it will require on an average about $2\frac{3}{4}$ barrels of cement and 0.8 cubic yard of sand to make a cubic yard of mortar; or from 0.7 barrel of cement and 0.2 cubic yard of sand to 1.1 barrel of cement and 0.32 cubic yard of sand, or an average of 1 barrel cement and 0.26 cubic yard of sand, to the cubic yard of rubble masonry. Lime mortar is often used for rubble work; this should never be used for foundation-walls.

Rubble masonry is used for foundation-walls of houses, piers, and abutments; for backing retaining-walls, filling between the

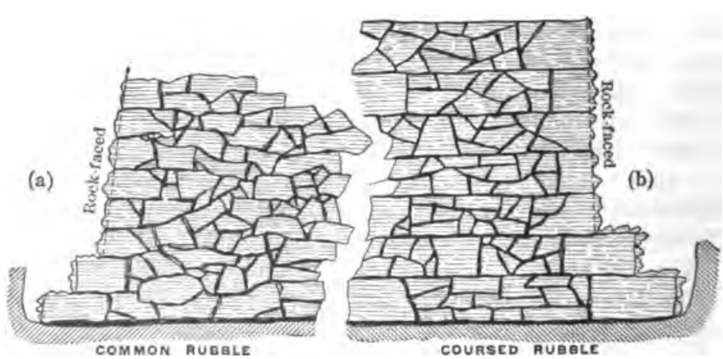


FIG. 217.

face-walls of piers, and for almost any wall where great strength is not required, and where the appearances on the face are unimportant. If, as in (*b*), Fig. 217, the stones are roughly squared and roughly

dressed on beds and sides, a better class of coursed rubble is obtained, the joints of the small stones being approximately horizontal and vertical, respectively.

506. Random or broken-range masonry is similar to the coursed rubble in construction; but all stones, whether large or small, are dressed to horizontal and vertical surfaces, and are cut to fit neatly. In this class of masonry large and small stones are laid without any special arrangement. The main point is to distribute the large stones over the face of the masonry, avoiding as far as practicable any large portions of the surface showing a collection of large or of small stones. The corner-stones should always be of large size. The depth of the stones on the face may vary from 12 to 3 or 4 inches.

507. In all classes of squared-stonework it is usual to prescribe definitely the least thickness or depths of stones. This may vary from 4 inches to 12 inches. Any depths beyond these are permissible, provided the breadths and lengths bear a certain relation to the depths. For the soft varieties of stone the widths or breadths may vary from 1 to 2 times the depths, and the lengths should not exceed 3 times the depths. For hard stones the widths may be as much as 3 times the depths, and the lengths as much as 4 or 5 times the depths. These proportions are required to prevent cross-breaking. It is evident that for a good class of broken-range work there is required a great deal of cutting and dressing, of trouble in assorting and laying the stones, and a considerable proportion of mortar, on account of the large percentage of small stones. Consequently this class of masonry will be expensive, unless the stone lies in well-defined and easily quarried strata of varying thicknesses, requiring also little cutting and dressing. For these reasons, although this masonry presents an attractive appearance, it is seldom used, except for houses, churches, and similar structures. If well built, it has considerable strength to resist crushing. It is not well suited to stand shocks or blows. In localities where suitable quarries are found, as in limestone sections of the country, it is used for abutments and retaining-walls. When laid in mortar 1 cement to 2 sand it will require from $\frac{1}{2}$ to $\frac{3}{4}$ barrel of cement per cubic yard of masonry. This class of masonry is shown in Fig. 235, under the head of Retaining-walls.

508. *Ashlar, Range-work, and Block-in-course Masonry.*—All of these classes of masonry are composed of well-dressed and squared stones. The difference between them is mainly in the dimensions

of the stone, thickness of the mortar-joints, and extent of bond secured.

Range-work, as generally understood, only differs from ashlar in having thicker mortar-joints, say over $\frac{1}{2}$ inch and less than 1 inch; whereas ashlar has joints from $\frac{1}{8}$ to $\frac{1}{2}$ inch in thickness, according to the quality of the masonry.

Block-in-course masonry only differs from ashlar in being built of smaller blocks, the depths not exceeding 1 foot.

Ashlar masonry has an inferior limit of 1 foot in depth, and ranges as high as 3 feet or more. In all three classes each course is of the same thickness throughout; all bed-joints are usually horizontal and all side joints vertical.

509. The strongest bond for these kinds of masonry is that in which a header is placed over the middle of each stretcher below, or headers and stretchers alternate in each course. Therefore, regarding the proportions of dimensions already given, which holds with respect to headers and stretchers alike, in this construction the area of the face should be composed of at least from one fourth to one third of header ends. Such relations and conditions can be made to exist in the very best ashlar masonry with thin joints not over an $\frac{1}{8}$ or $\frac{1}{4}$ inch in thickness. But in ordinary and massive masonry the stretchers (in ashlar masonry) have any lengths from 3 to 6 feet; the headers have any breadths from 1 to 3 feet or even more. It is therefore impracticable, even if desirable, to attempt the placing of headers exactly over the middle portions of the stretchers, or having an overlap or bond of one-third the length of the stretchers. It is therefore specified that the bond shall not be less than from 1 to $1\frac{1}{2}$ feet, and with this limitation the positions of the headers and stretchers can be arranged in each course according to the relative widths of headers and lengths of stretchers. With a limit for the depths of the stones, and inferior limits of 3 feet in length of stretcher and a width of header not less than the depth of the course, the general arrangement of the stones, provided the bond is always equal to or greater than 1 foot, in each course may well be left to the honesty of the contractor and fidelity of the inspectors. It is well to specify that no vertical joint will be permitted under or over a header. Although no valid objection could be raised to bonding on a broad header 3 feet or more in width, the privilege is liable to abuse. It may be allowed occasionally where the introduction of a short or long stretcher would break a zigzag bond forming a continuous series of narrow

steps for a considerable vertical distance on the face of the masonry.

510. The lengths of headers are of more importance than is contemplated in specifications, which usually prescribe only an inferior limit of about 3 feet. Evidently in an 18-inch or 2-foot course, with a breadth of stretcher 2½ to 3 feet, 3-foot headers would be of no value. In such cases they should never be less than from 4 to 6 feet. In other words, headers should never be less than 2 to 2½ times the depth of course, and in ashlar masonry never less than 3 feet, whatever may be the depth of the course.

In block-in-course masonry, the depths of courses varying from 6 to 9 inches, the length of the stretchers should vary from 18 to 27 inches, and the same for the headers; whereas the widths of the headers and stretchers vary from 6 to 18 inches, and the bond from 6 to 9 inches. Otherwise the same remarks apply as for ashlar masonry.

In walls not over from 2 to 3 feet in thickness the headers should extend entirely through the walls. Especially is this important in rubble and broken-range work. In thicker walls, from 3 to 5 feet, the headers should overlap from the two faces; and in still thicker walls interior headers should overlap well the tails of the headers from the faces.

In case of facing walls of ashlar backed with rubble the headers should extend well into the backing.

511. Ashlar and range-work masonry are generally called first-class masonry, and are used for both large and small piers and abutments for bridges, facing of important retaining-walls, and other large structures. A rougher range work or a first class coursed rubble is called second-class masonry; it is used mainly for abutments or other important structures on land, and often for smaller piers for bridges.

Block-in-course masonry is used for small piers and abutments mainly on land, and for houses, when stones of the proper thicknesses can be readily obtained, and may be either first or second class.

Ordinary or roughly coursed rubble is sometimes called third-class masonry. These names are confined usually to railway structures.

512. The backing or filling behind or between the face-walls of abutments, retaining-walls, or piers is either common rubble, coursed rubble, or concrete.

In the better first-class masonry the filling stones are but little smaller than the face-stones; the depths are only from ½ to 1 inch less. The beds should be fairly well dressed, but the vertical sides are only hammer-dressed to approximately plane surfaces. They should bond well with each other, and with the tails of the headers of the face-walls. The joints or spaces between the backing stones vary from 2 to 6 inches. These should be filled with mortar into which the smaller stones or spawls are pressed. When rubble backing is used the space is often filled with stones of all sizes, and the interstices filled with grout, a very liquid mortar. The filling is often made with concrete rammed in layers. Each course of masonry should be completed throughout before building the course or a part of it next above.

513. Good ashlar masonry will require from ⅓ to ½ barrel of cement per cubic yard of masonry when mixed 1 cement and 2 sand.

514. *Strength of Stone Masonry.*—There is want of both theoretical and practical information on the resistance to crushing of stone masonry. It is variously estimated at from one quarter to one half the resistance to crushing of the stone composing it. It depends on the strength and sizes of the stones used, the accuracy of the dressing, the proportions of headers and stretchers, the thickness of the mortar between the stones, and the strength of the mortar. The only reliable information we have on this subject is the actual loads borne by existing structures, some examples of which have been given in this volume. This information, however, does not tell us what such structures will support, but only what they do support. The distribution of the entire superincumbent load or weight is assumed to be uniform over the entire area of the base. This may or may not be true, and evidently cannot be true in the case of high structures exposed to even light winds. From the actual existing loads upon many structures in this and other countries, Mr. Baker gives the following as safe loads. The ton = 2000 pounds.

TABLE LII.

Concrete	5	to	15	tons	per	square	foot,	or	70	to	210	lbs.	per	square	inch.
Rubble	10	"	15	"	"	"	"	"	140	"	210	"	"	"	"
Squared stone	15	"	20	"	"	"	"	"	210	"	280	"	"	"	"
Limestone ashlar . .	20	"	25	"	"	"	"	"	280	"	350	"	"	"	"
Granite ashlar	30			"	"	"	"	"	420			"	"	"	"

515. *Pointing Masonry.*—For this purpose the best cement should be used. It should be either neat cement or 1 cement and 1 sand. The operation consists in cleaning out the joints of masonry to the depth of from 1 to 1½ inches, and refilling the space with the pointing mortar, mixed rather dry, and forced into the joint under considerable pressure.

As ordinarily done it is of little value, and is mainly used to give a neat finish to the joint. The object of pointing is the protection of the mortar-joint from exposure, in order to prevent disintegration from alternate freezing and thawing or other atmospheric influences.

516. *Measurement of Masonry.*—The method of determining quantities of masonry in walls of houses is usually by the perch. This varies from 16 to 25 cubic feet. The actual estimating is governed by local rules and customs. For general engineering structures the quantities are estimated in cubic yards, and on the solid content of the structure.

517. *Cost of Masonry.*—The cost of work, though varying between wide limits, is of more importance to engineers than is usually accredited to it; and it is largely due to the ignorance of engineers of such subjects that the cost as often given of work is so high. The builder has the right to keep the actual cost of his work a close secret; it is to a certain extent his private property, and depends upon his skill, judgment, and experience. He must know the fixed relations between the cost of labor, of the raw materials, and of the completed work. While engineers should not aim to expose the actual cost of work of a contractor, thereby possibly injuring his business, it is his duty to know, as far as practicable, what is a fair cost of any specified work, after allowing for a fair profit to the contractor. But if through a total ignorance of this subject he lets work to contractors at unreasonably high figures, he to that extent fails in his duty to his employers. In a work of this kind it is impossible to more than outline the elements and their relative values in making up the total cost.

Based on fair average conditions, these elements are: cost of the stone, sand, and cement; dressing the stone; hauling or transportation; handling material; plant, such as derricks, tools, machinery, etc.; superintendence, clerks, and general office expenses. The proportion of dressed to rough stone (the latter should include the rough tails of the face-stones as well as the backing proper):

TABLE LIII.

	Cost per cubic yard.
The cost of quarrying for granite face-stones	$3.75 to $4.00
" " " " " sandstone and limestone	2.50 " 3.00
" " " " " granite rubble	2.50 " 3.00
" " " " " limestone rubble	1.00 " 2.00
" " " " " sandstone rubble	0.35 " 1.00
The cost of cutting and dressing beds and joints—	Cost per square foot.
" " " " " " granite	$0.30 to $0.35
" " " " " " marble	0.20 " 0.25
" " " " " " limestone and sandstone	0.12 " 0.15
	Per cubic yard.
Hauling, per mile, including handling	$1.00 to $1.50
Mortar	0.40 " 1.00
Setting stones in piers	1.40 " 2.50

The above costs of labor and raw materials, while not to be taken as applicable at all times and under all conditions, will serve as fair guides in making approximate estimates. These should be modified by information easily obtained at the time.

Having determined upon the character of the stone and masonry desired, it is easy to compute the proportion of cut to rough stone. Assuming two fifths and three fifths respectively, and the masonry of rock-faced limestone ashlar containing 1000 cubic yards, then 400 cubic yards will be cut stone and 600 cubic yards of backing. With 52 square feet of cutting per cubic yard we have 20,800 square feet of cutting. Taking the mortar as given above, and two miles of hauling, the following limits of cost per yard result:

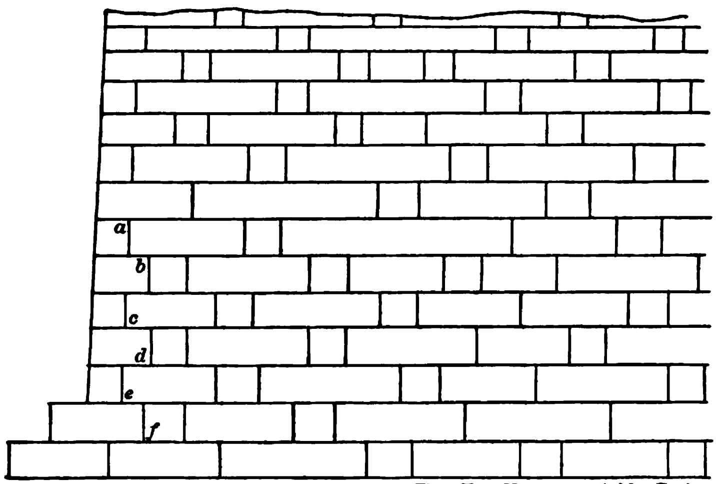
TABLE LIV.

1000 cubic yards	at $2.50 =	$2,500.00,	or at	$3.00 =	$3,000.00
400 cubic yards or 20,800 square feet cutting	at 0.12 =	2,496 00,	or at	0.15 =	3,120.00
Mortar per 1000 cubic yards	" 0.40 =	400.00,	" "	1.00 =	1,000.00
1000 cubic yards hauled 2 miles	" 1.00 =	2,000.00,	" "	1.50 =	3,000.00
1000 cubic yards laying	" 1.40 =	1,400.00,	" "	2.50 =	2,500.00
Totals		$8,796.00			$12,620.00
Add 15 per cent profit		1,319.40			1,893.00
		$10,115.40			$14,513.00
Aggregate cost per cubic yard		10.11½			14 51½

Mr. Baker gives for bridge masonry, first class, from $10 to $20, with an average of $14.

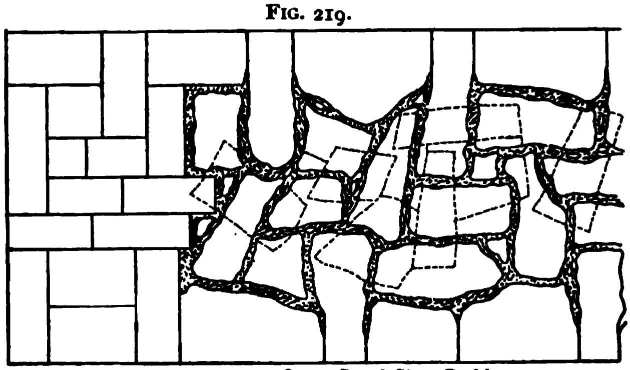
The purpose of the above calculation is rather to indicate the elements of cost and the mode of combining them into the total than to give perfectly accurate results; but within the limits

FIG. 218.



ELEVATION

FIG. 219.



PLAN

above deduced first-class masonry of limestone can be built with a good profit to the contractor.

518. Figs. 218, 219, and 220 are part side elevation, plan, and vertical cross-section, respectively, of a square-ended bridge pier.

These figures are designed to show side by side the general construction with close-dimension stone and the usual first-class railway masonry. The former is indicated by the close fit of the stones and the exact perpendicularity of the bed and side joints, as seen in the left-hand portions of the drawings; the latter, by the want of regularity in the dimensions and shapes of the stones and the greater or less intervals between the backing-stones, as well as of a perfectly symmetrical arrangement of the headers and stretchers in any one course. As good a bond can be secured by a preponderance in the number of headers in one course and of stretchers in another. It may not present as neat an appearance on the faces of the wall.

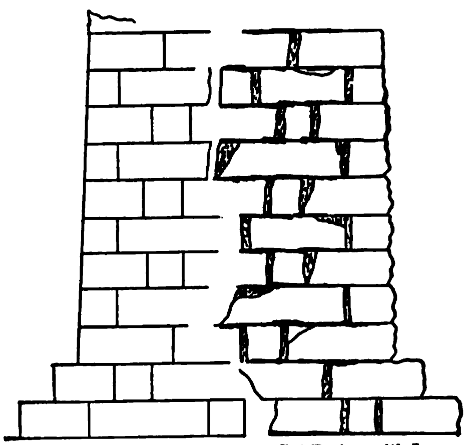


FIG. 220.

Some latitude left to the builders and inspector will, as a general rule, lead to better work. The builder and inspector alone can know the nature of the interior bond, as the apparent showing of headers on the face is no guarantee of their lengths. They are often only short blocks set in the wall in order to deceive, and to seemingly comply with what the contractor believes to be an onerous and useless requirement. An apparent stretcher is often a good header as well; in large piers many stones will be used from 3 to 4 feet in length on the face and extending an equal width into the wall. Such stones on an 18-inch course will contain only 24 cubic

feet, which is a little less than a stone $5 \times 2.5 \times 2 = 25$ cubic feet, often found in a 2-foot course.

In Fig. 218, *a, b, c, d, e, f, g* show zigzag joints with short bond, resulting from a series of headers placed exactly over the centre of short stretches, which should always be broken at intervals, as shown in course above *a*.

519. There seems to have been little effort, either on the part of theorists or practical men, to deduce a formula for determining the thickness of house walls, or any high thin wall which carries only a vertical load, and exposed only to the lateral pressure arising from the pressure of wind or from heavily-weighted floors. If a high thin wall is exposed to a wind-pressure of 50 pounds per square foot of surface, we can arrive at an approximate thickness at the base. The formula for the pressure of the wind is $P = \frac{AV^2}{200}$, in which P is the pressure, A the area in square feet of surface normal to the pressure, and V the velocity in miles per hour. If, then, A is 1 square foot, and $P = 50$ pounds, $V = \sqrt{10000} = 100$ miles per hour. Such a wind is a severe hurricane. In the most disastrous storms the wind-pressure is only registered at from 50 to 80 miles per hour, which corresponds with a pressure of 12.5 to 32 pounds per square foot. Except that a long wall may have a somewhat greater unit pressure on it than a very short one when the wind-velocity is the same, the length of the wall is unimportant. Assuming a wall 50 feet in height and 1 foot in length and weighing 150 pounds per cubic foot, which is about that of good granite or limestone, the weight of the wall, Fig. 221, will be

$$W = 150 \times 50 \times t;$$

its lever-arm will be, with respect to an axis at A, equal to $\frac{1}{2}t$; the total wind-pressure will be

$$P = 50 \times 50 = 2500 \text{ pounds};$$

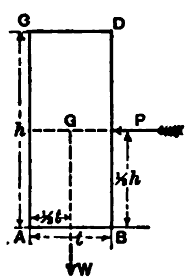


FIG. 221.

and its resultant acting midway between D and B, its lever-arm will be $= \frac{1}{2}h = 25$ feet. Then, from the balance of moments,

$$150 \times 50 \times t \times \frac{1}{2}t = 2500 \times 25 = 62{,}500 \text{ foot-pounds};$$

hence the thickness at base of the wall in the sketch $ABCD$, is $t =$ 4.1 feet. Such a wall under this pressure would be on the point of

overturning, assuming that the total pressure acts at the middle of the height. The weight of the wall above that point is

$$150 \times 25 \times 4.1 = 15{,}375 \text{ pounds,}$$

and the coefficient of friction of masonry on masonry (dry) is 0.65. The total frictional resistance therefore would be

$$15{,}375 \times 0.65 = 9993.75 \text{ pounds} > 2500 \text{ pounds;}$$

consequently there is no danger of sliding. No consideration has been given of the adhesion of the mortar when set. Assuming the adhesion of the mortar at 15 pounds per square inch, the adhesive resistance would be $4.1 \times 144 \times 15 = 8850$ pounds > 2500, the maximum intensity of pressure.

It is evident, then, that if the stability against overturning is secured there will be no tendency to sliding worthy of consideration. For safety the wall should be from $1\frac{1}{2}$ to 2 times the thickness calculated, or from 6.1 to 8.2 feet. But in house walls the floors give support at intervals of from 10 to 12 feet. This unsupported length should be taken as the height in the formula, which would then become

$$150 \times 10 \times \tfrac{1}{2}t^2 = 50 \times 10 \times 5; \quad \therefore\ t = 1.8 \text{ feet}$$

at the floor of the upper story. No mention is made of the support given by the partition walls, and an excessive wind-pressure has been used. It would seem, therefore, safe to use a thickness of from 12 to 16 inches at the extreme top for any height of wall; and adding, say, 4 inches for each 12 feet, the thickness at the base for a 50-foot wall would be from 28 to 32 inches, and for a 100-foot wall 44 to 48 inches, which would doubtless be good practice.

520. For piers of bridges such calculations would not be necessary under any ordinary conditions, as the top dimensions required for the proper rest and bearing of the bridge-spans will give, with the ordinary batter of $\frac{1}{2}$ inch all around, or 1 inch per foot of vertical height, a structure of ample stability against wind and ice pressure. The top dimensions vary from 6×20 feet to 12×35 to 40, which would give at depth of 40 feet from the top, and 1 inch batter, $9' 4'' \times 23' 4''$, or $15' 4'' \times 38' 4''$. At or about such a depth rounded ends are added, which would increase the length by the thickness, and there would result $9' 4'' \times 32' 8''$ or $15' 4'' \times 53' 8''$; and a batter of 1 inch for a distance below this of 60 feet would give $14' 4'' \times 37' 8''$, or $20' 4'' \times 58' 8''$, at the bottom of the

neat line. The rounded ends reach from a few feet below low water to a little above high water. Piers may have squared or rounded ends above this plane. Adding 3 feet all round for three or four offset courses, each 2 to 3 feet thick, and we have for extreme bottom dimensions 26′ 4″ × 64′ 8″, and a height over all of 108 feet. This general construction is typical of nearly all the modern bridge-piers for long spans over navigable rivers. The dimensions may be a little greater or a little less; the basis of calculation, however, will be the same.

The ends may be triangular, circular, or elliptical, as the engineer may deem best. For piers on land the ends are usually square to the sides, the main objects of curved or pointed ends being to give additional stability against shocks or blows, or severe, steady pressures arising from winds, ice and drift gorges. As they act as cut-waters or starlings, it is only necessary to place them at the up-stream end of piers, but for appearance and symmetry they are placed at both ends. In rivers where these gorges are very dangerous to the structure a well-defined starling or cutwater is provided at the upper end, the lower end being built square. This starling extends from a little above the danger line to a point 4 or 5 feet below low water, the extreme edge sloping downward at an angle of about 45° to the vertical. This forms a sloping cutting edge, upon which the ice or drift slides upward. Splitting and separating, it falls off sideways, and is carried past the piers by the current. It also presents a square obstruction to a rapid current, which would cause a deflection downwards of the current, often resulting in scouring and undermining the structures.

The pressure of ice acting upon a square surface opposed to its motion, when in thick sheets of great extent, has been regarded as equal to its resistance per square foot to crushing multiplied by an area equal to the product of the width of the obstructions and the thickness of the ice, this pressure acting with a lever-arm equal to the distance between the centre of gravity of the area pressed and the base of the pier, and that this overturning moment is resisted by the moment of the weight of the pier and of that of the load upon it. The other overturning moments are due to the force of the current on the pier, and the force of the wind on the pier and the superstructure acting with their respective lever-arms, which are the half height of the pier, and the height of the pier increased by the half height of the superstructure, approximately, for the wind-pressure, and one half

the depth of the water for the current; or, as all of these forces are assumed to act horizontally, they can be combined into a single resultant, whose lever-arm can readily be determined when the relative magnitude of the several pressures are known. The solution of the problem is then similar to that explained in paragraph 519 for determining the thickness of a wall to resist wind-pressure.

But giving unusual values to the external pressures and their lever-arms, reducing the weight of the structure and its load to a minimum, and assuming the concurrent action of all the pressures and weights under unfavorable conditions, a simple calculation will prove that the dimensions of piers required by other considerations, and as above given, will provide ample stability. The resistances of the materials to crushing are so far in excess of any crushing load that will be imposed upon them that this question need not be considered.

There is, however, another view of the subject which is not probably understood, but is evidently worthy of consideration, and which can be expressed in a simple formula. It is known that a solid mass of ice surrounding a pier will and does exert a very great lifting-power upon piles or piers enclosed in its grasp. This force has been estimated at 18,000 to 20,000 pounds per pile, or, say, 1 linear foot of ice field; and a pier weighing 1000 tons was lifted and held up under a passing train. This was attributed to adhesion of the ice to the pier during the expansion of the ice arising from a change in its temperature. This expansion may be resisted up to the point of crushing the ice, or the force may be considered as acting by impact rather than by a steady squeeze or pressure which is simply the force of expansion. With ice 2 feet thick acting on the end of a pier 15 feet wide, and allowing 20 tons to the square foot, the pressure would be 600 tons.

If, however, we assume the pressure to be in the nature of an impulse, we may use the following formula: $u = \frac{1}{2}\frac{W}{g}v^2$. The greatest distance that a squeeze can be transmitted or propagated through ice to an obstacle beyond is taken at 1200 feet, as the ice crushes and the residual force is expended in breaking it up. If the force is impulsive, the impulse cannot be transmitted to so great a distance; but assuming the limit of 1200 feet, which is the limit for the modulus of cohesion of ice under compression, then, with b representing the breadth of a field of ice, d the depth, and the weight per cubic foot of ice = 57.4 pounds. Then $W = (1200 \times b \times d)\ 57.4$,

$g = 32.2$; and substituting in the above expression, the greatest possible effect of ice field over 1200 feet $= u = \frac{(1200 \times bd)57.4}{64.4} v^2$, the velocity v being that which the field of ice has acquired at the moment of impact. This formula was used in calculating the dimensions of the ice barriers used to form the ice harbor at the head of Delaware Bay.

Should a long broad field of ice move with anything approaching a flood velocity of from 6 to 10 miles per hour or from 8.8 to 14.7 feet per second, the resulting value of u would be practically irresistible. But, fortunately, ice would be rotten before such a flood could be practicable, and even with very much lower velocities a large mass of ice would rarely acquire the velocity of the current before reaching an obstruction, such as a river pier. Assuming the velocity of ice at 1 mile per hour, or 1.47 feet per second, then with $b = 520$ feet, $d = 1.5$ feet, $v = 1.47$ feet,

$$u = \frac{1200 \times 520 \times 1.5 \times 57.4 \times 2.16}{64.4} = 1{,}802{,}000 \text{ foot-pounds of energy}$$

$= w's$; w' being the weight of the obstruction, and s being the distance through which it is moved. A large pier carrying two 520-foot spans would weigh, say, 5250 tons or 11,600,000 pounds $= w'$; then $s = 0.15$ foot, or, assuming the greatest effect $= \frac{1}{2}u$, $s = 0.075$ foot. In other words, the pressure would be sufficient to move such a structure through a distance of over 0.15 foot, or $\frac{3}{16}$ inch.

This is sufficient to indicate at least the tremendous strain to which many piers are subjected when exposed to such conditions and the necessity of a well-defined cutwater. For this reason open-work structures, such as screw-piles, were deemed best for landing-piers, lighthouses, jetties, bulkheads, wharves, etc., at the head of Delaware Bay.

The construction and stability of abutments for bridges will be considered under the head of Retaining-walls, the only difference being that the top dimensions are greater than for retaining-walls, since provision must be made to carry one end of the end bridge-span. This is called a bridge-seat, which is from 3 to 6 feet wide, and also a breast-wall about 2 to $2\frac{1}{2}$ feet thick.

BRICK MASONRY.

521. There is little opportunity of varying the character of the construction of brickwork, as all of the blocks are of the same

shape and size, being rectangular prisms, and with little or no variation in the dimensions in any one locality. The standard dimensions are $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$ inches.

If the walls are built with headers and stretches alternating in each course, that is, a header placed over the middle of each stretcher below it, the bond is known as the Flemish bond. It is easier to build and maintain a bond of about one fourth of a brick; but since the tendency for the back to separate from the front is less than that to split in a plane perpendicular to the face or length of the wall, it is evident that more stretchers are required than headers in order to give equal strength in both directions. The tendency has consequently been to use the English bond, that is, one or more complete courses composed entirely of stretchers, and then to introduce a complete course of headers, the relative number of courses being determined by the relative importance of transverse and longitudinal bond.

A proportion of two courses of stretchers to one course of headers will provide equal tenacity lengthwise and crosswise. Such an arrangement for the walls of houses is desirable if the joists of floors are so bedded as to throw an equal pressure on both the front and back of the wall. But as work is ordinarily executed it is impossible to form any idea of either the distribution of the pressure or the relative value of the bond, as no attempt is made to proportion the number of headers and stretchers, and it is often difficult to determine which bond has been adopted. It is more difficult to use the English bond and properly break the joints, as there are twice as many joints in a course of headers as in a course of stretchers.

In a tall brick chimney, owing to the short lengths of the sides and to its sustaining only its own weight, there is little danger of the back separating from the front, but there is great danger of splitting longitudinally; consequently it is only necessary to place the courses of headers at intervals of four or five courses.

522. Footing-courses in brickwork, unless the offsets are very narrow, should be laid entirely as headers; and even then the adhesion of the mortar must be relied upon to distribute the pressure where more than two footing-courses are used. But on this point the strength of brickwork is mainly dependent upon the strength of the mortar in all its parts. For, as has been stated, owing to the small bond or overlap possible, so long as the tenacity or adhesion of the mortar is less than the strength of the brick, fracture will follow the joints, and there is no advantage in having it of

greater strength. All experiments prove that with weak mortar nothing is gained by increasing the strength of the brick, but that the strength is increased as the strength of the mortar increases up to the limit above mentioned.

523. Leaving out the questions of certain advantages supposed to accrue from hollow walls, it is a doubtful question whether it is necessary to fill all interior spaces with mortar. That it will make a stronger wall cannot be questioned. But it is a most difficult, if not an impracticable, task to make brick masons thoroughly fill the wall with mortar. If specified, contractors will certainly charge more, and it is doubtful then whether it will be done. But the usual method of not even filling the vertical joints between face-bricks cannot be too severely condemned, and should not be allowed under any conditions. In the writer's experience, another matter of vast importance is grossly neglected, unless not only close inspection is required, and the work rigidly condemned when the requirement is neglected—that of keeping the bricks thoroughly saturated with water. The practice of the carriers of dipping the bricks in a barrel of water and at once loading them into hods and delivering them on the walls is of no benefit.

524. There is also much looseness allowed in the use of filling-in brick, which are often so soft and underburnt that they have little strength, and can add nothing to the crushing resistance of the wall. If all joints were thoroughly filled there might be some excuse for using them. It is true that it is not necessary they should be as well-shaped or as hard-burnt as the face-brick, but a reasonable degree of hardness should be required.

525. So far as good ordinary bricks are concerned, the walls built of them will have sufficient strength to resist crushing when used even in the highest houses. There are a number of examples in which the pressure reaches from 6 to 15 tons per square foot; and experiments indicate that the ultimate resistance to crushing of brickwork in lime mortar is 110 tons per square foot, and if laid in 1 to 2 cement mortar is 180 tons, giving, respectively, 11 and 18 tons, with a factor of safety of 10.

There is much distrust, however, of brickwork for high and heavy piers to carry long spans of bridges, although the pressures at the base would be usually within the limits mentioned above. Under the building laws of Chicago the load on brickwork is limited to 18,000 pounds per square foot when laid in ordinary cement and 25,000 in Portland cement, or from 8 to 11 tons, nearly.

526. The distrust of brickwork arises largely from the uncertainty of securing large quantities of good brick, and the difficulties in handling large quantities, and of putting them to satisfactory tests; in addition to the smallness in the size of the blocks composing a pier liable to abrasions, shocks, and continuous vibrations from passing trains. Moreover, a simple calculation will show that as much as 800,000, 1,000,000, or more pounds may be concentrated at the foot of one end post of a long span, which, even with large raising-stones and iron bolsters and bed-plates, will throw considerable pressure on a small area of the masonry below. These are dimensioned so that the pressure per square inch on the masonry below shall not exceed from 200 to 250 pounds, or, say, from 28 to 36 square feet, requiring stones from 5.3 to 6.0 feet square and from 18 to 24 inches thick. Such stones or bed-plates require both wide and long piers on top. It would be specially desirable to have the resultant pressure well back from the sides, and nearer the centre line of the pier in brick masonry, as the shocks and vibrations are somewhat severe near the top of the pier. Such considerations would require much wider piers in brick than in stone.

For the above reasons there seems to be few if any large brick piers carrying very long spans; in fact for even low piers and short spans, brick is seldom used except in those sections of the country where it is very difficult or exceedingly expensive to obtain stone.

The longest span resting on brick piers of which the writer has any knowledge was constructed by him across the Tombigbee River in Alabama. The rest piers under coping were 7×26 feet, carrying one end of a 275-foot span and one end of a draw-span 260 feet from end to end. The 275-foot span with its load would weigh approximately 1,400,000 pounds, and there is concentrated at the foot of one end post 350,000 pounds. The bed-plate is $4 \times 5 = 20$ square feet, giving a pressure on the coping of about 17,500 pounds per square foot, or 122 pounds per square inch. The coping-stone was brought from New York. By using two coping-courses the pressure was distributed over at least twice the above area, reducing the pressure to about 60 pounds per square foot on the brickwork. This brought the load per square foot well within very reasonable limits for good brick in cement mortar.

527. In the Southern States brick piers and abutments are very commonly used. Owing, however, to the unreliable nature of the foundation-beds, which are often carelessly prepared, they frequently split even under very slight want of uniformity in settling.

This can be easily prevented by the use of hoop-iron laid in the joints, and bent at its ends into the joints at right angles to it. These strips are laid breaking joints with each other, and have the effect of causing the entire structure to act as a unit.

528. For engineering purposes it is better, as is usually the case, to measure brickwork either in cubic yards or to reduce it to a basis of thousands by allowing so many brick to the cubic yard. It requires with bricks measuring $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$, and with joints from $\frac{1}{4}$ to $\frac{3}{8}$ inch in thickness, something under 500 bricks to the cubic yard; and allowing for waste, at least 500 will be required. For house walls it is measured usually by the perch or face measurement. This varies in different localities.

529. *Amount of Mortar Required.*—The quantity of mortar per cubic yard of brickwork varies with the dimensions of the bricks and the thicknesses of the joints allowed. Mr. Baker gives the following quantities, using the standard bricks $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$ inches:

TABLE LV.

Joints	$\frac{1}{8}''$ thick	from	0.10 to 0.15	cu. yd. mortar per cu. yd. masonry.
"	$\frac{1}{4}''$ to $\frac{3}{8}''$ thick	from	0.25 to 0.30	" " " " " " "
"	$\frac{1}{2}''$ to $\frac{5}{8}''$ "	"	0.35 to 0.40	" " " " " " "

And with fair averages of cement we may estimate that with 2 sand and 1 cement it will require about 2.75 barrels and 0.80 cu. yd. of sand per cubic yard of mortar; with 3 sand and 1 cement, 1.9 barrels cement and 0.90 cubic yard of sand; with 4 sand and 1 cement, 1.6 barrels cement and 0.95 cubic yard of sand. Hence for the $\frac{1}{4}$ inch joint there will be required 0.69 barrel of cement and 0.2 cubic yard of sand per cubic yard of masonry for 1 cement and 2 sand mortar.

530. A bricklayer and one helper will lay from 1000 to 2000 bricks per day, depending on the character of the work, which is about equivalent to from 2 to 4 cubic yards.

531. Cost of brickwork in buildings is from $10 to $13 per thousand; on an average, $11 per thousand, or $7 per cubic yard. In all matters of cost, a great deal depends upon the quality required, and cost of transportation. A similar calculation of the cost, when the elements are known, can be made as for stone masonry.

532. For all important purposes, such as facing of houses, arch-

rings, piers and abutments, sewers, etc., it is usual to specify that the bricks shall be of the best quality, hard-burned, compact, of uniform texture, and size, regular in shape, with well-defined angles and adjacent faces at right angles to each other; shall show no flaws or cracks, and shall give a clear, ringing sound when struck together, and must not absorb more than from 4 to 6 per cent of water, as determined by weighing them dry and after being soaked in water from 1 to 3 days. Sometimes, in addition, it is prescribed that they shall give from 4000 to 6000 pounds crushing strength.

533. For ordinary purposes it would seem that whether brick or stone should be used is purely a question of cost. Bricks weather and stand the effects of acid atmospheres better than many varieties of stone, and are little inferior to some of the harder stones. They resist fire better than granite or limestone. Brick walls are considered the best barriers to the spread of conflagrations. Less plant, in the way of machinery, derricks, etc., is required in building either walls or massive structures. It is easier to build arches or tunnel linings with bricks than with stone. For reasons already stated, it would not be desirable or economical to build very high piers.

534. The following general rules should be observed in building all kinds of masonry structures:

(1) All porous stones and bricks should be thoroughly saturated with water before bedding them on a layer of mortar. This applies especially to porous sandstones and bricks, and is absolutely essential in hot, dry weather.

Granites and limestones absorb such a small per cent of water that it is only necessary to *sprinkle* a little water over the mortar just before bedding a stone. This removes the dust from the bed of the stone and insures a better adhesion. Water poured on in quantities does more harm than good.

(2) All spaces between stones should be filled completely with mortar. Where these are over an inch in width the mortar should be first put in place, and chips of stone or spawls should be either driven or pressed into the mortar. Under no circumstances should the spaces be filled with stone and then an effort be made to fill under and around them with mortar. It cannot be done, except by grouting the work, which the writer does not recommend.

(3) All stratified stone should be bedded with the strata normal to the pressure upon them; that is usually horizontal, as the loads

and pressures are usually vertical. Exceptions will be noted under Retaining-walls.

(4) In any but rubblework the courses should all be laid normal to the loads or pressures—i.e., horizontal.

(5) Usually it is prescribed that the largest stones shall be laid in the foundation courses. This is frequently done to avoid labor and strain on machinery. It is certainly not advisable in second-class masonry, as a uniform distribution of large and small stones will conduce to a better and stronger structure. In first-class masonry, with no course less than one foot in thickness, it may safely be said that it is a matter of but little importance whether the depth of the courses decrease regularly from bottom to top, or whether some of the thicker courses are found high up on the walls. Appearances alone seem to require all of the thick courses at the bottom, and usually these are below ground or under water.

A blind following of this rule often places the thinnest, narrowest stretchers, the most inferior and shortest headers, immediately under a single course of coping-stones. It would seem wise, at least, to require maximum widths of beds for the courses immediately below the coping, or to use deeper courses; or not only to allow, but require, both conditions.

Fortunately with large dimension stones it is next to impossible to build a structure that will not be capable of standing any but the most unprecedently destructive agencies.

CONCRETE PIERS.

535. Concrete has been used for the foundations of all kinds of structures, from the smallest to the largest; usually, however, surrounded by the natural earth on land, or enclosed in iron or timber walls, well tied together, when under water. It has also been extensively used for the filling between the face-walls of large and high piers, the headers tying the face-walls well into the mass of concrete. The theory of such structures is evidently, first, that masses of concrete require some support until the cement has had time to set; and, second, that even after setting it requires some protection against exposure to atmospheric influences, and against disintegration from abrasion and shocks. For these reasons few, if any, large and high piers have been built entirely of concrete, from bottom to top, especially in deep rivers with rapid currents.

But many structures on land, including piers and abutments of greater or less magnitude, have been thus constructed.

536. It can be safely asserted that for small arches, culverts, retaining-walls, etc., concrete will fulfil every requirement of strength and stability, and that economy alone need be considered. The cement may either be the ordinary or the Portland, depending on the magnitude and the importance of the structures.

Good concrete can be made at a cost of from $4 to $6 per cubic yard. The contract prices range from $6 to $9 per cubic yard, according to the kind and cost of cement used, and the proportions of sand and broken stone to the cement. A fair analysis of the cost is as follows, per cubic yard of concrete:

Quarrying stone, $0.50; breaking stone, $0.50; 1.3 barrels Rosendale cement @ $1.15 = $1.50; hauling stone, $0.50; making and laying, $0.50; sand, $0.50. Total, $4 per cubic yard. For Portland cement add about $2, making $6. It will generally be found that where one element costs less, another will cost more.

As a rule, therefore, concrete will be cheaper than brick or stone masonry. The cost of rubble will usually be something less, certainly for cost of breaking if for nothing else, and is considered by many equally good—and doubtless it is for many purposes. But ease of handling, facility with which it can be made to conform to irregularities, rapidity with which it can be built, and the almost perfect homogeneousness of the mass, all conspire to render concrete the most useful building material used by engineers.

537. Where good broken stone, sharp clean sand, and good Portland cement are used, the writer can see no valid objection to concrete for piers and abutments, provided they are not built too rapidly and not loaded at too early a period. This opinion is based upon good and reliable precedents. A few examples are given below.

Concrete piers for a railway in the Island of Jamaica, carrying arches of 50-foot spans: Piers 48 feet high, 6 × 16 feet at top, 7 feet 6½ inches × 19 feet 2 inches at bottom; width at bottom of footing-courses, 11 feet. This is about the largest structure of its kind in the world. The piers up to the springing line are built of a casing of concrete blocks 18 × 9 × 9 inches, laid header and stretcher, and the filling between of cement concrete. All concrete was composed of 1 Portland cement, 3 sand, and 6 parts broken stone. Sand and cement were first mixed with water, and the mortar then mixed by hand with the broken stone, 1 cubic yard to the batch. This was

then deposited in place in small boxes and not afterwards disturbed. The joints between the blocks were filled with cement mortar. The construction of the arches will be described under the head of Arches.

Another example: Piers 80 feet high, carrying arches of 40 to 45 foot spans, in Spain and Scotland. In these piers layers of concrete and large stones alternated. The large stones sometimes weighed as much as 2 tons. Average proportions of concrete 70 per cent, and with the small stones 76½ per cent. 2680 cubic yards laid in 21 weeks,—an average of from 12 to 25 cubic yards per day. The concrete was deposited in casings of timber, which were not removed for 10 days from any given section. It would have required double the number of men to have used masonry instead of concrete. The cement was exposed in a dry place, turned over, and allowed to cool. Weighed 90 pounds per cubic foot. Residue on a No. 50 sieve, 2500 meshes, did not exceed 20 per cent. Stood a tensile strain of 500 pounds per square inch when 7 days old. Concrete composed of 1 cement, 5 ballast. The ballast consisted of broken stone or slag and sand. The ingredients were mixed first dry, and then with water.

Another cement had a residue of only 10 per cent on a No. 50 sieve; had a tensile strength of 350 pounds per square inch.

The following are the proportions of the ingredients used:

0.304 cu. yd. rubble stone, measured solid;
0.684 " " broken " measured in heaps = 0.342 cu. yd. solid;
0.358 " " sand, clean and sharp;
0.178 " " cement, or 3.9 cwts.

Where regular concrete has cost from $6 to $7, rubble and concrete, with from 20 to 30 per cent of rubble, has only cost from from $3.50 to $5 per cubic yard.

A third example is that of a concrete highway bridge, Philadelphia, Pa., which will be described and illustrated under the head of Arches. The structure consisted of two arch spans, two abutments, and one pier.

There is also another example of an arch with a very long span constructed for a sewer, and also of a concrete arch of 105-foot span, which will also be described.

538. A contract was entered into by certain parties with one of our largest bridge companies to erect complete the entire structure of the Louisville and Jeffersonville Bridge for a round sum of about

$1,000,000, the writer having made the necessary surveys, soundings, and borings, together with the estimates of costs, contemplating the use of pneumatic caissons sunk to a depth of 75 or 80 feet below the low-water surface, on these cribs filled with concrete extending up to or above the bed of the river, and upon these first-class masonry piers, Bedford (Ind.) limestone, with an iron and steel superstructure, the channel span of which was to have been 650 feet in the clear, and other spans from 450 feet down, as economy might indicate. In this plan the masonry piers would have been from 110 to 130 feet in height; the timber caissons and cribs with concrete from 70 to 50 feet. Through delays in securing authority to build the bridge and in raising the necessary money to proceed with the work, nothing was done for some time thereafter.

Subsequently, and without the writer's knowledge, a contract was made for building both the substructure and superstructure for a specified sum—about $1,000,000. It was contemplated in this contract to build the piers with concrete from bottom to top.

The conditions imposed were very general in regard to the character of materials to be used, such as that the cement should be of the Louisville brands or some equally good cement, and that the work was to be done in accordance with the best American practice.

Altogether, both the requirements of the specifications and the terms of the contract seemed vague and unsatisfactory. The writer felt called upon, in the capacity of consulting engineer, to protest against some of the provisions of the specifications.

(1) That there was no precedent justifying the construction of piers 160 to 180 feet in height entirely of concrete; (2) that to build such piers of any ordinary cement, whether of Louisville or any other Rosendale brands, was not in accordance with the best American practice; (3) that requirements as to fineness in grinding, weight and tensile strength, proportions and character of sand and broken stone, methods of mixing, placing, and ramming, were either omitted entirely or were not specific enough; (4) that it only required a very few weeks, at most, after assembling the members of an iron bridge, to erect and swing even the longest spans, and unless a sufficient time had elapsed after completing the piers, the enormous weight of superstructure and rolling load, amounting to from 3,000,000 to 4,000,000 pounds, might cause serious and irreparable injury to the piers.

The writer does not wish to be understood as depreciating the

many excellent qualities of our American cements, nor as reflecting upon the builders in any way. Opinions of manufacturers, èngineers, and contractors differ widely in regard to the suitableness of our natural cements for concrete structures. But all will admit that they are not comparable in strength and soundness with the Portlands, either American or foreign, and that, in piers of such magnitude, supporting very heavy loads, none but the very best cements should be used, and these only under rigid requirements as to fineness, weight, and tensile strength.

539. The writer therefore presented the following recommendations: (1) That both parties should consent to a modification of the contract as follows: All high piers, and especially those built in the river, to be faced with good first-class masonry; the filling to be of concrete composed of 1 cement, 2 sand, and from 4 to 5 of broken stone, the cement to be of the Louisville or some other equally good brand.

Or (2), if concrete piers were to be insisted on, that the cement should be of the very best Portland brand, coming up to good requirements as to weight, fineness, and tensile strength; that the proportions should be 1 cement, 2 sand, and 4 broken stone, which should be carefully and thoroughly mixed, and then placed in layers not exceeding 12 inches in thickness and properly rammed. Each section of from 2 to 4 feet should be enclosed in a strong casing, which should not be removed for at least ten days after the concrete had been deposited; and that no weight of superstructure should be placed upon the piers for at least sixty days after completion, or, better, ninety days.

(3) That while there was no precedent by which we could be guided, the writer was clearly of the opinion that with the use of the best Portland cement, good sand, and broken stone in the proper proportions and thoroughly mixed and rammed, with ample time for setting before being too heavily weighted, and with such portions of the pier liable to abrasion from passing drift and ice protected by a temporary sheeting for at least six months, better twelve months, there would be no valid reason why such a structure should not be as good as, if not better than, the ordinary sandstone masonry.

The first plan, that is, facing the piers with ashlar masonry, was adopted.

The above discussion is used to show the conditions under

which all-concrete piers would be permissible if not sanctioned by the common and best practice. Concrete will doubtless be more extensively used for these purposes when we become better acquainted with the strength, soundness, and durability of this valuable and useful material, and when structures of gradually increasing magnitude give the profession and the public greater confidence, and precedent exists to justify the claim that this mode of building is usual and good practice.

In Nova Scotia concrete has been extensively used for the piers of highway bridges. These have been subjected to the action of heavy drift ice and to the extremes of temperature. Out of 147 bridges, 44 of which have been standing for five years, only one failure has been reported, and this was attributed to poor workmanship. The aggregates were usually gravel; large rubble-stones were imbedded. The cement used was English Portland. The cost of concrete was from $5.50 to $6.50, and for arches $7.00 to $8.00, per cubic yard.

ART. XLII.

RETAINING-WALLS.

540. RETAINING-WALLS are walls usually of stone or brick masonry, of concrete, or of any combinations of these materials, the purpose of which is to support an embankment of earth either with a vertical or a sloping face. No earthy material will stand for any length of time with a vertical face. If an excavation be made into a mass of any kind of earth leaving the sides vertical, they may maintain this position for a greater or less time. After being exposed to air moisture, or frost, the most stable earth will begin to scale or fall off, forming for itself a more or less inclined or sloping face.

This is not a plane surface, but a curved one concave to the front, the upper portion maintaining a vertical or almost vertical surface, and flattening out somewhat gradually as the bottom is approached. The first caving or displacement is due to the destruction of the adhesiveness of the material, and little by little the particles fall away under the action of gravitation. This falling away

may result in a partial undermining of the mass, when large lumps or masses will fall down. Ultimately, however, some more or less definite slope will be reached which will be permanent except when acted upon by running water or some other external cause. The earth is then said to have attained its natural slope, and the angle which this slope makes with the horizon is called the angle of repose. Mere saturation with water has no effect except in altering or lessening the angle of repose; and when it is converted into quicksand or mud its angle of repose is practically zero, both its adhesion and frictional stability having been destroyed. As the adhesion is destroyed by atmospheric influences, it cannot be relied upon to give permanent stability. Certain materials, such as sand and gravel, when clean have practically no adhesive strength, and will assume rapidly the natural slope. The falling of any material being due alone to the action of gravitation, the only resistance to this is evidently the frictional resistance between the particles, when unsupported by artificial means, and if these two balance, the earth mass will have stability. This is called frictional stability.

541. Starting with the above facts only, many theories and formulæ have been evolved, by means of which the magnitude, direction, and point of application of the pressure against a wall supporting a mass of earth may be determined, and from this the proper weight and thickness of a wall to maintain the face of the mass either vertically or at a given inclination, both when the top surface of the bank is horizontal and level with the top of the wall, or rising at any given inclination to the horizon above and away from the wall. Experiments have been made in many ways and with many kinds of material, except with dry sand or gravel, and some similar materials, but little valuable and reliable information has been obtained.

The first assumption is that the pressure on a wall supporting a mass of earth is due to a certain triangular prism of earth which is assumed to slide bodily along a plane surface called the plane of rupture, and that the magnitude of the pressure is equal to the weight of that prism of earth. This plane of rupture does not coincide with the surface of the natural slope of the material, but is a plane bisecting the angle between the natural slope and a vertical plane, which may be taken as the surface of the wall; in other words, it makes an angle with the vertical equal to one half of the

complement of the angle of repose, or, in symbols, $\frac{90 - \phi}{2}$, ϕ being the angle of repose of the earth supported by the wall. This assumption is practically true for clean, sharp sand, but is not true for a tough and tenacious earth.

The second assumption is, that the point of application of the resultant pressure is at one third the height of the wall from the bottom, or two thirds of its height from the top.

The third assumption is that the direction of the pressure is parallel to the surface of the ground.

Although the assumptions in all of the more common theories may not be as specific as those above given in respect to the exact position of the plane of rupture, the amount of pressure against the wall, or in reference to the direction of the resultant pressure under all conditions as to the relative slopes of the ground surface and of the back of the wall, yet they are practically so.

The best-known theories are Coulomb's, Weyrauch's, Moseley's, and Rankine's.

542. Mr. Baker, in "Masonry Construction," says: "This is only another reason for the statement, already made, that theoretical investigations are of but little value in designing retaining-walls. The problem of the retaining-wall is not one that admits of an exact mathematical solution; the conditions cannot be expressed in algebraic formulæ. Something must be assumed in any event, and it is far more simple and direct to assume the thickness of the wall at once than to derive the latter from equations based upon a number of uncertain assumptions. . . . But the preceding discussion shows that the present theories of the stability of retaining-walls are not sufficiently exact to serve even as a guide for future investigations. Furthermore, the stability of a retaining-wall is not a purely mathematical problem."

The writer is not prepared to fully indorse the above quotation —certainly in regard to Rankine's theory, which Mr. Baker does not discuss, and whose theory, if the writer understands it, he does not state correctly, as he (Mr. Baker) states that in Rankine's theory "the thrust of the earth makes an angle with the back of the wall equal to the angle of repose of the earth;" and again, that the theory is not applicable "if the back of the wall inclines towards the earth to be supported." Mr. Rankine repeatedly states that the pressure or thrust of the earth is parallel to the surface

slope of the ground or bank of earth. His general formula is deduced on the supposition that the back of the wall inclines "towards the earth to be supported."

The writer admits that his (Mr. Baker's) remarks about all other theories with which the writer is acquainted have much force. He therefore does not propose to discuss those theories, and refers to Mr. Baker's full and able discussion of them, but proposes to discuss Mr. Rankine's theory, believing that it will serve as a good guide for future investigations. To the writer's mind it is based upon correct scientific principles, which need only the accurate determinations of the constants involved to make it a safe and reliable formula for all ordinary purposes.

543. Mr. Rankine does not consider the adhesion of the material nor the friction of the earth against the back of the wall, nor does he assume that the earth bodily slides on any plane of rupture—which are, to the writer's mind, serious defects in the other theories and formulæ. What he does assume or claim will be better expressed in as nearly his own words as the space which will be given to this subject will admit. He does not assume any sliding of the earth mass at all. This seems reasonable, as the greatest pressure would be exerted just before movement in the mass takes place, which can only occur when the wall has yielded or moved, when entirely new and different conditions and relations between pressures and resistances would arise and exist.

Adhesion of the earth mass is entirely out of the question, as the greatest pressure from a dry and tenacious mass will occur from the relatively loose mass of earth before being thoroughly compacted, since a compacted mass will usually stand with a vertical face.

544. If the mass becomes quicksand or a flowing mud from the presence of water, the effect is only to change its specific gravity and angle of repose, thereby changing the direction and magnitude of the pressure.

As we always seek the maximum possible pressure that can occur, adhesion should always be considered as *nil.*

If a retaining-wall does collapse or fail either by overturning or sliding, the earth, except in the case of sand or gravel, quicksand or mud, will not assume its natural or any other slope; which would seem to show that there is no well-defined plane along which the particles are in equilibrium, i.e., are just on the point of moving, and only awaiting an opportunity to do so.

The writer does not propose to discuss this subject in all its fulness and completeness, for which he refers the reader to Rankine's "Applied Mechanics and Civil Engineering." Only the statement of the principles and a few necessary relations and equations leading up to the subject of earth-pressure will be given.

545. *Internal Stress in General.*—If a body be conceived to be divided by an ideal plane traversing it in any direction, the force exerted between these two parts at the plane of division is an internal stress. And the problem is to determine the relations between the different stresses which can exist together in one body at one point.

In most practical questions respecting the stress in structures, the direction of the stresses chiefly to be considered is parallel to one plane, and the planes upon which they act are perpendicular to this plane, which is parallel to the stresses; the remaining stress, if any, being a principal stress, and perpendicular to the plane to which the others are parallel.

For a clear understanding of this subject of internal stress it will be necessary to demonstrate a few simple problems, the solutions of which depend entirely and solely on the simplest applications of the principles of the equilibrium or balance of forces in one plane. But little mathematics is necessary or used in these problems.

546. *A Pair of Conjugate Stresses.*—Theorem: *If the stress on a given plane in a body be in a given direction, the stress on any plane parallel to that direction must be in a direction parallel to the first-mentioned plane.*

If, in Fig. 222, we consider two planes XoX and YoY traversing a body, and let the stress acting on the plane YoY be in the direction XoX, then the stress on the plane XoX must be in the direction YoY. Consider the condition of a small prism $ABCD$, having its geometrical centre at the point o. This prism is in equilibrium under the action of the stresses between it and other portions of the body, acting on the ideal planes of divisions AB, BC, CD and AD. The planes represented in section by the lines AB and CD being parallel to the plane YoY, and AD and BC parallel to XoX, the resultant forces acting on the planes AB and CD are equal and directly opposed, and parallel to XoX, their common line of action traversing the axis o; they are therefore independently

balanced, and as the only remaining forces are those acting upon the planes AD and BC of the prism, they must be independently balanced. Their resultants must therefore be directly opposed, which could not be unless their directions were parallel to YoY. (Q. E. D.)

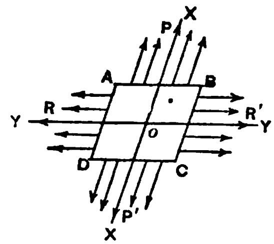


FIG. 222.

547. A pair of stresses, each acting on a plane parallel to the direction of the other, are said to be conjugate, as P and R, etc., in the above figure. (1) Their intensities may be equal; (2) they may bear a definite relation to each other; or (3) they may be unequal, and entirely independent of each other. The first condition exists in a perfect fluid; the third exists in a rigid solid body; the second exists in those materials intermediate between a fluid and a solid body, in which the ratio of the conjugate pressure bears a definite relation for each material when in a certain state or condition, such as wetness and dryness, but varies with different materials.

Again, in a perfect fluid the conjugate pressures must be of the same kind, and must be a pair of pressures or thrusts. In other materials they may be a pair of thrusts, a pair of pulls, or a thrust and a pull or tension.

548. *If the planes of action of a pair of conjugate stresses are both perpendicular to the plane which contains their two directions, then the obliquity of the stresses is the same on their respective planes of action.*

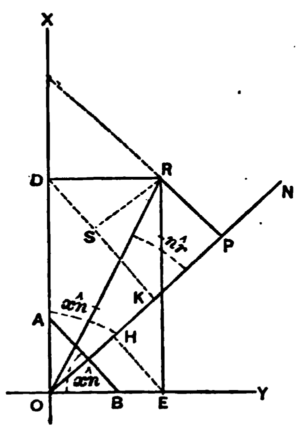


FIG. 223.

The above statements are proved in the following examples. The first problem is as follows:

The intensities and directions of the stresses on a pair of planes perpendicular to each other and to a plane to which the stresses are parallel being given, it is required to find the intensity and direction of the stress on a plane in any position perpendicular to that plane to which the stresses are parallel. In Fig. 223 let the stresses be parallel to the plane of the paper; the planes on which the given stresses

act OX and OY, respectively, and AB any plane perpendicular to the plane of the paper on which the intensity and direction is required. As the given stresses may have any inclination to the planes OX and OY, we can resolve each intensity into components respectively normal and parallel or tangential to the planes upon which they act. We have seen that in such cases the tangential components on the planes OX and OY are equal, and can be represented by one symbol. (See paragraph 326, Article XXXIV.)

Let the stress normal to OY have an intensity p, and parallel or tangential to OY have an intensity t, each acting on an area represented in section by OB. Then the total normal component of the oblique stress on the plane OY will be $= p \times \cdot OB$, and tangential component $= t \times OB$. Similarly, the normal component of the oblique stress on the plane OX will be $= p' \times OA$, and tangential component will be $= t \times OA$. Then draw ON normal to the plane AB, upon which it is required to find the magnitude and direction of stress when the stresses or their components on OA and OB are known. Let the angle which the normal ON makes with the axis OX, be

$$XON = \widehat{xn}.$$

Since the distribution of stress is supposed to be uniform, it is only necessary to assume that all of the planes have a length of unity. Then considering the condition of a small prism AOB, whose length is unity, bounded by the planes OA, OB, and AB.

$$OA = AB \sin \widehat{xn} \quad \text{and} \quad OB = AB \cos \widehat{xn}.$$

Since the angle

$$ABO = XON = \widehat{xn},$$

collecting the stresses on the planes OA and OB as given above, substituting the values of OA and OB in terms of AB, and noting that the total normal stress on OA is the normal component of the stress on OA + the tangential stress on OB; we have, in symbols,

$$p' \times OA + t \times OB = AB(p' \sin \widehat{xn} + t \cos \widehat{xn}) = OE. \quad (295)$$

This can be represented by $OE = DR$ in Fig. 223.

And, similarly, the total normal stress on OB is the normal component of the stress on $OB +$ the tangential stress on OA:

$$p \times OB + t \times OA = AB(p \cos \hat{xn} + t \sin \hat{xn}) = OD. \quad (296)$$

This can be represented by $RE = DO$ in Fig. 223.

In the rectangle thus formed, $ODRE$, the resultant of these stresses, which is the magnitude and direction of the stress on the plane AB, will be represented by the diagonal

$$OR = \sqrt{\overline{OD}^2 + \overline{OE}^2};$$

and the intensity of this stress,

$$\left.\begin{aligned} p_r = \frac{OR}{AB} = \frac{\sqrt{\overline{OD}^2 + \overline{OE}^2}}{AB} = \\ \sqrt{\{p'^2 \sin^2 \hat{xn} + p^2 \cos^2 \hat{xn} + t^2 + 2t(p' + p) \sin \hat{xn} \cos \hat{xn}\}}. \end{aligned}\right\} (297)$$

If we now decompose p_r into its components, normal and tangential, respectively, to AB, by drawing from R the line RP perpendicular to ON, we have the intensity of normal component $p_n = \frac{OP}{AB}$, and the intensity of tangential component $p_t = \frac{PR}{AB}$; OP being the normal component of the total stress OR, and PR being its tangential components; p_n and p_t being their respective intensities. *The mean intensity of a stress being equal to the total stress divided by the area over which it is distributed.*

Since the prism AOB is in equilibrium under three stresses, OD, OE, and OR, it is necessary that the algebraic sum of their components in any two directions shall be equal. If, then, perpendiculars be drawn from the points D and E to ON, intersecting it at the points K and H, respectively, we have three components, RP, DK, and EH, parallel to the plane AB, which must balance independently ;also three components, OP, OK, and OH, normal to the plane AB, which must likewise balance independently.

It is evident that $RP = DK - EH$, and that $OP = OK + OH$; but if not it can be easily proved, for if RP be projected on the line DK, the triangle DRS is equal to the triangle OEH, having all sides parallel, or all angles equal, and also one side

$DR = OE$. Therefore $SR = KP = OH$, and $OP = OK + KP = OK + OH$; also, $HE = DS$ and $RP = DK - DS = DK - EH$. Then

$$OP = OK + OH = OD \cos \hat{xn} + OE \sin \hat{xn}. \quad . \quad . \quad . \quad (298)$$

$$RP = DK - DS(\text{ or } HE) = OD \sin \hat{xn} - OE \cos \hat{xn}. \quad (299)$$

Substituting in these equations the values OD and OE from eqs. (295) and (296), and these resulting values of OP and RP in the values of p_n and p_t, we have

$$p_n = \frac{OP}{AB} = p \cos^2 \hat{xn} + p' \sin^2 \hat{xn} + 2t \sin \hat{xn} \cos \hat{xn}. \quad (300)$$

$$p_t = \frac{PR}{AB} = (p - p') \sin \hat{xn} \cos \hat{xn} + t(\sin^2 \hat{xn} - \cos^2 \hat{xn}. \quad (301)$$

From Fig. 223

$$PR = OP \tan NOR = OP \tan \hat{nr};$$

$$\tan \hat{nr} = \frac{PR}{OP} = \frac{p_t}{p_n} = \text{tangent angle of obliquity of } OR. \quad . \quad (302)$$

We have thus found the magnitude of the resultant stress OR, its intensity p_r, and the direction of the resultant with respect to the plane AB.

If we impose the condition that there shall be no tangential stress on AB, then $p_t = 0$ in equation (301), and

$$\frac{\sin \hat{xn} \cos \hat{xn}}{\cos^2 \hat{xn} - \sin^2 \hat{xn}} = \frac{t}{p - p'} = \frac{\tan \hat{xn}}{1 - \tan^2 \hat{xn}} = \frac{\tan 2\hat{xn}}{2},$$

and

$$\tan 2\hat{xn} = \frac{2t}{p - p'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (303)$$

For two values of $\hat{xn}$ differing by a right angle the values of $\tan 2\hat{xn}$ are equal; therefore there are two directions of the normal ON perpendicular to each other which fulfil the condition of having no tangential stress. Those two directions are called *prin-*

cipal axes of stress, and the stresses along them are called principal stresses which are conjugate to each other.

We can take these two principal axes of stress in any plane for the axes of co-ordinates. This is done in the following problems:

EQUAL PRINCIPAL STRESSES.—FLUID PRESSURE.

549. If a pair of principal stresses be of the same kind (i.e., both thrusts or both pulls) and of equal intensities, every stress parallel to the same plane is of the same kind, of equal intensity, and normal to its plane of action.

In the preceding paragraph we found that the intensity of any stress is equal to the total stress divided by the area over which it is distributed. If, in Fig. 224 we take OX and OY as the directions of the principal stresses, whose intensities are p along OX and p' along OY, the condition imposed is $p = p'$. Now consider as, before, a prism AOB. It is required to find the direction and intensity of the stress on any plane AB.

The total stresses on OB and OA are, respectively, $p \times OB$ and $p' \times OA$ on the axes OX and OY. Take, on OX, $OD = p \times OB$, and on OY, $OE = p' \times OA$, and, completing the rectangle $ODRE$, its diagonal OR will represent the magnitude and direction of the stress on AB, and its intensity

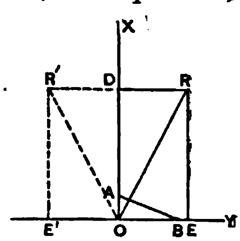


FIG. 224.

$$p_r = \frac{OR}{AB}; \quad \text{also,} \quad p = \frac{OD}{OB}, \quad \text{and} \quad p' = \frac{OE}{OA}.$$

And since $p = p'$, it follows from the figure and this condition that

$$\frac{OD}{OB} = \frac{OE}{OA} = \frac{OR}{AB},$$

and consequently

$$p = p' = p_r. \quad \ldots \ldots \quad (304)$$

The triangles AOB and EOR are similar, and OR must be perpendicular to AB. Therefore the stress on any plane perpendicular to the plane XOY is normal and of equal intensity in all directions. And, furthermore, every direction in the plane XOY is an axis of stress. In other words, in a perfect fluid the pressure at a given point is normal and of equal intensity in all directions, as the same thing can be shown whatever direction may be given

to the plane XOY with respect to the plane of the paper. The above principle only applies to a perfect fluid, which may be defined as that condition of a body in which it does tend not to preserve a definite shape, and therefore cannot exert tangential stress. In the gaseous condition of a fluid the stress is always a pressure or thrust. Liquids possess some little viscosity or tendency to resist distortion, and are capable of exerting a slight tension, but for all practical purposes the only kind of stress worth considering is pressure.

The above principles underlie the entire subject in applied mechanics known as Hydrostatics.

It might be called the circle of stress, since, if a circumference be described with a line representing the intensity of the stress or pressure in any direction at a point in a fluid, the radius of the circle will represent the intensity of the stress in other direction around that point.

If the principal stresses have equal intensities but are of different kinds, $p = -p'$, it is only necessary to lay off $OE' = OE$, but on the opposite of the origin O, OD remaining as before. The construction and proof is similar; p_r is still equal to p or p' in intensity. But the direction of OR' is not perpendicular to AB, but makes an angle $R'OX = ROX$ on the opposite side of OX.

The stress p_r agrees in kind with that one of the principal stresses p or p' to which its direction is the nearest. And when it makes an angle of 45° with each of the axes OX and OY, it is a shearing or tangential stress; so that a pull and a thrust on a pair of planes at right angles to each other and of equal intensities are equivalent to a pair of shearing stresses of the same intensity on a pair of planes at right angles to each other and making angles of 45° with the first pair of planes. The same conclusions would be reached by making $p = p'$ and $t = 0$ in paragraph 548.

Although the principles just established are only particular cases of the more general problem, they have been discussed, as they will assist in giving a clearer understanding of what will follow.

The above results were found by a different method in Art. XXXIV.

ELLIPSE OF STRESS.

550. As in fluid pressure it was shown that the semi-diameter or radius of a circle in any direction represents the intensity of the

stress in that direction, so in the following discussion it will be shown that the semi-diameter of an ellipse represents the intensity of the stress in its direction in a solid body. The difference is, however, broad between the two. In fluid pressure the intensities in every direction are equal, are of the same kind, and are normal to their planes of action. In the ellipse of stress the intensities are unequal, they may or may not be of the same kind, and in general are not normal to their planes of action. The first is comprehended in the second, and is only a special application of it. The general method of procedure is the same in both; only the simplest principles of equilibrium and balance of forces in the same plane are used in their solution. The writer mentions this in order to remove the common idea that the discussion of the ellipse of stress requires the use of the higher and complicated mathe-

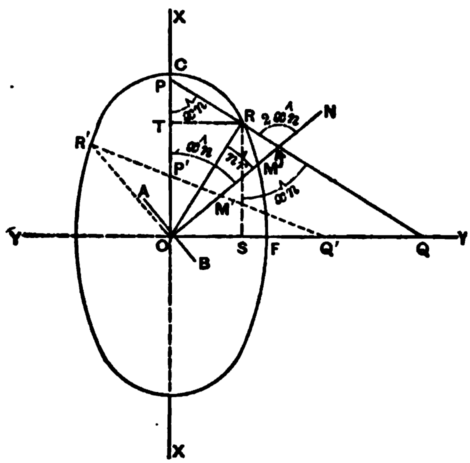


FIG. 225.

matical relations. These are nothing more nor less than the relations between the sides and angles of a triangle and the stresses which they represent, as in the parallelogram or triangle of forces; and, under the conditions assumed or proved, the equations expressing these relations are found by the ordinary test to be that of a simple ellipse. In Fig. 225 let OX and OY be the directions of the two principal stresses, OX being the direction of the greater stress. Let p_x be the intensity of the greater stress and p_y of the less. The kind of stress to which each of these belongs, pull or tension,

thrust or pressure, is to be simply distinguished by means of the algebraic signs. If a pull is considered as positive, a thrust must be considered as negative, and *vice versa.*

It is usual to consider the greater of the two principal stresses as positive. When the principal stresses are of the same kind, p_x is laid off above O on OX, and p_y on the right of O along OY, as indicated in Fig. 225 by the full lines on the right of OX. When the principal stresses are opposite kinds, p is laid off as before, but p' is laid off to the left of O. This is indicated by the dotted lines and the same letters with the dashes corresponding to similar parts of the two constructions.

This being premised, the following is the statement and solution of the problem:

A pair of principal stresses of any intensities, and of the same or opposite kinds, being given, it is required to find the direction and intensity of the stress on a plane in any position at right angles to the plane parallel to which the two principal stresses act.

Lay off on the axis OX a distance to represent the intensity $p_x = OC$, and on OY a distance to represent the intensity $p_y = OF$; on these as semi-axes construct an ellipse: then OR or p_r will represent the intensity and direction of the stress on the plane AB in any position perpendicular to the plane of p_x and p_y, or the plane of the paper. Also, draw ON normal to the plane AB, and letter the several points of intersection. Since the greater of any two quantities is equal to one half their sum plus one half their difference, and the less of two quantities is equal to one half their sum minus one half their difference, we can write the following two equations:

$$p_x = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2} \quad \text{and} \quad p_y' = \frac{p_x + p_y}{2} - \frac{p_x - p_y}{2}. \quad (305)$$

From which it is seen that the pair of stresses p_x and p_y can be considered as made up of two pairs of stresses, viz., a pair of stresses of equal intensity and of the same kind, whose common value is $\frac{p_x + p_y}{2}$; and a pair of stresses of equal intensity but of opposite kinds, whose values are $\pm \frac{p_x - p_y}{2}$.

The first pair being of the same kind and of equal intensity, it corresponds, so far as these portions of the principal stresses are

concerned, to the conditions of fluid pressure discussed in paragraph 549; therefore, arising from this pair of pressures of equal intensity and of the same kind, the pressure on any plane normal to the plane of these pressures is a pressure of the same kind, of equal intensity, and normal to its plane of action. Then lay off on ON, normal to the plane AB, a distance $OM = \frac{p_x + p_y}{2}$: this will be a part of the stress on AB required. For the second part of the principal pressures which are of equal intensity but of opposite kinds, $\pm \frac{p_x - p_y}{2}$, the principles discussed in paragraph 549 apply; that is, the stress on the plane AB will be of the same intensity, $\frac{p_x - p_y}{2}$, and its direction makes angles with OX and OY equal to the angles made by ON with OX and OY, but in the opposite directions. If, then, we draw through M a line PMQ, making $MQO = MOQ$, and $MPO = POM$, this line will give the direction of the stress; and as it will be of the same kind as that of the stress to which it is nearest, we lay off from M on MP a distance $MR = \frac{p_x - p_y}{2}$, and it will be the effect of the pair of stresses $\pm \frac{p_x - p_y}{2}$ on the plane AB. Then drawing from the extremity of MR the line $OR = p_r$, it will represent the resultant of the forces represented by OM and RM; in other words, the direction and intensity of the entire stress on AB. We have $PM = OM = MQ$, since the angle $MQO = MOQ$ and $MPO = POM$, and OM is a common side to the two triangles OMP and OMQ. Therefore

$$PR = MP - RM = \frac{p_x + p_y}{2} - \frac{p_x - p_y}{2} = p_y \quad . \quad (306)$$

and

$$RQ = MQ + RM = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2} = px \quad . \quad . \quad (307)$$

If we drop perpendiculars from R on OX and OY equal, respectively, to RT and RS, then

$$OR^2 = \overline{RT}^2 + \overline{RS}^2$$

and

$$\overline{RT}^2 = \overline{PR}^2 - \overline{PT}^2 = p_y^2 - p_y^2 \cos^2 \widehat{xn} = p_y^2 \sin^2 \widehat{xn}.$$

$$\overline{RS}^2 = \overline{RQ}^2 - \overline{SQ}^2 = p_x^2 - p_x^2 \sin^2 \widehat{xn} = p_x^2 \cos^2 \widehat{xn}.$$

Angle $\widehat{xn} = MOP = TPR = QRS$. Substituting,

$$OR^2 = p_x^2 \cos^2 \widehat{xn} + py^2 \sin^2 \widehat{xn}$$

$$\therefore\ OR = p_r = \sqrt{p_x^2 \cos^2 \widehat{xn} + p_y^2 \sin^2 \widehat{xn}}. \quad . \quad (308)$$

This equation could have been obtained direct from equation (297), paragraph 548, by making $t = O$, since the stresses on OA and OB, in Fig. 223, were there oblique to those planes, whereas in this case they are normal to those planes, and consequently have no tangential component.

It now only remains to find the obliquity, or the angle

$$NOR = \widehat{nr},$$

which OR makes with the normal ON to its plane of action AB. Since $MOR = \widehat{nr}$, and $RMN = 2\widehat{xn}$, we have

$$MR : OR :: \sin \overline{nr} : \sin 2\widehat{xn};$$

then $\sin \widehat{nr} = \sin 2\widehat{xn}\dfrac{MR}{OR} = \sin 2\widehat{xn}\dfrac{p_x - p_y}{2p_r}$, or the obliquity

$$\widehat{nr} = \text{arc}\sin\left(\sin 2xn\frac{p_x - p_y}{2p_r}\right). \quad . \quad . \quad . \quad . \quad . \quad (309)$$

This obliquity is always towards the axis of greatest stress. When p_x and p_y are of the same kind, MR is less than OM or MP, and OR falls onthe same side of OX with ON; hence $\widehat{xn} > \widehat{nr}$. When p_x and p_y are of opposite kinds, then MR' is greater than OM' or $M'P'$, and OR falls on the opposite side of OX from ON, and $\widehat{xn} < \widehat{nr}$. It is evident that the locus of the point M is a circle as it is always equal to $\dfrac{p_x + p_y}{2}$; and it can be shown that equation (308) is that of an ellipse whose semi-axes are p_x and p_y.

It must be remembered that both p_x and p_y are either essentially positive or negative; that is, if both are thrusts, or both pulls, they have the same sign, both positive or both negative, and $p_x + p_y$ or $-p_x - p_y$ means their numerical sum; but if one is a thrust and the other a pull, then $p_x + p_y$ means their numerical difference, since their essential signs reside in the quantities themselves, and are independent of the algebraic sign + or − connecting the two terms. It is in this sense that both OM and OM' are laid off equal to $\frac{p_x + p_y}{2}$; in the one case it is the numerical sum, in the other the numerical difference.

The ellipse of stress represents the relations amongst the intensities of the stresses in a solid mass which are parallel to one plane. Many of these relations have been deduced by other methods in the article on shearing stress, Art. XXXIV, such as that the sum of the normal stresses on a pair of planes at right angles to each other is equal to the sum of the principal stresses, and that the shearing stresses on a pair of planes at right angles are equal to each other; that the shear or tangential stress is most intense on a pair of planes at right angles to each other, and making angles of 45° with the axes of principal stress; and also that the converse of the ellipse of stress can be taken, namely, the intensities, kinds, and obliquities of any two stresses whose planes of action are perpendicular to the plane of their directions being given, the principal stresses and axes of stress can be found.

But in the present article we will only consider those applications which bear upon the questions of the pressure of earth.

The principal stresses p_x and p_y being represented by the semi-axes of the ellipse, they are respectively the greatest and least of the stresses parallel to the plane XOY. The foregoing is the complete discussion of the ellipse of stress. Its simplicity is self-evident.

551. To find the planes on which the obliquity of the stress is greatest, the angle of obliquity, and the intensity of that stress:

(1) When the principal stresses are of the same kind $MR < MO$, and angle $MOR = \widehat{nr}$ is the greatest when MR is perpendicular to OR.

$$\text{maximum } \sin \widehat{nr} = \frac{MR}{OM}$$

$$\text{or max. } \widehat{nr} = \text{arc} \sin \frac{MR}{OM} = \text{arc} \sin \frac{p_x - p_y}{p_x + p_y}. \quad . \quad . \quad . \quad (310)$$

To find the position of the normal ON to the plane AB in terms of maximum $\hat{nr}$: Since $PMN = 2\hat{xn}$, $\hat{xn} = \frac{1}{2}PMN$, and angle $PMN = MRO + ROM = 90 +$ maximum $\hat{nr}$; hence

$$\hat{xn} = \frac{90 + \text{max. } \hat{nr}}{2}, \text{ an obtuse angle, } \quad . \quad . \quad . \quad (311)$$

and for the position of the plane AB itself,

$$XOA = 90^\circ - \hat{xn} = \frac{90 - \text{max. } \hat{nr}}{2}, \text{ an acute angle. } \quad . \quad (312)$$

Eqs. (310), (311), (312) apply to two planes making equal angles on the opposite of OX.

Now to find the intensity of the stress for maximum $\hat{nr}$: since ROM is a right-angled triangle,

$$OR \text{ or } p_r = \sqrt{\overline{OM}^2 - \overline{MR}^2}$$
$$= \sqrt{\left\{ \frac{(p_x + p_y)^2}{4} - \frac{(p_x - p_y)^2}{4} = \sqrt{p_x p_y} \right\}}, \quad . \quad . \quad (313)$$

or it is mean proportional between the principal stresses. It might be obtained directly from the fact that POQ being a right-angled triangle, and in this case OR being perpendicular to PQ, $OR = \sqrt{PR \times RQ}$, and as $PR = p_y$ and $RQ = p_x$, $OR = \sqrt{p_x p_y}$.

(2) When the principal stresses are of the opposite kind it is only necessary to change $+p_y$ into $-p_y$, and equation (313) becomes

$$OR' \text{ or } p_r = \sqrt{-p_x p_y}. \quad . \quad . \quad . \quad . \quad . \quad (314)$$

But p_y being essentially negative, $\sqrt{-p_x p_y}$ is essentially positive, and the same as OR, eq. (313).

Angle $P'M'N = M'OR' + M'R'O$. But $M'OR'$ is 90°, and $M'R'O = \text{arc sin} \frac{p_x + p_y}{p_x - p_y}$; hence

$$\hat{xn} = \frac{1}{2}PM'N = \frac{1}{2}\left\{ 90^\circ + \text{arc sin} \frac{p_x + p_y}{p_x - p_y} \right\}, \quad . \quad . \quad (315)$$

and

$$xOA = 90^\circ - \widehat{xn} = \tfrac{1}{2}\left\{90^\circ - \text{arc sin}\,\frac{p_x + p_y}{p_x - p_y}\right\}. \quad . \quad (316)$$

In these equations remember that p_y being essentially negative, $p_x + p_y$ means the numerical difference and $p_x - p_y$ the numerical sum.

552. If the stress in every direction is a thrust or pressure, and the greatest obliquity be given, it is required to find the ratio of two conjugate pressures whose common obliquity is given. Let ϕ be the given greatest obliquity, then, from eq. (310),

$$\phi = \text{arc sin}\,\frac{p_x - p_y}{p_x + p_y}, \quad \text{or} \quad \sin\phi = \frac{p_x - p_y}{p_x + p_y}; \quad . \quad (317)$$

and if $\widehat{nr}$ be the common obliquity of a pair of conjugate pressures, $\widehat{nr}$ must not exceed ϕ.

Since with conjugate pressures the pressure on each plane is parallel to the other plane, their obliquities are equal, and equal to $\widehat{nr}$, it is evident from Fig. 226 that the angles between the normals to those planes is $90^\circ + \widehat{nr}$, and the angle between the planes themselves is $90^\circ - \widehat{nr}$. From eq. (317)

$$\sin^2\phi = \left(\frac{p_x - p_y}{p_x + p_y}\right)^2 = 1 - \frac{4pp'\cos^2\widehat{nr}}{(p + p')^2}, \quad . \quad . \quad (318)$$

in which p_x and p_y are the greatest and least principal stresses, and p and p' are the conjugate pressures, p being the greater of the two.

Eq. (318) is deduced as follows: In Fig. 226 let AB and $A'B'$ be two planes perpendicular to the plane of the conjugate stress p and p'. Since they are conjugate, the stress on each plane is parallel to the other plane, i.e., $p = OR$ is parallel to $A'B'$, and $p' = OR'$ parallel to AB, ON, and ON', their respective normals, and angle $NOR = \widehat{nr} = N'OR' = \widehat{n'r'}$. Since their obliquities are equal $\widehat{nr} = \widehat{n'r'}$, we can draw in Fig. 227 a single line ON to repre-

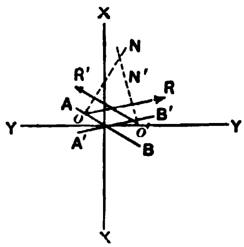


FIG. 226.

sent the normals to both planes; also draw a line OG making angle $= \hat{nr}$ with ON. Lay off on OG a distance $OR = p$, and also $OR' = p'$; bisect RR' at S and draw SM normal to RR'; also draw MR and MR'. It is evident that $MR = MR'$. We at once recognize the same construction as in the ellipse of stress, and $OM = \frac{p_x + p_y}{2}$; $MR = MR' = \frac{p_x - p_y}{2}$. Hence

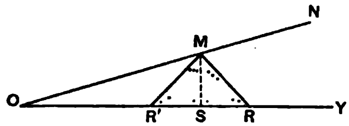


FIG. 227.

$$p_x = OM + MR, \quad \text{and} \quad p_y = OM - MR; \quad . \quad (319)$$

$$\text{angle } \hat{xn} = \tfrac{1}{2}NMR; \quad \text{and angle } \hat{xn'} = \tfrac{1}{2}NMR'. \quad . \quad . \quad (320)$$

These equations give the intensities and directions of the principal stresses p_x and p_y.

Since S is at the middle point of RR',

$$OS = \tfrac{1}{2}(OR + OR') = \tfrac{1}{2}(p + p'),$$

and

$$MS = \tfrac{1}{2}(p + p') \tan \hat{nr}.$$

But

$$MS = OM \sin \hat{nr};$$

hence $\frac{1}{2}(p + p') \tan \hat{nr} = OM \sin \hat{nr}$;

$$\therefore \; OM = \frac{p + p'}{2 \cos \hat{nr}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (321)$$

or

$$OM = \frac{p_x + p_y}{2} = \frac{p + p'}{2 \cos \hat{nr}}.$$

$$\frac{p_x - p_y}{2} = MR = \sqrt{\overline{MS}^2 + \overline{RS}^2} = \sqrt{\frac{(p + p')^2 \tan^2 nr}{4} + \frac{(p - p')^2}{4}}.$$

$MS = \frac{1}{2}(p + p') \tan \hat{nr}$; and

$$RS = OR - OS = p - \frac{p + p'}{2} = \frac{p - p'}{2},$$

which substituted give the last expression. Then

$$MR = \sqrt{\left\{ \frac{(p+p')^2 \sin^2 \hat{nr}}{4 \cos^2 \hat{nr}} + \frac{(p-p')^2}{4} \right\}}$$

$$= \sqrt{\left\{ \frac{(p+p')^2}{4 \cos^2 \hat{nr}} - \frac{(p+p')^2 \cos^2 \hat{nr}}{4 \cos^2 \hat{nr}} + \frac{(p-p')^2}{4} \right\}}$$

$$= \sqrt{\left\{ \frac{(p+p')^2}{4 \cos^2 \hat{nr}} - \frac{(p+p')^2}{4} + \frac{(p+p')^2}{4} \right\}}$$

$$= \sqrt{\left\{ \frac{(p+p')^2}{4 \cos^2 \hat{nr}} - pp' \right\}} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (322)$$

Dividing equation (322) by equation (321), we have

$$\frac{MR}{OM} = \frac{p_x - p_y}{p_x + p_y} = \sqrt{\left\{ 1 - \frac{4pp' \cos^2 \hat{nr}}{(p+p')^2} \right\}}.$$

Then

$$\sin^2 \phi = \left(\frac{p_x - p_y}{p_x + p_y}\right)^2 = 1 - \frac{4pp' \cos^2 nr}{(p+p')^2}, \quad . \quad . \quad (323)$$

the same as equation (318); and

$$\frac{(p+p')^2}{4pp'} = \frac{\cos^2 \hat{nr}}{\cos^2 \phi} = \frac{(p+p')^2}{pp'} = \frac{4 \cos^2 \hat{nr}}{\cos^2 \phi}. \quad . \quad (324)$$

The two roots of this equation are p and p';

$$\therefore \ (u-p)(u-p') = u^2 - 2u(p+p') + pp' = 0.$$

Assume the quadratic equation $u^2 - 2 \cos \hat{nr} u = - \cos^2 \phi$. Since $p + p'$ is proportional $\cos \hat{nr}$ and pp' to $\cos^2 \phi$, and solving with respect to u, we find

$$u = \cos^2 \hat{nr} - \sqrt{\cos^2 \hat{nr} - \cos^2 \phi} = p',$$

and

$$u = \cos^2 \hat{nr} + \sqrt{\cos^2 \hat{nr} - \cos^2 \phi} = p.$$

These values substituted in equation (324) for p and p' satisfy that equation, and are consequently their proper values. Hence we have for the ratio of the conjugate pressures

$$\frac{p'}{p} = \frac{\cos^2 \widehat{nr} - \sqrt{\cos^2 \widehat{nr} - \cos^2 \phi}}{\cos^2 \widehat{nr} + \sqrt{\cos^2 \widehat{nr} - \cos^2 \phi}}. \quad . \quad . \quad . \quad (325)$$

When $\widehat{nr} = 0$ the intensities are wholly normal, and they become the principal pressures or thrust, $p' = p_y$ the less and $p = p_x$ the greater, and $\cos \widehat{nr} = 1$. Hence

$$\frac{p'}{p} = \frac{p_y}{p_x} = \frac{1 - \sin \phi'}{1 + \sin \phi'}. \quad . \quad . \quad . \quad . \quad (326)$$

If $\widehat{nr} = \phi$, then

$$\frac{p'}{p} = 1, \quad \text{or} \quad p' = p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (327)$$

553. The principles and formulæ established and deduced in the foregoing discussion, in respect to internal stress in a solid or fluid body, are accepted and acted upon, without hesitation or doubt, in our everyday practice. Our experience confirms their reliability within any reasonable limitation of the directions and intensities of the stresses; and to guard against contingencies, and as a matter of precaution, factors of safety are used. We further know that when certain materials become more or less fluid, and flow with some degree of facility, that these principles are applicable. It does not seem very difficult to consider them as also applicable to those materials of an earthy nature lying between the hardest and most permanent bodies that we call solids, and those of the stiffest pastes or compact silts and mud, which we are accustomed to deal with as liquids having their own special specific gravities. The writer now proposes to apply these principles to the stability of any granular mass of earthy material, as will be found fully discussed in Rankine's works.

554. As to the actual and relative value of these principles and formulæ, and their relations to Coulomb's theory and the theories of other scientists, Mr. Rankine may speak for himself. I quote:

"Previous researches on this subject are based (so far as I am acquainted with them) on some mathematical artifice or assump-

tion, such as Coulomb's 'Wedge of Least Resistance.' Researches so based, although leading to true solutions of many special problems, are both limited in the application of their results, and unsatisfactory in a scientific point of view. I propose therefore to investigate the mathematical theory of the frictional stability of a granular mass, without the aid of any artifice or assumption, and from the following sole

"Principle. *The resistance to displacement by sliding along a given plane in a loose granular mass is equal to the normal pressure exerted between the parts of the mass on either side of that plane, multiplied by a specific constant.*

"The specific constant is the coefficient of friction of the mass, and is the tangent of the angle of repose. Let p_n denote the normal pressure per unit of area of the plane in question, q the resistance to sliding (per unit of area also), and ϕ the angle of repose; then the symbolical expression of the above principle is as follows:

$$\frac{q}{p_n} = \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (328)$$

"This principle forms the basis of every investigation of the stability of earth. The peculiarity of the present investigation consists in its deducing the laws of that stability from the above principle alone, without the aid of any other principle. It will in some instances be necessary to refer to Mr. Moseley's 'Principle of Least Resistance,' but this must be regarded, not as a special principle, but as a general principle of statics."

555. *Frictional Stability of Plane Joints.*—In a structure composed of a number of pieces connected only by touching each other at plane surfaces, it is necessary to stability that the obliquity of the pressure should at no joint exceed the angle of repose. This is accepted and acted upon by all engineers and builders as absolutely necessary in all blockwork structures of whatever material, and entirely regardless of the sizes of the blocks, whether large blocks in massive masonry, brickwork in walls, or in the roughest masses of small and loose broken stone. It is universally admitted in the case of piles of clean sand and gravel, and surely it is no stretch of the imagination to accept it as true in any mass of earthy materials either having no adhesiveness between its particles or grains, whether large or small, or in a mass whose adhesion or tenacity has been destroyed; and it is safe to assume in all cases

that there is no adhesion, and that the stability of the mass of earth, or gravel or shingle, or of any other material consisting of separate grains, is dependent entirely upon friction between its component parts.

556. Although the above principles and formulæ will not give the exact and actual pressure of a mass of earth on a wall sustaining it at any given slope, it is only due to the fact that there is always some adhesion between the particles composing it; but disregarding adhesion, the determined pressure approaches approximately the actual pressure just in proportion to the accuracy with which the angle of repose has been determined. If the angle of repose is too small, the least value of the ratio of the conjugate pressures will be too large; and consequently, although eliminating the adhesion would increase the apparent pressure on the wall, such an error in the angle of repose may greatly increase the pressure above the actual intensity; or if the angle of repose is greater than its actual value, the ratio of the conjugate pressures will be too small, and the determined pressure on the wall will be too small. This is based upon $\frac{p'}{p} = \frac{1 - \sin\phi}{1 + \sin\phi}$, which gives the least limit to the ratio. This is the limit assumed in practice, and is based upon the "Principle of Least Resistance."

In a certain class of problems the maximum ratio of the conjugate pressures is required, viz.,

$$\frac{p'}{p} = \frac{1 + \sin\phi}{1 - \sin\phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (329)$$

The same remarks apply, but in the inverse order; that is, as ϕ increases the ratio increases, and *vice versa.*

In certain cases it is necessary to combine the two conditions, and, as an error in ϕ which increases the ratio under its maximum value decreases the ratio under its minimum value, there will be a double error in the result. The only application of the combined actions will be explained under the head of Land Ties for Retaining-walls.

557. With these convictions, based not only upon theoretical investigations, but on analogy from experience and observation, the writer unhesitatingly says that Rankine's theory of the frictional stability of a granular mass should not be included amongst those theories characterized as "not sufficiently exact to serve even as a guide for future investigations." Rankine's formulæ give results

agreeing fairly well with the usual and best practice, and with the use of a small factor of safety, which is always employed in the application of all other formulæ used in engineering construction, will give safe and reliable results.

558. *Theorem I.*—It is necessary to the stability of a granular mass that the direction of the pressure between the portions into which it is divided by any plane should not at any point make with the normal to that plane an angle greater than the angle of repose. In the case of a mass of earth, the greatest obliquity of the pressure is to be limited to the angle of repose, which it must not exceed. The maximum value of the obliquity of the stress is found in eq. (310), max. ~~sin~~ $\widehat{nr} = \text{arc sin}\ \frac{p_x - p_y}{p_x + p_y}$, and the above condition requires max. $\widehat{nr} \leq \phi$; $\widehat{nr}$ being the angle between the normal and the stress, $\leq$ meaning equal to or less, but never greater, and ϕ being the angle of repose. The max. $\widehat{nr}$ being hereafter called θ', then $\frac{p_x - p_y}{p_x + p_y} = \sin\theta' \leq \sin\phi$; which simply means that in a mass of earth the ratio of the difference of the greatest and least pressures to their sum cannot exceed the sine of the angle of repose. Also, the ratio of the greatest and least pressures p_x and p_y is found from eq. (326) to be

$$\frac{p_x}{p_y} \leq \frac{1 + \sin\phi}{1 - \sin\phi}. \qquad (330)$$

If, however, we consider any two conjugate pressures p and p', whose common obliquity $\widehat{nr} = \theta$; and ϕ, now limited to the value of the angle of repose, that is, never exceeding it; p the greater and p' the lesser of the conjugate pressures,—we have, from eq. (325),

$$\frac{p}{p'} \leq \frac{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}. \qquad (331)$$

Eq. (331), just found, expresses the condition of stability of a mass of earth in terms of the ratio of a pair of conjugate pressures in the plane of greatest and least pressures.

559. Conceive a mass of homogeneous solid material to be

indefinitely extended laterally and downwards, and to be bounded above by a plane surface making a given angle of declivity θ with a horizontal plane. In Fig. 228 YOY represents a section of that surface with a vertical plane along its direction of greatest declivity, and OX a vertical plane normal to this latter vertical plane. Let w be the uniform weight of unity of volume of the substance. If, then, we take any plane parallel to YOY, or the upper surface, and at a vertical depth x below YOY, and there is no external force other than its own weight acting on the mass, it is evident that the only pressure which any portion of the plane BB bears is the weight of the material directly above it.

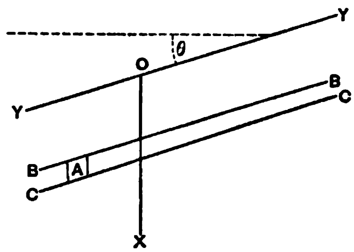


FIG. 228.

Hence in an indefinitely extended homogeneous solid bounded above by a sloping plane the pressure on any plane parallel to the sloping surface is vertical, of uniform intensity, and equal to the weight of the vertical prism, having for its base unity of area of the given plane, and for its height the vertical distance of this panel from the surface; or in symbols, since the area of a horizontal section of this prism is $1 \times \cos\theta$, the intensity of the vertical pressure is

$$p = wx \cos\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (332)$$

As this vertical intensity is balanced by an equal and opposite intensity, it follows, from the principle established in paragraph 546, that the stress, if any, on a vertical plane at that depth is parallel to the sloping surface, and is conjugate to the stress on a plane parallel to that surface.

If we now consider the condition of a molecule A included between the parallels BB and CC, we have seen that the intensity of pressure on its upper surface is uniform, the weight of the molecule, as equilibrium is assumed, must be balanced by an equal and opposite stress equal to the difference between the downward intensity on its upper surface and the upward intensity on its lower surface; consequently, as this stress is vertical, the stresses on its two sides must be equal and opposite, and parallel to the upper or lower surface of the molecule or to the surface of the upper

slope, YOY (see paragraph 546). And as these conditions must exist for each and every molecule included between the planes BB and CC, it follows that the state of stress at any given uniform depth below the upper sloping surface is uniform. In this paragraph three things are determined: (1) that the pressure on a plane parallel to the sloping upper surface is vertical and proportional to the depth (see eq. (332)) below the upper surface; (2) the direction of the pressure on a vertical plane is parallel to the upper sloping surface, that is, it is conjugate to the vertical pressure; (3) the state of stress at a given depth is uniform.

From the above equation (332), $p = wx \cos \theta$; and from the general ratio of $\frac{p'}{p}$, equation (325), we may write

$$p' < wx \cos \theta \frac{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi}}{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi}}, \quad . \quad . \quad . \quad (333)$$

or

$$p' > wx \cos \theta \frac{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi}}{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi}}; \quad . \quad . \quad . \quad (334)$$

which simply means that the intensity of the pressure parallel to the sloping upper surface cannot have a less value than

$$wx \cos \theta \frac{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi}}{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi}},$$

nor a greater value than as indicated in the first of the above equations, (333).

If the upper plane surface is horizontal, then

$$\theta = 0, \text{ and } p' < p \frac{1 + \sin \phi}{1 - \sin \phi}, \text{ or } > \frac{1 - \sin \phi}{1 + \sin \phi}; \; p = wx; \quad (335)$$

and if $\theta = \phi$, or the upper surface slopes at the angle of repose, $p = wx \cos \phi = p$.

It is evident that for all values of θ greater than ϕ equations (333) and (334) become impossible, which simply means that the angle of repose is the steepest possible slope. In this case $\theta > \phi$, $\cos \theta < \cos \phi$.

560. *Earth Loaded with its own Weight.*—In a mass loaded with its own weight the gravitation of the earth causes the vertical pressure, the vertical pressure causes a tendency to spread laterally, and the tendency to spread laterally causes the conjugate pressure; therefore the vertical and conjugate pressures stand to each other in relation of cause and effect, or active and passive, respectively; and by the *principle of least resistance* the passive or conjugate stress will be the least which is consistent with stability. And as this will always be the condition when a mass of earth is supported by a wall, or simply remaining at a slope of the upper surface equal to the angle of repose, as the side slopes of an embankment, it is only necessary to use the least values of p_y in the preceding paragraph, and the equations for practical use are the following:

For the vertical pressure

$$p = wx \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (336)$$

For the conjugate pressure parallel to the steepest declivity

$$p' = wx \cos \theta \frac{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi}}{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi}}. \quad . \quad . \quad (337)$$

For a horizontal ground surface

$$\theta = 0; \quad \cos \theta = 1; \quad p = wx; \quad \text{and} \quad p' = wx\frac{1 - \sin \phi}{1 + \sin \phi}. \quad (338)$$

For the ground surface inclined at the natural slope

$$\theta = \phi; \quad \cos \theta = \cos \phi; \quad \text{and} \quad p' = p = wx \cos \phi. \quad . \quad (339)$$

561. If the earth surface is acted upon by some additional load or pressure, the conjugate pressure may increase until it becomes the greatest possible, as indicated by equations (333) and (335). This condition has been discussed in case of loads upon soft foundation-beds (see paragraphs 398, 399, equations (276), (277)), and will be again used in determining the holding power of earth when acted upon by an external pull or thrust.

There is a third pressure, whose direction is perpendicular to the plane of p_x and p_y. It is a passive pressure, and also a principal pressure; it must therefore be the least possible consistent with

stability, and must be equal to the least pressure in the plane of p_x and p_y. This pressure, however, plays no part in connection with retaining-walls proper, but does in the case of wing or U abutments.

The greatest and least stresses, or principal stresses, in this plane are to be found from equation (305) by substituting for p_x and p_y in the fractions, i.e., the second members, the values of p and p' respectively, equations (336), (337), (338), and (339) of the preceding paragraph.

We are now prepared to apply the foregoing principles and formulæ to the pressure of earth against a vertical plane, or against the back of a retaining-wall, whether vertical or having an inclination to the vertical.

PRESSURE OF EARTH AGAINST A VERTICAL PLANE.

562. In Fig. 229 let OX represent a vertical plane in or in contact with a mass of earth whose upper surface YOY' is either horizontal or inclined at any angle θ, and is intersected by a vertical plane perpendicular to that of steepest declivity.

It is required to find the pressure exerted by the earth against the vertical plane from O down to X at a depth $OX = x$ beneath the surface; also, the direction and point of application of the resultant pressure.

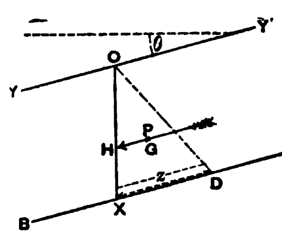


FIG. 229.

Let BB be a plane passing through the point X and parallel to YOY. It is only necessary to consider a length of the plane OX of unity in a direction perpendicular to the plane of the paper. Since the intensity of the vertical pressure is proportional to the depth of any plane below the surface YOY', and the conjugate pressure, within any given mass, having the same slope, same angle of repose, and the same specific gravity, varies with the vertical pressure, and it will be a uniformly varying stress, whose intensity at O is zero and whose intensity at X, the distance of x below the point O, is p_y; hence, to find the length of the prism having an oblique base of the area of unity in the plane OX, and whose weight per unit of volume is w, we have, letting z represent the length,

$$z \times w \times 1 \times \cos\theta = zw \cos\theta = p'.$$

Then lay off $z = XD$ (in Fig. 229) $= \dfrac{p'}{w\cos\theta}$. But from equation (334)

$$XD = \frac{p'}{w\cos\theta} = x\frac{p'}{p} = x\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}. \quad (340)$$

Hence if from the lowest point of the plane OX at X we draw or lay off a line whose length is $XD = x\dfrac{p_y}{p_x}$, i.e., the vertical height of the plane multiplied by the ratio of the conjugate pressure, it will represent the intensity of the pressure on the vertical plane at the point X. This line is laid off on BB', since, as shown in paragraph 546, the direction of the conjugate pressure is parallel to the surface YOY. This same construction could be made at any point between O and X, but since it is uniformly varying, it is only necessary to draw the line OD; then the ordinate of the triangle at any depth parallel to XD or YOY will represent the intensity of the stress at that depth. The total pressure on the wall will then be simply the weight of a prism of earth whose base is the area of the triangle OXD, and whose length is the length of the plane OX in a direction perpendicular to the plane of the paper. But as the length is taken as unity, the volume of the prism will be equivalent to the area of the triangle OXD, which, multiplied by the weight of a unit of volume of the earth ($= w'$), will give the weight of the prism which is equal to the total pressure P on the plane OX. The mathematical expression as determined directly will be given in another paragraph.

It will now be determined as follows: The total pressure on the plane

$$OX = P' = \int_0^x p'dx = \frac{p'}{p}\int_0^x p_x dx = \frac{p'}{p}\int w'x\cos\theta dx$$

$$= \frac{w'x^2}{2}\cos\theta\frac{p'}{p} = \frac{w'x^2}{2}\cos\theta\,\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}. \quad (341)$$

Since the prism of pressure has a triangular base, and the resultant pressure must pass through the centre of gravity of the prism at G, and must be parallel to the base XD, the point of application

of the resultant H will be found at a distance $XH = \frac{1}{3}x$ from the point X, or $OH = \frac{2}{3}x$ from the point O.

If the surface of the ground YOY is horizontal, $\theta = 0$, $\cos\theta = 1$, and equation (341) becomes

$$P' = \frac{wx^2(1 - \sin\phi)}{2(1 + \sin\phi)}. \quad . \quad . \quad . \quad . \quad . \quad (342)$$

If the surface of the ground slopes at the angle of repose, $\theta = \phi$, $\cos\theta = \cos\phi$, and there results

$$P' = \frac{wx^2}{2}\cos\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (343)$$

563. A very simple geometrical construction based upon the above principles and relations enables us to find the ratio of the conjugate pressures in terms of θ and ϕ. Let p' be the less and p the greater of two conjugate pressures having a common obliquity, or, what is the same thing, making equal angles with the normal to their planes of action, which is the angle θ, having for its maximum or limiting value $\theta = \phi$. If, then, in Fig. 230 we draw a line ON to represent both normals at once, and also a line OY making an angle of θ with ON, the lengths of lines representing p and p' will be found on this line; and since at the limiting value of $\theta = \phi$ we have $p = p'$, if we draw a line OR to represent $p = p'$, and therefore making an angle of ϕ with the normals ON, it is evident that p and p' will be, respectively, the secant and its external segment of a circle tangent to OR. If, then, we find a point M from which as a centre the circumference ACB is described tangent to OZ and cutting OY in two points P and Q, then will OP be the greater and OQ the less of the conjugate pressures, and OR will be their common value when $\theta = \phi$.

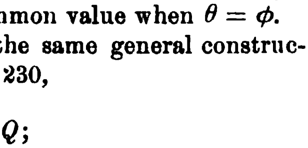


FIG. 230.

This, again, is easily recognized as the same general construction of the ellipse of stress. From Fig. 230,

$$OR^2 = OP \times OQ;$$

$$OQ = \frac{\overline{OR}^2}{OP} = \frac{\overline{OM}^2 - \overline{RM}^2}{OP} = \frac{\overline{OM}^2 - \overline{OM}^2\sin^2\phi}{OP} = \frac{\overline{OM}^2\cos^2\phi}{OP};$$

$$QS = SP = OS - OQ = OM \cos\theta - \frac{OM^2 \cos^2\phi}{OP},$$

since MS is perpendicular to OP. But

$$OP = OS + SP = OM \cos\theta + OM \cos\theta - \frac{OM^2 \cos^2\phi}{OP};$$

$$\therefore\ OP^2 = 2OM \times OP \cos\theta - OM^2 \cos^2\phi.$$

Solving this equation,

$$OP = OM(\cos\theta \pm \sqrt{\cos^2\theta - \cos^2\phi},)$$

the lesser value being OQ;

$$\therefore\ \frac{OQ}{OP} = \frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}},$$

and

$$\frac{OP}{OQ} = \frac{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}; \quad . \ . \ . \quad (344)$$

and $\frac{p'}{p}$ must be greater than $\frac{OQ}{OP}$ and less than $\frac{OP}{OQ}$.

STABILITY OF RETAINING-WALLS.

564. We are now prepared to apply the principles of the foregoing discussion to the stability of retaining-walls, whatever may be the cross-section of the wall, or the inclination of the back of the wall to a vertical plane.

In Fig. 231 the back of the wall slopes away from the mass of earth, as shown by the dotted line AB, or, what is more common and better, has a series of steps, as shown by the full lines. It seems useless to assume the back of the wall as a smooth and uniform surface along which an earth mass would be free to slide, if any sliding could occur, until the wall yields to the pressure—an assumption which is invalid, since the wall is always assumed to be stable under the action of the greatest possible pressure; and, as the back of the wall is or should be very rough, or built in steps, it is perfectly rational to consider that the prism,

regular or irregular, between the back of the wall and a vertical plane intersecting the base of the wall, or the prism ABC, simply rests upon the wall, thereby adding weight to it, and, consequently, increased stability. This weight is not added, the net weight of masonry alone being considered, as it will be on the side

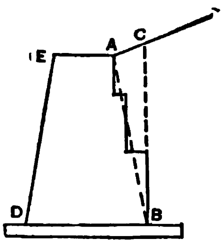


FIG. 231.

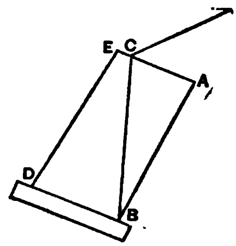


FIG. 232.

of safety. In Fig. 232, the wall leans backward against the pressure, and we can readily conceive that the back of the wall is imbedded in the earth as far as the vertical plane AC. The effect of this can be taken as simply relieving the wall of a part of its weight equal to the weight of a volume of earth represented by the triangle ABC per unit of length of the wall. For safety, then, the wall can be considered as composed of two portions: $BDEC$, having a specific gravity of masonry; and the portion ACB, having the specific gravity of the masonry less that of the earth behind it. This should be considered in determining the weight of the wall.

In either case, therefore, the pressure of the earth may be taken as if acting on a vertical plane, the form and slope of the back of the wall only affecting the actual weight and moment of stability. As will be seen hereafter, the sloping of the courses of masonry arising from the inclinations of the face and back of the wall has the effect of increasing the stability of the wall against both sliding and overturning.

565. The effects of the pressure on a wall is (1) to increase the crushing force on the front portion—this ordinarily is a matter not worth considering; (2) to cause one portion to slide on another; (3) to cause a bodily overturning of the wall on its base—that is, along the front or outer edge of the base, or that of some plane or section parallel to the base; this overturning tendency is equal to the product of the pressure and the per-

pendicular distance from the line of its action to the axis about which the turning actually takes place or is assumed to take place. This is called the moment of the pressures. These effects or tendencies are resisted, respectively, by (1) the strength of the masonry to resist crushing; (2) by the resistance called frictional resistance or stability, and is caused by the resistance to sliding of one portion of the masonry on another portion,—both of these resistances are usually sufficient to insure the stability of the wall against these tendencies; and (3) the resistance of the wall to overturning, which is the product of the weight of the wall acting through its centre of gravity and the perpendicular distance of its line of action from the actual or assumed axis, which is called the moment of the weight or stability.

566. *Stability requires, then, that the resistance to sliding shall be equal to or greater than the tendency to slide. This will be insured when the direction of the resultant of the weight of the wall and the pressure on it does not make with the normal to any plane of action an angle greater than the angle of repose of masonry on masonry. The stability against overturning will be insured when the moment of the weight is greater than the moment of the pressure with respect to the same axis.*

We have already found the equations by which the magnitude, direction, and point of application of the pressure of the earth can be determined (see paragraph 562). It only remains to find the weight of the wall, the centre of gravity through which its line of action passes, and the respective lever-arms of the pressure and the weight.

567. These quantities will first be determined in general expressions, and will subsequently be applied to the more common cases occurring in actual structures.

The general case is represented in Fig. 233, in which the surface YOY' of the ground has an indefinite slope, making an angle θ with the horizon. The wall $ABCD$ inclines backwards towards the earth.

The foundation-bed and the inclination of the masonry courses are inclined against the direction of the pressure and make an angle J with the horizon.

The prism, whose weight is equal to the pressure of the earth, is represented by the triangle ODX, its length in a direction perpendicular to the paper being taken as unity. This triangle is constructed by laying off DX from the base of the wall parallel to the

surface of the ground and equal to x times the ratio of the conjugate pressures, and then joining O and X. The rest of the construction is clearly shown in the drawing. The following notation is used:

$x = OD$ is the vertical plane upon which the pressure is supposed to act.

H, the point of application of the pressure P, at $\frac{1}{3}x$ from D.

HL, the direction of the pressure P parallel to the surface of the ground.

TS, the magnitude of the pressure P passing through the centre of gravity G' of the prism of pressure, which is on the line OE' bisecting DX, and one third of its length from E'.

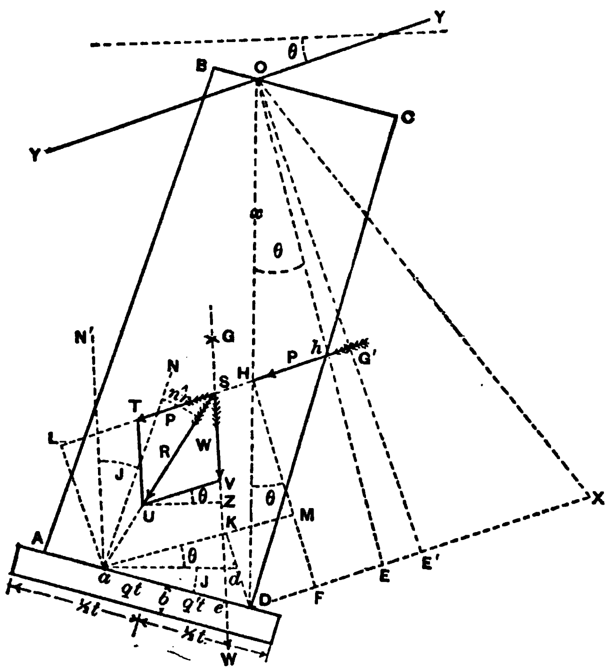


FIG. 283.

SV, the magnitude and direction of the weight of the wall acting vertically downwards through its centre of gravity G.

$SU = R$, the resultant of the weight W of the wall and the pressure P of the earth.

The point a, where the resultant R pierces the base AD, is called the centre of resistance.

The perpendicular distance aL from a to the line of action of of P is its lever-arm; and the perpendicular distance from a to the line of action of $W = ad$ is its lever-arm.

The line HF is parallel to aL, and the line aM is parallel to LH; KD is parallel and equal to MF; HM is parallel and equal to aL.

AD is the thickness of the wall at the base $= t$; e is the point where the line of action of W pierces the base AD; b is the centre of figure of the base; qt is the distance from b to a; $q't$ is the distance from b to e. These are expressed in fractions of the thickness, for convenience. e and a may be on the same side of b, in which case $ea = qt - q't$; or they may be on opposite sides as in the figure, and then $ea = qt + q't$. It will enter the formulæ as $qt \pm q't$.

OE is the altitude of the triangle ODX representing the prism of pressure.

UZ is horizontal, and makes an angle $= \theta$ with $UV = TS = P$. The several angles equal to θ are shown in the drawing.

aN is a perpendicular to the base AD; and the angle SaN must always be less than ϕ', the angle of repose of masonry on masonry, and this must be true for any point or plane parallel to AD. The angle of repose of the earth is ϕ, which must always be greater and than the angle θ.

The total pressure of the earth on the wall P is equal to the volume of the prism ODX, which for a length of unity is equivalent to its area. Area $ODX = \frac{1}{2} OE \times DX$. But from the triangle OED, $OE = (OD)$ or $x \cos\theta$; and $DX = (OD)$ or $x\frac{p'}{p}$, p' and p being the conjugate pressures. Hence area $ODX = \frac{x^2 \cos\theta}{2}\frac{p'}{p}$; and if w' is the weight of unity volume of the earth, then $\frac{w'x^2 \cos\theta}{2}\frac{p'}{p}$ is the weight of the prism and equal to P, or

$$P = \frac{w'x^2 \cos\theta}{2}\frac{p'}{p} = \frac{w'x^2}{2}\cos\theta\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}, \quad (345)$$

the same expression as already found in eq. (341). $w = w'$.

The lever-arm of P with respect to an axis perpendicular to the plane of P and W at the point a is aL.

$$aL = HF - MF = HF - KD = HD \cos\theta - aD \sin(\theta + J)$$
$$= \tfrac{1}{3}x \cos\theta - (\tfrac{1}{2}t + qt)\sin(\theta + J). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (346)$$

The moment of this pressure

$$= M = P[\tfrac{1}{3}x \cos\theta - (\tfrac{1}{2} + q)t \sin(\theta + J)]. \quad . \quad . \quad (347)$$

To Determine the Moment of the Weight of the Wall.—The weight of the wall will always be some constant n multiplied by its length, height, and thickness, and this by the weight of unity volume w. Hence $W = nhbtw \cos J = nhtw \cos J$, its length b being unity. The lever-arm of this weight with respect to an axis at a is $ad = ae \cos J = (qt + q't) \cos J$, and its moment is

$$nht^2w(q + q') \cos^2 J = Wt(q + q') \cos J; \quad . \quad . \quad (348)$$

and in order that the wall may be stable, it is necessary that

$$Wt(q + q') \cos J \geq P[\tfrac{1}{3}x \cos\theta - (\tfrac{1}{2} + q)t \sin(\theta + J)]. \quad (349)$$

Then to find the angle of inclination of the resultant R to the normal aN, the angle $SaN = SaN' - NaN' = eSa - NaN'$, and tan $eSa = \frac{Uz}{Sz} = \frac{P \cos\theta}{W + P \sin\theta}$ or angle $eSa = \text{arc tan } \frac{P \cos\theta}{W + P \sin\theta}$. Hence

$$SaN = \text{arc tan } \frac{P \cos\theta}{W + P \sin\theta} - J < \phi'. \quad . \quad . \quad (350)$$

Eq. (349) must be fulfilled to insure stability against overturning and eq. (350) stability against sliding. As the angle SaN has its greatest value at a plane or joint through S or rather at h, and the weight of the wall and consequently the frictional resistance at that joint is less, it is never necessary to examine into this condition lower down than the joint next below the point of application of P. But as it is clear that this condition can always be fulfilled by inclining sufficiently the bed-joints, in other words, increasing the angle J, eq. (350) can always be satisfied, and it is only necessary to see that eq. (349) is fulfilled. This is done simply by finding the value of t, the thickness of the wall, when all of the other quantities in the equations are given. It is to be noted that the axis of moments is taken at a instead of at A, where the turning would naturally occur. This is done as it evidently reduces the moment of the weight, since da is less than dA, giving a shorter

lever-arm, and at the same time it increases the moment of the pressure P, as La is greater than LA. Taking, then, the axis at a has the same effect as a factor of safety. The best practice takes the point a not farther from the centre b than three tenths to three eighths of the thickness AD or t. In other words, q is arbitrarily taken at three tenths to three eighths. The value of q' depends upon the form and inclination of the wall. The application of eq. (349) is simple, but it is useless to apply it as an illustration, since the conditions taken in the general case rarely occur in practice. The usual and more common case is the following, which materially simplifies the equations:

CONSTRUCTION AND STABILITY OF RETAINING-WALLS.

568. As seen in Fig. 233, the resultant pressure R acts towards the front of the wall. For this reason it is usual to build the front portion of ashlar or range-work either regular or broken, or rather of what is called good second-class masonry, and in some cases of brickwork. The rear portion or backing is almost universally built of rubble composed of large and small stones roughly laid in courses corresponding in thickness or depth with the face-work, and well bonded into it. For important works cement mortar is always used. Sometimes lime-paste and cement-paste are mixed in equal portions, or with an excess of the one or the other—according to the magnitude and importance of the work and the notion of the chief engineer. The face of the wall is usually built on a batter of $\frac{1}{2}$ inch to each vertical foot—sometimes as much as 1 inch. The back of the wall is always built rough, and commonly in the form of a series of steps.

The top of the wall, though theoretically it might have no thickness, is rarely made less than from 2 to 3 feet, and the requisite thickness at base and at intermediate points is secured by the steps, the number and widths of which are regulated accordingly. In abutments for bridges, in addition to the 2 feet of thickness at the top, it is necessary to provide at a depth of a few feet from the top a rest or bridge seat for one end of the end span of the bridge. This varies in width from 3 to 5 feet.

Abutments, being only short walls, are usually provided with stems or wings, which not only increase the stability of the face-wall, but also prevent the flow of the earth around the front, and in arch or short-span bridges favor the more easy flow of the

water through the opening, as well as preventing the water from getting behind the wall and destroying the earthwork. Retaining-walls proper are usually long walls which may or may not have wings at their ends. The principles governing the stability of abutments and retaining-walls are the same, however, and will be considered together.

THE "U" ABUTMENT

ELEVATION
FIG. 235. FIG. (a)

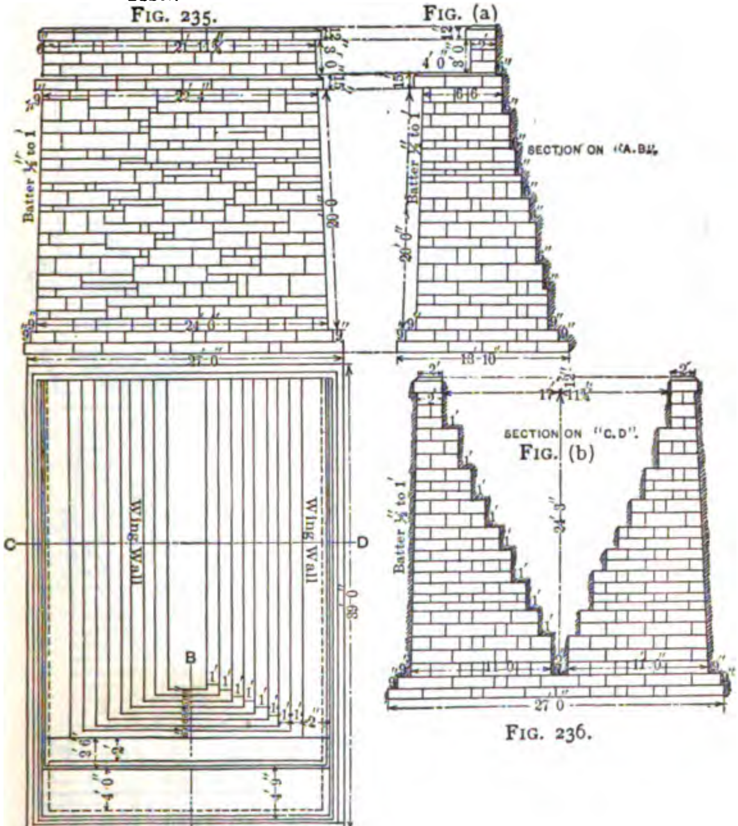


FIG. 236.

PLAN A FIG. 234.

Figs. 234, 235, and 236 show the plan, elevation, and cross-sections through the face-wall and wings, respectively, of the U abutment, which is shown as constructed of broken-range or random-course masonry.

Figs. 237, 238, and 239 are the plan, elevation, and sections of

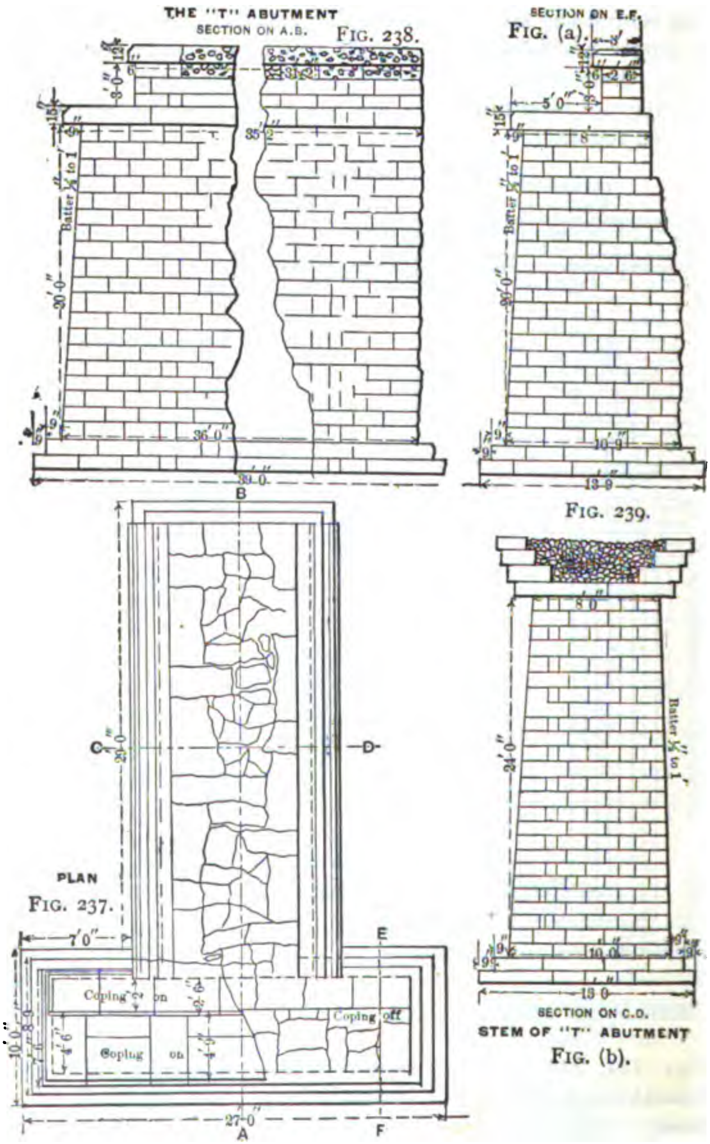
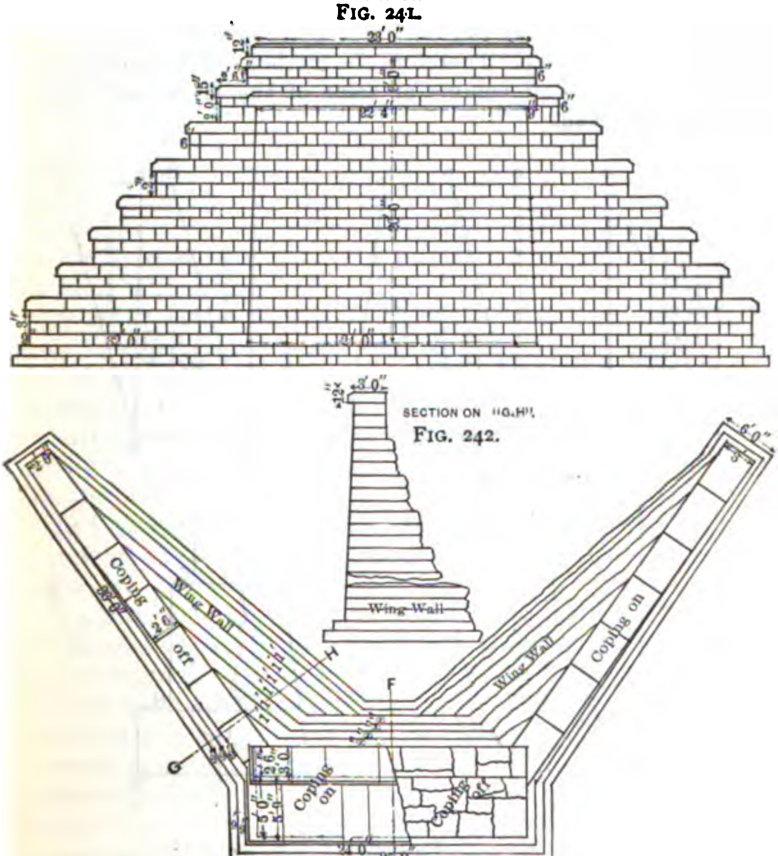


FIG. 237. FIG. 238. FIG. (a). FIG. 239. FIG. (b).

the T abutment, built of good range or second-class masonry, in the case shown.

Figs. 240, 241, and 242 are plan, elevation, and sections of the wing-abutment built of first-class ashlar.

WING ABUTMENT
ELEVATION
FIG. 241.



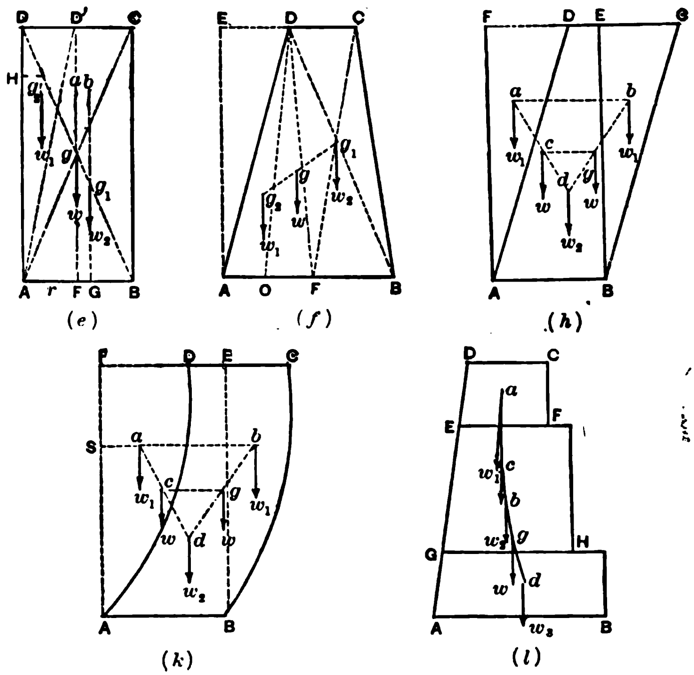
SECTION ON "G.H".
FIG. 242.

FIG. 240 PLAN

In all cases the backing is intended to be of rubble. Except for securing good bond and avoiding the use of an unnecessary amount of small stone,—which is really a matter of no great importance with good mortar, as the only effect is to reduce somewhat the weight of the wall,—the backing can be left pretty much to the

builder, as in order to avoid unnecessary care in the construction and in the selection of the material he nearly always builds the walls of a greater thickness than called for on the plans. The front of the walls are usually rock-face.

569. In determining the moment of stability of the wall it is necessary to know both the form of the cross-section and to be able to locate the position of the centre of gravity, as well as the unit weight of the masonry. Therefore the methods of determining the centre of gravity and volume of some of the usual and simpler forms will be given.



FIGS. 243.

In Figs. 243 (*e*), (*f*), (*h*), (*k*), (*l*) are represented the more common types. (*e*) is a vertical-faced rectangular wall; (*f*), a wall of a trapezoidal cross-section; (*h*), a wall with face and back inclined, with plane surfaces, against the pressure; (*k*), a wall with curved faces inclined against the pressure; and (*l*), the usual form of cross-section, with a batter-face and steps behind forming a

series of trapezoids the one over the other—practically they are rectangles, and will be so considered. The walls proper are drawn in full lines, and all lettered $ABCD$. The dotted lines are simply auxiliary lines used in determining the positions of the centres of gravity of the walls proper. The positions of the centres of gravity of the walls are indicated in all cases by the letter g. a, b, c, and d are the centres of gravity of the auxiliary triangles, curved segments, rectangles, etc., formed.

570. Any geometrical figure having a centre of figure,—that is, an interior point so situated that all straight lines drawn through it and terminating in the boundary-lines are bisected,—the centre of gravity will be found at that point. For instance, the centre of gravity of a circle is at its centre; of a rectangle or parallelogram, at the intersection of its two diagonals; of a regular polygon, at the centre of the inscribed or circumscribed circles; and similarly for many other forms.

In all of these cases it is seen that the figure of the wall can be made by simply transposing a part of a regular figure from one position to another, and the centre of gravity of the proper figure is determined, as Mr. Rankine calls it, by "transposition." The sole underlying principle is that explained for finding the centre of gravity or centre of parallel forces, or centre of gravity of loads, in Art. 22, paragraph 204.

In Figs. 243 (*e*), which is a rectangle or parallelogram, the centre of gravity g is at its centre of figure, i.e., at the intersection of its diagonals.

To determine the centre of gravity of a trapezoid (f): Draw DF parallel to CB, dividing the figure $ABCD$ into a triangle and a parallelogram. The centre of gravity of the triangle is on the line DO bisecting AF, and at one third of the length of this line from O at g_2; the centre of gravity of the parallelogram is at its centre of figure, g_1. By the principle of the lever the line of action of the weight of the trapezoid must intersect the line g_1g_2 at a point such that $w : w' :: g_1g_2 : gg_1$. The point g can then be found either by calculation or graphically; the latter is sufficiently accurate and simple, the former is long and tedious. In this proportion w' is the weight of the triangular portion and w that of the entire trapezoid; w, w', and g_1g_2 being known, gg_1 and gg_2 can be found.

In Fig. 243(k), with curved front and back, it would be necessary to know the character of this curve in order to find the centre of gravity of the segment DAF. If it is a parabolic segment, the

centre of gravity is found at a point a two fifths of FA from FD, and at a distance of $Sa = \frac{3}{8} \times FD$ from the line AF. If this segment is taken away from the rectangle $ABEF$, whose centre of gravity is C, then the centre of gravity of the remaining portion will be found on the prolongation of the line ac at a point d so situated that $w_2 : w :: ac : ad$, w_2 being the weight of the segment $ABED$, and w the weight of the rectangle $ABFE$. Then, if the segment FAD is transposed to the position EBC, its centre of gravity b is known, and the centre of gravity d is also known; then the centre of gravity g of the curved wall is found on the line joining d and b, and at point g, so situated that $w : w_1 :: db : dg$, from which g can be located, the centre of gravity of $ADCB$.

In Fig. 243(l), a and b being the respective centres of gravity of the rectangles $DEFC$ and $EFHG$, the common centre of gravity of their sum is on the line ab; then, joining this point C and the centre of gravity of the rectangle $AGHB$ at d, the centre of gravity of the entire wall is found on the line cd at a point g, found by applying the principle of the lever as above.

571. It is not a matter of so much importance in retaining-walls to find the exact position of the centre of gravity itself as it is to determine the effect of changing the form of the cross-section or the inclination of its faces upon the position of the line of action of its weight, as this increases or decreases its lever-arm and its moment of stability. If, in Fig. 243(e), which is a vertical-faced rectangular wall whose centre of gravity is at g, we should batter the face AD, thereby removing the triangular portion DAD', we would reduce the weight of the wall by the weight of the triangular portion DAD'; but as its centre of gravity of the wall would be removed from g to g_1, thereby increasing its lever-arm with respect to an axis at A or r, by a distance $= FG$, there would be a certain value for the base of the triangle DD', for which the moment of the weight of the trapezoidal wall $AD'CB$ would be the same as the moment of the weight of the heavier rectangular wall $ABCD$. The centre of gravity of the trapezoidal wall will be found on the prolongation of the line g_2g, and at distance from g_2 found by the principle of the lever, as in the preceding cases; but as we only desire to know the horizontal distance from g to $g_1 = FG$, we can assume that the weights w_2, w, and w_1 act at points g_2, a, and b on the same horizontal line. Then it is required to find ab.

$w : w_2 : w_1 :: g_2b : g_2a : ab$. g_2a is known, since we know the positions of the centres of gravity g_2 and g of the triangle and rec-

tangle, respectively; w and w_2 are known, as they are the weights of the rectangle and trapezoid, respectively (all walls are taken as unity in length, and the weights are simply proportional to the areas); w_1, the weight of the triangle, is the difference between w and w_2; and $w_2 : w_1 :: g_2a : ab$.

$$\therefore\ ab = \frac{w_1}{w_2} \cdot g_2a.$$

If moments be taken about r, distant from A equal to $Ar = (\frac{1}{2} - q)t$, then the moment of the rectangle about an axis at r is equal to wqt, and the moment of the trapezoid is

$$w_2(qt + ab) = w_2(qt + \frac{w_1}{w_2} \cdot g_2a).$$

Since $w = w_1 + w_2$, $w(qt) = (w_1 + w_2)qt$; and, in order that the moments of stability shall be the same,

$$(w_1 + w_2)qt = w_2\left(qt + \frac{w_1}{w_2}g_2a\right);$$

$$\therefore\ w_1qt = w_1g_2a; \quad \therefore\ g_2a = qt = Fr.$$

This simply means that if we so determine the base DD' of the triangle DAD' that its centre of gravity is vertically above the centre of resistance or axis of moments r, the trapezoidal wall will have as much stability against overturning as the heavier rectangular wall, whereas an amount of masonry represented by the triangle will be saved. In order that g_2 may be vertically over r, DD' must be equal to $3Ar = 3Hg_2 = 3(\frac{1}{2} - q)t$.

This is self-evident from the figure, since if g_2 is vertically over the axis of moments, the lever-arm of the weight of the triangle DAD' is zero, its moment is zero, and it could not add anything to the moment of the weight of the trapezoid. It is also evident that as the weight has been reduced the obliquity of the resultant pressure is increased, and consequently the frictional stability of the wall is diminished; and unless the bed and courses of the masonry are inclined against the pressure, the wall may give way by sliding.

In Figs. 243 (h) and (k) increased stability is secured by increasing the lever-arm of the weight by the distances cg, the weight remaining the same.

In Figs. 243 (h) and (k) we have seen the method of determining the position of the centres of gravity of the inclined walls $ABCD$.

We desire now to find the distances cg through which the centres of gravity have moved in passing from a rectangular wall $ABEF$ to the inclined walls. We have

$$w : w_1 :: ad : cd :: ab : cg;$$

$$\therefore cg = \frac{w_1 ab}{w};$$

$$w_1 : \tfrac{1}{2}AF \times FD \text{ (in } (h)\text{)} = \tfrac{1}{2}x \,.\, FD;$$

$$w : AB \times AF = tx;$$

and

$$ab : t :: cg = \frac{1}{2}\frac{x \,.\, FD \,.\, t}{tx} = \frac{FD}{2} : q't.$$

In words, the lever-arm is increased by one half the base of the triangle formed by the inclination of the face of the wall with the vertical, or the height of the wall $AF (= x)$ multiplied by the tangent of the angle of inclination.

In figure (k), when the curve AD is a portion of a parabola,

$$w_1 : \tfrac{2}{3}AF \times FD = \tfrac{2}{3}x \,.\, FD; \quad w : tx; \quad ab = t.$$

Hence

$$cg = \tfrac{2}{3}FD = q't,$$

or equal to two thirds of the ordinate of the parabola at the top of the wall.

While these forms of walls materially increase the stability with a given quantity of masonry, or give the same stability with less masonry, they are rarely used, unless it is desired to give clearance in excavations, or when building walls along water-fronts, as their slopes or curves conform more nearly to the form of vessels and boats, which enables these to rest a little more easily alongside the wharves.

572. As the almost universal rule is to build retaining-walls with a slight batter on the face and with a rough sloping surface or steps on the back, which, so far as stability against overturning is concerned, is practically the same as a vertical-faced rectangular wall, the specific gravity being taken as the average of the first-class masonry face, the rubble backing and the earth resting on the steps, that is about 140 pounds per cubic foot, which would be a reasonably light wall, the formulæ will only be applied to determining the thickness of a vertical-faced rectangular wall, with horizontal bed joints. We will assume that the earth is of fair average,

moist, and moderately compacted, weighing 100 pounds per cubic foot. Then, in formula from equation (349) and Fig. 244,

$$Wt(q+q')\cos J = P[\tfrac{1}{3}\times\cos\theta - (\tfrac{1}{2}+q)t\sin(\theta+J)];$$

$$q=\tfrac{3}{8};\quad q'=0;\quad J=0;\quad x=20;\quad \theta=30^\circ;\quad \phi=35^\circ.$$

Substituting,

$$Wqt = P[\tfrac{1}{3}\times 20\cos 30^\circ - (\tfrac{1}{2}+\tfrac{3}{8})t\sin 30]. \quad (351)$$

$$P=\frac{w'x^2\cos\theta}{2}\times\frac{\cos\theta-\sqrt{\cos^2\theta-\cos^2\phi}}{\cos\theta+\sqrt{\cos^2\theta-\cos^2\phi}}$$

$$=\frac{100\times(20)^2\times\cos 30^\circ}{2}\cdot\frac{\cos 30^\circ-\sqrt{\cos^2 30^\circ-\cos^2 35^\circ}}{\cos 30^\circ+\sqrt{\cos^2 30^\circ-\cos^2 35^\circ}};$$

$$\cos 30^\circ \doteq 0.866;\qquad \cos 35^\circ = 0.819;$$

$$\cos^2 30^\circ = 0.749956;\quad \cos^2 35^\circ = 0.670761.$$

Substituting,

$$P = 8833 \text{ pounds},\quad W = 140\times 20\times t,$$

$$\tfrac{1}{3}\times 20\times 0.866 - (\tfrac{1}{2}+\tfrac{3}{8})t\sin 30^\circ = 5.773 - 0.438t.$$

Substituting in equation (351),

$$140\times 20\times\tfrac{3}{8}t^2 = 8833\,(5.773-0.438)t;\quad 1050t^2+3868.85t=50992.91.$$

$$\therefore\ t = 11.68 \text{ feet}$$

as the thickness of the wall to be safe against overturning.

In this case the earth rises from the top of the wall at an angle of 30°; and if the earth is well drained and remains in the condition supposed, the thickness thus determined would be ample. It would hardly be good practice, however, to have a thickness less than $1\tfrac{1}{2}\times 11.68 = 17.5$ feet, or even equal to the height of the wall, say 20 feet, in order to provide for the earth becoming saturated with water.

Using equation (350) to determine whether a wall is safe against sliding,

$$SAN = \text{arc tan}\,\frac{P\cos\theta}{W+P\sin\theta} - J < \phi';\quad \cos\theta = 0.866;$$

$\sin \theta = 0.5$; $J = 0$; $\phi' = 36.5°$, the angle of repose of masonry, or $18\frac{1}{4}°$, the angle of repose of masonry on moist clay; and $P = 8833$ pounds. In the first case

$$W = \frac{2}{3} \times 20 \times 140 \times 11.68 = 21{,}803,$$

and in the second

$$W = 20 \times 140 \times 11.68 = 32{,}704;$$

$$SaN = \text{arc tan } 0.292 < \phi'; \quad \text{arc tan } 0.292 = 16° \ 17' < 36.5°.$$

In the second,

$$SaN = \text{arc tan } 0.206 < \phi'; \quad \text{arc tan } 0.206 = 11° \ 39' < 18\tfrac{1}{4}°.$$

There is therefore no danger from sliding either of masonry on masonry or of the entire wall on its foundation-bed. In the first case only two thirds of the weight of the wall is taken, as the obliquity of the resultant pressure would be greatest at the joint next below its point of application, which, being at one third of the height of the wall from the bottom, only the weight of two thirds of the wall is above that point. (See Fig. 244.)

573. In the more usual case the surface of the ground is horizontal and level with the top of the wall. (See Fig. 244.)

In this case $\theta = 0$; $\cos \theta = 1$; $\sin \theta = 0$. Substituting,

$$P' = \frac{100 \times (20)^2}{2} \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{100 \times 400}{2} \frac{1 - 0.5736}{1 + 0.5736} = 5420 \text{ lbs.};$$

$\frac{1}{3} \times \cos \theta - (\frac{1}{2} + q)t \sin (\theta + J) = \frac{1}{3} \times 20$ is the length of the lever-arm, and the moment is $5420 \times \frac{20}{3}$. The moment of the weight of the wall remains the same. Hence

$$140 \times 20 \times \tfrac{5}{8}t^2 = 5420 \times \tfrac{20}{3}. \quad \therefore t = 5.85 \text{ feet.}$$

Referring to Fig. 244, t is the thickness of a vertical face-wall $AFGD$ of uniform thickness from bottom to top, having a uniform specific gravity.

574. If we assume a wall supporting the pressure of water, we must change the value of w' from 100 to 62.5 pounds. Then $P_2 = \frac{62.5 \times (20)^2}{2} \cdot \frac{1}{1} = 12{,}500$ lbs., since $\phi = 0$. Sin $\phi = 0$ also.

$$140 \times 20 \times \tfrac{5}{8}t = 12500 \times \frac{20}{3}; \quad \therefore t = 8.9 \text{ feet.}$$

If, instead of water, the material is quicksand or flowing mud, make $w' = 125$ lbs.; then $t = 17.8$ feet.

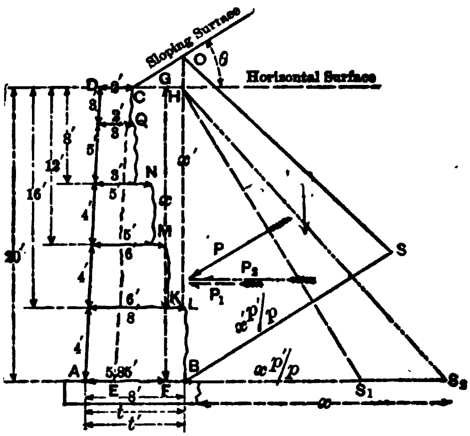


FIG. 244.

NOTATION.—$BS = x'\dfrac{p'}{p}$; $BS_1 = x\dfrac{p_1}{p_2}$;

$BS_2 = x$, fluid pressure;
$P = OBS$, prism of pressure for sloping surface;
$P_1 = OBS_1$, same for horizontal surface;
$P_2 = HBS_2$, same for pressure of water;
$ABLKMNQCDA$, actual masonry wall.
$AECD$, ashlar masonry;
$ECQNMKLB$, rubble;
$CQNMKLH$, earth resting on steps of back of wall.

575. The thickness of the wall at base AF for the different materials are by the formulæ as follows:

(1) For a surface slope, with ordinary earth, $AF = t = 11.68$ ft.
(2) For a surface horizontal, with ordinary earth, $AF = t = 5.85$ "
(3) For fluid pressure, water, $AF = t = 8.9$ "
(4) For fluid pressure, quicksand or mud, $AF = t = 17.8$ "

Doubtless many walls of the above thicknesses have been built and have served their purposes, and may now be standing. But it would hardly be considered safe practice to use less thicknesses than the following: For case (1) from 15 to 16 feet; (2) from 7 to 8 feet; (3) from 10 to 12 feet; and (4) from 18 to 20 feet.

If in Fig. 244 we had determined the exact weights of the ashlar, rubble, and earth or shingle resting on the steps of the wall,

and the common centre of gravity of these portions, the unit weight might differ somewhat from 140 pounds; and the lever-arm of the weight and moment of stability would be also different from those used. The pressure P would likewise vary with the unit weight of the earth and its assumed angle of repose. If the wall is built of uniform thickness, the lower limits given in cases (1), (2), (3), and (4), and if built in steps, as shown in Fig. 244, the higher limits of thicknesses at the bases AF and AB, respectively, will give safe values. In all ordinary cases, if the thicknesses as determined by the formulæ, with fair average values for θ, ϕ, w, and w', be multiplied by $1\frac{1}{4}$ to $1\frac{1}{2}$, they will be found ample to secure stability.

576. We will now consider, in a somewhat general manner, since the conditions and data for any exact discussion are entirely wanting, the stability of what are called *surcharged retaining-walls.* A wall is surcharged when the surface rises from the top of the wall at the natural slope for a certain height, and then his horizontally. It is but natural to assume that the pressure of such a bank would be intermediate in magnitude and direction between that of a bank having a horizontal surface level with the top of the wall and one rising at the angle of repose and extending indefinitely at that slope. Mr. Rankine gives an approximate formula for determining the thickness of a surcharged wall. It would at least be unwise, in the writer's opinion, to reduce the thickness from that determined by eq. (351) after making $\theta = \phi = 35°$, and multiplying the result by $1\frac{1}{4}$ to $1\frac{1}{2}$. For ordinary earth the usual practice is to maintain a slope of an excavation, which is supposed to rise at the angle of repose to the top of the excavation, and then passing to the natural slope of the ground, by a simple masonry wall of only a few feet in height constructed at the foot of the slope; or, it may be, to build an earthen terrace rising at a slope equal to the angle of repose for a certain height and then extending horizontally; and in order not to occupy too much area with the base of the slope, or in case it is near a running stream which would wash the slope away, a foot-wall is constructed. Whatever may be the purposes of the masonry foot-wall, the above two cases are typical of the so-called surcharged walls.

577. Wherever nature provides such slopes the requisite conditions for stability are found to exist, as a rule, and such a thing as a landslide is rare. It is only when man attempts to imitate nature by constructing embankments, or by disturbing the natural state of

equilibrium by making excavations through masses of earth, that settling or slides occur, which continue until natural conditions are again restored.

So far as terraces or embankments are concerned, good drainage is or can be provided; only a limited amount of water can fall on their tops or slopes, and this can be readily drained off. Surface water, to a large extent, can be kept away from it by proper side ditches, culverts, drains, etc. It is not very difficult, therefore, to maintain embankments of limited extent in a reasonable condition of dryness. The slopes of embankments are usually flat—seldom less than $1\frac{1}{2}$ to 1; and, if not flat enough in the beginning, will soon find their proper slope without any special damage to either slopes or walls supporting them. If proper openings are left in the walls, and shivers of rock or gravel are provided near and against the back of the wall for either a part or the whole of its height, it would seem that walls whose thicknesses have been determined by the foregoing methods would have both permanence and stability.

578. The more difficult cases of surcharge occur in the sides of excavations. Simply cutting off a few feet from the natural slope, or a small triangular prism having only a few feet in base and height, will often initiate a slide that will continue for years, and no artificial means seem adequate to prevent or stop it. Again, excavations of great depths may be made at the foot of natural slopes, or through them, without causing any serious or noticeable effects of any kind, as seen in thousands of excavations through all kinds of material. These things are mentioned as showing that in certain conditions the balance of natural forces is often of the most delicate character, and also, that nature often leaves large masses in conditions having great surplus of stability. In the one case a very small cause produces great and far-reaching effects; in the other, great causes seem to produce little or no disturbance. These conditions are unknown and indeterminable.

It is probable that an immediate substitution of even a very slight artificial support might obviate much difficulty caused by the removal of a small prism of earth at the foot of a slope, as evidently such removal can only cause a disintegration of a small and adjacent mass, which in turn admits of an additional loosening, this extending ultimately to great distances and over large areas. When under the influence of vibrations, shocks, or an unusual seepage or flow of water, the movement of even a small section may

result in an almost irresistible sliding or flowing of large masses. Too long delay in providing artificial supports or resistances is therefore, doubtless, a fruitful source of much trouble and expense.

579. In deep excavations particularly, seepage-water, which originally either sank to greater depths or, finding easier channels of escape in other directions, changes its course when an excavation is made near it, as the line of least resistance may be changed, and in addition from the same cause an increased velocity of flow takes place in towards the excavation. This is manifested by the escape of the water along any portion of the slope, or if one stratum is more porous than another, the entire flow passing through it washes or scours the material out, causing an undermining of the slope. It is then that it is determined to build a masonry wall over the face of the porous strata to hold it in place. This wall is calculated on the basis of a fluid pressure having the specific gravity of the material itself. It may be from 100 to 150 pounds per cubic foot, thereby providing a thickness equal to or even greater than the height. The construction of such a wall may be effective, and often is.

But probably it is as often a failure as it is a success. The reason in many cases is not difficult to discover. The very construction of a solid masonry wall laid in cement prevents the escape of the water; this being confined in the material, converting it into flowing mud or quicksand, tends to change the direction of the pressure to a more or less horizontal one, the intensity of which is not due to a vertical pressure of an earth mass with a horizontal surface level with the top of the wall, but practically to a vertical height measured from the base of the wall to the top of the surcharge. If a calculation is made upon such a basis, the entire inadequacy of such a wall to resist the pressure will be readily seen. In fact the wall will result in vastly more harm than good.

580. That a dry wall, or at least one built very open, would be vastly more effective, is self-evident. It would serve to hold the material in place and allow the free passage of the water at the same time, which could be properly conveyed away by suitable drains in front of the wall. Walls for this purpose should be built thoroughly, bonding the work both horizontally and vertically, and not in regular horizontal courses, headers being freely used with their lengths in a vertical direction, which should also be extended in pits sunk below the foundation-bed of the wall.

581. With walls of this kind and for this purpose *counterforts*

are useful in localizing the seepage-water, thereby preventing any large quantities of water with increasing velocity from flowing along and parallel to the wall, and scouring out the material behind it. Counterforts are projections built in rear and perpendicular to the main walls, to which they are well bonded. They are placed at any desired distances apart. The result is a wall in sections alternately thick and thin. Simply as a question of stability, there is but little masonry saved as compared with a wall of uniform thickness and equal stability. They are, however, advantageous when unusual pressures are concentrated only at certain points.

If such projections are built on the front of the wall they are called buttresses, which are used for giving greater stability, as also for architectural effect, relieving the monotonous uniformity of a long wall.

582. While it is not very difficult to sustain a fluid or semi-fluid when its condition of internal equilibrium is established, and not greatly disturbed at any time, as is evidenced by the numerous high dams of earth, masonry, or concrete used for water-storage purposes, it is of the greatest difficulty, and even impracticable in many cases, to hold a fluid or semi-fluid in the act of seeking its condition of equilibrium when this natural condition is once disturbed. This is evidenced by the difficulty of dealing with even thin layers of quicksand underlying other and firmer strata.

When once such a mass moves even for very short distances, the work performed can only be balanced by some resistance or obstruction which must also move, if only for an infinitesimal distance.

If the wall has dimensions and weight sufficient to withstand this pressure, some portion of the earth mass itself must move, which will probably move upwards along the back of the wall, and flow over it. This no doubt often occurs, but usually the wall itself will be overturned or pushed forward. This tendency may often be resisted by connecting the two walls on the opposite side of an excavation by strong inverted arches under the road-bed.

When such conditions as above discussed are found to exist, all attempts at holding the slopes will often fail, and there is but one of two things to do: (1) Allow the material to flow in, and remove it as it accumulates; or (2) abandon that portion of the work altogether, and seek a new route around it. The latter plan will often be found the safest, and ultimately the most economical.

LAND TIES FOR RETAINING-WALLS.

583. Many cases often arise where there is no special difficulty in constructing a wall of reasonable dimensions and of sufficient stability so far as the wall itself is concerned, but the difficulty arises in securing a stable foundation-bed. If constructed on piles, the pressure will cause these to lean forward, carrying the wall with them, or, owing to the small coefficient of friction between the wall and the soft, soapy character of the foundation-bed, the wall will slide bodily forward. These tendencies may be resisted by driving the piles with an inclination against the pressure, so that the resultant shall coincide in direction with the axis of the piles; or the piles may be tied back to the material behind them by bolting long sticks of timber to them, which reaching well to the rear, and connected by crosspieces, will present considerable bearing surface against the earth, thereby tying the piles well back into firm earth. If this cannot be found at a reasonable distance, piles are often driven in front of the crosspieces. These ties and crosspieces may be connected with the main piles direct, or to the crib or wall resting on and bolted to the piles. This is a common construction along many river fronts, where long lines of timber bulkheads or wharves are constructed. These constructions will be further considered under their proper heads.

The case here considered is that of a retaining-wall so situated that there is great danger of sliding from its proper position.

584. In Fig. 245 let $ABCDEFGH$ be a retaining-wall, or a portion of one, supporting a bank of earth, having a horizontal surface level with the top of the wall; andlet $KLMN$ be a thick iron plate, or a wall of solid timbers, imbedded in the earth well in rear of the wall. This plate may either extend to the surface of the ground OR or may be entire within the mass, its upper surface being $OK = RL$ distance below the surface of the ground, as shown in the drawing. The plate is connected by a long iron tie-bar ab to which is made to pass through the wall at a distance from the bottom equal to one third of its height, as this is the point at which the resultant pressure of the earth is supposed to act. This rod is held by nuts and large washers against the wall and plate, as indicated. When the plate extends to the surface, and is simply imbedded in the earth, the pressures on its two faces are equal and directly opposed; therefore they balance each other. These are the least pressures, and can be represented by the weight of a

triangular prism $ONTUMR$, and its equal acting on the other face of the plate, $OT'NMU'R$. This pressure, from equation (342), is

$$P = \frac{wx^2}{2} \cdot \frac{1 - \sin \phi}{1 + \sin \phi}.$$

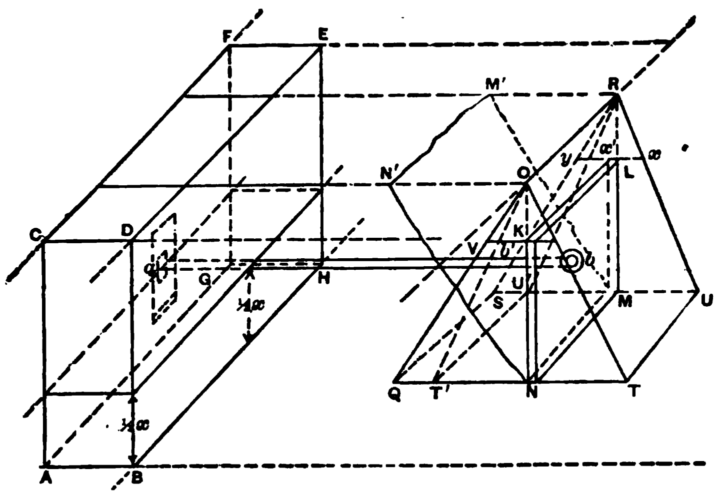


FIG. 245.

If, then, the wall of masonry tends to slide forward, causing a pull on the rod ab, which is communicated to the plate, the pressure of the earth can be increased until

$$P' = \frac{wx^2}{2} \frac{1 + \sin \phi}{1 - \sin \phi},$$

represented by the weight of the triangular prism $OQNMSR$. As the first pressure, P, is in the direction of the pull, and that of the greater, P', is in the opposite direction, the holding-power of the earth is represented by their difference, $P' - P$, equivalent to the weight of the prism $OQT'SU'R$, or

$$P' - P = \frac{wx^2}{2}\left(\frac{1 + \sin \phi}{1 - \sin \phi} - \frac{1 - \sin \phi}{1 + \sin \phi}\right)$$

$$= \frac{wx^2}{2}\left(\frac{(1 + \sin \phi)^2 - (1 - \sin \phi)^2}{1 - \sin^2 \phi}\right). \quad . \quad . \quad . \qquad (352)$$

As it would require a very thick plate or strong frame of timber to distribute the pull over its entire surface, if it extended to the surface of the ground, it is usual to limit the size of the plate, and to entirely imbed it in the earth, its upper edge KL being at a distance of $OK = RL = x_1$ below the surface. The holding-power of the earth on the plate $OKLR$ would be $P_1 - P_2$, or, similarly to equation (352),

$$P_1 - P_2 = \frac{wx_1^2}{2}\left(\frac{(1+\sin\phi)^2 - (1-\sin\phi)^2}{1-\sin^2\phi.}\right);$$

and the holding-power on the plate $KLMN$ would be represented by the prism of trapezoidal section $VQT''b'SU'X'Y$, or, in symbols,

$$(P' - P) - (P_1 - P_2) = \left(\frac{wx^2}{2} - \frac{wx_1^2}{2}\right)\left(\frac{(1+\sin\phi)^2 - (1-\sin\phi)^2}{1-\sin^2\phi}\right)$$

$$= \frac{w(x^2 - x_1^2)}{2}\frac{4\sin\phi}{\cos^2\phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (353)$$

It must be borne in mind that these pressures are only exerted over a unit of length LK of the plate, and to obtain the total pressure the above values must be multiplied by the number of units in LK.

To find the depth of the centre of pressure of the plate below ground, proceed as follows, taking an axis of moments at O or OR:

The pressure on the plate, or rather the holding-power, is

$$H = \frac{w(x^2 - x_1^2)}{2}\frac{4\sin\phi}{\cos^2\phi};$$

its unknown lever-arm or distance from the centre of pressure b to the surface of the ground is x_0. Hence its moment is $\frac{w(x^2 - x_1^2)}{2}\frac{4\sin\phi}{\cos^2\phi}x_0$. The moment of the pressure upon the plate, if it extended to the surface of the ground, would be $\frac{wx^2}{2}\frac{4\sin\phi}{\cos^2\phi}\cdot\frac{2}{3}x$; and of the portion of the whole plate from the face to KL, x_1 distance from the surface, would be $\frac{wx_1^2}{2}\frac{4\sin\phi}{\cos^2\phi}\cdot\frac{2}{3}x_1$. Since, when the plates extend to the surface, the centre of pressure is two thirds of the depth of the plate from the surface, and since

the moment of the whole is equal to the sum of the moments of its parts, we have

$$\tfrac{2}{3}x \cdot \frac{wx^2}{2}\frac{4 \sin \phi}{\cos^2 \phi} = x_0 \cdot \frac{w(x^2 - x_1^2)}{2}\frac{4 \sin \phi}{\cos^2 \phi} + \tfrac{2}{3}x_1 \cdot \frac{wx_1^2}{2}\frac{4 \sin \phi}{\cos^2 \phi};$$

hence

$$x_0 = \frac{x^3 - x_1^3}{3} \div \frac{x^2 - x_1^2}{2} = \frac{2}{3}\frac{x^3 - x_1^3}{x^2 - x_1^2}. \quad . \quad . \quad (354)$$

If the pull is sufficiently great, the effect would be to cause a mass of earth represented by ONN' in section to slide upward along some plane, curved, or irregular surface, as NN'.

The principles of the above discussion are often applied in tying bulkheads on water-fronts; in holding walls or piers built on or near sloping banks; and, in a crude way, in connecting the guys or stiff legs of derricks to what is commonly called "a dead man," which consists of a log of wood or posts imbedded in the ground.

It is also a clear illustration of Mr. Rankine's theory of the stability of a granular mass, whether as applied in retaining-walls, land ties, or in the supporting power of earth or soft soils for houses or other structures, which was fully discussed under the head of Foundations on Soft Materials.

585. As a practical application of the foregoing, assume a plate 6 feet high and 5 feet long, imbedded in earth to support or hold a wall of masonry 20 feet high; required the holding-power of such a plate, the earth weighing 100 pounds per cubic foot, and the depth of the centre of the plate such that the distribution of the pull on the rod over the plate shall be proportional to that of the earth resistance. $\phi = 35°$; $\sin \phi = 0.5736$; $\cos \phi = 0.819$. Substituting in equation,

$$H = \frac{w(x^2 - x_1^2)}{2}\frac{4 \sin \phi}{\cos^2 \phi} = \frac{100(x^2 - x_1^2)}{2} \times 3.43 = 26{,}754 \text{ lbs.}$$

As the pressure on the plate is uniformly varying, its resultant would have to pass through the centre of gravity of the trapezoidal prism in order that the pull on the rod and the resistance may be directly opposed. The centre of gravity of the trapezoid $QVKN$ can be found graphically, as already explained. The exact calculation is tedious, and it will be near enough if we make $x = 16$ feet, or the bottom of the plate 4 feet above the level of the base of the wall; and since the plate is 6 feet high, $x_1 = 10$. Substituting in the above equation, the value H is found as above, 26,754 pounds

per foot of length; or over a plate 5 feet long the total holding-power is $= 26{,}754 \times 5 = 133{,}770$ pounds. The depth of the centre of pressure of the plate will be $x_0 = \frac{2}{3}\frac{x^3 - x_1^3}{x^2 - x_1^2} = 13.2$ feet below the surface of the ground, which is about on a level with the centre of pressure of the wall.

To carry safely this pull would require 1 bar about $4\frac{1}{4}$ inches diameter, or, better, 2 bars of 3 inches diameter each.

The writer believes that the foregoing principles and formulæ, as applicable to determining the stability of retaining-walls, are fully as reliable as any of the other theories, and that an intelligent use of the formulæ will lead to safe and reliable results. All other theories have been discussed and enlarged upon in nearly every book that has been written in the last forty years upon the subject of retaining-walls. Those given above are rational, easily understood, and easily applied. It has not, therefore, been deemed necessary to introduce a discussion of Coulomb's, Moseley's, or Weyrauch's formulæ in this work.

ART. XLIII.

RESERVOIR WALLS, DAMS, AND WEIRS.

586. In the preceding article we have seen that the general formulæ for the stability of retaining-walls will also apply to walls for supporting the pressure of water, or reservoir walls, by making both θ and $\phi =$ zero. We make $\theta = 0$ because the surface of still water is always horizontal, and $\phi = 0$ since it is assumed that there is no friction between the molecules of water, these having perfect freedom of motion amongst themselves; the natural slope of water becomes horizontal, and its angle of repose zero.

In the discussion of retaining-walls the pressure of the earth was assumed to act upon an ideal vertical plane passing through the rear edge of the base; and any material between this plane and the back of the wall, whether earth or water, was assumed to add weight to the wall if the back sloped away from the pressure or was built in steps, but that this prism of earth or water lessened the weight of the wall if the back inclined towards the pressure, that is, leaned to the rear. Under these assumptions the formulæ of the preceding article apply alike to the pressure of earth and water.

It is, however, preferable in many cases to discuss the subject of water-pressure by assuming that the pressure acts directly on the surface of the wall whatever may be its inclination, and not upon an ideal vertical surface, as the rear surface of a reservoir wall is seldom vertical or inclined towards the pressure, but almost invariably inclines away from the pressure, or, in other words, has a batter on both front and rear surfaces.

587. It was shown in paragraph 549, (1) that in a fluid the pressure is normal to its plane of action, whatever may be the inclination of that plane; therefore that every direction was a principal axis of stress; (2) that at a given depth or at a given point the intensity of the stress or pressure was equal in every direction; and (3) that the intensity of the pressure in any direction is equal to the weight of a column of water whose base is unity in area and whose length or height is equal to the depth of the point below the surface, i.e., $p_x = p_y = p = p' = wx$, p_x and p_y being the principal stresses, and p' and p being the conjugate stresses, in any direction, of the preceding principles and formulæ found in Article XLII.

588. Applying these principles to any wall supporting water, as shown in Fig. 246, we can readily find the dimensions of the wall to sustain the pressure.

Let OY be the water surface assumed to be level with the top of the wall. It is usually from two to four feet below the level

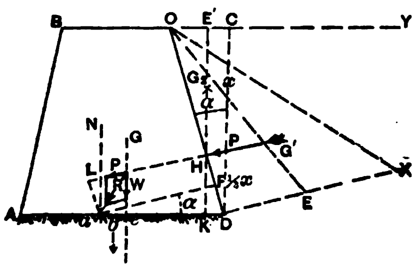


FIG. 246.

of the top of the wall. This assumption, then, is on the side of safety, as it gives a greater pressure than actually exists. $ABOD$, a vertical cross-section of the wall, which has a length of unity; ODX, the prism of unity in length whose weight is equal to the pressure of the water against the back of the wall OD. In this case DX is drawn perpendicular to OD at D to represent the direction of the pressure at that point, and its length $x\frac{p'}{p} = x$, or

the vertical depth of the point D below the top of the wall, since $p' = p$. The line from OX forms the third side of the prism, since the pressure is uniformly varying. The resultant pressure P passes through G', the centre of gravity of the triangle ODX, which is one third of the median line OE from E; and the centre of pressure is consequently at H, or $\frac{1}{3}OD$ from D, the direction of P being also normal to OD and parallel to DX. Let G be the centre of gravity of the wall, Ge the line of action of the weight, b the centre of figure of the base supposed to be horizontal, a the centre of resistance. Forming the parallelogram on P and W, R will be the resultant pressure. The lever-arm of P is $aL = HD - DF = \frac{1}{3}OD - aD \sin \alpha$, $OD = x \sec \alpha$, $CD = x$, and $aD = bD + ab = \frac{1}{2}t + qt = (\frac{1}{2} + q)t$. Substituting,

$$aL = \tfrac{1}{3}x \sec \alpha - (\tfrac{1}{2} + q)t \sin \alpha. \quad . \quad . \quad . \quad . \quad (355)$$

The lever-arm of

$$W = ab + be = qt + q't = (q + q')t. \quad . \quad . \quad . \quad (356)$$

Hence $W(q + q')t = P[\frac{1}{3}x \sec \alpha - (\frac{1}{2} + q)t \sin \alpha]$;

$$P = wOD \times \tfrac{1}{2}Dx = \frac{wx^2}{2} \sec \alpha.$$

Substituting,

$$W(q + q')t = \frac{wx^3}{6} \sec^2 \alpha - \frac{wx^2}{2}(\tfrac{1}{2} + q)t \tan \alpha, \quad . \quad . \quad . \quad (357)$$

W being the entire weight of the wall, w the weight of a unit volume of water, usually one cubic foot, or $w = 62\frac{1}{2}$ pounds. Eq. (357) expresses the condition that the wall shall not give way by overturning.

That the wall shall not give way by sliding, the angle RaN must be less than the angle of repose of masonry on masonry or of masonry on earth. This condition is expressed by eq. (350) by making $\theta = \alpha$ and $J = 0$.

$$RaN = \text{arc tan} \frac{P \cos \alpha}{W + P \sin \alpha} \leq \phi'. \quad . \quad . \quad . \quad (358)$$

When the back of the wall is vertical, $\alpha = 0$, $\sec \alpha = 1$, and $\tan \alpha = 0$, $\cos \alpha = 1$, $\sin \alpha = 0$.

Eq. (357) becomes

$$W(q+q')t = \frac{wx^3}{6} = \frac{wx^2}{2} \times \tfrac{1}{3}x. \quad . \quad . \quad . \quad . \quad (359)$$

Eq. (358) becomes

$$RaN = \text{arc} \tan \frac{P}{W} \leq \phi'. \quad . \quad . \quad . \quad . \quad (360)$$

The factor $\frac{wx^2}{2} = P$, eq. (359), is the same as P in eq. (351) by making $\theta = \phi = 0$. This, taken with the value of $P = \frac{wx^2}{2} \sec \alpha$, leads to the rule for finding directly the pressure of water on any plane surface immersed in it, as follows:

The pressure of a fluid upon any plane surface immersed is equal to the area of the surface multiplied by the depth to which its centre of gravity is immersed below the surface, and this product by the weight of a unit of volume of the fluid.

It will not be necessary to apply the above equations, as this has been done in paragraph 574.

Putting equation (357) under the following form:

$$W(q+q')t + \frac{wx^2}{2}(\tfrac{1}{2}+q)t \tan \alpha = \frac{wx^3}{6} \sec^2 \alpha. \quad . \quad (361)$$

The term $\frac{wx^2}{2}(\frac{1}{2}+q)t \tan \alpha$ is unintelligible as it stands. By referring to Fig. 246 we find $OC = CD \tan \alpha = x \tan \alpha$. Hence area $OCD = \frac{x^2}{2} \tan \alpha$, the weight of this prism of water $= \frac{wx^2}{2} \tan \alpha$, and then the factor $(\frac{1}{2}+q)t$, which is equal to aD, is not the lever-arm of this weight. The product, therefore, means nothing. But since the centre of gravity of the triangle OCD is found at G_2 on the median line DE', then $KD = \frac{1}{3}OC = \frac{1}{3}x \tan \alpha$. If, then, we subtract from the above equation

$$\frac{wx^2}{2} \cdot \tfrac{1}{3}x \tan^2 \alpha = \frac{wx^3}{6} \tan^2 \alpha,$$

we have

$$W(q+q')t+\frac{wx^2}{2}\tan\alpha[(\tfrac{1}{2}+q)t-\tfrac{1}{3}x\tan\alpha]$$

$$=\frac{wx^3}{6}\sec^2\alpha-\frac{wx^3}{6}\tan^2\alpha=\frac{wx^3}{6}. \quad . \quad . \quad (362)$$

Analyzing this equation, in the term

$$\frac{wx^2}{2}\tan\alpha[(\tfrac{1}{2}+q)t-\tfrac{1}{3}x\tan\alpha],$$

the factor $\frac{wx^2}{2}\tan\alpha$ is the weight of the prism resting on the slope OD, which is the vertical component of the total pressure P, and $(\frac{1}{2}+q)t-\frac{1}{3}x\tan\alpha$ is the distance aK. The product is the moment of the weight of the prism OCD, which adds to the moment of the weight of the masonry; and the second member, $\frac{wx^3}{6}=\frac{wx^2}{2}\times\frac{1}{3}x$, is the product of the horizontal component $\frac{wx^2}{2}$ of the total pressure P, by $\frac{1}{3}x$, its lever-arm. This is as it should be, since evidently the horizontal component of P alone tends to overturn the wall or cause it to slide on any joint, and its vertical component adds stability to the wall. Equations (361) and (362), of course, give the same values for t. Equation (362) is the form in which the relations are more commonly found in books.

589. There is little, if any, danger from sliding under the pressure, except when built on soft, soapy soils. These conditions and the remedy for them were fully discussed in the preceding article on Retaining-walls. It is only necessary, then, to see that the conditions of resistance to overturning are fulfilled as expressed in equations (361) or (362).

Large W in all of the preceding formulæ represents the actual weight of the wall, whatever may be its form of cross-section. Theoretically, the cross-section would be a triangle; but some thickness is always given at the top of the wall, which will vary from two to twenty feet, according to the material of the wall and the purposes for which it is built, or from practical considerations of convenience and use.

590. As has been shown, a wall under pressure from earth or water, or any force whatever, may fail (1) by overturning around the axis at A, or at any point within the wall (if built of compressi-

ble materials), such as a, which is called the centre of resistance; (2) by sliding on any joint AD or any other joint parallel to it; and (3) by crushing the material at the base AD. Conditions (1) and (2) have been discussed and provided for in the preceding paragraph, and it only remains to consider condition (3).

Stability against Crushing.—Under ordinary circumstances, with low walls and relatively small lateral pressures, no danger will arise from the pressure, as the resistance of the material will far exceed the crushing pressure. In very high walls, however, this pressure may be very great, and must be provided for either by the proper selection of the material of which the wall is constructed, or the area of the base must be sufficiently large, that the intensity of the pressure may be kept within limits of the strength of the material used.

As the principle involved is the same for both low and high walls, it will be discussed in this place.

With vertical-faced rectangular walls the pressure on the base arising from the weight of the wall is uniformly distributed over the base. The total pressure is the weight of the wall; the unit of pressure, or the intensity, is equal to the weight divided by the area of the base; and since a wall of unity in length is usually considered, the weight of the wall is $W = wxt$, the area of base is $= t$; and hence the intensity of the pressure is $= \frac{W}{t} = wx$, in which $W =$ total weight, $w =$ weight of a cubic foot, $x =$ height in feet, and $t =$ thickness. This condition is represented in Fig. 247(a). If the pressure is composed of a uniformly varying pressure and a uniformly distributed pressure, it can be represented by Fig. 247(b).

In this case it is evident that the total pressure is $W + W' = wxt + \frac{1}{2}wtx'$, in which x is the least height of the structure and $x + x'$ its greatest height, t being the thickness. The mean intensity of this pressure is $wx + \frac{wx'}{2}$; the least intensity at the point B is wx, and the greatest is $wx + wx'$, at the point A. If the pressure is uniformly varying, it can be represented by a triangle, Fig. 247(c), in which the total pressure $W = \frac{1}{2}wxt$, its mean intensity is $\frac{1}{2}wx$, its least 0 at B, and its greatest wx at A.

In either case the ordinate at any point represents the intensity at that point.

In Fig. 247(a) the dotted lines show a wall with inclined faces. Where this batter is not very great, it is usual to assume that the

total weight W is uniformly distributed over the base AB. This probably is not true when the slopes are very flat, a greater intensity of pressure being on the central portion. The point of actual greatest intensity, or the intensity itself, cannot be determined.

591. In all of these cases the varying intensity of the pressure on the bases AB arises from the form given to the cross-section of the wall. The same conditions may, however, arise, whatever may be the form of the cross-section, if the wall is acted upon by a lateral pressure, as the tendency of this pressure is to overturn the wall, and in so doing is to lift the wall from its base on the side

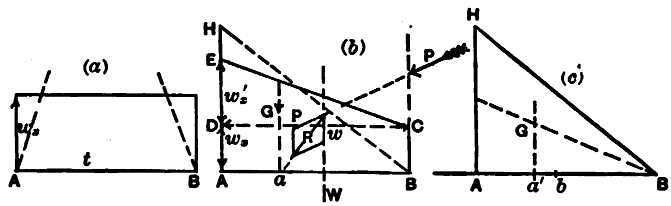


FIG. 247.

adjacent to the pressure, and to transfer the greater portion of the weight to the opposite side of the base. The extent of this transference will depend upon the direction of the lateral pressure and the relative magnitudes of the pressure and the weight of the wall. In determining this effect of the lateral pressure it is only necessary to take the case in which the base of the wall is rectangular in form.

In Fig. 247(b) let $p = wx + wx'$ = greatest and $p' = wx$ the least pressure, however it may be caused: then the area of the trapezoid $ABCE$ will represent the total weight or pressure of the wall AB,

$$= \frac{p + p'}{2} t = W; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (363)$$

and as a is the centre of pressure, and the moment of the pressure to the right of a must be equal to that to the left of a, then the centre of gravity of the trapezoid $ABCE$ must be found on the vertical line Ga. The distance Aa of the centre of gravity of a trapezoid from the larger of the parallel sides is

$$y = \frac{(2p' + p)t}{3(p + p')} = Aa. \quad . \quad . \quad . \quad . \quad . \quad (364)$$

Finding the values of p and p' from equations (363) and (364), we find the least intensity pressure

$$p' = \frac{6Wy}{t^2} - \frac{2W}{t} = BC,$$

and for the greatest intensity

$$p = \frac{4W}{t} - \frac{6Wy}{t^2} = AE. \quad . \quad . \quad . \quad . \quad . \quad (365)$$

So long as the pressure can be represented by a trapezoid extending entirely over the base AB, every portion of this base will be under some pressure.

When the smaller pressure $p' = BC = 0$, the trapezoid becomes a triangle ABH, Fig. 247(c), and the distance Aa from A to the vertical line Ga through the centre of gravity $= y = \frac{1}{3}t$. This value substituted in p' gives for the least intensity

$$p' = \frac{2W}{t} - \frac{2W}{t} = 0,$$

and for the greater intensity pressure

$$p = AH = \frac{2W}{t}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (366)$$

or, since the mean intensity of the pressure is $\frac{W}{t}$, it follows that if the greatest intensity is more than twice the mean, there will be a tension at one edge of the base, namely, B. The weight and the area of the base of the wall must therefore be so proportioned that under the greatest lateral pressure P the intensity of the pressure at the outer toe A of the wall shall not be greater than twice the mean intensity $= \frac{2W}{t}$.

If in equation (365) we make $Aa = y = Ab - ba = \frac{1}{2}t - qt$, the equation (365) becomes

$$p = \frac{W}{t} + \frac{6Wq}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (367)$$

Hence

$$q = \frac{pt - W}{6W} = \frac{1}{6} \quad \text{when} \quad p = \frac{2W}{t},$$

which is the condition that there shall be no tension at any point of the base. The meaning of equation (367) is that the resultant pressure must not intersect the base farther from the centre of figure than one sixth of the thickness t; in other words, the centre of resistance must be found in the middle third of the thickness. When this condition is fulfilled there is little danger of the material crushing.

Equations (357), (362), (358) and (367) express the conditions for permanence and stability which should be fulfilled in the design and construction of all important walls or dams. The usual practice in retaining-walls is to allow a greater value for q than one sixth, as high as three tenths to three eighths being sometimes used.

MATERIALS AND CONSTRUCTION OF RESERVOIR WALLS AND DAMS FOR STORAGE OF WATER FOR DOMESTIC USES, IRRIGATION, OR POWER.

592. Reservoir Walls or Dams are constructed of earth, timber, masonry, or concrete, or of a combination of these materials.

593. *Earthen Dams.*—For the ordinary reservoirs which are intended to store water for use in cities, the enclosing walls have been commonly built of earth, either alone or in combination with masonry or concrete. As there is much difference of opinion and practice in regard to the best materials or combination of materials that should be used, a few illustrations and examples will give a better idea of the practice, and more useful information, than any attempt at a theoretica' or scientific discussion of the subject.

It may, however, be stated that any discussion of the stability of earthen dams, or any application of the preceding formulæ for stability, is of little value: (1) for earthen dams rarely, if ever, give way by overturning bodily, or by sliding as a whole; (2) long, relatively flat slopes are of necessity given to the embankments; (3) the dimensions and weights are ample always to fulfil the preceding conditions of stability. The essential considerations are: (1) that as the storage of water is the object sought, the dams should be made of water-tight materials and construction; (2) the greatest danger to earthen dams arises either from the water flowing over the top of the dam or percolating through the banks or under them.

1st. The first danger can always be provided against by raising the dam high enough above the surface of the water in the reservoir, if this surface can always be regulated, as is the case where

the water is pumped into the reservoir. The height above the surface is sufficient when waves formed by the force of high winds do not or cannot break over the top of the dam. From 3 to 6 feet will usually be ample for this purpose.

2d. If these dams are built across running streams, proper waste weirs should be constructed on the crest of the dam having sufficient length and depth to carry the greatest discharge of water when the enclosed water has reached the intended height, and proper provisions should be made to carry this flood discharge clear from the outer slope of the dam. It is risky, whatever may be the precautions taken, to carry the overflow water down the slope of an embankment. It is far better and more satisfactory to cut channels or canals entirely clear of the dam, carrying them around the ends.

3d. The greatest source of danger to earthen embankments has its origin in the percolation of water through the material or following along the pipes and conduits through or under the embankment. Even a very small and insignificant seepage of water may ultimately result in the total destruction of the embankment.

At first this seepage may only extend through a portion of the embankment, and, spreading up and down and sideways, may convert a considerable portion of the material into a soft mud or paste, the pressure of which tends to open crevices, destroys the conditions of equilibrium by necessitating flatter slopes, and the consequent settling and spreading result in a disintegration and destruction of the embankment, and the outflowing water causes death and destruction along its pathway.

594. *Foundation-beds.*—In order to prevent water from finding an outlet under the embankment it is essential that all loose, disintegrated, and porous materials should be removed from the entire area upon which the embankment is to rest, until a firm, compact soil is reached, unless the depth is very great, in which case the upper loose and porous strata should be removed to a depth of four or five feet, or to a reasonably good material. Then along or near the centre line or axis of the dam a broad trench should be cut to a reasonable depth, or till some more or less compact stratum is found, this trench narrowing as it deepens; from the bottom of this a narrower trench should be carried down to and into a perfectly reliable material, whatever may be the depth required, finishing at the bottom with a width of not less than 4 to 6 feet. This trench should then be filled with good strong concrete or the best clay-

puddle. The former seems to be the better practice. The upper and broader trench is often filled with puddle.

In the Western States, particularly in California, many very high earthen dams have been constructed.

Experience seems to indicate that it is not advisable to build earthen dams on a rock foundation-bed owing to the difficulty in securing a water-tight joint between the two, and that the best materials on which to build are a sandy or gravelly clay, fine sand, or loam. Such structures should never be built on shale or slate or solid rock. If a proper bed cannot be found after cleaning off the soil to a depth penetrated by the roots of grasses, bushes, and trees, then a trench should be dug as above described until a proper bed is reached, or at least to a great depth, and this filled with puddle or concrete; this should extend up into the embankment above for several feet. When building on a firm material close to the surface the bed should be well scored with longitudinal trenches, so as to give a good bond between the bank and its bed.

The above-described preparation of the foundation seems to be universal.

595. *Material for an Earthen Embankment.*—Although the banks are built of many kinds of earth, it seems to be universally recognized that the best material is a mixture of gravel, sand, and clay, in about the following proportions:

Coarse gravel................	1.00	cubic	yard.
Fine "	.35	"	"
Sand......................	.15	"	"
Clay......................	.20	"	"
	1.70	"	"

When mixed loosely and spread in thin layers this will make about 1.3 cubic yards, and when compacted in the embankment will make about 1.25 cubic yards.

The proportions are based upon the voids existing in the original cubic yard of gravel and those remaining after each of the preceding coarser grades has been added and mixed, only clay enough being used to give cohesiveness to the mass after the addition of the sand and to fill the voids then existing.

Gravel alone has been used in dams having a depth of 50 feet which have retained water without serious leaks. The best practice is to mix finer material and clay with it. Clay has also been used in

many places in the West. When used alone, however, it is likely to become more or less saturated with water, and in warm, dry climates it is liable to dry and crack. Sand and gravel lack cohesiveness, but have great stability; while clay has cohesiveness, but is wanting in permanence and stability. The combination given above possesses the qualities of weight, stability, cohesiveness, and imperviousness, having an angle of repose between that of fine sand and shingle, a mixture of sand and clay.

596. If such a combination of materials can be obtained in sufficient quantities and at a reasonable cost, the entire embankment should be made of it. But this is seldom the case. Inferior materials are necessarily used, in which case it is the practice to use a middle wall or core of the best material, filling on either side with the inferior; or the water slope of the embankment should be faced with a thick layer of puddle, composed of the best materials mixed in the proper proportions. These two constructions are typical of all dams made entirely of earthy materials.

For the puddle core is often substituted a masonry or concrete wall. Each of these has its advocates. On the one hand it is claimed that a masonry, concrete, or even a puddle core, having an inferior material forming the up-stream half, is an element of weakness, as the water, percolating through the earth, is checked by the impervious core, and collected and retained in the earth, thereby transferring the entire pressure to the core, whereas it should be resisted by the up-stream face of the dam; and they claim that it is better to let this seepage-water continue through and find egress on the outer or down-stream slope. Consequently, if there is not sufficient material available to make a homogeneous puddle embankment throughout, that it is better to build the up-stream third or half of good material, backing this up with the inferior material, which should be well bonded into the other portion of the embankment. By this construction the pressure is resisted on the up-stream slope, the leakage is reduced to a minimum or prevented entirely, and such water as may seep through finds a ready and easy means of escape.

On the contrary, the advocates of the puddle or masonry cores argue that for the same expenditure of money more puddle can be put in the form of a central wall or core than can be done on the water slope; that it is not exposed to the danger of slipping when the water is suddenly drawn off; that the water does not escape from the puddle quick enough to drain the bank, and consequently a

head of pressure exists which causes the slope to cave; that the puddle in the core is not exposed to the injurious effects of frost, nor to drying out and cracking; that masonry or concrete core walls are not elements of weakness; that accidents and breaks are not due to their presence, but rather to defects in design or construction, such as having too little thickness; that although when embankments are properly carried up there will be little or no subsequent settlement, but to provide for such settlement the wall should have sufficient stability to resist the differences in the earth pressure on its two faces; that the thickness at any point should be from one sixth to one fifth of the vertical depth at that point from the top; that the advantage of a puddle wall consists in its producing almost a homogeneous embankment throughout; that such cores, whether of puddle or masonry, should be founded on and bonded to an impervious bed, or extended downward to a considerable and safe depth; and, finally, that such walls well executed are entirely consistent with and justified by good practice.

The statement above, that solid impervious cores are objectionable, as stopping the percolation of water, is entirely consistent with the opinion expressed by the writer, when discussing the stability of surcharged retaining-walls (see paragraphs 580, 581, and 582), to the effect that when water once gets into a mass of earth, it is far better to provide ample and ready means of escape, rather than obstructing or attempting to hold it.

Having decided upon the material or materials to be used, and upon the position of the puddle, whether on the water slope or in the centre as a core wall, the next step is the determination of the proper form and dimensions of the cross-section.

597. The width at top varies from 6 to 25 or more feet. The water slope should rarely be less than 2 horizontal to 1 vertical, especially on its upper portion; the lower part of the slope may be $1\frac{1}{2}$ to 1. Benches are sometimes made on this slope. The outer slope may be, and usually is, from 2 or $1\frac{1}{2}$ to 1. The best English practice seems to be to make the water slope 3 to 1, and the outer slope 2 to 1, with the puddle wall in the centre.

With the top width, height, and side slopes given, the width at base is easily calculated. Assuming top width $= 20$ feet, height $=$ 50 feet, water slope 2 to 1; outer slope $1\frac{1}{2}$ to 1, the width at base will be $20 + 2 \times 50 + 1\frac{1}{2} \times 50 = 195$ feet. Figs. 248 and 249 give cross-sections of earthen dams, showing character of underlying

strata, and materials used in the embankments, and also deep trenches cut into the natural soil and filled with concrete or puddle. Dimensions are shown on the drawings.

Fig. 248 shows cross-section of a dam constructed of several different kinds of material. The core is constructed of a selected material, though not a true puddle. The cut-off concrete and

CROSS SECTION.
ASHTI DAM, INDIA.

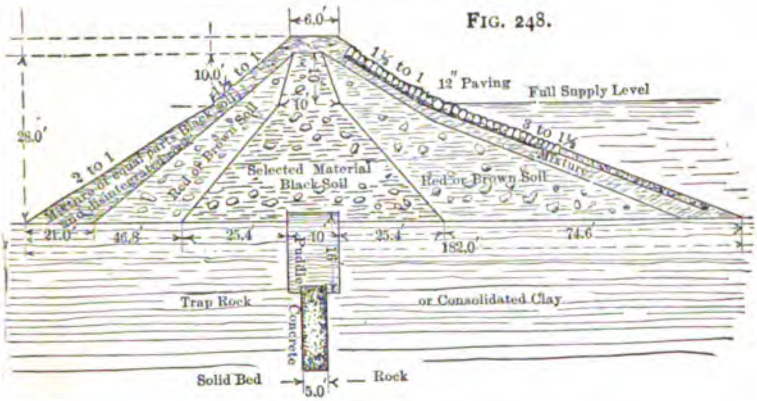


FIG. 248.

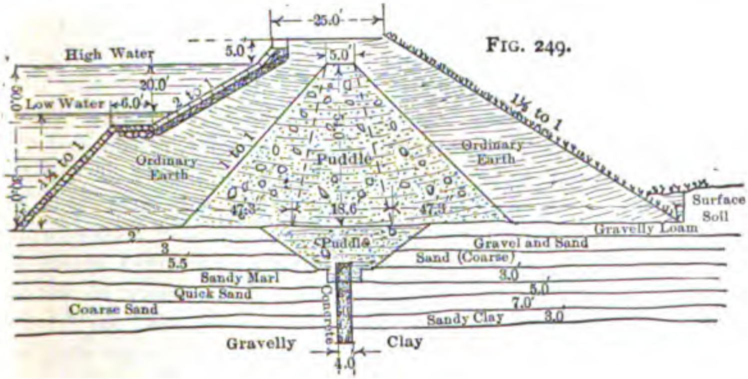


FIG. 249.

puddle wall is shown below the dam proper. A special mixture of black soil and disintegrated rock is placed on the outer slope. The water slope has also a mixture placed on it, which is covered with a layer of gravel and sand. The upper portion of this slope

is paved with stone. This embankment is composed of 4 or 5 different materials, each of different weight per unit of volume, and likely to settle unequally. It is regarded as a good example of an earthen dam.

Fig. 249 represents a cross-section of an actual dam used, with the underlying strata and cut-off wall. As actually constructed the puddle core was 5 feet thick at top, 54 feet above the base, and the side slopes 1 to 1, as shown by the full lines, giving a base of 113.0 feet. This core was covered on the sides and top by an ordinary earth, the outer slope of which was sodded, and had a uniform slope of 1½ to 1. The inner or water slope was 2 to 1 for a vertical reach of about 24 feet below high-water surface, on which was placed a layer of broken stone one foot thick, and then paved with granite blocks 10 to 14 inches thick, and of varying lengths and widths. At the foot of this slope was a bench or berm of 5 to 6 feet in width, paved with a double layer of flat stones. The lower portion of this slope was 1½ to 1, and paved with cobblestones well rammed.

The puddle core was made of coarse and fine gravel, sand, and clay in about the following proportions: 8 loads of mixed gravel (7 loads of coarse and 3 loads of fine gravel, when mixed, made about 8 loads in bulk), 1 load of sand, and 2 loads of clay. These materials were mixed by spreading the gravel in a thin layer, then the clay evenly upon this, after mashing the lumps, and on top of the clay the sand. The mixing was done by dragging a harrow over the mass until thoroughly mixed. This mixing was done on the bank. The layer was then moistened and rolled with a 2-ton roller. "Such a core packs down as solid, resists penetration or abrasion of water nearly as well, and is nearly as difficult to cut through as ordinary concrete, while rats and eels are unable to enter and tunnel it." (See Fanning's "Water-supply Engineering.") A core of the above dimensions would stand the water-pressure itself; but it was considered necessary to protect the surface above water-line with a frost-covering of earth: this necessitated the ordinary earth filling on its two sides and top. The small puddle wall enclosed by dotted lines (Fig. 249) shows about the least dimensions consistent with good practice, and could be adopted where good material is scarce and expensive. the batter on the sides being 1 in 8, giving 18′ 6″ base when 54 feet high. Such a puddle wall, while stopping leakage, could not sustain much pressure.

597a. TABLE LVa.

HIGH EARTHEN DAMS, CALIFORNIA.

Names.	Height above Ground or Bed, feet.	Top Width.	Wet Slope.	Dry Slope.	Length of Crest.	Puddle Walls.		Depth of Water.
						Average Thickness.	Depth below Bed.	
Old Dams.								
Pilarcetos..........	95	26	$2\frac{3}{4}$ to 1	2 to 1	650	20	46	90
San Andreas	95	25	$3\frac{1}{2}$ to 1	3 to 1	650	20	48	90
Crystal Springs......	50	30	$3\frac{1}{2}$ to 1	3 to 1	560	25	98	45
San Leandro........	110	40	3 to 1	3 to 1	600	40to50	40	100
Proposed Dams.								
San Mateo	170	100	5 to 1	4 to 1	600	40	20	160
San Pablo	128	30	$3\frac{1}{2}$ to 1	$3\frac{1}{2}$ to 1	1000	40	......	108

Crystal Springs Dam.—The puddle trench of this dam was cut through 23 old creek beds, full of gravel and water, without reaching rock. The height of the embankment was only 50 feet.

Usually a trench was cut to rock along centre line of dam. For small dams this trench was 20 feet wide, and for large dams 50 feet wide. The site of the dams were cleared of roots, stumps, loose rock, loose material and loam, and extended to solid ledge or to a good foundation stratum. These materials, as a rule, should not be used in the dams. The slopes on the sides of the valley were cut into steps to provide foothold.

Puddle of the best stiff blue clay was put on carefully in layers of 3 inches in thickness, wetted slightly, spaded up and rammed down to a homogeneous mass, so that the contact between the clay and bed-rock was complete. The trench was entirely filled with this material. Longitudinal grooves were also cut into the rock in order to make a tongue-and-groove bond. Sometimes concrete is used to fill these grooves. This puddle wall was brought up above the natural bed of the valley.

On the water side of this wall selected material was used for the embankment. If good clay was handy the entire dam was made of it.

Then all of the loose material and gravel in the bed of the creek was removed in order to prevent any water sweeping underneath and reaching the puddle pit. Layers of about 8 inches in thickness were then placed, commencing at the slopes, up and down stream, and sloping on a 5-per-cent grade towards the puddle trench. The surface of each layer was moistened or

watered before placing the next layer above, and the new layer also moistened. Each layer was then rolled with a 6000-pound roller, which reduced the thickness to 5 or 6 inches, the inner toes of the layer coming nearer and nearer to the puddle wall. When within 10 or 15 feet of it selected clay was provided, and used in layers rarely exceeding 3 inches in thickness. This selected material was spread over and across the puddle trench. Successive layers were then put on, each layer was spaded up and rammed and beaten down, so as to form a homogeneous mass with the underlying layer. The wet slope was protected by a heavy layer of broken stone 2½ feet in thickness.

The waste-way was built around the end of the dam, the capacity of which was three times the maximum flow. The common types of head-gates are used. Earthen dams should be built of the full height intended, and not left to be spliced on top and slopes at some subsequent time.

The plan of the San Mateo dam, 170 feet high, was changed from an earthen to a massive concrete dam, as the site was found to be underlaid with solid rock at reasonable depths.

597*b*. In Figs. 250 and 251 are given, respectively, the cross-section of a reservoir embankment and the plan of a reservoir, which

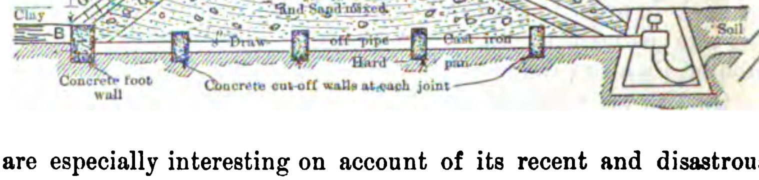


FIG. 250.

are especially interesting on account of its recent and disastrous failure. Fig. 251 shows the plan of the reservoir and the position of the break in the embankment at *A*; while Fig. 250 is designed to show a cross-section of the embankment at or near the line of the washout, in which is shown the arrangement of the different materials forming the embankment, the kinds of material used. and the position of the overflow-pipe and also the underdrain-pipe.

All of the materials used seem to be exceptionally good, and well suited to the purpose.

It is claimed that the construction was carefully executed. The failure of the wall cannot be definitely explained or accounted for, but has given rise to much discussion and criticism.

Briefly described, the excavation was made to a depth of 30 feet on one side and 6 feet on the other, in a stiff, gravelly clay or hard-pan. The embankment walls were constructed on hard pan at the bottom of this excavation. It was considered a good foundation-

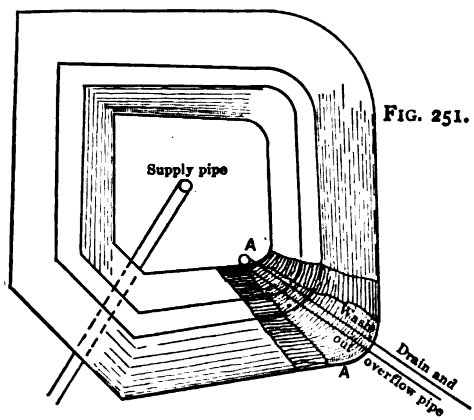


FIG. 251.

bed. The top width of the embankment was 12 feet, its height 45 feet, its base 147 feet. Although the entire bank was made of good material, it will be necessary to class the dam as among those having a puddle facing and no well-defined central core wall, as the water slope was faced with a good layer of puddle 6 feet thick. The different materials, and where placed, of the filling are fully shown on the drawing. In addition, on the puddle facing a layer of broken stone 6 inches thick was placed, and on this a paving of granite blocks 8 inches thick. The drain-pipe passing under the bank was encased at its inner end in a large concrete mass, as shown at *B*, Fig. 250; also, the end of the overflow-pipe was encased in a mass of concrete, as shown at *C*, at the level of the high-water surface. This pipe was simply puddle around where it passed through the embankment. The lower drain-pipe had cut-off walls of concrete at each joint, as shown, to prevent or limit the seepage-water along the line of the pipe. The reservoir had been constructed and in use for a considerable period. No evidence of any

weakness, defects, or leaks had been discovered until, almost without notice, a large breach was made at or near one corner, as indicated at *A*, in Fig. 251. The immense volume of water, pouring out suddenly, washed away a considerable portion of the bank, destroyed property in the neighborhood, and in addition caused the drowning of several persons.

The precaution was taken to spread a layer of good clay over the entire bottom of the reservoir in order to prevent any leakage under the bank or along the drain-pipe. It will be noted that the slopes, both inner and outer, are only $1\frac{1}{2}$ to 1. This is steep, even for the outer one, but is unusually so for the inner or water slope.

598. The failure was attributed by some engineers to the steepness of the slope, both directly and indirectly: (1) as not giving sufficient stability to the dam acting as a wall under pressure, and (2) as being too steep to hold the puddle facing and paving, especially if any percolation of water through the puddle facing should occur, softening the earth filling underneath, and decreasing the frictional resistance between the two. Others claimed that leakage along the line of the overflow-pipe not only contributed to, but caused, the failure and giving way of the embankment. Unless this seepage-water followed the overflow-pipe for its entire length, it would collect in the earth-mass and undoubtably tend to bring about the other above-mentioned causes of failure. The claim that the puddle lining did not slip at other portions of the dam would not prove that it did not do so where the breach occurred, especially as adjacent to the overflow-pipe the unfavorable conditions would be more likely to exist, as the accumulation and retention of the water would be local, and confined near the pipe. This and other similar experiences seem to indicate that—

(1) It is usually, if not always, impossible to keep water absolutely from finding its way into an earth mass.

(2) That where there is no scarcity of water, and more or less waste is not a matter of great moment, it is better to make a stable and open backing, in order to allow for the free escape of the seepage-water.

(3) That when it is important to limit the leakage, the embankment should be made not only homogeneous, but of the best puddling material throughout,—practically a puddle embankment,—in order that the resistance to percolation may be as nearly uniform entirely through the mass as practicable, and small in amount.

(4) That the slopes should not be too steep, in order that the change in its condition of stability, due to the presence of water in limited quantities, may not demand steeper slopes than those actually existing, say from 2 or 3 to 1 on the water slope, and 1½ or 2 to 1 on the outer slope.

(5) That the selection between ashlar or rubble masonry or concrete cores, as well as a puddle core, is mainly one of convenience and economy, with probably the advantage in favor of the puddle core.

(6) That the puddle should always be protected by a frost covering, above the low-water surface, of earth and broken stone, to prevent the disintegrating effects of frost at the exposed surface. This portion of the slope should also be paved with courses of stone or brick, in order to prevent the injury arising from wave action on the slope. The lower portion of the slope may be paved with cobble or rough stone rammed into the bank, or a layer of cement concrete laid over with a thin layer of cement mortar. The sheet of concrete may be 10 or 12 inches in thickness. It should be bonded into the slope with ribs or in steps.

(7) All earth dams, whether built of a homogeneous material throughout, or of combinations of materials, with or without core walls, should be built in thin layers, which should be somewhat thicker near the edges than at the centre of the bank. These layers are in some cases as much as 9 to 12 inches in thickness; the best construction requires that the layers shall not be more than from 3 to 5 inches thick. The surface of each layer should be moistened by sprinkling, and then well rolled with a grooved or ribbed roller. A smooth-surfaced roller fails to bond the layers into each other, leaving smooth and continuous seams through the bank.

(8) A good precaution is to cover the entire bottom of the reservoir with a layer of puddle, over which is spread a layer of sand or gravel. The puddle layer usually forms a continuation of the facing puddle, and sometimes extends under the bank to the puddle core, when this latter is used.

(9) When a masonry core is used, the cut-off trench under the embankment can be filled with masonry or concrete up to or above the base of the embankment, and the masonry core built on this. With a concrete core the cut-off trench is also filled with concrete. In these cases the thickness of the core should be from one fifth to one sixth of the height of the core above that point. The thick-

ness of a puddle core should be from one half to one third its height.

(10) When pipes are to be carried through the embankments it is best to place them mainly in the natural bed under the embankment, with only their upper surfaces in contact with the earth. As little puddle as practicable should be used under them, and this should be of a uniform thickness, in order to avoid unequal settling and springing of the pipes. Cut-off walls should be used around and between these, and over the pipes good puddle should be placed and compacted. Often the pipes rest on or pass through cut-off walls of concrete or brick. After being laid and tested the whole is filled in with concrete or masonry, and sometimes simply well filled with puddle. Wherever masonry is used around and for the support of pipes, the exposed surfaces should be as rough as practicable, that they may bond better with the puddle placed and compacted around. These pipes either open direct into the reservoir, or into masonry gate-chambers. Better still, the pipes can be laid in a trench, entirely below the base of the dam, in rock or firm soil. This trench should be lined and roofed with concrete. If practicable, all pipes, waste-weirs, and channels for discharging flood-waters or draining reservoirs should be carried around and entirely clear of the dam.

LOOSE-ROCK DAMS AND LOOSE-ROCK AND EARTH DAMS.

599. *Loose-rock Dams.*—Of late years broken-stone dams have been used to a considerable extent in the Western States, for reasons already stated. These dams are simply constructed by piling rocks into a mass, using some degree of care in order to give a slope to the mass at which they will safely stand, first placing the largest stone in a layer or course without reference to any even top surface, and filling the interstices with smaller stones; upon this another course is laid in the same manner, the stones on the outer edges being laid to the required slope. Such dams are called hydraulic-mining dams. When the interstices are well and closely filled, although there may be much leakage, they will, notwithstanding, back the water up behind them until the leakage becomes equal to the flow of the stream. This, of course, limits the height of the dam. The courses, as in earth dams, should be kept a little lower towards the centre of the mass than at the edges.

Such dams can be constructed in running water by simply de-

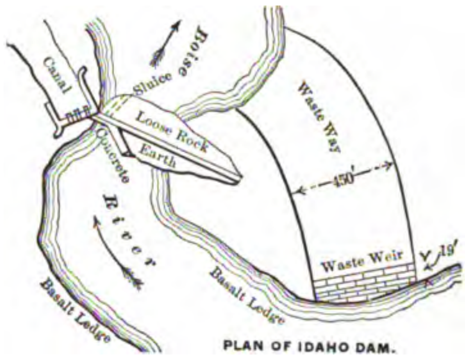
positing the stone in the water, as nearly in courses as practicable, until the mass reaches above the water surface, care being taken to make the thickness of the under-water portion sufficient. The interstices can be fairly well filled by dumping smaller stone or large gravel over each course as formed. The upper slope should not be less than ½ to 1, and the lower or outer slope less than 1 to 1.

Where desired to further limit the leakage, a sheathing of plank can be laid over the water slope. For greater protection, logs can be built into the rubble mass at intervals, both in vertical and horizontal planes, their ends projecting beyond the slope a foot or more. Stringers or caps are bolted to the ends of the projecting pieces in a vertical plane at right angles to these stringers, which are about 4 feet apart horizontally; in one example a double course of 3-inch plank was spiked to the stringers, tarred paper being placed between the layers of plank. Subsequently the outer course of plank was calked, and the whole surface painted with paraffine paint. The Walnut Grove Dam was thus constructed. It rested mainly on solid rock forming the bed of the stream; a small portion of the wall is said to have rested on a bed of loose earth and gravel from 5 to 12 feet deep. This no doubt contributed to its ultimate destruction in a great flood. Other causes of failure were due to the careless manner in which the loose stones, between reasonably well-built slope walls, were laid, and also from the want of an ample waste-way. As this dam gave way, the dimensions will be given: Length at top 420 feet, width at top 15 feet and at bottom 138 feet, and 110 feet in height. The slopes were therefore less than ⅝ to 1, on the average. The actual slopes were: Outer slope 1 to 1½, and water slope 1 to 2¼. The outer slope was increased to 1 to 1 near the bottom. A culvert was built through the dam about 14 feet from the bottom. A valve tower reached from the top of the dam to the inner end of the culvert. This tower was 8 × 8 × 90 feet.

600. *Loose-rock and Earth Dam.—Idaho Dam.*—This dam was constructed of loose rock faced with earth. The earth facing was given to prevent leakage.

The site of the dam is at a point where solid basalt outcrops across the channel of the Boise River, on which the dam rests. A basalt ledge 12 feet in height borders the river bank just above the dam, on which a waste-way is constructed having a width of 450 feet. The waste-way is to be excavated in gravel, and is to be carried 8 feet below the crest of the dam. It has a length of 720 feet, and dis-

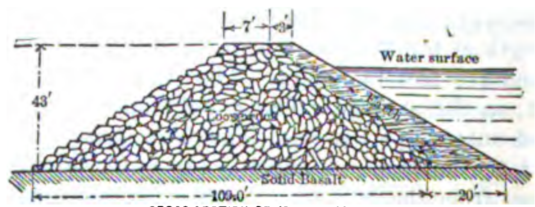
charges back into the river 100 feet below the dam. Near the discharge end a waste-weir, built of rubble masonry, is placed entirely across the waste-way. The top of this weir is at the proper depth below the crest of the dam. It has a height of 8 feet and a bottom width of 19 feet; the upper slope is 1 to 6, and its lower slope has an ogee-shaped curve. Fig. 252 shows the plan of the dam, waste-way,



PLAN OF IDAHO DAM.

FIG. 252.

canal, and headworks, and the position of these with respect to the course of the river and the site of the dam. Fig. 253 shows a ver-



CROSS SECTION OF IDAHO DAM.

FIG. 253.

tical cross-section of the broken-stone dam with its earth facing. The length on the crest is 220 feet; maximum height, 43 feet; top width, 10 feet, of which on the inner slope 3 feet is earth facing—this facing being 20 feet thick at the base. The lower slope of the rock work and the slope of the earth facing are $1\frac{1}{2}$ to 1; the inner rock slope is about 1 to 1. The construction is clearly shown on the drawing.

601. Loose-rock dams should be founded either on solid rock, hard-pan, or very stiff clay. Materials that will scour out cause

unequal and great settlement and disarrangement of the dam. When carefully constructed on a good foundation-bed, such dams have great stability and permanence, provided water is not allowed to flow over their tops. The fall and shocks from the falling water will press the stones out of position, and endanger, if not destroy, the dam.

It is claimed that they cost less than masonry or earthen dams. This will clearly depend upon the cost of materials and transportation. If this is not great, masonry will be cheaper on account of the reduced number of cubic yards required when cement is used. In the dam described the number of cubic yards of stone is very great, owing to the flat slopes required.

Sometimes the up-stream face is built of good coursed rubble with a very steep slope; and the lower slope, though flatter, is also well built, forming two strong walls, between which ordinary dry rubble is placed. All these dams should be on exceptionally good and solid foundation-beds.

MASONRY DAMS.

602. *Masonry Dams and Weirs.*—The distinction between the terms *dams* and *weirs* is rather one of use than of construction, though the form of cross-section and character of the structures are different in many of their details.

A dam proper is defined as a masonry wall which is intended simply to impound and hold water, the water not being allowed to flow over its top.

A weir is defined as a masonry wall over whose top the water is allowed to flow freely. Dams and weirs will be discussed separately.

603. Masonry dams are regarded as rigid structures: the least unequal settlement in any portion of them tends to produce cracks. If a masonry dam is built on clay or even hard-pan, there are two risks to be run: (1) unequal settlement; (2) leakage between the dam and its bed. Either of these may cause settling, and endanger the safety of the structure. With proper precautions, however, low masonry dams can be built on these materials; but in high dams another danger arises from the great unit pressure on the foundation-bed, which may reach as high as 10 to 15 tons per square foot, or from 155 to 233 pounds per square inch—much greater pressures than are safe on any kind of earthy material. It ~~may~~ may therefore be said that all masonry dams should be founded on

solid rock. If this is not practicable, or if the expense attending it is prohibitive, excavations can be made to a safe depth, and sufficient spread of base provided with good rubble or concrete, or piles can be driven into the gravel or earth beds. In India the cylinder or well foundations are used in unstable material, and occasionally open cribs or caissons have been sunk for the foundations. Both piles and the cylinders should be well protected from scour by placing sheeting around them, or should be surrounded with concrete and the cylinders also filled with concrete. Such methods of obtaining good foundations are, however, more applicable to low dams or weirs than to very high masonry dams, which should always rest on rock direct or on cribs and caissons sunk to rock. No cost or labor should stand in the way of securing the absolute safety of the foundation, and consequently that of the dam itself.

604. Masonry dams and weirs may be constructed of ashlar or rubble or wholly or in part of concrete.

Ashlar Masonry Dams.—While ashlar masonry is capable of bearing heavier loads, it is not well suited for the construction of dams, since continuous horizontal joints should be avoided, and the great care required in cutting and selecting such stones, as well as laying the same so as to break joints both vertically and horizontally, requires so much labor, time, and cost, that we rarely find dams constructed of ashlar masonry. This, if used at all, is only employed as a facing, the interior being purposely built of uncoursed rubble masonry or of concrete.

605. *Uncoursed Rubble.*—Rubble masonry has been used almost exclusively in the construction of both masonry dams and weirs. While it has sufficient resistance to crushing, it is admirably suited to the breaking of joints in all directions without the expenditure of any special labor for this purpose. It, however, requires a great deal of mortar and special care in filling perfectly all interstices, which is absolutely necessary to be done. It will cost much less than ashlar masonry. It is advantageous to have a great variety in the shapes and dimensions of the stones that they may interlock or bond in every direction.

Many large dams are, however, built of rather small stones—such that one or two men can handle; the only advantage in this class of construction is to avoid the expense of derricks, engines, and other kinds of machinery; or they may be built of large stones, one, two, or more cubic yards in volume, and weighing many tons.

These require strong and expensive machinery to handle them. The smaller interstices between the stones may be filled with rubble or concrete.

In either class of masonry all facing stones should be set with the best Portland cement mortar; the filling or hearting between them can be laid in ordinary cement mortar. The mortar for the facing should be composed of 1 cement to 2 of sand, but for the rubble or concrete filling 1 cement, 3 sand may be used. Works of this kind should not be grouted under any circumstances.

606. *Concrete Dams.*—The same distrust of concrete that has limited its use for other engineering purposes has also resulted in but a limited use in the construction of dams, the more so on account of the general belief that it is impracticable to make it impervious. But with the more uniform product now obtained, a better acquaintance with its valuable properties, and the improved methods of thoroughly incorporating its ingredients, there would seem to be no valid objection to its use on the score of suitableness for the purpose, and the controlling factor in its selection should be that of economy. It is doubtless a less difficult matter to construct a solid mass free, certainly, from any large or continuous open channels or interstices than it is with rubblework. The greatest care and the closest inspection is required to secure good rubblework when built in large masses. Mechanics and laborers will not take the pains necessary; each gang or shift of men try to make the best record for a day in order to please their employer, even at the risk of slurring over the work. The same causes, however, operate in the use of concrete, but not to the same extent, as mixing by machinery is more likely to produce a uniform product; and the same may be said of hand mixing. The main source of a defective product in making concrete is the use of inferior materials. The broken stone should be thoroughly washed to remove dust and dirt; the sand should be clean and sharp; the water should also be clean (dirty water will not make good concrete: but little attention is given to this matter). Too much water should not be used, as it will always result in a porous mass; if too wet, the concrete cannot be properly consolidated. A layer of concrete should never be deposited on another layer whose surface has a skim of partially set cement, which leaves the surface smooth. Such a skim always exists where the mixing and ramming have been properly done. In such case the surface should be roughened before placing the new layer. It is not necessary that any layer should be exactly

of the same thickness throughout; a few breaks, resembling low steps, will prevent any continuous seams through the mass.

607. One or two illustrations of masonry and concrete dams are given below. It should be noted that a cross-section of a dam is

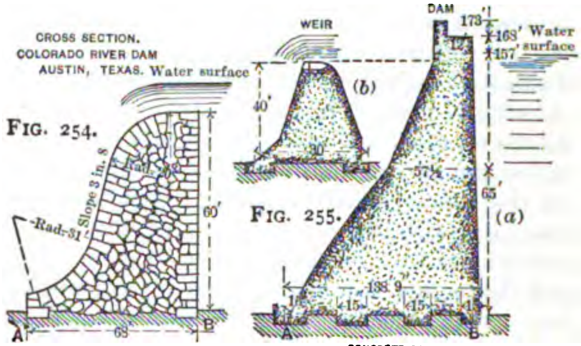


FIG. 254. ASHLAR FACE, RUBBLE BACKING

FIG. 255. CONCRETE DAM

designed simply to be stable under the pressure of the water against its inner slope, whatever may be the depth of the sustained water; whereas a weir has not only to be stable under this water-pressure, but its outer slope has to be formed in such a manner that the fall of the overflowing water shall neither destroy the dam by the shocks and vibrations produced by it, nor undermine it by the action of the same forces. The entire length of the structure may be designed as a dam proper or as a weir, or partly dam and partly weir. The construction and design must be regulated accordingly.

In Fig. 254 is shown a cross-section of a part weir and part dam constructed across the Colorado River at Austin, Texas. The section shown is that of the weir portion. The structure was only recently completed at a cost of \$300,000. It was supposed to have been founded on solid rock, but owing to the existence of some undiscovered underlying seams of earthy material, a portion of the dam, or more accurately the walls of the head-gates, was undermined, resulting in its destruction. This necessitated the construction of a coffer-dam around this portion of the wall, and the further excavation of the underlying bed, in order to get below the seams of loose material. This work is now being prosecuted; the additional cost will amount to \$50,000, or more.

As seen in the drawing, the weir is constructed with a facing of cut blocks of coursed granite. Its interior filling is of rubble masonry. Its length is 1275 feet along the crest-line; of this 1125 feet is designed as a weir, and 150 feet as a dam. Its maximum height is 66 feet. Its upper face is vertical. The lower face has an easy ogee-shaped curve, calculated to pass the overflow water with such ease as will reduce the erosive action at its base to a minimum. The maximum flood to be passed is estimated at 250,000 cubic feet per second from a drainage area of 50,000 square miles. The cross-section is heavier than required by theory, if the structure be regarded simply as a dam. The lower curve is extended to deliver the flood-water away from the toe of the dam and against the back-water below. The top width is 5.0′. The total top width is 16.0′, and maximum width at bottom 68.0′.

There has been considerable and acrimonious discussion in regard to the design and construction of this dam or weir. It is given as a good type of this kind of construction and as one of the very latest. Its purpose is both for water-supply and water-power to the city. See Supplement for additional facts.

608. In Fig. 255 is shown a cross-section (*a*) of a concrete dam and (*b*) of a weir. The elevation of the water surface is shown at 157 feetabove the bottom; the total height of the dam is 173 feet. Being built across a valley, the height decreases as the sides of the valley rise. The weir portion being on the higher ground, its maximum height is 40 feet above its bed. Its top is shown as on a level with the water surface of the reservoir. The vertical scales of the two sections are not the same. The dam and weir are both founded on rock, into which longitudinal trenches were cut and filled with concrete, thereby bonding the dam into the rock, so as to prevent leakage between the two.

This structure was built throughout of concrete. Its length along its crest is 1230 feet. A parapet 5 feet high is built on the top. The top width is 12 feet and bottom width 138 feet 9 inches. At either end are wasteways cut through the solid rock, their aggregate length being 920 feet.

The maximum capacity of the reservoir is 300,000 acre-feet; its available capacity 157,000 acre-feet, the acre-foot being equivalent to 43,560 cubic feet.

The inner face of the dam is almost vertical, having only a very slight batter. The outer or lower slope has the form shown in the drawing. This is one of the many theoretical sections adopted in

the construction of very high dams. The almost universal practice is to make the inner face nearly vertical, or, at any rate, with a very slight inclination. This may be straight or curved, concave up-stream. Sometimes a more decided batter is adopted near the bottom of the wall. The outer or lower slope has several different curves, depending on the formulæ used, which are often modified in a somewhat arbitrary manner to suit supposed special conditions. All, however, approximate to a continuous curve from bottom to top, concave down-stream, the curve being quite steep near the top and flattening as it approaches the outer toe of the dam.

Theoretically, the thickness at the top may be zero; hence the cross-section of the dam is approximately that of a right-angled triangle: but in fact the top width is always many feet. Consequently the upper section of almost all dams is rectangular for a certain distance below the top, and then it commences to spread out.

609. At all points the three conditions given in paragraphs 565, 566, 567 must be fulfilled, namely: (1) That the wall shall not give way by the sliding of one portion on another. This condition is satisfied when equation (350) is satisfied; or for horizontal joints, equation (358). (2) That the wall shall not give way by crushing of the materials with which it is constructed. That this condition be fulfilled it is necessary that the value of p in eqs. (365) and (366) shall not exceed the safe crushing resistance of the material. The safe resistance to crushing for the various materials is taken as follows, per square inch: Granite, 155 pounds; limestone, 150 pounds; sandstone, 130 pounds; brick, 120 pounds. Or per square foot, respectively, 22,320, 21,600, 18,720, 17,280 pounds. (See Wilson, "Irrigation Engineering.")

These are limiting pressures within one foot of the edge of the masonry. In the heart of the wall it should not exceed 200 pounds per square inch, or 28,800 per square foot. It is evident, from an inspection of Figs. 254 and 255, that when the reservoir is empty the resultant pressure, which is simply the weight of the wall, passes nearer the inner edge B than the outer edge A. But when the reservoir is full the resultant pressure is the resultant of the weight of the wall and the pressure of the water, which will pass nearer the outer edge A than the inner edge B. The condition of safety against crushing must then be fulfilled both when the reservoir is empty and when it is full. Up to a height of 200 feet it may be safely stated that, with any well-designed dam constructed

of masonry, thére is little danger of the dam giving way by crushing; and although it is well to apply the formula, that the actual pressure on the base may be known, it is not necessary.

The resistance to crushing of concrete is taken anywhere between 60 and 150 pounds per square inch, or 8640 to 21,600 pounds per square foot.

As already stated, there is no danger of a dam giving way by sliding if it is safe against overturning. The form of cross-section is therefore determined mainly with reference to its stability against overturning. This condition requires that the moment of the pressure tending to overturn the wall shall be less than the moment of the weight which resists this overturning. Therefore for all depths the equations (349) and (357) or (362) must be satisfied at any and all horizontal sections of the wall.

Since theoretically the thickness at the top should be zero, while practically it is many feet, it is not necessary to apply the formula until a depth is reached requiring a greater base than the top width. Then as the depth increases the resultant pressure will be more and more inclined, requiring an increased and increasing width of base in order that the centre of pressure shall not be outside of the outer edge, which is the position, theoretically, of the axis of rotation; but, as often stated, this axis should be taken at from one eighth to one third of the thickness of the wall from the outer edge. The absolute thickness of the wall will then depend upon this distance of the axis from the outer edge. Many theories and formulæ have been worked out, based upon certain conditions and assumptions, and while no two agree, the general cross-sections obtained are somewhat similar.

The authors of the formulæ are Krantz, Rankine, Wegmann, Molesworth, Delvere, and others. For full discussion of these subjects the reader is referred to the works of these authors.

It is necessary to increase the heights of the dams from 1 to 10 feet to prevent the waves from dashing over them. The top width varies from 5 to 15 feet.

610. *Weirs Classified.*—Weirs are classified, according to their construction, as follows: (1) With clear overfall to the bed of the stream; (2) with clear overfall to a water-cushion; (3) with clear overfall to aprons constructed to resist and break the fall; (4) with rollerway in lower faces; (5) with heavy profile and ogee-shaped curve.

(1) With clear overfall the danger and objections arise from

erosive and scouring action, which will undermine and destroy the dam if the material upon which the weir is constructed is of an earthy or gravelly character; and even solid rock will in time be greatly affected, though it may not result in any serious damage to the weir.

(2) With water-cushion the erosive action is greatly limited by the overflow falling into a water-basin.

What should be the relation between the volume and height of overflow and the depth of the water-cushion to prevent scouring out of loose material, is probably not known. Often special constructions are made to form these pools or cushions below weirs. They can be made entirely effective for the purpose.

(3) With aprons the overflow water is made to fall on a strong platform, usually of timber, which may rest on the natural bed of the river, or upon beds constructed of broken stone or piles. These receive the force of the fall, and they are made long enough, either with a gentle slope or horizontally, so as to greatly retard the velocity of the water, and at the same time lead it well away from the weir. They are sometimes inclined or curved upwards, which still further retards or breaks the velocity.

(4) With rollerways the weirs are constructed with long, flat slopes, or series of steps, by which means the velocity of the water is retarded and at the same time the water is conducted away from the weir. These require the use of large quantities of material, and consequently great expenditure of money. Masonry aprons or rollerways are often used.

The ogee curve may be considered as a modification of rollerway construction. It reduces the material required to a minimum, and produces about the same effect as the rollerway. The object of the ogee-curved face is to cause the water to slide rather than fall over and clear of the weir. So long as the water slides, it preserves a bluish color, and when it commences to fall it becomes whitish or milky. The following is the construction of the ogee curve: In Fig. 259 make $GB = \frac{5AG}{2}$ and $AE = \frac{AB}{3}$; then bisect AE with a perpendicular HC, which prolong until it intersects the normal from A to GB, at the point C. Draw CE and prolong it until it intersects the normal to BG, drawn from the point B, at the point D; and with C and D as centres and CE and DE as radii, describe the respective curves AE and BE. This construction is simply that of two connected parallel straight lines

with a reverse curve. Fig. 260 shows a good example of this form of outer face of a weir used in connection with the Croton Dam, New York, the construction of which is novel and somewhat peculiar.

611. The Croton dam and weir were constructed for water-storage purposes. It is one of the largest masonry weirs, and was founded on unstable material for a portion of its length and on solid rock for the remaining portion.

As seen in the drawing, the weir portion of the dam is composed of a series of cribs of different widths, forming a series of

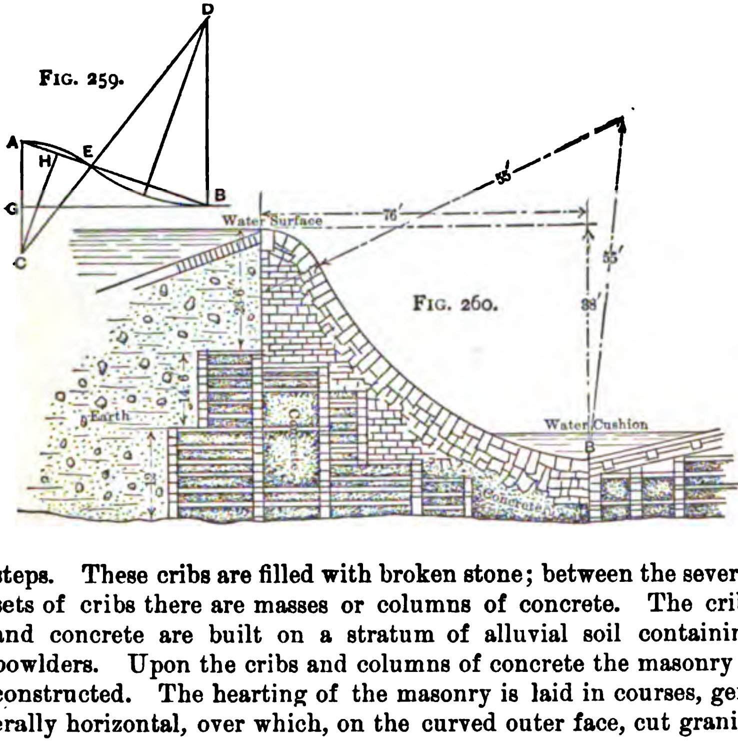


FIG. 259.

FIG. 260.

steps. These cribs are filled with broken stone; between the several sets of cribs there are masses or columns of concrete. The cribs and concrete are built on a stratum of alluvial soil containing bowlders. Upon the cribs and columns of concrete the masonry is constructed. The hearting of the masonry is laid in courses, generally horizontal, over which, on the curved outer face, cut granite ashlar is laid.

All of the masonry is laid with hydraulic cement mortar. As shown, the up-stream face is vertical for a depth of 23 feet from the top, at which point the masonry rests on a concrete column between two sets of cribs. The upper of the two cribs is above the masonry face, and forms two steps. This face is backed by

earth having a long, flat stope, which is paved on its upper surface.

The crest of the dam is formed on a convex curve having a radius of 10 feet, and the outer slope is formed by a concave curve having a radius of 55 feet, thus forming an ogee curve similar to that which will be taken by the water flowing over the crest. The total depth along the inner face is 50 feet. The maximum thickness at the base is 76 feet. The lowest point of the masonry weir at B is only 38 feet below the line of the crest at A.

Extending outwards from B a series of cribs are constructed so as to give a rising surface, by which a water-cushion of $2\frac{1}{2}$ feet in depth is formed. Of these the two cribs adjacent to the masonry are filled with concrete; the other three are filled with

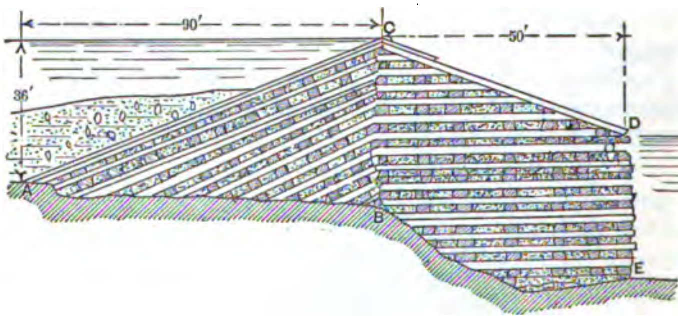


FIG. 261.

broken stone. There are other cribs, extending to a further distance of about 300 feet, not shown in the drawing, at the end of which the water falls to the bed of the stream, a depth of about 15 feet. This construction, therefore, combines the following advantages: (1) The water slides along the ogee curve, breaking any clear fall and consequent shock; (2) the shock and velocity are further reduced by the water-cushion and upward slope of the apron, together with its great length; (3) the total fall is divided into two falls of less height, namely, 38 and 15 feet, respectively; and (4) by this arrangement the timbers of the crib are submerged under water at all times.

612. The new Croton dam will have a total length of 2180 feet, divided as follows: An earthen dam 530 feet long; extreme height above foundation-bed 120 feet; top width 30 feet at 20 feet above high water. The upper or water slope is 2 to 1, paved with from

$1\frac{1}{2}$ to 2 feet of cobblestone laid on $1\frac{1}{2}$ feet of broken stone. The outer or lower slope is 2 to 1, broken by three benches 5 feet wide; the entire slope sodded. The core of this dam is rubble masonry, extending below the base of the dam to solid rock, having a total height of 225 feet, with a thickness at top of 6 feet and at base of 18 feet.

The main dam, 630 feet long, has a maximum height above its foundation-bed of 248 feet, and 163 feet above the river-bed; its top width is 18 feet and its bottom width 185 feet. It is built of rubble masonry, and faced above ground with coursed work bedded in Portland cement. This dam is connected with the core wall of the earthen dam, and the earthen dam itself with heavy masonry wing walls. The crests of these two portions are straight.

The weir portion, 1020 feet long, curves up-stream from the masonry dam nearly at right angles to it. The maximum height is 150 feet, and its extreme width at base will be 195 feet. Its up-stream slope is nearly vertical, while its outer slope conforms to an ogee curve, but is broken into a series of steps varying from 2 to 10 feet in height. It is constructed of uncoursed rubble backing and coursed facing. The crest of the weir is about 14 feet lower than the crest of the dam.

Both the masonry dam and weir are founded on solid rock. The capacity of the reservoir is 92,000 acre-feet.

The overflow water passes over the weir into a canal or channel excavated in the hillside, and returns into the river well below the toe of the dam.

613. In Fig. 261 is shown a weir constructed of logs and square timbers forming a large crib, which is filled with broken stone or gravel, the entire upper surface enclosed in a sheeting of plank. It is necessary, when such dams are built on earth, gravel, or soft rock, to carry the water some distance below the toe of the dam by means of a rollerway, as shown on the drawing, from which the water should fall either into a water-cushion of back or dead water, or if this cannot be secured, it should fall on a timber or masonry apron.

It is unadvisable to let the water have a clear fall over the crest, even when the weir is founded on solid rock, unless it can fall into a water-cushion. There are many designs of timber dams, but Fig. 261 shows both the usual design and character of construction.

Unless there is a constant overflow of water, and all the timbers

can be kept constantly wet or are so well puddled around that the air can be kept away from them, they will rot at an early period, and under such circumstances are only to be regarded as temporary structures.

Constructed as above indicated, such dams are cheap, easily constructed, and will be stable and permanent. When founded on gravel or loose material, sheeting-plank should be driven to a considerable depth below the base of the dam along the line of both inner and outer lower edges. The thickness of the dam proper, *ABC*, along the line *AB* is about 95 feet, while the height of the crest along *BC* is about 35 feet. The rollerway *BCDE* was only added after a clear overfall had worn away the bed of the stream near the outer toe. Its length, *DC*, is 50 feet. The dam abutted against heavy masonry wings at its ends. The upper face has an inclination of 21° with the horizontal on both sides of the crest.

614. *Construction across Running Streams.*—In small running streams, and in many larger ones, the water can be carried around the site of the dam in artificial channels, either in excavation or between embankments, and subsequently returned to its proper channel. The water may be confined to a portion of its natural bed while the dam is being constructed across another portion. In this case sluices or conduits must be built of sufficient capacity, low down in the dam, to carry the entire flow of the stream. These can be built simultaneously from both banks, and the water subsequently diverted from the middle to one or both sides, and the dam then completed. It may even be necessary to build a temporary dam above the site of the intended structure in order to divert the greater portion of the water through some artificial channel.

In construction and use of sluices proper gates should be provided in order that they may be closed when the dam is completed and ready for the storage of water.

Movable dams, head-gates, sluice-gates, etc., will be explained under the head of Improvement of Rivers and Canal Construction.

615. *Springs in Foundations.*—No fixed rule can be given for dealing with springs. If but one spring is discovered, its inflowing channel may be traced back to some elevated position, and the flow conducted around and clear of the dam in pipes. If it is a well-defined natural channel, it may simply be conducted clear of the dam in masonry or concrete conduits or culverts. But usually a number of small springs are found distributed in channels at different points of the bed. Such cases present great difficulties. By

care and slow approaches they may be smothered or choked down. The tendency will be to collect these different channels into one large flow, which, seeking an escape along the line of least resistance, will often cause much trouble in closing a dam.

Springy soils are always sources of great difficulty, cost, and danger; and where practicable it is better to change the site if by so doing such uncertain foundations can be avoided.

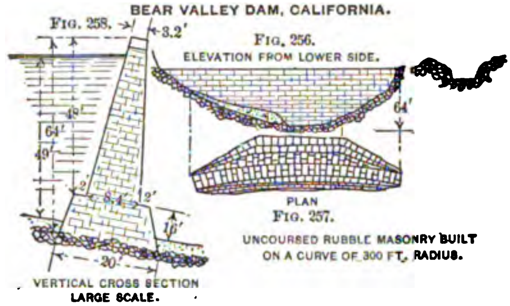
616. *Curved Dams.*—The dams heretofore considered have been supposed to provide stability by their weight and sufficient spread of base. As is readily seen, they require large quantities of material and consequent cost. There has been much theorizing on the subject of reducing the weight and quantity of material by giving the form of a large horizontal arch to the dam convex up-stream, thereby transmitting the pressures and strains along and around the dam to natural or artificial abutments at its ends. But the true theory of the arch being so little understood, few engineers have been bold enough to attempt to apply the principles of the arch to the construction of curved dams.

Since the pressure of water is normal to the up-stream face of a dam, the advantage to be derived from the curved plan arises from the fact that we can decompose the pressure into two components, one perpendicular to the span and the other parallel to it. The thickness and weight can therefore be reduced by proportioning them on the basis of the component perpendicular to the span, while the parallel component produces a compression on the up-stream face tending to bring all parts of the dam to act as a unit. It is evident that it would be safe to give a lighter cross-section to the dam when built on a curve than when in a straight line; but to what extent this can be done with safety is uncertain, if not impossible to determine.

There have been built but few dams on the curved plan. Experience has proved that their construction is safe, but with what margin or factor of safety is not known. Figs. 256, 257, and 258 show the elevation, plan, and vertical cross-section of one of the boldest designs of the curved dam, namely, the Bear Valley Dam, California. It is not given as an example to be followed, though it has proved its efficiency, but to show that at least a very great reliance can be placed on the stability of curved dams. As shown in the drawing, the top width is 3.2 feet, at 48 feet below it is 8.4 feet, and at the extreme base it is only 20 feet, while its extreme height is 64 feet. The length of its crest is 450 feet. The radius

of curvature is 300 feet. It is built of uncoursed rubble. The lines of pressure fall from 13 to 15 feet outside of its base.

The Zola dam has a maximum height of 123 feet; width at top 19 feet and at base 41.8 feet. It is only 205 feet long. Its radius



of curvature is 158 feet. It is built of uncoursed rubble. Its stability depends upon its arched form and the excellency of its construction, as is the case with the Bear Valley dam. It is recommended to increase the thickness near the end.

ART. XLIV.

EQUILIBRIUM OF CHAINS, CORDS, RIBS, AND LINEAR ARCHES.

617. THE following discussion of several useful principles is introduced in this place on account of the relations of these principles and the formulæ expressive of them to the stability of arches. The principles are, however, directly applicable to and valuable in connection with other and useful engineering problems.

Equilibrium of a Cord.—Referring to the discussion of the funicular polygon and to Figs. 68 and 69, it is evident that as the number of sides of the polygon is increased, and consequently the number of loaded points, the more nearly does the polygon of external forces approximate to a continuous line, straight or curved, and the more nearly does the funicular polygon approach the condition of a cord continuously loaded. Also, the number of radiating lines, representing the stresses on the several bars or sides of the funicular polygon, increase accordingly.

In Fig. 262, let $RABR_1$ be a cord or chain—in other words, a

funicular polygon of an indefinite number of small sides; A and B two points on this cord, AP and BP the directions of the two sides at those points, and CP the resultant of the forces between the points A and B. Since the portion of the cord between A and

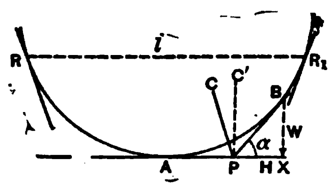


FIG. 262.

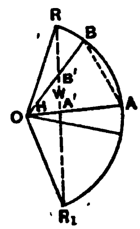


FIG. 263.

B is in equilibrium under the action of the forces AP, BP, and CP, they must meet at one point and be proportional to the sides of a triangle respectively parallel to their lines of action, and the directions AP and BP must be tangents to the cord. If, then, in Fig. 262 we draw lines tangent to the cord at the points R, R_1, A, and B, they will be the directions of the pulls at those points. Then, in Fig. 263, from a point O draw lines OR, OB, OA, and OR_1, making their lengths proportional to the magnitudes of the pulls, and join the extremities of these lines by a straight, broken, or curved line: the straight line joining A and B will represent the direction and magnitude of the resultant of the forces on the portion of the cord (Fig. 262) between A and B, since the three forces are proportional to the sides of a triangle.

If now the points A and B are taken nearer and nearer together, the line AB (Fig. 263) approaches nearer and nearer a tangent to the curve, and when they become consecutive points the straight line will be tangent to the curve; and hence, as this line is the resultant of the forces between A and B, the direction of the force or load at any point B is represented by a tangent to the line $RBAR_1$ (Fig. 263), which is called the line of loads.

If, then, a line of loads be drawn such that while its radius vector is parallel to a tangent to a loaded cord at a given point its own tangent is parallel to the direction of the load at the given point in the cord, the radius vector from the fixed and common point O will represent the pull on the cord at the given point, and a straight line drawn between any two points on the line of loads will represent in magnitude and direction the resultant load

between the corresponding points on the cord. The supporting pulls or forces at the points R and R_1 are represented by the extreme radii OR and OR_1.

A loaded cord is stable, but permits of oscillation.

618. If the forces are parallel and vertical, then the line of loads becomes the straight line RR_1 in Fig. 263, and A and B fall on the points A' and B' of this line; CP becomes $C'P$, and vertical, in Fig. 262. If, then, A be taken as the lowest point of the cord, so that AP is horizontal, the following will be the algebraic expression of this condition:

Let $H = OA'$ = horizontal pull or tension on the cord at A;
" $P = OB'$ = pull or tension on the cord at B;
" $W = A'B'$ = resultant load on portion of the cord AB;
" α = angle XPB (Fig. 262) = angle AOB (Fig. 263) = the inclination of the cord at B.

Then

$$W = H \tan \alpha; \quad P = \sqrt{W^2 + H^2} = H \sec \alpha. \quad . \quad (368)$$

Taking the origin at A, Fig. 262, and the co-ordinates of B, $AX = x$, $XB = y$, then

$$\tan \alpha = \frac{dy}{dx} = \frac{W}{H}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (369)$$

a differential equation, from which the form assumed by the cord can be determined when the distribution of the load is known.

619. In the preceding case the distribution of the load was not given. If, however, the load is uniformly distributed along a straight line, so that if A, Fig. 262, is the lowest point of the cord, the load suspended between A and any point B is proportional to the horizontal distance between them, $AX = x$, and the total load between those points is $W = wx$, w being the intensity of the load, i.e., so many pounds or tons per lineal foot. The general solution of the following problems would require that the points of support R and R_1 should be at different levels. For simplicity, and as corresponding with the usual construction, they will be considered at the same level.

Required to find the form of the curve $RABR_1$ and the relations between the load W and the tensions at A and B, and the coordinates $AX = x$ and $BX = y$.

Substituting the value of $W = wx$ in equation (369), we have

$$\frac{dy}{dx} = \frac{wx}{H}.$$

Hence, recollecting that if $x = 0$, that $y = 0$ also, and integrating,

$$y = \frac{wx^2}{2H}, \qquad \qquad (370)$$

which is the equation of a parabola whose focal distance or modulus is

$$m = \frac{x^2}{4y} = \frac{H}{2w}. \qquad \qquad (371)$$

For a parabola, the inclination α to the horizon is expressed by

$$\left.\begin{aligned} \tan\alpha &= \frac{dy}{dx} = \frac{x}{2m} = \frac{2y}{x}, \\ \text{and} \quad \sec\alpha &= \sqrt{1 + \frac{dy^2}{dx^2}} = \sqrt{1 + \frac{x^2}{4m^2}} = \sqrt{1 + \frac{4y^2}{x^2}}. \end{aligned}\right\} \qquad (372)$$

Then, in the triangle PBX, Fig. 262, we have

$$BX : PX : PB :: W : H : P :: \tan\alpha : 1 : \sec\alpha$$

$$:: \frac{2y}{x} : 1 : \sqrt{1 + \frac{4y^2}{x^2}} :: wx : \frac{wx^2}{2y} :: wx\sqrt{1 + \frac{x^2}{4y^2}}. \quad (373)$$

Since

$$W = BX = wx, \quad H = \tfrac{1}{2}AX = \tfrac{1}{2}wx \text{ cotan } \alpha = \frac{1}{2}\frac{wx^2}{y},$$

and

$$P = \sqrt{w^2 + H^2} = \sqrt{w^2x^2 + \frac{w^2x^4}{4y^2}} = wx\sqrt{1 + \frac{x^2}{4y^2}}.$$

These relations result from the uniform distribution of the load, which makes the resultant wx pass midway between A and X; and

equilibrium requires that the direction of the pull at B shall also bisect AX, which, being a property of the parabola, confirms the accuracy of equation (370).

Calling the length of the span $RR_1 = l$, and the co-ordinates of the points R and R_1, respectively, y_1 and x_1 and y_2 and x_2, we have, from the properties of a parabola,

$$x_1^2 = \frac{2Hy_1}{w} \quad \text{and} \quad x_2^2 = \frac{2Hy_2}{w};$$

$$\therefore\ x_1^2 : x_2^2 :: y_1 : y_2; \quad \text{and since} \quad y_1 = y_2, \quad x_1 = x_2 = \tfrac{1}{2}l.$$

Substituting in equation (371),

$$m = \frac{x^2}{4y} = \frac{l^2}{16y_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (374)$$

which gives the modulus m in terms of the length of the span l and the height of the points of support above the lowest point of the cord.

To determine the inclination of the cord at the points of support,

$$\tan \alpha_1 = \tan \alpha_2 = \frac{2y_1}{x_1} = \frac{4y_1}{l}, \quad . \quad . \quad . \quad . \quad (375)$$

after substituting for $x_1 = x_2 = \frac{1}{2}l = x$ in the first of equations (372).

Now, required the values of the horizontal tension H and the tensions R and R_1.

$$R = R_1 = H \sec \alpha = H\sqrt{\frac{1 + 4y_1^2}{x_1^2}} = H\sqrt{1 + \frac{16y_1^2}{l^2}}, \quad (376)$$

after substituting value of sec α from the second of equations (372). From equation (371), $H = 2wm$. Substituting value of m from eq. (374), we have

$$H = 2wm = \frac{2wl^2}{16y_1} = \frac{wl^2}{8y_1}. \quad . \quad . \quad . \quad . \quad . \quad (377)$$

The expression for the half length of the parabola $RABR_1$ or ABR_1 with $x_1 = \frac{1}{2}l$,

$$s = \sqrt{y_1^2 + \frac{x_1^2}{4}} + \frac{x_1}{4y_1} \,.\, \text{hyp log} \frac{y_1 + \sqrt{y_1^2 + \frac{x_1^2}{4}}}{\frac{1}{2}x}$$

$$= m\{\tan \alpha \sec \alpha + \text{hyp log} (\tan \alpha + \sec \alpha)\}. \quad . \quad . \quad (378)$$

The total length of the cord is $= 2s$.

The following approximate value of s is near enough for ordinary purposes:

$$s = x_1 + \frac{2y_1^2}{3x_1} = \frac{1}{2}l + \frac{4y_1^2}{3l_1} \quad \text{or} \quad 2s = l + \frac{8y_1^2}{3l} \text{ (nearly)}. \quad (379)$$

To determine approximately the elongation of the cord $d(2s)$ required to produce a given small depression dy of the lowest point A of the cord. We have, from equation (379),

$$2ds_1 = \frac{16y_1}{3l}dy. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (380)$$

Or, conversely, to determine the depression from the given elongation,

$$dy = \frac{6l}{16y_1} \,.\, 2ds_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (381)$$

whether the elongation is caused by heat or by tension.

The principles and formulæ established in the preceding paragraph are applicable to the suspension bridge, and are approximately accurate and correct, when the platform is suspended from the cables or chains by vertical rods or bars, and can be readily applied when the suspending rods are sloping by introducing the angle of slope—in this case the axis of the parabola is parallel to the direction of the suspending rods.

620. *Extrados and Intrados.*—When a cord is loaded with parallel vertical loads, as in Fig. 264, and ordinates are drawn proportional in lengths to the intensity of the load at any number of points along it, the straight or curved line joining the lower extremities of these ordinates is called the *extrados* of the load; the curve of the cord itself is called the *intrados*. In Fig. 263(*a*)

$RABR_1$ is the intrados and X_3OX_2 or X_1OX is the extrados, according as it is curved or straight. The load between any two points, as A and B, is evidently proportional to the plane area ABX_3O or $ABXO$, bounded by the intrados, extrados, and the extreme ordinates AO and BX_3 or BX. The algebraic processes of determining the equation of the intrados when the horizontal tension H and the equation of the extrados are given, and other similar problems, are of great intricacy, and will not be introduced in this volume. For this discussion the reader is referred to Rankine's Applied Mechanics, page 174, and following pages.

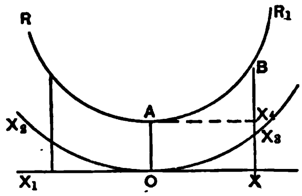


FIG. 263(*a*).

When the vertical load is of uniform intensity, as in paragraph 619, the intrados is a parabola and the extrados is obviously an equal and similar parabola situated at a uniform depth below the intrados.

621. *Catenary* is the name given to a curve in which a cord or chain of uniform material and sectional area hangs, when loaded, with its own weight alone.

If the horizontal tension be taken equal to the weight of a portion of the area between the intrados and extrados in Fig. 263(*a*), and this area is made equal to the square of a certain line whose length is m, then that line is called the *parameter* of the intrados.

If, then, w be the weight of a unit length of the cord or chain, W the total weight of any length s, then $W = ws$, and the horizontal tension $H = wm$.

The inclination α of the curve at any point B to a horizontal line is expressed by the equations

$$\cos \alpha = \frac{dx}{ds}; \quad \sin \alpha = \frac{dy}{ds}; \quad \tan \alpha = \frac{dy}{dx} = \frac{\sqrt{1 - \frac{dx^2}{ds^2}}}{\frac{dx}{ds}}. \qquad (382)$$

From these equations and from eq. (369) of paragraph 618 we deduce

$$\tan \alpha = \frac{W}{H} = \frac{ws}{wm} = \frac{s}{m} = \frac{dy}{dx}. \quad . \quad . \quad . \quad . \qquad (383)$$

By certain reductions not necessary to make in this volume we find the length of the arc

$$AB = s = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right), \quad . \quad . \quad . \quad . \quad . \quad (384)$$

and the ordinate y is found by integrating

$$\frac{dy}{dx} = \sqrt{\frac{ds^2}{dx^2} - 1} = \frac{s}{m} = \frac{1}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right);$$

hence

$$y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}} - 2\right) = \sqrt{s^2 + m^2} - m, \quad . \quad (385)$$

which is the equation of the catenary.

Since $H = wm$ and $W = ws$, then

$$P = w\sqrt{m^2 + s^2} = \frac{wm}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right) = w(y + m), \quad . \quad (386)$$

in which $H =$ the horizontal tension, i.e., the tension at the lowest point of the curve, $w =$ the intensity of the load or weight of unit lengths of s, $m =$ the parameter or length of arc whose weight is equal to the horizontal tension, $W =$ total length of weight s ($= AB$), e the base of the Napierian system of logarithms, $x = AX_4$, the abscissa of the point B, and $y = BX_4$, its ordinate.

From eq. (386) we see that the tension at any point B is P, equal to the weight of a portion of the cord whose length is the ordinate plus the parameter.

In this case the axis of x is taken tangent to the curve at its lowest point. If it is taken at a depth $AO = m$ below the vertex, and x' and y' be the new co-ordinates of B,

$$x = x' \text{ and } y' = y + m = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right) \text{ (from eq. (386))}, \ (387)$$

which shows that the intrados for a horizontal extrados becomes identical with a catenary having the same parameter when the least ordinate AO is equal to the parameter. The following are some of the geometrical properties of the catenary:

(1) The radius of curvature at the vertex is equal to the parameter, and at any other point is $v = m$, see[2] α.

(2) The length of the normal at any point, measured from the point to a horizontal line at the depth of the parameter m below the vertex, is equal to the radius of curvature at that point.

(3) If a parabola be rolled on a straight line, the focus of the parabola traces a catenary whose parameter is equal to the focal distance of the parabola.

622. *Linear Arches or Ribs.*—If we consider a cord or chain capable of bearing a thrust or compressive stress, and invert the cord by revolving the curve in Fig. 264 through 180° of arc, while not changing the direction, amount, or distribution of the loads, except that they act inwards on the cord, i.e., towards its centre of curvature, instead of outwards, then it becomes what Mr. Rankine calls a *linear arch* or *equilibrated rib*, on which at each point acts a thrust or compressive stress, exactly equal to the pull or tension as determined in the preceding paragraphs.

Linear arches do not exist, but all the propositions and equations respecting cords or chains are applicable to the lines of resistance of real arches and arched ribs when a thrust is substituted for a pull, and the direction of the thrust at each joint is tangent to the line of resistance, or curve connecting the centres of pressure at the joint.

The principles of paragraph 619 are applicable to linear arches under parallel loads, and the quantity H is a constant thrust in direction perpendicular to that of the loads. The form, therefore, of eduilibrium for a linear arch, under a uniform load, is a *parabola* similar to that described.

623. In the case of a linear arch under a vertical load, the intrados is the curve of the arch itself; the extrados is the line drawn through the upper extremities of the ordinates, whose lengths represent the intensities of the load, as would be shown by inverting the entire figure (see Fig. 264(a)). And from paragraph 621 it appears that the figures of all such arches may be deduced from that of the catenary by inverting and altering its horizontal and vertical co-ordinates in given constant proportions for each case.

The following principles, though applicable to cords or chains, have their principal value in the applications of them to linear arches or ribs.

624. *Circular Arch.*—If we assume a hollow cylinder of any

material and of a length equal to unity to be acted upon at all points of its exterior surface by a pressure of uniform intensity and normal to the surface, as indicated in Fig. 264(a), we readily find the following equations and principles:

Let p denote the intensity of the external pressure, in units of force per unit of area; r the radius of the ring, unity in thickness; T the thrust exerted around the ring, or per unit of length: then $p_x = p_y = p$. Obviously, as the pressure is of equal intensity all around, the form of the ring should be similar to itself all around;

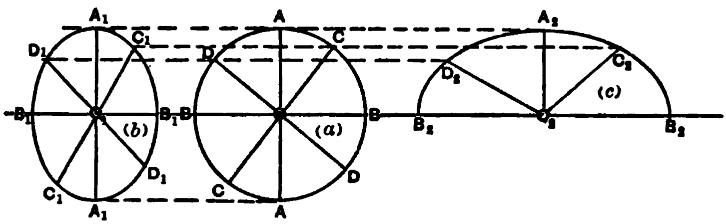


FIG. 264.

in other words, a section of the cylinder in the plane of pressures should be a circle. If, then, we cut the ring by any diametrical plane BB or CC, since the total pressures on the two halves are balanced by the thrusts at B and B or C and C, each equal to T or $2T$ at both points, we can, so far as equilibrium is concerned, remove one half of the ring, provided we substitute a pressure equal to T at the points B and B or C and C; and since the pressure is similar to a fluid pressure, both the horizontal and vertical intensities are equal to the normal intensity p. Since this intensity acts normal to the diameter BB or CC, then the total pressure $p \times 2r$ must be equal to $2T$. Hence

$$2T = 2pr \quad \text{and} \quad T = pr, \quad . \quad . \quad . \quad . \quad . \quad (388)$$

which means that the thrust on a circular ring under a uniform normal pressure is the product of the pressure on a unit of circumference by the radius.

625. If, then, we consider the condition of a quadrant AB, the thrust in a horizontal direction at A is equal to that in a vertical direction at B, and the total vertical and horizontal pressures exerted on the quadrant are equal: $V = H = T = pr$.

This principle and the resulting equations apply as well to thin hollow cylinders, boilers, pipes, etc., under a uniform intensity of pressure exerted by a fluid confined within, r being the radius

of the internal cylindrical surface, and T being a tension or pull instead of a thrust.

In either case the uniform distribution of the thrust or pull is only true when the thickness of the ring is small as compared with the radius.

626. In the preceding paragraph it is to be noted that every pair of conjugate pressures were not only uniform in amount and direction, but also equal to each other; which is practically true when the external pressures are great and the diameter of the arch or ring is relatively small, as would be the case with a cylindrical ring immersed to a great depth in water.

If while the pressures are uniform in amount and direction, relatively to two planes or diameters conjugate to each other, but these pressures are not equal to each other as in the preceding paragraph, then all the forces or pressures, acting parallel to any given direction, will be altered from those which act in a fluid mass by a given constant ratio, so that they may be represented by parallel projections of the lines which represent the forces that act in a fluid mass. Hence the figure of a linear arch, which sustains such a system of forces or pressures, must be the parallel projections of a circle, that is, it must be an ellipse. The constructions in Fig. 264 show clearly the meaning of a parallel projection and the method of constructing them. Fig. 264(b) shows an ellipse which is the parallel projection of a circle; figure (a) when the vertical dimensions remain the same and the horizontal dimensions are shortened or contracted; and figure (c) when they are lengthened or expanded in a given constant ratio, denoted by γ.

Then will r be the vertical and γr the horizontal semi-axis of the ellipse; and if x and y are the vertical and horizontal co-ordinates of any point in the circle, respectively, and x' and y' those of the corresponding point in the ellipse, then $x' = x$, and $y' = \gamma y$.

If CC and DD be any pair of diameters of the circle at right angles to each other, their projections, C_1C_1 and D_1D_1, will be a pair of conjugate diameters of the ellipse.

Let P_x be the total vertical pressure and P_y the total horizontal pressure on one quadrant, AB, of the circle. Then

$$P_x = P_y = T = pr.$$

If, then, P_x' and P_y' be the total vertical and horizontal

pressures, respectively, on one quadrant of the ellipse, as A_1B_1 and A_2B_2, and T_x' be the vertical thrust on the rib at B_1 or B_2, and T_y' the horizontal thrust at A_1 or A_2, by the principle of transformation

$$T_x'=P_x'=P_x=T=pr; \quad T_y' = P_y'=\gamma P_y=\gamma T=\gamma pr; \qquad (389)$$

which means that *the total thrusts are as the axes to which they are parallel.*

Again, let $P' = T'$ be the total pressure parallel to any semi-diameter of the ellipse, O_1D_1 or O_2D_2, on the quadrant D_1C_1 or D_2C_2, which force is also the thrust of the rib at C_1 or C_2, the extremity of the diameter conjugate to O_1D_1 or O_2D_2: making O_1D_1 or $O_2D_2 = r_1$, then

$$P' = T' = \frac{r_1}{r}P = pr_1; \quad . \quad . \quad . \quad . \quad . \qquad (390)$$

or, the total thrusts are as the diameters to which they are parallel.

Since the intensities are equal to the total pressures divided by the areas over which they are distributed, these areas being, respectively, for the conjugate, horizontal, and vertical pressures on the elliptical arch of unity in length, cr and r_1, then the intensities p_x' and p_y' are as follows:

$$p_x' = \frac{P_x'}{\gamma r} = \frac{p}{\gamma}; \quad p_y' = \frac{P_y'}{r} = \gamma p. \quad . \quad . \quad . \qquad (391)$$

If r' and r are the axes or horizontal and vertical semi-diameters of the ellipse, respectively, then $\gamma = \frac{r'}{r}$, and equations (391) become $p_x' = \frac{rp}{r'}$ and $p_y' = \frac{pr'}{r}$. Hence

$$\frac{p_x'}{p_y'} = \frac{rp}{r'} \div \frac{pr'}{r} = \frac{r^2}{r'^2}; \quad . \quad . \quad . \quad . \quad . \qquad (392)$$

which means that *the intensities of the principal pressure are to each other as the squares of the axes of the elliptic arch to which they are parallel.*

Then, to adapt an elliptic arch to uniform vertical and horizontal pressures, the ratio of the axes of the arch must be the square

root of the ratio of the intensities of the principal pressures. Since

$$\gamma = \frac{r'}{r}, \quad \gamma^2 = \frac{r'^2}{r^2} = \frac{p_y'}{p_x'}; \quad \therefore \gamma = \sqrt{\frac{p_y'}{p_x'}}. \quad . \quad . \quad (393)$$

And, similarly, we could prove that the intensity of the pressure in the direction of a given diameter is directly as that diameter and inversely as the conjugate diameter; or, that the intensities of a pair of conjugate pressures are to each other as the squares of the conjugate diameters of the elliptic rib to which they are respectively parallel.

The elliptic arch is obviously the form for a tunnel lining, since without material error we may consider the pressures as not only conjugate, but as principal stresses, respectively vertical and horizontal, and each as uniform in its own direction; and, as a practical fact, tunnels are usually a portion of an ellipse.

627. It is evident that if the pressure be normal at every point of the arch the thrust must be constant at every point, since it can only vary by the application of a tangential pressure on the arch; and the radius of curvature is inversely as the pressure, or

$$T = p\rho = \text{a constant}; \quad . \quad . \quad . \quad . \quad . \quad (394)$$

which is expressive of the condition of a circular arch under a uniform normal pressure.

The radii of curvature of the ellipse are expressed by the following equations, using same symbols as before. At the points A_1 or A_2

$$\left.\begin{aligned} \rho_y &= \frac{\gamma^2 r^2}{r} = \gamma^2 r; \\ \text{and at } B_1 \text{ or } B_2 \qquad \rho_x &= \frac{r^2}{\gamma r} = \frac{r}{\gamma}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (395)$$

Equation (394) is used in connection with determining the thrust around a curved dam.

The circular and elliptic arch, or some modified form of the latter, such as the basket-handle arch, which is formed by several circular portions of different radii which can be made to approximate very closely to the actual ellipse, are the only forms in which arches are usually constructed.

628. The hydrostatic arch is formed so as to be in equilibrium under a normal pressure, but not of uniform intensity, as in a fluid the intensity of the pressure varies as the depth of the point of the arch at which the pressure is exerted below the water surface.

The geostatic arch is the true form of an arch acted upon by the pressure of a mass of earth, in which not only are the conjugate pressures unequal in intensity, but they are also inclined to each other in direction.

The mathematical discussion of both of these subjects is intricate and difficult. The reader can find these subjects fully discussed in Rankine's Applied Mechanics.

629. A few of the more simple and important relations and equations will be of service for a clearer understanding of the theory of arches and tunnels.

From equation (394) it is seen that under the action of a normal pressure of equal intensity the total thrust on the ring or arch is constant for all points, and, likewise, that the radius of curvature, which varies directly as the thrust and inversely as the pressure, is also constant; i.e., the curve of the arch is a circumference of a circle.

The Hydrostatic Arch.—If, while the direction of the pressure is normal to the ring at every point, the intensity of the pressure varies with the depth of the point below the surface, and is equal to the intensity of the vertical pressure at that same point, we have a condition of actual fluid or water pressure on the ring.

The linear arch suited to this condition of pressure is called the hydrostatic arch. Then, from equation (394), the radius of curvature at any point is

$$\rho = \frac{T}{p}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (396)$$

and varies inversely as the intensity p of the pressure, it will also vary inversely as the depth below the water surface.

If, in Fig. 265(a), YOY_1 represents the surface of the water; BAB a linear arch in equilibrium under the pressure of water; x_0 the least depth, or depth of the crown $A = OA$ below the surface; r_0 = radius of curvature at the crown; $OY_1 = XC = y$, and $OX = Y_1C = x$, the vertical and horizontal co-ordinates of any point C on the arch, the origin O being taken at the intersection of a vertical line (drawn upwards from the crown A) and the surface of the water YOY_1, the length of the arch perpendicular to the

plane XOY being unity; the radius of curvature at any point C of the ring $= r$; $w =$ the weight of a unit of volume (1 cubic foot) of the liquid or water, the pressure of which is equivalent to the load on the arch: Then the intensities of the external normal pressure on the arch are equal to the weight of a unit of volume of the liquid by its depth below the surface.

Hence at A the intensity of the normal pressure is $p_0 = wx_0$,
and at any point C the intensity $p = wx$.

Since the intensity of the pressure varies directly as the depth, while the radius of curvature varies inversely as the depth, the thrust of the arch is a constant quantity,

$$T = p_0 r_0 = w x_0 r_0 = pr = wxr, \left(\text{since } \frac{r_0}{r} = \frac{x}{x_0} \text{ and } \frac{p_0}{p} = \frac{x_0}{x}\right);$$

from which follows the relation

$$xr = x_0 r_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (397)$$

The property of having the radius of curvature inversely proportional to the depth of the point below the water service enables the curve to be drawn by joining a series of short circular arcs. The shorter the arcs the closer the approximation to the true curve.

At some point B or B_1, Fig. 265(a), the curve becomes vertical,

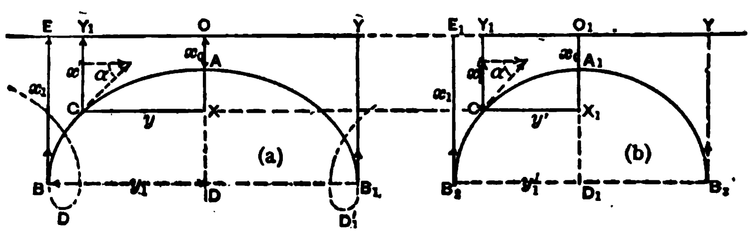


Fig. 265

but continues to curve downward to some point D or D_1, where it reaches its greatest depth and has its least radius of curvature. From these points it curves upward and upward, and if continued would form a series of similar curves. Obviously the only portion of the curve or arch suitable for supporting a load is from B around through A to B_1, the curve being vertical at the points B and B_1

and horizontal at A, its highest point or crown, where its radius of curvature is greatest.

630. Considering, then, the portion of the curve BAB_1 in Fig. 265(a), it is evident that the entire vertical pressure on the semi-arch ACB must be sustained by the thrust T of the arch at B; and the entire horizontal pressure by an eqnal horizontal thrust T at A. The vertical intensity of the pressure at any point C, at a depth x below the surface, on an infinitely small horizontal area dy is $wxdy$, and the total vertical load above the semi-arch ACB will be

$$W_1 = \int_0^{y_1} wxdy$$

between the points A and B, whose co-ordinates are, respectively, x_0, y_0, and x_1, y_1. Hence

$$T = W_1 = wxr = wx_0 r_0 = w\int_0^{y_1} xdy;$$

$$\therefore xr = x_0 r_0 = \int_0^{y_1} xdy = \text{area } OACBE. \quad . \quad . \quad (398)$$

Or, in words, the area of the figure, bounded by the surface of the water Y_1OY, the curve of the semi-arch ACB, and the vertical ordinates at its extremities, namely, the shortest and longest ordinates, respectively OA and EB, is equal to the constant product of any vertical ordinate of the curve by the radius of curvature at its extremity, or $xr = x_0 r_0 = x_1 r_1$.

631. Similarly, the vertical pressure on any portion of the arch, as from A to C, is

$$W = w\int_0^{y} xdy,$$

which is supported by the vertical component of the thrust on the arch ring at the point C, Fig. 265(a), which is $T \sin \alpha$; α being the angle of inclination of the linear arch at the point C to the horizon. Hence

$$T \sin \alpha = W = w\int_0^{y} xdy = wxr \sin \alpha = wx_0 r_0 \sin \alpha = \frac{wx_0 r_0}{\sqrt{1 + \frac{dy^2}{dx^2}}};$$

$$\therefore xr \sin \alpha = x_0 r_0 \sin \alpha = \int_0^{y} xdy. \quad . \quad . \quad . \quad (399)$$

Or, in words, the area between the shortest vertical ordinate $OA = x_0$ and the other vertical ordinate $x = Y_1C$, the curve AC and the surface Y_1OY, i.e., the area $OACY_1$, is the continued product of any vertical ordinate by the radius of curvature and the sine of the angle of inclination of the linear arch at the point whose vertical ordinate is x.

632. The horizontal pressure on the semi-arch ABC is equal to that exerted by a fluid on the vertical plane AD submerged in it, its upper edge being at a distance $OA = x_0$ below the surface of the water and its lower edge at a distance $OD = x_1$ below the same surface. Since this is a rectangular surface whose length is unity and height $x_1 - x_0$, its area is $= x_1 - x_0$, and its centre of gravity is at a point $x_0 + \frac{1}{2}(x_1 - x_0) = \frac{x_1 + x_0}{2}$ below the water surface; hence, from the principle of paragraph 588, Art. XLIII, the total pressure on this surface

$$= w\frac{x_1^2 - x_0^2}{2} = w\int_{x_1}^{x_0} p dx. \quad . \quad . \quad . \quad . \quad . \quad (400)$$

This pressure is balanced by the thrust (horizontal) T_0 at the crown A; hence

$$T_0 = wxr = wx_0r_0 = w\frac{x_1^2 - x_0^2}{2} \quad \text{or} \quad xr = x_0r_0 = \frac{x_1^2 - x_0^2}{2};$$

hence

$$x_1 = \sqrt{x_0^2 + 2x_0r_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (401)$$

From this equation we find, when the depth of the crown below the water surface and the radius of curvature at the crown are known, the depth of the point $B = x_1$ below the surface; that is, the point where the curve becomes vertical can be determined.

Similarly, the horizontal pressure on the portion of arch between C and B is equal to the pressure of the water on a portion of the plane XD, whose area is $x_1 - x$, and whose centre of gravity below the surface is $\frac{x_1 + x}{2}$; hence total pressure

$$= w\frac{x_1^2 - x^2}{2}.$$

This must be balanced by the horizontal component of the thrust of the arch at $C = T \cos \alpha$. Hence

$$T \cos \alpha = wxr \cos \alpha = wx_0 r_0 \cos \alpha = w\frac{x_1^2 - x^2}{2}. \qquad (402)$$

Hence

$$x_0 r_0 \cos \alpha = \frac{x_1^2 - x^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad (403)$$

which gives, for the value of any vertical ordinate x,

$$x = \sqrt{x_1^2 - 2x_0 r_0 \cos \alpha} = \sqrt{x_0^2 + 2x_0 r_0(1 - \cos \alpha)}$$
$$= \sqrt{x_0^2 + 4x_0 r_0 \sin^2 \frac{\alpha}{2}}, \quad . \quad . \quad . \quad (404)$$

after substituting the value of x_1 from eq. (401), and recollecting that $1 - \cos \alpha = 2 \sin^2 \frac{1}{2}\alpha$.

From equation (403) we can write for any ordinate x',

$$\frac{x_1^2 - x'^2}{2} = x_0 r_0 \cos \alpha';$$

and combining $\frac{x_1^2 - x^2}{2} = x_0 r_0 \cos \alpha$ by subtraction, we find

$$x'^2 - x^2 = 2x_0 r_0(\cos \alpha - \cos \alpha');. \quad . \quad . \quad . \quad (405)$$

or the difference between the squares of any two vertical ordinates varies as the difference of the cosines of the respective inclinations of the arc at their lower ends.

From equation (405), since at the crown the cosine of the inclination is unity,

$$1 - \cos \alpha = 2 \sin^2 \tfrac{1}{2}\alpha = 1 - \frac{1}{\sqrt{1 + \frac{dx^2}{dy^2}}} = \frac{x^2 - x_0^2}{2x_0 r_0}. \quad (406)$$

The several properties of the hydrostatic arch thus determined can be expressed by one formula,

$$x_0 r_0 = xr = \int_0^{y_1} xdy = \frac{\int_0^y xdy}{\sin \alpha} = \frac{x_1^2 - x_0^2}{2} = \frac{x_1^2 - x^2}{2 \cos \alpha}$$
$$= \frac{x'^2 - x^2}{2(\cos \alpha - \cos \alpha')}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (407)$$

These different properties can be expressed in words, by simply reading the meaning of the different factors in each term, all of which are equal to the depth of any point below the liquid surface multiplied by the radius of curvature at that point $= xr$.

632a. *Geostatic Arch.*—A linear arch which is suited to sustain a pressure similar to that of earth is called a geostatic arch, as was fully explained in Art. XLII, in which the conditions of pressure in a mass of earth in equilibrium were shown to be that of two conjugate pressures in a given vertical plane, the intensity of the vertical pressure being simply proportional to the weight of the material per unit of volume and to the depth of the conjugate plane (which is parallel to the surface of the earth-mass) below the surface, and acting on a unit of area of the conjugate plane, whether inclined or horizontal. The intensity of the conjugate pressure, acting parallel to the surface, and on a unit area of the vertical plane, bears a certain constant ratio to the vertical intensity, but not equal to it, i.e., $\frac{p_y'}{p_x'} = \gamma^2$, a constant. (See equation (393).)

If the slope of the earth mass has an inclination θ to horizon, then the intensity of the vertical pressure, at a given depth x', will be

$$p_x' = w'x' \cos \theta; \quad . \quad . \quad . \quad . \quad . \quad . \quad (408)$$

w' being the weight of a unit of volume of the earth.

The conjugate pressure, then, from equation (393),

$$= p_y' = \gamma^2 w'x' \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (409)$$

Let us suppose a hydrostatic arch, whose vertical and horizontal co-ordinates are x and y, to be subjected to the pressure of a material whose weight w per cubic foot is $w = w' \cos \theta$. Then, at γ given point in this arch, whose depth is $x = x'$ below the surface, we have, from equations (408), (409),

$$p_x = p_y = wx = \gamma w'x' \cos \theta = \gamma p_x' = \frac{p_y'}{\gamma}. \quad . \quad . \quad (410)$$

If, as in Fig. 265(*b*), we transform by parallel projection the hydrostatic arch (*a*), so that while the vertical ordinates remain equal the conjugate ordinate of any point in (*b*) shall bear to the horizontal co-ordinate in (*a*) of the corresponding point the ratio γ, then

$$x = x' \text{ and } y' = \gamma y. \quad . \quad . \quad . \quad . \quad . \quad (411)$$

The total vertical and horizontal pressures in the hydrostatic arch between two given points will be, respectively,

$$P_x = \int p_x dy, \quad P_y = \int p_y dx; \quad . \quad . \quad . \quad . \quad (412)$$

and the total vertical and conjugate pressures on the arc between the two corresponding points in the transformed arch (b) will be

$$P_x' = \int p_x' dy', \quad P_y' = \int p_y' dx'. \quad . \quad . \quad . \quad (413)$$

Substituting in equations (412), (413) the values of the terms in equations (410), (411), namely,

$$p_x' = \frac{p_x}{\gamma}, \quad p_y' = \gamma p_y, \quad dx' = dx, \quad dy' = \gamma dy,$$

there results

$$P_x = \int p_x dy, \quad P_x' = \int \frac{p_x}{\gamma} \gamma dy = \int p_x dy; \quad \therefore P_x = P_x'. \quad (414)$$

$$P_y = \int p_y dx, \quad P_y' = \int \gamma p_y dx; \quad \therefore P_y' = \gamma Py. \quad . \quad . \quad . \quad (415)$$

The relations between the total pressures in the same direction are the same as between the co-ordinates, equation (411), of the two curves (a) and (b), Fig. 265.

Consequently the transformed arch is such a parallel projection of the original arch under forces represented by lines which are the coaresponding parallel projections of the lines representing the forces acting on the original arch; and therefore it is *in equilibrio.*

633. Then, since total vertical pressure in the two arches is the same, the total vertical thrust at the points B, B_1, B_2, and B_3 (Fig. 265), will be equal, and equal to the pressures

$$P_x = P_x' = T_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (416)$$

But since the total horizontal pressures are different, having the ratio $\frac{P_y'}{P_y} = \gamma$, the ratio of the horizontal thrusts in (b) to that in (a) must have the same ratio; or, calling the crown thrust in (b) at

root of the ratio of the intensities of the principal pressures. Since

$$\gamma = \frac{r'}{r}, \quad \gamma^2 = \frac{r'^2}{r^2} = \frac{p_y'}{p_x'}; \quad \therefore \gamma = \sqrt{\frac{p_y'}{p_x'}}. \quad . \quad . \quad (393)$$

And, similarly, we could prove that the intensity of the pressure in the direction of a given diameter is directly as that diameter and inversely as the conjugate diameter; or, that the intensities of a pair of conjugate pressures are to each other as the squares of the conjugate diameters of the elliptic rib to which they are respectively parallel.

The elliptic arch is obviously the form for a tunnel lining, since without material error we may consider the pressures as not only conjugate, but as principal stresses, respectively vertical and horizontal, and each as uniform in its own direction; and, as a practical fact, tunnels are usually a portion of an ellipse.

627. It is evident that if the pressure be normal at every point of the arch the thrust must be constant at every point, since it can only vary by the application of a tangential pressure on the arch; and the radius of curvature is inversely as the pressure, or

$$T = p\rho = \text{a constant}; \quad . \quad . \quad . \quad . \quad . \quad (394)$$

which is expressive of the condition of a circular arch under a uniform normal pressure.

The radii of curvature of the ellipse are expressed by the following equations, using same symbols as before. At the points A_1 or A_2

and at B_1 or B_2

$$\left.\begin{aligned} \rho_y &= \frac{\gamma^2 r^2}{r} = \gamma^2 r; \\ \rho_x &= \frac{r^2}{\gamma r} = \frac{r}{\gamma}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (395)$$

Equation (394) is used in connection with determining the thrust around a curved dam.

The circular and elliptic arch, or some modified form of the latter, such as the basket-handle arch, which is formed by several circular portions of different radii which can be made to approximate very closely to the actual ellipse, are the only forms in which arches are usually constructed.

628. The hydrostatic arch is formed so as to be in equilibrium under a normal pressure, but not of uniform intensity, as in a fluid the intensity of the pressure varies as the depth of the point of the arch at which the pressure is exerted below the water surface.

The geostatic arch is the true form of an arch acted upon by the pressure of a mass of earth, in which not only are the conjugate pressures unequal in intensity, but they are also inclined to each other in direction.

The mathematical discussion of both of these subjects is intricate and difficult. The reader can find these subjects fully discussed in Rankine's Applied Mechanics.

629. A few of the more simple and important relations and equations will be of service for a clearer understanding of the theory of arches and tunnels.

From equation (394) it is seen that under the action of a normal pressure of equal intensity the total thrust on the ring or arch is constant for all points, and, likewise, that the radius of curvature, which varies directly as the thrust and inversely as the pressure, is also constant; i.e., the curve of the arch is a circumference of a circle.

The Hydrostatic Arch.—If, while the direction of the pressure is normal to the ring at every point, the intensity of the pressure varies with the depth of the point below the surface, and is equal to the intensity of the vertical pressure at that same point, we have a condition of actual fluid or water pressure on the ring.

The linear arch suited to this condition of pressure is called the hydrostatic arch. Then, from equation (394), the radius of curvature at any point is

$$\rho = \frac{T}{p}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (396)$$

and varies inversely as the intensity p of the pressure, it will also vary inversely as the depth below the water surface.

If, in Fig. 265(a), YOY_1 represents the surface of the water; BAB a linear arch in equilibrium under the pressure of water; x_0 the least depth, or depth of the crown $A = OA$ below the surface; r_0 = radius of curvature at the crown; $OY_1 = XC = y$, and $OX = Y_1C = x$, the vertical and horizontal co-ordinates of any point C on the arch, the origin O being taken at the intersection of a vertical line (drawn upwards from the crown A) and the surface of the water YOY_1, the length of the arch perpendicular to the

at the crown, and the vertical thrust at the springing

$$= T_1 = P_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (422)$$

To pursue further a purely mathematical discussion of the theory of the arch would lead us into a labyrinth of higher analysis, requiring the use of elliptic functions, transcendental equations, and the like; the results of which, while doubtless true from a purely theoretical point of view, would not be of any practical value, since ideal or linear arches have no real existence, and the necessary modification of the actual conditions of loading, supporting, and constructing a real arch would render the data and assumptions used so uncertain that the results would be unreliable, and the large number of difficult equations to solve and relations to satisfy would impose an amount of labor almost prohibitive. For such complete discussion see Rankine's Applied Mechanics.

636. The preceding discussions, however, lead to the following general conclusions:

(1) That if an arch is subjected to the action of a vertical load uniformly distributed along a horizontal line the curve of the arch should be that of a parabola (see paragraph 619). This never occurs with masonry arches.

(2) If the load is uniformly distributed along the curve, then the curve of the arch may be that of the common catenary (see paragraph 621). In this case the depth of the load at the crown is fixed by the equation of the curve, being equal to the modulus m. The condition of the vertical load in this can be represented by a mass of some substance, such as masonry, having a uniform specific gravity, whose bounding surfaces are respectively the curve of the arch, the least and greatest vertical depths of the curve from a horizontal surface, and the horizontal surface itself. The length of the arch being unity, and the least depth the modulus m, the intensity of the load at any point is proportional to the depth at that point.

By transforming the curve so that, while the horizontal abscissa of each point remains the same as in the original curve, the vertical ordinates representing the intensity of the loading at each point are changed in a constant ratio, we can alter at will the depth of the highest point of the arch below the surface; in other words, as m is the depth equal to OA of the inverted catenary or arch below the horizontal extrados (see Fig. 264), and we desire to change that to a depth $= y_0$, one half, one third, or two or three

times m, it is only necessary to alter the vertical ordinates of each point in the same ratio, and the resulting curve will be the transformed catenary. The result of this is to alter all vertical forces in the same ratio, but all horizontal forces remain the same, both in the common and the transformed curve. It is suited for the support of any load whose pressure is wholly vertical, and whose extrados is either a horizontal plane or another transformed catenary.

The common or true catenary differs but little in the portions near the vertex from a parabola, which for many practical purposes may be used for it. The transformed catenary may be made to approach very near to the curve of a circular arc.

(3) If the load is of uniform intensity, and normal at every point, the curve of equilibrium is a circle. A hollow vertical cylinder immersed in water is an example of this case, or a horizontal semi-arch immersed at a great depth is approximately under this condition.

(4) The hydrostatic arch is useful in practice from the fact that every arch, after completion and the removal of direct support, such as is provided by the centring, spreads, or tends to spread, at the haunches, thereby causing a horizontal pressure against the backing or spandrel filling, the reaction of which will keep the arch in equilibrium by causing the intensity of the horizontal and vertical pressure at each point to be equal, which is the condition of equilibrium of the hydrostatic arch, the thrust being uniform through this arch.

(5) The geostatic or elliptic arches are to be used where the intensity of the conjugate pressure is either greater or less than the vertical intensity, and in which it may be either inclined or horizontal.

Many semi-elliptic arches may be treated as if they were hydrostatic arches; others correspond more nearly to the geostatic arch.

ART. XLV.

CONSTRUCTION OF MASONRY ARCHES.

637. THE effort will now be made to apply to the actual construction of arches the preceding principles, and to explain certain other conditions that follow from those preceding; also the

manner in which these must be introduced or applied in order that the equilibrium of the arch may be secured.

Whatever may be the form of the curve assumed for the intrados of the arch, the direction and condition of the loading, the magnitude and directions of the resulting thrusts at the crown, springing and any other point between the two, *there are certain general principles of equilibrium that must exist; and where the primary and essential conditions of equilibrium between the forces, internal and external, acting upon the arch do not exist* (whether the external forces or pressures arise from the natural conditions in which the arch is placed and surrounded, or as a result of applying known or predetermined loads or pressures at one or more points of the arch), *then in such cases equilibrium must be established either by changing the form of the arch or by applying forces, pressures, or tensions, of proper magnitudes, directions, and points of application.*

With the ideal linear arch these secondary or applied forces necessary to produce equilibrium may or can be determined, by more or less difficult mathematical relations and equations, with a greater or less degree of accuracy. Their determination with respect to actual arches must of necessity be only roughly approximate, and as usual a good margin of safety must be provided. Here it may be safely stated that it is not good engineering practice, from any point of view, to construct arches with the minimum possible of depth or thickness of ring-stones.

638. Regardless for the present of the exact form of curve adopted for the intrados, arches may be divided into two types: (1) those in which the arch springs or rises vertically from its abutment; (2) those in which the arch springs obliquely, or in which a tangent line to the curve at the springing makes an acute angle with the horizon.

In either case the curve at the crown or in other words a tangent to the curve at the crown, is assumed to be horizontal. Where this is not the case the actual curve will be a parallel projection of the former, and the thrusts or applied forces will be altered in the same ratio in which the ordinates of the transformed curve have been altered, and it will not be necessary to consider further such cases.

639. In Fig. 266 is shown the type of arch in which it springs vertically from the abutments.

In Fig. 267 is shown the type in which it springs obliquely from the abutments.

Only the semi-arch is considered in each case, and the same definition of terms apply to both.

The first type may be either a full semicircular, elliptic, hydrostatic, or geostatic arch.

The second may be a portion or a segment of a circular, elliptical, or geostatic arch.

BDEF is the abutment; *KACBmLK*, the semi-arch ring; *ACB*, the soffit or intrados; *KLm* is sometimes called the extrados, but not in the sense used in paragraph 620; *KbaA* is one half of the keystone, which, together with *abed*, . . . *ghlk*, and *klmB*, are called voussoirs; *A* or *K* is the crown; *B* or *m*, the springing lines; *AO*, the rise; *BO*, the half span; and a certain distance on either side

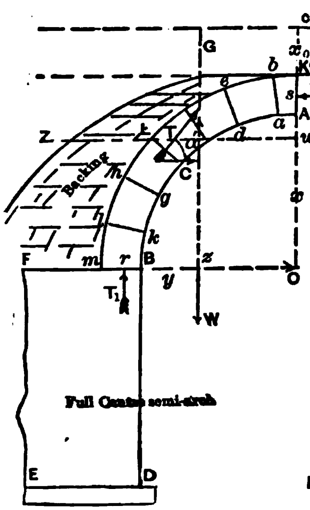


FIG. 266.

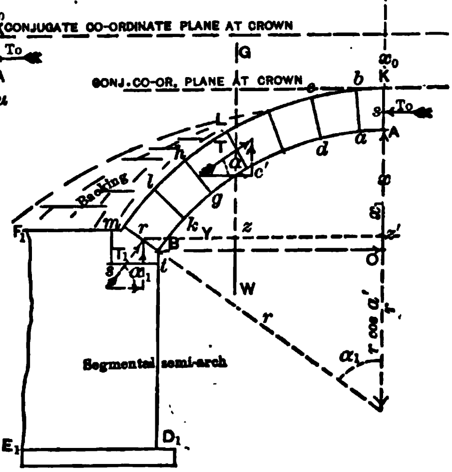


FIG. 267.

of some line *hg* in the two drawings are the haunches. In Fig. 266 $AO = BO =$ rise $=$ half-span $=$ radius of curvature of all points of the soffit, for a full centre or semicircular arch. But the radius varies at each point for an arch of any other curve. In Fig. 267 the rise, half span, and radius of curvature vary according to the curve of the soffit, and the length of the arc or segment of the arch. *mB* is called the skew-back when meaning the inclined surface from

which the arch springs; often it is applied to the block *Bmts* forming the capstone of the abutment upon which the arch rests. See Fig. 267.

The thrusts at the crown *A*, springing *B*, and any intermediate point *C* are indicated by the arrows, and called, respectively, T_0, T_1, and T; the angles which these make with the horizontal are represented by the letters α, α', etc. The horizontal components of these thrusts are $T \cos \alpha$, $T_1 \cos \alpha'$, etc. The vertical components are $T_1 \sin \alpha'_1$; $T \sin \alpha_1$, etc. The radial lines *ab*, *de*, *hg*, *lk*, and *mB* are the joints. The backing or spandrel filling is the rough masonry or concrete resting above the abutment and arch-ring. The spandrel-wall is built over and flush with the faces or ends of the arch to any given height; intermediate and parallel walls are also sometimes built. The radius of curvature at the crown is commonly the radius of curvature of the soffit, more properly, however, that of the linear arch; to this the curve of the soffit should approximately conform, which should be confined to the middle third of the arch-ring. The remark applies to any other point.

640. *Conditions of Stability.*—It is evident that arches can fail in either one of three ways, as is the case with all blockwork structures, as follows:

(1) By rotating of one portion of the arch on another around some axis. This axis may be either at some point as *g* on the intrados or *h* on the extrados, or, as is usually assumed, at some point on the joint between the intrados and the extrados.

(2) By sliding on some joint, as *gh*.

(3) By crushing of the material composing the arch-ring.

641. So long as the resultant pressure at any joint pierces the surface of the joint between the intrados and the extrados there could be no rotation either around an axis on the soffit or on the extrados if the material of which the arch-ring is composed was incompressible; but to provide for the compressibility of the material, it is necessary that the resultant should not pierce the surface of any joint too near its edges: and for perfect safety it is usual, in the construction of arches, to control the direction of the pressure, or to sufficiently increase the depth of the arch-ring that this point shall be in the middle third of the joint. When this is done and the magnitude of the resultant pressure does not produce an intensity of stress at any point exceeding a safe resistance of the material to crushing, the stability will be secured. The stability against sliding is assumed to be secured, for as a general

rule radial joints will prevent the angle between the resultant pressure and the normal to the joint exceeding the angle of repose, if not the direction of the joint can be so changed that this condition will be fulfilled.

642. *Joint of Rupture.*—That joint at which the tendency to open at the extrados is called the joint of rupture. This joint may be at the crown, or springing, or at any intermediate point. Mr. Rankine defines the joint of rupture as the point of the arch where the conjugate component of the thrust along it is a maximum. This definition will be explained and better understood later.

642a. *Conjugate Thrust.*—"The total conjugate thrust of an arch is the conjugate component, horizontal or inclined, as the case may be, of the entire pressure exerted between one semi-arch and its abutment, whether directly applied at the point from which the arch springs, or above that point, through the material of the spandrel.

(1) "When a linear arch is of such a figure as to be balanced under a load of which the pressure is wholly vertical, its conjugate thrust is exerted simply at the point from which it springs, and is equal to the conjugate component of the thrust along the arch, which is a constant quantity throughout its whole extent." This is the common theory of constant horizontal thrust, as discussed later.

(2) When an arch springs vertically from its abutment, the point of springing sustains the vertical load of the semi-arch only, and the conjugate thrust is exerted wholly through the spandrel.

(3) In other cases the conjugate thrust is exerted partly at the point of springing and partly through the spandrel. In the common theories of the arch, conjugate horizontal components of the thrust through the spandrel are not considered.

643. If, then, in Fig. 266, the crown A is horizontal and the intensity of the pressure at that point is vertical, we find, from paragraph 627, that the horizontal thrust along the arch at that point is $T_0 = p_0 r_0$, T_0 being the horizontal thrust, p_0 the intensity of the vertical loading at that point, and r_0 the radius of curvature. If x is the vertical and y the horizontal co-ordinate of any point C in the arch, then cotan $\alpha' = \frac{dy}{dx}$, and $\alpha' = \text{arc cot} \frac{dy}{dx}$ is the inclination of the arch at C to the horizon.

Let P_x = total load on the portion of the arch between the ori-

gin or crown A and the point C. If, then, at C we draw a vertical line $= P_x$, and a line T tangent to the curve, completing the triangle of forces, we find, for the value of T,

$$T = \frac{P_x}{\sin \alpha'} = P_x \operatorname{cosec} \alpha' = P_x \frac{ds}{dx}, \quad . \quad . \quad (423)$$

in which ds denotes the increment of the arc AC.

The horizontal component of T is

$$T \cos \alpha' = P \quad \operatorname{cotan} \alpha' = P_x \frac{dy}{dx}. \quad . \quad . \quad (424)$$

It is evident that for equilibrium T_0 must be equal to $T \cos \alpha'$. If it is not, then a horizontal pressure must be applied to the arch between A and C, which will make up the difference, in order to produce equilibrium. This difference must be applied so as to act inwardly or outwardly, according as $T_0 > T \cos \alpha'$ or $T_0 < T \cos \alpha'$, respectively. This difference is called the conjugate pressure necessary to producee quilibrium. Calling it P_y, we have

$$P_y = T_0 - T \cos \alpha' = T_0 - P_x \operatorname{cotan} \alpha' = T_0 - P_x \frac{dy}{dx}. \quad (425)$$

And if this equation be fulfilled for every point of the arch it will be stable or balanced.

644. If $T_0 > T \cos \alpha'$, then P_y is positive, and an inward pressure must be applied equal to the difference. If we find a joint between A and C where $P_y = 0$, i.e., $T_0 = T \cos \alpha$, no inward pressure need be applied between that point and the crown. But the value of P_y will be positive as we approach the springing line B, at which point $T \cos \alpha' = 0$ and $P_y = T_0$, and we must apply an inward conjugate pressure P_y equal to T_0, the crown thrust. This is provided for by building up a mass of rubble masonry or concrete on the top Fm of the abutment, which, by its resistance to compression, supplies the requisite conjugate pressure to produce equilibrium; or the form of the arch must be changed to a curve which would be in equilibrium under the loading assumed, and then the conjugate component of the thrust would be constant and equal to the thrust of the crown down to the springing, the conjugate thrust being only exerted at the springing.

645. If, on the contrary, $T \cos \alpha' > T_0$, then P_y in eq. (425) becomes negative, and an outward horizontal pressure, or, more conveniently, a tension must be applied in order to produce equilibrium. In Fig. 266 and from eq. (425), it is evident that P_y can only

become negative at some point not very far from the crown; and when reliance cannot be placed on the adhesion and tenacity of the mortar, iron cramps, bolts, or hoop-iron should be used to fasten together the spandrel masonry and the arch-stones. By such means stability can be given to arches whose curve does not correspond with the required conditions of loading. It is better to avoid these negative values of the intensity p_y of the total conjugate pressure P_y.

646. It can be shown mathematically (see Rankine's Applied Mechanics, pages 200, 201, and 202) that for every circular linear arch in which the depth of the loading above the crown is less than one third of the radius there will be negative values of the intensity p_y at and near the crown, showing that outward horizontal pressure, or tension, is required to preserve equilibrium.

In all such cases there will be some point or a certain value for the angle α' where $p_y = 0$. It is evident that at this point the conjugate tension P_y attains a negative maximum; and from eq. (425), for this maximum to be reached, the horizontal component of the thrust at this point, $T \cos \alpha'$, must attain a positive maximum greater than T_0. The exact determination of this point can only be made by satisfying a transcendental equation, and can therefore only be determined by approximation and successive substitutions.

It is this point, or where the angle α' satisfies the transcendental equation referred to above, where the intensity p_y is nothing, or where the conjugate tension $-P_y$ is a maximum. Below this point only is it necessary to provide a conjugate pressure from without by the construction of a solid backing, whether the pressure is exerted all the way down through the spandrel or only at the springing. The joint at which p_y is zero, or where the conjugate component of the thrust is a maximum, is called the joint or point of rupture.

By expanding or contracting the dimensions of a circular arch, it can be transformed into an elliptic arch, which will be in equilibrium under the forces applied to the circular arch in the preceding discussion.

All of the preceding principles and equations apply to the segmental arch, Fig. 267, as well as to Fig. 266.

647. It is evident, from the foregoing discussion, that an arch constructed as shown in Figs. 266 and 267 tends to fail by settling or sinking at the crown and spreading outward at the haunches, or that portion of the arch below the joint of rupture; and, with the

completed arch composed of its two halves supporting each other at the crown, the tendency is to break into four parts, the two upper segments, extending from the crown to the joint of rupture, falling inwards or downwards, opening at the intrados at the crown and on the extrados at the joint of rupture, and the two segments of the arch below the joint of rupture, revolving outwards, opening on the intrados at the springing.

This mode of failing is characteristic of all flat or segmental arches. In such arches, therefore, it is necessary to carry the backing up to or above the highest possible position of the joint of rupture, which is rarely found above that joint which makes an angle of 45° with the horizon. Above this point it is generally rounded off to the crown, as indicated in Figs. 266 and 267.

648. In pointed arches, the conditions are just reversed. The tendency is for the two upper segments to rise, opening on the extrados at the crown, on the intrados at the joints of rupture, and for the two lower segments to revolve inwards, opening on the extrados at the springing. Therefore a sufficient weight must be placed above the crown, or the lower segments must be tied to a spandrel backing.

649. The exact determination of the position of the joint of rupture is difficult, and only possible by a succession of approximations in the more general cases of arch construction; and the same remark applies to the determination of the exact distribution of the horizontal conjugate pressure P_y, that is, its intensity p_y at each and every point of the arch for any given condition of loading and form of arch adopted.

Mr. Rankine states that the joint of rupture in most examples of circular arches which occur in practice will be found between those joints which make angles of 35° and 45° with the horizon, and, consequently, if the backing is carried up to that joint which makes an angle of 45° with the horizon, its height will be sufficient to insure stability, provided that portion of the arch-ring between this point and the crown, which may be left for a time unloaded, shall be stable when under the action of its own weight, that is, the line of pressures must lie within the limits of the middle third of its thickness; and he gives the following approximate rule, if the thickness of the arch has been determined upon finally, as follows:

Make the depth of the lowest point of the extrados of the unloaded portion of the arc below its highest point a mean proportion between the thickness of the arch-ring and the radius of the intrados.

If x is this depth $= KU$, Fig. 266, assuming the backing as not extending above L, AK the thickness of the arch-ring $= t'$, r' the radius of the extrados OK, and r the radius of the intrados AO, then

$$t' = r' - r; \quad x = \sqrt{t'r} = \sqrt{r'r - r^2}. \quad . \quad . \quad . \quad (426)$$

650. In order to obtain a clear understanding of the application of the preceding principles and formulæ it will be necessary to

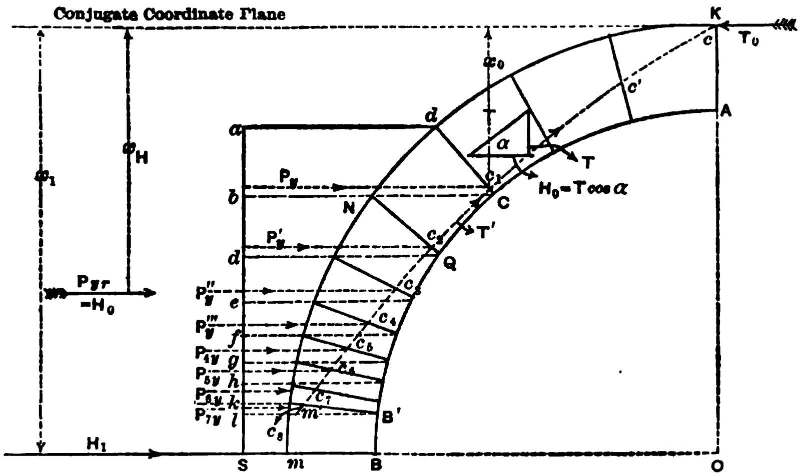


FIG. 268.

divide the arch-ring (Fig. 266) from the joint of rupture LC down to the springing line or joint mB into a series of voussoirs, so that a horizontal line touching any joint, as LC, at its intersection with the soffit will touch the next joint below, NQ, at its intersection with the extrados; and similarly for the other joints, as shown in Fig. 268. Then the sum of the vertical projections of the surfaces of the joints will form a continuous surface *as* of a length perpendicular to the plane of the paper of unity, or for convenience call it the line *as*.

At the joint of rupture dC, if the thrust T pierces this joint at one third of dC from C, the pressure on this joint at the outer edge d will be zero, and the joint will be on the point of opening at d if, as is usual, the adhesion and tenacity of the mortar are disregarded. The mean intensity of the pressure on the joint is $\frac{T}{dC}$, its least

value is zero at d, its maximum is $\frac{2T}{dC}$ at C. The horizontal component of T is $T \cos \alpha$. This is distributed over the vertical surface ab; its mean intensity is $\frac{T \cos \alpha}{ab}$, its least is zero at a, and its greatest is $\frac{2T \cos \alpha}{ab}$ at b.

If at this joint $T \cos \alpha < T_0$, it is evident that there will be an unbalanced horizontal force equal to $T_0 - T \cos \alpha$, and rotation outward of the portion of the arch-ring would take place around some axis. This must then be balanced by the application of a pressure acting from without to within. This pressure is $P_y = T_0 - T \cos \alpha$. This pressure must be distributed over ab in exactly the same manner that $T \cos \alpha$ was distributed, that is, total P_y must be directly opposed to $T \cos \alpha$; which means that p_y, the intensity of P_y on ab, must be zero at a, and have its maximum value at b, which shows that at the point of rupture d the intensity of the conjugate pressure necessary to produce equilibrium must be zero.

At the joint NQ there is a thrust T' and a horizontal component $T' \cos \alpha'$ of different amounts from those at dC. For while T' may be greater, $T' \cos \alpha'$ may be less than the corresponding pressures at dC; and, as seen from the position of the line of pressure $cc_1c_2c_3$, etc., c_2 is farther from Q than c_1 is from C. Therefore the intensity of the pressure at N is greater than zero; also, $T' \cos \alpha'$ which is distributed over bd in some manner, will have a sensible value at b greater than zero; but for this joint $P_y' = T_0 - T' \cos \alpha'$, which is the conjugate pressure necessary to balance the unbalanced force $T_0 - T' \cos \alpha'$, and must be distributed over bd in the same manner that $T' \cos \alpha'$ is distributed over bd. Here the intensity p_y' is greater than zero over the entire surface bd. In the same manner the thrust at the other joints below should be treated. We note here two conditions: (1) that for each joint below dC and including it there is an independent value for the conjugate pressure to be supplied from without, namely, P_y, P_y', P_y'', etc.; and (2) that the intensities of these pressures, namely, $\frac{P_y}{ab}, \frac{P_y'}{bd}, \frac{P_y''}{de}$ = respectively p_y, p_y', p_y'', are all positive, except for the least intensity $p_y = 0$ at the one point d.

651. The next question what is the aggregate of all of these pressures $P_y + P_y' + P_y''$, etc., that must be applied from d

down to m', or, in other words, what is the resultant conjugate pressure P_{yr} that must be applied from withont to maintain the arch *in equilibrio*. It is evident that, so far as equilibrium is concerned, we might consider that portion of the arch above the joint of rupture, from d to K, removed, and the horizontal component of the thrust at d balanced, and balanced alone, by the sum of the externally applied pressures $P_y + P_y' + P_y''$, etc., $= P_{yr}$, and this sum could only reach its maximum value where the intensity p_y is zero. Below this p_y is positive, and above it it becomes negative. Therefore the conclusion is self-evident that the aggregate of these conjugate pressures will have its maximum when distributed over the entire surface from the springing to the joint of rupture dC, at which point the horizontal component of the thrust H_0 must also be a maximum, and also the intensity of the conjugate pressure p_y is zero. If the horizontal component of the thrust is a maximum at any other joint, it would only be necessary to apply external pressures up to that joint. So then we are forced to the conclusion that the point of maximum horizontal component of the thrust in an arch-ring is the joint where the intensity of the externally applied force necessary to produce equilibrium $p_y = 0$, and that if the backing is built up to that joint, equilibrium and stability would be absolutely secured.

652. These relations can be expressed algebraically as follows:

$$P_{yr} = P_y + P_y' P_y'', \text{ etc.},$$
$$= H_0 = \max T \cos \alpha, \ = \max P_x \cot \alpha = \max P_x \frac{d_y}{d_x} \quad . \quad (427)$$

for the total amount of external pressure required to produce equilibrium, P_x being the vertical component of the thrust, or the total vertical load from the crown to that joint (the joint of rupture) where the horizontal component of the thrust is a maximum.

The intensity of the conjugate pressure is expressed by

$$p_y = \frac{dP_y}{d_x} = - \frac{d}{d_x}\left(P_x \frac{d_y}{d_x}\right),$$

which for the joint of rupture becomes

$$p_y = 0 = - \frac{d}{d_x}\left(P_x \frac{d_y}{d_x}\right). \quad . \quad . \quad . \quad (428)$$

This equation when solved locates the point of rupture, and the

corresponding value P_x can then be computed, which substituted in eq. (427) gives H_0; and where the conjugate pressures are horizontal, which is generally the case, the value of α obtained from $\cot \alpha = \frac{d_y}{d_x}$ which satisfies the eq. (428) is called the angle of rupture, being the angle which the tangent line at the joint of rupture makes with the horizon. When the joint of rupture is at the crown, the value of P_x is nothing, and $\cot \alpha = \cot 0 = \frac{d_y}{d_x} = \text{infinity}$. Eq. (427) gives no result, but in such cases we have seen that the maximum value of

$$H_0 = T_0 = p_0 r_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (429)$$

This is the case when we are considering the hydrostatic, geostatic, or circular arch under uniform normal pressure.

The preceding principles and relations seem, to the writer's mind, to be entirely satisfactory and conclusive in respect to that portion of the arch below the joint of rupture, the only difficulty being in the location of the joint of rupture, which can be accurately determined for the ideal linear arch, but will vary with the values given to W, x, and Y in the equation $H = \frac{W_y}{x}$, which will be discussed and applied in a subsequent paragraph.

653. Theoretically the same principles are applicable to that portion of the arch above the joint of rupture. As we have passed the point of maximum horizontal component H_0 of the thrust in the arch-ring, where the intensity $p_y = 0$ and above which it becomes negative, it is evident by this fact alone that we no longer need the application of an externally applied thrust, but that we require an externally applied pull or tension, as an outward thrust or pressure is impracticable unless supplied by some support such as the centring, which is temporary and must be removed. We must therefore either rely upon the adhesion of the mortar, or use iron rods or cramps between the masonry of the arch-ring and the spandrel-walls above, and only that portion of these walls above the line at which they become self-supporting is available for the purpose; and it is evident, except in small arches with relatively high spandrels, that the application of external tensile supports is impracticable, as the supporting mass must be itself reliably self-fixed and permanent.

654. The condition, then, of that portion of the arch above the joint of rupture is simply as follows: It rests on and is supported by the arch and its backing below the joint of rupture. This is assumed to be rigid; and even though the line of pressure passes nearer C than $\frac{1}{3}dC$, Fig. 268, thereby producing an actual tension or tendency to open at the edge d, the opening or rupture cannot take place by the outward rotation of the arch-ring below the joint dC: therefore the arch must be stable, no matter how near the line of pressure is to the inner edge C at the joint of rupture or the outer edge K at the crown, unless by compression or actual crushing, near C or K, the material of the arch-ring yields, and consequently the crown sinks, the joint dC opening at d and the joint KA at A on the intrados. The practical limitation to this increase of pressure near the edges K and C is that its intensity shall not exceed the safe unit resistance of the material of the arch-ring, which commonly is taken at one eighth to one tenth the ultimate strength; or, as more commonly practised, it is considered safe to so limit the centres of pressure at the crown and joint of rupture, as well as at intermediate joints, that the least pressure shall not be less than zero, and the greatest more than twice the mean pressure. This condition requires that the line of pressure shall not be nearer the extrados K at the crown than $\frac{1}{3}KA$, or nearer the intrados C at the joint of rupture than $\frac{1}{3}dC$; or, in other words, that the line of pressure between the crown and the joint of rupture shall be found within the middle third of the arch-ring. And if twice the mean intensity at any joint, which is the maximum pressure that can exist at any point in it, is less than a perfectly safe resistance of the material, then the arch will be stable. Mr. Rankine adopts this method in regard to the portion of the arch-ring above the joint of rupture, as preferable to any means of tying this portion of the arch up to the masonry above or obliquely above, while not abandoning the existence or activity of horizontal conjugate pressures. And if in any proposed arch we cannot construct the line of pressure entirely within the middle third of the arch-ring above the joint of rupture, the thickness of the arch-ring is increased until the line does lie in the middle third. Other theories will be explained later.

655. In this connection it is to be noted that when the arch springs vertically from the abutment (Fig. 266) the value of P_y, the conjugate thrust for the horizontal springing joint, is zero, and the resultant, $P_{yr} = P_y' + P_y' + P_y''$, etc., $= H_0$, is exerted entirely

through the spandrel backing above the springing. But when, as in segmental arches (Fig. 267), the springing joint is inclined, one of the conjugate pressures P_y is exerted through the abutment and the others through the spandrel above. In all other theories this value of P_y at the springing is the only horizontal thrust recognized, and is taken as equal to the assumed thrust at the crown, no horizontal conjugate thrust exerted by the backing above being considered, which involves the practical ignoring of the existence of a joint of rupture. It is true that in some cases, by a series of arithmetical calculations, the value of the maximum horizontal component of the thrust around the arch is determined, which of itself locates approximately the joint of rupture, and this maximum component is assumed as the least crown thrust consistent with stability, and when found is assumed to be constant throughout.

656. We have now seen the manner of determining the total amount of the externally applied pressure exerted by the masonry of the backing below the joint of rupture, including that component exerted through the abutment at the springing joint, and that this total, the resultant of which we have called P_{yr}, is equal to H_0, the maximum horizontal component of the thrust in the arch-ring. It will then be easy to find an equation for the value of P_y, if any exists, at the springing.

If, as before, p_y is the intensity of the conjugate pressure at any point x below the conjugate co-ordinate plane, that is, the plane of the surface, supposed to be horizontal, of the water, earth, or other material over the arch, then $p_y dx$ is the pressure on any elementary area; and if x_0 is the depth of the joint of rupture and x_1 of the springing below the same plane, then the total conjugate pressure which is exerted between the depths x_0 and x_1, not including the component $H_1 =$ the P_y at the springing, if any, will be $\int_{x_0}^{x_1} p_y dx$, which, added to H_1, is the total $P_{yr} = H_0$. Hence

$$H_1 + \int_{x_0}^{x_1} p_y dx = H_0, \quad \text{and} \quad H_1 = H_0 - \int_{x_0}^{x_1} p_y dx, \quad . \quad (430)$$

which gives that horizontal component, if any, at the springing.

Now we desire to find the point of application of the resultant $P_{yr} = H_0$. Taking moments of the three horizontal forces or pressures in equation (430), with respect to an axis in the conjugate

co-ordinate plane from which x_0 and x_1 are measured, and calling the distance of P_{yr} or its equal H_0 below this plane x_H, we have

$$H_1 x_1 = H_0 x_H - \int_{x_0}^{x_1} x p_y dx;$$

hence

$$H_0 x_H = H_1 x_1 + \int_{x_0}^{x_1} x p_y dx;$$

$$\therefore\ x_H = \frac{H_1 x_1 + \int_{x_0}^{x_1} x p_y dx}{H_0}. \quad . \quad . \quad . \quad . \quad . \quad (431)$$

It must be borne in mind that the component H_0 is applied at the joint of rupture, and is only used for P_{yr}, which is applied at x_H below the co-ordinate plane, for convenience and because the two are numerically equal. We now know the magnitude, the direction, and the point of application of the resultant of the conjugate pressure necessary to produce equilibrium below the joint of rupture. Therefore the pressure is completely determined.

657. In many cases the position of the joint of rupture, the intensity of the horizontal conjugate pressure, and the point of application of the total conjugate pressure can be determined readily when the form of the arch and the character of the external load are known. In other cases, however, the solution of the problem algebraically is difficult and intricate. For these cases the graphical methods are preferred. A few examples will further elucidate the foregoing principles and equations.

Example I: Circular Arch under Normal Pressure of Uniform Intensity p.—This is similar to fluid pressure in having the intensity equal in all directions and at all points on the arch. (See paragraphs 624, 625, 627, 636.)

Since the pressure is normal at every point there can be no tangential components, and consequently the thrust is the same at all points.

$$T_0 = T = H_0 = pr = p_0 r_0, \quad . \quad . \quad . \quad . \quad (432)$$

since both the radius of curvature and intensity are the same at all points. Also, $p_x = p_y = p$. The point of maximum horizontal component is evidently at the crown, since the crown thrust is equal to the total thrust in the arch-ring at any other point, and must therefore be greater than any component of that thrust. For

convenience, take the horizontal conjugate plane co-ordinate tangent to the crown of the arch: then x_0, the distance of the joint of rupture, which is at the crown, below that plane, is zero, i.e., $x_0=0$. There will be two cases.

Case 1. *Full-centre or Semicircular Arch.*—$x_0=0$; $x_1=r$; and since the arch springs vertically from the abutment, the conjugate component at the springing is zero. Hence

$$H_1 = 0; \quad P_{yr} = H_0 = T_0,$$

and equation (431) becomes

$$x_H = \int_{x_0=0}^{x_1=r} \frac{xp_y dx}{H_0} = \int_{x_0=0}^{x_1=r} \frac{xpdx}{pr} = \frac{r^2 p}{2pr} = \frac{r}{2}. \quad . \quad (433)$$

That is, the point of application of the resultant of the conjugate pressures $P_{yr} = H_0 = T_0$ is at a point half way between the crown and the springing, and is exerted entirely through the backing from crown to springing. (See Fig. 266.)

Case 2. *Segmental Arch* (see Fig. 267).—Inclination at the springing $= \alpha_1$; $x_0 = 0$; $x_1 = r - r \cos \alpha_1 = r(1 - \cos \alpha_1)$.

In this case H_1 has a value equal to

$$T_1 \cos \alpha_1 = pr \cos \alpha_1 = H_1; \quad H_0 = pr;$$

and substituting in equation (431),

$$x_H = \frac{H_1 x_1 + \int_{x_0}^{x_1} xp_y dx}{H_0},$$

it becomes

$$x_H = \frac{x_1 pr \cos \alpha_1}{pr} + \int_{y_0=0}^{x_1=r(1-\cos\alpha)} \frac{xpdx}{pr} = \frac{x_1 pr \cos \alpha_1 + \frac{1}{2}px_1^2}{pr}$$

$$= \frac{pr[r \cos \alpha_1(1 - \cos \alpha_1)}{pr} + \frac{pr[r(1 - \cos \alpha_1)^2]}{2pr}$$

$$= r[\cos \alpha_1(1 - \cos \alpha_1) + \tfrac{1}{2}(1 - \cos \alpha_1)^2] = \frac{r}{2} \sin^2 \alpha_1. \quad . \quad (434)$$

We then have the total conjugate pressure $P_{yr} = H_0 = T_0 = pr$; and its point of application vertically below the crown $= \frac{r}{2} \sin^2 \alpha_1$. The horizontal component of the conjugate pressure at the springing is $H_1 = H_0 \cos \alpha_1 = pr \cos \alpha_1$. The thrust in the arch-ring is constant throughout. Joint of rupture is found at the crown. In

this case a part of the conjugate pressure is exerted through the spandrel, and a part, H_1, through the abutment at the springing.

Example II.—If by transformation the circular arch be converted into the elliptic arch, in which vertical ordinates remain the same while the horizontal ordinates are altered in the ratio of γ, we have a semi-elliptic arch under conjugate uniform vertical pressures and conjugate uniform horizontal pressures whose intensities have the ratio of γ^2, i.e., $\frac{p_y}{p_x} = \gamma^2$. (See equa. (393), paragraph 626.)

Then with a rise $a = r = x_1$ or vertical semi-axis, and a horizontal semi-axis $= \gamma a = \gamma r$; $H_1 = 0$, since the arch springs vertically from its abutment, $p_y = \gamma^2 p_x$. The total crown-thrust is the horizontal intensity p_y multiplied by the rise, that is, $p_y a = a\gamma^2 p_x$; and since the total vertical pressure and thrust at the springing are equal to P_1, the total horizontal or crown thrust is $\gamma P_1 = T_0$. Hence we have $T_0 = \gamma P_1 = p_y a = a\gamma^2 p_x$. The maximum component H_0 is at the crown and $= T_0 = a\gamma^2 p_x$. Substituting in eq. (431),

$$x_H = \frac{\int_{x=0}^{x_1=a} x p_y dx}{H_0} = \frac{\int_{x_0=0}^{x_1=a} x\gamma^2 p_x d_x}{a\gamma^2 p_x} = \frac{a^2}{2a} = \frac{a}{2}, \quad . \quad (435)$$

or the point of application of the resultant conjugate pressure is $\frac{1}{2}a$, the rise below the crown. This would doubtlessly be the condition of a tunnel lining if it were possible to believe that at very great depths below the surface the pressure is due to the total mass of earth above, as in such a case the height of the tunnel would be inconsiderable as compared with the depth below the surface, and consequently it would be an inappreciable error to consider the intensity of the conjugate pressure at the crown and springing to be the same, that is, a conjugate uniform pressure from crown to springing, and not varying with the depth as would be the case when the pressure height is limited, as will be seen in the discussion of tunnels.

658. In the preceding examples conditions of pressure have been assumed that rarely exist in practice. They are approximated to in those cases of circular arches submerged in water to such great depths that their diameters are insignificant as compared with these depths, and also, as already stated, in the case with elliptic

arches imbedded to great depths in a mass of earth. In this connection, an interesting example of a tunnel-lining is the Sydenham Tunnel, Eng.; it was of horseshoe or elliptic section, constructed in clay, which swelled or flowed and never reached a condition of stability until the walls were thickened and the section was practically converted into a circle. This will be alluded to in another paragraph.

We will now apply the principles to a few arches under loads or pressures actually occurring.

Hydrostatic Arch, Example IV.—In this form of arch (see Fig. 265(a)), since it is under a normal pressure but of varying intensity, that is, under actual fluid or water pressure, it is evident that the conjugate co-ordinate plane must be above and not touching the crown, as in the preceding examples, since from the equation characteristic of this arch, $xr = x_0 r_0$ or $\frac{x}{x_0} = \frac{r_0}{r}$, if $x_0 = 0$, r_0 is infinite. At any point below the surface in a fluid, the intensity of the pressure is the same in every direction, and equal to the weight of a unit of volume (i.e., w = weight of a cubic foot of water) multiplied by the depth x'. Then $p_y = p_x = p = wx$, and equation (430) becomes, since $H_1 = 0$,

$$H_0 = \int_{x_0}^{x_1} p_y dx = \frac{w(x_1^2 - x_0^2)}{2} = T_0, \quad . \quad . \quad (435a)$$

as the crown-thrust is the total pressure on the vertical plane AD. This is evident, as the arch springs from the abutments vertically, and the only pressure on the abutment is the weight of the water on the semi-arch AB, and consequently the horizontal component of the conjugate thrust H_1 at the springing B is zero, and the conjugate pressure P_{yr} is exerted through the backing or spandrel above the springing. The intensity p_y is positive for all points between the crown and springing.

Then equa. (431) becomes, after substituting the value of H_0 from equa. (435a),

$$x_H = \frac{w\int_{x_0}^{x_1} x^2 dx}{H_0} = \frac{w\frac{1}{3}(x_1^3 - x_0^3)}{w\frac{1}{2}(x_1^2 - x_0^2)} = \frac{2}{3}\frac{x_1^3 - x_0^3}{x_1^2 - x_0^2}, \quad . \quad (436)$$

which corresponds with the value found in equa. (354), paragraph 584 for the position of the centre of pressure on a plane whose upper edge is immersed at the depth x_0 below the surface, whether in water or in a mass of earthy material.

659. *Geostatic Arch, Example V.*—Referring to paragraph 632*a*, if we first construct a hydrostatic arch, in which the normal intensity of the load or vertical pressure is $\gamma p_x' = \gamma w' x' \cos j$ (instead of wx as in the preceding example), as the weight of a unit volume of the material now loading the arch is $w = \gamma w' \cos j$ if the conjugate co-ordinate plane is inclined, or $w = \gamma w'$ if it is horizontal; then, as in the hydrostatic arch,

$$p_x = p_y = wx = \gamma w' x' \cos j = \gamma p_x';$$

$$H_0 = T_0 = \gamma p_x = \gamma \int p_x dy; \quad . \quad . \quad . \quad . \quad . \quad . \quad (437)$$

the thrust at all points of the arch being the same, H_1 (as before) $= 0$, and x_H having the same value as in equation (436).

But if, as in an earth-mass, the intensity of the conjugate thrust is not equal to the intensity of the vertical pressure or load at any given point below the surface, though bearing a fixed ratio to it, say γ^2, so that

$$\frac{p_y'}{p_x'} = \gamma^2, \quad \text{or} \quad p_y' = \gamma^2 p_x' = \gamma^2 wx \cos j, \quad . \quad . \quad (438)$$

then the hydrostatic arch would not be the proper form for equilibrium. But if we transform this arch so that, while the vertical co-ordinate of any point remains the same, the horizontal or inclined co-ordinate of the new arch at the point is altered in the ratio c, then $x = x'$ and $y' = cy$. The effect will be simply to alter the horizontal pressures in the same ratio, while the vertical pressures remain the same, and there results for the new arch

$$P_x = P_x'; \quad P_y' = cP_y; \quad . \quad . \quad . \quad . \quad . \quad (439)$$

$$H_0 = T_0 = \gamma P_x = \gamma^2 w \cos j \frac{x_1^2 - x_0^2}{2}; \quad . \quad . \quad . \quad (440)$$

and from equation (431),

$$x_H = \frac{\int_{x_0}^{x_1} \gamma^2 p_x' dx_1 x}{H_0} = \frac{\int_{y_0}^{x_1} \gamma^2 w x^2 \cos j dx}{\gamma^2 w \cos j \frac{x_1^2 - x_0^2}{2}}$$

$$= \frac{\gamma^2 w \cos j}{\gamma^2 w \cos j} \cdot \frac{2}{3} \frac{x_1^3 - x_0^3}{x_1^2 - x_0^2} = \frac{2}{3} \frac{x_1^3 - x_0^3}{x_1^2 - x_0^2}, \quad . \quad . \quad (441)$$

which gives the position of the conjugate horizontal resultant pressure at the same depth as in case of fluid pressure. See also Land Ties for Retaining-walls, paragraph 584.

660. *Semi-elliptic Arch*, under the same conditions of loading as the semi-circular arch, Example III, that is, with conjugate uniform and horizontal pressures, is simply the transformed circular arch. See Fig. 264 (b and c), in which

$$x_1 = AO = A_1O = A_2'O_2 = a,$$

the semi-vertical diameter, and the half span or semi-horizontal diameter is $y_1 = O_1B_1$ or O_2B_2 (as the case may be) $= ca$, the pressure being normal and of uniform intensity.

The total horizontal thrust $= H_0 = T_0 = x_1p_y = ap_y$; and from $p_y' = \gamma^2 px$,

$$H = T_0 = ap_y = \gamma^2 ap_x = \gamma Px,$$

in which P_x is the total pressure on the abutment in the semi-circular arch $= pr$. As the semi-elliptic arch springs vertically from the abutment, the vertical pressure is the only one at that point $= cP_x$ and $H_1 = 0$, as there is no conjugate component of the thrust at the springing. Then equation (431) becomes

$$x_H = \frac{\int_0^{x_1=a} xp_ydx}{H_0} = \frac{\frac{1}{2}p_ya^2}{ap_y} = \frac{a}{2}. \quad . \quad . \quad . \quad (442)$$

In those cases in which it is desired to substitute the elliptic arch for the hydrostatic arch or the geostatic, the same equations may be used for most practical purposes. To be more exact, however, we may make H_0, in eqs. (437)–(440), equal to γP_x, γ being the ratio of the half spans of the elliptic and hydrostatic arches and P_x the total vertical pressure on the semi-hydrostatic arch, i.e., the total vertical pressure on either abutment.

It is to be noted in all of the examples that the joint of rupture is at the crown, which is therefore the point of maximum conjugate thrust. There is consequently an outward thrust at all points of the arch-ring, which at any point is equal to the difference between the thrust T_0 at the crown and the conjugate horizontal component of the thrust at that point $H_0 = T \cos \alpha'$, which difference is P_y in all of the foregoing equations, the special value of which is H_1 for the springing line.

H_1 is zero for all arches which spring vertically from the abutment. But at all points between the springing and the crown the outward intensity of pressure is an active force, equal and opposed to the inward positive intensity p_y, which represents the resistance of the backing to the spreading of the arch-ring. This total resistance P_{yr} is never negative; consequently the ideal linear arch must be backed up from the springing to the level of the crown. That this may not be necessary in actual masonry arches arises from the fact that the adhesion of the mortar to the stone or the tenacity of the mortar in the joints may be, and is, considerable with good cement mortar, and it may be in many cases that the backing is not necessary, or at any rate only required for a certain portion of its height.

It is evident that an iron rod or bar passed through the arch-ring at the two points on either side of the crown, indicated by the value of x_H, and well secured on the extrados, would supply the place of the backing. This is merely mentioned as indicating the kind of outward thrust that must be resisted in order to secure equilibrium.

It is wise, however, to build the backing to the highest point indicated by theory. It will at least be on the safe side.

How far the preceding principles and equations, which are only strictly applicable to the ideal linear arch under assumed conditions and intensities of loading, are modified by the actual loading weights of the arch-ring itself, and other conditions occurring in actual constructions, may not be possible to determine.

But we can construct a linear arch which will be in equilibrium under the assumed conditions, and then proportion an actual design of the arch-ring, so that the linear arch shall be at all points within the middle third of the thickness of the actual arch-ring, and properly back it up with masonry. The stability of the actual arch will be secured, provided the joints are so arranged that the angle which the tangent line to the linear arch, which now becomes the line of pressure of the actual arch, at any joint does not make an angle with the normal to the joint greater than the angle of repose of masonry on masonry; and if the intensity of the thrust on the arch-ring does not exceed a safe limit, say one tenth of the resistance to crushing of the material, we may conclude that the arch will be stable. The angle between the resultant pressure and the normal at any joint can always be kept within safe limits by giving a proper direction to the joint itself, so that stability of

friction or resistance to sliding can always be secured. Radiating joints will generally secure resistance against sliding. If it is found that resistance to crushing is not secured, the thickness or depth of the arch-ring must be increased until it is, the effect of which will be to give greater resistance to spreading at the haunches, and rotating around any axis at any joint, or the tendency to open at any joint.

In the following applications of the foregoing principles to the construction of arches the resistance to sliding or crushing will be considered as provided for, except where specially mentioned. And only safety against failure by rotation or opening at the joints is to be provided.

PRACTICAL APPLICATIONS OF THE FOREGOING PRINCIPLES.

661. *Let it be required to find the semi-horizontal diameter or axis of a semi-elliptic arch suited to support an earthen embankment carrying on its upper horizontal surface a railway and its load, and also the thickness of the arch-ring.*

The weights of the arch-ring, spandrel-walls, and backing, of the earthen embankment itself, and of the track and train-load constitute the total load. The preceding principles are all based upon the symmetrical distribution of the load with respect to a vertical plane through the crown and the axis of the arch. It is therefore necessary to assume the rolling load as an equivalent uniform load extending over the top of the arch. Also, that the weight of the embankment is uniformly distributed over the length and span of the arch, notwithstanding the spread of the base due to the side slopes.

To arrive at and properly combine the above three loads of train, earth, and masonry is practically impossible, and all that can be done is to make a rough approximation by considering all three to have heights proportional to their respective specific gravities and of a homogeneous material. To arrive at the proper distribution of the loads the length of the arch must be known. This will be determined by the width of the embankment at the top, the depth to the crown of the arch, and the side slopes.

With side slopes $1\frac{1}{2}$ to 1, height above crown $x_0 = 20$ feet, and top width 14 feet, the width at base or length of arch would be $14 + 60 = 74$ feet. Then, assuming a rolling load of 6000 pounds per foot of span over the entire length, then per square foot of

surface it would be $\frac{6000}{74} = 81$ pounds. The intensity of the pressure of the earth on the arch $= wx_0 = 100 \times 20 = 2000$ pounds. And if the rise of the arch is $x_1 - x_0 = a = 18$ feet, the intensity of the pressure arising from the weight of the masonry, with an average of, say, 8 feet in height of masonry and at 160 pounds, will be $160 \times 8 = 1280$ pounds. Then the total average intensity of the load will be $2000 + 1280 + 81 = 3361$, or, reducing this to a homogeneous material of an average height of 28 feet, we have $\left(\frac{3361}{28} =\right)$ 120 pounds per cubic foot, and for safety assume the weight per cubic foot to be 150 pounds.

According to the theory of transformation (paragraph 626) we will first construct a hydrostatic arch, loaded with a liquid, whose weight per cubic foot bears a certain predetermined ratio $\gamma \cos j$ (or θ) to the actual weight of 150 pounds, i.e., $w = \gamma w' \cos j$, in which j is the angle of inclination of the conjugate co-ordinate plane (that is, the upper surface of the material) with the horizon. In this case this plane is horizontal; hence $j = 0$, $\cos j = 1$, and $w = \gamma w^1$.

For reasons which will now be apparent, the ratio of the intensities of the principal conjugate pressures of the material assumed as pressing on the arch was taken $= \gamma^2$, ie., $\frac{p_y'}{p_x'} = \gamma^2$, which becomes for a horizontal upper surface $\frac{p_y'}{p_x'} = \gamma^2 = \frac{1 - \sin\phi}{1 + \sin\phi}$ or $\frac{1 + \sin\phi}{1 - \sin\phi}$. (See discussion of the theory of earth pressure.) Then $\gamma = \sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}}$ or $\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}$. Using first value of γ,

$$w = w'\sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (443)$$

Now, assuming a hydrostatic arch loaded with this kind of material, we have

$$p_x = p_y = wx = w'x\sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}} = \gamma p_x' = \frac{p_y'}{\gamma}. \quad . \quad (444)$$

Then construct the hydrostatic arch, having the following data given: Depth of the crown A below the surface $x_0 = 20$ feet; depth

of springing-line $B = 38$ feet; rise of arch $AD = a = x_1 - x_0 = 18$ feet; the angle of repose of the material an average, for earth on earth, $\phi = 30°$; sin $\phi = 0.5$. Then

$$p_x = p_y = w'x\sqrt{\tfrac{1}{3}} = 0.58\, w'x; \quad w = w'\sqrt{\tfrac{1}{3}}. \quad . \quad . \quad (445)$$

As was seen in paragraph 634, the exact determination of the semi-horizontal diameter of this arch cannot be found without great difficulty; therefore, using the approximate eq. (420) for this semi-diameter or half span,

$$y_1 = \frac{19}{20}(x_1 - x_0)\sqrt[8]{\frac{x_1}{x_0}}.$$

Substituting the above values of $x_0 = 20$ and $x_1 = 38$ feet, then

$$y_1 = \frac{19}{20} \times 18 \times \sqrt[8]{1.9} = 21.2 \text{ feet.}$$

Then, in Fig. 269, laying off the diameter or span = 42.4 feet and the rise = 18 feet, we can construct the hydrostatic arch, approximately, by using three centres for each half. The data given are $x_0 = 20$ feet; $x_1 = 38$ feet; $x_1 - x_0 = a = 18$ feet, the rise; $y_1 = b = 21.2$ feet, the half span. Then the radii of curvature at the crown and springing, respectively, are: $r_0 = \frac{a}{2}\left(1 + \frac{b^2}{a^2}\right) = 23.7$ feet (nearly), and $r_1 = \frac{a}{2}\left(1 + \frac{a^2}{b^2}\right) = 14.5$ feet (nearly). These values should satisfy the equation $x_1 r_1 = x_0 r_0$. A closer approximation must be ob-

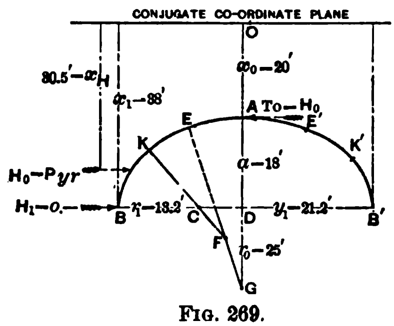


FIG. 269.

tained by altering some of the above values, making $r_0 = 25$ and $r_1 = 13.2$ feet; then $38 \times 13.2 = 25 \times 20 = 500$. Take on AD prolonged a distance $AG = r_0 = 25$ feet, and on BB' a distance $BC =$

13.2 feet C and G will be two of the centres. Then with C as a centre and a radius $CF = AD - BC = 4.8$ feet describe an arc, and with G as a centre and $GD = 7$ feet as a radius describe another arc intersecting the first at F; then F will be the third centre. With the centre G and radius AG describe AE, from F describe KE, and from C describe KB, and similarly for the other half of the arch $AE'K'B'$.

Then, from eq. (435a),

$$H_0 = T_0 = w\frac{x_1^2 - x_0^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (446)$$

in which $w = 0.58 \times w' = 0.58 \times 150 = 87$ pounds, $x_1 = 38$, and $x_0 = 20$; hence $H_0 = T_0 = 87 \times 522 = 45{,}414$ pounds. The point of application of the maximum conjugate thrust H_0, equal and opposite to the resultant conjugate pressure P_{yr}, is found from eq. (436).

$$x_H = \frac{2}{3}\frac{x_1^3 - x_0^3}{x_1^2 - x_0^2} = 30.5. \quad . \quad . \quad . \quad . \quad . \quad (447)$$

662. If, then, to avoid confusion from the various lines of construction, we reproduce the curve of the arch, as shown in Fig. 269, and proceed to construct an elliptic arch by transformation, Fig. 270, so that for each point the vertical co-ordinate shall remain the

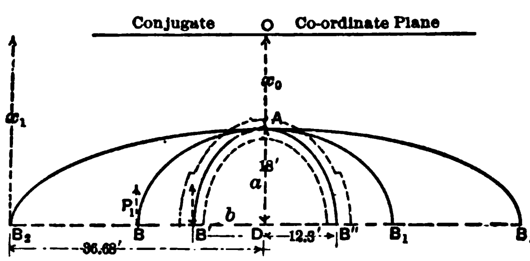


FIG. 270

same, while the horizontal co-ordinate is reduced in the ratio of γ, i.e.,

$$x = x'; \quad y' = \gamma y = y\sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}} = 0.58y. \quad . \quad . \quad (448)$$

Then the semi-horizontal diameter of the ellipse will be $0.58y = 0.58BD = DB'$; and if upon the semi-axes AD and DB' we construct the semi-ellipse $B'AB''$, this will be the semi-elliptical linear arch, which will be stable under the actual pressures assumed.

662a. All vertical intensities and total pressures will be the same as in the hydrostatic arch; that is, the total vertical pressure at the springing carried by the abutment on either side $= 28 \times 87 \times 21.2 = 150 \times 0.58 \times 21.2 \times 28 = 51{,}632$ pounds $= P_1$. And similarly for the general value of the vertical pressure from the crown to any point whose vertical ordinate is x, i.e., $P_x = P_x'$. Theoretically, in the hydrostatic arch the total vertical pressure on the semi-arch should be equal to $P_1 = T_0 = 45{,}414$ pounds. Practically it will always be a little greater, as indicated above. The above value of P_1 is based only on an estimated average depth of the arch below the surface $= 28$ feet, and $DB' = 0.58 \times 21.2 = 12.3'$.

663. But the horizontal intensities and horizontal total thrusts at any point will be altered in the elliptic arch by the ratios γ^2 or γ, and consequently, in the case considered, where the horizontal co-ordinate is altered in the ratio γ,

$$y' = y\sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}} = y\sqrt{\tfrac{1}{3}} = 0.58y\,. \quad . \quad . \quad . \qquad (449)$$

Also, for the vertical and conjugate horizontal pressures in the ratio γ, we have for the elliptic arch the thrust at the crown

$$= T_0' = H_0' = \gamma P_x = \gamma^2 w' \frac{x_1^2 - x_0^2}{2} = \tfrac{1}{3} \times 150 \times 522 = 26{,}100 \text{ lbs.},$$

which is very nearly equal to $0.58T_0 = 0.58 \times 45{,}414 = 26{,}340$, which corresponds with the theory of transformation of linear arches.

The point of application of the resultant of the total pressure of the backing necessary to produce equilibrium is the same as in the original hydrostatic, namely, $x_H = 30.5$ feet.

From eq. (393) it is seen that the ratio of the semi-axes is as the square root of the ratio of the intensities of the principal pressures, that is,

$$\frac{B'D}{AD} = \frac{y'}{a} = \sqrt{\frac{p_y'}{p_x'}};$$

and from eq. (443),

$$\gamma = \sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}} = \sqrt{\frac{p_y'}{p_x'}}, \quad \therefore \gamma = \sqrt{\frac{1}{3}} \text{ in this case.}$$

Hence

$$\sqrt{\frac{p_y'}{p_x'}} = \frac{y'}{a} = \gamma = \sqrt{\frac{1 - \sin\phi}{1 + \sin\phi}} = \sqrt{\frac{1}{3}}. \quad . \quad . \qquad (450)$$

In words, the semi-horizontal diameter, $B'D = y'$, of the ellipse $B'AB''$, Fig. 270, must not be less than $\sqrt{\frac{1}{3}}$ times the semi-vertical diameter $AD = x_1 - x_0 = a$, i.e., $y' = b > a\sqrt{\frac{1}{3}}$. In this case $a = 18$ feet; $\sqrt{\frac{1}{3}} = 0.58$; $\therefore a\sqrt{\frac{1}{3}} = 10.44$ feet. The actual value of $b = B'D = 0.58 \times 21.2 = 12.3 > 10.44$. Theoretically b could be reduced to 10.44 feet; but if less than this value, the sides of the arch would be forced inwards by the external conjugate pressure.

664. With the crown-thrust $T_0' = 26,100$ pounds, and a safe resistance of 20,000 pounds per square foot, the depth of the keystone should be $\frac{26100}{20000} = 1.3$ feet or 16 inches. Using Trautwine's empirical formula, depth of keystone in feet

$$\frac{\sqrt{r + \frac{1}{2}\text{ span}}}{4} + 0.2 \text{ feet} = \frac{\sqrt{25.5}}{4} + 0.2 = 1.46 \text{ feet or 18 inches,}$$

in which r is the radius of a circle which would touch the arch at the three points B', A, B''; $r = \frac{\overline{B'D}^2 + \overline{AD}^2}{2AD} = \frac{\overline{12.3}^2 + \overline{18}^2}{2 \times 18} = 13.2$ feet; the half span $= 12.3$ feet.

Rankine's empirical formula is depth in feet $= \sqrt{0.12r}$, in which $r = \frac{a^2}{b} = 26.4$ feet for the elliptic arch. Substituting, we have for the depth of keystone $d = 1.79$ feet or 22 inches.

It is not necessary in small arches to increase the thickness of the arch-ring gradually from the crown to the springing line, but it is safest to do so in most cases. The total vertical pressure on the semi-arch, which is supported by the abutments at B' and B'', would certainly be not less than $T_0 = 45,414$ pounds, and may be as much as $P_1 = 51,632$ pounds, which with a safe resistance of 20,000 pounds per square foot would require $\frac{51632}{20000} = 2.58$ feet, or about $30\frac{1}{2}$ inches. This is doubtlessly more than necessary; an increase of from $\frac{1}{4}$ to $\frac{1}{2}$ would be ample in any case.

If, then, a masonry arch be built as indicated by the dotted lines around $B'AB''$, the arch would be considered stable.

This form of elliptic arch with so great a rise as compared with the half span is more common in tunnels than for other purposes.

It is more than probable that no greater thrusts will exist in arch-rings, or any greater thickness of the arch stones will be required, when the depth of the crown is more than from 20 to 40

feet below the surface of the ground than when within these limits, even for a wide tunnel, as in case of a double track; as in ordinary firm material the earth would, if not disturbed, arch itself over the masonry lining, and the portion exerting a pressure on the arch would therefore be limited in height to not more than above stated. If an open cut is made in which a masonry tunnel is built, and subsequently filled over with earth, or an arch filled over with a high embankment, the pressure will doubtlessly be due to a much greater depth than given above, temporarily at least.

665. In the preceding example the horizontal co-ordinate y was contracted or made less than that of the hydrostatic arch. The transformation may, however, be made by expanding or lengthening the values of y for the hydrostatic arch B_2AB_1, so that

$$y' = y\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} = y\sqrt{\frac{1.5}{0.5}} = y\sqrt{3} = 1.73y;$$

hence in the case assumed

$$y' = 1.73y = 1.73 \times 21.2 = 36.68 \text{ feet.}$$

In this case $\gamma = \sqrt{3}$, $\gamma^2 = 3$, and then

$$H_0' = T_0' = \gamma P_x = \gamma^2 w' \frac{x_1^2 - x_0^2}{2} = 3 \times 150 \times 522 = 234{,}900 \text{ pounds.}$$

At 20,000 pounds resistance per square foot this arch would have a crown-thickness of 11¾ feet, which would be out of the question. If, however, 30,000 pounds, be allowed, the thickness would be 7.8 feet, which is certainly greater than is desirable.

If such an arch is only about 4 feet below the surface, which is often the case, then $x_0 = 4$ and $x_1 = 22$ feet. Then $H_0' = T_0' = 3 \times 150 \times 234 = 105{,}300$ pounds, and depth of arch-ring at crown equal to from 3 to 5 feet, varying with the allowed unit-resistance. This degree of expansion or lengthening the span of the arch is given by Mr. Rankine's theory as the extreme limit.

666. The Little Juniata arch bridge, which has somewhat the same form as the one now considered, has a span of 50 feet, rise 16.2 feet, the soffit 6 feet below base of rail. The soffit is described with three different radii, respectively 38.67 feet, 20.65 feet, and 10.5 feet, and has a thickness of arch-ring of 2 feet 4 inches from crown to springing.

667. In any case the horizontal thrust at the crown can be determined in accordance with the preceding principles. Whenever

the pressure is vertical and the crown of the arch is horizontal, the condition of the arch at that particular point is that of a normally pressed arch; the intensity of the pressure being the weight of the column of material vertically above the crown, and the radius that of a curve parallel to the soffit at the crown and passing through the centre of pressure at the crown, assumed as at 1 foot above the soffit in the following examples.

Then the horizontal thrust will be $T_0 = r_0 p_0$; and with the data given in paragraphs 661 to 665 we have for the true hydrostatic arch under water-pressure $w = 62\frac{1}{2}$ pounds; $p_0 = wx_0 = 62\frac{1}{2} \times 20 = 1250$ pounds; $r_0 = 26$ feet; $T_0 = p_0 r_0 = 32{,}200$ pounds.

For the hydrostatic arch subject to a fluid pressure whose weight per cubic foot is γ times that of the actual earth and masonry load. Then for $\gamma = \sqrt{\frac{1}{3}}$, $w = \gamma w' = 0.58 \times 150 = 87$ pounds; $p_0 = 87 \times 20 = 1740$ pounds; $r_0 = 26$ feet; $T_0 = 1740 \times 26 = 45{,}240$ pounds.

For the elliptic arch $B'AB''$ under a pressure of material weighing 150 pounds per cubic foot, $p_0 = 150 \times 20 = 3000$ pounds; $r_0 = \frac{b^2}{a} + 1 = \frac{(12.3)^2}{18} + 1 = 9.4$ feet; $T_0 = 3000 \times 9.4 = 27{,}200$ pounds.

For the elliptic arch B_2AB_3, $p_0 = 3000$ pounds; $r_0 = \frac{b^2}{a} + 1$ $= \frac{(36.68)^2}{18} + 1 = 76$ feet; $T_0 = 3000 \times 76 = 228{,}000$ pounds.

The above results differ but slightly from the corresponding values determined by the general solutions of these problems in paragraphs 661 to 665, and agree as nearly as could be expected with the approximate value of the dimensions used.

If in this transformation the ratio of the semi-horizontal to the semi-vertical diameter of the elliptic arch be kept within the limits of $\gamma = \sqrt{\frac{1}{3}}$ and $\gamma = \sqrt{3}$, and the proper thickness be given to the arch-ring, then, according to Rankine's theory of earth-pressure as applied to the construction of underground arches and tunnels, the stability of the arch will be secured when its line of pressure remains within the middle third of the arch-ring, whether the curve of the soffit is elliptic or circular. The smaller limit $\gamma = \sqrt{\frac{1}{3}}$ is often, if not generally, used in the construction of tunnels. It should never be less, as there would be great danger of the sides being pressed inwards.

668. Arches and arch-culverts for railways, whether under low or high embankments, are usually of circular sections, and when of small span the full centre or semicircular arch is used. But often for want of height, especially with long spans, it is desirable to use the elliptic arch or circular segmental arch, in which the rise varies from one third to one sixth of the span, instead of one half, as in the full centre arch. It is evidently unwise, and generally unnecessary, to use the superior ratio $\gamma = \sqrt{3}$.

If in the example of the flat arch B_1AB_1 we make $\gamma = \sqrt{1.56} = 1.25$, instead of 1.75, the crown thrust would be $T_0'' = 98{,}000$, and a thickness of arch-ring at the crown of 3.5 to 4 feet would answer, the span being 53.0 feet.

In the Juniata arch, the crown of which is only 6 feet below the surface, the crown thrust would be $H_0 = T_0 = 40{,}000$ pounds approximately, which would call for a depth of about 2.0 feet for the keystone, the actual depth being 2 feet 4 inches.

669. The thickness of the arch-rings thus far given are for first-class ashlar masonry. For an inferior grade of masonry or brickwork it should be increased by one-fourth part.

The preceding examples illustrate the mathematical theory of arches. When applied to the full centre or semicircular arch or the circular segmental arch, as shown in Figs. 266 and 267, the principles and applications of it are the same. The equations are much more complicated and difficult of application. The work is long and troublesome, and the uncertainty as to the accuracy and reliability of the data necessary, and consequently that of the results, has made engineers look with disfavor and distrust upon the employment of mathematical formulæ for the solution of problems connected with the practical construction of masonry arches. Much labor and thought has, however, been devoted to the true theory of the stability of arches, but mainly with a view of solving the problems required by the graphical methods.

GRAPHICAL METHODS.

670. In all of the graphical methods, except the one used by Rankine, the horizontal thrust is assumed to be constant from springing-line to crown. Its magnitude is usually found by considering the half arch as acted upon by only three forces, namely, (1) the horizontal thrust at the crown, (2) the vertical or inclined

thrust at the springing, and (3) the entire weight of the semi-arch, including the weight of the arch-ring, the backing, and any load fixed or moving (reduced to its equivalent uniform load), such as the earth above, the weight of trains, etc. Referring to Figs. 266, 267, these three forces are indicated by T_0, T_1, and W; the points of application T_0 and T_1 are assumed at different points between A and K and M and B, respectively; and the line of action of W is to pass through the centre of gravity of the aggregated weights above—a point difficult to determine. All three points of application are therefore arbitrarily assumed or approximated to within certain limits. Then moments are taken about the point of application of T_0 or T_1, the position and magnitude of W being known. To find the value of T_0 the axis of moments is taken at r, the point of application of T_1. Then calling the lever-arm of T_0 with respect to the assumed axis x, and that of W as y, we have only two active moments, which for equilibrium must be equal. Hence

$$T_0 x = Wy; \quad \therefore T_0 = \frac{Wy}{x}. \quad . \quad . \quad . \quad . \quad . \quad (451)$$

If y is taken as one fourth the span, or $\frac{1}{2}ro$, and x is taken as the rise os in Fig. 266, and equal to sz' in Fig. 267, we have approximately $T_0 = \frac{bW}{2a}$, a and b being respectively the rise and half-span. Applying this to the semi-elliptic arch, Fig. 270, $B'AB''$, and emthe data used in the example paragraphs 663 and 667, $b = 12.3 + 1 = 13.3$, $a = 18 + 1 = 19$, and maximum possible $W = P_1 = 150 \times 12.3 \times 38 = 70{,}110$ pounds. These values substituted give $T_0 = 24{,}538$ pounds, which agrees approximately with the value of T_0 in the elliptic arch.

It is evident that W, T_0 and T_1 will vary materially with the points of application of T_0 and with the nearness of the approximation in finding the magnitude of W, the position of its centre of gravity.

In Rankine's method the above conditions are only used for that portion of the arch between the joint of rupture and the crown.

The relation $T_0 = \frac{Wy}{x}$ will be further discussed in another paragraph.

671. The following is an outline of Rankine's graphical method of determining the joint of rupture and thrusts at crown and other points, and the conditions upon which stability will be secured.

First, however, a few remarks on some of the many theories will be appropriate. Examining the above equation in connection with Figs. 266 and 267, it will be observed that every different value of y or x will give a different value for T_0, W remaining the same; and in order to maintain the equality, if T_0 varies either x or y or both must likewise vary. We could therefore construct any number of lines of resistance.

Hence the problem is to determine which of the infinite number of lines of resistance is the true one; to do this many hypotheses have been made.

(1) *Theory of Least Resistance.*—That the true line of resistance is the one which gives the least absolute pressure at any joint; or in other words, of the lines of resistance that is the true one which gives the least crown thrust T_0 consistent with equilibrium.

(2) *Winkler's Hypothesis.*—In this hypothesis it is assumed that "for an arch-ring of constant cross-section that line of resistance will be the true one which lies nearest to the axis of the arch-ring, as determined by the method of least squares."

(3) *Navier's Principle,* which is as follows: The condition of an arch of any form at any point where the pressure is normal is similar to that of a circular rib of the same curvature under a normal pressure of the same intensity; hence the thrust at any normally pressed point of a linear arch is the product of the radius of curvature by the intensity of the pressure at that point. It is to be noted that this principle is adopted and followed by Rankine.

Mr. Rankine does not undertake to determine the exact point of application of the thrust at the crown or the true position of the line of resistance, but states that if the line of resistance is found within the width of the middle third of the arch-ring, the stability of the arch is assured. And in fact he limits the application of this rule to that portion of the arch-ring between the crown and the two joints of rupture, one on each side of the crown. His method is as follows.

672. Given a linear arch or rib of any figure, $Acc'c''$, under a vertical load distributed in any manner, it is always possible to determine a system of horizontal or sloping pressures which, being applied to that rib, will keep it in equilibrio. These may be called the conjugate pressures; they are assumed to be horizontal, and the load is assumed to be symmetrically distributed on each side of the crown of the arch.

In Fig. 271 (A) let $ADcc'c''$ be any linear arch. The thrust at

the crown must be determined by the equation $T_0 = p_0 r_0$. Then from any point o, Fig. 271(B), draw a vertical line ob to represent the line of loads, and a horizontal line oa to represent, to any scale, the calculated horizontal thrust at the crown T_0. On the line of loads ob lay off to the same scale distances oh, oh', oh'', etc., to represent the loads from the crown down to the points c, c', and c'' respectively. Also from o draw lines oc, oc', etc., parallel to the curve of the arch at those same points. Then draw the horizontal lines from h, h', etc., to intersection with these oblique lines at the points c, c', etc., respectively. A curve drawn through the points c, c', and a will be the locus for the ends of the lines

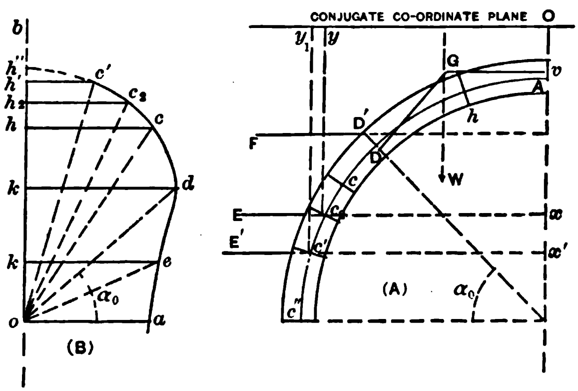


FIG. 271.

representing the thrusts at any points of the arch. The more numerous and the closer together the points c, c', etc., in Fig. 271(A), and the corresponding lines oc, oc', etc., in Fig. 271(B), the more accurately can the line c, c', . . . a be drawn. In Fig. 271(B) the limiting curve starting at a gets farther and farther from the vertical line up to some point d, where the horizontal distance kd is a maximum, and then it begins to draw nearer and nearer to the vertical line near b. It will not, however, reach or intersect the vertical ob until the tangent to the curve of the arch becomes vertical, which can only be the case when the arch springs vertically from its abutments, as at c'', Fig. 271(A), in which case the vertical oh'' is equal to the entire weight from A down to c''. Any line, then, in the diagram of forces Fig. 271(B) radiating from o and terminating in the limitimg curve will represent the thrust at that point of the arch at which a tangent line to the curve of the arch

is parallel to the radiating line in question. If we take the radiating line *oe*, Fig. 271(B), and find the point on the arch-rib, Fig. 271(A), at which the tangent line is parallel to *oe*, then *oe* will represent the thrust in pounds or tons. If a scale of 20,000 pounds to the inch has been adopted for the scale of loads, and *oe* measures $1\frac{1}{2}$ inches, then the thrust at this point will be 30,000 pounds. The horizontal $k'e$ will be the horizontal component of this thrust, and the vertical ok' will be the load between the crown A and the point, and similarly for any other point on the arch-rib.

It is evident that there is some point corresponding to a maximum horizontal thrust. This point is found by locating that point on the limiting curve, Fig. 271(B), farthest from the vertical *ob*. This point is at d; kd is the horizontal component of the thrust, *od* the thrust itself, and *ok* the vertical load from the crown to that point. Now find on the curve, Fig. 271(A), the point D at which the tangent line is parallel to *od*, Fig. 271(B); then D is the point of maximum horizontal thrust, and, is by the principles of paragraphs 650 to 652, the point or joint of rupture, and the angle of rupture is the angle *doa* which the thrust or tangent line makes with the horizon. This horizontal component H_0 of the thrust is equal to the maximum total conjugate pressure P_{yr} of eq. (427) exerted through the spandrel, between the joint of rupture and the springing, and whose point of application is the x_H of eqs. (433) to (436) and (441), (442).

To supply this conjugate pressure and maintain the equilibrium of the arch the backing must be built up solid to the horizontal line FD', passing through the joint of rupture D.

673. We have now found the horizontal thrust at the crown; the horizontal component of the conjugate thrust at any point of the arch-rib; the maximum horizontal thrust, that is, maximum $H = H_0 =$ maximum total P_{yr}; the position of the joint of rupture, and the inclination of the tangent at that point of the rib which is the angle α in eq. (427), paragraph 652. It only remains to find the mean intensity of the horizontal pressure in any thin layer of the spandrel. If we assume the points c_2 and c', Figs. 271(B) and (A), to be very close together, so that while the co-ordinates of c_2 are x and y, those of c' will be $x + dx$ and $y + dy$ (dx being the thickness of the layer of the spandrel EE' in question), then in Fig. 271(B) the thrusts at c' and c_2 of the arch-rib are the radiating lines oc' and oc_2. The horizontal components of these thrusts are H' and H_2, represented by $h'c'$ and h_2c_2; hence $h_2c_2 - h'c' =$

$H_1 - H' = -dH$ is the horizontal pressure to be exerted through the layer $c'E'Ec_1$, Fig. 271(A), and the intensity of this pressure is p_y; hence $p_y dx = -dH$, and $p_y = -\frac{dH}{dx} = -\frac{d}{dx}\left(P\frac{dy}{dx}\right)$. The determination of this intensity at any point between D and c'', or between D and A, the crown, enables us to determine the kind and direction of pressure to be supplied from without by compression or by tension. The negative sign is prefixed to dH simply to indicate that if the horizontal lines diminish in going downward (that is, the general symbol H) as it does from the point d to a [see Fig. 271(B)], then pressure from without must be supplied; that is, masonry backing must be built from c'' to the horizontal line $D'F$ to resist the tendency of the haunches to spread or revolve outwards. If, on the contrary, H increases on going downward, or diminishes on going upward, as between the points d and h'', then tension must be supplied from without (or pressure from within). In other words, the arch-ring from D to A in Fig. 271(A) must be tied to the spandrel above by bolts unless the adhesion and tenacity of the mortar can be relied upon to hold that portion of the arch-ring from sinking. The pressure from within can only be supplied by direct support, which is the case so long as the centring remains in position; but almost invariably on removing the centres the arch settles more or less at the crown, and unless the proper form is given to the curve of the soffit, or the arch-ring has a sufficient thickness or depth, the extent of the settling will be to endanger the stability of the arch. Ordinarily a sinking of from 1 to 3 or 4 inches will not indicate insufficiency of strength.

674. The algebraic equations and relations of the above solution are the following:

Horizontal thrust at the crown $T_0 = p_0 r_0$. . . . . . (452)

Total horizontal pressure exerted on that portion of the arch below any point C

$$= H = P\frac{dy}{dx} = P \text{ cotan } \alpha. \quad . \quad . \quad . \quad . \quad (453)$$

The thrust on the arch-ring at any point T

$$= \sqrt{H^2 + P^2} = P \text{ cosec } \alpha. \quad . \quad . \quad . \quad . \quad (454)$$

The intensity of the horizontal pressure to be exerted through any layer of the spandrel

$$p_y = -\frac{dH}{a_x} = -\frac{d}{d_x}\left(P\frac{d_y}{d_x}\right). \quad . \quad . \quad . \quad . \quad (455)$$

In these equations p_0 is the vertical pressure on a unit of area at the crown, r_0 the radius of curvature at the crown, P the total vertical load between the crown and any point C, α the angle which the tangent line to the arch-rib at the point makes with the horizon $= c'oa$, coa, or eoa, etc., and α_0 the angle doa of the tangent line or its parallel at the joint of rupture. All of these equations have been fully discussed in paragraphs 642, 643, 644, etc., to 652.

Having found as above the position of the joint of rupture and insured the stability of that portion of the arch by the masonry backing, or if it is desirable to find the point d, Fig. 271(B), more accurately than by scaling from the limiting curve for thrusts, a number of the values of H from eq. (453) can be calculated. Then between the maximum value of H thus obtained and the next values on either side introduce other lines of thrust, calculate another series of values for H, and take the maximum of this series. If this is not close enough, take this maximum value and the next on either side, introducing thrust-lines still closer together. The last maximum obtained will correspond with the joint of rupture, and the radiating line od, corresponding to this maximum H_0 represented by kd, will be parallel to the tangent at the joint of rupture. Finding this point D on the arch-ring, the joint of rupture is located.

Or, again, if the relations between P, x, and y can be expressed by equations, substitute these in eq. (455), making $p_y = 0$, and solving, we find the position of D and the values α_0 and H_0. There would generally be two roots to this equation, one of which corresponds to the crown of the arch, the other to the point D, i.e., the joint of rupture. These can be easily distinguished from each other. If there is only one root, it corresponds to the crown of the arch, which would be the case in the hydrostatic arch.

675. In whatever manner the joint of rupture may have been found, that portion of the arch from that joint to the springing-line of the arch is kept in equilibrium by the lateral pressure, $P_{yr} = H_0$, between the arch and its spandrel and abutment. The line of pressure will be a curve similar to the linear arch, which should also be parallel to the intrados of the arch. The thickness of the ring should then be proportioned to the inclined thrusts at the

various joints of the arch. It is in reference to that portion of the arch (see Fig. 271(A)) between D, the joint of rupture, and the crown A that Mr. Rankine is charged with a failure to sustain and apply his theory, abandoning, as is charged, "the principal characteristic of his theory, viz., the recognition of the horizontal components of the external forces, etc." (See note to Baker's "Masonry Construction," page 490.)

To the writer's mind there is no inconsistency or abandonment of principles. As seen above, Mr. Rankine has fully provided for the portion of the arch above the joint of rupture by iron bolts, bars, or clamps; when these are necessary will be indicated by the change of sign from positive to negative in the value of the intensity of the horizontal or conjugate pressure p^y; and while he states that the linear arch is limited to cases in which equilibrium is secured solely by pressures from without, in other words, where p_y has no negative values, that the relations between the form of the arch and the external loads or forces are such that the joint of rupture is at the crown: for the value of p_y is zero at the joint of rupture, and if it is at any other point, it follows that it must pass from positive to negative at that point. He, however, only states that no negative values are to be allowed when lime mortar is used, for he clearly recognizes the adhesive and tensile strength of cement mortar in supplying the necessary tension from without—not below the joint of rupture, but above it. And as if to impress this point he says: "When tenacity to resist horizontal or oblique tension is given to the spandrels of an arch, and to the joints between them and the arch stones, by means of cement, hoop-iron bond, iron clamps, or otherwise, the conjugate pressure denoted by p_y must not at any point exceed a safe proportion of that tenacity; that is to say, about one eighth. By this means stability may be given to arches of seemingly anomalous figures; but such structures are safe only on a small scale." He therefore still recognizes the existence of horizontal components of external forces, but does not recommend tying an arch-ring to masonry situated above it and supported by it directly—at least to a certain but possibly unknown extent. (See Rankine's C. E., page 417). And on page 421 he says that "it is below those joints" (that is, the joints of rupture where p_y is zero and changes from positive to negative) that conjugate pressures (not tension) from without are required to sustain the arch, and that consequently the backing must be built with squared joints. The reverse conditions exist in pointed arches, and a weight must

be placed over the crown equal to that which would have been distributed over the two arches conceived to be extended to their own crowns or highest points.

He therefore, for the reasons stated above, prefers to insure the equilibrium of that portion of the arch above the joint of rupture, not by the application of external pressures from within or external tension from without, but by making the arch-ring thick enough to insure stability. This will be accomplished when the line of pressure is made to conform to a linear arch balanced under vertical forces only. This is equivalent to assuming a constant horizontal component of the thrust from the crown to the joint of rupture, and consequently it is held in equilibrium (see Fig. 271(*a*)) by only three forces, viz, the thrust T_0 at the crown; the weight W acting through the centre of gravity of the mass of the arch and its load above the joint of rupture, the aggregate weight of which is represented by W; and the thrust T' at the joint of rupture. If, then, the line GW is the vertical line through the centre of gravity of the load, and it be possible to draw from any point on this line two lines, one of which, Gv, is parallel to a tangent to the curve of the soffit at the crown, and the other, GD, parallel to a tangent to the soffit at the joint of rupture, so that the points v and D in which these lines pierce the joints at the crown and at the point of rupture, respectively, are within the middle third of the depth or thickness of the arch-ring, the stability of the arch will be secure; and if the second point be the centre of resistance at the joint of rupture, the first will be the centre of resistance at the crown, and the crown or highest point of the true line of pressure. If, however, the pair of points fall outside of the middle third of the arch-ring, the depth of the arch stones must be increased until the required condition is fulfilled. Both steps in Rankine's graphical method are simple and easy of application, the first determining the thrust at every joint from the crown to the springing, determining the position of the joint of rupture and the tangent to the true line of pressure at that joint; and the second determining the position of the line of pressure above the joint of rupture, and the proper thickness of the ring-stones in order that this line shall be within the middle third of the arch-ring, without resort to the uncertain application of special means of applying a system of external tensions in order to secure stability.

676. *Line of Pressure.*—As we have already seen, the location and construction of the true line of pressure is practically impos-

sible unless the assumption made by the particular investigator is true. Even then the uncertainty of the exact relations, magnitudes, and positions of the lines of action of the forces producing equilibrium leave us in doubt as to the extent of variation of this hypothetically true line of pressure from the actual and existing true line. Mr. Rankine only considers it necessary to construct a line of pressure in that portion of the arch above the joint of rupture, as below that point the externally applied pressures will take care of the line of pressure. And, moreover, instead of attempting to determine the true line, he is satisfied if with the given conditions he can construct any line of pressure which will lie wholly within the middle third of the arch-ring, and if this cannot be done with the form and dimensions of the proposed arch, then one or both of these must be altered so that the condition shall be fulfilled.

In Fig. 272 is shown a semi-arch, from which it is seen that if the crown-thrust is varied in position or magnitude, or in both respects, an infinite number of lines of pressure may be drawn. If it acts at the middle of the joint a, it is evident that if T_0 (or H) is increased the tendency is to raise the line of pressure nearer to the extradosal curve; if sufficiently, it will touch or even pass outside of the extrados, and will consequently tend to or actually cause the arch-ring to open at the intrados at some point S_2 or S_6. This condition will rarely if ever occur in flat arches. If, however, T_0 is decreased, the line of pressure will finally touch or pass below the intrados, and the tendency will be to open at the extrados at S_2 or S_4. If, however, the point of application of T_0 is raised towards C, while its magnitude is decreased, the line of pressure will be depressed, approaching more and more nearly the intrados. One such line is shown in $Cc_1c_2c_3D$. This condition gives the least crown-thrust consistent with equilibrium of the arch as a whole, and can be found from the equation $H = \frac{Wy}{x}$. If, on the contrary, the point of application is lowered towards C', while the magnitude is increased, the line of pressure will be raised towards the extrados, as seen in $C'Ss_1S_2F$, or some line touching the extrados at some other point than F. This is the condition for maximum crown-thrust. It would not be admissible for the thrust to be concentrated so near the edge of the arch-ring. It is therefore usual in all theories of the arch to limit the variation of the centre of resistance at the crown, that is, the point of application of the thrust T_0 or H, to points within the middle third of the arch-ring,

that is, somewhere between the points m and m', or for extreme variation from the centre a at the points m or m'. According to Rankine it is immaterial which one of these limits is taken. For the magnitude of the thrust, however, he uses $T_0 = p_0 r_0$.

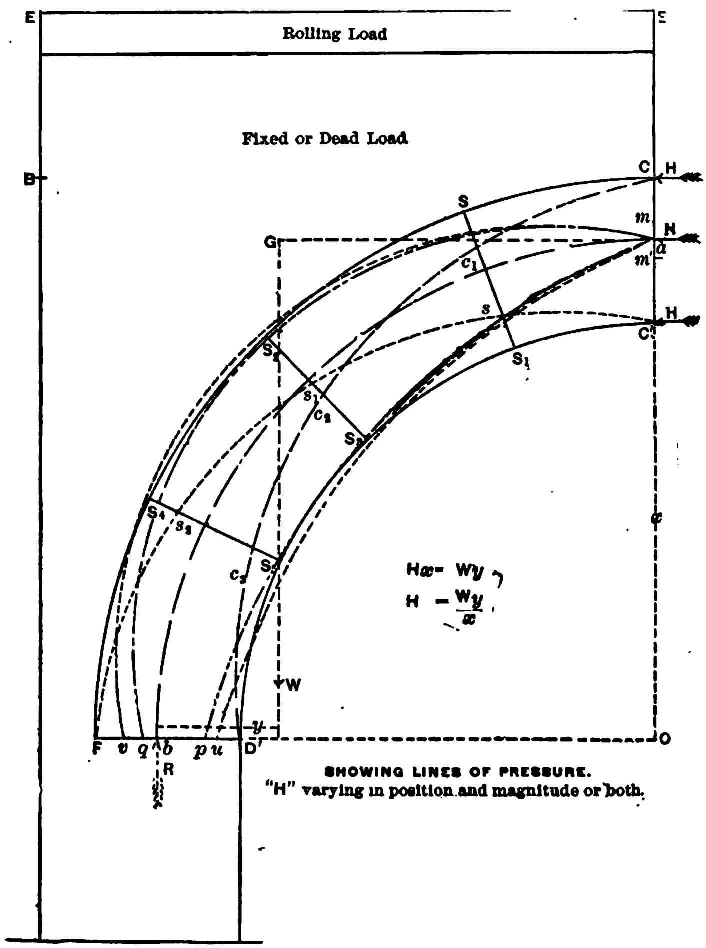


FIG. 272.

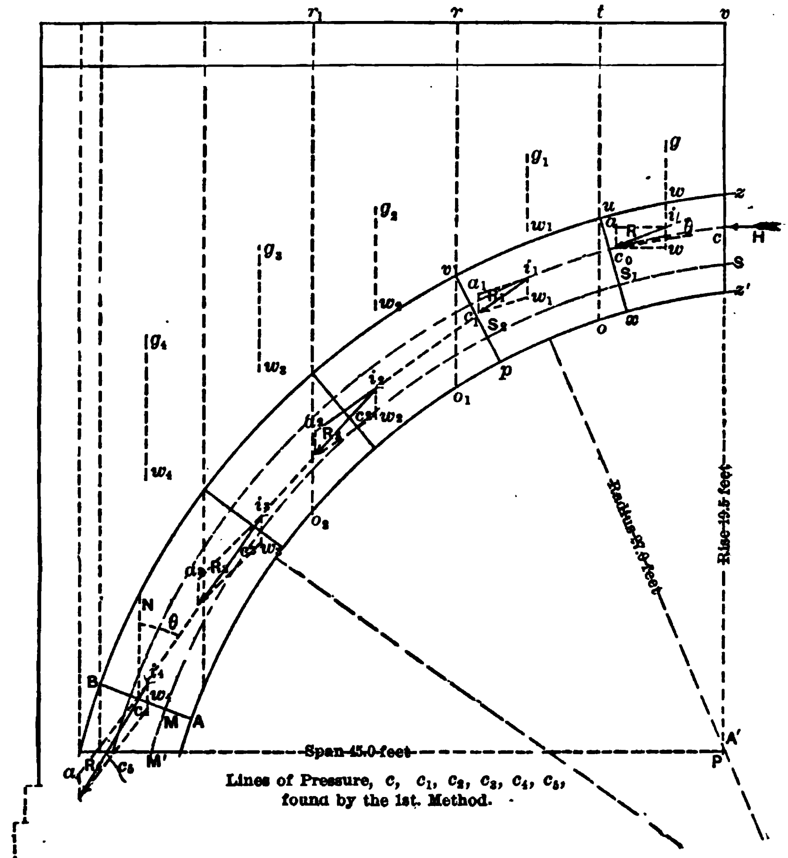
677. In the methods now to be explained the crown-thrust is determined from the equation $T_0 = \dfrac{Wy}{x}$, which will give the least

crown-thrust consistent with equilibrium. Its point of application is, consequently, according to the principle above established (see paragraph 676), placed above the middle of the ring at the crown, and to insure that no point of the surface at the crown joint may be under tension, it must not be more than one sixth of the thickness above the centre a; and, that it may be the least possible consistent with this condition, it is taken at the point m, one sixth of the depth from and above a (Fig. 272).

678. The ordinary method of constructing the line of pressure is clearly shown in Fig. 273. The point of application of the crown-thrust H is taken at the point C, the upper limit of the middle third of the crown thickness of the arch-ring, that is, one sixth of the thickness zz' above the centre point. Then, to determine the magnitude of this crown-thrust, it is usual to consider that only three forces hold the semi-arch in equilibrium, viz., the crown-thrust H acting horizontally at the point C, the thrust or reaction at the abutment acting at a point M, which is one third of the thickness of the ring from the intrados at A on the springing-line. Then $H = \frac{Wy}{x}$, in which W is the weight of the entire half arch and its load acting through the centre of gravity of the entire mass, including a uniformly distributed moving load. This should only be determined in this way when the joint AB at the springing is evidently the joint of rupture, that is, when $z'oo_1o_2A$ is about 45° of arc. In all other cases it is either necessary to find the position of the joint of rupture by Mr. Rankine's methods, or by the following, which is really the same as his method, though the form of the equation used is different (see paragraph 674): Take the equation $H = \frac{Wy}{x}$, and divide the mass above the arch-ring and the arch-ring itself into a series of small volumes by vertical planes. These volumes are represented by a series of figures, $z'vto$, $otro_1$, etc., which are approximately trapezoids, the areas containing the same number of units as the volumes, since the length of the arch perpendicular to the plane of the drawing is always taken as unity. But since the weights are made up of masses having each a different specific gravity, it is necessary to consider the mass above the arch-ring as reduced to the same specific gravity as the arch-ring itself, in order that the ordinates of the trapezoids may represent elementary weights or intensities to a given scale, including a similar reduction for the uniformly distributed load, so that the line $vtrr_1$ is the

limiting line of the elementary loads, and not the top of the spandrel-wall of the arch itself. Then the areas of the trapezoids multiplied by the weight of a unit of volume of the material of the arch-ring will be the weight of a certain portion of the entire structure included between the two consecutive vertical planes, such as vz' and to, to and ro_1, etc. It is not necessary at this stage of the work that the actual dimensions of the voussoirs should be known or considered; but if they are, the vertical planes should be taken through the edge of the joints at the extrados, as shown in the drawing, Fig. 273. For each joint the forces to be considered are the unknown crown-thrust H, the thrust at the joint under consideration, and the weight of that portion of the arch and its load between the crown and the joint. Taking the first joint ux, then, in the equation $H = \frac{Wy}{x}$, W is the weight of the volume $z'vtux$, considered to be a trapezoid; y is the perpendicular distance from the line gw to the point c_0; and x is the perpendicular distance between the line of action of H prolonged and the same point. Substituting, we find $H = t$, for example. Then, for the joint $v'p$, W is the weight o_1vrvz'; y is the distance between the line of action of the resultant of the two loads gw and g_1w_1 and the point c_1; and x is the distance between the line of action of H prolonged and the same point. Substituting, we find, say, $H = t'$; and similarly for the other joints all the way down to the springing AB. For the joint AB, W is the weight of the entire semi-arch; y the distance from the line of action of this weight to c_4; and x the vertical distance cA' from H to the same point, say, $H = t_4$. Then from these several values of H select that one which is the greatest. This greatest value will be the least value of the thrust at the crown consistent with the stability of the arch (see paragraph 655). The joint to which it corresponds will be the joint of rupture. If t is the greatest, ux will be the joint of rupture; if t', then the joint vp; or, finally, if t_4, then the joint AB, since in any case the joint of rupture is that position in the arch-ring at which the horizontal component of the thrust is a maximum. Above this joint Rankine's method of constructing the line of pressure is the same as the one to be described, though he finds the crown-thrust in his general theory in a different manner, that is, by first finding the joint of rupture and then the thrust, and not finding the maximum thrust and locating from it the joint of rupture. The result is the same, though the process is different. This may or may not be the same

crown-thrust as found from the equation $T_0 = p_0 r_0$. The probability is, however, that they will be, or that there will be no material difference, as all of the quantities entering into the problem are to a greater or less extent interdependent. Having, then, determined the point of application, magnitude,



Lines of Pressure, c, c_1, c_2, c_3, c_4, c_5, found by the 1st Method.

FIG. 273.

and direction of the crown-thrust, the construction of the line of pressure is very simple. Referring to Fig. 273, prolong the line of action of the horizontal thrust to the point of intersection with the line of action of the weight w of the trapezoid $z'vto$; then on these lines, respectively, lay off to the same scale the distance

$iw = w$ and $ia = H$, complete the rectangle, and the resultant pressure on the joint ux is represented by the diagonal R; and the point c_0, in which this resultant pierces the joint ux, is the centre of pressure or resistance. Then prolong R to intersection with g_1w_1, and lay off R and the weight w_1 of the trapezoid $otro_1$ on these lines, respectively, and complete the parallelogram; then its diagonal R_1 is the resultant pressure on the joint $v'p$, and c_1 is the centre of pressure. Prolong R_1 to intersection with g_2w_2, and forming this parallelogram, its resultant R_2 is the pressure on the next joint and c_2 is the centre of pressure; and so on to the joint AB at the springing. The line drawn through $cc_1c_2c_3c_4$ is the line of pressure. If this line is found throughout its length within the middle third of the arch-ring, as shown in the drawing, its stability is secured so far as rotation is concerned. That sliding at any joint may not occur, the angle θ between the direction of the resultant pressure R_3 and the normal, Nc_4, to the joint AB must not exceed the angle of repose ϕ of masonry on masonry. This same condition must be fulfilled at all other joints as well.

679. Scheffler's method is similar to the above-described method up to the point of finding the least crown-thrust. By the construction of a table, which will be explained below, the necessary calculations are simplified, and the actual determination of the centres of pressures graphically is made very clear and easy. It may be as well to mention that both in the preceding method and in Scheffler's the horizontal components of the external forces are not considered.

The crown-thrust is determined by a series of arithmetical trials, for the purpose of determining what particular values of W, x, and y will give the maximum value of H, and this is taken as the value for the least crown-thrust (see paragraph 655). This could be done as well without attaching any significance whatever to the joint of rupture, or without any reference to its position, and consequent height or thickness of the backing. In other words, the stability of the arch is to be secured by giving such a form and such a thickness to the arch-ring proper that the line of pressure will remain at all points within the middle third of the ring, or at any rate well within its thickness—not nearer, Scheffler says, than one fourth of its thickness to either intrados or extrados; then stability of the arch is secured. That is, what is known as the backing is really embodied in the arch-ring, as the crown-thrust in both of these methods is the same as the horizontal component of the

thrust at any other point, although it is taken as the maximum component of a long series of horizontal components. The method is much used, and will therefore be described somewhat fully.

680. The first step is to reduce the load on and that of the arch-ring to an ideal homogeneous mass of the same specific gravity throughout. This is taken as that of the arch-ring. Vertical ordinates are then laid off from and above the soffit to any scale, to represent these elementary weights at as many points as may be deemed necessary. The upper extremities of these ordinates are connected by a line, straight or curved, as the case may be. If this limiting line of loads $d_1d_2d_3$, etc., is straight and horizontal, then the total load on the semi-arch is represented by the weight of the area, or volume, unity in length, $DsKfaBkcd_1d_2 \ldots d_6ED$. If the limiting line of loads is curved, the volume will be the equivalent area enclosed between this curved line and the soffit. In either case the ordinates between the two will represent the intensities at any point of the soffit.

This area is divided by a series of vertical lines into a series of trapezoids, as in the preceding example. The smaller the trapezoid the more accurate will be the result. Only six trapezoids are used in Fig. 274. The position of the centre of gravity of the first trapezoid, Bcd_1q, is found by the proper method from the vertical cB. It is easy then to write the horizontal distances of the centres of gravity of the other trapezoids by a series of additions.

681. Form a table as shown below. Under column 1 is placed the number of the joint, not counting the crown joint. These joints are numbered 1, 2, 3, 4, etc. In the second column is placed the average heights of the trapezoids $= \frac{cB + d_1q}{2}, \frac{d_1q + d_2h}{2}$, etc.; in column 3 the widths cd_1, d_1d_2, etc.; in column 4 the areas of the trapezoids, which are the product of the numbers in columns 3 and 2; in column 5 the distances of the centres of gravity of the separate trapezoids, viz., $\frac{1}{2}d_1c$, $d_1c + \frac{1}{2}d_1d_2$, $d_2c + \frac{1}{2}d_2d_3$, etc. If the widths of the trapezoids are the same, we would simply add the constant width to the preceding distance in the column. In this case this is done for each trapezoid, except the one above the springing, for which we would add, for the distance of the centre of gravity of d_5d_6ED from cB, $\frac{1}{2}(d_4d_5) + \frac{1}{2}d_5d_6$ to the horizontal from G to cB.

In column 6 are found the moments of the weight of each trapezoid separately with respect to an axis at P; these are the

products of the numbers in columns 4 and 5. In column 7 the successive sums of the numbers in column 4 represent the areas of the first trapezoid; of the sum of the 1st and 2d, of the sum of the 1st, 2d, and 3d, trapezoids, etc.; that is, the total areas or their equivalent loads from the crown down to any joint, as 1, 2, 3, etc.

In column 8 are the sum of the numbers in column 6, taken 1st and 2d; 1st, 2d, and 3d; 1st, 2d, 3d, and 4th, etc. Now from the principle of moments it is evident that if we divide the numbers in column 8 by the corresponding numbers in column 7 we will find the distance of the line of action of the weight w_1 of the 1st trapezoid; the distance of the line of action of the weight of the 1st and 2d trapezoid; of that of the 1st, 2d, and 3d, etc., from the line cB. These quotients are recorded in column 9.

In column 10 are recorded the horizontal distances from P to the corresponding points in joints 1, 2, 3, etc.; that is, from P to p, P to p_1, P to p_2, etc.; and in column 11 is recorded the difference between these numbers in column 10 and those in column 9. These distances will be the lever-arms of the weights above any joint and the points p, p_1, p_2, etc., respectively. These distances are the values of y in the formula $H = \frac{Wy}{x}$; the values of x are the vertical distances, respectively, of the points p, p_1, p_2, etc., below the prolongation of the line of action of the horizontal crown-thrust H, which are scaled off and recorded in column 12. Substitute in $H = \frac{Wy}{x}$ the corresponding numbers in columns 7, 11, and 12, respectively, for W, y, and x: the resulting values of H are recorded in column 13. The greatest value in this column is what is called the least crown-thrust, and the corresponding number in column 1 is the corresponding joint; and as it is the point of maximum horizontal component of the thrusts around the ring, that joint will be the joint of rupture. This may be near the crown, at the springing, or at any intermediate point. It is to be observed that areas and areas by distances are called, respectively, weights and moments of weights. The weight of the unit of volume is omitted, as it would appear in both cases, and would consequently only make large numbers without affecting the result. But when using column 7 for the values of W in the above equation the weight of a unit of volume must be introduced if the areas are in square feet, and y and x from columns 11 and 12 are in feet. Then the

numbers in column 7 must be multiplied by 160, the weight in pounds of a cubic foot of stone, or by whatever is the weight of a unit of volume of the material of the arch-ring, and these products substituted for W. The numbers in column 13 should be multiplied by the same quantity.

TABLE LVI.

1	2	3	4	5	6	7	8	9	10	11	12	13
No. of the Section.	Dimensions of Trapezoidal Sections. height	width	area (Products of numbers in columns 2 and 3.)	Horizontal distance of the centre of gravity of each section from P.	Moment of each section about an axis at P (product of numbers in columns 4 and 5).	Areas of the loads above any joint (obtained by successive additions of numbers in column 4).	Moments of the sum of the loads above any joint, about an axis at P (obtained by successive additions of numbers in column 6).	Horizontal distance from the line of action of resultant load above any joint from P (the quotient of numbers in columns 7 and 8).	Horizontal distances from the points p, p_1, p_2, p_3, etc., to P.	Horizontal distances of the lines of action of the resultant load above any joint from the points p, p_1, p_2, etc. ($= y$).	Vertical distances of the points p, p_1, p_2, etc., below the line of action of the crown-thrust ($= x$).	Values of H found by substituting in $H = \frac{wy}{x}$ the corresponding numbers in columns 7, 11, and 12.
	ft.	ft.	sq. ft.	ft.	cu. ft.	sq. ft.	cu. ft.	ft.	ft.	ft.	ft.	cu. ft.
1	7.8	7	54.6	3.5	191.1	54.6	191.1	3.5	6.7	3.2	1.4	124.7
2	8.9	7	62.3	10.5	654.15	116.9	845.25	7.23	13.4	6.17	3.3	218.5
3	10.85	7	75.95	17.5	1329.12	192.85	2174.37	11.27	20.1	8.83	6.2	274.6
4	13.5	7	94.95	24.5	2325.27	287.8	4499.64	15.63	27.0	11.37	10.0	327.2
5	17.75	11.0	71.0	30.0	2130.0	358.8	6629.64	18.47	30.8	12.33	13.7	322.1
6	18.5	2.4	44.4	33.2	1474.08	403.2	8103.72	20.1	32.9	12.8	16.8	307.2
			403.2		8103.72							

The above table applies to the arch shown in Fig. 274, of which the span is 73 feet, rise 18.5 feet, thickness of arch-ring 3 feet, ordinate $cB = 7.5$ feet, $d_1q = 8.1$ feet, $d_2h = 9.7$ feet, $d_3K = 12.0$ feet, $d_4L = 15$ feet, $d_5D = 18.5$ feet, $d_6E = 18.5$ feet. The average of these, taken two and two in succession, will be the mean heights of the rapezoids, viz., $\frac{7.5 + 8.1}{2} = 7.8$ feet, $\frac{8.1 + 9.7}{2} = 8.9$ feet, and so on. These are the numbers recorded in column 2. The widths cd_1 to d_3d_4 inclusive $= 7.0$ feet each, $d_4d_5 = 4$ feet, and $d_5d_6 = 2.4$ feet. These numbers are recorded in column 3. In column 4 products of corresponding numbers in columns 2 and

3. In column 5 first distance $w_1P = \frac{1}{2}cd_1$; then successive additions of 7, to and including section 4; then add $\frac{7+4}{2}$ and $\frac{4+2.4}{2}$ in succession. Column 6 products of corresponding numbers in columns 4 and 5, and in column 7 the continued sum of the numbers in column 4. The last term should equal the sum of all

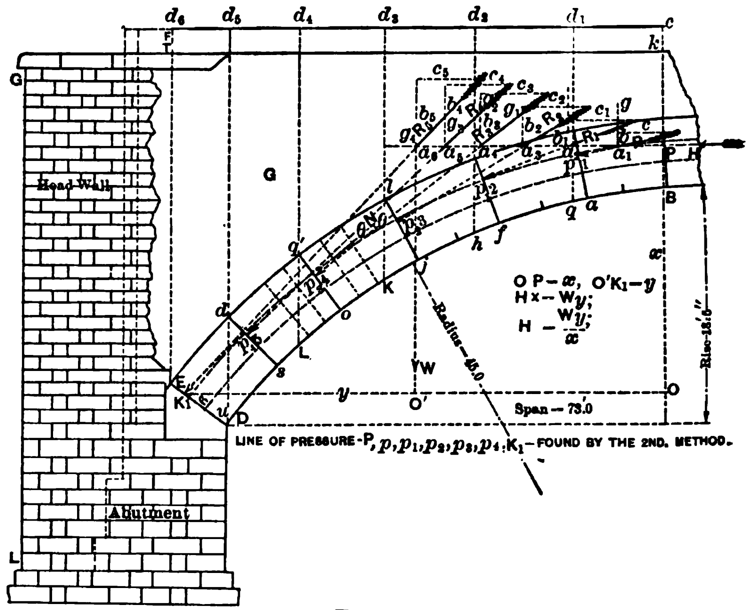


FIG. 274.

numbers in column 4. In column 8 are the continued sum of numbers in column 6. The last term should be equal to the sum of all numbers in column 6. In column 9 are the quotients arising from dividing the numbers in column 8 by those in column 7. The numbers in column 10 will depend upon the exact position of the joints 1, 2, 3, etc., with respect to the verticals, and will have to be determined after locating the joints and the positions of p, p_1, p_2, etc., approximately at least. These latter may be taken either at or near the centre.

In general it may be taken a little less than the continued sum of the numbers in column 3. Assume a decrement of 0.3 at each joint, we write column 10 as follows: $7 - 0.3 = 6\ 7, 14 - 0.6 = 13.4$, etc. The numbers in column 11 are the differences of the numbers

in columns 9 and 10. In column 12 are distances scaled from the horizontal line of crown-thrust prolonged. And, finally, column 13 is run out by substituting corresponding numbers in columns 7, 11, and 12 in $H = \frac{Wy}{x}$ for W, y, and x, respectively; and we find the maximum horizontal component of the thrust at the joints which is the crown-thrust to be used in practice, and not the least, as found in the table opposite joint No. 1, which we see is the number opposite joint 4 in column 1 of table, and joint $q'o$ in Fig. 274. The $q'o$ is the joint of rupture, and the least crown-thrust is $H = T_0 = 327.2 \times 160 = 52{,}352$ pounds. For the joint 3, $H = 274.6 \times 160 = 43{,}936$ pounds; and for joint 5, $H = 322.1 \times 160 = 51{,}536$ pounds. If now the number of sections between 3 and 5 be increased, as shown by the dotted joints on either side of joint 4, and the calculations repeated, we would find more accurately the maximum thrust and the true joint of rupture.

682. Assuming maximum H as above found, we can proceed to construct the line of pressure, prolong the line of action of H, and mark points on it the distances from P as found in column 9, these points being on the line of action of the total load above the corresponding joints. The first intersection is a_1. Then laying off vertically to the scale adopted for the loads the line $b = 54.6 \times 160 = 8736$ pounds, and from its upper extremity to the right the horizontal line $c = H = 52{,}352$ pounds, and drawing R, the third side of the triangle of which b and c are the other sides, then R will be the resultant pressure for the joint, and p will be the centre of pressure. Then on the same horizontal line the point a_2 distant from p 7.23 (see column 9), erect the vertical $b_1 = 116.9$ (see column 7) $\times 160 = 18{,}704$ pounds, and $c_1 = c = 52{,}352$ pounds, draw R_1, it will be the resultant for joint 2, and p_1 will be the centre of pressure. Then at a_3 11.27 feet from P, erect $b_2 = 192.85 \times 160 = 30{,}856$ pounds, and draw horizontally $c_2 = c_1 = c = 52{,}352$ pounds; the closing line R_2 will be the resultant for joint 3, and p_2 the centre of pressure. Continue this process, using horizontal distances from P as found in column 9, and volumes in column 7 multiplied by 160, the weight of a unit of volume, i.e., a cubic foot, which is laid off vertically, and the constant horizontal thrust H, as already shown; and if the line of pressure joining the points $P, p, p_1, p_2, p_3, p_4, p_5, p_6$ lies wholly within the middle third of the arch-ring, the stability of the arch will be secured, provided the joints are so arranged that sliding cannot take place, and the intensity of the thrust on any joint does

not exceed a safe resistance of the material to crushing. If the line of pressure falls without the middle third, either the form of the arch must be changed or the thickness of the ring increased, or both.

It is to be noted that the trapezoids do not exactly represent the loads on the sections of the arch-ring. For instance, at the joint 1 there is a small wedge in excess of the load between the joint and the vertical, while at the joint 4 this excess wedge $q'oL$ is quite large. This can in part be remedied by moving the joint either in position or in direction, or by deducting the weight of the wedge from that of the trapezoids, which in turn would necessitate a change in the position of the lines of actions of the loads. The method can only be approximately correct on this account as well as on the assumption of a constant horizontal component of the thrust at all points of the arch-ring. The safety must therefore be provided by a sufficient thickness of arch-ring, and backing up the arch as high as the joint of rupture.

BRICK ARCHES.

683. The same general principles of stability apply to brick arches as to those of stone. It is usual to make the thickness of the arch-ring from one-eighth to one-fourth part more.

Probably the main objections to the use of brick arises from the difficulty of securing at a reasonable price bricks uniform in dimensions, hardness, and strength; the inability to secure good and sufficient bond necessitates, to a great extent, the construction of the arch with a series of rings, only held together by the adhesive and tensile resistance of the mortar; the great number of joints and the varying thickness of these joints required to cover increasing lengths of arch-rings, and lastly the non-homogeneous nature of the material composing the arch.

Many of these difficulties and objections can be in whole or in part removed, and at the present time it may safely be said that the choice between stone and brick is rather that of convenience and cost than of strength and durability. The main point of good bond can be secured by maintaining thin joints in each ring, but so regulating their thicknesses that at reasonably short lengths headers can be introduced, bonding the rings together and producing unity of action between them. This construction is shown in the left-hand half of the arch (see Fig. 275), while on the right is shown the construction with separate rings only connected with the

mortar. If good bricks, good cement, and good workmanship are required and secured, the brick arch will furnish a permanent, stable, and strong structure. The conveniences attending the use of bricks render them the almost sole reliance for tunnels, sewers, and similar structures. Brick arches can be materially strengthened

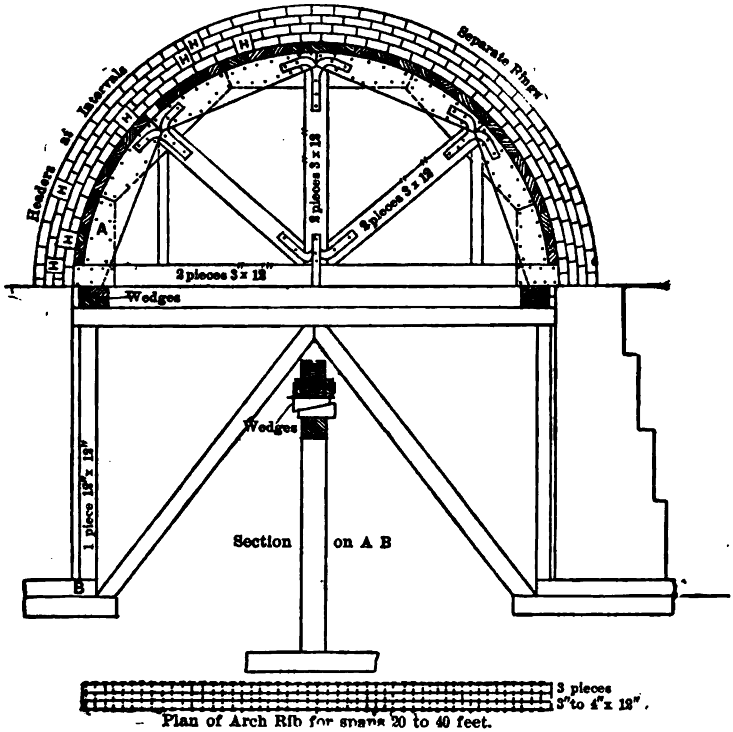


FIG. 275.

by the use of hoop-iron placed in the joints. This application of iron is similar in its effects to that of the wire netting or iron ribs in connection with concrete arches, which have been fully explained and illustrated.

SKEW-ARCHES.

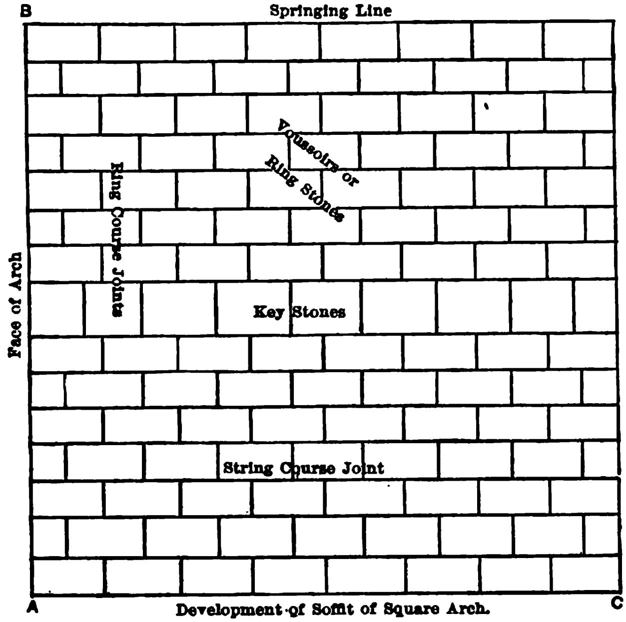
684. Although the construction of skew-arches is avoided as much as possible, yet the engineer is often called upon to construct them, and consequently the general principles in their design and construction should be understood.

The square arch being one in which the axis of the arch is perpendicular to the plane of the face of the arch, the skew-arch is one in which the axis is inclined to the faces or ends of the arch. The angle of skew is the angle which the axis of the arch makes with a normal to the plane of the face. In a square arch the planes of the face and those of any parallel vertical planes are perpendicular to the planes of the faces of the abutments, whereas in a skew-arch similar planes make, respectively, obtuse and acute angles with each other, the obtuse angles being at diagonally opposite corners, and the same for the acute angles.

Skew-arches of short span may be constructed similarly to square arches, but the direction of the thrust being oblique to the beds of the voussoirs, would tend to force the arch stones at the springing and at the acute angles outward. To obviate this tendency the bed-joints of the voussoirs must be normal to the direction of the thrust. This requires that the bed-joints or string-course joints of the voussoirs shall be curved. In order, then, to design a skew-arch with true or proper string-course joints, it becomes necessary to develop on a drawing to a large scale the soffit of the arch, and upon this drawing lay off two sets of lines, the one representing the line of thrust which will be parallel to the ring-course joints, that is, parallel to the faces of the arch, and another set of lines perpendicular to these at every intersection.

685. A circle or wheel making one revolution on a horizontal plane will trace a straight line equal in length to its circumference. This straight line is called the development of the circumference; and similarly, if a right cylinder be revolved once on a horizontal plane, a rectangle will be traced whose area is equal the cylindrical surface of the cylinder. The half of this would be the development of the semi-cylindrical surface. This is shown in Fig. 276, in which the line AB is the development of soffit-line on the face of the arch and AC is the springing-line. The continuous straight joints perpendicular to AB or parallel to AC are the string-course joints, and the discontinuous lines normal to these are the ring-course joints. The respective distances apart of the joints in each set are arbitrary, depending upon the widths and lengths of the voussoirs to be used. Having due regard to good bond, the development of the soffit of the square arch is only a convenience in laying out the dimensions of the voussoirs; and when their lengths and widths materially differ, it enables the builder to locate more easily the stones for each string-course.

686. To make the development of the soffit of the skew-arch is somewhat more difficult. Although it is only necessary to develop the half of the soffit, the whole soffit is developed in the drawings.



Development of Soffit of Square Arch.
Showing lines of pressure or ring and string course joints.

Fig. 276.

Take, as example, a skew-arch whose span is 22.5 feet on the skew, length parallel to its axis 32.0 feet, rise 10 feet, angle of skew 30°. In Fig. 277 $GLQK$ is the horizontal projection of the soffit on the skew, RAE is the horizontal projection of any vertical section on the skew, and TAB the horizontal projection of a vertical section on the square; the angle $BAE = 30°$.

If, now, a series of lines, aa', bb', cc', be drawn parallel to the axis HF and prolonged until they meet the circular arc MND, Fig. 277(a), described with a radius of 10 feet from any point C on the axis HF prolonged, MND being the vertical section of the soffit revolved on the horizontal plane, the lengths of the various ordinates, x_2y_2, x_1y_1, etc., can be readily calculated or determined graphically. These ordinates are equal to the ordinates of the corresponding points on the vertical section on the skew. If, now, we draw any line RO, Fig. 277(b), parallel to QA, and draw

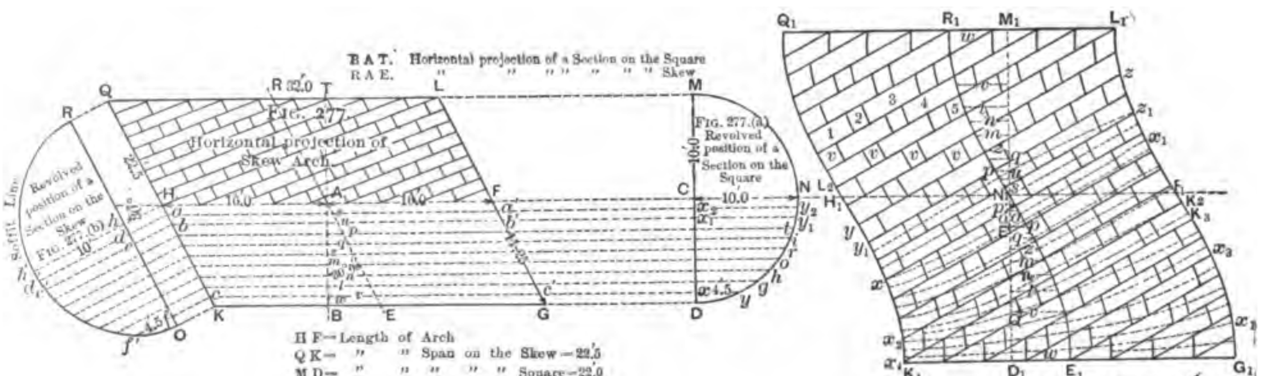


Fig. 277. (c).

Development of the Soffit of Skew-arch.

$Q_1H_1K_1 = R_1N_1E_1 = L_1F_1G_1 = ohR$ (Fig. 277(*b*)) = lines of pressure, also of all ring-courses or heading joints, being lines on soffit of arch parallel to the face.

Full lines yz, y_1z_1, etc., = spiral string-course joints, and are perpendicular to lines of pressure at crown alone.

Dotted lines x_1x_4, x_2x_3, etc., = true string-course joints, being perpendicular to lines of pressure at every intersection, but rarely used.

$M'N'D' = MND$ = development of section on square.

$R'N'E' = Rho$ = development of section on skew.

v, v, v, v = voussoir of arch giving shape and position with dimensions to scale to be numbered 1, 2, 3, etc., and built into wall as shown on development.

a series of normals to it from the points H, a, b, c, etc., and on these make hh', dd', ee' and ff' equal, respectively, to CN, x_2y_2, x_1y_1, and xy of Fig. 277(a), then will the curve $Rh'd'e'f'O$ be the revolved position of a vertical section of the soffit on the skew.

Prolonging the axis FH, and taking $H_1F_1 = FH$, at its middle point N_1 draw a line $M_1N_1D_1$ perpendicular to it, making $M_1N_1 = N_1D_1$, equal to the lengths of the quadrant $MN = ND$ of Fig. 277(a). Then will $M_1N_1D_1$ be the development of the vertical section of the soffit on the square. (See Fig. 277(c)).

Then from N_1, in both directions, on N_1M_1 and N_1D_1 lay off a series of distances, $N_1p' = Nu$, $N_1o' = Np_1$, $N_1q' = Nq$, so on, and from these points of division lay off normals, respectively, equal to the distances between the lines AB and AE, Fig. 277, viz., s, u, p, q, m, n, l, t, w ($= BE$), BE being equal to D_1E_1, and through the extremities of these ordinates draw the curved line $E_1N_1R_1$, reversing at N_1. This line will be the development of the vertical section on the skew. Curved lines $Q_1H_1K_1$ and $L_1F_1G_1$ parallel to and at distance from it equal one half the length of the arch will be the development of the soffit on the faces or ends of the arch. The figure $K_1H_1Q_1R_1L_1F_1G_1D_1K_1$ is the development of the entire soffit of the arch. On this development draw any number of curved lines parallel to $K_1H_1Q_1$, shown by heavy, broken lines. These will be lines of pressure, some portions of which will also be the ring-course joints. Then draw a series of lines—dotted in the drawing—which shall be perpendicular at each intersection to the lines of pressure. The equation of these lines is logarithmic. They can be drawn approximately with the free-hand.

It will be noticed that these string-course joints, commencing at or near K_1, get farther and farther from the springing-line $K_1D_1E_1G_1$, —so much so towards G_1 that short string-course joints have to be introduced to prevent the blocks or voussoirs becoming too large,—and that these lines are asymptotes to the springing-line. A similar set of joints commencing at L_1 and separating towards Q_1 can also be drawn, but it is only necessary to draw them on one half of the development. It is evident that each voussoir in the true skew-arch in either half is of a different size and shape. Each has to be cut and numbered to correspond with its position on the development. They might be cut straight on the ring-course joints and likewise on the string-course inclined at different angles for each block or voussoir. The skew-arch with true joints is seldom used. As a substitute the ring-course joints are left as before, and from

the obtuse angles K_1 and L_1 straight lines are drawn, L_1L_2 and K_1K_2, so that they shall be normal to the ring-course joints at one point, that is, at the crown H_1 and F_1; and from K_2 to G_1 is divided off into suitable widths for the voussoirs, also from K_2 to L_1 and similarly from L_2 on the other face of the arch; from these points of division lines are drawn over the entire development parallel to K_1K_2 and L_1L_2, as shown by the full straight lines, which are then the string-course joints. These lines intersect the springing-lines. There is required, then, a series of skew-backs or wedge-shaped stones from which the arch springs. The ring-course joints are then drawn either straight or curved, so as to limit the lengths of the voussoirs and produce a good bond. In this construction the stones are more easily cut, and while it will be convenient to number and have a place for each stone, it is not necessary. It is not very difficult to build a skew-arch of bricks.

The skew is often obtained by building the arch of a series of ribs which project beyond each other in opposite directions on the two sides so that the proper obliquity or skew will be obtained. This can be done by making both arch and its abutments of these ribs, or the abutments may be built with a plane face, and the skew obtained by arched ribs set with the proper projections on the abutment.

CENTRING FOR ARCHES.

687. No arch becomes self-supporting until keyed up—that is, until the crown or keystone course is laid. Until that time the arch-ring, which should be built up simultaneously from both abutments, has to be supported by frames called centres. These consist of a series of ribs placed from 3 to 6 or more feet apart, supported from below. The upper surfaces of these ribs are cut to the form of the arch, and over these scantlings or laggings are placed from 2 to 6 inches thick, which form an open or closed sheeting upon which the arch stones rest. The ribs may be of timber or iron. They should be strong and stiff. It is good economy not to spare bolts and braces, and, if possible, to strengthen them by direct supports at close intervals from springing of the ribs. Any deformation that occurs in the rib will distort the arch, and may even result in its collapse on removing the centres. Only good judgment and experience can be relied upon in designing, constructing,

and supporting the ribs. Rigidity is essential. A great variety of designs have been used.

Just at what point the weight of the arch comes on the centre is not known. It is stated generally that the stones begin to bear on the centre when that joint is reached at which one voussoir would slide on the one below it. It is well to provide for all and every unfavorable condition; and even after the arch is keyed there will be a greater or less pressure on the centering, as is evidenced by the fact that all arches settle more or less on its removal. When the centring should be removed is a question that never has and perhaps never will be agreed upon. Some engineers recommend the almost immediate removal after completion of the arch, on the view that it is better to let the arch take its own conditions and position of equilibrium while the mortar is soft.

688. This certainly is not the usual practice. In tunnel-work it is not uncommon, however, to build a section of 15 to 20 feet of the lining, and as soon as the excavation ahead will permit remove the centring, carry it forward, and proceed with the building of another section. As a rule, the centring is rarely taken away under ten days or two weeks, and often for a very much longer period of time.

689. Some form of wedge, or supports in boxes of clean dry sand are placed between the ribs and their main supports. These have the double purpose of bringing the ribs to their proper potions by driving the wedges in or out, and thereby adjusting all ribs to their proper and relative positions; and also to enable the gradual removal of the centre by lowering it a little at a time, in order to prevent shocks or a too sudden throwing of the arch on its bearings or supports. Simple queen wedges are commonly used, but a form of compound wedge shown in Fig. 278 possesses many advantages.

For short spans, or where direct intermediate supports can be obtained, there is probably nothing better than the form of rib shown in Fig. 275, which also shows the two more common methods of building brick arches.

As seen in the figure, the rib is composed of a series of segments, usually made of three pieces of plank from 2 to 4 inches thick, framed so as to break joints, the width of the planks at the joints varying from 6 to 8 inches, and between these from 10 to 12 or more inches. A tie-beam, likewise made of two pieces of plank, tie the ends of the rib together, and when the span is over 8 or 10

feet intermediate struts and braces are introduced. The whole is thoroughly bolted together with rods, bolts, and straps. The direct supports are capped. Queen wedges are placed between the cap and the tie-beam of the rib. The construction is clearly shown in all of its details on the drawing.

Another form of centre, which is of the braced bowstring girder type for long spans, is shown in Fig. 278. The compound

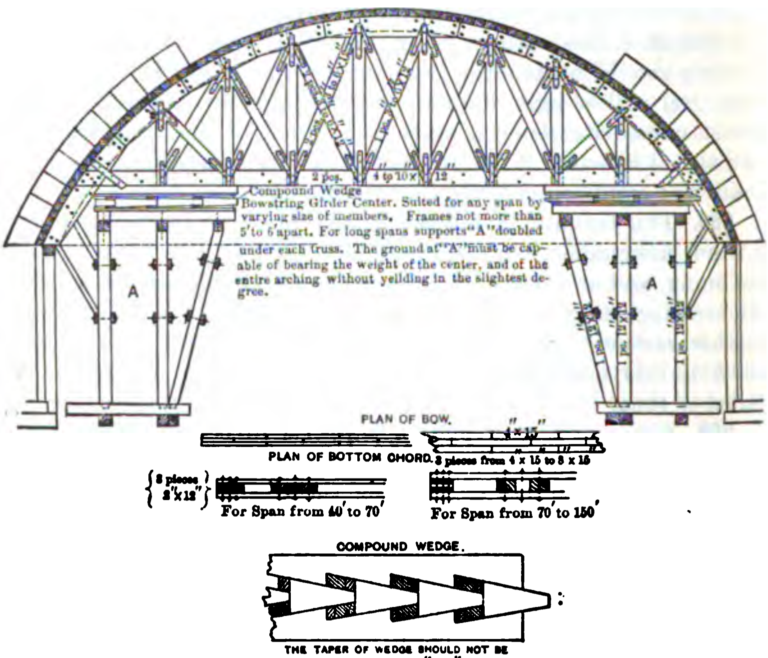
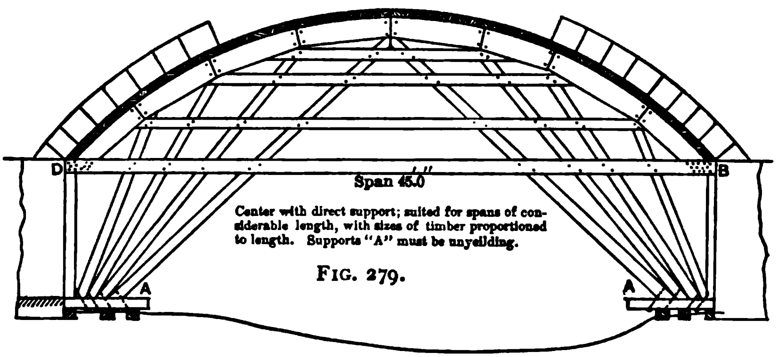


FIG. 278.

wedge is also shown in connection with this design. The rib is, as before, built of several pieces well bolted and braced, and as constructed has considerable inherent strength in itself. The exact form of bracing adopted admits of great variety in its details. The proper rule is to make the arch rib strong in itself, even to the extent of carrying the entire load, and the design and construction of the bracing to be also stiff and strong, as, whatever may be the construction, it will always be uncertain in what proportion the load will be distributed between the two. The drawing also shows the arch-ring as partly built up from the two abutments.

In Fig. 279 is shown another form of centre, in which the several joints of the ribs are supported directly by a system of braced



Center with direct support; suited for spans of considerable length, with sizes of timber proportioned to length. Supports "A" must be unyeilding.

FIG. 279.

struts. This will make a good form of centre so long as the unsupported length of any strut is not too great.

In all cases the arch is built up simultaneously and symmetrically from the two springings to the crown. It is not unusual to load the centre with its entire load, or a large portion of it, simply laid on top dry, and subsequently laying and bedding the voussoirs in mortar, or, sealing the edges, and filling the joints with a cement grout. This latter method subjects the centre to its load, and brings all parts into bearing, before the final adjustment and bedding the voussoirs in their proper positions.

690. *Abutments and Piers for Arches.*—There is no fixed rule for the top width of piers and abutments for arches. To a great extent the practical requirements of construction will regulate the thickness, as in the case of piers for trussed bridges. In full-centre arches, where they spring vertically from the abutments, there is no tendency to overturn either piers or abutments from the arches proper. In such cases the piers between the two arches of a series need not be much greater than the sum of the thicknesses of the arch-ring, that is, from $1\frac{1}{2}$ to 2 times. For the abutments sufficient room for the arch-ring and the thickness of the backing will be usually sufficient. Mr. Rankine says that for the piers of a series of arches each pier should have sufficient stability to resist the thrust from a moving load on one of the arches springing from it, which may be roughly computed by multiplying the travelling load per lineal foot by the radius of curvature of the intrados at the crown in feet, and then proportioning the thickness in accordance with the thrust applied

at its top and the height of the pier. By the principles already established, and from some of the best existing examples, the thickness of piers varies from one tenth to one fourth of the span, while for the thickness of the abutments those values approaching the one-fourth limit are most suitable, the thickness of the pier ranging from one seventh to one sixth the span.

Abutments, however, are usually backed up with earth, which often extends to great heights above the arches. This necessarily neutralizes the thrust of the arch in part or entirely, and may frequently require the thickness necessary to resist the inward pressure of the earth, as the side walls of tunnels, or those of retaining-walls. In another paragraph Mr. Rankine gives the thickness of the abutments at from one fifth to one third the radius of curvature at the crown.

Mr. Trautwine's rule is as follows: When the clear height of the abutment above the ground does not exceed one and a half times its thickness at the ground surface, in other words, if the thickness at the ground is taken at two thirds the height, then the thickness at springing is in feet

$$= \frac{\text{radius in feet}}{5} + \frac{\text{rise in feet}}{10} + 2 \text{ feet.}$$

Mr. Baker gives, as the formula used by German and Russian engineers,

$$t = 1 + 0.04(5s + 4h),$$

in which s is the span and h is the height of the abutment, i.e., the springing-line, above the top of the foundation; presumably this means the clear height above the ground.

The French rule seems to give an average thickness for the abutments from one third to one fourth the span, reaching as high as one half in some cases, and rarely less than one fourth the span.

STONE AND BRICK MASONRY ARCH BRIDGES.

691. In Fig. 280 is shown longitudinal and cross-section of one of a series of seven brick arches for a bridge of 818 feet in length, constructed over the Raritan River, N. J. Each of these has a span of 75 feet. They are built on a skew of 80°. The arches are of brick, with a granite ring-course at their ends. The rise is 15 feet, or one fifth of the span, which is small for this kind of

masonry. The joints, owing to the small difference in length between the intrados and extrados, can be easily broken without special trouble. For this purpose the bond is secured by using five courses of stretchers and then a course of headers. The stone masonry for the piers is of rock-faced Stockton stone, in courses not less than 15 inches in thickness, alternating header and stretcher. The beds are not less in width than the thickness of the course, with vertical joints dressed not less than 8 inches. The total cost of this structure was $99,000.

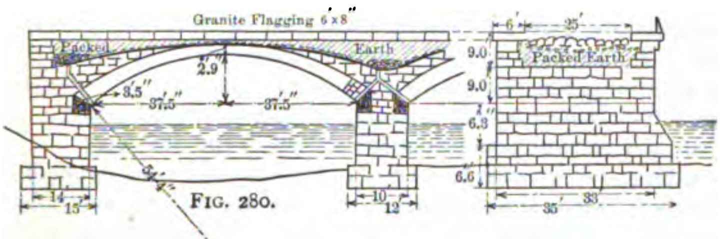


FIG. 280.

The centre portions of the pier are built of good rubble. The ice-breakers have no projections of over ¾ inch on the faces. The skew-backs, of Stockton stone have sufficient area to take the whole bearing of the arch on one stone; each of these are dressed to a bed of 3 feet. The best hydraulic cement was used. The piers have a thickness of 10 feet at the springing and 12 feet at the base. The abutments are 14 feet at springing and 15 feet at the base.

The arches are built of the best hard-burned brick, laid 5 courses of stretchers and 1 of headers, with thin joints. The stone masonry ring-courses are 2 feet in thickness, alternating 1½ and 2 feet in length. These are of rock-faced granite, with 1-inch chisel-draught. The brick arch, between granite ring-courses at each end, is 2 feet 5 inches thick at the crown and 3 feet 5 inches at the springing. The spandrel-walls are built of coursed rock-faced masonry, with beds dressed 1¼ times the thickness of the course, vertical joints dressed for 6 inches from the face, and with ¾-inch chisel-draughts. The filling or backing of the piers and over the arch is good rubble. The upper surface of the backing is sloped so as to carry the water to drains over the piers, and is covered with a layer of mortar 1 inch thick. Over this is a 3-inch layer of gravel used for drainage purposes. Above this, to the level of the subgrade of the roadway, is placed earth rammed in layers. The roadway is 25 feet wide;

this is formed of Belgian blocks of trap-rock, and 6 inches thick, resting on a 3-inch layer of clean sand. The joints between the blocks are filled with sand. The sidewalks, 5 feet 6 inches wide, are formed with granite flagging bedded on top of the spandrel-wall. An iron railing completes the structure. The grade of the roadway over arches is 0.7 per cent.

The calculated horizontal thrust at the crown was 36,000 pounds and at the springing 56,400 pounds per foot of length; and as the thickness or depth of arch at the crown and springing are, respectively, 29 and 41 inches, the areas per foot of length are 348 and 492 square inches, giving as the thrust on the brickwork at crown and skew-back, respectively, 103.5 and 114.5 pounds per square inch, and assuming the crushing strength of brick at 1750 pounds per square inch, there results a factor of safety of 15.2 nearly. The details and general dimensions are shown in the drawings.

A large brick arch has been contracted for over Jones Falls, Baltimore, with the following general dimensions and design. Span, 130 feet; thickness at crown, 5 feet. It is a skew-arch, built in ribs set with projections on top of the piers. The ribs are 4 feet wide. The piers are built with plane faces and set at the proper skew.

692. Much study has been given by German engineers to the theory and practice of arch construction, with satisfactory results in many respects, both in regard to simplicity of design and economy in construction. Their arches do not spring from vertical-faced abutments at points high above the ground, but both the curves of the intrados and extrados, or curves parallel to them, extend well into the ground, and they spring from natural abutments, or artificial ones constructed of Portland cement, in which are placed frequently large blocks of stone set on radial lines. The curve of the intrados is usually segmental, or of the basket-handle form. The apparent arch, or that portion above the ground, has a small rise in proportion to the length of the span—generally only 1 in 10. The beton is composed of 1 cement, 3 sand, and 1 broken stone, and in arches of short span the large blocks of stone are imbedded to as much as 30 per cent of the mass. For spans under 98 feet in width the arches are themselves constructed of beton, composed of 1 cement, 3 sand, 4 broken stone. Sometimes the end ring-courses are cut stone, the concrete being used for the body of the arch. The spandrel is of rock-faced broken-range masonry, the mortar being 1 cement and 1½ sand. In large arches the

usual cut-stone voussoirs are used backed with rougher masonry. The joints between the voussoirs are from 0.6 to 0.8 inch in thickness, and dry cement is packed in them with thin iron tamping-tools. Instead of solid spandrel-walls, columns arched over are used to support the roadway above the haunches of the arch. The haunches are built of rubble-stone and cement mortar, or of a concrete composed of 1 cement, 2 sand, and 3 gravel. The arches are covered with a layer of cement mortar about 1 inch thick, which itself is covered with a layer of asphalt 0.3 inch in thickness, on the surface of which the water is carried to outlets near the springing. The thickness of the arch-ring is reduced to a minimum consistent with safety.

Under the ordinary methods of constructing masonry arches of long spans cracks are not uncommonly developed on striking the centres, caused by a sinking at the crown of 1, 2, or more inches. To obviate this trouble several means have been suggested, such as rounding the joint of the voussoir above the joint of rupture, or leaving the middle portion flat and rounding the two portions of the surface on either side of it. Either of these methods would require special voussoir stones having great strength, as a large pressure would have to be distributed over a relatively small surface. Such methods in metallic structures would be practicable, as it is easy to concentrate at the point of rotation a sufficient bearing surface, but they fail in masonry structures. M. Liebbrand, of Stuttgart, proposed to substitute for the rounded joint a thin plate of lead 0.8 inch in thickness. These plates to be placed in the joints at the crown and the points of rupture, but only covering the middle third of the joint, which should be left open above and below. Lead has the property of spreading laterally without losing its cohesion. The effect of this would be to limit within a small area the varying positions of the line of pressure caused by permanent or moving loads on the arch.

693. If from any cause the curve of pressure approached too near to the edges of the plate, causing a pressure greater than the resistance of the plate, it would yield, thereby increasing the area and reducing in proportion the pressure per unit of area.

The important questions to be determined were (1) the compressive resistance of lead and the proportionate lateral yield under different pressures, and (2) the durability of lead when used in joints.

As a result of numerous experiments it was found that sheets

of soft cast lead from $\frac{5}{8}$ to $\frac{3}{4}$ inch in thickness would support, without yielding, about 1700 pounds per square inch; that with a pressure of 4200 pounds per square inch on cubes of $3\frac{1}{8}$ inches on the edges the lateral yield or increase of area was in the ratio of 1 to 1.3, decreasing the unit pressure in the same ratio, or to about 3231 pounds. Increasing the pressure to 12,780 pounds per square inch on the original surface, the horizontal section increased so rapidly that the unit pressure only increased in the ratio of 1 to 1.27, at which it remained practically constant.

As to the question of durability, it is known that the Romans used sheets of lead between cut stones; and arched bridges built from forty to sixty years ago had sheet lead inserted in the joints for the purpose of securing a uniform distribution of the pressure.

The only question, then, to be considered is the great reduction of the available area of joint surface at the crown and joints of rupture, whereby the adjacent stones will be subjected to a very high unit pressures over about one third of their depths. The only remedy for this is either to provide particularly strong stones, such as basalt, granite, or silicious sandstone, or to subject the stones in those positions to unusual unit pressures, the pressure per square inch varying in some actual structures from 700 to 1704 pounds per square inch, and it is claimed that a stone possessing a resistance of from 10,650 to 11,360 pounds per square inch will be sufficiently strong, giving an average factor of safety of 10.

694. The following are examples of some of the German arches, with the cost based upon an area equal to the horizontal projection of the arch included between the foundations:

TABLE LVII.

Bridge.	Arch Span.	Cost per sq. ft. projected area.
Over the Enz	147.6 feet	\$3.20½
" " "	67.57	2.09
" " Glatt	68.22	1.46½
" " Murr	139.7	2.43½

In Fig. 281 is shown a half section of the arch over the Enz. The span along AB at the foundation level is 147.6 feet; the apparent opening along CD is 98.8 feet, with a rise of 9.2 feet; the width between parapets, 12.8 feet, of which 8.2 feet is roadway. The foundation-bed is sandstone, and covered with a bed of sand and gravel. Portland cement was used in the foundation and

masonry. The specified test for this cement was that briquettes of 1 cement and 3 sand should show a tensile strength of 113 pounds per square inch when seven days old, one day in water and six days in air, if quick-setting, and 138 pounds for slow-setting. The reverse rule is adopted usually in this country. The

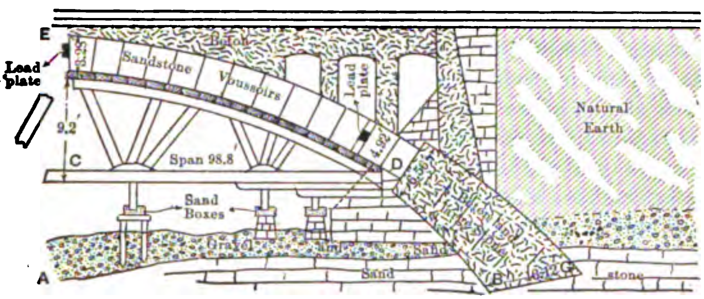


FIG. 281.

buried portion of the arch is made of beton containing rough stone laid with radial joints. The centring for this portion of the arch was a bed of stone covered with a coating of dry mortar. The timber centre rested upon piles and temporary piers. The direct supports rested upon sand, in boxes 10 inches square and 4¾ inches deep. The sand was washed and dried and kept from again becoming wet. The centre was loaded with all of the voussoirs before commencing the arch work. This latter work was completed in two days after commencing. The entire arch was built of sandstone having a resistance to crushing of 13,277 pounds per square inch. Thickness of arch at crown, E, = 3.28 feet; at springing, D, 4.92 feet. The buried portion was increased in thickness to 6.56 feet at D, and at GB to 13.12 feet. The soft lead plates are 1.64 feet wide by ⅜ inch thick, and are placed as shown at the crown E, and the second joint above the springing D. The masonry joints were ⅝ inch, secured by blocks of wood 2 inches × ⅝ inch. After laying a row or string-course of voussoirs, the joints were washed and calked with tow to a depth of 1⅛ inches, and finished with a pointing-mortar of 1 cement and 1½ sand driven in with a calking-tool. The last five joints near the key were made at the same time. Eight masons and eight helpers, working on the two halves in two gangs, laid the whole arch in seven and a quarter working days.

695. Five days after completion the sand-boxes were tapped, but

the centre was only lowered 0.117 inch. In twenty-eight days it was lowered again, the crown sinking $1\frac{1}{8}$ inches up-stream and 1 inch down-stream; and in thirty-five days the centre was struck entirely. The sinking at the crown was found to be $1\frac{5}{8}$ inches up-stream and $1\frac{1}{8}$ inches down-stream. The total sinking in the subsequent four weeks, with all of the masonry completed, was $2\frac{1}{4}$ inches up-stream and $2\frac{1}{16}$ inches down-stream. As far as observed no further settlement had occurred. The lead plate at the crown was found in perfect contact with the voussoirs over the entire width of the plate. At the intrados its thickness had decreased 0.03 inch, and at the extrados about 0.1 inch. The pressure, therefore, was not uniformly distributed, the line of pressure meeting the joint above the middle point, but within one sixth of the width of the plate. The greatest pressure upon the edge of the lead sheet did not exceed 1628 pounds per square inch, and the pressure on the adjoining voussoirs could not have exceeded 398 pounds per square inch. On examining the leaden plates at the springing-lines on one abutment the joints at the extrados had opened 0.01 on the down-stream and 0.05 inch on the up-stream face, and on the opposite abutment 0.007 and 0.014 inch at the corresponding positions. On the intrados the joints had closed on one abutment 0.12 on the down-stream and 0.06 up-stream; and on the opposite one the intrados closed 0.9 and 0.11 inch at corresponding points. The leaden sheets were free along their upper portions, so much so that only 1.15 feet, instead of the full width of 1.64 feet, was in contact with the voussoirs. Calculation showed that the curve of pressure met these joints about $5\frac{1}{4}$ inches below the middle of the sheet of lead. The pressure on the lower edge of the lead plate reached 1846 pounds per square inch, but the plate did not yield. The maximum pressure on the adjoining voussoirs was 327 pounds per square inch. No fissures or cracks were observed in any part of the masonry.

After completion of the work, the tow in the lead joints was cleaned out and cement mortar packed in.

The total time consumed in the construction of this arch was ten months, and the total cost $6079. The cost per square foot of the projection of that portion of the arch above ground was $5.15, whereas for the actual span of 147.6 feet it was $3.20½ per square foot.

The cost of the arch bridge shown in Fig. 280 was $99,000. This included a trussed draw-span of 130 feet and a plate-girder span of 62 feet in length. Estimating the entire length as an arch,

and taking the projected area, the cost would be about $3.46 per square foot.

696. The following example of a concrete arch is given as showing an application of the same principle used in the case of the lead-plate joint:

Span, 105 feet; rise, 13.1 feet; thickness at crown, 19.7 inches; at the springing, 27.5 inches; abutments, 8.2 feet in thickness to low-water mark, 11.5 feet below. Concrete in abutments, 1 cement, 2 sand, 6 gravel, and $\frac{1}{2}$ part of broken limestone; for haunches of the arch, 1 cement, $1\frac{1}{4}$ sand, 3 gravel; near crown, 1 cement, 1 sand, 3 gravel. At the joints, at crown, and at springing, asphalt plates, $\frac{3}{8}$ and $\frac{13}{32}$ inch respectively, made of several layers, were placed. After the arch had settled 1.9 inches the asphalt plates were reduced to a uniform thickness of $\frac{1}{2}$ inch. The total sinking of the arch was 4.7 inches. It was claimed that by this construction cracks in the arch, as well as movements of the abutments, were prevented.

697. The two following examples are of arches used under earthen embankments respectively 40 and 154 feet high. The span in each case is 20 feet; rise, 10 feet. The height of the abutments above ground is 10 feet in the first case, and assumed to be the same in the second case. Therefore the height of the earthen bank above the soffit of the arch, that is, including the arch-ring, in the two cases, is 20 feet for the first and 134 feet for the second arch. The engineers who designed these arches gave for the first an arch-ring 23 inches in thickness, and the same from crown to springing; and for the second 3 feet at the crown and 4 feet at the springing.

By Trautwine's rule:

$$\text{Thickness at crown in feet} = \frac{\sqrt{\frac{1}{2}\text{ span} + \text{radius}}}{4} + 0.2\text{ foot}$$
$$= 1.32\text{ feet} = 16\text{ inches.}$$

By Rankine's rule:

Depth of key in feet $= \sqrt{0.12r} = 1.1$ feet $= 13\frac{1}{4}$ inches.

From the rules of the French engineers the following are taken: Depth, in feet, at the crown $= d$;

$$d = 1\tfrac{1}{12} + \tfrac{1}{23}\text{ span in feet} = 1.955\text{ feet} = 23\tfrac{1}{2}\text{ inches.}$$

With a rise of $\frac{1}{4}$ of the span, $d = 1 + 0.05$ radius $= 18$ inches. With a rise $\frac{1}{10}$ of the span $d = 1 + 0.02$ radius; and for elliptical or basket-handle arches with a rise $\frac{1}{3}$ of the span, $d = 1 + 0.07$ radius.

The numerical results obtained above are for the semi-circular

arches above mentioned. From which we see that both Trautwine's and Rankine's rule give less thickness than actually used. The first French rule gives about the actual thickness used for the arch under the lower embankment, but much less than that used under the higher. These formulæ do not seem to contemplate any increase of thickness with an increase of load above the crown, and the question naturally arises, was there any necessity for the engineer making the arch-ring under the high embankment, 134 feet above the crown of the arch, as much as 3 feet at crown and 4 feet at the springings.

698. Apply the principle that the thrust at the crown is the intensity of the pressure or load above multiplied by the radius of curvature, i.e., $T_0 = p_0 r_0$, and allowing 150 pounds as the load per unit of length of the arch, then, for the first case,

$$p_0 = 150 \times 20 = 3000 \text{ pounds}; \quad r_0 = 10.$$

Hence $T_0 = 30{,}000$ pounds. Allowing 200 pounds per square inch as a safe resistance, then the area per unit of length of the arch would be $\frac{30000}{200} = 150$ square inches, and depth of keystone $= \frac{150}{12}$ $= 12.5$ inches. Therefore both theoretical and empirical formulæ show that a depth of keystone of $23\frac{1}{2}$ inches is unnecessary for a span of 20 feet and a load above the crown of 20 feet. Applying the same principle to the load of 134 feet, $p_0 = 134 \times 150 = 20{,}100$; $r_0 = 10$. Hence $T_0 = 201{,}000$, which, allowing 200 pounds, the area per foot of length, will be $\frac{201000}{200} = 1005$ square inches, or a depth of 7 feet, whereas the actual depth is 3 feet, and the empirical rules give only from $13\frac{1}{4}$ to $23\frac{1}{2}$ inches. If, however, as the writer believes, it is safe practice to limit the load to that due to a height of 40 feet, whatever may be the actual height of the earth above the crown, the last example would reduce to $p_0 = 150 \times 40$ $= 6000$ pounds, $T_0 = 6000 \times 10 = 60{,}000$, the area per unit of length $= \frac{60000}{200} = 300$ square inches, and the depth at the crown = 2.1 feet. The above discussion shows the inconsistencies or differences between the practice, the theory, and the empirical rules as usually given and acted upon. The empirical rules seem to be based upon the thickness of long-span arches with a depth of material of probably not over 8 or 10 feet, which, so far as the writer has any information, is usually the condition for arches of long span. It is clearly advisable to increase the thickness of the arch-ring as the depth of the load increases up to the limit above given; and while these empirical formulæ give no doubt an excess thickness of arch-ring for low embankments, it would

hardly be good practice to use any thinner arch-ring than those given by the empirical rules.

For the reason that while with depths under 8 or 10 feet above the crown the load is light, and therefore the theoretical formulæ give a less thickness than the empirical formulæ, the former take no cognizance of the effects of vibrations and the tendency to change the position of the line of pressures, while the latter presumably do provide for both of these contingencies. The greater the intensity of the dead load, the less the proportionate effect of moving loads in causing vibrations and changes in the position of the line of pressures, and consequently the more the reliability of the theoretical formulæ up to the above specified depth of load.

699. The following list gives a few examples of various lengths of spans for arches, with which the engineer can form an idea, when designing an arch of any given span for any specific purpose, of how far from, or how near to, he may be conforming to the practice of other engineers.

TABLE LVIII.

	Span.	Rise.	Radius at Crown.	Thickness	
				At Crown.	At Springing
Cabin John Aqueduct, Washington, D. C.	220	57	184	4	6
Grosvenor Bridge, Chester, Eng.	200	42	140	4.6	7
Elyria, Ohio	150	24	129	3.75	4.5
Turin, Italy	148	18	160	4.92	
Neuilly, France	124	6.92	281	2.67	3.6
Cresheim Sewer Arch (sandstone)	116	21.17	90	3.5	4.5
Etherow (Eng.) Railroad	100	25		4	4
Leeds (Eng.)	100	15		3	7
Oise River (France) Railroad	77	6.40	119	4.80	
Westminster, London	76	38	38	7.60	14
Chestnut Street, Philadelphia (brick in cement)	60	18	34	2.5	
Bewdly (Eng.) Turnpike	60	20	33	2.2	
Filbert Street, Philadelphia (brick in lime mortar)	50	7		2	
Ougnon River, France	45	3.83		3.83	
Philadelphia & Reading R.R.	44	8	34	2.5	
Arbois, France	43	6.13		2.97	
Chesapeake & Ohio Canal (rubble in cement)	40	15	21	3	

All of the arches are circular. All dimensions given in feet.

TABLE LIX.

	Span.	Rise.	Radius at Crown.	Thickness.	
				At Crown.	At Springing
Gloucester, Eng.	150	35	98	4.5	
London, Eng. (granite)	120	32	112	4.5	8
Trilport (France) Railroad	81	28		4.45	
Bow Bridge (Eng.) Turnpike	66	13.75	47	2.5	
Chesapeake & Ohio Canal Viaduct	54	9	45	2.5	
Avon Viaduct, Eng. (brick in cement)	50	15	28.3	2	

The above arches are elliptic in form, all dimensions in feet.

Some examples of skew arches have been described and illustrated. The largest skew arch known to the writer is to be built of brick; span 130 feet, rise , radius at crown . The skew is to be obtained by a series of projecting ribs instead of the regular skew form. The thickness at the crown is 5.0 feet. and at the springing . The contract has been let; the cost will be .

The Masonry-arch Bridge on the Eastern Railway, France.—This structure is 960 feet long, and is composed of seven elliptical arches 127 feet span and 46 feet in height. The spandrels over the haunches are each pierced by three arches, and the pillars between them, as well as the piers, are pierced with longitudinal openings 5 feet wide for the purpose of lightening the work. The piers are hollow, the inner space being 4 × 10 feet; the width between parapets, 14 feet; the proportion of superficial area in solid to that hollow is 19,378 to 38,756, the total being 58,134 square feet. Total cost, $107,000. Average cost of masonry, $7.10 per cubic yard. Nearly the whole of the masonry is built of stone of small size.

The following table, giving the dimensions of six Galician bridges and two French bridges, is taken from *The Engineering News*, December 7, 1893, from an article written by Mr. John C. Trautwine. The Jaremize Bridge, of 213 feet span, is the largest masonry arch bridge for railway traffic in the world, and is only exceeded in length by one single-arch bridge-span for any purpose, namely, the Cabin John Aqueduct (see Table LVIII). The conditions were favorable for the construction of these masonry arches.

Good stone and in large quantities, as well as timber for the centres, was accessible and relatively cheap. The length of these arches is only 14.7 feet, being built for a single-track railway.

Numerous experiments were made on arches varying in width of span from 4.43 to 18.28 feet, and also five arches each 75.4 feet span, 6.56 feet long between faces, and 15.1 feet rise. Two of these were concrete, one of rubble, one of brick, and one of cut stone.

TABLE LX.

	Span, feet.	Thickness.		Maximum Unit Pressure in pounds per square foot.
		At Key in feet.	At Springing in feet.	
Jaremize Bridge, Galicia......	213.2	6.89	10.2	56320
Jamna " "	157.4	5.6	8.5	51405
Worochta " "	131.2	4.6	7.2	43827
" " "	113.5	4.25	6.7	36044
Zeniec " "	72.2	2.6	4.3	40550
Tablonica " "	82.0	3.6	5.2	36864
Viaduct du Gour Noir, France.	196.8	5.58	13.8	62259
Pont du Ceret, France........	147.6	4.59	13.1	55296

For arches over 131.2 feet cut stone was used; from 49.2 to 131.2 feet span, rough coursed work; for smaller arches, rubble of flat stones. The mortar, 1 Portland cement, 3½ sand.

In order to avoid too great loading of the centres during the erection of the larger arches, the innermost ring was first built entire, and the second ring was not begun until from two to three weeks later. It was then commenced in at least four different places between the skew-backs, so that the closing of the ring took place simultaneously at not less than three points. This plan was followed in all arches of greater span than 52.5 feet.

The cost of the arches in Table LX was, in the order of their numbers, \$33,888, \$18,868, \$14,632, \$16,432, \$7,140, \$6,064, or per square foot of projected area \$10.81, \$8.11, \$7.60, \$9.85, \$6.73, \$5.03.

The Cresheim sewer arch, 116 feet span, the third longest arch in the United States, has a width between outsides of parapet walls of 10 feet. Settled only about 3.3 inches after removing centre. Cost \$15,875, or \$12.41 per square foot of projected area. Including abutments, however, the span would be about 140 feet, and cost per square foot on this basis about \$10.31.

700. Arches under 15 to 20 feet are usually called culverts. They frequently have high embankments over them. There is

more danger of such arches being subjected to excessive loading and shocks during the operation of embanking over them. For these reasons, and for the additional one that there is no wisdom in affecting very thin arch-rings, culverts from 5 to 15 feet span have usually a thickness of from 10 to 18 inches at the crown, and should be well backed up near the haunches. The abutments have a thickness of from 3 to 6 or 8 feet at the springing, and from 4 to 8 or 10 at the base.

Mr. Rankine gives as the thickness of the abutments from one third to one fifth of the radius of curvature at the crown. This doubtless applies to arches of long span, as few engineers would spring an arch of 15 or 20 feet span, full centre even, with a radius of curvature at the crown of 7½ or 10 feet from an abutment having only 2 or 3 feet thickness.

ARCH CULVERTS.

701. The dimensions of arch culverts have been given in the preceding paragraph. The lengths of such culverts have to be determined from the relative total heights of culverts and embankments. It will, in general, be the top width of the embankment increased by three times the height of the embankment above the top of the parapet or spandrel walls. In such cases wing walls, either straight to the front, splaying out at angles of about 30°, or parallel to the face of the arch, must be provided. Their lengths must be sufficient to prevent the possibility of the earth flowing around them into the waterway. In general the splay or inclined wings are used, as these afford a larger and freer entrance and exit to the water flowing through them, neither damming up the water nor endangering the safety of the embankments by the water getting behind the walls and abutments. These wings usually leave the face of the arch two or three or more feet from the face of the abutment, thereby leaving square shoulders on either side of the arch, which is likely to catch drift or ice, thereby forming a gorge and obstruction to the flow which may endanger the safety of the arch or the embankment.

This can be avoided by commencing the wings at the face of the abutment and carrying them up vertically to the springing-line, but throwing the face of the wing on a warped batter outwards to its extreme end.

Above the springing the wing can be thrown, by an offset, behind the extrados of the arch. This prevents any obstruction to

the flow for a height equal to the height of the springing-line above the bed of the stream.

When the foundations of culverts do not spring from a rock bed, the bottom of the culvert is usually paved over its entire surface and extending to the extreme outer edge of the wing-walls.

The paving-stones should be set on edge, breaking joints, and slightly lower in the centre than at the sides. Also, at the ends of the arch apron-walls should be built entirely across, and often deeper than the foundation-beds of the abutments and wing-walls. All embankments over and around culverts should be carefully constructed. The earth should be placed in layers, and compacted for at least 8 or 10 feet from the abutments and wings, and for an equal depth over the arch-ring. This work should be done on both sides at the same time, to avoid not only shocks and excessive pressures on the arch, but to avoid unequal loading of the sides. With these precautions, the remaining portion of the earth may be filled in as usual. It is far better to make the openings much too large than the least too small. If possible, the dimensions of some natural and contracted channel in which the greatest flood-water is confined should be found and measured, and this should be the minimum waterway provided for.

BOX-CULVERTS.

702. Culverts under 5-feet span are usually built of two side walls from 2 to 4 feet in thickness, and covered over with long, broad stones from 10 to 16 inches in thickness, leaving square or rectangular openings, generally 2×2, 2×3, 3×3, 3×4, or some such dimensions, for the passage of the water from small streams or ditches. Their bottoms are usually paved; and if constructed on loose or porous earths, sand, or gravel, either apron-walls should be built at their ends or sheet-piles should be driven to prevent the possibility of scouring action under the culverts. Often double culverts are used. These are constructed of three walls covered over with flat stone, leaving two openings side by side. Masonry box culverts are clearly shown in the drawings, Figs. 282 (*a*), (*b*), (*c*), and (*d*). When the opening is wide, one or two courses at the top of the side walls are made to project inwards, over which the covering stones are placed, allowing the use of shorter covering stones.

703. Timber culverts are often used. These may consist of

two side walls built of solid timbers, and covered over also with solid beams. Or a series of frames may be constructed and planked on all sides.

These should be paved, or have a course of plank under the walls. Such culverts under high embankments should not be used if it is practicable to avoid them; and when so used, it is advisable to make them of sufficient dimensions, that at some subse-

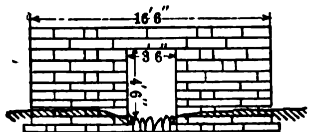


FIG. 282.
ELEVATION OF END.

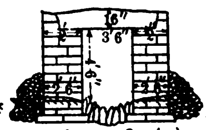


FIG. 282.(a)
CROSS SECTION.

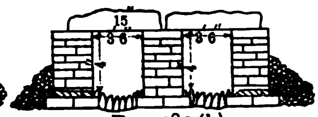


FIG. 282.(b)
CROSS SECTION, DOUBLE CULVERT.

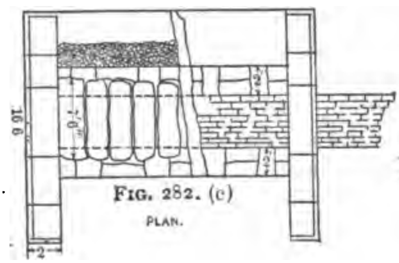


FIG. 282. (c)
PLAN.

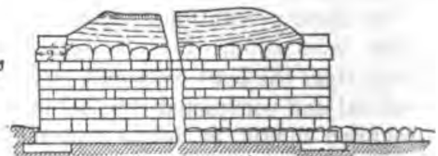


FIG. 282. (d)
LONGITUDINAL SECTION.

quent time masonry culverts can be built inside of them, still leaving ample waterway.

If necessary for this, three or more timber walls should be built, leaving two or three openings side by side. For low embankments, or where the top of the culvert is level with the subgrade of the road, there is no special objection to their use, as they can be always readily reached for repairs, and in some respects are better than rough masonry walls subjected to violent shocks and vibrations. Timber culverts are clearly shown in Figs. 283 (*a*) and (*b*) for solid walls, and Figs. 284 (*a*) and (*b*) for frames and sheeting.

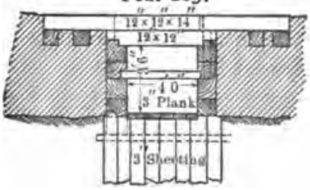
VITRIFIED-PIPE CULVERTS.

704. For openings under 2 feet square the vitrified salt-glazed earthen pipes may be used instead of culverts. These should be well bedded on beds of sand or fine earth; and to prevent scour their ends should be protected by small timber or masonry head-

WOODEN CULVERTS.

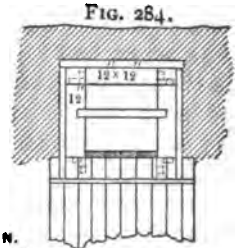
SOLID WALL OPEN CULVERT, CROSS SECTION.

FIG. 283.



FRAMES & SHEETING, CROSS SECTION.

FIG. 284.



ELEVATION.

12" FIG. 283.(a)

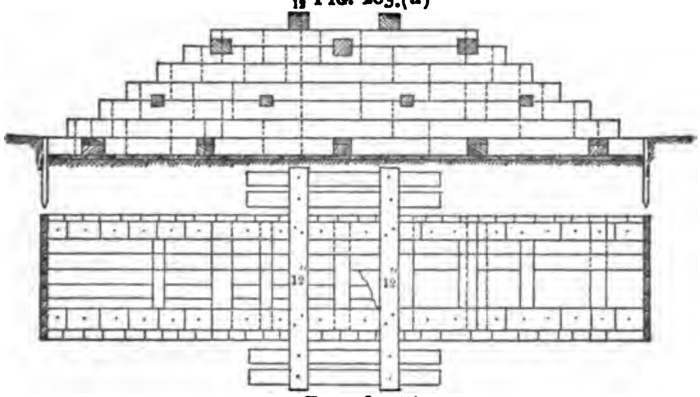


FIG. 283.(b)

PLAN.

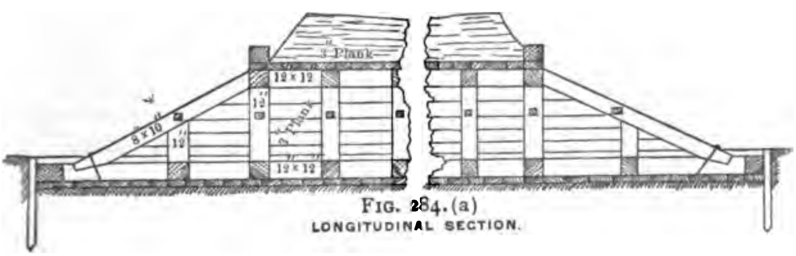


FIG. 284.(a)

LONGITUDINAL SECTION.

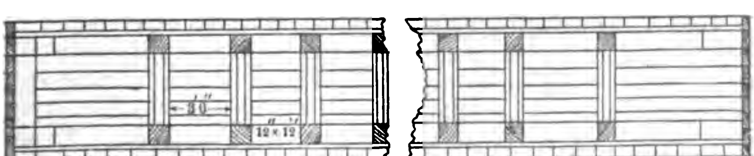


FIG. 284.(b)

PLAN.

walls. These pipes have been used to a very great extent, often with entirely satisfactory results. There is great danger of their breaking under heavy weights or shocks, and especially of opening at the joints when badly laid; and in some cases they may do more harm than good. But with the best pipes, carefully handled and bedded, good results may in general be expected.

For very small openings, very rough and roughly-laid stone—for an opening, well defined, of 12 inches square, more or less, covered over with loosely-piled broken stone—will answer every purpose to prevent the accumulation of surface-water in hollows during the rainy seasons, where no continuous drain is necessary.

It is the part of wisdom and good practice to be liberal in the use of some of these small and economical forms of drainageways. If some means of passing water under embankments in all hollows is not provided, it is perfectly certain that the water will find or make a passageway for itself, not only resulting in much expenditure of money either for repairs or in damage and destruction of property and in damages for loss of life.

Many serious disasters have occurred from giving too little waterway, either in the dimensions of single culverts or in the employment of an insufficient number of culverts, properly distributed along the line. The water from heavy falls of rain, being dammed up by the embankment, finds its way into the interior of the mass, which may not of itself disturb its stability, but when a heavy train is run over the embankment the excessive load causes a settling and spreading of the material, resulting in the derailment and destruction of the train. Therefore the importance of providing ample openings, both in number and dimensions, cannot be too strongly urged as both good practice and economical. Experience proves that it is not always safe to assume that the highest recorded flood will never be exceeded, and a margin of safety should be provided.

ARCHES OF CONCRETE.

705. The increasing employment of concrete, especially in the construction of arches, renders any information on the resistance of concrete to transverse strain and thrusts of considerable value and importance. The following are the results of some experiments made by Mr. A. F. Bruce, an English engineer. Experiments were made with hard and soft sandstone, whinstone gravel,

sand from sand-pits, river sand, and sand prepared by crushing sandstone and whinstone. The proportions used were such that in every case all voids were filled, (1) the voids in sand with cement and (2) those in the stones with mortar. These voids were carefully ascertained, and found to vary in sand from 33 to 34 per cent, in gravel 34 per cent, in broken sandstone 40 to 42 per cent, and in whinstone from 46 to 50 per cent.

The bars for determining transverse strength were 30 inches long and 4 inches square. The blocks were left in the moulds for five days; they were left in water until a day before they were to be tested. Then they were taken out and weighed. After being allowed to dry they were again weighed, the difference in weight being taken as a measure of the porosity. Only one or two were broken the same day, in order that the growth of strength with age might be determined. The ends of the bar were held down by clamps, giving a clear span of 27 inches. The clips were removed from a testing-machine and a bar substituted, by means of which and a stirrup connection passing beneath the bar to be tested, and by the usual method, the bar was broken. The modulus of rupture was then calculated from the usual formula,

$$mWl = nfbd^2; \quad m = \tfrac{1}{4}; \quad n = \tfrac{1}{6}; \quad f = \frac{3Wl}{2bd^2};$$

and substituting dimensions of bar $l = 27$, $b = 4$, and $d = 4$ inches, the modulus of rupture becomes for this particular case $f = 0.633w$.

706. In order to ascertain the strength of concrete in resisting a thrust, a number of arch ribs were built. The centring at first had 10 feet span with 5 feet 8 inches radius, but in later experiments the span was reduced to 3 feet, the radius remaining the same. The ribs were made 2 inches square at the crown and increasing in thickness towards the haunches, the top being level. The ribs sprung from firm and substantially built abutments. The concrete was kept moist while setting by throwing wet bags over it. They were broken by weighting the crown, for a length of 18 to 24 inches, with lead weights of 1 cwt. (112 lbs.) each, using ordinary weights for the smaller increments required. The haunches were weighted to balance the effects of the weight at the crown. The unit crown-thrust, i.e., the thrust per square inch of cross-sections was found by the formula $T_0 = p_0 r_0$ for total thrust, and for the intensity per square inch $t_0 = \frac{T_0}{A} = \frac{p_0 r_0}{A}$, in which $p_0 =$ the weight

per foot run, r_s = radius of curvature, and A, the area of cross-section in square inches. The number of tests for transverse strength was about 300; that for thrust was more limited.

The following table gives some of the results.

TABLE LXI.

SHOWING THE RESISTANCE OF ARCHES TO FRACTURE, THE MODULUS OF RUPTURE, AND THE RATIO BETWEEN THE TWO.

The modulus of rupture, f, and the resistance to thrust, s, expressed in pounds per square inch.

1 Cement to—	Age in Weeks.														
	1.			2.			4.			8.			12.		
	s	f	s/f	s	f	s/f	s	f	s/f	s	f	s/f	s	f	s/f
3 Sandstone, 2 " crushed	....	....	.. ..	552	276	2.0*									
5 Sandstone, 2½ " crushed	....	...	....	....	...	....	....	....	. . .	908	202	4.5			
7 Sandstone, 3 " crushed	190	36¼	5.2				426	110	3.9*	925	171	5.4			
3 Whinstone, 1½ " crushed, 1½ Sandstone	405	202	2.0*				1846	429	4.3						
6 Whinstone, 2½ Pit sand	..	...	....	...	...	...	435		6.1				1160	170½	6.8
5 Whinstone, 1 " crushed, 2 Sandstone	....	....	....	....	....	...	....	....	....	912	104¾	8.7	1058	176	6.0
3 Gravel, 2 Crushed sandstone	....	....	....	834	220	3.8	2022	300	6.7						
5 Gravel, 3 Pit sand	....	....	....	....	..	...	663	114¼	5.8				990	140	6.1
7 Gravel, 3 Pit sand, bad quality	...	....	...	245	42¼	5.8	431	63	6.7						

* Yielded prematurely owing to insufficient loading on one haunch.

The ratio s/f shows the tendency to increase with age, that is, the power to resist crushing stress grows faster than the transverse strength. The conclusion to be drawn is that where concrete is to be used to resist crushing, which is almost universally the case, we can safely use the modulus of rupture determined by experiment as a safe working stress in concrete arches.

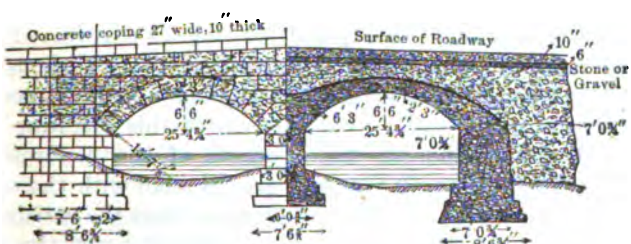
EXAMPLES OF CONCRETE-ARCH BRIDGES.

707. The first concrete-arch bridge ever constructed in the United States is the highway bridge over Pennypack Creek, Philadelphia, Pa., Mr. C. A. Frik, engineer. The structure consists of two arched spans of 25 feet 4¾ inches each, with a rise of 6 feet 6 inches, supported by concrete abutments and a central pier.

The arch-rings are 2 feet 3 inches deep at the crown, covered on top with a $\frac{1}{2}$-inch layer of Portland cement. The backing and spandrel walls are also made of concrete well rammed. The spandrel walls and faces of the arch are moulded in imitation of ashlar masonry. For the foundation work Dykerhoff's, and for the piers, abutments, and arches Manheimer's, Portland cement was used. As a further precaution, wire nets, with $1\frac{1}{2}$ mesh and diameter of wire $\frac{1}{4}$ inch, were placed about 2 feet vertically and horizontally throughout the concrete. The width of roadway or length of arch from out to out of the parapet walls is 34 feet $3\frac{1}{2}$ inches, width of macadam roadway 26 feet, and on either side a paved gutter 2 feet wide. The cost was estimated as follows:

680 cubic yards of concrete @ $9.30.......	$6,324 00
Macadam, etc...........................	2,338 00
Total cost........................	$8,662 00

In Fig. 285 is shown a vertical section through one abutment, arch, spandrel, and one half of central pier, on the right; and on the left an elevation of one abutment, half of central pier and arch-ring and spandrel wall. The abutments are 7′ $0\frac{3}{4}$″ thick at the springing line, 8′ $6\frac{3}{4}$″ at base. The central pier is 6′ $0\frac{5}{8}$″ thick at springing and 7′ $6\frac{5}{8}$″ at base. The arches are segmental, having a radius of 15′ $7\frac{1}{8}$″. The height of the base of coping of parapet wall is 11′ 11″ above springing line over the central pier, with a grade of 2.64 per 100 each way from the centre. Dimensions are shown on the drawings.



HALF ELEVATION FIG. 285. HALF SECTION

An arch built entirely of concrete, over the river Danube at Munderkingen, and recently completed, is worthy of more than a passing notice. The arch has a span of 164 feet; rise, $\frac{1}{10}$ span = 16.4 feet; width from face to face of arch, 26.3 feet. It is built on

a skew of 25°, and has a grade of 3 per cent. One end of the arch springs from a natural limestone cliff, which constitutes the abutment. The other end springs from an abutment resting on a pile foundation, consisting of vertical and inclined piles driven into fine sand. At both abutments and at the crown steel joints are introduced to insure both the proper distribution of stresses and to allow of alterations in the form of the arch, arising from changes of temperature or from other causes.

These hinges do not extend continuously the full width of the arch; but at each hinged joint are placed twelve separate semicylindrical convex pieces, 1.64 feet in length, bearing on similar pieces of concave form. The pressure on these bearings is estimated at 8500 pounds per square inch, whereas the pressure on the concrete must not exceed 425 pounds per square inch. To bring the pressure within the proper limit, the bearings are bolted to a base consisting of a top and bottom plate 0.6 inch thick, between which three eye-beams are placed. The dimensions of this distributing-plate or bearer are 20 × 32 × 8 inches.

The thickness of the arch at the crown is 3.28 feet, and at the springing 3.61 feet. There are arched openings above the abutments 8.2 feet span, and also arched openings in the spandrel, to lighten the weight. Arrangements are made for taking up the expansion and contraction of the roadway.

The concrete is made of Portland cement. This was composed of 1 cement, 2½ sand, and 5 of river-sand and broken stone. It was mixed in a machine resembling a foundry rattler, containing about forty steel balls, 4.8 inches in diameter. The dry materials were placed in the machine, which was revolved for two or three minutes. Water was then added, and the machine revolved for two or three minutes more, until the ingredients were thoroughly mixed. The contents were then emptied through openings smaller than the diameter of the balls. The capacity of this machine was 52 cubic yards per ten hours. The strength of the concrete is claimed to be from 25 to 30 per cent greater than with the same materials mixed by hand. At the end of seven days the resistance to crushing was about 2900 pounds per square inch; at the end of twenty-eight days, about 3600 pounds,—about eight times the working pressure allowed. The centring was given a rise of 8 inches above the intended level of the crown. The arch settled 7¾ inches.

Another example of concrete piers and arches is found on a railway in Jamaica. The construction of the piers was described

in paragraph 537. These were in height 48 feet; in thickness at top, 6 feet; at bottom of neat-work, 7 feet 6 inches; and of footing-courses, 11 feet. Length of top, 16 feet; at bottom, 19 feet 2 inches. The span of arches was 50 feet; rise, 22 feet $2\frac{1}{4}$ inches; arch-ring, 2.0 feet in thickness. The arches, spandrel, and backing were built of concrete in mass up to a point where the radial lines make angles of 60° with the horizon. The upper portion of the arch-ring between these points was built of concrete voussoirs bedded in 1 to 2 mortar, except the keystone, which was of concrete in mass. All concrete was made of 1 Portland cement, 3 sand, and 6 broken limestone; concrete filling over arches, 1 cement, 6 sand, 12 broken stone. Spandrel-wall above keystone, 1 foot in thickness; parapet wall, 4 feet 7 inches high and 1 foot

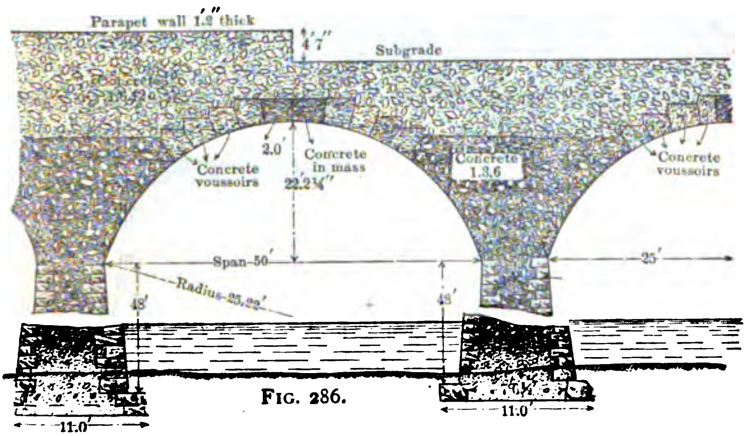
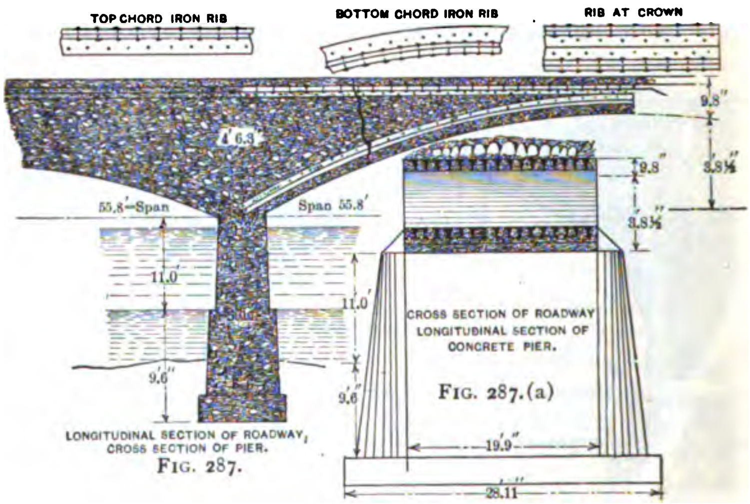


FIG. 286.

2 inches thick. This construction is shown in Fig. 286, which represents one half section perpendicular to axis of arch and one half elevation.

708. *Concrete and Iron Arch Bridge.*—The drawings Figs. 287 and 287(*a*) show sections of a concrete arch bridge constructed over the river Neutra, Hungary. There are six arches, each of 55.8 feet span; the rise is 3 feet $8\frac{1}{2}$ inches; thickness at the crown 9.8 inches, and at the springing-line 4 feet 8.3 inches. Total width of roadway, out to out of parapet walls, 19.66 feet. The concrete for the foundations was composed of 1 part Portland cement, 5 parts Roman cement, and 30 parts of sand and gravel, or 1 to 6 concrete. This was deposited in place by means of a

funnel-headed tube. The concrete above water was made of 1 part Portland cement and 10 parts sand and gravel or ballast; in the main the upper parts of piers and abutments were made 1 to 8. All concrete was rammed in layers of 8 inches in thickness. There were thirteen iron beams imbedded in the concrete. These were made of angle-irons and stiffening-plates. No lateral iron members were used; but at the abutments the iron chords or beams were anchored to vertical members. These were connected by horizontal channel-bars. The top chords were made continuous. The arches were erected on centring, supported on pile-bents 8.2 feet apart. The centring was covered with heavy tarred paper to



LONGITUDINAL SECTION OF ROADWAY, CROSS SECTION OF PIER.
FIG. 287.

FIG. 287. (a)

prevent warping from the moisture. When the false-work was completed the iron chords were placed, and the concreting commenced. The concrete for the arches was made of 1 part Portland cement and 6 parts of sand and gravel for a layer of 10 or 12 inches in thickness; for the spandrel, concrete 1 to 8 was used. The concrete was carefully rammed in layers at right angles to the radial lines of the arch, and special care taken in ramming the concrete which projected below the iron chord. The spandrels were built with a slight grade toward the piers, for the purpose of drainage. One arch was usually completed in one day. All centring was left in place for a period of thirty days after the completion of the last of the series of arches. They were then struck from the two centre

arches, and thence towards the ends. The first arch built had stood supported by its centring for 43 days, and the settlement on removing the centre was only 0.079 inch; the one supported for 36 days settled 0.551 inch; and the last built, supported for 31 days, settled 0.591 inch.

The thickness at the crown and springing is less than that of the arch shown in Fig. 285, though having less rise and more than twice the length of span.

The total concrete used was 1346 cubic yards, and total iron 88,180 pounds. The cost was $13,700. This is the equivalent of iron at 4 cents per pound and concrete at $7.56 per cubic yard. The bridge was tested with a uniformly distributed load of 82 pounds per square foot of roadway, and subsequently a very much heavier load was tried. The maximum temporary deflection was 0.138 inch, and maximum permanent set 0.031 inch. The maximum crown-thrust was calculated to be 410 pounds per square inch. The maximum tension on the upper chords 2.6 tons per square inch. The strain on the anchor-bolts 4.07 tons per square inch. With Portland cement concrete mixed 1 to 6 and 28 days old, the calculated strain at the crown is within one fifth of the ultimate breaking strain.

709. *The Monier Method of constructing Arches.*—As has been already stated, flat arches give way by sinking at the crown and spreading at the haunches. In other words, the arch opens on the intrados at the crown and on the extrados at some point between the crown and springing, thereby causing a tensile strain on the lower portion of the arch-ring at the crown. To take up this tension a wire netting is built in the concrete near the line of the soffit, and in some cases a little below the line of the extrados. This is specially intended to take up the tension when the arch is not symmetrically loaded. This netting is constructed on the centring by first running a series of wires from abutment to abutment over a smaller set placed parallel to the axis of the arch about 3¼ inches apart, on the top of the larger wires another layer of smaller and then a second layer of larger wire. The diameter of the larger wire is 0.39 inch and of the smaller 0.28 inch. The wires are loosely tied at their intersections by means of small wire. As many layers are used as may be deemed advisable. The completed net is to be raised about 1.9 inches above the centring. Cement is then spread over the centring from the abutments towards the crown, and lightly compacted with the trowel. Then

a series of thicker layers are placed until a thickness of 9.5 inches is formed. These layers were rammed with iron tamping-bars.

When a thickness of 9.5 inches is secured, another netting of wire is woven. This only extends over about one eighth of the span towards the crown, and over this netting a finishing layer of cement mortar 1.9 inches thick is placed. The proportions of the mortar were 1 part Portland cement and 3 sand. The concrete for the piers and spandrel-walls and backing, 1 Portland cement, 4 sand, and 6 broken stone.

Experiments made on arches thus constructed, having a span of 32.8 feet, rise 3.28 feet, length along axis of arch 13.2 feet, and thickness of arch-ring at centre or crown of 6 inches and at springing of 8 inches. After some tests with light loads, a load of 110 tons was placed on half of the span, equivalent to 1000 pounds per square foot, under which the maximum deflection was 0.12 inch, from which it almost recovered by the following day. Under a load of 2000 pounds per square foot the arch sank without entirely giving way.

In Switzerland an arch of this type was constructed having a span of 122.1 feet, rise 11.5 feet,—a ratio of 10.6 to 1; arch length or width of roadway, 12.8 feet; thickness at crown 7.9 inches, and at the abutments 25.6 inches. This was only intended for light traffic. It was tested with a load consisting of four loaded carts, nine horses, and twelve men,—in all 40,000 pounds. The maximum settling or lifting did not exceed 0.16 inch.

Another arch of same type had a span 65.6 feet, rise 8.2 feet, thickness at crown 7.9 inches.

This principle has been applied to the construction of concrete sewers. In all such applications of these methods lightness, strength, economy, and rapidity of construction are the objects aimed at. The objection to all such combined structures lies, first and foremost, in the difficulty of making two distinct materials built together or rigidly connected with each other, act together as a unit, and in determining what portions of the total stresses are borne by the separate materials. Other objections are raised: (1) that the wire will corrode when in contact with the wet mortar; (2) that the adherence of the mortar to the wire would be very feeble; (3) that the difference in the expansion and contraction of the wire and concrete would tend to loosen the bond between the two or to cause cracks in the concrete. The first two objections are answered by saying that experience and experiment both

show them to be more fanciful than real; and as to the third objection it is claimed that the coefficients of expansion for concrete and iron are practically the same, being for 1° C. 0.0000143 and 0.0000145, respectively.

The general construction and appearance on a drawing of the Monier arch is the same as shown in the drawings Fig. 287, substituting for the iron ribs or chords a wire netting over the entire soffit. The upper netting usually follows a curve more or less nearly parallel to the soffit instead of being horizontal and level with the roadway, the spandrel filling being above the upper netting. Therefore no drawing of the Monier arch is necessary for a clear understanding of the descriptions.

Another type of this arch is the employment of iron or steel eye-beams bent to the curve of the arch and imbedded in concrete, in somewhat the same manner as above described.

ART. XLVI.

TUNNELS.

710. **Tunnels** are underground excavations for roadways, canals, and in general for any purposes where it is necessary or economical to limit the amount of material removed, and at the same time impracticable or undesirable to use the surface above for the passageway or other purposes intended. The cost of tunnelling is so great and time required in the operation is so long, that it may be stated that tunnels should be avoided except under very special conditions.

To what extent a line of communication may be or should be lengthened in order to avoid a tunnel of a given length, or at what depth of excavation, where the choice is granted, it is desirable or wise to abandon an open cut and resort to tunnelling, depends on so many and varying conditions, that no general rule can be laid down governing the matter. It cannot be decided purely from considerations of economy in the first construction, though for ordinary purposes this will be a very potent factor. For roads and canals an increase in the length of line involves a permanent increase in the expenses of operation and maintenance, and a corresponding increase in the length of time consumed in the transportation of passengers and freight—which is an important item in these days of demand for rapid transit. The maximum depth of

excavation admissible in open cut will depend largely upon the question of cost. The relative cost, however, varies greatly with the character of the material to be excavated, and upon the cost of the lining, temporary and permanent, required in tunnels. The selection should be made, where the choice is given, after a careful comparison of all items of cost, both immediate and remote.

It may be roughly estimated, that when an open cut would exceed 60 to 70 feet in depth it will commonly be found desirable, economical, and expeditious to resort to a tunnel. In many cases no choice is given: the tunnel must be built. In such cases the method of construction may, however, still be a matter of grave consideration, that is, whether an open excavation may be made at first, in which is constructed the tunnel lining, over which the material is placed up to the original surface, and restored to its original condition, or whether the tunnel shall be excavated, regardless of the depth below the surface, without disturbing the original surface or endangering any structures near the line of the tunnel. This has application especially to underground passages along the street lines of cities, the necessity for which is becoming more apparent every day, as a solution of the important question of Rapid Transit.

711. To avoid confusion of terms, underground excavations can be divided into (1) tunnels proper, or those in which the transverse dimensions are sufficiently large to admit of the passage of trains, vehicles, or canals, and (2) drifts or headings, where of smaller dimensions, when such excavations are horizontal or inclined to the horizon. Drifts or Headings include excavations for sewers, conduits, subways—according to the purposes for which they are used. The term Shaft is applied exclusively to an excavation, usually of small dimensions, made vertically or nearly so. The lining for any of these excavations, when necessary to use it, may be of timber, iron, stone, or brick masonry or concrete, as may be determined from considerations regarding strength, durability, and permanency. Timber linings are commonly used for temporary purposes, and are generally followed up closely with some more durable lining, and should never be left for any great length of time, though this is often done. As the masonry lining is constructed, the timber is removed in whole or in part. Those portions of the supporting frames and the poling-sticks or sheeting, outside of the masonry lining, are sometimes left in place, and all spaces around and those between the masonry and the undisturbed earth are well filled with rammed

earth, broken stone, or concrete. Iron and stone linings are sometimes used, but rarely; and the same may be said of concrete.

The permanent or masonry lining is almost universally built of brick. Bricks have sufficient durability and strength when hard and well burned, and afford much greater facility in constructing the lining, owing to the confined and cramped positions in which the men have to stand and work.

712. Whether any lining, either of a temporary or permanent nature, will be required, depends entirely upon the character of the material to be handled. This should be carefully determined by borings or by sinking test shafts, and with these there must always be more or less uncertainty in regard to the uniform continuity of the material between any two bore-holes or shafts. The form and dimensions both of the excavation and lining must also depend upon the character of the material. Much is, however, left to chance; and the engineer must be prepared to meet with the unexpected, and be ready to adapt his mode of excavation and kind of lining to the circumstances and conditions as they arise. Watchfulness, good judgment, familiarity with many different methods, and a promptness in coming to some decision are the essential characteristics of a good tunnel-engineer.

713. Many elaborate and costly surveys have been made to locate the centre line of long tunnels, in mountainous districts, on the surface of the ground, mainly for the purpose of determining the exact distance and relative direction of its two ends and to be enabled to locate accurately the proper positions for borings, test and working shafts, or to determine the proper and most advantageous location and alignment of the centre line of the tunnel. A straight line should always be adopted, if practicable. A slight grade from some interior point to the two ends is advantageous, both in the prosecution of the work and for purposes of temporary and permanent drainage.

714. *Materials.*—In many respects solid rock is the most advantageous material through which to tunnel, especially the softer varieties, provided they are strong and durable. The excavation is limited to the minimum dimensions required for the purpose intended; no exact or uniform surface has to be maintained; the form of the cross-section can be made such as will afford the greatest convenience in the prosecution of the work; the drainage can be more readily provided for; and no artificial lining of either a temporary or permanent character is required.

715. Stratified rock may or may not present any serious difficulties, depending upon the thickness and inclination of the layers. If the stratification is thin, divided by layers of porous earth, and especially if much inclined to the horizon, there will be more difficulty and danger in prosecuting the work than when the layers are thick and horizontal, or nearly so, a lining of some kind will often be required, especially for the roof; and much inflow of water will be likely to occur, requiring ample drainage facilities.

Many kinds of rock will disintegrate and decay on exposure, though offering no special difficulty or danger in the prosecution of the work. Tunnels through such materials should be lined. The lining may consist only of a thin facing wall of brick, mortar, or concrete, as the object is not so much to support a load or pressure as it is simply to protect the surfaces from the disintegrating effects of atmospheric or gaseous influences.

716. Clay and ordinary earths offer no special difficulties, unless containing an excess of water. These materials will admit of easy excavation, and will stand independently for a limited time, or with light sheeting and framing. It is unwise, however, to take advantage of this temporary stability, as great damage and danger may result. It is not difficult to hold such materials at first; but should caving commence, or any cracks or fissures start, it may be a matter of extreme difficulty to stop a movement, however slight, when once started.

717. Experience shows that in most, if not all, stiff and compact soils even a thin layer of mortar is often sufficient to maintain the slightly inclined or vertical surface of an excavation, as can be seen in cisterns constructed in many parts of the country, provided it is put on without delay, before the cohesive strength of the material has been disturbed. And it is equally true that arched roofs in such materials would be greatly strengthened by the same simple means, but to what extent is at least uncertain, and could not be relied upon.

The thin masonry or timber linings used in tunnels at very great depths are standing proofs that the pressure on them cannot be due to a pressure proportional to the depth below the surface. It might be safely asserted, but for the fact that whatever care and expedition is used there will be some disturbance of the material over and on the sides of tunnels, that there would be but little pressure upon the lining, and even with the little settling which usually takes place the pressure will not exceed

that due to a, relatively speaking, mass of loose earth of a very limited extent. Above this limited height it seems evident that the earth is self-supporting. In Fig. 288 is shown the general cross-section of a tunnel, and tunnel lining *ABCDEF*. The slopes *CH* and *EL* represent the natural slope of the material. According to the ordinary theory of earth pressure, the vertical pressures could only be due to the weight above and enclosed by these slopes. Conceiving the ideal vertical planes *KC* and *ME* tangent to the sides of the tunnel lining, the weight of the mass enclosed by these planes and the extrados of the lining, and the upper surface of the earth will represent the maximum possible vertical pressure upon

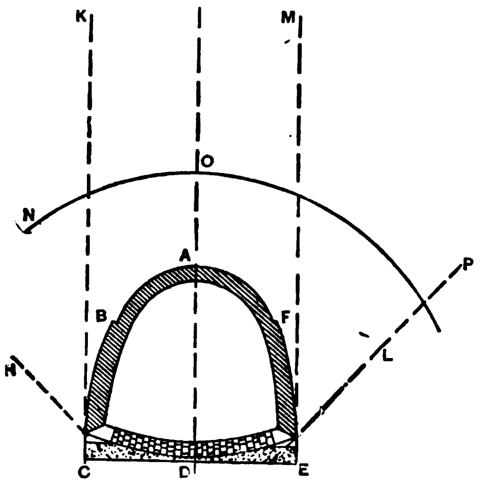


FIG. 288.

the masonry. It is evident that this full pressure could only be exerted when this mass was a fluid. Even with a loose-earth filling the friction along the planes *KC* and *ME* would relieve the arch of a considerable portion of the pressure, and much more in an undisturbed mass possessing a considerable cohesive resistance. The pressure would then only be due to some mass of earth bounded by the curved line *NOP*, whose height above the tunnel would vary with the character of the material above and within the width of the tunnel. These considerations, and the additional fact that all materials, whether in mass, or in columns, bars, or other sizes and shapes, usually give way in detail, by scaling, chipping, or rending along the edges or surfaces, fully explain why the thin tunnel

linings, and arches as well, constructed on the Monier method, or with bars, are capable of resisting such great apparent or actual strains and pressures.

718. As the materials, such as sand, gravel, and loose stone, have little or no stability due to adhesion, and have a tendency to assume at once, when free to move, the natural slope, it becomes necessary to provide immediate support on all sides, as well as at the head or front of the excavation, to prevent an inflow of the material at and near the surfaces, which, when once initiated, may extend to great distances from and above the tunnel, producing both immediate and permanent pressures of great intensity. These conditions are aggravated by the presence of greater or less quantities of water, always found in strata of sand and gravel at even small depths below the surface; but in such materials, where the mass is supported at once, there will be some point above which, owing to its great frictional stability, the mass will be self-supporting.

719. In any of the foregoing materials the process of tunnelling presents no great difficulties, but requires a varying degree of care and closeness of the temporary lining with the increasing tendency of the material to caving and flowing, also special methods of tunnelling must be adopted to give immediate support and to provide efficient and sufficient means of drainage.

720. *Quicksand and Mud.*—The most troublesome and treacherous materials with which the engineer has to contend in constructing all kinds of works, especially tunnelling, are quicksand and mud. These materials, fortunately, exist where tunnelling is necessary only in relatively thin layers or pockets; but wherever encountered, special methods of work are required. These methods will be explained in another paragraph.

Certain seamy clays partake, to some extent at least, of the characteristics of these materials. Though firm and compact between the seams, there is always great danger of sliding in large masses on the seams, especially when this soapy material is softened by the seepage of water along them. They can, however, be worked by the same general methods as ordinary earth if the precaution is taken to provide immediate and strong temporary linings, followed closely up with the permanent masonry lining.

721. *General Processes of Tunnelling.*—In almost all materials drifts or headings are run in advance of the full-sized excavation. The dimensions of the cross-section of the heading vary in dimensions from 6×6 feet to 8×8 feet—sometimes more and sometimes

less. There is nothing gained by using small headings; they are relatively more expensive and require more time than when of the sizes above given.

The positions of the heading with respect to the section of the tunnel also vary. In some materials it is deemed advisable to run them near the upper portion of the tunnel section, in others at the bottom; and, again, in some materials they are run at the centre of the depth along the centre line in either case. When two headings are run they may be at the bottom, one on each side at the ends of the transverse diameter; or near the top, and vertically above the positions just mentioned.

722. All headings are run for one or all of the following purposes: (1) As pilots, in order to discover the nature of the material in advance of the tunnel proper; (2) to provide faces from which the excavation can be advantageously enlarged to the full size of the tunnel; (3) to enable the temporary supports, or even the permanent linings, to be constructed at those portions of the tunnel requiring immediate support; (4) entirely for drainage purposes; (5) to provide good and firm supports for other portions of the temporary lining and the construction of the permanent lining. As in most cases it is necessary or advantageous to run a top drift or heading in any event, bottom drifts are rarely used, unless the nature of the material requires efficient drainage. It is easier to work sideways and downwards than inwards and upwards. Where drainage is the object in view, the bottom drift should be made as small as economy may justify.

Therefore in solid rock the heading is run near the top, at the crown; and as no lining is required, the work can be carried on simultaneous sideways and downward on the two sides, while the core or central portion is being worked from the lower and rear face, or in whatever manner may be found most convenient and economical.

In materials requiring drainage the bottom drift should be always kept well ahead of the main excavation, and the top drift also kept in advance, though not necessarily more than 15 or 20 feet. As in such materials a temporary lining is required, each section must be lined as the work progresses, and the excavation below the top drift must be regulated in such a manner that the portions of the lining above shall be always firmly and continually supported. All forms of temporary lining should be as far as practicable arched, so that the main supports can be placed as near

the outside of the excavation as possible, leaving the centre portions of the tunnel unobstructed by braces and props.

The two following descriptions will illustrate the more modern methods of tunnelling, and of the construction of the temporary and permanent linings :

THE BALTIMORE BELT-RAILROAD TUNNEL.

723. This tunnel is built following certain street lines. Owing to objections that would arise to obstructing the traffic along the streets, and the great depth of the tunnel below the street surface, it was determined to adopt the regular tunnelling methods, although it would have been more economical in much of the work to have adopted the open-cut method. The open-cut method along the streets of cities has the difficulty of contending with and taking care of the many gas and water mains, sewers, etc.; and in narrow streets, unless elaborate and expensive timbering or walls are used, there is great danger of damage to the adjacent buildings. The same danger exists to a great extent from tunnelling; but unless the top of the tunnel is to be very close to the surface of the street, all things considered, fewer difficulties and less danger may arise when tunnelling than when in open cut. The cost, however, may be greater. Both methods are and have been successfully adopted. It will commonly be found economical and expeditious to judiciously combine the two methods. This was done on the Baltimore & Potomac and Union tunnels in the city of Baltimore.

The Baltimore and Ohio road in passing through the city of Philadelphia adopted the open-cut method for the entire length. In the work now to be described the tunnel method was, however, adopted for sufficient reasons. The depth from the street surface to the crown of the tunnel varied from only a few feet to as much as 40 to 70 feet. The material encountered varied in character, but in the main it was a rather wet, sandy clay. It was evident there would be much inflow of water, and it was subsequently found that the flow constantly filled pipes of 5 inches in diameter. As a bottom drift was therefore necessary, it was decided that two bottom drifts, one on each side, as shown in the several sections, should be run. In this case not only would the proper drainage be provided, but the side walls of the lining could also be constructed. These walls served the two purposes of

affording firm supports for the portions of the work above, and preventing any incipient movement of the material outside, which might result in damage to the adjacent buildings. The drifts proper were 8 × 8 feet, but the excavation at the bottom was continued until a firm bed was reached.

In driving these drifts, frames were constructed, composed of two batter-posts resting on boards, and a cap-piece framed to their tops. These frames were placed about 4 feet intervals. The excavation was advanced in the usual way, by driving sheeting-plank over the top and against the side posts as the work proceeded, with slight upward and outward inclination so that the next frame could be placed, and leaving sufficient space to insert the next section of sheeting. In some cases the head or front end of the drift was also sheeted. The sheeting was driven close together in order to prevent as far as practicable any inflow of water and earth or sand. The side walls of the lining were built in these drifts as close as practicable to the posts of the timber frames, and when necessary a foundation of 1 or 2 feet of concrete was used, upon which the walls rested.

The timber framing was left in place; the open spaces between these and the masonry was filled either with dry rubble or rubble laid in mortar.

It was attempted to drive the top heading by using a series of timber segments forming a polygonal frame, and supporting these at the joints by props resting on the core of earth enclosed between the side walls, or rather drifts. These timber segments were 12″ × 12″ × 6′, forming a series of ribs or frames placed close together; and as each rib was supported independently, the props were close together also, the effect of which was to form a series of small and separate galleries, which caused much inconvenience to the workmen. The excavation was pushed ahead by the use of iron poling-pieces 5′ × 8″ × ¼″, stiffened by ¾-inch angle-irons.

The crown at this point was about 20 feet below the surface of the ground; the material was made earth, and undoubtedly the pressure was very great. The progress of the work was not satisfactory by this system of timbering, and when the pressure of the material came upon the arch the sides bulged and the top flattened.

These were undoubtedly trying conditions for an arch of comparatively fresh mortar. The timber segments were built in, and the filling between them and the brick arch was only the excavated earth. It was difficult to keep the iron poling-pieces in line.

Altogether, the entire method was deemed unsatisfactory, and was abandoned after using it for only a short distance.

Resort was then had to the well-known and often-used method of long poling-bars. The top heading was run in the usual manner, with frames and sheeting. After this heading had been completed for a certain distance the poling-bars were placed against the under side of the cap and supported by vertical props, either resting on the earth core, or in unreliable material on large crosspieces

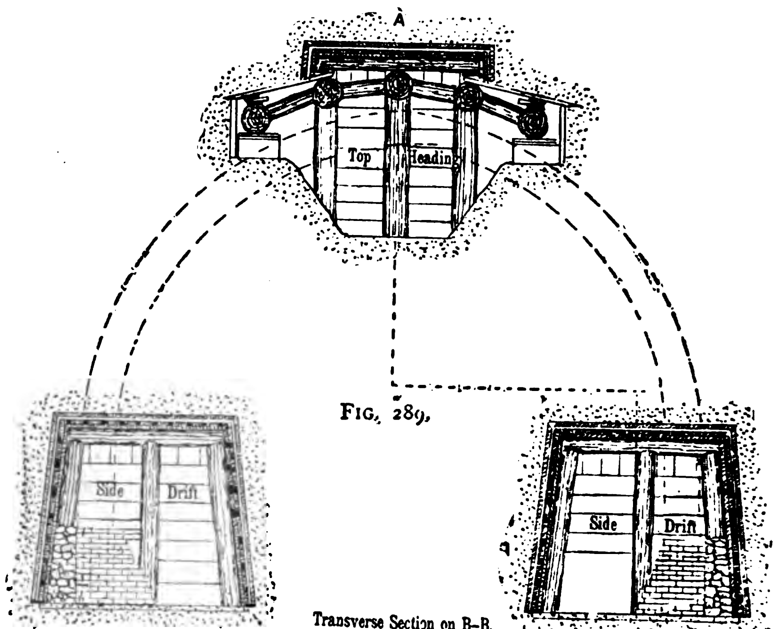


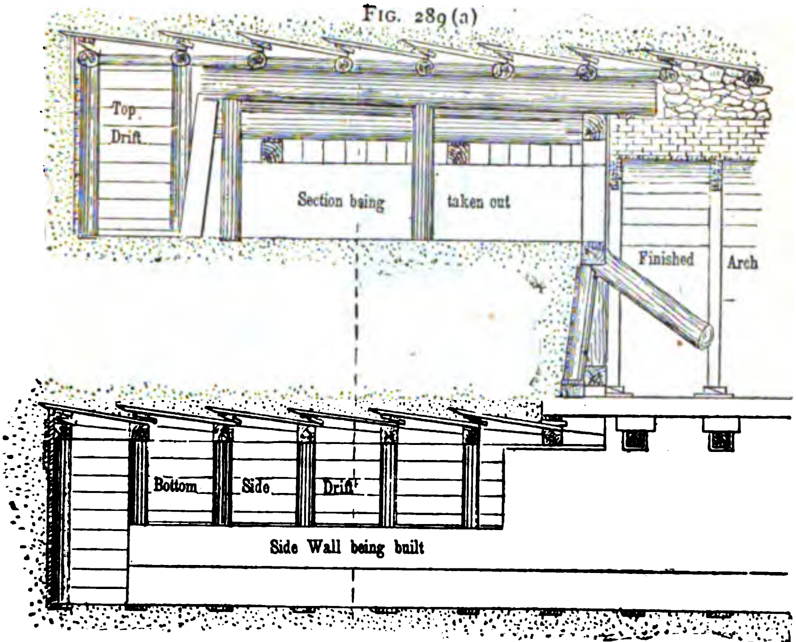
FIG. 289.

WORKING FOR PROPS TO FOURTH AND FIFTH BARS.

or sills of oak 22 × 22 inches in cross-section, and of sufficient length to reach across the tunnel, with their ends supported on the top or roof of the side drifts, which was strengthened by additional props when required. The excavation was then carried on sideways and downwards, the side posts and sheeting on the sides of the top heading being removed. As the work progresses additional poling-bars were inserted and supported at the proper intervals. Figs. 289 and 289(a) are, respectively, a cross-section and longitudinal section of this work, showing only the side drifts and top heading.

The side walls in the drifts are shown partly built; the curved dotted lines show a section of the completed lining in rear of the heading, also shown in longitudinal section resting on the arch centring in Fig. 289(a). In Figs. 290 and 290(a) are shown, respectively, a cross and longitudinal section of the tunnel and lining when the excavation has been carried down to a plane only a little above the tops of the side drifts—the core of earth still left between the side drifts.

FIG. 289 (a)



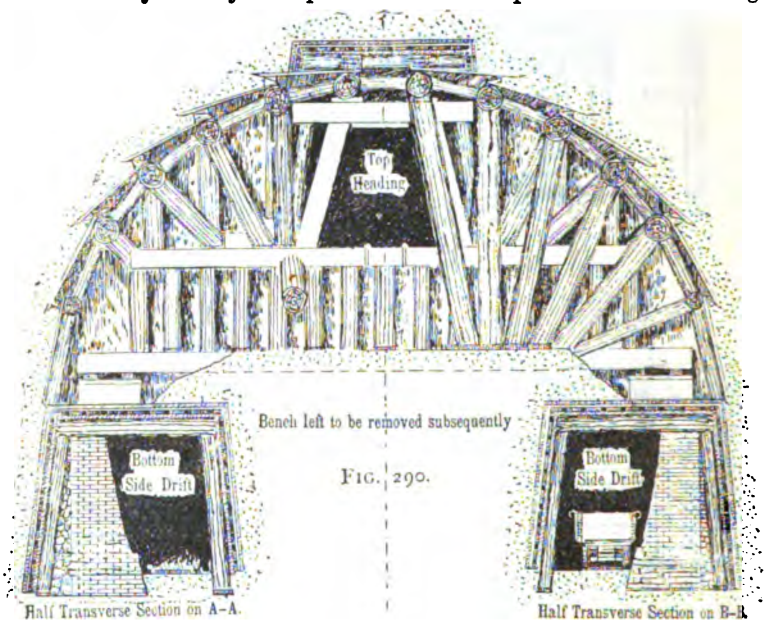
Longitudinal Section A-A.

The poling-bars are of oak, 10 to 12 inches in diameter and 20 to 24 feet long.

The masonry arch was built in sections about 18 feet in length. The centring was moved forward and supported on the side drifts by blocking, and the lagging placed, after which the brickwork was commenced. The props supporting the poling-bars were moved as they were reached by the brickwork. As seen in the drawings, all the timbers of the lining above and outside of the masonry arch were left in place, and the spaces between them and the masonry filled with rubble laid in cement mortar.

That portion of the lining constructed by means of the timber segments and iron poling-pieces having yielded when the pressure came upon it, was repaired with difficulty, and the work carried on in the manner already described.

A portion of the brick lining was supported on timber centring, held at the joints by iron plates. In other portions iron centring



SECTION WITH EXCAVATION COMPLETED

was used. This was made of two 6 × 6 inch angle-irons bolted together, forming an arched rib . Six of these ribs were used, spaced 4 feet intervals, centre to centre. They were made in two halves, bolted together at the crown, and held erect by spacing-rods. The rearmost rib is held fast by the completed masonry, and in turn holds the others while the lagging is being placed. The masons' scaffolds were supported by, or rather suspended from, them. This and the earth core or bench were of very great convenience in laying the brickwork, the materials being brought on the upper level and convenient for handling. This bench is kept 50 to 75 feet in rear of the front end of the masonry. A travelling derrick hoisted the materials from the lower to the upper tracks on which the material cars run. It also afforded a firm support for

props and the large cross-sills, as its sides were supported by the frames of the bottom drifts, which is not the case when simple trenches are cut alongside, as is often done in tunnelling. In this tunnel, owing to the quick striking or removing the centres, it was found that the masonry lining flattened at the crown and bulged at the sides. This was attributed to a want of sufficient time for the mortar in the rubble filling to set. Earth packing was tried, but

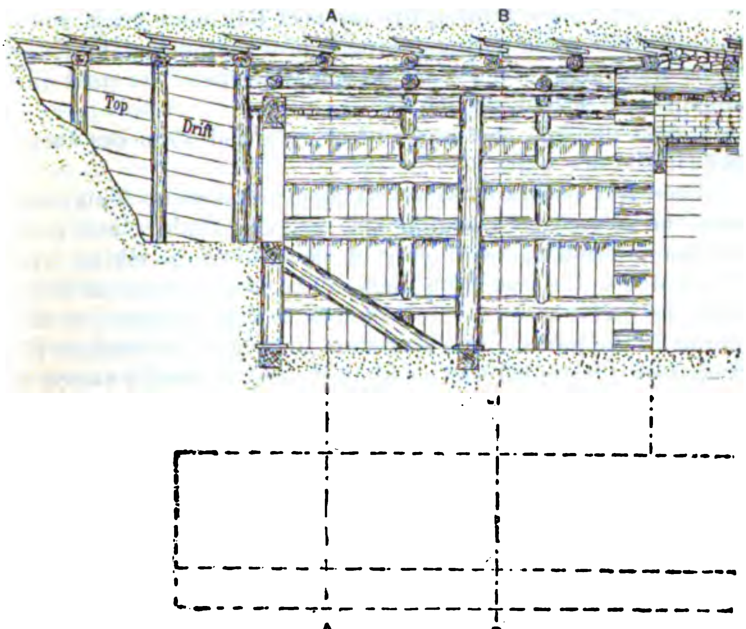


FIG. 290 (a)

gave still worse results, and then dry rubble filling, which gave good results, only using a little mortar with the small chips or spawls.

The side walls were not built more than 20 feet in advance of the arch as a rule, but occasionally they were as much as 90 feet. The drifts were kept but little in advance of the masonry lining, as it was the intention to keep the masonry as close to the head of the excavation as practicable—a rule which should always be followed in any kind of unstable material. A wise precaution was taken in this work, namely, sinking large pipes 6 inches in diameter in

advance of the headings. These driven wells collected large quantities of water, which was pumped out.

This represented so much water that would otherwise have found its way into the drifts. These wells could be driven from 60 to 70 feet into the soil in a week's time, with four five men. As the tunnel reached these wells the pumps were moved to those in advance. The pipes were left in, and used in connection with the air-compressors.

The arch-ring is built of five layers of Baltimore brick, and well bonded. For some portions through heavy ground eight rings or layers were used. New York Rosendale cement was used in the proportion 1 cement, 2 sand.

Refuge niches, 7 feet high, 3 feet wide, and 15 inches deep, are built in the side walls.

There was necessarily some sinking of the surface. This resulted from the necessity of changing and removing timbers, and more or less compression and springing of the timbers in taking up the strains due to the load upon them. The crown of the arch would settle as much as 2 to 6 inches, due to the compression of the mortar in the joints. The maximum sinking of the surface of the street over the tunnel was about 18 inches; it usually varied from 1 to 12 inches. Some damage was done to the water and gas mains and sewers, as well as to the concrete trench of the cable-railway. These were not serious, but required immediate repair, and in some cases considerable lengths of new mains had to be constructed. The sewers were from 2 to 4 feet in diameter; water-mains, 3 to 20 inches; gas-mains, 1½ to 20 inches. But little, if any, damage was done to adjacent buildings, although the line of natural slope passed beneath them.

The temperature in the tunnel was usually from 75° to 80°, and in some cases it was as high as 95°. The temperature was lowered by the exhaust-air from air-compressors used in connection with some of the machinery, or drawn direct from the open air through pipes provided for the purpose.

The bottom drifts were of sufficient size to allow two miners and a helper in each, and working in 10-hour shifts an advance of 50 to 90 feet was made per month.

It required about 125 mason-hours for an 18-foot section. In laying the brick arch, from 5 to 8 men worked together on an 18-foot section, which required about 22 two-shift days' work for completion.

The cost of the tunnel, ready for the track, was about $225 per lineal foot. Howard Street Tunnel is 8350 feet long, and cost $1,750,000. The land damages were $1,000,000. Total cost of entire work, $6,000,000. Maximum cross-section 27 feet wide, 22 feet high. Excavation from 3 to 5 feet larger.

724. The necessity for transferring, in large cities, a great portion of the traffic from surface tracks to underground tracks, particularly that calling for uninterrupted and rapid transportation, renders the subject of tunnels along street-lines one of great importance and interest to engineers; and while the questions involved are of the greatest importance and intricacy, in regard to the best plans and modes of prosecuting the work, which would lead us away from the purposes of this volume, a short description of some of the designs will be appropriate. The description and drawings of the Baltimore Belt Tunnel were given in great detail, not so much from the point of view regarding rapid transit, but because it was a good illustration of a modern method of tunnelling through a somewhat unstable material.

The following is a brief description of two designs presented to the New York City Rapid Transit Commission for constructing the underground railway along the lines of the streets of New York City, having special regard to the conditions existing there.

725. One of the plans contemplates a tunnel, excavated without disturbing the surface of the street; it provides for four tracks, at a depth of 20 feet below the street-level.

The only open excavations required will be narrow trenches close to and along the curb-lines for the purpose of building the side walls of masonry. The drifts required are to be run transversely to the street-lines, from curb to curb. This is to be done in such a manner as not to disturb the surface, or the gas, water, or sewer mains.

The roof is to consist of a series of transverse steel beams resting on the side walls and on intermediate steel columns, or rather on longitudinal girders resting directly on the columns. A single line of girders and columns at the middle of the width is to be used, the columns placed 4 feet apart, centre to centre. The transverse beams are to be 15-inch **I** beams, weighing 50 pounds per lineal foot. These beams rest at one end on the masonry walls and at the other on the centre girder, where they abut against each other, and are spaced 16 inches apart on centres. These transverse beams are to be covered on top with steel plates $\frac{3}{8} \times 16$ inches. The longitudinal girders are made of two 12-inch channels, weighing 20

pounds per foot, placed back to back and 7 inches apart. At the points where the transverse beams rest two pieces of 7-inch channels are riveted between the longitudinal channels.

The supporting columns are made of 2 to 7 inch channels, weighing $14\frac{1}{2}$ pounds per foot, with two $\frac{3}{8} \times 12$ inch cover-plates, riveted to the channel flanges. The columns are finished top and bottom with angle-irons, in order that they may fit fully and squarely the girders at the top and a cast-iron base at the bottom. This iron base is 16 inches square at top and 2 feet square at bottom, where it rests upon a masonry base. The cover-plates of the roof are to be protected from corrosion by injecting coal-tar on the upper surface during the construction. Excepting the difficulties necessarily attending the excavation and placing the beams and columns, the method and design are the simplest conceivable. The load on the roofing beams was estimated as follows: Weight of sand 7 feet deep at 130 pounds per cubic foot; 20 inches of pavement at 150 pounds per cubic foot; and a moving load of 150 pounds per square foot. These aggregate 55,400 pounds per lineal foot of a carriageway 44 feet wide. The deflection of the transverse beams at their middle points under this load would not exceed 0.35 inch.

The material of the lining or ironwork would weigh about 3100 pounds per lineal foot of roadway.

This is Mr. William E. Worthen's plan.

726. The plan of Mr. William B. Parsons contemplates four tracks; but instead of all these being on the same level, two of the tracks are placed above the other two, forming a two-story or double-deck system; the lower tracks supported on a bed of concrete, the upper on a system of transverse beams, supported at one end on the masonry side walls and at the other on steel columns resting on intermediate masonry walls. This was also roofed over with steel beams. These beams were placed immediately below the street paving or under the cable subways; so that, while the upper tier of tracks are only 10 feet 10 inches below the sidewalk level, or several feet less depth than in the Worthen plan, the lower tracks are about 24 feet 8 inches below, or 4 feet 8 inches lower than that contemplated in the Worthen plan. This work was to be carried on in open cut, but leaving at least one fourth of the width of the street always open to the traffic; and, in addition, this plan contemplated placing the pipes, mains, and wires in a specially constructed subway beneath the tracks of the cable-railway.

These three plans just described show about the general devel-

opment of the plans and designs for tunnels intended for rapid transit. For a full discussion of this subject, with drawings, see *Eng. News*, Oct. 17 and 24, 1891.

727. *St. Clair Tunnel.*—As illustrative of a very difficult work carried to a successful completion, we give the following description of the construction of the St. Clair Tunnel, under the St. Clair River, which with the Detroit River forms the connection between Lakes Huron and Erie. Both rivers are wide and deep. Aside from the cost, the difficulties in the way of securing good foundations for the piers and the obstruction to navigation rendered impracticable the construction of a bridge.

The first effort to secure a satisfactory passage was made in an attempt to tunnel under the Detroit River.

This tunnel was to be 8000 feet long, of which 3000 feet would be under the river. A tunnel 18½ feet in diameter, lined with brick masonry 24 inches in thickness, was contemplated. This tunnel was to have been drained by a tunnel 5 feet in diameter, constructed immediately below the main tunnel. The smaller or drainage-tunnel was driven an aggregate of about 1700 feet from the two ends; but owing to the difficulties of construction, caused principally by sudden irruptions of sand and water under a pressure greater than that due to the head from the river above, the work was abandoned entirely and finally.

728. In 1885, about twelve years after the above-mentioned failure, the enterprise was again undertaken, but on a different site. A point on the St. Clair River was selected, and a number of borings were made on a parallel line 50 feet south of the centre line of the proposed tunnel. This was done to ascertain approximately the character of the river-bed, without, however, making holes that might be of subsequent inconvenience in prosecuting the work. These borings were few in number, and the pipes, 6 inches in diameter, were simply driven by means of a pile-driver through the sand and gravel into the clay. The information obtained by these borings was not satisfactory, as it was found that the underlying strata were formed of fine sand and gravel, and below these a soft, tenacious clay permeated with water indicated an unstable kind of material for tunnelling through. It was, however, decided upon to run a drift or heading at a depth of about 75 feet, or 60 feet below the water surface, at which point the crown of the tunnel should be located. Shafts were immediately sunk in the two banks, and drifts commenced from them. On one side the

drift was only driven about 20 feet, and on the other about 186 feet; but owing to the difficulties encountered and the dangers incurred, this work was also abandoned. Gas, water, and quicksand were encountered; an explosion of gas occurred at one time. The drift was driven in the usual manner, i.e., with frames and sheeting. Nothing farther was done until the year 1888.

729. The importance, or rather the necessity, from a business standpoint, was evidenced by the third attempt to drive the tunnel, which was carried to completion. The estimated cost was $2,500,000. The interest on this sum was guaranteed, bonds issued and sold, and the work undertaken in a manner to insure success.

A proper beginning was made by an elaborate system of borings. The borings were made on the centre line of the tunnel, and at intervals of 20 feet all the way across the river. Sufficient information had already been obtained to settle the question as to the location of the tunnel. It could not be driven, or at least it was so believed, above the clay; and the depth of the rock below the surface, as well as its loose, porous, and shaly character, and being of the same formation which furnishes gas at many points in the vicinity, precluded the plan of tunnelling through the rock. All things considered, it was decided to drive the tunnel in the clay stratum at about the middle of its depth, leaving a thickness of about 12 feet both above and below it. This being settled, the borings were only extended to the clay, mainly to be sure of locating the tunnel properly with respect to the clay stratum, and to avoid holes through the upper portion of the clay into the tunnel. For these borings the 6-inch pipes were mainly sunk by means of the water-jet, or aided by the pile-driver. There were 110 borings, sunk about 40 feet each below the water surface, consuming about 70 days, and at a total cost of, for all borings on and off of the centre line of the tunnel, and averaging 45 feet in depth and 121 in number, $5000, or a cost per foot of bore-hole of $0.91.

The writer has made a number of borings from 50 to 90 feet in depth for 50 cents per foot, with good profit to the contractor, perhaps not in so great a current or with as many other difficulties to contend with as on the St. Clair River. In Fig. 291 is shown a cross-section of the river, with water, sand, clay, and rock lines; also a longitudinal section of the tunnel, with its grades, heights, etc. The heights were 21 feet on the steep grades at the junction with the lighter grades, and 20 feet on normal lines. There was a continuous downward slope from the United States shaft to the shaft on the Canada side.

The lowest point of the tunnel below surface grade of the railroad is 100 feet.

Shafts were sunk on each shore—one 58 feet in depth and the other 90 feet. At this depth the inflow of material was more rapid than it could be removed. The shafts were then abandoned and filled up.

It was now determined to carry on the work from the two sides of the river in open cut as far as might be practicable, or might not prove more expensive than the tunnel. Owing to the repeated mishaps and failures, contractors either would not or dared not undertake the work except on such liberal allowances for contingencies as would justify this mode of construction; and consequently the work had to be undertaken by the railway company itself.

The total length is about 8500 feet, made up of the approach on

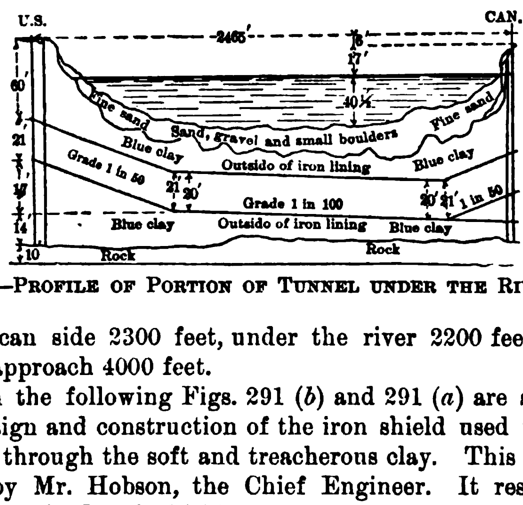


FIG. 291.—PROFILE OF PORTION OF TUNNEL UNDER THE RIVER-BED.

the American side 2300 feet, under the river 2200 feet, and the Canadian approach 4000 feet.

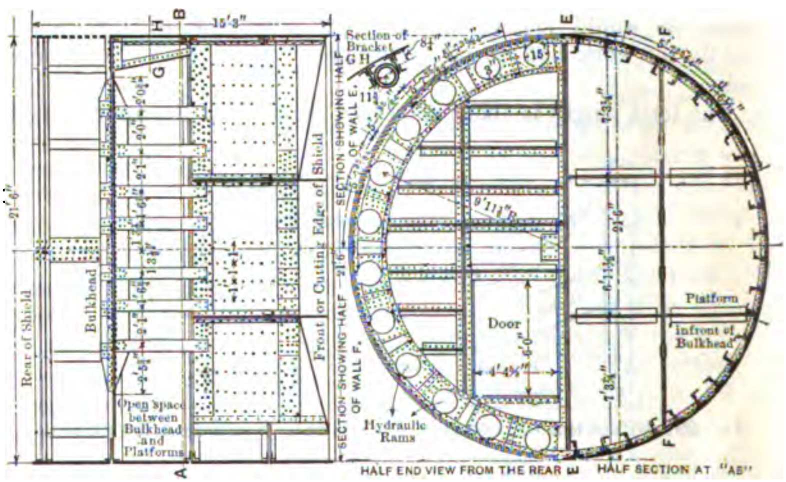
730. In the following Figs. 291 (*b*) and 291 (*a*) are shown the general design and construction of the iron shield used in driving the tunnel through the soft and treacherous clay. This shield was designed by Mr. Hobson, the Chief Engineer. It resembles in some respects the Beach shield.

The shield consists of a cylindrical shell of steel 15 feet 3 inches long, and 21 feet 6 inches in external diameter.

The steel plates are one inch thick, and planed to size 5 feet $7\frac{35}{64}$ inches long and 4 feet broad. Twelve of these plates make one complete ring, and three of these rings, together with one 3 feet 3 inches wide, make the entire shell in circumference and length. The plates were simply abutted against each other both along longitudinal and circumferential joints. Over the longitudinal joints,

for the rearmost ring of the shield, splice-plates were placed both inside and outside; the inner plate was 2 feet $11\frac{1}{4}$ inches long and $\frac{1}{2}$ inch thick, the outer plates 4 feet long and $\frac{5}{8}$ inch thick, each 1 foot wide. These outer splice-plates were the only projections on the exterior surface.

This form of joint for the rear ring was required in order to leave room for the hydraulic rams, which were placed around and close to the inner cylindrical surface of the shell, and also as the iron lining always extended into the rear end and close to the same surface, thereby leaving no interval between the shield and the



VERTICAL LONGITUDINAL SECTION OF SHIELD. FIG. 291(b).

VERTICAL TRANSVERSE SECTIONS. FIG. 291(a).

permanent lining of the tunnel. All of the rivets at these joints were countersunk. To the inner surface and along all the edges of the other plates forming the forward rings angle-irons were riveted; the rings were formed and connected to each other by riveting through the free and projecting legs of the angle-irons. These joints were not favorable so far as resisting tension was concerned, but the angles acted as stiffening ribs to the plates in resisting compression from without and distortion or deformation.

The front edge of the shield was planed to a cutting edge on an angle of 20°. The bulkhead in the shield was placed at the first circumferential joint, i.e., 4 feet from the rear edge of the shield; its edge was riveted between the angle-irons forming this joint.

It was made of ½-inch steel plate. To the rear face of this plate there were riveted 7 horizontal and 3 vertical stiffeners, only extending to points 9 inches from its circumferential edge. In the lower part of the bulkhead there were two openings, each 6 feet high and 4½ feet wide, through which all of the material excavated in front of the shield passed. A sliding door was constructed for these openings, which could be lowered over the openings in case there was danger of an inflow of material, such as water or quick-sand, from the front of the shield. It was not found necessary to close these doors in this work.

To admit the hydraulic rams used to force forward the shield, 24 holes 13½ inches in diameter were cut through the bulkhead near its circumference; to reinforce the plates through which these holes were cut, plates 16 inches wide and ½ inch thick were riveted to the bulkhead all around its edge. To receive each ram two gusset-plates were bolted to the bulkhead and to the shell of the shield. These transmitted to the outer shell the thrust of the rams. The tail of the cylinder of the hydraulic jack was fitted in a ring, bolted between the gusset-plates. The head of the cylinder was flanged, and rested against the bulkhead. That portion of the cylindrical shell in front of the bulkhead was strengthened, and deformation prevented by placing three vertical and two horizontal partitions. These latter also served as platforms for the workmen at the front face of the tunnel. These partitions and platforms were placed so that between their rear ends and the bulkhead there was an open space. Across this space, however, were placed a series of flat plates 7 × ½ inches, firmly riveted to the vertical partitions at one end and the bulkhead at the other. These were intended to prevent the giving way of the bulkhead in case they should be subjected to a great pressure arising from an inrush of material from the front. The horizontal partitions then extended from 4 feet in front of the bulkhead to the cutting edge of the shield, or about 7 feet in length. The vertical partitions were sloped back from the cutting edge. The workmen standing on the horizontal partitions of the platform excavated and shovelled the material to the rear, so that it fell through the open space and down in front of the doorways in the bulkhead, through which the material was carried, and thence out to the end of the tunnel. The doors slid in grooves faced with rubber bands when the chains suspending them were loosened. A very just criticism upon this arrangement is that no provision was made to force the doors down to their

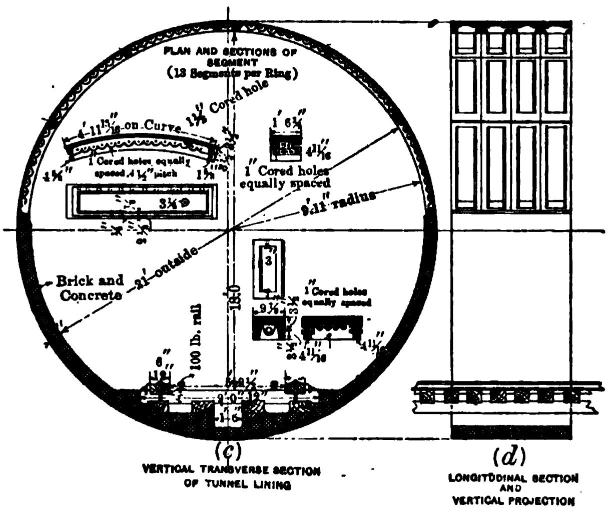
bearings in case they were jammed by the pressure arising from the inflow of material—the weight of the door itself being the sole reliance. As a safety, precaution holes were bored with an auger at least 8 feet in advance of the excavation, so that if a pocket of quicksand or water existed proper provisions could have been made to meet the danger, and at any rate ample time would have been given the workmen to escape through the openings, and also for closing the doors.

731. For a distance of 1716 feet on one side of the river and 1994 feet on the other the work was carried on without the aid of compressed air, but when the tunnel reached the banks of the river provision was made for the use of compressed air by building solid bulkheads of brick masonry in cement mortar. These walls were 8 feet in thickness along the axis of the tunnel, and through them two large air-locks, 17 feet long and 6 feet in diameter, were built for the passing of men and ordinary tools, materials, etc. Also, a lock 25 feet long and 10 inches in diameter was constructed in order to pass sections of pipe, etc., too long for the main locks. The compressed air was furnished by two 20×24 inch Ingersoll compressors at each end, and a 6-inch wrought-iron leading pipe. Only a moderate pressure of 10 pounds was at first used, which was increased to as much as 28 pounds above the atmospheric pressure, when bad material was encountered. It is probable that the work could have been carried on more economically and rapidly had compressed air been used at an earlier stage of the work, as in many cases caving and sloughing increased the quantity of material to be removed as much as 50 per cent, which the compressed air would have prevented.

732. Shields for underground excavations have been used in one form or another for a long period of time. Mr. Brunel, in tunnelling through mud under the Thames River, England, used a shield made of iron plates. At the front end was a bulkhead composed of a series of cast and wrought iron frames standing vertically and side by side. Each frame stood on cast-iron feet resting on a plank flooring; to these feet the frames were connected by hinged arms. The frames were also similarly connected to each other. Each frame was covered in front by poling-boards. They were 3 feet broad, and had three stages or platforms in their height, from which men could work. The poling-boards were 3×6 inches in thickness and width respectively. Each of the frames worked independently. The work was carried forward by

removing in every alternate frame one of the poling-boards, excavating about 6 inches ahead, and replacing the board and holding it against the material with two small jack-screws. When this had been done in front of 6 of the 12 frames, the frames themselves were lifted and pressed forward by two large screw-jacks, which rested against them at one end and the completed masonry lining in rear. Then the remaining six alternate frames were operated in the same manner.

The shields for the Hudson River tunnel were, in their general



FIGS. 291 (c) AND (d).

construction, similar to those described for the St. Clair tunnel, differing, however, in many details of construction.

733. But few tunnels up to this time had been lined throughout with iron instead of brick masonry. One of these, which was then in course of construction, was the Hudson River tunnel. The lining for this is 19 feet 6 inches in outside diameter and $1\frac{1}{4}$ inches thick.

In the St. Clair tunnel an iron lining was used also, 21 feet 2 inches in outside diameter and 2 inches thick. The iron lining, in soft and yielding materials, saturated with water, is considered both more reliable and permanent than one of brick. (See Figs. 291 (*c*) and (*d*).) This lining is made up of cast-iron segments $4\frac{1}{1}\frac{5}{8}$ inches in length and $18\frac{1}{2}$ inches in width, with a thickness of 2 inches.

The flanges for the circumferential joints are 2⅜ inches thick, and for the radial or longitudinal joints 2¾ inches thick at the base and 1⅝ inches thick at outer edge. Through each of the former flanges are cored twelve 1-inch holes, spaced 4½ inches, and bosses ⅞ inch high and 3 inches in diameter are cast on the inner surface of the flange to compensate for the weakening caused by the holes. Each longitudinal flange has four bolt-holes, spaced 3 inches apart. Also a 1½ hole was cored through the shell of the segment, through which cement grout could be injected to fill in the space between the lower half of the lining and the clay, requiring about a 3-inch layer. The loss of strength due to these holes was compensated for by a 3¼-inch boss. A complete ring required 13 segments and a key-piece 9⅜ inches long. To bolt the segments of each ring required 56 bolts 5⅞ × ⅞ inches, while to connect the rings together required 157 bolts 8 × ⅞ inches. All bolts were of mild steel, with square heads and hexagonal nuts; these were screwed up against a wrought-iron washer 1⅞ × ⅛ inches.

The joints were made tight against leakage by means of $\frac{3}{16}$-inch oak packing-pieces, the iron surfaces having been previously planed. These packing-pieces absorbed water, swelled, and made a tight joint. They were only placed in the longitudinal joints. In the circumferential joints coarse canvas covered with asphalt was used; over which on the inner edge a groove was left in the abutting surfaces ½ inch wide and 2 inches deep for a lead calking, if found necessary. This has been put in around many of the joints. If the timber packing rots or does not prevent leakage, it can be removed and the joint calked with lead. All segments varying from the specified dimensions more than $\frac{1}{32}$ inch were rejected; also, the variation in weight allowed was between the limits of 1000 and 1050 pounds. The lining required 9333 pounds of iron per lineal foot, or 56,000,000 pounds in all. The iron segments were heated to about 400° F. and dipped in a bath of melted pitch before using. The wooden packing-pieces were also soaked in pitch.

The temporary track in the tunnel was laid on a bed of earth. This was subsequently removed, and the whole tunnel thoroughly cleaned. The lower half was then lined with brick, laid in cement mortar to a thickness equal to the depth of the inner flanges, and brickwork was also laid for the support of the timber of the permanent track. Over the whole was then placed a 1-inch layer of neat cement mortar. This protected the lower half of the iron work from corrosion, especially that caused by the drippings of brine

from the refrigerator-cars. The upper half of the iron lining was painted with asphalt. The stringers, ties, and guard-rails were treated with 16 pounds of creosote per cubic foot. The iron rails weigh 100 pounds per yard.

Fig. 291(*c*) shows vertical transverse sections of tunnel and iron lining, with some details of segments, track, etc. Fig. 291(*d*) shows longitudinal section and view of a portion of the lining track, etc.

734. To push the shield into the clay 24 hydraulic jacks were provided, each of which was mounted between gusset-plates firmly riveted to the outer shell of the shield (see Fig. 291(*a*)). Each jack had two cylinders—one 8 inches in diameter for forcing the shield forward, the other $2\frac{5}{8}$ inches in diameter for drawing back the larger plunger in order to make room for placing a new ring of the lining. The effective area of the larger was $49\frac{1}{2}$ square inches, and of the latter $3\frac{5}{8}$ square inches. The water pressure averaged about 2000 pounds per square inch, so that the larger plunger exerted a pressure of 99,000 pounds and the smaller 7250 pounds. The cylinders were made of cast steel, as was the head which received the thrust. The plunger was of cast iron. The jacks were made as small as possible, and were placed as close to the inner lining as practicable so as to bring the pressure on the solid iron in the tunnel lining, and not endanger the flanges of the segments of which it was formed. The extreme length of stroke was 26 inches, the usual stroke was limited to 23 inches, which gave ample room for placing a new ring of the iron lining. In this manner the shield was pushed forward, the jacks transmitting the pressure at one end to the lining of the shield and at the other to the permanent iron lining. The shield was pushed forward, the material at the front excavated and removed, and a new ring added to the permanent lining. It is not necessary to describe the details of the cocks, valves, and pipes connected with the jacks for regulating and controlling their action. The pumps, as well as the other plant, were placed near the entrance to the tunnel on each side of the river. The water-pipe from the pumps to the jacks was $1\frac{1}{4}$ inches internal diameter; it had to be lengthened, as the shield advanced, under pressure. This required a special device, which consisted of two half-lengths of pipe having a kind of hinged or link motion. The maximum length of this pipe on one side of the river reached 4000 feet, between the pump and the jacks.

Practically the only resistance to the forward motion of the shield was the friction of the cylinder against the clay. As the

pressure required to overcome this resistance varied from 450 to 2000 tons, and the cylindrical surface in contact with the clay was 1030 square feet, the frictional resistance per square foot was from 850 to 3880 pounds, or 6 to 27 pounds per square inch.

The tunnel was lighted by electricity, by means of incandescent lamps.

735. Ventilation was effected by two Root blowers at each end, each having a capacity of 10,000 cubic feet per minute. These delivered air into a galvanized-iron pipe two feet in diameter, which was supported under the roof of the tunnel and terminated at the air-locks. Pipes were carried beyond the air-locks as the work advanced, so as to deliver the fresh air near the shield at all times. As no lanterns or torches and but little powder and steam were used in the tunnel, the ventilation was unusually perfect.

736. The drainage of the tunnel proper, both during construction and subsequently, has proved to be a simple matter, but required the use of pumps, owing to the down grade from both sides. The iron lining is almost impervious.

The greatest difficulty arises from the necessity of preventing the surface and seepage-water along the slopes of the open-cut approaches from running into the tunnel. This was, however, controlled by collecting the water in shafts or wells and pumping the same into pipes or drains constructed to carry off the water. Pumping capacity is provided sufficient to take care of a rainfall of 3 inches per hour.

737. As fast as the material in the tunnel was excavated it was loaded on four-wheel cars holding about 1 cubic yard each, and drawn by horses to the mouth of the tunnel. At this point a derrick lifted the car-body from the truck and landed it half-way up the slope of the open cut; another derrick lifted it from this level to the top of the slope, where the earth was dumped into a flat car, which was then hauled by horse or locomotive power to the proper dumping-ground.

738. *Locating the Centre Line of the Tunnel.*—At the far end of each open cut a brick pier, 3 feet square and bedded at a depth of 12 feet below the surface, was constructed. This was capped with a cut-stone block 2 feet thick, on which was placed a theodolite, over which a comfortable house was built. The object-glasses were 2½ inches in diameter. These two instruments at the extreme outer edge of the work were in full view of each other. Each instrument was adjusted to bisect the object-glass of the other; dis-

tance apart 2100 yards. An ordinary 7-inch Stackpole transit was placed at the mouth of the tunnel, and adjusted until it was exactly on the line of sight of the large theodolite. The small transit was then sighted on the object-glass of the theodolite for a back-sight and reversed to prolong the line into the tunnel. There was therefore no difficulty in keeping the line straight until the brick bulkhead was reached. To carry the line through and beyond the bulkhead into the air-chamber the following plan was adopted: A cast-iron pipe 12 inches in diameter and 25 feet long was built into the bulkhead and projecting on either side, and placed on the centre line of the tunnel; this pipe was closed at each end with a hinged cover of heavy plate-glass, and provided with valves at each end, so that the pipe could be converted into an air-lock. The glass plates were covered with iron plates for protection to them when not in use. In the pipe near each end a set of cross-wires were mounted on a ring, adjustable in the same manner as the cross-hairs in any transit. To adjust these cross-wires a transit was set up in rear of the bulkhead and adjusted exactly on the centre line of the tunnel; both the iron and glass cover on the rear end of the pipe were removed so that the sight could be taken directly on the wires in the pipe, only the iron cover was removed from the front end of the pipe; and the cross-wires were illuminated by an electric light held up in front of the front glass cover. The vertical wires were then adjusted to the same and proper vertical plane. The transit was then carried through air-locks in the bulkhead and set up in front of the iron pipe; the front glass was now removed after closing the rear glass on the opposite side of the bulkhead, in rear of which an electric light was held; again the cross-wires were in an unobstructed view; the transit was now shifted until its line of collimation was in the same vertical plane with the adjusted vertical wires in the iron pipe, and the line was then prolonged by reversing the transit. The position of the centre of the shield and of the plane of the bulkhead was tested every day, and if found out of line or position, it was forced into its proper place by using only a certain number of the jacks, leaving the others idle for the time. It is to be noted that by the method of alignment used errors that might arise from sighting through glass were avoided, as the sight was taken on the naked wires. It was important that when the shields met there should be no error whatever in the alignment or levels from the two ends. To insure against such errors the shields were stopped when about 100 feet apart, and a 6-foot drift

was run between the two, lined with timber. Proper measurements were made through this drift to verify the accuracy of the preceding work. As a result, when the two shields met the error in levels was only $\frac{1}{4}$ inch, and in the line it was inappreciable. The levels were run by the ordinary method. In this tunnel the conditions for an accurate alignment were very favorable, as the view along the surface of the ground was unobstructed from end to end. Had an intervening hill existed, three theodolites would have been necessary in order to have pursued the same method—one at each end and one on the higher intervening ground. Had the country been rugged and mountainous, the exact method would have been impracticable. In this case an accurate triangulation would have been necessary to have established the directions of the line at the two ends, or to have located the proper positions of shafts, if these were to be used. Where intermediate shafts are used in order to run the drifts on the proper line, the usual method is to establish accurately the line over the top of the shaft, then to place a straight-edge across the shaft on the line, and by the use of heavy plumb-bobs suspended with long chords to transfer the line to the bottom of the shafts. As this line can only be a few feet in length, errors of exact transference, and the greater chance of error in working from a short line in both directions, have to be met and overcome. Or the line can be transferred by a transit having what is called a "diagonal eye-piece" for sighting vertically upwards, then revolving the transit on its horizontal axis, and thus transferring the line of the straight-edge to the bottom of the shaft. These shafts, called permanent or working shafts, may have any diameter desired, —the larger the better. They have been made as much as 50 feet in diameter. They may be rectangular or circular in form. When rectangular, they are usually timber-lined; when round, lined with either brick or iron.

They are expensive to construct, and increase the cost of excavating the tunnel proper on account of the time and labor required to lift the material to great vertical distances. They aid greatly in the rapidity of excavating the tunnel, and where large sums of money are to be expended the loss of interest may be greater than the cost of construction of the shaft; and, on the whole, they may prove to be the most economical method of prosecuting the work.

739. *The Stampede or Cascade Tunnel on the Northern Pacific Railway.*—This tunnel is 9850 feet from portal to portal, $16\frac{1}{2}$ feet in the clear width, and 22 feet in clear height. The materials

through which it was driven varied from very hard rock and rock of medium hardness to a shaly substance which swelled upon exposure to air and moisture, bringing to bear a tremendous pressure upon the lining. While a full account of the construction of this tunnel would be interesting and instructive, space will not allow it, as tunnelling operations have been already discussed. A few points of interest will be mentioned, especially as an accurate account of the cost was kept and published. As the contract required the work to be executed in 28 months, and 6 months was consumed in driving 900 feet by the use of hand-drills, it required an average progress of 13.57 feet per day for the remaining 22 months, which, it may be stated here, was accomplished.

The usual method of tunnelling in rock, especially where a limited amount of water was encountered, was adopted, namely, driving a heading near the top—in this case an 8-foot heading. With the exception of about 500 feet inward from each portal, where the rock was hard, the entire tunnel was timber-lined. The average number of men employed was 350, and their wages varied from $2.50 to $5.00 per day. A bonus of 25 to 50 cents for each foot was paid for any progress made over the required daily average. Work was carried on night and day. There was required of the different grades of powder or explosive used a total of 309,625 pounds. The rock was of a basaltic character, with a considerable dip, which, together with its shaly condition, rendered it necessary to keep a strong lining close up to the front of the excavation. To allow for the timbering, the excavation had to be 19½ feet wide and 23 feet 10 inches high, or an equivalent of 15.7 cubic yards per lineal yard more than the clear section of the tunnel. The heading was taken out to the full width of the tunnel at a point 8 feet below the top. While hand-drilling was used, there were employed in each heading 17 men with 23 helpers or clearers, and the daily progress made was 4 lineal feet in 24 hours. When air-drilling, 5 drills were used at each end. Below this heading a bench was left, upon which rested the supports for the lining; the excavation of the bench was carried forward at the same time, its rear face or breast being 10 to 20 feet in rear of the face of the heading. In blasting, from 20 to 23 holes, averaging about 12 feet in depth, were made in the face of the headings, and about 18 of the same depth in the face of the benches. Each drill would make in medium-hard rock 6 or 7 holes in 5 hours ; but in very hard rock not more than one third of the aggregate depth would be made.

The 10 drills, 5 in each end, made a progress of 6.9 feet per end, or a total of 13.8 feet. In each end about 400 pounds of powder were used at each blast. This would break from 8 to 12 lineal feet of rock, but in hard rock only about 4 feet in the headings and 6 feet in the benches. The timber-posts were 12 × 12 inches, the wall-plates and the five segments in the arch-lining were also 12 × 12 inches; the sills were 8 × 12 inches. The poling-boards were 4 × 6 inches. The vacant spaces above the timber lining were packed full of cord-wood. The frames were from 2 to 4 feet apart, depending upon the character of the material excavated. The materials were hauled out with mules until the length of the haul exceeded half a mile, when special platforms were constructed, under which cars were run and loaded, and the hauling was done for the remaining work with locomotives. In some cases the 12-inch timbers were reduced to a thickness of 4 inches under the great pressure due to the swelling of the shale.

As it was evident that the material would be self-sustaining if only it was covered so as to prevent exposure, masonry walls were not considered necessary. But it was determined to use concrete walls, composed of 1 cement, 2 sand, and 5 broken stone; these had a minimum thickness of three feet. Every alternate post was removed, and struts inclining to the adjacent posts on either side substituted; this space was then built up with concrete, the intermediate posts were then removed, and the vacant spaces filled with concrete. Above, and resting on the walls, a brick arch was constructed to support the roof.

The total cost of the tunnel was $1,160,000, or $118 per linear foot.

The following table gives an itemized statement of cost to the company and to the contractor of the work done during one month. During this month everything was favorable for rapid prosecution of the work.

During this month an advance of 246 feet was made in one end of the tunnel and 258 feet in the other, or a daily rate of 8.2 and 8.6 feet, making a total of 16.8 feet per day. During this month the cost to the company was $52,485.50, or $104.15 per linear foot, and to the contractor $37,427.57, or $74.26 per linear foot, giving a profit of $29.87 per linear foot to the contractor. This was based on the following itemized statement of costs. Allowing for interest on cost of plant at $50,000 × $0.06; depreciation in value of plant for the entire work 75 per cent, and for one month one

twenty-eighth of this, $50,000 × $0.75 × $\frac{1}{28}$; and an additional 10 per cent allowance for contingencies on all expenses, there results:

TABLE LXII.

Pay-rolls for labor at the two ends	$22,897.75
160,000 feet B. M. @ $10	1,600.00
1562 pounds iron @ $0.6	93.72
127 cords of wood @ $3	381.00
480 tons of coal @ $4	1,920.00
1800 caps (estimated) @ $0.1	18.00
28,800 feet fuse @ $0.1	288.00
228,000 pounds Giant powder @ $0.16	3,648.00
Allowance for interest, $100,000 @ 6% for 1 month	500.00
Depreciation of plant, $100,000 × $0.75 × $\frac{1}{28}$	2,678.60
	$34,025.07
Add 10 per cent	3,402.50
Total cost to contractor	$37,427.57

Cost per linear foot @ 504 feet = $74.26.

Cost to the company:

504 linear feet @ $78	$39,312.00
1683 cubic yards excavation @ $4.50	7,573.50
160,000 feet B. M. @ $35	5,600.00
Total	$52,485.50

Cost per linear foot @ 504 feet = $104.14.

Profit to contractor per linear foot, $104.14 − $74.26 = $29.88.

A rapid but expensive mode of tunnelling in rock is to drive a heading at the bottom, and at short intervals to drive vertical shafts, say 6 × 8 or 8 × 8 feet, from it upward to the top of the tunnel section, and from these latter run each way a top heading. This plan was adopted in the Arlberg tunnel in the Alps.

740. The following notes on a few of the more important tunnels will be interesting:

The Box Tunnel, England: 3200 yards long, 30 feet wide, 24½ feet clear height above the roadway. Partly through clay and partly through limestone. It is straight, and has a continuous grade of 1 in 100 from end to end. Lined for about one quarter of its length with masonry side walls and a brick arch. There were 7 brick-lined shafts, 25 feet in diameter, with a maximum depth of 300 feet.

The Brislington Tunnel: 1100 yards long, driven through hard rock. A heading 7 × 8 feet was driven from end to end before commencing excavation for full section.

Woodhead Tunnel, England: A fraction over 3 miles long. Clear width, 14 feet 3 inches; clear height, 18 feet 3 inches. This has been paralleled by a twin tunnel, leaving walls 21 × 12 feet between the two, with arches between them. Driven in rock, leaving vertical side walls, and no invert used.

Kilsby Tunnel, England: Driven through clay and sand; lined with brick masonry from 18 to 27 inches thick; 2400 yards long; cross-section 27 × 32½ feet. Much difficulty arose from the existence of pockets of quicksand. Estimated cost, $436,500; actual cost, $1,697,500. Four years consumed in the construction. Cost per linear foot, $235.

The Netherton Tunnel, England: 3036 yards long, 27 feet wide, 24½ feet high; brick lining in side walls and arch 22½ inches thick, with an invert 13½ inches thick. The invert was forced up as much as 5 to 8 inches from the external pressure. The versin was then increased to 2½ feet, and thickness of invert to 22 inches. Shafts were used 9 feet in diameter, brick-lined; maximum depth 344 and least 66 feet; average progress sinking shafts, 2 feet in 24 hours. The material excavated was a blue marl.

Sydenham Tunnel, England: 2100 yards long. Shafts were used 9 feet in diameter, and from 50 to 186 in depth. The material was London clay, which swelled, squeezing in the shafts, and in places crushing the masonry linings of shafts and tunnels. Eight rings of brick, 36 inches in thickness, were used at first. Subsequently rebuilt with 10 rings, and finally with 12 rings, equivalent to 4 feet 6 inches in thickness; even this had to be rebuilt in places. In the shafts the 9-inch lining had to be replaced with a lining 18 inches in thickness. Whatever may have been the directions and magnitude of the pressures upon the lining of this tunnel, the weak portion of the lining was evidently in the invert, which failed by rising first at its centre and then at its edges. The result was a sinking at the crown and pressing inwards of the side walls, showing clearly a tendency of the clay to flow. To meet these conditions the form of the section was changed by lowering the invert and flattening the arch at the crown, until finally the section of the tunnel was almost a perfect circle. The thickness of the lining was finally 4½ feet for the side walls and arch, and 3 feet for the invert. Lime mortar was used, as the lining could thereby the

better adjust its form to that required for equilibrium, without crushing the bricks.

The Bletchingly Tunnel, England, was also driven through blue clay. 3972 feet long; clear width, 24 feet; clear height, 21 feet. The versin of the invert was made 3 feet. The thickness of the lining varied from 22½ inches to 3 feet. Time occupied in the construction, nearly two years. Net cost, $476,185; and per linear foot, $120 nearly.

The Saltwood Tunnel: Similar in every respect, except that versin of invert was made 3½ feet.

The last three examples indicate clearly the necessity of a sharply curved and thick invert in tunnelling through this kind of material.

The Hauenstein Tunnel, on the Central Swiss Railroad: 2729 yards long; built for two tracks; clear width, 26 feet; clear height above tracks, 20 feet. Driven through limestone, sandstone, and shale. No invert used in the harder materials. The masonry lining entirely of limestone. The heading was driven at the bottom, but with a gentle rise inwards for drainage. The progress in sinking the shafts was from 1¼ to 1¾ feet per day; that on tunnel proper from about 2¼ to 3½ feet per day. The progress was less than 2 feet per day when tunnelling down grade.

The Mont Cenis Tunnel is 7 miles and 1044 yards long. From the end on the French side the grade rises at 117.22 feet per mile to the centre, or a total rise of 444.9 feet. From the centre to the Italian end it falls at the rate of 2.64 feet per mile, or a total of 10.04 feet, for the purpose of drainage. The tunnel is lined with masonry throughout. The side walls are of stone masonry, the arch of brick; clear width at bottom, 25.84 feet; maximum, 26.24 feet; clear height, 20.65 feet. No invert was used, but drainage was effected by a covered channel under the centre of the bottom. The drilling was done by machinery. The power applied was compressed air, the escape of which through the exhaust-valves served to ventilate the heading. There were no shafts used. The maximum depth of the tunnel below the summit is about one mile. The entire tunnel was driven from the two ends. In 12 years, between 1857 and 1868, an average progress was made of 209 feet per month, about 104.5 feet at each end. The entire cost, $9,439,400, and per linear foot $235.44½. The lining was about an average of 2 feet in thickness.

The Hoosac Tunnel: 4¾ miles long; clear width, 24 feet at

bottom, 26 feet maximum, for two tracks; clear height above tracks, 18.71 feet. Driven through mica-slate with quartz; very hard in some places. Compressed-air drills used. Some portions required lining, others did not. General thickness of lining from 2 to 2.66 feet, and was constructed of brick masonry, but only used when the rock was soft and showed a tendency to disintegration.

These examples have been selected as showing the methods of tunnelling in a variety of materials, the difficulties encountered, and the manner of overcoming them, and with the previously described modern tunnels, give a full view of methods, difficulties, and costs.

741. Although it was seen that in the Baltimore Belt Railway Tunnel the segmental timber lining proved unsatisfactory, the following example shows the successful use of that kind of temporary support. For a clearer understanding of the extremely unfavorable conditions under which the work now to be described was prosecuted, the following facts and circumstances will prove interesting as well as instructive. Though the line is now operated, a portion of it is of a temporary character, while the actual work on some difficult portions of the line is now in process of construction.

The Everett and Monte Cristo Railway extends from Everett City on Puget Sound, Washington, to Monte Cristo, a distance of about 60 miles, with a difference of elevation of about 2800 feet. The maximum grade allowed is 3 feet per 100, compensated at the rate of 0.00333 per degree of curvature. The maximum curvature is 13°, or radius of 441 feet, nearly. Transition curves 140 and 200 feet long are used. The annual rainfall is from 60 to 120 inches. This added to the extreme roughness of the ground, having steep slopes, precipices, dense forests, and undergrowth, rendered the location exceedingly slow and difficult.

It required six weeks for two locating parties to locate about six miles in one portion of the line, or a rate of half a mile per week, often averaging only 100 feet per day. In some places the men had to be suspended by means of ropes; in other portions the line was entirely inaccessible and had to be located by triangulation.

There were seven tunnels required in a distance of about nine or ten miles.

The sections in earth were 16 cubic yards per linear foot, and in rock 13¼ cubic yards. In driving one of the tunnels a small air-compressor operating two Ingersoll-Sargent drills was used; but in the main the drilling was by hand. The rock in the open cuts

shattered badly when blasted, and was divided by seams of clay; and when the lower portion of the slopes was removed, immense landslides occurred.

Tunnel.	Length in feet.	Time of Completion in Days.	Average Progress per Day in feet.	No. M feet B. M. in Lining.
1	817	186	4.39	276.8
2	138	117	1.18	87.4
3	260	132	1.97	109.0
4	149	118	1.26	77.8
5	81	73	1.11	42.0
6	278	100	2.76	60.8

With the exception of tunnels Nos. 2 and 7 no special difficulties were encountered. It was intended to timber-line all of the tunnels. In case of No. 2 it was supposed that the tunnel would be entirely in solid rock and overlaid by sand, but the heading ran into the overlying sand, caused by a dip in the rock. The sand poured into the tunnel in large quantities; and as this could not be remedied, it was determined to complete the short remaining distance in open cut by carrying a large portion of the material through the completed tunnel.

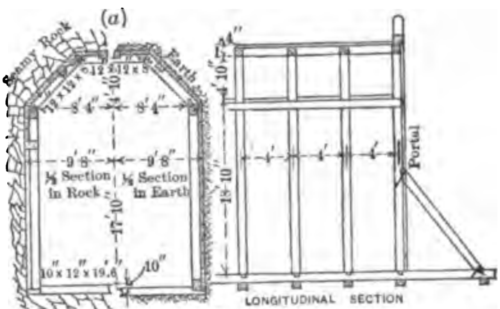


FIG. 292.

Figs. 292 and 293 show the two forms of timber lining in cross and longitudinal section. Fig. 292(a) shows on the left the lining in the seamy rock and other hard material, and the right-hand half the lining in softer material. In rock no longitudinal sill is used at the bottom of the vertical posts, these simply resting on cross sills; and only the top and a small portion of the sides near the top were sheeted with plank, these resting on small blocks let into the posts. The clear height above the top of the rail is 23 feet, the clear width between the posts of the sides 16 feet 8 inches, and from outside of sheeting or width of excavation 19 feet 4 inches.

All timbers in the arch-frames, posts, segments for roof, and longitudinal pieces were 12 × 12 inches in cross-section, except the bottom cross sills, which were 10 × 12 inches; the sheeting plank

or poling boards 4 inches thick; the length of the side posts was 17.0 feet, of the arch-segments 8 feet; from the top of the posts to the under side of the top horizontal segment of the frame, 4 feet 10 inches.

The main frames or ribs were placed 4 feet apart on centres; $12'' \times 2\frac{1}{2}'' \times \frac{1}{4}''$ plates of iron were placed over the joints in the arch-segments, and bolts $\frac{7}{8}'' \times 8''$ were driven to hold the foot of the lower segments to the top longitudinal pieces, and in addition longitudinal pieces were notched over the arch-segments. Dimensions and details of construction are shown on the drawings.

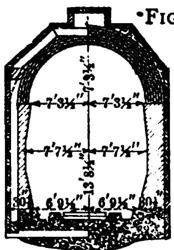


FIG. 293(a).

Showing inflow of Material into Tunnel No. 7.

FIG. 293(b).

742. Great difficulty was experienced in driving tunnel No. 7 through a mixture of clay and sand. After advancing some distance a considerable quantity of water began to flow into the tunnel. A great pressure came upon the timber lining, which showed signs of weakness. The timber used in the lining was a soft pulpy hemlock, and under the pressure (it is stated) the wall-plates, originally 12×12 inches in cross-section, were compressed to only 4 inches in thickness. To relieve this condition of things longitudinal pieces were placed under the top segments of the lining frames and supported direct with posts. But as the material above had evidently loosened and some movement in mass had already taken place, the enormous pressure exerted by it proved too great and the framing failed. The material poured into the tunnel, filling it to the spring-line of lining for a distance of over 200 feet, and flowing out 190 feet beyond the portal, large bowlders being carried in the flowing mass, a large crater 75 feet in diameter and 40 feet deep was formed, having a volume of 3500 cubic yards. This crater was subsequently partly filled with water. These conditions are shown in Fig. 293(*b*). This led to the construction of a temporary road, using 18° to 24° curves. The road has been operated for some time. Work is now in progress repairing the damage and completing the tunnel. The same form of lining is used for temporary purposes, but a masonry and concrete lining is built inside of the timber lining as the work advances. Fig. 293(*a*) shows on one side the timber lining and the portion of the

masonry inside of it, and on the other side the full thickness of the masonry lining in which the timber frames are imbedded. The drawing before the writer shows that the bottom is of concrete, the arch-ring of six courses of brick, and the side walls apparently of stone masonry. (See *Engineering News*, Oct. 5, 1893.)

The depth of the concrete at the bottom is about 3 feet; the side walls at the bottom 2 feet 6½ inches and at springing about 26 inches, and the same for the arch-ring. The arch is a full semicircle, with a radius of 7 feet 3½ inches. The inner face of the side walls are slightly curved, giving a maximum clear width of 15 feet 3 inches; the clear width at the bottom is 13 feet 7 inches, and at the springing 14 feet 7 inches. The drawing, Fig. 293(*a*) shows dimensions and general design.

743. A similar form of temporary lining was used in a tunnel 6112 feet long, driven through disintegrated material and bowlders. The arch segments of this lining were made up of five pieces 12 × 12 inches in cross-section, instead of three pieces, as in the above example. The radius of the soffit of the arch was 7 feet 10 inches. These shorter segments are better arranged to receive the thrust along their longitudinal axes. There were, however, more joints or planes of weakness. They were placed only 2 feet on centres. In other respects the general construction in the two cases was the same. The side walls were coursed rubble masonry from 20 to 30 inches in thickness; the arch-ring was of brick, in 4 or 6 layers, depending upon the character of the material, or from 18 to 26 inches in thickness. The headings were driven near the crown on either side, so that the inclined segments of the timber lining, adjoining the top or horizontal segment, could be placed in them. These were connected by a cross excavation, in which the top segment was placed, completing the support of the crown.

The excavation was then carried on sideways and downwards. The centres for the archwork were composed of ribs placed about 4 feet centres, which were moved forward as the brickwork was completed.

A good form of this general type of timber linings in tunnels is to make the segments of the top portion of the frames of three pieces 4 × 12 or 14 inches, bolted together, instead of a single piece 12 × 12 inches. The upper surface of these segments can be rounded, over which the sheeting-plank are placed. A strong and easily handled roof can be made in this manner.

Tunnel under Chicago River: A tunnel has recently been com-

pleted under the Chicago River for the West Chicago Cable Railway. The tunnelling was done throughout the entire length of 1576 feet by the open-cut method.

The special features are: (1) the unusually large dimensions of the tunnel, and of the open excavation necessitated in the approaches; (2) constructing the tunnel inside of coffer-dams, of very great lengths and widths across the river; (3) tunnelling under some very high buildings, calling for great care, and strong, stiff, temporary and permanent supports.

To prevent damage to the buildings, it was originally intended to sink iron cylinders and fill them with concrete, and to support the side walls of the buildings on arches turned between the cylinders. The interior columns and walls were to be carried on temporary trusses during the construction, and supported by piers rest-

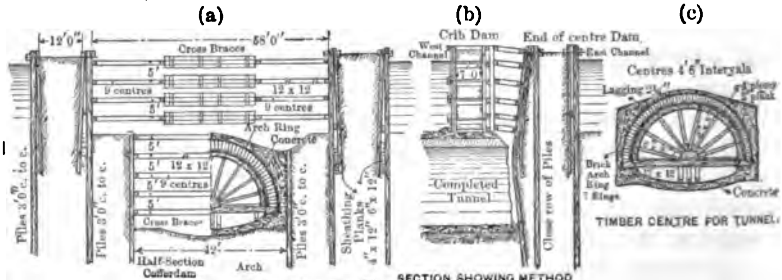


FIG. 293½.

ing on the tunnel masonry when completed. This plan was abandoned, and it was deemed safer to tear down and rebuild some of the walls, and to support others temporarily on jacks. The side walls were supported on piles, and the interior columns on piers resting on the completed arch masonry, the iron columns resting directly on masonry pedestals, which in turn rested on thick beds of concrete.

One of the buildings was supported on a heavy fire-proof floor of steel girders and brick arches, which was built of sufficient strength to carry fallen ironwork and other débris in case of the destruction of the building by fire. The approaches to the river were constructed on grades of 5.46 and 10 per cent respectively, and reduced to 1.8 per cent under the river proper. It was also necessary to support the railway tracks in such manner as not to interfere with the safety of a large traffic. This was effected by

driving four rows of piles between the tracks, upon which was constructed a strong floor or platform on which the tracks rested. The excavation was then carried on underneath the tracks, inserting strong braces as the excavation proceeded. The pressure was so great that 12 × 12 inch timbers crushed into oak wale-pieces to the depth of from ¼ to ⅝ inch. The soft swelling clay also bulged up at the bottom in places. This was stopped and equilibrium restored by throwing a large quantity of broken stone into the cut, which of course increased the cost and labor of carrying on the work. Otherwise no special difficulties were encountered in the excavation, tunnelling, and subsequent refilling over the completed arch.

The special feature of this work was the construction of the tunnel across the river. To accomplish this a coffer-dam was constructed out from the west shore to the middle of the channel, and the tunnel constructed in a similar manner to that of any structure in a coffer-dam. A cross-section of this cofferdam is shown in Fig. 293½, (*a*). As seen, it is a simple double-wall coffer-dam with clear width between walls of 58 feet, and braced across, as shown. Inside of this a single-wall coffer-dam of piles was made, with clear width only sufficient to allow of building the concrete and masonry. When the tunnel end reached the channel end of the coffer-dam a crib-wall was built over the end of the completed tunnel, as shown in Fig. 293½, (*b*).

This crib-wall was intended to form the end wall of another section of coffer-dam to be built out from the other or east shore. The side walls of the old dam were then pulled down, and a new dam built out connecting with the crib-dam and east shore; in this new dam the tunnel was carried on to completion from shore to shore. This construction, building two independent dams abutting at the centre, is somewhat similar to the method adopted in building the piers of the Tower Bridge, London, yet to be described.

The clear width of the tunnel is 30 feet; the clear height at crown about 16 feet; the masonry arch rests upon and is imbedded in concrete; the arch springs from a stone footing-course; the arch-ring is built with seven rings of brick. For a distance of 250 feet each way from the centre of the river, the two outer rings are laid in asphalt mortar, with very thin joints, and this mortar is used for the outer ring throughout the length of the arch; the other rings are laid in natural cement mortar. The concrete is composed of 1 Portland cement, 3 sand, and 6 parts of broken stone. The work

required 8000 barrels Utica natural cement and 21,000 barrels of Portland cement; the cost, including land damages, was $1,800,000.

In building the masonry tunnel under the platform carrying the railways, the masonry was built around the supporting piles. After a sufficient length of arch was built, the tracks were supported by short posts; the piles were then removed and the holes in the masonry filled.

Even where the river is deep and wide the writer believes tunnels could often be built within a series of pneumatic caissons sunk end to end, and by using short sections overlapping the ends of two adjacent caissons, or by attaching rubbing tubes to the ends of the caissons, and, when inflated by compressed air, or by water-pressure under sufficient head, the ends of the original caisson could be removed and the arch completed. It would no doubt in many cases save time and expense, and avoid the necessity of going to such a great depth below the bed of the river in order to maintain a considerable depth of material over the top of the arch, which is at all times a menace to the work and to the safety of the workmen. The idea is certainly worth consideration.

ART. XLVII.

SEWERS.

744. SEWERS are small tunnels, usually constructed in open cut, and after construction of the sewer, which is usually built of two or more courses of brick, the excavation is filled over and the street resurfaced and paved. The cross-sections of sewers vary from the regular tunnel section, to the oval or egg-shaped with the small end down, or they may be circular. The greatest diameters vary from 3 or 4 feet to 6, 10, or even 20 feet.

745. They may be constructed of a series of brick rings with simple earth tamped around them, or they may be packed around with rubble masonry or concrete, or they may be built entirely of concrete. In some of the larger sewers the thickness of the brick-work at the crown is as much as from 12 inches for a sewer 6 feet in diameter to 18 inches in 12- to 20-foot sewers; the thickness increasing from 17 inches to 4 feet or more at a point equal to or a little greater than half the height or vertical diameter of the sewer below the crown.

There are instances recorded in which sewers were built of but a single course of brick, even when the sewers were a considerable

depth below the ground. The engineers who designed and constructed them did not recommend the use of such thin walls. This is only mentioned as showing the great strength that can be developed; but such constructions would scarcely be considered good practice.

As ordinarily constructed, the only difficulty arises from the necessary trench to be excavated, often to great depths, requiring considerable timbering to support the earth; the interruption to the street traffic; the great expense incurred; the destruction of the paving; the settling of the new earth used to fill the trench, and consequent trouble in maintaining the surface of the street in good condition for a greater or less period of time.

746. In view of these conditions and facts, the following description of a sewer constructed by the tunnel method proper is worthy of consideration and comparison, in regard to convenience, economy, and rapidity of construction, with the prevailing methods.

In Cologne, Germany, a sewer 940 feet in length, on a slope of 0.4 per cent, has been recently constructed under streets varying from 18 to 28 feet in width, by a somewhat novel method. The cross-section of the sewer is egg-shaped, 72 inches high and 48 inches wide near the top. The walls are 15 inches thick, built of brick in layers. The drawing, Fig. 294, shows the dimensions, forms, and details of construction.

The invert block is from 25 to 30 feet below the surface of the street. The depths of the cellars of the adjoining houses varied from 10 to 15 feet. The underlying material was formed of about 6 feet of filling, 1½ to 8 feet of clay, and below these layers of sand and gravel. It was deemed unsafe and inadvisable to use the open-trench method of construction. Three permanent manholes were to be provided. At these points the usual timber-lined shafts were sunk to the sewer level, and for convenience and rapidity of construction four shafts in addition were sunk. From these headings or small tunnels were driven. The lining frames were made of old iron rails weighing 26.4 pounds per yard of length. Pieces of rail of sufficient length were bent to a curve, the length of the bent portions being

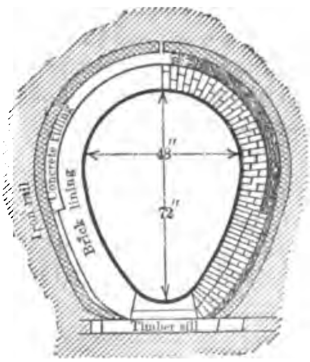


Fig. 294.

about 3 feet 6 inches at one end; the other end was bent for a length of a few inches, forming a base, the bent pieces conforming roughly to the shape of the sewer-section. These pieces of iron were bolted to a timber sill, two of them forming a frame, their upper ends meeting over the vertical axis of the sewer, as shown in the drawing.

There were two sizes of these bent iron frames, the one was two or three inches higher and the same wider than the other. These were set up alternately, and spaced about 3 feet on centres.

The alternate frames being a little larger facilitated the placing of the sheeting-boards around the rails.

The poling-boards were $2\frac{1}{2} \times 6$ or 8 inches, and 6 feet long. The tunnelling operations were carried on in the usual manner, the frames and sills were left in place, and the small spaces between them and the masonry lining were filled with concrete. The rate of tunnelling was about 8 feet per day. The shafts were sunk with four men at the rate of about $3\frac{1}{2}$ feet per day.

The clear dimensions of the shaft were $6\frac{1}{2} \times 6\frac{1}{2}$ feet, the total 10×10 feet outside of frames. The invert-stone block rested on a 6-inch bed of concrete. The iron frames cost $3.75 each; the total cost of the work about $14.50 per linear foot.

For the direct connection of sewers from the adjacent houses was substituted a 12-inch sewer-pipe laid about $11\frac{1}{2}$ feet on an average below the ground, having a grade of $1\frac{1}{2}$ per cent, to which the house sewers were connected. From this 12-inch sewer special drop-pipes 9 inches in diameter conducted the sewage and water to the main sewer near the manholes, these pipes entering the main sewer below the water surface. This was done to prevent shocks from overfall at the manholes. The four temporary shafts were filled subsequently with a cheap concrete.

747. The following are costs of large sewers in many cities, taken from a report of Mr. Rudolph Hering to the Taxpayers' Association of Baltimore, which shows that in comparison the usual methods are little if any more economical than the tunnel method described. The report also discusses the relative advantages of the day-labor and contract system of building works of public improvements. The conclusion on this point reached by him will be endorsed by most engineers of experience, that there may and do arise conditions which will make the day-labor system economical; that often a better grade of workmanship and better quality of materials may be obtained; but that, as a rule, under

properly drawn specifications and an intelligent and faithful inspection of the work, it may be stated that all public works can be more economically and satisfactorily executed under the contract system, provided competent and capable contractors are employed.

It seems that in the case of one of the sewers, executed partly in cut and partly in tunnel, the sewer in the former being 3 feet in diameter and in the latter 7 and 8 feet, with an average depth of cutting, cost $50 per foot, whereas under the contract system in other cities the following prices are paid :

For an 8-foot sewer in St. Louis, where no special difficulties existed, the construction of the sewer itself was $10 per running foot and executed both in open cut and tunnel in part. An 8-foot sewer in Philadelphia, excavation 15 feet deep, requiring some cradling (that is, forming a wide and firm timber, masonry, or concrete bed upon which the sewer rests and in which it is partly bedded), cost $16.30 per foot.

A 7-foot sewer in earth excavation, 4 feet deep, built in cradle, $34.44; 6½-foot sewers with cradle backing, from $15 to $18.85. The same in other cities for $10.90 to $20.

A 5-foot sewer excavation in clean, sharp sand, 33 feet deep, cost $17.91.

In earth excavation from 11 to 16 feet deep, 3-foot sewers in Philadelphia cost from $2.50 to 5.84; and at depths of 8 to 8.5 feet, 3-foot sewers in New York cost $9.90 to $13; without cradling, $3 to $5.

In Washington, sewers 8 feet in diameter and 5 feet below surface cost about $18.19; 6-foot sewers, $17.50 in clay.

A 9-foot sewer in excavation 24 feet deep, in quicksand requiring cradling with timber foundations, cost over $57 per foot. Another 8-foot sewer, excavation 18 feet in sand and gravel saturated with water, cost $39.

A 3-foot sewer with labor at $3 and material at $2 a foot has aggregated a cost of $25.00.

In Boston 3-foot sewers, built in cradle, cost from $7 to $11 per foot. A sewer 3 × 4 feet in tunnel cost the contractor, without profit, $29, in tunnel.

A sewer 6½ feet in diameter, built on platform and in stone cradle; excavation 24 feet; actual cost, without profit, $28.25 per foot. Sewers 4½ feet diameter, excavation from 10 to 20 feet, partly in cradling, cost $9.78 and $13.25, without profit.

The above prices in the main are simply for the construction of the sewer itself, without appurtenances.

The prices in Baltimore for the cheapest sewers were $7.50 per foot, and for the larger sewers $49 to $50. The conclusion of the committee from Mr. Hering's report was that at least 40 per cent of the money spent in constructing the sewers by day-labor could have been saved under the contract system, and the work executed in an efficient and reliable manner.

748. The cost of vitrified-pipe sewers varies mainly with the cost of the pipe itself, with some little additional cost in handling and laying, as the cost of excavation remains practically the same for the small as for the large sizes, within limits. The list price of sewer-pipe ranges from $0.27 to $0.30 *per foot* for the 6-inch pipe; $0.40 to $0.50 for the 8-inch; $0.75 to $0.90 for the 12-inch; $1.60 to $1.80 for the 18-inch; $3.10 to $3.30 for the 24-inch. There, however, was a discount of from 60 to 75 per cent in car-load lots.

The cost of Portland-cement pipe will not materially vary from the above.

Cast-iron pipes: $0.96 for the 12-inch pipe *per foot;* $1.40 for the 16-inch pipe; $1.88 for the 20-inch pipe; $2.80 for the 24-inch pipe; $5.12 for the 36-inch; and $6.16 for the 48-inch pipe per linear foot. These prices vary between $1\frac{1}{4}$ to $1\frac{3}{4}$ cents per pound, the thicknesses of the pipes varying from $\frac{1}{2}$ to 1 inch. The larger sizes are in 6-foot lengths, and the smaller sizes in 12-foot lengths. The vitrified pipes are in lengths of from 2 to 3 feet.

749. There has been a considerable amount of discussion of the relative efficiency and cost of pipes of different sizes, especially in reference to the minimum sizes. Various estimates of cost have been given. For instance, cost for 6-inch pipes: Excavation and refilling trench per linear foot, 60 cents; cost of pipe and laying, 15 cents; total cost per linear foot, with an allowance of 10 cents for manholes, 85 cents per lineal foot.

For the 8-inch pipes: Excavation and refilling, 60 cents; manholes, 10 cents; pipe and laying, 20 cents; or a total of 90 cents, or an increase of about 6 per cent. And similarly for other dimensions of pipe.

750. The proper slopes for sewers should be such that the velocity of flow shall be at least 2 feet per second when the sewer is running one-half full. The hydraulic slope necessary for different pipes has been given as follows. The hydraulic slope should

be distinguished from the slope of the sewer bottom. Even on light slopes the latter may be greater than the former.

TABLE LXIII.

Hydraulic	slope	for	6-inch	pipe,	0.7	per	cent.
"	"	"	8-inch	"	0.4	"	"
"	"	"	10-inch	"	0.3	"	"
"	"	"	12-inch	"	0.2	"	"
"	"	"	15-inch	"	0.15	"	"
"	"	"	18-inch	"	0.12	"	"

A velocity of from $2\frac{1}{2}$ to $3\frac{1}{2}$ feet per second will provide better sanitary conditions.

751. While it is beyond the proper scope of this volume to undertake any discussion of the important subject of sanitary engineering, a few general principles are of importance even to the ordinary engineer.

(1) A sewer, large or small, should be perfectly tight from one end to the other, in order to prevent absolutely any leakage into the surrounding soil.

(2) Its fall should be continuous from entrance or head to outlet, in order that the content may have no greater tendency to stop its movement at any one portion of its length than at any other portion.

(3) It should be well ventilated in order that the noxious gases may be diluted by a large quantity of fresh air; and proper means for escape of foul air should be provided, in order that an increase of pressure may not force these gases back into the houses through the traps.

(4) It must be provided with easy and ample means of inspection, and proper provisions for flushing and cleaning when required.

(5) Its size and form must be so proportioned that the ordinary dry-weather flow will keep it clear of silt or organic deposits.

(6) The proper dimensions for sewers depend upon the amount of sewage proper, the usual rainfall of the district in any given period, the slope of the surface which affects the rapidity and amount of surface water finding its way into the sewer, and the proper proportion of this water to be carried in the sewer relatively to that which will or ought to be directly conducted by and discharged from the surface gutters.

(7) A provision of 1 inch of rainfall per hour, only one half of which is supposed to reach the sewer, is not an unusual one made.

752. Mr. Waring states that if the supply of water is 10 gallons per head per day for the whole population, the quantity of sewage to be removed will be about 100 pounds daily for each inhabitant. Of this the closet-flow will be about one third. And that in Brooklyn (1879), aside from rains, the sewage equals one and one-fourth times the water-supply, or 50,000,000 gallons per day, or 3,125,000 gallons per hour, escaping during eight hours of maximum discharge. To what extent sewers should be proportioned to the quantity of rainfall is an unsettled question. It is a safe rule to estimate all sewage, except rainfall, at 8 cubic feet per head of population per day, one half of which will be discharged between 9 A.M. and 5 P.M., equal to a flow of 500 cubic feet per hour for each thousand of the population. If a pipe or sewer is too large it will silt up to a great extent during ordinary dry-weather flow, and during floods the upper portion of the length of the sewer may be full, whereas that towards its outlet may be only partially so, owing to the increase of velocity acquired; consequently it is capable of carrying larger quantities of water admitted to it at intervals of its length than would seem possible if this fact is neglected. A well-constructed 18-inch sewer is ample for the drainage of an ordinary village area containing 700 or 800 houses, even with a fall of 1 in 1000. In another instance an 18-inch sewer, having a fall of 1 in 1062, drained an area containing 1600 houses. Its ordinary current was 2½ miles per hour. This would carry brickbats thrown into it to its outlet. During ordinary rains it was not more than half full at a point a mile from its outlet, and at the outlet the stream was not more than 2½ inches deep.

A sewer 15 inches in diameter, having a fall of 1 in 120, or 1 inch in 10 feet, would drain nearly 200 of the largest city houses, and a 9-inch drain, with the same inclination, would carry the house-drainage and storm-water from 20 such houses.

The rate of fall necessary for the removal of ordinary road silt from sewers is as follows:

TABLE LXIV.

For 6-inch pipes, a grade of	1 in 60	For 24-inch pipes, a grade of	1 in 400
" 9- " " " "	" 1 " 90	" 30- " " " "	" 1 " 500
" 12- " " " "	" 1 " 200	" 36- " " " "	" 1 " 600
" 15- " " " "	" 1 " 250	" 42- " " " "	" 1 " 700
" 18- " " " "	" 1 " 300	" 48- " " " "	" 1 " 800

To attain a velocity of 3 feet per second, a 4-inch drain should have a minimum inclination of 1 in 92; a 6-inch drain, 1 in 137; a 9-inch drain, 1 in 206; and to attain this they must run at least half full. The discharges, respectively, would be 7.85, 17.66, and 39.76 cubic feet per minute.

Accuracy in form and joints and smoothness of interior surface are important. The material should be a hard, vitreous substance, and the pipes should be salt-glazed. Such pipes are preferred to brick in sewers on account of their being non-absorptive.

The forms of cross-sections for sewers are the circular, elliptic, or egg-shaped, and the ordinary tunnel shape.

The circular gives the greatest sectional area for a given amount of wall surface, and consequently the greatest capacity for discharge. Where provision is made for the removal of heavy rains, the elliptic form is preferred. This gives capacity for discharge and at the same time, owing to its diminishing cross-section towards the bottom, prevents the stream from being too shallow and sluggish in its ordinary flow. The tunnel section is only to be adopted when the minimum flow is large and where great capacity is required to carry the storm-waters or a largely increased flow from any cause.

753. Proper traps should be used in all connections between the receptacles for surface-water at corners of streets and the sewers, to prevent the escape of foul air along and close to the sidewalks; and also a place should be provided below the mouth of these connecting pipes for the deposit of sand or other heavy material carried in by the flood-waters.

Man-holes should be provided for ventilation and for purposes of examination; and lamp-holes should be provided over pipe drains at every change of direction either vertically or horizontally, so that by suspending a lamp in the sewer at these points the portion of the pipe between them and the nearest manhole can be examined.

The manhole is a shaft or chimney built up from the sewer to the surface of the street, of sufficient size for the entrance of a man. The lamp-holes may be small shafts in which a length of vitrified pipe or a cylinder of other material, large enough to allow a lamp to be lowered through them. Both man-holes and lamp-holes are covered generally with cast-iron caps.

Great care is necessary, in building sewers or laying pipes, that the foundation shall be firm and unyielding, otherwise they will

open along the joints; and earthenware pipes are likely to be cracked or broken under a heavy pressure if any portion of their length settles out of line. All sewerworks should be first-class in every respect; and they should be tested in order to discover leaks, if any exist, before refilling the trench.

SEWERS OF THE CITY OF DULUTH, MINN.

754. The city of Duluth extends along the shore of Lake Superior for about 6 miles and inland for about 1 mile. The average slope of the ground towards the lake is about 9½ per cent, but reaching as high as from 16 to 32 per cent in portions of the city. The original system was the separate one, the surface water being carried off independently.

Owing to the growth of the city it became desirable or even necessary to provide for the underground removal of storm-water, the roadways being greatly washed by the surface water.

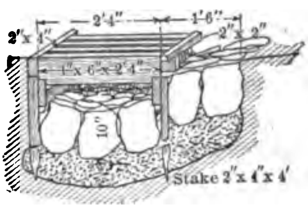
Messrs. Rudolph Hering and Andrew Roeswater were requested to examine and recommend the proper construction of pavements, sewers, and drainage-works. They reported the necessity of keeping stones and gravel from entering the sewers, as these were deposited at the foot of the hills, where the velocity of flow is so greatly reduced as not to be able to carry these heavy deposits into the lake. The usual custom is to build catch-basins at the inlets to the sewers, for the purpose both of preventing silt from depositing and interfering with the discharge of sewage on flat grades, and also of preventing excessive wear on steep grades, and the importance of catch-basins at street corners and at alleys where intercepting sewers are required is strongly urged.

"The great velocity of the water flowing down the gutters makes it necessary to build the basins in such a manner that this velocity will be checked when flowing through the basins, and not be increased again until the water has escaped therefrom. The capacity of the basin is governed partly by the maximum amount of water which at a low velocity of, say, 2 feet per second will flow through it, and partly by the greatest amount of solid matter which is expected to be deposited during a single storm. To prevent the possible failure of a basin by becoming choked up with débris, and therefore causing the water to flow past it down the avenue and overcharging the next basin, we think it desirable, as a matter of safety, to proportion the basins of double capacity.

"To check the velocity, we advise the placing of a fender across

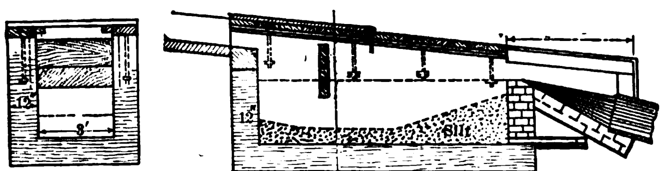
the inlet in the path of the stream, as shown in Fig. 295 (*a*), and in order to prevent the water from permanently remaining in the basin, a small drain should be laid from the bottom to the outlet pipe likewise, as shown in the drawing.

"As unimproved streets and avenues paved with gravel are likely to cause a larger amount of gravel to enter the catch-basins than streets having improved pavements, we advise increased size in such cases. At the upper end of the graded avenues, to which drains have been built, it may be found expedient to build a specially large catch-basin of timber for a temporary purpose, which later can be replaced by a smaller one of permanent construction.



(*a*)
FIG. 295.

"It is evident that to secure the effective working of catch-basins they must be cleaned out after every storm. To prevent the entrance of a considerable amount of the larger refuse, we recommend that the gutters of the gravelled streets be covered with gratings. The probably best dimensions for the basins are shown in Fig. 295 (*b*), but experience alone can definitely determine these



(*b*) (*c*)
FIGS. 295.

dimensions. Fig. 295 (*b*) shows a vertical longitudinal section, (*c*) a cross-section of catch-basin and outlet."

For that portion of the report on pavements, see Pavements for Streets.

THE SALT LAKE CITY INTERCEPTING AND OUTFALL SEWER.

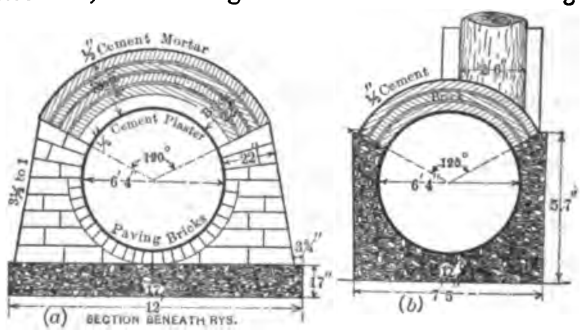
755. The following is a description of the sewer system and construction of sewers designed by Mr. A. F. Doremus, City Engineer of Salt Lake City. This city has now a population of about 60,000 people. It is designed for the drainage of about 3000 acres, including the residence and business portion of the city. The total length of the intercepting and outfall sewer is about 8 miles.

The diameters, lengths, and quantities of materials required in the construction of the several sizes per lineal foot is given in the following table:

TABLE LXV.

Sewer diameters, in inches.......	38	42	48	54	64
Lengths of sections, in feet......	5,200	5,300	5,600	9,800	17,500
Concrete per lin. ft., in cu. yds..	0.295	0.326	0.375	0.623	0.739
Brickwork per lin. ft., in cu. yds.	0.109	0.118	0.132	0.222	0.261
Cement plaster inside, sq. ft. per lin. foot....	1.105	1.222	1.396	1.521	1.862

The grade on the portions of the line where the diameters of the sewers were 38 and 42 inches is 0.05, and on other portions 0.04, in 100. Following the street-lines, the sewer was laid 22 feet from the centre line, and curving at street intersections through arcs of



FIGS. 296.

90°, with a radius of 127.32 feet. Fig. 296 (*b*) is usual section, and shows combined brick and concrete construction for the largest diameters, 54 to 64 inches, in which are a concrete base and side walls, surmounted by a brick arch composed of three rings. The design and construction of 38, 42, and 48 inch sections were the same, except that the arch-ring consists of only two layers of brick, and the thickness of concrete base and side walls was reduced from 12 inches to 8 inches in thickness.

A special design, Fig. 296 (*a*), was used under the several railway crossings. This was required, as the extrados or top of the brick arch was only about 2 feet 2 inches below the rails. These portions of sewers were built on a bed of concrete 17 inches thick and 12 feet wide, upon which masonry walls were built. The lower half was lined with paving-brick, and the top with a brick arch composed of five rings of brick. In all cases the sewers were lined with a cement coating ½ inch thick: and over the top of the brick

arch a similar layer of cement mortar was placed. Manholes were placed at intervals of 800 feet. Over the sewers of smaller diameter the manholes are placed centrally, the bottom diameter being equal to that of the sewer itself, and the top diameter 2 feet 6 inches. For sewers of the larger diameters the manholes are to one side, as shown in Fig. 296 (*b*), and have a uniform diameter of 2 feet 6 inches.

Vitrified branches are placed at all street intersections, and elsewhere on the line at intervals of 800 feet. Where the sewer attains its largest size of 64 inches it has an estimated capacity of 35,000,000 gallons daily.

The specifications call for the best hard-burned bricks, laid in cement mortar, 1 cement, 2 sand. The concrete, 1 Portland cement, 4 sand, and 5 parts screened gravel; the ½-inch coating, 1 cement 1 sand.

Much trouble was encountered while constructing the sewers from the inflow of water, requiring the construction of drain-boxes. A layer of gravel was placed as a foundation for the concrete base over a considerable portion of the length, and in some very wet places plank foundations were used. Bids were called for on the basis of combined concrete and brick sewers, and upon all brick-work, the brick section being of the same thickness all around. The following gives quantities and cost:

TABLE LXVI.

Kind of Work.	Quantities.	Price per Unit.	Total Cost.
Earth excavation....	73,700 cu. yds.	$0.40	$29,480
Earth excavation and refilling....	116,000 " "	0.49	56,840
Solid-rock excavation............	10 " "	6.00	60
Solid-rock excavation and refilling.	50 " "	7.00	350
Embankment	500 " "	0.25	125
Tunnelling in earth..............	25 lin. ft.	11.25	281
" " solid rock..........	350 " "	17.00	5,950
Concrete masonry...............	23,700 cu. yds.	6.69	158,553
Brick "	8,400 " "	9.99	83,916
Cut-stone "	50 " "	20.00	1,000
Rubble "	200 " "	6.50	1,300
Dry rubble "	200 " "	4.50	900
Cement plaster..................	65,370 sq. yds.	0.33	21,572
Riprap..........................	500 cu. yds.	2.00	1,000
Lumber.........................	200,000 ft. B. M.	30.00 per 1000	6,000
Aggregate cost...............	....	....	$367,327

For sewers constructed with brick throughout the aggregate cost was $15,000 more than that given above. The combined concrete and brick construction was decided upon. The cost of sewer Fig. 296(*b*) was $20 per linear foot. The cost of masonry alone in 64-inch sewer, Fig. 296(*a*), was $8 per linear foot. Owing to the large sizes of the blocks in the city and wide streets, which were 660 feet square and 132 feet wide, respectively, the outlay for laterals is lessened, but the house-connections will be more expensive.

ART. XLVIII.

CONSTRUCTION OF HIGHWAYS.

756. THE construction of highways will be discussed under the two heads of country or ordinary roads and of city streets.

Country Roads.—The importance of good location and easy grades as regards the proper and economical construction and maintenance of good roads, and the general methods of making excavations, embankments, and bridges, have been fully discussed in other portions of this volume. The questions now remaining to be discussed refer to the proper methods of draining, dimensions, and form of the upper surface of the formation, and the construction of the permanent way, that is, the kind, quality, and placing of the surfacing or covering material in order to fulfil the requisite conditions of a good road.

As in many if not all other kinds of construction, neither economy in the first cost nor the greater number of miles unwisely and badly constructed should be the controlling factors in the location and construction of highways. These considerations have resulted, it may be safely said, in the reckless waste and squandering of a sufficient amount of money in so-called road construction to have given, if it had been judiciously spent, every State in the Union a system of good main highways instead of the abominable roads now found in every State and county.

757. Unquestionably good highways are of vastly more advantage to any community than the best-devised system of railways, and it is a source of encouragement that a realizing sense of the value and importance of good common roads is being developed in all parts of the country. It is not too much to say that the development and construction of good roads is coeval with epochs of the greatest progress, advancement, and highest prosperity of the older

countries and governments, while the neglect or indifference in such constructions have marked periods of retrogression, decline, and want of prosperity.

758. It would be beyond the scope of this volume to name and discuss the value of good roads in increasing the actual or marketable value of adjacent property, and in the vastly greater price received for agricultural products by reason of the greatly diminished cost of transportation both in power required and in time consumed; to say nothing of the wear and tear on vehicles and teams arising from ruts, mud-holes, and generally rough roads. Such things represent the expenditure in actual waste of hundreds upon hundreds of thousands, even millions, of dollars. We have only to show in what way this waste can be avoided while adding to the comfort, convenience, and health of communities.

759. Though much can be accomplished even on the badly located and existing roads, and these must to a great extent be improved as they stand, yet in any community a change in the location of a few of the worst portions of the lines and an adjustment of the grades can be made at reasonably small cost, inconvenience, or damage to the property-owners. As far as practicable this should be done before expending any great sum on attempts to improve such portions of roads.

760. All new roads should be located with due regard to good alignment, easy grades, and good drainage, and not following boundary lines regardless of length of road, high and difficult ground, with steep grades and costly construction.

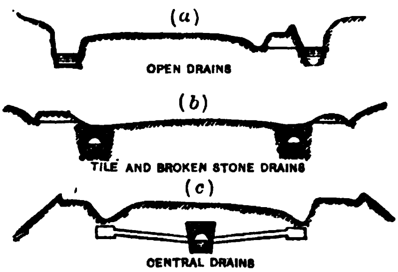
A road should have sufficient width to accommodate the traffic both present and prospective. It should provide space for a wheelway for vehicles, and also a footway on each side. This refers to a proper width of right of way. The whole need not be improved on the first construction and opening of a roadway.

Wide roads are preferable. They expose a larger surface to the drying action of the sun and wind, and require less supervision and repairs than narrow ones. Their first cost is greater in almost direct proportion to their width, and the cost of maintenance depends more upon the kind and character of the traffic than upon the width. On a narrow road the traffic is confined to one track, causing a greater degree of wear and tear.

The width of right of way varies from 25 to 66 feet. The width which is macadamized varies in different countries. In Belgium it is on an average 8¼ feet, in France from 16 to 22 feet,

and in the United States 16 feet. The least width admissible is about 8 feet for a single track, This, however, requires at frequent intervals turn-out tracks. That for a continuous double track is 16 feet. Where macadamizing is only used for a single track, the other track should be kept in condition for the traffic, and will often be preferred for probably one half the year.

Figs. 297 show the usual forms and widths of wheelway and



FIGS. 297.

foot-paths, with the methods of drainage, both with open ditches and tile covered with broken stone.

No general rule or practice can be given either in regard to width or drainage of mountain roads. Owing to the necessarily great number of sharp curves, they should be wide—not less than from 16 to 24 feet. If the traffic is large, the greater width should be used; and in general it will be necessary to macadamize for a double track.

On hillsides and mountain roads it is generally considered better to give the entire surface a single sideways slope, inclining inwards, all the water draining to this side. But since the traffic will commonly hug the inside, there is great danger, especially in hard rains, of washing and wearing into ruts. It is safer, especially when ice may form, and on steep grades.

It is, however, better in general to have the centre of all roads somewhat higher than the sides, so that the water may shed rapidly both ways from the centre line, and not follow along the line on grades. When the surface is kept in good order this centre rise or crowning may be very little. The crowning should bear some certain proportion to the width of carriageway. Earth roads require the greatest and asphalt roads the least rise.

There is no dispute in regard to making all road surfaces convex transversely, but there is a difference of opinion and practice

as to the proper elevation of the centre above the sides, and as to the form of the surface, whether it should be composed of two straight lines intersecting at the centre, which are rounded off at and near the centre, or whether the transverse section shall be circular, elliptic, or parabolic arcs.

For country roads a suitable convexity can be obtained as follows: Having the centre rise, give seven eighths of this at one fourth the width from the centre, and five eighths of the rise at the half-width point, between centre and side.

If the rise is excessive, vehicles will keep to the centre line. If they run to one side, the tractive force is increased by an undue proportion of the load being concentrated on one line of wheels. Straight-line sides wear hollow and retain water.

The proper convexity should be given to the formation—that is, the natural soil or foundation,—and not obtained by varying the thickness of the covering material.

761. *Earth Road.*—While a great deal of attention and study has been given to the cost of covered or paved roads, seemingly but little has been given to the proper construction and maintenance of earthen roads. In the United States the main dependence for a large proportion of country roads must be upon roads simply surfaced with the ordinary earthy material found along their lines. Hence the importance of understanding the best construction for the existing local conditions. Unfortunately there is little reliable data. It has generally been assumed that it is idle to do more than roughly mark out and grade an earthen road, as it will be bad under any circumstances in wet and freezing weather, and will be worked into a reasonable condition by the traffic in good weather. The conclusion is an entirely fallacious one. By earthen roads is meant those which are formed entirely of clay, sand, and gravel and sand, and the ordinary mixed earths.

762. *Clay Roads.*—All mud and perishable material should be removed, as also weather-worn and loose material, and the surface well compacted and rolled to the proper surface. When it is necessary to make embankments or fills of clay, all lumps should be broken up, and the material thoroughly dry or at least but slightly moist, and it should be rolled, or otherwise compacted in thin layers. The ribbed or corrugated rollers alone, or followed by a smooth roller, are preferable, as preventing the formation of well-defined layers with smooth, regular seams between them. Rollers with smooth surfaces are, however, commonly used. In whatever

manner and degree of care clay roads are constructed, thorough and efficient drainage is essential, in accordance with some of the types shown in Figs. 297. Owing to the great retentiveness of clay, the narrower the roadway the more effective will be any system of drainage. In the maintenance of clay roads neither holes nor ruts should be filled with sods or turf. They will decay, and form the softest mud. Nor should they be filled with field or broken stone, as the wear will not be uniform, resulting in the formation of hard ridges, or else allowing water to enter the clay, forming mud, and resulting in a worse condition of road than that which it was intended to repair.

Trees and close hedges should not be allowed near the sides of clay roads. It requires all the sun and wind possible to keep its surface dry and firm.

The crowning or centre rise for earth roads is taken at about $\frac{1}{40}$ the width of the street. That gives a side slope of about one in twenty ($\frac{1}{20}$) from centre to side. This is sometimes on grades made 1 in 12.

After removing loose and perishable material, and thoroughly compacting the soil, the surface of clay roads can be greatly improved by covering with a layer of from 3 to 5 inches of sand, gravel, shell, coal-slack, cinders, furnace-slag, etc., where these materials are found in quantities, and where their cost is trivial. This layer should be well compacted by rolling. Of late a ballast of burnt clay has been used and recommended for both railways and highways in the absence of other materials. This material is obtained from the road-bed itself. It is excavated to the depth of two or more feet, and left in irregular piles to be dried out by the sun and winds, this taking from one to two weeks. The burning is then effected by forming cone-shaped piles, intermixed with some easily combustible materials, straw, and pieces of dry wood. The fires are continued until the mass assumes a red color. The burnt clay, after cooling, is replaced on the road-bed. The mass will then settle into a firm, dry layer, and forms a good, solid road-bed.

Wet clay, vegetable mould, etc., are unfit for any kind of embankments.

763. *Sand Roads.*—There is no disadvantage arising from vehicles using the same track on sand roads. On the contrary, it is claimed as an advantage to have all the vehicles run in the same track, and narrow roads with frequent turnouts will answer every purpose unless the volume of the traffic is sufficient to justify

continuous double tracks. The reasons calling for wide and open clay roads do not exist in the case of sand roads.

Wet sand roads are preferable to dry ones. Ditching or artificial drains are not desirable beyond what is necessary to carry off surplus water in case of hard rains, and when used should not have any great depth, as this would cause too rapid drying and caving or scouring out of the material. Such roads should be protected by an abundant vegetable growth on each side of the wheelways.

Overhanging trees are favorable in shading the road, and the fall of leaves and twigs on the road is of advantage rather than otherwise.

A coating of 5 or 6 inches of clay will improve such roads. A coating of a few inches of loose straw will under a few days' traffic result in the formation of a hard, firm surface. Roads covered with a deep layer of dry sand are very objectionable in every respect.

764. *Roads of Ordinary or Mixed Earths.*—Mixed earths differ so greatly in the character and proportions of their ingredients, that it is difficult to classify them or state definitely what conditions will exist in different kinds of weather. A good earth for roads must either possess the property, after being rolled and compacted, of shedding the water rapidly from its surface so as to prevent percolation into or collection of water in its interior, or, if the water enters, it must be of such a character that it will not remain in the mass, but will readily drain off. These types are represented by a good clay on the one hand, and a clean sand or gravel and sand on the other; and only those earths which fulfil one or the other of these conditions are suitable.

Certain mixtures of sand or gravel and sufficient clay to act as a binder will make a good firm road and a water-tight surface; whereas such mixtures overlaid with a more porous surface would hold any water reaching it, in which case the clay would become mud and result in the destruction of the road-bed. A small proportion of sand in a clay will prevent cracking and splitting of the mass. In case of cracks the water finds ready entrance, and will injure if not destroy the road.

Any ordinary earths with proper construction and ample provision for drainage will make fairly good roads, even in unfavorable weather.

Sufficient supervision, quick repairs of seemingly small defects

and irregularities of surface, and occasional rolling will maintain a good surface at comparatively small cost.

A judicious use of road-grading machines will be of advantage on all classes of earth roads. The use of ploughs is to be condemned: they break up and disintegrate an already compacted surface. Scraping-machines are not intended to drag the soft and disintegrated material from the ditches and road surface and pile it up in the centre of the road, but to remove all such material and deposit it where it cannot get back on the surface or in the ditches.

Cost of Earth Roads.—There is little reliable data upon which to base a statement of the cost of dirt roads. This is due to a want of systematic method or continuity in their first construction, often amounting to no more than clearing the roadway of trees, stumps, large bowlders, and a skimming excavation or a fill here and there. Much of the subsequent work called maintenance is nothing more nor less than original construction to a very great extent, and of this under the prevailing systems no knowledge is obtained or records kept. The general result is a large expenditure of money and poor apologies for roads. The cost will also vary so greatly with local conditions, even in the same county and on the same road, executed in a disjointed, haphazard manner by different foremen or overseers, that it is impossible to obtain an average cost. It is, however, variously estimated anywhere at from $400 to $3000 per mile.

765. The conditions governing the construction of good roads and the proper construction of them were well exhibited at the World's Columbian Exposition, under the auspices of the National League for Good Roads, General Roy Stone, President.

The conditions entering into and determining the proper manner of road construction are: (1) the grade of the road; (2) the nature of the soil; (3) the volume and nature of the traffic.

The grade determines the crown and to some extent the depth of the road-bed.

The soil determines the depth and class of road-bed, and also questions of drainage.

The traffic determines the width of the roadway and of the road-bed, the depth and class of road-bed, and the quantity of materials for the service required.

There were nine sections of road, each of which represented a class of road adapted to certain conditions and requirements. These were laid out in three strips, each containing three sections.

The widths of roadway in each strip were the same throughout. Strip 1 was for a roadway 2 rods or 33 feet, Strip 2 was $2\frac{1}{2}$ rods or $41\frac{1}{4}$ feet, and Strip 3 for 3 rods or 50 feet in width, the width being the full width from fence to fence. In each strip there were three sections: (1) Roads of steep grades on firm, dry soils and for light traffic; (2) roads of flat or minimum grades on wet and yielding soils and for heavy traffic, with provisions for subdrainage; and (3) roads of moderate grades on firm, dry soil and for moderate traffic.

The road-beds $2\frac{1}{2}$ and 3 rods wide are intended to provide for gutters or drains and sidewalks on either side, having symmetrical cross-sections. The general construction in these two cases is the same, the crowning alone varying.

The 2-rod roadways had no sidewalks, and in their places were substituted deep ditches. The cross-section was not symmetrical. The stone road-bed occupied one side of the side line with a width of 8 feet, and alongside of this was an earth road 16 feet wide, economy of construction and maintenance being the reason for adopting these dimensions and construction. This provides a single-track broken-stone road, suitable for heavier traffic and for use in wet weather; and an earthen track, which may be useful at least for a large portion of the year, and for certain kinds of travel more desirable than the broken-stone road, especially if this latter is not kept in good repair.

There were three classes of construction depending on the grade, soil, and traffic: (1) The macadam road-bed 8 inches in thickness; (2) the macadam road-bed 6 inches in thickness; (3) the Telford road-bed 12 inches in thickness.

The general description of these different modes of construction is as follows: In each case a proper rolling of the earth-bed called the subgrade, and also of each successive course of stone up to the grade surface proper, is essential for the proper construction, economy of maintenance, and permanency of the road-bed. A steam-roller weighing 10 tons is recommended for economy, rapidity, and efficiency, but the 5-ton horse roller may be used.

Where the binding materials are screenings, sand, or gravel, they should be sprinkled before rolling, but with a clay binder it is worked dry.

The construction, then, is as follows, after the subgrade, as the earth-bed is called, is properly formed and rolled:

786. For the 33-feet roadway, width measured from fence to

fence: Roadway, 25 feet wide; broken-stone portion, 8 feet; earth portion, 16 feet; side ditches, 3 feet wide at top and 1 or more feet at bottom, the depth of the ditches from 1 to 4 feet, according to nature of the soil.

767. For steep grades, firm, dry soils, and light traffic: Crown of wheelway, 3½ inches; slope of side track, ½ inch per foot; first or bottom course of stone, 3 inches thick, the second or top course also 3 inches thick. The stone broken to 1½ inches. The road surface finished with a layer of screenings not less than ½ inch in thickness.

For roads 41½ feet wide between fences: Roadway, 25 feet wide; broken stone wheelway, 12 feet; sidewalks, 8½ feet, with 6 inches of macadam laid as already described; crown of wheelway, 5 inches; the depth of gutters below centre grade, 12 inches; slope of sidewalks, 4 inches.

For roads 50 feet wide between fences: Roadway, 30 feet; wheelway, 16 feet; sidewalks, 10 feet, with 6 inches macadam; crown of wheelway, 6 inches; depth of gutters below centre grade, 16 inches; slope of sidewalks, 5 inches.

768. For moderate grades, firm, dry soils, and moderate traffic: The same widths are used in the order as above. The thickness of the macadam is increased to 8 inches in two courses of 4 inches each, and finished with a thin layer of stone screenings. The following are the specific differences for the three different widths of roadway:

For the 33-foot roads: Crown of wheelway, 3 inches; slope of side track, ½ inch per foot.

For the 41½-foot roads: Crown of wheelway, 4 inches; depth of gutters below centre grade, 10 inches; fall of sidewalk, 4 inches.

For the 50-foot roads: Crown of wheelway, 5 inches; depth of gutters below centre grade, 12 inches; slope of sidewalk, 5 inches.

769. For flat grades, wet and yielding soils, and heavy traffic: For these conditions some special means of subdrainage is necessary. Ditches are dug on either side of the broken-stone road-bed and underneath it. These ditches have a bottom width of 1 foot at a depth of 4 feet below the grade of the road, with side slopes of 3 inches per foot rise. Tile pipes 4 inches in diameter are laid, with open joints. The ditches are then filled with broken stone, coarse cinders, or gravel, over which the road-bed is constructed. For the 33-foot roads open ditches were used in place of the sidewalks.

For the grades, soils, and traffic referred to in this paragraph the Telford road is recommended.

General dimensions in the same order, with the following modifications:

For the 33-foot roads: Crown of wheelway, 3½ inches; slope of side track, ½ inch per foot.

For the 41½-foot roads: Crown of wheelway, 5 inches; depth of gutters below centre grade, 12 inches; fall of sidewalks, 4 inches.

For the 50-foot roads: Crown of wheelway, 6 inches; depth of gutters below centre grade, 12 inches; slope of sidewalks, 5 inches.

The construction of the Telford roads is as follows:

770. The first or bottom course consists of large stones roughly shaped, and about the same relative dimensions of granite paving-blocks. This course is about 8 inches in depth. The broadest surface is placed on the earth formation, and the wedge-shaped spaces between these stones are then filled with the smaller pieces and chips of stone. The projecting corners of the large stones are then broken off, and the whole work compacted with bars and hammers. A course of stone broken to 1½ inches is used to fill the spaces remaining between the large stones of the bottom course; and, finally, a third course of broken stone 1½ inches in size and 4 inches in depth is placed over the bed thus formed. A surface coat of screenings laid as before completes the road-bed.

In all cases special attention is given to proper drainage ditches. In flat, wet countries extra depths may be given to the drainage ditches, and in such cases it is best to raise the grade of the road a foot or more above the natural surface adjoining, the material from the side ditches being used to effect this purpose.

The open ditches may be used instead of the tile and broken-stone drains in and under the road-bed.

Where rock is scarce or expensive the field stone obtainable along the route may be substituted for the bottom course of large stone in the Telford pavement. It is not a desirable substitute, and in such cases the top course of broken stone should be at least 4 inches in thickness after being rolled and compacted.

771. The following tables give the quantities of material and cost for the several modes of construction above described. The stone used in these specimen roads, which were rather intended to show methods than to prescribe or advocate any particular variety of stone, was a dolomite (limestone) of excellent quality, which has been extensively used on the streets and boulevards of Chicago. It wears rapidly under heavy traffic. For the Telford roads it has sufficient hardness, but breaks badly. The granite furnished by the Hudson River Supply Company was superior to the limestone.

TABLE LXVII.

QUANTITIES AND COST PER MILE.

Width of broken-stone roadway	8 feet	12 feet	16 feet
For the 6-inch macadam:			
Broken stone, cubic yards	940	1,410	1,875
Binder, cubic yards	156	235	315
Total cost of grading, ditching, and of material, rolling, etc.	$3,050	4,575	6,100
For the 8-inch macadam:			
Broken stone, cubic yards	1,250	1,875	2,500
Binder, cubic yards	210	315	420
Total cost	$3,750	$5,635	$7,500
For the Telford road:			
Large stones, cubic yards	1,300	1,960	2,610
Small broken stone, cubic yards	780	1,175	1,565
Binder, cubic yards	156	235	315
Cost	$5,150	$7,750	$10,350
Cost of subdrainage	1,585	1,585	1,585

The use of ordinary field stone instead of the regular Telford stone for the bottom course would make the cost less by from $1000 to $1500 per mile. On steep grades paved gutters would be required in some cases. The cost of these are not included in the above estimates.

772. The sidewalks were gravelled to the depth of about 3 inches over a width of 4 feet along the centre line, and the remaining portions on either side of this footpath were sodded, leaving room for planting trees on the edge of the gutter. The gravelled footpath is slightly rounded.

773. In estimating quantities of material an excess of 20 per cent over the actual compacted thicknesses is allowed for the broken stone and binders, and for the Telford stone an allowance of 25 per cent is made.

The estimates of cost are based on the following prices per square yard of completed road, including the cost of excavation:

TABLE LXVIII.

6-inch macadam	$0.65 per square yard.
8-inch "	0.80 " " "
12-inch Telford	1.10 " " "
12-inch field stone	0.90 " " "
Tile subdrain	0.30 per lineal foot.

774. An improved construction for country roads was also exhibited. One of these was a Telford road with 6 inches of coarse stone, 4 inches of medium, and 2 inches of crushed stone. The cost for a 15-foot roadway was placed at \$3000 per mile where the cost of stone is \$1.00 per cubic yard. Another road was built of burnt clay with a 6-inch course of coarse material, 4 inches medium, and 2 inches of fine. The cost of a 15-foot roadway was stated at \$2000 per mile, with coal for fuel not exceeding \$2.00 per ton. Burnt clay is being used for ballast on Western railways, but is a new material for road construction.

775. One specimen of asphalt paving cut from a pavement in actual use for over six years' constructed on an 8-inch bed of concrete on which $2\frac{1}{2}$ inches of asphalt were placed. The travel over this pavement had been 204 tons per square foot per day, or a total of 495,720 tons. The cost, with five years' guaranty, was \$2.90 per square yard. The cost of repairs during the second five years has been 46 cents per square yard.

The total amount of Trinidad asphalt pavement in the United States is 781 miles.

A cement and gravel pavement in use for two years under heavy traffic was also shown.

Specimens of wood paving, used in New South Wales, made of blocks of Black Butt or Eucalyptus. This wood is very dense and heavy. The blocks were $8\frac{1}{2} \times 6 \times 2\frac{3}{4}$ inches.

COUNTRY ROADS.

776. While much attention and study has been given to the construction of good roads covered or paved in some manner with hard materials, little has been bestowed on the proper construction and maintenance of country or ordinary roads. In a country embracing the great area of the United States, and with even those States but thinly settled which have the largest population, it is hardly to be expected that in the near future any uniform and systematic methods of location, grading, paving, and draining of our roads will be adopted as have been in those European states which, covering an area but little greater than one of our largest States, have a population equal or nearly so to that of all of them, and while it is to be hoped that gradually properly constructed and paved or broken-stone roads may be extended from

one or more centres until all of the principal county and State roads will compare favorably with the best foreign highways, yet it must not be forgotten that for a great many years the large mass of our people must put up with what are called dirt roads. Therefore the proper construction and maintenance of dirt roads is the question whose solution should engage the thoughts of our engineers and taxpayers. At present both time and money are absolutely thrown away. Our country roads are fairly good when the weather is favorable, but this is mainly due to Providence, and not to our labors or money expended. The locations are bad, the grades are so steep in many places which could be easily avoided, that it is almost impossible for teams to ascend them, or to hold back in descending. In unfavorable weather they are abominable if not impassable; and on such roads, up to date, time and money have been wasted sufficient to have nearly all of our roads in fair condition at all times, and a large portion of them up to the best standards.

777. However much of the wear and destruction of roads may be attributed to the unfavorable character of the soil, the kind and quantity of traffic, the kind of vehicles employed,—two or four wheeled, broad or narrow tire, wheels of large or small diameter, relative length of axles between front and rear wheel bearings, etc., —we can safely assert that the most potent factor among the many destructive agencies is that of the weather, considered in connection with defective drainage or no drainage at all.

778. Effects of weather are manifested in the rains, strong, dry winds, and the expanding and contracting influences of freezing and thawing.

779. The effects of the rains are twofold: (1) They wash the surface; (2) they soak into the underlying material or foundation. The washing of the surface removes the binding material, cuts the roadway into a series of transverse gutters or ruts extending to the sides, and on steep down grades longitudinal ruts are formed, either of which results is destructive of the roadway, and renders it unsafe for travel. Where simple gutters or drains are cut on the sides, the flow of water widens and deepens them, encroaching upon and destroying the roadway—so much so that on many roads it is absolutely dangerous to drive along them, especially after dark, unless familiar with the conditions.

The water may reach the foundation either by passing through the road surface, or by seeping or flowing under it from the sides.

The nearer a roadway approaches to a non-porous surface and a porous foundation the better will be the road. If the surface is free of water, it will be but little affected by the action of frost. A porous foundation allows a rapid disappearance of any water reaching it from above or from the sides by either sinking downwards or laterally, provided no scouring takes place, which, however, can be readily prevented by means of intercepting drains which will carry it to some watercourse. If the road traverses a sandy or gravelly section, no large or heavy stones, no extra depth of stone, and commonly no drains are required, as good natural drainage will usually exist; and, moreover, a damp or wet sand is more favorable for traction than when dry. If, on the contrary, the foundation is clay or loam, any water that reaches it will remain in it. It will become soft and result in the disintegration or destruction of the surface. In such soils drains must be placed to carry off the water, or it must not be allowed to reach it all, being intercepted by means of a drain excavated on the up-hill side, and transverse drains under the surface of the road-bed.

As the water washes the finer or binding material from the road surface, it is desirable that all water falling on the road should be shed off as rapidly as possible in order to prevent accumulation and flow over any portion of the road. On grades over 5 to 6 per cent it is difficult to prevent the water from flowing along the road for a considerable distance before reaching the side drains or gutters, resulting in ruts or gullies in the road.

If the natural soil is sandy, the wear and solidity of the road may be increased by covering it with from 4 to 6 inches of clay or loam, upon which a thin layer of 2 or 3 inches of rather coarse gravel is placed. The clay bed should be formed with a crown of 1 inch per foot. Under the influence of rain and the traffic the gravel will be worked into the clay, forming a uniform, compact mass, which will resist well the washing or scouring effects of water. All ruts should be filled as soon as they begin to show themselves.

780. Straw, marsh-grass, sorghum waste, and the like, placed on sandy roads will prove beneficial. It is not necessary to turnpike sandy roads; but little levelling or grading is necessary. The growth of grass and trees should be encouraged, as wet sand is better than dry.

781. Gravelly soils or layers of gravel on either a sandy or clayey foundation make good roads. When placed on a sandy foundation, a layer of clay should be used with it as above de-

scribed; when used on a clayey foundation, it will get all the clay required. Before placing the layers of gravel the foundation should be rolled and properly crowned. Each layer, not exceeding 3 or 4 inches in thickness, should be rolled or travelled over, after which another layer may be placed and treated in the same way, so that it may be packed from the bottom, and not simply compacted on top with a layer of loose material below.

782. Roads on clayey soils must be thoroughly drained. A crowning of 1 inch in 20 is usually sufficient. They should be ploughed and harrowed so as to make the surface homogeneous and uniform. A road machine can be made to do good work if properly handled, but is likely to make the side slopes too steep with a ridge of soft earth in the centre. After being worked and shaped, roll the surface well, using heavy rollers. The surface should be even, and rolled down hard.

783. *Drains.*—When the surface of the ground is level, one drain on each side will be required. In many cases open ditches will answer every purpose, and are, of course, less expensive. If there is found a tendency to scour in the ditches, undermining its sides, the only remedy is to pave them with any kind of rough stone available. Such a pavement is comparatively inexpensive, and will prove ultimately to be economical. Sodding will answer the same purpose, or the sides of the roads and gutters may have grass-seed sown on them. The gutters in such cases may be made rather broad and shallow. In heavy, stiff soils cross-drains of broken stone covered over may be necessary to insure the water passing freely to the side drains, and in some cases a centre drain may be required, filled with broken stone and covered over. Where a centre drain is required, with or without side drains, the impervious soil should slope towards the drain. In this case the necessary rise or crowning must be made with the covering material.

Covered drains where used may be made by placing rough stone in the bottom of the trench, which should be from 2 to 3 or more feet deep, or common tile or vitrified pipes laid with open joints may be used. Over these small broken stone or a rather coarse gravel should be placed, filling the trench to the top. The pipe should be laid true to the proper grade. If any sag occurs it will fill up and choke the pipe. These drains must lead to some watercourse or to the surface of the ground at some point from which it will flow freely and readily away from the lowest point of the road.

Ordinary earth roads thus constructed will amply repay all

money expended on them, and will secure fairly good roads throughout the year. The first cost will be greater than that now expended in what is called opening or constructing new roads, which are mere excuses for roads, and are sources of endless inconvenience, discomfort, and expense, and prove ultimately more costly than properly constructed roads. If a whole road or system of roads cannot be properly constructed at once, let at least a part of it be so constructed. The great benefits and advantages derived will soon convince the people that it is the part of wisdom to improve all portions rapidly.

784. *Road Coverings.*—The above-described road constructions apply principally to the ordinary or dirt roads, but thus prepared they are in good shape and condition for a covering of hard material. This covering has to serve the double purpose of forming a solid and impervious surface, thereby keeping the water from the foundation below, and also of forming a surface which, affording a good and firm foothold for the teams, will, by its smoothness, permit loaded vehicles to be hauled over it with the least expenditure of power.

The materials which may be employed to secure a water-tight and solid covering are gravel, broken stone, and stone or brick pavements. The material to be used should be selected by considering the volume and character of the traffic ; the comparative cost, including both the first cost of construction and the annual and permanent cost of repairs and maintenance—the latter estimated against the interest on the increased cost of the better material and methods of construction.

Stone or brick pavements are used only in cities or in very thickly settled districts. For country roads in general we must rely mainly on gravel or broken stone, and in many sections of the country but little opportunity will be given to choose between the two, as both will not be found in sufficient quantities for extended road improvements.

785. If gravel is hauled in carts or wagons, and simply dumped on a roadway, the coarser pieces will roll down to the outer edges of the heap, while the finer will collect at the centre. The result will be a roadway wanting in uniformity, both as regards smoothness and compactness. It is therefore advisable to screen the mixture into three, or at any rate into two, grades. The larger should be spread into a layer, the smaller and finer into a layer over the first, thereby filling the voids and binding the whole together; and

if this is then well rolled, a much better roadway will be secured at only slightly greater cost.

786. Too much sand is a disadvantage rather than an advantage in the gravel for road-making. The best gravel for the purpose consists of angular fragments of stone of varying sizes, and a fine stone dust, with a small quantity of clay, in the shape of fine powder. If the larger stones be removed entirely, the remaining materials, if properly laid, will make an excellent road for a somewhat light traffic. This gravel is found in banks more or less difficult to work. Of the rounded water-washed gravel, that which has an excess of quartz in any shape is not so valuable, as it is very brittle, and does not give a good bond.

A slaty gravel makes a better road than the harder quartz, as sufficient powder will be formed by the action of wheels to bond the whole together and form a smooth surface. It should always be recollected that in order to make a good road of gravel it must be placed in layers, the sizes of the pieces in each layer being as uniform as possible, the coarser placed at the bottom and the finest in the top layer. If it cannot be rolled, then it is necessary to keep all ruts filled as fast as they may be formed; for if the mass becomes firm and compact, with ruts existing, it will be difficult to form a good, even surface.

787. For broken-stone roads, stones of a hard and tough character should be selected. Hornblende-granite or syenite makes a good road. Some of the conglomerates, amygdaloids, and felsites are suitable. Slates, sandstones, schists, and the like have but little value for road-making purposes. The same may be said of micaceous granites. Trap-rock meets all of the requirements of good roads better than any other kind of stone. Any of the above are, however, to be preferred to mud.

788. To recapitulate: First make a firm bed; lay the stone on this in layers, each separate layer composed of nearly equal-sized fragments; roll each layer; use stone-dust or good gravel for a binder, using only water enough to facilitate the forcing of the dust into the voids. Do not use any clay or loam with the stone, as either material absorbs water and destroys the bond, and makes mud in wet weather and dust in dry weather. On sandy or gravelly soil the layer of large stone at the bottom may be omitted, nor is it necessary to use as thick layers of the smaller stones. This may be laid direct on the sand to a depth of from 4 to 6 inches with

good results. This depth must depend upon the kind and volume of traffic.

CONSTRUCTION OF ROADS OVER SWAMPS.

789. Many soils called swampy are capable by proper drainage of becoming firm. This may be effected by open drains or subsoil drains, conveying the water to some near watercourse, or carrying it clear of the section to be drained.

But swamps referred to in this place are not supposed to admit of such treatment, either on account of the extensive and costly system of drains required, or the fact that they are but little, if any, above the surface of the adjacent watercourses, and are consequently in a state of continuous saturation, often to an unknown depth. There are but two general solutions to such problems: (1) To construct a permanent way, regardless of the cost, similar to that required for railway structures, such as dumping sand, gravel, or broken stone along the line, until a firm and solid roadway is formed; or the construction of a trestle-work. (2) To construct some form of road-bed, floating, as it were, on the soft and yielding material, spreading the bearing surface until the unit pressure is not greater than the bearing resistance of the material.

This can be effected in a variety of ways. (1) By the simple corduroy-road, which consists of a series of small trees or logs, as straight as can be obtained, and from two to three times the width between the wheels in length, laid close together, and at right angles to the axis of the road. These logs may be round, or, if of sufficient size, they can be split before laying. This necessarily makes a rough and uneven road-bed, but very cheap, when suitable timber is convenient. But it is often resorted to as a temporary expedient. A grillage made of two or three courses of different-sized sticks, carefully laid and bound together, can be built into a fairly good road-bed, and if covered with brush or marsh-grass, over which is placed a layer of dirt, a very satisfactory road can be constructed.

In many of the extensive swamps of the Southern States the swamp is covered with a crust or matting of roots of the cane or undergrowth. This crust is capable of bearing considerable loads when distributed over logs, planks, or even brush. The surface, however, is uncertain and treacherous, owing to weak spots or places in the crust; but with a little care almost any of the above-

mentioned expedients will enable vehicles to pass over in safety. A very satisfactory, convenient, and cheap method is adopted in the swamps around and near Mobile. A number of sawmills have been established on the river-bank, access to which has been made safe and convenient, both for highways and branch railways, by spreading a broad layer of sawdust and other mill-waste to the depth of three or more feet. This forms a springy, and rather spongy road-bed, but will carry safely heavy loads at slow speeds. The sawdust gradually sinks into the swamp, and requires refilling on top to maintain the grade. It seems to keep constantly wet, absorbing and drawing up the water from the marshy soil. The material will doubtlessly on this account be durable. The construction costs little beyond that of hauling the sawdust from the mills.

In the West piles of logs are placed at intervals on the swamp and burnt to charcoal, which is then spread over the surface to the depth of one or two feet, and has been found to give a good, durable road-bed.

It would seem always economical and safer to place a layer of logs or waste planks directly on the swamp, covering this with small brush or marsh-grass, and upon this placing the covering material, whether earth, charcoal, or sawdust, as this will prevent extended and unequal sinking in places, thereby destroying or materially impairing the road-bed.

790. *Paved Roads.*—Simply for convenience, the term paved roads will include all kinds of roads upon which any kind of covering, to obtain a smooth, hard, and firm road surface, is placed, without regard to the character of the materials or methods of construction; that is, as distinguished from the common or dirt roads already described. The earthy material in those cases was used to form the road-bed or surface. Now we are to consider these same and other materials simply as forming a foundation upon which to place and support the road covering.

Macadam and Gravel Roads.—Although broken-stone and gravel roads may be used either for city or country purposes, they are now about the only kinds of covering that are used for country roads, and will therefore be described under the head of COUNTRY ROADS.

791. *Foundations.*—Whatever may be the covering or road-surfacing material, and whatever may be the cost of this surfacing, roads possessing permanence, stability, and cheapness of maintenance can only be secured by means of good foundations. With

these secured the condition of the road surface will depend upon the material used and the manner of placing it. The following conditions are necessary to be complied with in forming good foundations:

(1) The entire removal of all vegetable, perishable, and soft and weather-worn materials.

(2) The thorough compacting, by rolling or otherwise, of the bed of natural soil thus prepared, so as to form a uniform and firm bed, conforming more or less closely to the transverse curve of the finished road-bed.

(3) Efficient surface drainage, and when necessary subsurface drainage.

The labor, cost, and difficulty in complying with the above conditions, and necessary modifications of them, will depend entirely upon the character of the natural soil.

Sand and gravel soils which permit of the free passage of water present no difficulty and require no precautions beyond those already described for sand roads. They give naturally secure and solid foundations.

Clayey soils, on the contrary, as well as mixed soils which retain water, offer the greatest difficulties. And although, when properly surfaced and compacted, fairly good broken-stone roads may be secured by sufficiently draining the clay bed by means of open transverse tile-drains or even shallow trenches filled with broken stone, leading with sufficient slope into the side ditches, and the bed then covered with layers of broken stone, yet the best roads on a clay bed can only be secured by excavating to a depth of 18 to 20 inches below the finished road surface, and placing over this a layer of sand, shells, slag, cinders, etc., of such thickness as will permit of the road covering, after being rolled or compacted, bringing the whole to the proper surface. The bed, the filling, or foundation and covering should all be compacted separately. With many if not all of these filling materials more or less water will find its way to the clay bed, and to that extent offer the same objection as to placing the broken stone directly on the clay. But when placed in layers of not over 4 inches in thickness and rolled, three such layers will reduce under 10-ton rollers to 8 inches, the compactness thus secured, especially with slag, will retard the flow of water through them and the working up of the slag into the layer, both of which conditions will occur if only broken stones of usual sizes are placed on the bed.

However, on country roads, either the broken stone will be laid directly on the clay, or at best an intermediate layer of sand, slag, cinders, etc., only will be available in certain localities. With efficient drainage fairly good roads can be secured in either case, especially the latter.

792. *Broken-stone Roads.*—There are two general types of broken-stone roads, namely, the telford and macadam. The preparation of the foundation-bed is essentially the same in both.

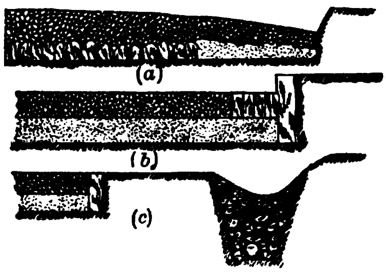
Telford Road.—Upon a graded bed a layer of stone about 8 inches in thickness is laid by hand. The stones are set on the largest face, and with the longest dimension transversely to the line of road, the breadth of the upper edge not being over 4 inches. The open spaces between the stones are filled with chips, forced in tightly with a light hammer. This forms a solid stone course over either a part or the whole surface, the upper surface of which has the proper convexity. In the telford proper this curve is obtained by using thicker stones near the centre, getting thinner towards the sides, and only a portion of the base is thus covered, the crowning being about 4 inches in the breadth of 15 feet from the centre. The middle 18 feet of the roadway is covered with hard stones to the depth of 6 inches. First a layer of 4 inches is laid and opened to traffic. After this has been compacted by vehicles and horses, ruts being filled as formed, the second layer of 2 inches is placed. These stones are broken as nearly cubical as possible, and the largest dimension not over $2\frac{1}{2}$ inches. On each side of the middle 18 feet a filling of broken stone or gravel is placed; the thickness of this is regulated so that the total crowning will be 6 inches. Over the entire surface thus formed is spread a layer of clean gravel $1\frac{1}{2}$ inches thick, which serves as a binder. A half section of such a road is seen in Fig. 298(*a*). There is much labor and expense connected with the construction of this type of road.

A modified form of this road is constructed by placing on the large stone layer a course of stones not exceeding 3 inches in size, and rolled. This is covered with a layer of sand $\frac{1}{2}$ inch thick, and rolled; over this a layer, of stones not exceeding 2 inches in any dimension, about 4 inches thick is placed and rolled; this is followed with another layer of sand, also rolled; and finally another layer of clean, sharp sand is laid and rolled until a firm smooth surface is obtained. The surplus sand is then swept off.

793. *The Modern Macadam Road* is constructed as just described for the modern telford road, omitting the lower course of

large stone, the total depth of broken stone varying from 4 to 12 inches in thickness, laid and rolled in layers as described.

Among the advantages of broken-stone roads of any kind may be mentioned—



FIGS. 298.

(1) Comparatively low first cost; (2) freedom from noise; (3) easy traction when in good condition; (4) good foothold for animals.

Among the defects (1) muddy in wet weather; (2) dusty in dry weather; (3) difficulty of keeping the surface clean; (4) and relatively high cost of maintenance.

Broken-stone roads, whether telford or macadam, when placed directly on a clay soil, are likely to become rather poor roads on account of the surface water reaching the clay. The mud thus formed will be certain to rise and fill the voids, the stones settling and sinking into the clay. Much of the stone will be reduced to a powder under the traffic, thus forming mud in wet weather and dust in dry weather. But by carefully observing the rules already given the macadam road will be found satisfactory for country roads. If a layer of sand or gravel is placed over the clay, while adding to the cost, it will go a long way in giving excellent results. The broken stone should not be screened. The hardest stones are not the best: a judicious mixture of hard and soft stones are preferred. The binding sand or gravel should contain no loam or clay. The stones should not be too large. Too much binding material and an excessive amount of water in rolling are injurious. Thorough rolling is essential. (See Figs. 298 (*b*) and (*c*), transverse sections.)

794. *Quality of the Stones.*—It may be stated generally, that for country roads the stones used will necessarily be of the kinds and qualities found in the locality, and that, whether good or bad,

the roads will be benefited by the judicious and liberal use of them, though giving only a poor equivalent for the labor and money expended in many cases. When, however, a choice is made, the stones should be hard and tough, and capable of resisting the disintegrating action of the weather.

The hardest stones are not necessarily the best. Igneous and siliceous stones, though hard and, it may be, tough, do not consolidate well, the sandy material produced by rolling or by the traffic having little value as a binder. With limestones, while neither so hard nor tough, the rolling or traffic produces a material possessing good qualities as a binder. Such roads, with good construction and proper cross-section, will be practically impervious to water, will not disintegrate so rapidly, and will wear better than many harder varieties of stone. A hard stone without toughness will be so brittle that it crushes to powder under heavy loads.

Basalt or trap and granites of the best quality—that is, the syenites—are considered the most efficient rocks and, ultimately, the most economical. Granites containing much mica and gneiss break up and wear away rapidly. "The carboniferous and transition limestones are fairly durable, and make smooth and pleasant roads for light traffic and pleasure drives. The quartzose grits and siliceous grits mixed with limestones form excellent roads." Slates make an inferior road: they either crumble and disintegrate, or form mud.

River and field stone have been used. They make, usually, a rough road, and wear irregularly.

795. The rock is broken for road purposes either by hand or by machinery. The fragments should be as nearly cubical as possible. The greatest dimension for granite or trap stones should not exceed $1\frac{1}{2}$ inches; for limestone, 2 inches. The smaller-sized stones compact better, require less binding material, and form a smoother surface. All sizes below the maximum are admissible.

Hand-broken stones are much better in many respects; but the great saving in cost and the increased daily output have led to the extended use of crushing-machines. The two general types of crushers are: (1) Those which crush the stone against a fixed block by blows or pressure from a hinged lip or arm, such as the Blake crusher, of which there are many designs; and (2) by a crushing or grinding pressure between a hollow conical surface in which turns a solid mass of iron having ribbed or corrugated surfaces, such as the Gates crusher. The cost of crushers varies

from $200 to $7000, according to capacity and steam-power required. The price of any required crusher can be readily obtained from the manufacturer.

796. The cost of quarrying and crushing stone varies greatly, according to kind, quality, and lay of stone in the quarry, and may be taken as extreme limits at from 80 cents to $1.50 per cubic yard. Cost of hauling must be added.

797. The quantity of broken stone required per unit of length varies with the width of the road and thickness or depth of covering. The width varies from 8 to 60 feet, the thickness from 4 to 16 inches. This depends upon the traffic and grade of road. For grades flatter than 1 in 100, 10 inches thick; from 1 to 4 per 100, 8 inches; and over 4 in 100, 6 inches; many roads having only a thickness of 4 inches of broken stone. Over such roads loads averaging 6000 pounds have been hauled, and ordinary teams hauling a net load of 3000 pounds.

In other cases from 4 to 6 inches have not proved satisfactory. This was, no doubt, due to bad construction, unrolled and undrained foundation-beds, and unrolled stone layers. Layers of such thicknesses on sand beds or good layers of sand or gravel over the natural soil will provide a fine and lasting surface.

The thickness of covering is that after compacting and rolling. The volume after rolling may be taken at from two thirds to three fourths of the loose volume. If, then, we wish to know the number of cubic yards of broken stone required to give a finished road 9 inches in thickness, the unrolled thickness will be $\frac{4}{3} \times 9 = 12$ inches. A road-bed 16 feet wide would require 16 cubic feet per foot of length, or $16 \times 5280 \div 27 = 3130$ cubic yards per mile. Or divide 36 by the thickness ($\frac{36}{12} = 3$), this gives the number of square yards covered by one cubic yard, or per mile $\frac{5280 \times 16}{9} = 9390$ square yards, and $\frac{9390}{3} = 3130$ cubic yards. The finished road will contain about 2350 cubic yards, including the necessary quantity of binding material, which will be from 20 to 25 per cent of this volume. Whether this is the crushed stone itself or is added in the form of sand or gravel will depend upon the kind of stone, manner of breaking, etc. If sand or gravel is to be added, it should not be mixed with the stone, but, after rolling a layer of three or four inches of stone, it is spread over the layer and rolled into, aided by the use of brooms and water.

798. The binding material is essential to the proper consolida-

tion of the broken stone, and necessary to secure an impervious, firm, and stable surface. A little clay or earth is sometimes used, with sand or fine gravel binders. An excess of clay is sure to form mud and defeat the purpose of the binder. Water, when applied, should be by the use of a sprinkler.

799. Rolling saves wear and tear of horses and vehicles. Although after long use a fairly good consolidation may be effected by the traffic, yet such roads will become soft and muddy in wet weather and dusty in dry weather. While to a certain extent this will be the case when rolled, these effects are greatly reduced. Rollers are made varying from 3 to 20 tons in weight. Such rollers cost from $150 to $100 per ton of weight. A 10-ton roller weighs only about 450 pounds per linear inch, whereas the weights from wheels of vehicles may be as much as 800 to 1000 pounds per inch. This indicates the advantage of heavy rollers.

Steam-rollers are preferable to those drawn by teams. Steam-rollers are the cheapest in the end, enable more rapid construction, require less stone, and give a better road surface. Rollers with one ribbed roller in front, followed by a roller with a smooth surface, are effective, and better than those commonly used.

The rolling of broken-stone roads is of the greatest importance. The layers should be about 4½ inches thick. The sides should be first rolled, and made so firm that when the rollers pass over the crown or centre the spreading of the stones laterally will be prevented, and the compacting more effectively secured. The rolling should be continued until the stones cease to creep or sink under the roller, and the surface is smooth and hard. Cost of rolling varies from 2 to 14 cents per square yard. The cost of telford roads varies from $9000 to $12,000 per mile; the cost of macadam roads from $3000 to $5000 per mile. The cost per square foot of surface varies from 50 cents to $1.25 for macadam and $1.70 for telford.

800. *Wear of Broken-stone Roads.*—The wear of poorly constructed roads is necessarily very great, and cannot properly be charged to maintenance. The deterioration of roads arises from three main causes—action of atmosphere, wheels, and horses' feet, the latter being the greatest and the former the least. It varies with the quantity and character of the traffic for roads constructed in the same manner and of the same kind of materials. The new material required annually is on an average about 70 cubic yards per mile, caused by legitimate wear only. Before applying new

material the old surface should be broken in shallow transverse grooves, not over 2 inches deep, at intervals of from 4 to 6 inches, the new layer spread and then rolled. Wet or damp weather is the best time to repair, but when water is convenient any but cold or freezing weather is suitable.

The cost of maintenance will vary from 6 to 50 cents per square yard per annum.

The rise of crowning of broken-stone roads is about 1 in 60.

Cross-sections of broken-stone roads, with several kinds of side drains, are shown in Figs. 298 (*a*), (*b*), and (*c*).

800½. *Gravel Roads.*—Road coverings of gravel make good and serviceable roads. The same remarks apply in regard to the selection of the gravel as were made in regard to broken stone—namely, that kind of gravel found in the locality will have to be used in the construction of roads, even though but a poor equivalent for the money expended be obtained. Rounded gravel, such as is obtained from seashores or rivers, does not make a good road, even when mixed with the usual binding material, as no efficient mechanical hold or bond can be made and maintained throughout the mass. Such a surface is loosened and the pieces worked out both by the action of the traffic and the weather. If, however, about one half this gravel is broken into angular fragments and mixed with the other half, and this mixture with about one eighth of its volume of clay,—sand is unsuitable,—the mass can be well compacted and a good road obtained.

Pit gravel usually contains too much earthy matter, and requires screening through a netting of small meshes to get rid of a portion of the earthy material, and all pieces over about 1½ inches in diameter should be either broken or removed by means of a screen with large meshes.

A ferruginous clay is considered the best binding material. Clay in wet weather forms mud, and in dry weather cracks, resulting in a loose surface.

Constructing Gravel Roads.—The soil must be excavated, rolled, and otherwise prepared by sufficient provision for drainage, as already described. Then a layer of the prepared gravel, not over 4 inches in thickness, is spread uniformly over the proper width, and rolled with a roller weighing not less than 2 tons, having a weight per linear inch of 150 pounds or more. The rolling should be continued until the gravel ceases to creep or rise in front of the roller. The surface should be moistened by sprinkling in advance

of the roller. It is not well to use too much water. Successive layers are thus treated until the proper thickness is obtained. It is not recommended to place layers of stones having large dimensions at the bottom and smaller sizes in the top. Each layer before rolling should be a mixture of the several sizes.

The repairs of gravel roads should be kept up continuously, filling ruts when first formed by raking any loose gravel into them. When new material is added it is better to apply it in thin layers of 2 to 2½ inches than in one thick layer.

Gravel roads cost from 15 to 75 cents per square yard, which will be from $700 to $2000 per mile, according to width of road, etc.

EXTRACTS FROM THE SPECIFICATIONS FOR THE CONSTRUCTION OF THE HUDSON COUNTY BOULEVARD.

801. *Fills.*—All fills shall be made with good, clean material, and shall be deposited in layers, the full width of the portion of the road, not exceeding 6 inches in thickness, and compacted by horses and vehicles before placing another layer. The best material shall be reserved for the top layer.

Excavation and Surfacing.—Where there is a surplus of clayey soil in excavations, a supply of clean sand or gravel shall be provided, of suitable quality for paving, and shall be mixed with the clayey surface as directed.

Where there is no curbing the side slopes shall be 1½ to 1.

Rolling.—The surface of the roadway shall be rolled thoroughly with a roller weighing not less than one gross ton for each foot of width of face.

All soft or low places shall be brought up to proper surface in thin layers, well rolled. The whole surface of the road shall present when finished a hard, smooth, uniform surface, over any part of which water will flow readily toward the gutters.

Pipe Culverts may be either of the best vitrified salt-glazed pipe, socket-jointed, sound, smooth, and perfect in shape, and laid on 1½-inch plank to true line and grades. Joints to be filled with best hydraulic cement mortar, one cement and one sand, each joint wiped and pointed inside and out. Or cast-iron pipes are to be used when too near the surface, for the full protection of the vitrified pipe.

Box Culverts to be built of good, sound stone when required.

Covering-stones not less than 6 inches thick, and of sufficient length to bear a foot on each side wall.

Waterways to be paved with spalls or cobblestones.

Blind Drains.—Where subsoil drainage is required the drains shall consist of large and clean spalls, the larger at the bottom of the trench, and topped off with the smaller stones, over which is placed a layer of salt hay or small brush, over which is to be rammed the back filling.

Paving Gutters.—Where there is danger of wash the gutters shall be paved with Belgian blocks of stone. They shall be paved 4 feet wide with curb, and 6 feet without curb. Where curb is set the surface of the paving shall conform to the section of the road, and abut against the curb. Where no curb is set, the paving shall be turned up for 2 feet of its width on the slope. The trenches are to be excavated to a depth of 12 inches below finished grade, and filled with clean, sharp sand or gravel, upon which the blocks are to be bedded and well rammed. After laying and ramming the blocks a layer of clean, sharp sand or gravel shall be spread over the paving, and all interstices well filled.

Curbs.—Curbs shall be of the best bluestone, not less than 4 feet long, 20 inches deep, and of a uniform width of 5 inches. The ends shall be dressed to the depth of 1 foot, so that the joints shall not exceed $\frac{3}{8}$ inch.

Preparing Subgrade for Paving.—The street for the full width between outside gutter-linés (except the space 16 feet wide in the middle) shall be excavated to a subgrade which, when properly rolled and compacted, shall be parallel to and 12 inches below grade-line of the cross-section of the finished roadway.

All soft or spongy places developed shall be removed, and their places filled with good material. The subgrade surface shall be rolled thoroughly with a roller weighing not less than one gross ton for each foot of width of face. If necessary to consolidate and pack the surface, it shall be sprinkled in advance of the roller. No ploughing will be allowed within 4 inches of the subgrade.

Telford Foundation.—The telford foundation shall consist of sound, hard, and durable, frost-proof quarry stone, and shall be from 8 to 9 inches deep, 4 to 8 inches wide, and 6 to 14 inches long. They shall be laid by hand, with their broadest edges down and longest side across the road, on the subgrade as prepared, the courses breaking joints. Between adjoining telford and gravel or earth driveway, a line of stones, 10 inches deep, shall be laid parallel

to the curb-lines. These stones, the telford paving, are to be wedged tightly with smaller stones until the whole surface is firmly wedged together. All projecting points and edges are to be broken off with hammers, so that the top of the pavement shall present a fairly smooth and regular surface. This shall be thoroughly rolled with a steam-roller weighing not less than 15 tons, until the whole mass is firmly bedded into the earth subway.

Macadamizing.—Upon this telford foundation shall be spread a layer of close-grained trap-rock, broken to uniform size, not wider than 2½ inches in any dimension. The stone must be free from dirt or dust, and broken into fairly uniform and regular cubes. This layer shall be thoroughly rolled with the roller above described for the telford foundation. It shall be sprinkled in advance of the roller. The thickness of this layer must be such that when rolled and compacted its upper surface shall be 1½ inches below the surface of the finished roadway. Upon this shall be spread a layer of stone of the same quality, of fairly uniform size, not over 1½ inches on any edge. This layer is to be rolled with the same roller, clean, coarse trap-rock screening, which would pass through a 1-inch screen being spread evenly in front of the roller, and copiously sprinkled in such quantity as shall be necessary to fill all the interstices in the broken stone.

The rolling and sprinkling shall continue, more stone being added if necessary until the upper layer has become thoroughly compacted, and its surface at the proper cross-section and grade for the finished roadway. Upon this surface shall be spread a layer of fine screening, which when rolled shall be one half inch in thickness, which shall be thoroughly wetted and rolled until, by the continued use of water, a wave is produced before the roller.

Gravel or Earthen Driveways.—The space 16 feet wide in the centre of the roadway shall be excavated to a subgrade dependent upon the character of the bottom. If the bottom be of good material the subgrade shall be 4 inches below the grade of the finished road section. If the bottom be soft or spongy the subgrade will be made 8 inches below the grade of finished road section. In the latter case the material excavated shall be replaced by a layer of sand or shale mixture, which formed the top-dressing of the roadbed as graded under former contract, or by a layer of other good, firm material, well rolled and compacted to ultimate resistance by a roller of the same weight as specified for rolling the subgrade of the telford foundation. The top of said layer, when rolled, shall

be 4 inches below the finished grade surface. The subgrade, in good material, shall also be rolled by the same roller to ultimate resistance.

This forming of the subgrade of centre roadway shall proceed at the same time as the laying of the telford foundation, and care shall be taken that the earth shall be well packed and compacted against the outer line of stones in the foundation. Where the bottom shall require subdrainage the telford foundation shall be continued beneath the centre roadway, across the whole road. Where this is done a layer of hard-pan shall be spread over the top of the telford foundation, 2 inches thick, and well rolled.

Top-dressing of Centre Roadways.—The top-dressing of centre roadway shall consist of Shark River gravel or Roa Hook gravel, containing no stone over 1 inch in any direction. But if this last shall be used, 20 per cent of pure clay, free from loam or vegetable mould, shall be thoroughly and evenly mixed with it, before spreading, as a binder. The gravel roadway shall be sprinkled and rolled in layers of 2 inches by the same roller used for rolling the macadam, until the whole surface shall present a smooth, even cross-section from gutter to gutter. (See *Engineering Record*, April 19, 1894.)

ART. XLVIIIa.

CITY STREETS.

802. The same construction and pavements of city streets apply equally to suburban roads; and while macadam or gravel roads may be used in cities on streets only subjected to light traffic, greater care is taken in preparing the natural soil, better stone is selected, and occasionally special methods and materials are used to secure a firm, smooth, and impervious covering. Otherwise the construction of such roads for cities is similar to that already described for country roads.

803. Concrete pavements may be classed as macadam with cement mortar as a binding material. No special difference exists between this and ordinary concrete. On a good well surfaced and rolled bed a thickness of from 5 to 8 inches is used, varying with the kind of traffic. Excellent results have been obtained with this material. A road of this kind should not be opened to traffic under two to three weeks after completion. The best Port-

land cement should be used, and good sound, hard stone; otherwise a smooth surface cannot be maintained.

Coal-tar has been mixed with heated stone which has been properly broken. This mixture is spread and rolled in the usual manner. After being rolled to the proper contour, a 2-inch layer of a similar mixture with smaller stones is placed on top. Sometimes a 6-inch layer of broken stone is laid, and upon this is poured a boiling mixture of creosote oil and pitch sufficient to fill all interstices. While still warm, a thin layer of small stones is placed and rolled. More small stone or chippings is added, and rolled until a smooth, firm surface is secured. Dry weather is essential for such construction. One great property of a good covering, namely, imperviousness, is secured; and the facility with which it can be kept clean is in its favor. Limestone or other soft varieties is used in these roads.

804. *Wood Pavements.*—Pavements of wood have been extensively used both in the United States and in Europe. In general it may be said that while they have proved satisfactory in Europe the contrary has been the case in the United States. This condition can evidently be ascribed in a large measure to defective material and construction, foundation-beds, green timber, imperfect filling of the joints with impervious cement, and want of proper maintenance.

It affords good foothold for horses; it is not noisy, is fairly durable, and requires less tractive force than stone pavements. It can be used on all grades up to 5 in 100. It does not become muddy, produces but little dust, and, finally, is cheap in first cost.

The disadvantages are: It is difficult to keep clean; the wood absorbs water containing more or less foul and decaying matter; and it has the further objection of difficulty in removing to get at pipes, etc., and in replacing satisfactorily. It has been condemned in many cities as a fruitful source of breeding certain diseases; the detritus, consisting of small fibrous pieces of wood, is considered harmful to both eyes and lungs of people exposed to its effects.

Many forms of wood blocks with interlocking devices have been used. The present practice is, however, reduced to the rectangular or cylindrical blocks, placed with the fibre vertical, and with their joints filled with water-proof cement.

The rectangular blocks are 3 inches wide, 9 inches long, and 6 inches deep. The cylindrical blocks are commonly 6 inches in diameter and 6 inches long.

The kinds of wood best suited seem to be cedar, yellow pine, cypress, and juniper. Hard woods, such as oak, do not make the best pavements,—they become slippery; the softer close-grained woods are preferred. Not considering the character of the foundation, wood pavements deteriorate through decay; hence some of the methods used for preserving timber should be used. The best is, no doubt, creosoting. This costs from $12 to $18 per 1000 feet B. M. The blocks should be sound, clear heart, free from knots, shakes, or other defects, and well seasoned. It is claimed that blocks filling these conditions need not be creosoted or otherwise treated. Independent of other considerations, the creosoted blocks contract and expand but little, whereas untreated block pavements will expand about 1 inch in 8 feet; this is caused principally by the absorption of moisture.

Wood pavements properly laid on an unyielding foundation, with water-proof filling in the joints, under some conditions have great durability, their life varying from five to nineteen years in London, and only from three to ten years in the United States.

The cost of wood pavements in the United States varies from $1 to $3.50 per square yard, according to locality, character of foundation, treated or untreated blocks; in England, from $3 to $3.50, including foundations, but not excavation.

The cost of maintenance per square yard varies from 5 to 50 cents.

Foundations for Wood Pavements.—The blocks are laid on the natural soil, on beds of sand and gravel, on a layer of telford paving or on a layer of concrete, and sometimes a double layer of plank.

The layer of sand and gravel placed on the natural soil varies from 3 to 6 inches in thickness. This should be rolled with a 2000 to 3000 pound roller. When a layer of concrete is used, it should not be less than 6 inches thick, over which is placed a cushion-coat of sand 1 inch in thickness; or a coating of asphaltic mastic is laid on the concrete, or the concrete is covered with a layer of roofing-felt.

The plank foundation is laid on the natural soil or on a layer of sand and gravel, over which a thin layer of sand or cushion-coat is placed.

The telford foundation is formed by a layer of large stones 6 inches thick; this is well filled with small stones, over which a layer of wet gravel is compacted with a heavy roller; then a 2-inch layer

of small stones covered with wet gravel, well rolled; then a thin layer of sand. On this a course of plank 1 inch thick has been laid.

All of the above foundations are laid on a well-rolled bed of the natural soil.

On the foundations the blocks of wood are placed. The blocks are laid breaking joints; transversely the joints are continuous. They should be rammed to the proper bearing with a rammer weighing 50 to 80 pounds. The sand cushion should be dry when the blocks are laid. The blocks are set on the cushion-coat of sand, asphaltic mastic, or plank, with the fibre vertical, in parallel courses, the length of block transverse to the street-line. The overlap or bond of the blocks should be at least 2 inches. At street intersections the courses are laid diagonally, no joint exceeding $\frac{3}{8}$ inch in width.

The gutters can be formed by three courses of blocks laid parallel to the curb. To allow for expansion, the course nearest the curb is omitted, and the space filled with sand.

With cylindrical blocks the courses are laid in parallel rows across the street, and in close contact. The split blocks are used close to the curb, and around manholes and the like. All blocks should be brought to the uniform and regular street surface by the proper use of sand.

The vertical joints are filled in one of the following ways: Screened gravel is rammed in the joints with steel bars, and the surface covered with a layer of gravel and sand about three fourths of an inch thick. The joints may be simply filled with sand. Bitumen is poured into the joints, which fills any slight interstices under the blocks and seals the joint, and over this is poured a grout made of neat Portland cement. This is repeated until the joints are filled. After this has set a thin layer of dry sand is spread over the surface. The grout is sometimes made of 1 part cement and 2 of sand.

With cylindrical blocks clean screened gravel from one quarter to one half inch in diameter is placed in the joints to the depth of 2 inches. Over this is poured hot paving cement to a depth of 2 inches. Then the joints are filled with gravel, and paving cement poured in until the joints are full. Over the surface is spread a layer of dry sand 1 inch thick. The paving cement may be composed of 1 ton of residuum from the direct distillation of coal tar

and 50 gallons of creosote oil. These are melted together in iron boilers, and should be boiling hot when poured into the joints.

The gravel used must be clean and sharp, free from sand, clay, or other objectionable matter. The rise at centre or crowning 1 in 100.

Special specifications and requirements can be obtained from the engineers of Western and Southern cities. The following is a sample:

805. The joints are sometimes as much as 1½ inches in width. These joints are then filled with clean-dry gravel from ¼ to 1 inch in size, well rammed. After repeated filling and ramming the pavement is flooded with hot composition, using not less than 1½ gallons per square yard. Over this a thin layer of small gravel is spread and swept in the joints until they are thoroughly filled. The whole surface is then covered with hot composition, not less than one half gallon per square yard, and immediately covered with dry roofing gravel, or the gravel screened from that already used, to the depth of 1 inch. All gravel must be lake-shore, entirely free from sand or pebbles over one half inch in size, and dried and heated sufficiently to prevent chilling the composition.

The cylindrical blocks used are of cedar, from 4 to 8 inches in diameter and 6 inches long. These are laid on a foundation of 2-inch hemlock plank, laid on centre and end transverse stringers, firmly bedded in the sand layer of 3 inches in thickness upon which the plank rests. The sand is spread on the natural soil, rammed, and rolled down. The natural bed is graded, thoroughly flooded, rammed, and rolled to make it uniform and solid. The above is taken from specifications for laying cedar-block pavements in Chicago.

ASPHALTUM AND COAL-TAR PAVEMENTS.

806. Some asphaltic mixtures or compounds, either natural or artificial, have been used for a great many years in forming road and street pavements.

Asphaltum, a natural bitumen or mineral pitch, is found either alone or mixed with other substances. It is considered to be a product of the decomposition of vegetable and mineral substances. It is of a brownish-black color, and varies in consistency; it melts a little below the boiling-point of water, and burns with a smoky flame. Deposits are found in many parts of the world, especially in or near tropical regions.

It is found in a state of great purity in the interstices of the older rocks. Its chemical composition varies in different localities. It has the lustre and general appearance of pitch, and is amorphous in structure.

Pure asphaltum is unsuitable for paving purposes on account of its brittleness at low temperatures and its softening at high temperatures. Moreover it has little resistance to wear and tear under traffic.

The main source of supply in the United States is found in the island of Trinidad, W. I., where there is a lake having an extent of more than 100 acres, and of unknown depth. This is known as "lake or live asphalt." This lake is surrounded by deposits found on the land, and known as "overflow or land asphalt or pitch." The former or lake pitch is found to be superior to the latter or land pitch, and it is commonly specified that only the lake pitch is to be used for paving purposes. The natural bitumen is mixed with vegetable and earthy matter. It liquefies when heated over a slow fire, and by this process these substances are removed, the vegetable and lighter impurities rise to the surface, and are skimmed off, while the heavier and earthy substances sink to the bottom. The pure bitumen is then drawn off.

For street paving this is mixed with the residuum of petroleum and with sand. The mixing is done at a temperature of about 300° Fahr., and while hot it is spread upon the foundation and compressed under heavy rollers.

807. Asphalt is the name given to mixtures of pure asphaltum with calcareous or silicious substances. It may be natural or artificial.

Natural asphalt consists of either limestone or sandstone impregnated with pure bitumen, and known as bituminous limestone, bituminous sandstone, rock asphalt, etc.

The artificial asphalt is a mechanical mixture of bitumen, sand, and crushed limestone.

808. *Bituminous Limestones.*—The asphalt used for paving purposes in Europe is obtained from the bituminous limestones found in France, Germany, Sicily, and Switzerland. These limestones contain from 7 to 12 per cent of bitumen. For carriageways in Europe the natural rock is reduced to powder, heated until softened, then spread upon the foundation while hot, and tamped until well compacted. For sidewalks and floors a liquid asphalt is made by heating a mixture of the powdered rock with asphaltum,

and to the heated mixture is added coarse sand or gravel. The proportions are natural stone 60 per cent, natural bitumen 4 per cent, sand or gravel 36 per cent.

In Paris there are two methods of constructing asphalt pavements. In one method, after grading, sprinkling, and rolling the natural ground, it is covered with a layer of cement concrete from 4 to 6 inches in thickness. After this sets, and is well dried, the asphalt is spread and surfaced with a wooden float. This asphalt is made by mixing sufficient bitumen with the natural-rock to make the bitumen from 15 to 18 per cent. This mixture is heated for about six hours in order to secure a uniform product. The thickness of the asphalt is about 1¼ inches, applied in two layers. This covering will not soften at a temperature of 140° Fahr.

In the second method the natural rock is used without the addition of bitumen.

The quarried bituminous limestone is reduced to a powder by means of suitable machinery. It is then put in a roaster. When sufficiently heated, to about 300° Fahr., it is put into a special wagon and carried to the place where it is to be used. It is said that it can be carried in sheet-iron carts with double sides as far as 9½ miles without losing more than 30° or 40° Fahr. The hot material is emptied on the concrete foundation, spread by hot rakes in layers of sufficient thickness to allow for the compression, namely, about 3 inches for a finished coat of 2 inches in thickness. It is first rammed while hot, by means of cast-iron rammers 6 to 8 inches in diameter; this is followed by a roller, heated by an internal furnace, drawn by two men. The final compression is effected by the carriage-wheels. During the rolling a small quantity of hydraulic cement is strewn over the surface. The rolling is continued until the asphalt is cold.

Limestones containing over 10 per cent of bitumen become soft in summer. If they contain much less they will not have sufficient binding power to sustain heavy traffic.

809. In the United States numerous beds of bituminous sandstones are found, namely, in Utah, California, and Kentucky. These have been used in many Western cities. The Gilsonite of Utah is pulverized and mixed with petroleum; this mixture is heated; the temperature must be kept below 500° Fahr. Broken stone or gravel is added to the heated mass. It is then ready for use. About 80 per cent of gravel makes a durable pavement. The bituminous rock of some portions of California is heated and mixed

with sand, the proportions being from 1 of rock to 3 to 8 of sand by bulk. Other varieties are used without mixing sand. The Kentucky sandstones also are used in their native condition. They are crushed to powder and heated to about 250° Fahr. While hot and in a plastic condition they are spread upon the foundation, tamped, and rolled.

810. *Artificial Asphalt.*—The large proportion of asphalt pavements in the United States are made of artificial asphalt, which is a mixture of bitumen, sand, and limestone. The bituminous-limestone pavements become hard, smooth, and slippery under traffic, and are therefore objectionable in a cold and frosty latitude. The sand mixed with the Trinidad asphaltum prevents the forming of a smooth, slippery surface to some extent.

The refined or purified Trinidad asphaltum is very brittle at ordinary temperatures, and is a rather poor cementing substance. To give it the proper qualities, it is mixed while at a temperature of about 325° Fahr. with from 17 to 20 pounds per hundred of the residuum from the distillation of petroleum—a thick, heavy paraffine-oil. It is a substance entirely different from asphaltum, and has a different specific gravity. Thorough agitation is necessary to produce and maintain a uniform product. This mixture is the asphalt cement used in paving.

The wearing surface of pavements is composed of—

Asphalt cement.................	12 to 15 per cent.
Sand...........................	70 to 83 " "
Pulverized carbonate of lime......	5 to 15 " "

The sand should be clean and free from clay. With suitable sand, the limestone may be reduced in quantity or omitted entirely.

The sand and asphaltic cement are heated separately to about 300° Fahr. The powdered limestone, while cold, is mixed with the hot sand, to which is then added the cement in proper proportions. This mixture is spread on the concrete foundation in two layers. The first or cushion coat contains from 2 to 4 per cent more asphaltic cement than given above; it is spread so that the thickness after rolling will be about one half inch. The second or surface coat is made of the proportions given, and is brought to the pavement at a temperature of about 250° Fahr. It is spread on the cushion-coat by means of heated iron rakes, thus forming a uniform and regular layer such that, after compression, the required thickness of 2 or more inches will be obtained. The layer is then rolled, a small quantity of hydraulic cement is scat-

tered over the surface, and the rolling continued until the surface is smooth and firm. One ton of the refined asphaltum makes about 2300 pounds of asphaltic cement, which will yield about 3.4 cubic yards of the sand and limestone compound. One cubic yard of this material weighs about 4500 pounds, and will coat 12 square yards when 2½ inches thick, 18 when 2 inches, and 27 square yards when 1½ inches thick.

This material can be used on grades as high as 4 in 100, though it is not considered advisable to use it on grades steeper than 2.5 in 100.

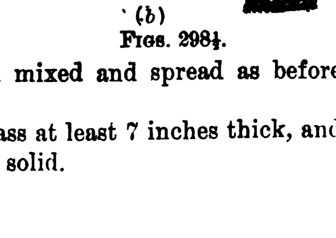
There are many special specifications and conditions which differ from each other to a greater or less extent.

811. Gutters are formed with granite blocks or bricks, and sometimes by spreading a coat of hot pure asphalt, smoothed with hot irons.

In streets having railways along them it is necessary to place a course of granite blocks next the rails, alternating header and stretcher, thus bonding into the pavement. These should be solidly bedded. The asphalt pavement is then laid. Clean, fine gravel, heated, is forced into the joints between the blocks until they are nearly filled. No. 6 paving cement, at a temperature of 300° Fahr., is then poured into the joints, and additional heated gravel then rammed into the joints.

ASPHALT PAVEMENT ON GRAVEL AND CONCRETE BASE, FIG. 298½(*a*).

812. A bituminous concrete base is sometimes used, which is formed with broken stone well rolled in a 4-inch layer, coated with coal-tar paving cement. On this a layer of small stone and coal-tar is placed. The stone is heated to 250° Fahr., thoroughly mixed with the tar, and spread over the first layer to such a depth that after compression the thickness will be about 1½ inches. It must be rammed and rolled while hot. This forms the foundation.



Figs. 298½.

The surface or wearing coat is then mixed and spread as before described. (See Fig. 298½(*b*).)

This pavement must be a solid mass at least 7 inches thick, and thoroughly rolled until it is firm and solid.

It is usual, owing to the great number of inferior asphalts and mixtures that have been used, to require the contractor to keep this kind of pavement in repair for a series of years. This can be included in the contract price per square yard, or the contract can be made for a price covering only the cost of construction and an agreed upon price for maintenance. On either basis the contractor is pretty sure to take care of his own interests.

The time of guaranty varies from 5 to 15 years. The average cost of maintenance is placed at about 10 cents per square yard per annum. In London it is given at from 19 to 25 cents. In Washington, D. C., it is placed as low as 1½ to 2 cents.

The cost of construction of asphalt pavements in the United States is from $2.50 to $4.50 per square yard. In Europe, from $3.00 to $4.50.

813. *Coal-tar Pavements.*—Many coal-tar pavements have been patented and employed in this country. As a rule they have proved unsatisfactory, their failure being due to the presence of volatile oils, which are oxidized on exposure, thereby destroying the binding or cementing qualities of the material.

They are also affected by changes in temperature, become soft in summer and brittle in winter.

These pavements are made of sand, small gravel, and stone dust, cemented by some form of coal-tar.

A mixture of bitumen and coal-tar has been used, and has given more or less satisfactory results.

Vulcanite Asphalt Pavement.—The Filbert vulcanite pavement is constructed as follows:

A foundation is made by using a layer of broken stone 5 inches thick, the largest dimension of the stones being under 3 inches. This layer is spread upon the prepared bed and well compacted with a steam-roller. This is covered with a hot paving-cement, composed of No. 4 tar distillate, about one gallon to the square yard of surface. The second or binder course consists of smaller stones, 1½ inches in greatest dimension, cleaned, screened, heated, and mixed with No. 4 tar distillate in the proportion of one gallon of tar to one cubic foot of stone. This is spread upon the base to the depth of 2 inches, rammed, and rolled while hot. Upon this is laid a wearing surface 1½ inches thick. This contains 25 per cent asphalt and 75 per cent tar, and clean sharp sand and pulverized stone, the sand and stone in the proportion of 2 to 1. To 21 cubic feet of this mixture is added 1 peck hydraulic cement, 1 quart of flour of sulphur, and 2 quarts of air-slaked lime.

To this mixture are added 320 pounds of paving-cement. These materials are heated to about 250° Fahr., thoroughly mixed, and spread on the binder course to a depth of 2 inches. While hot it is rolled with a steam-roller, hydraulic cement being scattered over the surface. The rolling is continued until the surface is cold and firm.

This pavement is cheap, is not so slippery as asphalt, and makes a more durable gutter surface. It is difficult to obtain a uniform quality of coal-tar. Frequent repairs are needed, which are rendered more difficult on account of the entire mass uniting into a more or less homogeneous solid.

The combination asphalt and coal-tar pavements will cost from $2.00 to $2.50, according to quality and proportions.

814. *Asphalt-block Pavements.*—The asphalt blocks are formed of a mixture of crushed limestone, varying from dust to grains of $\frac{1}{4}$ inch, and 10 per cent of asphaltic cement. This is pressed into bricks or blocks $4 \times 5 \times 12$ inches and $6 \times 8 \times 2\frac{1}{2}$ inches in size.

When cool these can be handled like blocks of other material, and are laid and bedded in a similar manner. They are practically water-proof, and are smoother than stone blocks and not so noisy. They wear rapidly under traffic, and are better suited for residence than for business streets.

The natural soil is graded and compacted by rolling. Upon this is placed a layer of bank gravel, containing no pebbles over 1 to $1\frac{1}{2}$ inches in size, of sufficient thickness to give 5 inches depth after rolling. Over the gravel a 2-inch layer of fine sharp sand is spread for a cushion-coat. Upon this the blocks are laid and bedded.

When laid, the surface is covered with clean, fine sand. A plate of iron large enough to cover four blocks is then set, and rammers weighing 45 pounds are raised and dropped on the plate. The ramming is continued until a regular uniform surface with the proper grade and crown is secured, and all blocks are firmly bedded. Fine dry sand is then spread over the surface and swept into the joints. The cost is from $2.00 to $2.50 per square yard.

CARRIAGEWAY PAVEMENTS FOR LARGE CITIES.

815. The following is taken from a paper recently read before the Society of Arts of London, England, by Mr. Lewis H. Isaacs, F.R.I.B.A., Assoc. Inst. C. E., etc., on the subject of street-pavements. These requirements must be satisfied: (1) It must be a

sanitary pavement, and as noiseless as possible. (2) It must be safe for horses, affording sufficient foothold with the minimum of traction. (3) It must be as free from dust and mud as possible. (4) It must be economical, not only as regards first cost, but also with respect to its maintenance and cleansing. (5) It must be durable. (6) It must be easily cleaned, and non-absorbent of moisture. (7) It must admit of being readily taken up and quickly relaid for repairs at all seasons. (8) It must admit of the construction of tramways with it.

816. Great stress is laid on the firmness and solidity of the foundation, which really constitutes the measure of the paving, and can be no better than its foundation, the surface material being only a covering or roofing, the best foundation being concrete, composed of 1 part Portland cement and 6 parts clean, sharp river ballast. Under granite blocks blue lias lime concrete in the proportion of 1 to 5 of lime and ballast would answer; but lime will not do under asphalt on account of its tendency to blow.

The cement should be well burned and slow-setting, weighing at least 114 pounds to the British bushel, and leaving not more than 10 per cent residue on a No. 50 sieve (2500 meshes per square inch).

Next in importance to the foundation is the cross-section of the carriageway, more convexity being required on flat than on sloping surfaces; and also varying with the material, gritstone requiring more camber or centre rise than granite, and wood more than asphalt. He assumed that from 1 in 30 to 1 in 40 would be sufficient.

817. *Paving Materials.*—For light traffic and on grades causing granite to be dangerous gritstone blocks are the best. It is not durable enough under heavy traffic. The blocks must be cut with the grain parallel to its length, otherwise it wears rapidly. The sizes of the blocks vary from 8 to 10 inches in depth, 5 to 7 in width, and 7 to 10 inches in length. The width need not be uniform, as the horse can obtain a foothold on the block itself, and will not slip back to the joint, as on granite. On sharp grades, both with gritstone and granite, the narrower the courses the better. Special foundations are not required with gritstone unless the natural soil is unreliable; then lias lime concrete is employed. Where furnace ashes can be obtained a cushion-coat 1 inch thick is placed on the foundation. The side joints are filled for one half the height with gravel, the upper half filled with boiling pitch and tar.

818. Granite blocks form the most durable and economical pavement. Unless a proper kind of granite is selected they will wear smooth; they become slippery in certain conditions of the atmosphere. In common with most other pavements they require more tractive power than wood or asphalt, and are especially objected to on account of the noise produced. They are non-absorbent, produce little dust or dirt, and are therefore comparatively cleanly.

The sizes of the blocks in general use are 9 × 4, 9 × 3, 7 × 4, 7 × 3, 6 × 4, 6 × 3, or 5 × 3 inches. The cubes, which are only 5 to 6 inches deep, should not be used in leading thoroughfares where the traffic is heavy. The 3-inch cubes are preferred to 4-inch, as giving better foothold. The blocks should be dressed truly square in order to give stability and firmness, and to admit of reversing the blocks in relaying. An exactly uniform top surface is not desirable; a certain amount of roughness prevents too great slipperiness. If the foundation is lime concrete, a grouting of lime and sand is used in the joints; if a cement concrete is used in the foundation, the joint filling of cement and sand grouting should be employed.

819. Gutters are formed of two or more courses of cubes. In very flat streets better channels are made by using blocks of granite 12 to 15 inches wide, and of the same depth as the cubes, as affording less frictional resistance. On streets with steep grades Mr. Isaacs recommends the laying of a second row of these wide granite blocks parallel to and at a distance from the gutter course equal to the average gauge of the wheels. This is in the nature of a stone tramway, requiring a minimum tractive effort, while the granite blocks between give a good foothold.

This would be laid in England on the left going uphill, as the traffic keeps to the left; in this country it would be on the right.

820. *Wood Pavements.*—He states that wood pavements are rapidly growing in favor in London.

The foundation is formed with 6 inches of Portland-cement concrete floated to a good surface, on which are placed 5-inch creosoted yellow-deal blocks. This pavement lasts from seven to ten years. The cost is $1.92 per square yard, or with creosoting under pressure, $2.04 per square yard. Cost of maintenance, 24 cents per annum. The usual precaution of leaving a space between the blocks and curb, to allow for expansion, is required.

Instead of joints from ¼ to ⅜ inch between blocks, it is recom-

mended to lay them close together, sides and ends touching, as insuring longer life.

Wood blocks are said to be unsuitable for use when laid close to tram-rails. It is not advised to use them on grades steeper than $3\frac{1}{2}$ to 100.

The great and almost universal objection is on account of their real or assumed unhealthfulness and offensive odors emitted.

821. Rock-asphalt pavements are highly recommended on account of durability, imperviousness, healthfulness, cleanliness, noiselessness, and readiness with which they can be repaired. Such a pavement in London, after eight years' use, subjected to a traffic of 500,000 tons a year, was only reduced in bulk $\frac{5}{18}$ and in weight $\frac{1}{7}$. A $2\frac{1}{4}$-inch rock-asphalt pavement on a 6-inch concrete foundation costs about $3.48 per square yard, with a two-year guaranty from the contractor.

822. Taking the first cost of wood, granite, and asphalt pavement, and adding the annual cost of maintenance for a period of fifteen years' use, he figures the annual cost per square yard of granite at 18 cents, wood 33 cents, asphalt 42 cents.

823. Macadam roads in London are constructed by laying on the natural earth bed a layer of builders' refuse, clinkers from furnaces, broken pottery, and other hard materials 12 inches in thickness, reduced by rolling to 8 or 9 inches. On this is laid 5 inches of Thames ballast, then three 3-inch courses of macadam stone, reduced by rolling to 6 inches, and over this a thin layer of watered and rolled sand. This road costs $1.44 per square yard; and when under heavy traffic, repairs cost annually $1 per yard.

The comparative merits of different pavements are given in the following table:

TABLE LXIX.

Rank.........	1st.	2d.	3d.
Public hygiene........	Asphalt	Granite	Wood
Noiselessness.........	Wood	Asphalt	Granite
Safety for horses......	Wood	Asphalt	Granite
Cleaning.............	Asphalt	Granite	Wood
Durability............	Granite	Asphalt	Wood
Economy	Granite	Wood	Asphalt
Facility for repairs....	Asphalt	Wood	Granite
Facility for tramways..	Granite	Wood	Asphalt

824. The following are comparative tables of costs, construc-

tion, repairs, and durability of several kinds of pavements, as determined in Minneapolis, Minn.:

Cedar-block paving on plank......71⅝ miles
Asphalt........................ 2½ "
Granite pavement on sand........ 9½ "

Cedar-block pavement on plank costs 85 cents per square yard. It is considered preferable to lay the blocks on a concrete base and to use tarred joints, increasing the cost to $1.50 per square yard.

The granite blocks on good sandy soil are considered satisfactory.

Brick pavements cost from $1.80 to $1.90 per square yard.

TABLE LXX.

TABLE OF RELATIVE COST DURING TWENTY YEARS ON BUSINESS STREETS.

	Cost per Sq. Yard.	Cost of Removal.	Years.	Cost per Year.
Cedar blocks, no tar....................	$0.85	$0.79	6	$0.29
" " on concrete, tarred joints....	1.45	0.74	8	0.30
Granite..............................	1.88	1.88	20	0.25
Asphalt (repairs kept up for 10 years, after which time add 8 cents..........	2.75	....	..	....
Small Minnesota bricks.................	1.80	1.18	10	0.33
Large " "	1.90	1.23	10	0.38
Galesburg bricks.......................	2.15	1.48	10	0.40½
Ohio bricks............................	2.70	2.03	10	0.52

PAVEMENTS FORMED OF BLOCKS.

825. It may be stated that, except on perfectly firm and reliable natural beds, all kinds of block pavements require the construction of artificial foundations, such as sand, gravel, concrete, or some combination of these. This is due both to the relatively high first cost, as well as to the importance of maintaining each block in its proper position. If the foundation-bed is at all yielding or irregular, excessive settlement will take place, even if the bed is of a perfectly uniform material, as the traffic on streets can never be uniformly distributed over the entire surface. In fact it is difficult, if not impossible, to maintain a good road surface with any kind of covering or paving on a yielding and unstable foundation; whereas with a good foundation a fairly good road surface may be maintained even with inferior paving material. Many

natural sandy and gravelly soils are sufficiently firm and uniform to admit of laying the blocks on the properly graded, surfaced, and compacted soil. But as a rule an artificial bed of some kind must be made on the natural soil upon which to lay paving blocks. And in general that kind and quality of foundation which is the firmest will ultimately prove the most economical, regardless of the first cost.

Probably the greatest source of annoyance and cost of maintenance arises from the necessity of tearing up pavements for the purpose of laying new water and gas mains and branch pipes, or of repairing those already existing. This requirement has made it necessary to consider the ease and rapidity of removing and relaying pavements—one of the prime essentials in what are termed the best pavements. A pavement thus patched up can never be brought to as good a condition as in the original construction. The tendency now is to provide underground conduits of sufficient dimensions to carry all necessary pipes, telephone, telegraph, and electric lighting and power wires, thereby avoiding not only the frequent tearing up of pavements, but also removing unsightly and dangerous networks of poles, wires, etc.

Stone-block Pavement.—Several types of stone blocks have been used in street pavements.

826. *Cobblestone Pavements* consist essentially of good-sized river-jacks or small bowlders from 4 to 8 inches in diameter. These are set on a bed of good, sharp sand r fine gravel not less than 10 inches thick, on clay, or other yielding natural soil. This bed should be 18 inches thick. The stones are set with their longest diameter vertical and small end down.

They are rammed with heavy rammers until well settled in the sand bed, and over the surface a layer of sand and gravel is spread, and is worked into the joints by the vehicles and horses. Few roads are now built of this material; but on very steep grades it may even now be considered as the most suitable, notwithstanding its many serious defects.

827. *Belgian-block Pavement,* which consists of more or less cubical-shaped blocks of stone obtained from quarries of hard stone, such as granite, was and is used as a substitute for cobblestone pavements. These pavements are made in the same general manner as for the cobblestone. They offer less resistance to traction, but are rough and noisy.

828. *Granite-block Pavements.*—The form and sizes of Belgian

blocks being ill suited for forming good stable pavements, led to use of blocks of those forms and dimensions which would enable a better and stronger pavement to be made, while at the same time giving a more certain and reliable foothold. The blocks should be as narrow as is consistent with sufficient depth and length for stability and bond, and should be laid on an impervious and unyielding foundation, with the joints between the blocks filled with an impermeable cement. When thus constructed they make the most durable and economical surface for heavy traffic. They are suitable for all grades, durable, yield little dust, require but little cost for maintenance, and give a good foothold. They are exceedingly noisy, and become slippery in certain conditions of the weather,

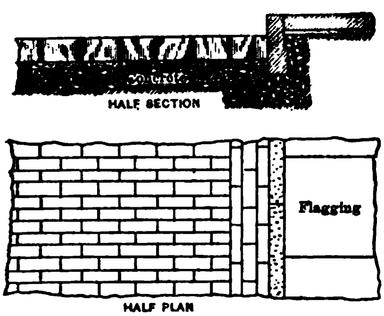
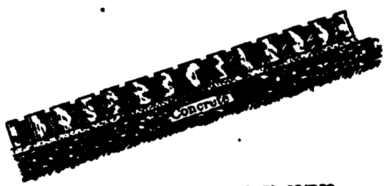


FIG. 299.



GRANITE BLOCKS ON STEEP GRADES

FIG. 300.

especially with blocks of too great width. They affect injuriously the horses, and are wearing on vehicles.

Granite and the harder varieties of sandstones have been found to make the best pavements.

The best dimensions have been found to be: Width, 3 to 4 inches; length, 9 to 12 inches; depth, 7 to 9 inches. They are laid in courses across the street, breaking joints longitudinally. The depth and width of the blocks in each course should be the same.

The blocks should be well squared, not tapering in any direction, with no irregular projections. They are laid with their longest side perpendicular to the curb-line, except at street intersections, where they may be laid at an angle to the curb. The gutters are formed by laying three or more courses adjacent to the curb, with their greatest length parallel to it. See Figs. 299.

As it is impossible to measure each block, and uniformity in dimensions is required, it is usual to state that a certain number of

transverse courses shall not occupy more than a certain length along the street, say four courses to 14 inches. The narrow 3-inch blocks seem to be preferred.

829. *Foundations.*—While a good bed of sand and gravel will make a good foundation, and many granite-block pavements are bedded on such layers, the best practice, especially on business streets with heavy traffic, seems to justify nothing less strong, solid, and impervious than a good cement-concrete base. The thickness of this concrete base should not be less than 4 inches, and need not be more than 9 inches, the average being from 6 to 8 inches. This is laid upon a properly prepared bed. Over the concrete is placed a thin layer of fine, clean, and perfectly dry sand for a cushion-coat. Upon this the blocks are set, and placed as close together as practicable. The blocks should then be well rammed to a firm bearing. The original layer of sand not being over about ¾ inch in thickness, it may be necessary sometimes to remove a block after ramming, and sprinkle enough sand to keep its upper surface at the proper level with respect to the other blocks. The joints are sometimes filled with sand or fine gravel, or with lime or cement paste. These materials will not insure an impervious joint. This can only be accomplished with a bituminous cement, which alse diminishes the noise and strengthens the pavement. This kind of joint is made by first filling the space from one half to two thirds full of gravel, pouring over this hot pitch, and then more gravel and more pitch in succession, until the joint is entirely filled; lastly fine gravel is sprinkled over the joint. This cement is composed of coal-tar, pitch, and creosote oil, in about the proportions of 100 pounds pitch to 4 gallons tar and 1 gallon creosote. It should be poured in while boiling hot. About 4 gallons are required per square yard; cost per gallon, about 8 cents.

Murphy's Grout Filling is composed of Portland cement, iron slag, and sand. It is claimed to be cheap, water-proof, and durable. It is applied practically in the same manner as other filling material. Steel brooms are used to sweep it into the joints.

830. Sandstone blocks are used in some cities. They form a good wearing surface, though not as durable as granite; are less noisy, do not wear slippery, and when of good stone stand any kind of traffic. Limestone blocks are not considered suitable.

831. On very steep grades granite blocks are either set with wide joints open and filled to within an inch of the top with creosoted wood, strips, or a mortar of gravel and cement; or the

blocks may be set inclining somewhat against the grade, forming a series of shallow ridges, thereby securing the desired foothold. See Fig. 300.

832. The wear of granite blocks under heavy traffic will not exceed ¼ inch per year. After wearing down several inches these blocks can be removed and used to pave streets having less and lighter traffic running over them.

The cost of maintenance need not exceed from 2 to 4 cents per square yard per annum.

The cost of granite-block pavement on a foundation of sand or gravel varies from $1.50 to $3.50 per square yard, and from $2.50 to $4.50 on concrete. In Europe the cost varies from $3.00 to $4.00 per square yard, Belgian-block from $1.80 to $2.50, sandstone-block from $1.25 to $3.00, cobblestone from 40 cents to $1.50.

The life of granite-block pavement may be taken at from 15 to 20 years. On the basis of wear of ¼ inch per year blocks 9 inches deep will have worn down to 5 inches in 16 years. If, then, the smoothed upper surface be roughened these blocks could be used on streets with light traffic, thereby prolonging the life 15 to 20 years more. The life of granite pavements, therefore, may be taken at from 15 to 40 years.

In Fig. 301 figures (*a*) and (*b*) are the representations of two

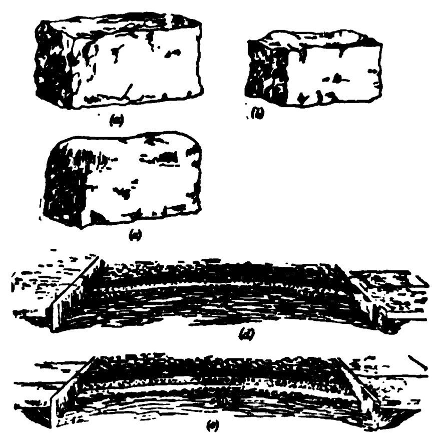


FIG. 301

sizes of granite block when prepared for laying. Figure (*c*) shows the form assumed under traffic. Figure (*d*) shows a section of a street paved with granite blocks on a bed of sand placed on the

natural soil. Figure (*e*) shows a section of a street paved with granite blocks on a concrete foundation, covered with a cushion of sand on which the blocks are laid.

BRICK PAVEMENTS.

833. Pavements made of bricks have been used in some foreign countries for fifty or more years. It is, however, only within the last twenty years that this kind of pavement has been employed in the United States. The first was laid in Charleston, W. Va., in the year 1872. Since that time there has been a great development in the use of brick pavements.

834. There are a great many advantages to be secured by the use of good bricks—relatively small tractive effort required, good foothold suited to all grades, easily cleaned and repaired, practically non-absorbent, not noisy, easily and rapidly laid, and durable under any ordinary traffic.

In many localities, however, they have proved unsatisfactory and costly, but it may be safely stated that where such has been the case it was owing either to weak and imperfect foundations, to soft and porous bricks used in the construction, or to manufacturing the bricks from an inferior quality of clay, particularly if containing lime, which is destructive regardless of the quality of the other ingredients or of the perfect and proper burning. The trouble, however, arises from the employment of inferior grades of brick on account of their cheapness. As in all other kinds of pavement, cheapness in materials and construction will generally result in discomfort, and ultimately in greater cost than if good materials and foundations had been used in the first place.

835. The qualities required are hardness, toughness, imperviousness, and consequently resistance to the wear and injurious effects of exposure to air, moisture, and frost; and it cannot be too strongly impressed that these qualities are of little value if the foundation is yielding and unreliable.

Much attention has been given to the proper composition of the clay from which paving bricks should be made; but it will probably be found, as was the case in the manufacture of iron and steel, that, however interesting and instructive this line of investigation may be, the engineer will have to rely on the qualities of the resulting product rather than upon the exact proportions of the ingredients in the natural mass, except in so far as the presence or

absence of certain ingredients are known to be absolutely harmful or helpful. In a careful and extensive reading upon this subject in books and magazines—and the present amount of literature upon it is very great—the writer finds that, while considerable attention has been given to the questions of resistance to crushing, cross-breaking, chemical composition, resistance to abrasion as determined by grinding and abrading machines, the simple properties, as regards absorption of water both to determine the degree of porosity and the presence of excessive quantities of free lime, together with the uniform hard or vitreous appearance on the faces and on the fractured surfaces, are considered of more importance. The burden of satisfying these conditions and requirements is thrown upon the manufacturer, leaving him free to a great extent to manufacture the bricks of such materials and in such manner as he may deem best.

836. *Clay.*—Pure clay is a hydrated silicate of alumina. Its analysis varies from silica 46.3, alumina 39.8, water 13.9 per cent to silica 97.6, alumina 1.4, water 1.0 per cent, with almost every combination between these limits, and small percentages of several other substances.

The impurities in clay are uncombined silica, lime, magnesia, ferric oxide, potash, and soda. Both pure alumina and silica are infusible. Alumina gives plasticity and adhesiveness to the clay and strength to the product, but it shrinks, warps, and cracks on drying. Silica prevents distortion, cracks, etc.; the more silica the less shrinkage, but the greater brittleness, and the less plasticity and adhesiveness.

Lime and magnesia, while infusible themselves or with alumina, fuse in the presence of silica, as do also several other common impurities in clay, and produce what are called vitrified brick.

Potash has the most active fluxing effect, then soda, lime, magnesia, and iron in the order named.

837. To cause vitrifaction, a clay should contain 3 to 3½ per cent of soda, 5 of lime or magnesia, or 8 per cent of iron, or some equivalent combination. Lesser proportions produce a fire-brick unvitrified and porous. From an economical point of view larger proportions are desirable, as a lower temperature and less fuel are required. As much as three times the above quantities render the clay too fusible.

Finely divided lime and magnesia in small quantities are not especially objectionable, as silicate of lime or magnesia is formed in

the burning. Iron, while not essential, fluxes with silica, and cements and gives strength to the product. Potash and soda fuse at a lower temperature.

The material selected, after being properly prepared, is moulded into the proper sizes and shapes.

838. Except for special purposes, the bricks are rectangular prisms, with opposite sides truly parallel, and adjacent faces at right angles. The angles are often rounded. The dimensions vary according to locality and the requirements of specifications. They do not, however, differ materially in this respect from the ordinary building brick. The sizes are $3 \times 4 \times 9$ inches, $4 \times 5 \times 12$, $2\frac{1}{2} \times 4\frac{1}{2} \times 8\frac{1}{2}$. In Canton, Ohio, there are three grades:

(1) Standard bricks, $2\frac{1}{2} \times 4 \times 8\frac{1}{2}$ inches, weighing 7 pounds each, requiring 58 bricks to the square yard.

(2) Repressed, $2\frac{1}{4} \times 4 \times 8\frac{1}{2}$ inches scant, weighing $6\frac{1}{2}$ pounds each, requiring 61 to the square yard.

(3) Metropolitan, $3 \times 4 \times 9$ inches, weighing $9\frac{1}{2}$ pounds each, and requiring 45 bricks to the square yard.

839. The time occupied in burning will vary considerably, according to circumstances. Burning for water-smoking, from 2 to 4 days; burning proper, from 4 to 6. Cooling, from 3 to 5 days. If the drying is too rapid the bricks will warp and crack; too rapid cooling produces brittle bricks, while slow cooling anneals the bricks and makes them tough.

The bricks are burned in two kinds of kilns. In the up-draught or clamp kiln only about 40 per cent of the bricks are fit for paving, while in the down-draught kilns from 80 to 90 per cent of the bricks are burned sufficiently hard for paving purposes.

There is a difference of opinion as to the extent of burning necessary to make the best brick. Burning to vitrifaction is considered desirable by some engineers, while others maintain that it is neither necessary nor desirable, and that the burning should be stopped just at the point of fusion. The degree of burning is a rather ill-defined point, and the difference is one of degree of temperature rather than of a well-defined change in physical properties, and doubtless the proper condition is one of incipient fusion, which is known and understood as vitrifaction, and should be as nearly uniform throughout the body of the brick as possible. This is best indicated by the absorption test with half bricks, and by the general appearance of the fractured surface.

840. *Quality of Bricks.*—The absorption of a very small per

cent of water is one of the first essentials in good paving-bricks. Bricks approaching fire-clay in composition may absorb from 4 to 5 per cent of water, while others will disintegrate with an absorption of from $2\frac{1}{2}$ to 3 per cent.

Weight is a fair relative measure of the quality of paving-bricks.

Transverse strength is sometimes taken as one of the tests of quality, only in so far as it may be taken as a measure of the resistance to crushing and wear. The test of abrasion is also sometimes made.

Tests for Quality.—(1) For absorption, a whole or a part of a brick is immersed in water after being carefully dried and weighed. It remains in water from 1 to 3 days, when it is taken out, the surface water carefully and quickly removed, and again weighed. The gain in weight is the weight of water absorbed. This gain, divided by the original weight, gives the ratio of absorption. This should not be more than about 2 per cent; as much as 4 per cent is sometimes allowed, and in other instances not more than 0.5 per cent.

The modulus of rupture for transverse strength is taken at 1600 pounds per square inch, as determined by the usual formula,

$$f = \frac{3}{2}\frac{Wl}{bd^2}.$$

Resistance to crushing is taken at from 8000 to 12,000 pounds per square inch.

Lime Test.—Bricks after immersion in water for 10 days should show no signs of blowing.

Abrasion Test, or test for toughness, is made in an ordinary foundry rumbler, with about 100 pounds of foundry shot. The wear should not amount to more than $\frac{1}{4}$ pound per brick, several pieces being tested at a time. The Government test is to grind two bricks of each class on a horizontal stone 14 feet in diameter, making 28 revolutions per minute, or place two bricks of each class in a rumbler, for half an hour, making 28 revolutions per minute, The percentage of loss is determined by weights before and after treatment. According to a number of experiments, after 8 hours' grinding the loss was found to vary from 0.102 to 0.364 per cent.

The weight of bricks varies from $5\frac{5}{8}$ to $7\frac{1}{4}$ pounds.

The following table gives the effect of different temperatures on the weight and dimensions:

TABLE LXXI.

	On leaving Mill.	On leaving Drier.	Burned, not Vitrified.	Vitrified.
Weight	6.46 lbs.	5.79 lbs.	5.08 lbs.	5.08 lbs.
Length	$8\frac{1}{2}$ in.	8.0 in.	$7\frac{7}{8}$ in.	$7\frac{3}{4}$ in.
Breadth...........	$4\frac{7}{16}$ "	$4\frac{3}{16}$ "	$3\frac{7}{8}$ "	$3\frac{3}{4}$ "
Thickness	$2\frac{1}{2}$ "	$2\frac{1}{4}$ "	$2\frac{1}{8}$ "	2 "

As was stated, the main reliance seems to be on the tests for absorption and the presence of lime, though the right to apply any of the other tests is reserved. The general requirements are that the bricks shall be well burned, hard, tough, sound, and free from clefts or cracks, or other defects; square and straight on the edges; when broken the fracture shall be smooth, uniform in texture, straight, not conchoidal, nor granular; and such other specific requirements as may be demanded.

841. *Foundations.*—The same considerations as repeatedly given require a solid, firm, and impervious base for brick pavements, and it is due to defective foundations to a great extent that such unfavorable reports have been made on the durability and stability of brick pavements.

In all cases the natural soil should be graded and surfaced, and well consolidated with heavy rollers. Upon this a foundation is laid, the character of which varies, but may be classified under two kinds: (1) a layer of sand and gravel, or similar materials; (2) a layer of concrete.

Out of a list of forty-four cities, the writer has classified the character of the prepared foundations, as nearly as they admitted of it, as follows:

Fourteen have layers of sand, on broken stone or gravel, from 1 to 4 inches in thickness, with a subcourse of brick laid flatwise, or from $2\frac{1}{2}$ to 6 inches of sand with a course of brick laid flatwise, and some on from 4 to 6 inches of sand alone.

Twenty have beds of broken stone, gravel, or cinders, from 6 to 8 inches in thickness.

Ten have foundations of from 6 to 9 inches of cement concrete.

These foundations were thoroughly consolidated by rolling or ramming.

Over these a cushion-coat of sand, fine gravel, fine broken stone or concrete made with fine broken stone, from 1 to 2 inches in thickness, was spread, upon which the bricks were laid, rolled, or rammed.

842. One or more examples of actual construction under special specifications will be interesting and instructive.

Richmond, Ind.—Foundations either of broken stone or concrete. The natural soil excavated to level of required subgrade, this surface, after filling all soft or spongy places with slag, broken stone, or brick, thoroughly compacted with a 10-ton roller. (1) Broken-stone foundation: Stone to pass $2\frac{1}{2}$-inch ring, covered with coarse sand or gravel, then rolled; thickness after rolling, 8 inches. (2) Concrete foundation: Concrete composed of broken stone, in sizes from $\frac{3}{4}$ inch to $2\frac{1}{2}$ inches. Hydraulic cement used to show a tensile strength of 60 pounds after 24 hours, 1 hour in air, 23 hours in water. Thickness of concrete bed not given, presumably 8 inches. Curbing of limestone in blocks 3 feet long, top width 5 inches, crandall-dressed to depth of 3 inches on back and 12 inches on front face.

On either of these bases was placed a cushion-coat of 2 inches of sand. Upon this bricks were set on edge, length transverse to axis of street, overlap or bond 3 inches. Paving rammed twice with double rammer weighing 80 pounds; blows delivered on a drag 14 inches square.

Joints filled with paving-pitch of Trinidad asphaltum and coal-tar cement and still-wax. Surface covered with $\frac{1}{2}$ inch clean sand.

Bricks containing lime must not "blow" after soaking in water for 10 days; must not gain more than 2 per cent in weight when immersed in water for 24 hours.

Modulus of transverse strength 1600 pounds per square inch.

Chester, Pa.—Subgrade rolled with 10-ton roller to crowning of $\frac{3}{8}$ inch to 1 foot, covered then with two $3\frac{1}{2}$-inch layers of fine gravel, rolled separately. Cushion, 2 inches of bar sand. Joints filled with grout composed of 1 Portland cement and 3 sand. Surface covered with $\frac{1}{2}$ inch of wet sand.

Grand Rapids, Mich.—Foundation of concrete on 6 inches of broken stone. Cushion of sand, or the broken stone covered with 4 inches of sand instead of concrete.

Cincinnati, Ohio.—Subgrade rolled with steam-roller weighing 200 pounds per linear inch. Foundation of concrete, 6 inches thick. Cushion-coat on concrete 2 inches thick after rolling. Bricks from 8 to 10 inches long, 4 to 5 wide, and 3 to 4 thick.

These bricks are indented both top and bottom. On top indents cover at least 10 per cent of surface and $\frac{3}{8}$ inch deep. On the bottom openings cover not more than half the surface, and must not

reach nearer the top surface than 2½ inches. This is filled with rough mortar, composed of 1 cement and 4 coarse sand.

The bricks must not absorb more than 2 per cent of water in 72 hours. Bricks laid in courses transversely to axis of street, course breaking joints or overlapping 3 inches. Rammed two or more times with 40-pound rammers, using a flatter. Joints filled with No. 6 paving cement.

McKeesport, Pa.—Subgrade rolled with a roller weighing 1 ton per foot. Foundation of 5 inches of coarse sand and gravel, well rolled and covered with proper thickness of sharp sand to bring surface to proper elevation. Bricks, $9 \times 5 \times 2\frac{1}{2}$ inches, or $10 \times 6 \times 3$ inches. Joints filled with hot pitch and fine sand.

Bloomington, Ill.—The road-bed to be carefully graded and shaped to an elevation of at least 11 inches below the established grade surface. After grading and shaping, it is thoroughly rolled and compacted with a steam-roller. Upon this bed is placed a 3-inch layer of cinders and properly rolled, then there is spread a layer of sand of sufficient thickness to grade the surface to a uniform shape, regular and smooth surface, for receiving the bottom course of brick.

Upon this surface a course of brick is laid upon their flat or broad surface, with their lengths parallel to the axis of the street, laid as close together as practicable, and breaking joints in all directions. Dry and screened sand is then spread over this course of brick, and swept in so as to fill all joints. Upon this is spread a cushion-coat 1 inch thick of screened sand. Upon this the top layer or paving course of brick is laid. This is laid in courses across the street, breaking joints parallel to the axis of the street, and as close together as practicable.

The bottom course of brick must be of good quality, known as "sidewalk brick." The top course must constitute a good quality of "paving-brick," maintaining uniformity and regularity in shape to such a degree as will be consistent with a first-class pavement, and render satisfaction to the engineer in charge. The top surface is covered with screened sand, and rolled with a roller weighing at least two tons, the sand being continually brushed into the joints as the rolling proceeds.

Charleston, W. Va.—On the graded surface is placed a layer of clean, coarse sand, 3 inches thick, then a layer of oak boards 1 inch thick, dipped in hot coal-tar; on this a layer of clean sand 1½ inches thick; on this cushion is laid on edge a layer of common red bricks,

herring-bone fashion, and covered with dry, clean sand, and sweep this sand in the joints until filled.

Columbus, Ohio.—Foundations are of either broken stone or concrete. Broken stone not larger than 2½ inches in size is placed on a prepared subgrade. Sufficient sand is used to fill interstices, when the mass has been thoroughly compacted by rolling to a thickness not less than 8 inches. If more material is required the rolled surface is loosened to the depth of 2 inches to receive the new material. When concrete is used, the layer is to be 6 inches thick, composed of 1 cement, 2 sand, mixed dry, then made into mortar with as little water as possible. Broken stone not over 2½ inches is used, so as to have an excess of mortar. Bricks, hard-burned, uniform in size, free from cracks, flaws or breaks.

The bricks are set on edge on a 2-inch sand-bed, breaking joints at least 2 inches, rammed with paving-rammer weighing 75 pounds. Joints filled with No. 6 coal-tar paving-cement, or Murphy's grout filling. The surface is then covered with a half-inch dressing of clean, coarse sand. No test required for quality other than approval by those in charge.

Chattanooga, Tenn.—Laid on cement concrete foundations cost $2.18 to $2.35 per square yard. Size of bricks, $2\frac{1}{4}'' \times 4'' \times 8\frac{1}{2}''$, corners slightly rounded, used on grades as steep as 8½ per cent, somewhat excessive; crowning 7″ for 36′ roadway. Subgrade excavated and then consolidated with rollers weighing 5 tons, or with rammers. This is 11 inches below finished surface. On this 6 inches concrete, this coated with a thin layer of cement to fill little cavities and bring surface to true gauge. On this 1 inch sand, then bricks; bricks hard, burnt almost to melting-point. No physical tests required. Laid on edge at right angles to direction of street, except at crossings, where they are laid at an angle of 45°, covered with coal-tar paving-pitch, over which dry river-sand is sprinkled.

For heavy traffic, best bricks on concrete foundation and interstices filled with a composition of not less than 10 per cent of Trinidad asphalt, No. 6 coal-tar paving-pitch, and a proper proportion of still-wax. For lighter traffic the joints can be flushed with paving-pitch alone, or a grout of 2 Portland and 1 natural cement, and 3 sand. This costs about 10 cents per square yard. Sprinkle pavement just before applying grout, then a coating of fine sand. Close for traffic several days to let cement harden. A sand-filler is only used for light traffic. Cost with pitch filling $1.85, with

grout filling $1.78, per square yard, including excavation and curbing.

The Bucyrus pavement is prepared by sprinkling and rolling subgrade with a 10-ton roller. When concrete is not used, either broken sandstone or limestone, or two 3-inch courses of clean gravel, are used. Size of bricks, 8″ × 4″ × 2½″ to 9″ × 4½″ × 3″.

Sometimes a bed of unscreened gravel or crushed sandstone or limestone is spread on subgrade and rolled to 4 inches thickness; on this a 2-inch layer of sand, on which is laid common hard-burned brick close together, with length parallel to direction of street; on this 1½ inches sand, on which is placed the vitrified brick. Fine screened sand is then cast over the surface, then rolled twice with a 5-ton roller, and afterward with an 8-ton roller until smooth. Sandstone curbing, 4′ × 5″ × 20″.

Louisville.—Concrete foundation, 6 inches thick; 90 per cent of cement to pass a sieve of 2500 meshes, and cement must have tensile strength of 50 pounds after twenty-four hours in air. A 3-inch layer of sand is spread for the brick. Absorption test alone required. A half brick thoroughly dried must not absorb as much as 4 per cent in weight after seventy-two hours of immersion in water. The paving is covered with sand swept into the joints, surface rammed, and joints filled with Nos. 5 or 6 paver's cement; the surface then covered with a ½-inch layer of sand.

Parkersburg.—Excavation to subgrade, 14 inches. Earth consolidated with a roller weighing five tons. Then two layers, of 4½ inches in thickness, of disintegrated-rock sand. Roll each layer with a 5-ton roller. A cushion-coat, 2 inches in thickness, of river-sand is used, on which the bricks are laid. Portland-cement grout is poured into the joints. The surface is then covered with sand, which is swept into joints; finally, a ½-inch finishing-coat of sand is placed.

Paver's pitch or Murphy's grout is used. Rollers weigh from 250 to 300 pounds per linear inch, or a 10-ton roller. Bricks varying in specific gravity from 2.2 to 2.4; absorption from 2 per cent to 0.46 per cent.

Resistance to crushing, 12,000 pounds per square inch. A concrete composed of 175 pounds cement, 5 cubic feet of bank sand, and 12½ cubic feet of broken stone or slag will cover 3½ square yards of base 6 inches in thickness.

Canton, Ohio.—Excavation to a depth 14 inches below paved surface; bed surfaced and rolled. Foundation of crushed stone, flushed with sand and rolled to 8 inches thickness with 10-ton

roller. Or a concrete base 8 inches thick is used, cushion-coat of sand 1 inch in thickness, and rolled with 5-ton roller; joints filled with paver's pitch or sand. Finally, a finishing-coat of sand 1 inch in thickness is spread.

The following table gives the itemized cost of three grades of bricks.

TABLE LXXII.

Cost of—	Standard Brick, per sq. yd.	Repressed Brick, per sq. yd.	Metropolitan Brick, per sq. yd.
Grading	$0.12	$0.12	$0.12
Rolling and surfacing	0.03	0.03	0.03
Crushed stone	0.40	0.40	0.40
Sand and rolling	0.05	0.05	0.05
Sand cushion	0.05	0.05	0.05
Bricks	0.58	0.67	0.61
Laying	0.07	0.07	0.07
Paver's pitch	0.20	0.20	0.20
Rolling and sand	0.02	0.02	0.02
Contractor's profit	0.15	0.16	0.16
Total cost per square yard	$1.67	$1.77	$1.71

Out of a list of 47 cases the joint filling was as follows: 12 have tar filling, 5 cement grout, 27 sand, 2 tar and sand, 1 tar and grout.

Cost varied from $1.30 to $2.75 per square yard. Average, $1.80 per square yard.

843. Pavements are thus destroyed: (1) Crushing by wheel weight; (2) abrasion by friction of wheels and slipping of horses' feet; (3) impact of passing weights and from shoes of horses. Estimated life under light traffic, from 35 to 40 years; medium, 20 to 25 years; heavy, 10 to 15 years.

For Light Traffic.—Broken stone, sand, and gravel; sand with a layer of bricks on flat sides; or 6 inches of concrete, will make a good foundation.

For Medium Traffic.—Broken stone or gravel 9 inches thick; 6 inches stone or gravel, with bricks on flat sides, bedded in sand; or 6 inches of concrete.

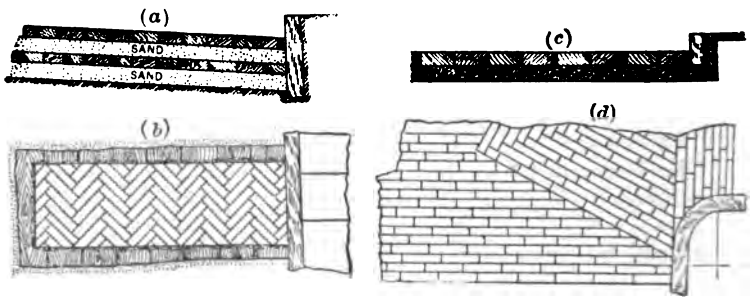
For Heavy Traffic.—Not less than 12 inches of broken stone or gravel, or 9 inches of concrete, have proved satisfactory.

All soils retentive of water must be thoroughly drained. Cushion-coats of sand, clean, sharp, and dry.

In streets occupied by railway tracks, stone paving blocks, laid header and stretcher on concrete foundations to depth of cross-ties, filled in between paving and rail with Portland-cement grout. With girder rails, the space between head and base of rail filled with white-pine strips, to give good bearing for pavement.

The centre rise or crowning of brick pavements varies in different cities, but may be taken at 1 in 20.

In Figs. 302, figure (*a*) shows section of brick pavement with plank and sand foundation; (*b*), plan of same; (*c*), brick on concrete foundation. Figure (*d*) shows plan of brick pavement at street intersection.



FIGS. 302 (*a*), (*b*), (*c*), (*d*).

844. The following abstract from the report of Messrs. Hering and Rosewater on the pavements, sewers, and drainage of the city of Duluth, Minn., will be interesting and instructive (for report on Sewers, see Art. XLVII, paragraph 754):

The difficulties of street paving and drainage arise especially from the very steep grades found in this city.

It was recommended that the proper prevention for washouts on steep grades was the use of the best permanent pavements, or by reducing the quantity of storm-water running down the avenues. To accomplish the latter purpose, the construction of catch-basins, and direct conveyance of the water from these to the natural watercourses is advised.

"The sectional forms of streets, avenues, and alleys at intersecting points are more or less dependent upon the grades and upon the relative proximity to the business or residence sections of the city. At the intersection of avenues with streets in the business portion of the city the limits of gradients for crossings must be less than in the residence portion. In both cases certain pre-

scribed limits of form are essential to the requirements of traffic and drainage.

"Upon all streets and avenues in the city, except in the business section, the curb grades should be unbroken between intersecting curbs.

"In the business section of the city the lines of curb grades, when exceeding 8 per cent, should be broken at the street or property line.

"All street-crossings, from the curb on one side to the curb on the other, irrespective of location, should be designed on the basis of a 23-per-cent slope, except when the slope of intersecting streets or avenues is less, in which case the grades shall be continued unbroken over the crossings.

"In determining the height of the block-corners the elevation of the curbs at the two opposite points should be added to the rise of the two walks from their respective curbs to the corner, on the basis of 2½ per cent inclinations, which sum divided by 2 will give the desired elevation. In other words, the block-corner shall have an average of the elevations determined from the two opposite curbs, after allowing the usual rise in the walks of ¼ inch per foot, across their width.

"Upon steep grades, to facilitate the entry of vehicles to the alleys, made difficult by the side slope of the avenues, it will be best to reduce the alley width across the sidewalks from 20 feet to 10 feet. After allowing for 1 foot slope across this latter width, increase the grade of a portion of the curb and walk on each side of the alley opening, and if necessary provide steps, otherwise the entry to the alleys will be hazardous."

"Upon steep avenues already graded and paved the accumulation flow of water at the gutter-lines is so great that stones ranging in diameter from 8 inches down to ordinary gravel move toward the inlet with such velocity that only the largest of these stones are successfully kept out of the sewers by the intercepting inlet-bars. To prevent this there are two remedies feasible. One of these is to cover the gutters along their entire lines with wooden gratings, as shown in Fig. 295(*a*). The cost of such coverings will be from 15 to 25 cents per linear foot.

"The other remedy is to reduce the depth of volume and velocity of flow on the surface by eliminating the crown and gutters of the avenue, and making them practically flat. The first remedy is suggested for those streets already paved or curbed and

guttered, and the other for avenues not yet improved, and in connection with catch-basins.

"Where streets or avenues have inclinations exceeding 16 per cent they are practically useless as thoroughfares." The lots in the centre of blocks can be reached by building a wall across the avenue, cutting down on one face and filling up behind the other, so that practicable grades can be made on each side of this dividing wall. The rear of the lots on one side can be reached from one direction, and those on the other from the opposite direction. Steps are provided to descend from the top of the wall to the lower level for foot-passengers. This plan is recommended on streets having from 16 to 32 per cent natural grades. Also, the width of the roadway should be reduced to 30 feet, and the sidewalks widened proportionally, with a saving of 28 per cent in the cost.

After impressing the importance of a solid and unyielding foundation, whatever kind of paving material is used, the report says:

"The selection of economical paving material should be based upon a due consideration of the bed and gradients of the streets or avenues to be paved, and of the available facilities for efficient drainage. With clay or rock beds and light gradients, either sand, gravel, broken stone, or concrete can be used with reasonable assurance of good results." The order of preference for foundations is: 1st, concrete; 2d, broken stone, forming a bed of compact layers; 3d, gravel; and 4th and last, sand. On steep grades foundations of concrete or broken stone should be used; on low or filled-in ground broken stone or gravel will do well. Sand should only be used as a foundation upon the more level roads having a compact natural bed. Where there is danger of moisture penetrating the road-bed, a line of blind drains should be laid below the curbs, discharging into the corner inlets.

To prevent the foundation from sliding on steep gradients, they should be anchored at suitable intervals by concrete ribs or bars, of from 6 to 12 inches in depth below the concrete base, placed across the avenue.

"The character of the wearing surface or paving material to be selected, and the method of its preparation, depends largely upon the price of the material available for such purposes, its adaptability to the gradients of the surface to be paved, its relative durability, its liability to derangement after being laid, the cost of maintenance, and upon the liability to interfere with the drainage. To meet the requirements of travel a paved surface must be favorable to the easy movement of horses and vehicles. It must be able

to resist the disintegrating effects of the elements for a reasonable number of years; for sanitary reasons it must be capable of ready cleansing, and, lastly, it should admit of being easily repaired at reasonable cost.

"To be favorable to the easy movement of horses and vehicles the pavement must afford a secure foothold to the horse, must admit of such rapid transit as the gradients of the roadway will allow, with the minimum jarring of the wheels, and in connection with the most effective foothold, must develop the least resistance to traction. In other words, for light vehicle traffic the pavement must admit of the best attainable speed. It must be as noiseless as possible, and produce the minimum amount of jarring to the vehicles."

For the gradients of the streets of Duluth, reaching, as they do, 32 per cent, the following pavements are recommended within the limits named:

Stone.—On grades of 10 per cent or less:

Rectangular stone blocks of the usual size and pattern, with sand joints, to be laid on a foundation of concrete or broken stone. Cost, per square yard, \$2.75 to \$3.25.

On grades exceeding 10 per cent:

(1) Rectangular stone blocks, ranging from 3½ to 4 inches in width, and chamfered on the upper edges, as shown in Fig. 303(*a*), in order to secure good foothold for horses. Cost, if laid on broken stone, per square yard, \$2.75 to \$3.25; if laid on concrete, per square yard, \$3 to \$3.50.

(2) Kidney or cobble stones (see Fig. 303(*b*)), selected of nearly equal sizes, not exceeding 3 inches in thickness, laid on a concrete foundation, and set in cement mortar. Cost per square yard, \$2 to \$2.50.

Brick.—On grades up to 6 per cent:

Best paving-brick, laid on concrete. Cost, per square yard, \$2.25 to \$2.75.

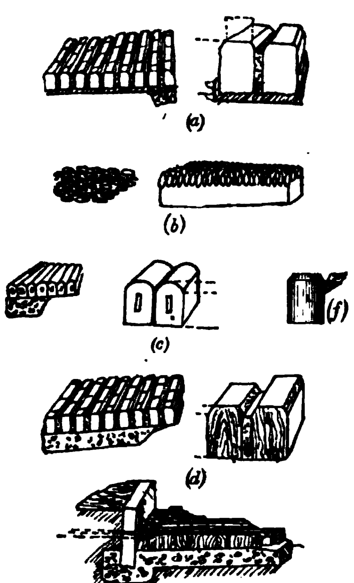


FIG. 303.

On grades exceeding 6 per cent:

Best paving-brick of the special form shown in Fig. 303(*c*), laid on a concrete foundation. Cost per square yard, \$2.50 to \$3.

Sheet Asphaltum.—On grades less than 4 per cent:

Laid on concrete foundation. Cost per square yard and maintenance for five years, $2.25 to $3; or, if maintained for 10 years, $3 to $3.50.

Wood.—On grades less than 6 per cent:

Ordinary round cedar blocks: laid on concrete. Cost per square yard, $1.80 to $2; laid on broken stone and boards, $1.50 to $1.60.

Rectangular pine blocks of usual form and size, laid on concrete foundation. Cost per square yard, $2.25 to $2.40.

On grades above 6 per cent:

Round cedar blocks, from 4 to 5 inches in diameter, and chamfered to a diameter of 3 inches at top (see Fig. 303(*f*)), laid on concrete. Cost per square yard, $2 to $2.25.

Rectangular pine blocks with chamfered edges (see Figs. 303 (*d*), (*e*)), or plain with a width of 3 or 3½ inches, and set with 1 to 1½-inch joints, both laid on concrete. Cost per square yard, $2.40 to $2.60.

In order to realize the advantages of the chamfered blocks and wide joints for the purpose of affording a foothold on steep grades, it is important that the pavements be frequently swept and the grooves between the blocks be kept open.

While macadam and telford pavements possess unexcelled merits as roadways, the expense of maintenance places them, together with gravel-surfaced roads, in the list of temporary pavements. In view of the constant tendency of their component parts to wash into the gutters and inlets, their use is not advisable for steep grades. However, if the upper layer of either of these pavements is treated with coal-tar, binding the parts together, it is possible that they will meet satisfactorily all the demands of a temporary pavement for gradients up to 16 per cent.

For grades exceeding 10 per cent the following are suggested:

(1) Rectangular stone blocks with chamfered edges; (2) kidney or cobble stones; (3) best hard and tough paving-brick, of special form; (4) cedar blocks with chamfered edges; (5) pine blocks with chamfered edges or wide joints. The chamfered stone blocks considered best, but most expensive; the pine blocks the least desirable, because the least durable.

845. The following costs are taken from an extended list during the years 1889 to 1892, both inclusive. The table gives the least and greatest, and also the average.

Table LXXII (A).

TABLE OF COST OF STREET PAVEMENT AND SEWERS.

Class of Work.	Min.	Max.	Average.	Min.	Max.
Earth excavation, cu. yds.	$0.10	$0.60	$0.32	$0.18	$0.30
Rock " "	0.50	5.00	1.52	0.40	1.50
Borrow " "	0.20	0.61	0.35½		
Gravel and rolling, "	0.40	1.25	0.91½		
Loam, cu. yds.	0.40	1.50	0.63		
Telford macadam, sq. yds.	0.65	1.50	0.75½		
Cedar block on concrete, sq. yds.	1.47	1.80	1.68½		
" " " sand, "	0.88	1.25	1.04½		
Rough stone paving, gutters, sq. yds.	0.50	1.28	0.92	0.48	0.60
" " " in cement, sq. yds.	1.20	2.50	1.32		
Pine timber in place, per 1000 ft.	15.00	25.00	18.36	15.00	20.00
Oak " " " "	31.00	45.00	39.00		
Piles in place, per lin. ft.	0.16½	0.41	0.25½	0.20	0.50
Granite curb, per lin. ft.	0.88½	1.40	1.22½		
Sandstone curb, per lin. ft.	0.60	1.00	0.77		
Tar, per gallon	0.10	0.14	0.12		
Removing and relaying sidewalks, lin. ft.	0.04	0.20	0.10		
Cast iron, per pound	0.04	0.25	0.04½		
Earth excavation in sewers, cu. yds.	0.20	1.00	0.56		
Rock " " " "	0.50	12.50	3.61	1.20	3.00
24-in. pipe in concrete, lin. ft.	1.25	3.00	2.47	1.57	2.00
21-in. " " " "	1.00	4.00	2.46		
20-in. sewer-pipe in concrete, lin. ft.	1.00	2.50	1.94	1.35	1.80
18-in. " " " " "	0.85	2.00	1.65	1.20	2.00
15-in. " " in sand, lin. ft.	0.30	1.10	0.66	0.79	1.85
12-in. " " " " "	0.20	0.75	0.41	0.65	1.25
10-in. " " " " "	0.16	0.75	0.38	0.55	0.70
9-in. " " " " "	0.16	0.62	0.33½		
8-in. " " " " "	0.15	0.70	0.31	0.40	0.68
6-in. " " " " "	0.10	0.25	0.19	0.25	0.41
Sewer-brick, per 1000	17.00	30.00	19.60	7.00	18.00
Cut-stone masonry, cu. yds.	13.00	25.00	21.12		
Common rubble-masonry, cu. yds.	0.80	7.50	4.17½	3.00	7.50
Portland cement rubble-masonry, cu. yds.	6.00	7.50	5.93½		
Pene-hammered work, sq. ft.	0.10	1.00	0.56½		
No. 1 intakes	30.00	60.00	45.09		
" 2 "	20.00	60.00	39.23		
" 3 "	40.00	50.00	45.00		
" 5 "	30.00	70.00	51.66		
" 3 catch-basins	40.00	80.00	60.00	75.00	155.00
Manholes	25.00	95.00	52.48	25.00	30.00
Lampholes	1.00	30.00	13.37		
Inlets, ½ bends	0.25	3.00	1.23		
Portland cement concrete, cu. yds.	10.00	10.00	10.00		
American " " "	7.00	7.00	7.00		
Lampholes, lin. ft.				0.15	0.40
Flush-tanks				55.00	160.00
Bluestone flagging, sq. ft.				0.13½	0.15
Crosswalk, lin. ft.				0.80	1.25
Cobble gutter, sq. ft.				0.03	0.13
20-inch bluestone curb				0.25	0.48
Cost of brick pavement on concrete per sq. yd.				1.37	1.93
" " asphalt " " " " " "				2.25	2.80

Various constructions of macadam and telford pavements from \$0.58½ to \$0.91 per sq. yd.

845½. *Creosoting Plant.*—The growing demand for creosoted timber for many purposes, such as piles, bridge timbers, paving-blocks for streets, etc., has led to the construction of many plants. The following is a brief description of the plant for the Wharf Company, Galveston, Texas:

The treating cylinder is made of steel having a tensile strength of 60,000 pounds per square inch. It is 9 feet in diameter and 100 feet long, exclusive of hemispherical doors at the ends. The following method applies especially to the treatment for paving-blocks. The cage-cars, each of which holds 5000 feet B.M. in blocks, six in number, are coupled together and run into the cylinder on a track the level of which is the same inside and outside of the cylinder, the connection between the two being made by movable beams. The inside track consists of 40-pound rails supported on brackets fastened to the shell of the cylinder. The six cars containing 30,000 feet B.M. of blocks are run into the cylinder, the movable stringers removed, and the end doors closed and bolted. The doors fit into an annular groove in which a gasket of heavy belting is fitted in order to make a tight joint.

The timber is seasoned by passing steam superheated to 700° Fahr. through 6000 linear feet of 1-inch heating-coils inside of the cylinder, while a vacuum of 15 to 20 inches is maintained outside of the coils. After seasoning the creosote-oil is admitted until the cylinder is filled. This is accomplished by means of a 15-inch pipe leading from an elevated storage-tank, the capacity of which is 71,500 gallons. This pipe leads into a 12-inch branch which delivers the oil to the cylinder through four 8-inch valves in its bottom. After thus filling the cylinder, which requires six minutes, the valves are closed. An additional quantity of oil is then forced in by means of a pressure-pump. To determine this additional quantity, the free or unoccupied space in the cylinder is found by subtracting from the interior capacity the volume of displacement of blocks and cage-cars reduced to gallons. To this space add the number of gallons to be forced into the blocks on a basis of 10 gallons per cubic foot of blocks, which gives the total number of gallons to be delivered to the cylinder. This is indicated by a glance at a gauge connected with the storage-tanks. The pumps are then stopped, the oil withdrawn from the cylinder, and pumped into the storage-tank to be used again, the train of cars is run out, and

another train run in. The daily output is 60,000 feet B.M. in blocks.

These blocks are 5×6 inches in section, 6, 8, or 10 inches long, of heart yellow pine. A guaranty of 10 years' wear is given.

TRAMWAYS.

846. Tramways, as usually understood, consist of two parallel lines of broad stones placed at a distance apart equal to width between the wheels of vehicles. These wheelways were placed either in the middle of the street or on one or both sides of the middle, the obvious advantage being a reduction of tractive effort while giving a good foothold for horses. The friction of the wheelway surface is only about $\frac{1}{130}$ of the load, or about one half that of the best block pavement. The rest of the surface is covered or paved in any of the ways already described.

Such tramways, however advantageous in many respects, have practically fallen into disuse in this country. They can, however, be advantageously used on steep grades, as described in paragraph 819.

It is not necessary or desirable that the surfaces of the tramway stones should be perfectly smoothed on top. They are expensive to construct.

846½. The nearest approach to tramways now used to any great extent is the street-car tracks. Street-car tracks are constructed commonly with rails having a flat surface wide enough to carry easily the wheel-tires of vehicles, with a raised head at one side to guide and support the car-wheels.

These rails are commonly spiked or bolted to continuous longitudinal timbers placed under them. These timbers are bedded either in a layer of concrete or broken stone, or cinder or ballast. To preserve the gauge of the track, cross-ties are framed into the longitudinal timbers, usually near the bottom; also transverse tension rods are used to tie the rods together The usual and desired pavements are laid. The methods of uniting these to the tracks have already been explained.

Such construction was required with the slow-moving, easily-stopped horse-cars, and were constantly used by ordinary vehicles. With the rapidly moving cable-cars and cars run by electricity it is not safe for ordinary vehicles to use the same track. The two methods of carrying traffic seriously conflict with each other,

resulting in delays and inconvenience, besides the danger to life incurred; and the most important question of street improvement and use is to devise some means of removing all so-called street-cars from the surface, either by raising their tracks well above the streets, or on elevated railroads, as they are called, or by transferring them to tunnels constructed under the streets.

847. Using the ordinary formed or girder rail, street-car tracks, whatever may be the motive power, can be constructed similarly to railway tracks, that is, with rails supported on cross-ties imbedded in ballast of any kind. The tram-rail has not sufficient stiffness, and must be supported by a continuous timber.

848. The ordinary cable road requires a special construction owing to the character of the motive power and the method of communicating motion to the cars. The power is exerted through an endless cable kept in constant motion by powerful steam-engines located permanently at one end of the line. This cable is placed in an underground conduit and supported at intervals on pulleys or rollers.

849. *Cross-section of Conduit.*—There are several types of conduit, both as regards the figures of the cross-section and the character of materials and construction adopted.

850. The essential requirements are an underground tube or conduit having sufficient clear space to allow an unobstructed motion to the cable, ample room for the passage of the grip-frame and for the necessary supports and pulleys required to guide and maintain the cable in its proper relative position.

The circular cross-section is not well adapted for the purpose, as greater depth than width is desirable.

The egg-shaped section, with the small end up, is well suited, as it gives a good depth with a reasonable average width, and affords a large bottom space for the deposit of mud or dust. A section having the form of a circular sector with the curved base down answers the same purpose. Both of these sections taper towards the slot-beams, between which the grip-bars and frames pass.

An irregular cross-section adapted to give free motion to all moving parts, with special spaces for pulleys and their supports, is conducive to economy of material in the side walls and bottom of the conduit, but is probably more expensive to construct.

851. *Materials used in Construction.*—Solid cast-iron tubes with a slot left in the top have been used. These proved both expensive and unsatisfactory.

Wrought-iron tubes, formed of angle-irons bent to the curve of the sides of the conduit, converging toward the axial line at the top in order to support the slot-beams, fastened at their bottom extremities to transverse ribs of iron, and lined throughout with iron plate, the angle-iron ribs being spaced from 2 to 4 feet apart, have been used. Concrete or ballast is packed on the outside. In some cases a similar construction has been made of wood. In the above three cases the rails are supported on the ends of brackets formed of iron or timber, and fastened to the walls of the tube or conduit. Generally these brackets consist of inclined struts resting against the base of the tube, and horizontal ties fastened to the slot-beams. The rails either rest on longitudinal stringers placed on the brackets, or if of the girder type they rest directly on plates or chairs fixed to the brackets. The distances apart of the brackets, or yokes, as they are called, depend upon the depths of the stringers or rails; usually, however, they are from 3 to 4 feet.

At the present time the common practice is to use concrete for the bottom and side walls of the conduit, the inner surface moulded to proper form by a temporary timber lining. At intervals of 3 or 4 feet iron open-work frames or yokes are built in the concrete, which support both the slot-beams and rails, thereby keeping slot beams and rails in their proper relative positions.

The outer surface of the concrete is built against the natural earth forming the sides of the excavation. A small semicircular drain along the axis of the conduit between the yokes will serve as a place of deposit for mud and dirt. The entire inner surface smoothed over facilitates the cleaning out of the conduit.

As substitutes for the yokes as described, old rails (say 60 lbs. per yard) have been bent so as to reach from rail to rail while passing under the bottom of the conduit, and special ribs of iron are cónnected with these and conforming to the curve of the inner surface of the conduit, their upper ends supporting the slot-beams and fastened to the outer and upper ends of the bent rail, upon which track rails are supported; the whole framework imbedded in concrete as already described. Such cheap expedients can hardly be recommended.

852. *Cable.*—The cable is made of wire, and is from $1\frac{1}{4}$ to $1\frac{1}{2}$ inches in diameter; it moves with a speed varying in different cities, and in the same city, according as the track is along business

or thickly-settled districts, or in the outskirts, from 4 to 13 or more miles per hour. The cable passes from the power-house into the conduit, where on straight and level portions of the line it rests on grooved wheels or pulleys turning upon horizontal axes, and placed at reasonable distances apart so as to prevent too great sag or depression of the cable, and returns to the power-station along another conduit. In curving around the corners of streets it is made to keep the curve by a series of grooved wheels turning on vertical axes placed on the side and at the proper elevations in the conduit. In passing over a depression along the line either the cable must bear against and slide along the slot-beams, causing great wear on it, or it must be held down by some kind of a frame turning on a pin or pivot, so that it can be easily pushed aside, and be so acted upon by a spring or other device that it will return to its place as soon as the grip-frame passes on. The grip-frame is so adjusted that it lifts the cable above the pulleys on the straight portions of the line, and thus allows the bottom bar or jaw of the grip to pass over each pulley, the cable immediately dropping back on the pulley, and it passes readily around the horizontal pulleys on curves, sliding along guide-bars, which prevent lateral bending.

853. *The Grip and Grip-frame.*—There are several types of grips and grip-frames connecting the car with the cable, and thereby imparting the cable's motion to the car or train of cars. Only two of these grips will be described, and these briefly. In one of them there is an outside frame fixed in position with respect to the car. This consists essentially of two vertical plates held together at their upper extremities by a plate parallel to the length of the car, and at their lower extremities by a thick bar, with a grooved plate on its upper surface, on each side. At the extreme ends of the lower bars are placed friction rollers, on which the cable glides along when the cars are standing still; also near the ends are conical spools which, when lifted, throw the cable off the lower bar, or jaw, as it is called. This whole construction as described may be called the grip-frame. Sliding vertically in this frame is a thin plate, which is lowered or raised by a lever operated by the gripman standing on the grip-car. Fastened to the lower extremity of the plate is another horizontal bar or jaw grooved on its bottom surface. The gripman, in order to start the car, moves the lever so as to lower the sliding plates and jaw and clamp the cable

tightly between the upper and lower bars or jaws; the car then takes up the motion of the cable. The length of the movable jaw may be 18 inches or more. Care and skill are required in order to avoid imparting the high speed of the cable to the car too suddenly. To stop the car the gripman operates the lever so as to lift the sliding plate and jaw, and puts on the brakes. The cable then glides inoperatively along the lower or fixed jaw, and always in position to be gripped, unless purposely thrown off the jaw by lifting the conical spool. A similar grip-bar is used without the friction rollers, in which case the cable rests directly on the upper surface of the fixed jaw. In the other type of grip the lower part of the frame is formed similarly to a series of plyers placed side by side and acting simultaneously; the jaws of these plyers turn on a long pin, and the gearing is such that the gripman closes the jaws around the cable, they being bored or cored so as to grip tightly the cable when their lower edges are in contact, or nearly so; by a reverse movement the jaws revolve outward, and the cable is free to glide between them, or if sufficiently opened the cable drops free of contact with the apparatus. In this type of grip it will be noted that there is no sliding frame, and when the jaws are full-opened the cable falls out, and has to be "picked up" before a start can be made.

854. The track rails can be of the shallow street-tramway type, resting on longitudinal stringers supported on the yokes, or they can be made of the girder-rail type, either with or without the tram-plate, and resting on the yokes, without the use of stringers. The rails vary in height from 5 to 7 inches.

855. In the last few years street-cars run by electric power have been brought into use. There is nothing especially distinctive so far as the construction of the road-bed and tracks is concerned, as compared with that of ordinary railroads. The motive power is supplied by means of a current of electricity generated by dynamos located at some convenient or central station; this current is transmitted along overhead wires, and connection is made with the cars by what is called a trolley-wheel and pole attached to their tops, by means of which and connecting wires the current is carried to the electric motors placed under the cars, causing the motion of the motor armatures, to which by suitable gearing the wheels are connected and made to revolve. The return circuit has been supposed to be through the rails of the track, but has been found to be practi-

cally through a ground circuit, and along the network of water and gas mains, the effect of which has been to corrode and destroy the pipes by the action known as electrolysis. This can be prevented by a return wire, or even a more perfect and efficient rail connection. Some such means of directing the return current will doubtless be required in the near future. The inconvenience, obstruction, and danger resulting from the use of overhead wires has led to a demand for placing all wires in subways or underground conduits. At present the invention or design of some form of subways that will effect the purpose desired and satisfy all the conditions required in dealing with this subtle force is engaging the earnest thought and talent of engineers, and offers an inviting field for investigation, as it is clear that electric and telegraph wires must be placed underground. These wires and their supports are unsightly, obstruct the streets, especially in connection with the rapid and effective working of firemen, and, besides, the currents are exceedingly dangerous to life.

856. A natural extension of the principles involved in the construction and operating of electric street-car lines is to that of electric railways, in which the electric locomotive is substituted for the steam locomotive.

Several lines have been projected, and in some cases put under construction; electric locomotives have been constructed, and doubtless in the near future such railways will be in operation.

Great speed is claimed, with economy of construction of the roads. The engine will be lighter for hauling the same loads at given speeds. Much steeper grades can be economically overcome, thereby diminishing greatly the cost of construction of the road-beds and tracks.

With steeper permissible grades a great many sharp curves can be avoided, as compared with those on ordinary railways.

At present it seems that there is nothing in the way of the future development of electric railways other than the relative economy in the combined cost of constructing and operating the road.

ART. XLIX.

FRAMED STRUCTURES.

857. THE general definition of framed structures and the general principles of their construction have been given and discussed in other parts of this volume.

We have seen the methods of finding the bending moments and shears at any section of a beam considered as a unit, whether of a solid rectangular or other form of cross-section, whether of cast iron, of solid section, or built up of plates and angles, as in built iron or steel beams.

In the next article these principles will be applied to determining the stresses in trussed beams, or simply trusses, and the proper designs, dimensions, and connection of the parts to supply the resistances required. In this article solid and built beams will be discussed.

In solid beams of rectangular or circular sections the entire beam at any section is considered as resisting the bending moment and shear, these resistances consisting of a series of stresses—compressions and tensions—distributed in accordance with established laws.

In flanged beams, while, strictly speaking, the same actual conditions exist, and all fibres unite in resisting both the bending moment and shear, yet it is usual to consider that the entire bending moment is resisted by the flanges alone, and that the entire shear is resisted by the web, which is usually a solid plate connecting the flanges.

So far as determining the stresses in the members of trussed beams, such as roof and bridge trusses, the same assumptions are made. The flanges or chords are supposed to resist the bending action, and the web members, which consist of a series of ties and struts, resist the entire shearing action, not, as in beams, by a uniform resistance to shear distributed over the area of the cross-section of a solid web, but by direct stresses of tension and compression along certain well-defined lines or members.

In the practical application of the principles to the determination of dimensions, certain members act as beams and certain members as columns. Therefore, all of the principles, as applicable to beams and columns, will necessarily be found in the complete discussion of bridges. For this reason practicable examples of the

strength of beams and columns have been reserved for this portion of the work.

It will be as well, however, to dispose of one kind of framed structures known as trestles, or viaducts, before entering upon the general discussion of stresses and strains in trusses.

TRESTLES AND VIADUCTS.

858. Trestle-work is generally applied to timber structures composed of a series of frames or rows of piles placed transversely to the line of a road, over and connecting which a series of beams called stringers are placed ; on top of these the cross-ties are laid; and finally, on the ties the iron rails for the track and the timber guard-rails are placed. There is no special limit to distances from centre to centre of bents: the usual distance or length of span is from 12½ to 15 feet; sometimes as much as 25 to 30 feet when the heights of the bents are very great. For longer spans regular trussed bridges would be used, and the bents or supports are then called timber piers.

Viaducts are trestles in which the bents and stringers are of iron. Much longer spans are used; the heights may or may not be greater than in case of timber trestles, and commonly two bents are well braced together in both horizontal and vertical planes, forming what are called towers. Every alternate space is thus braced.

The spaces may all be of the same length, or alternately long and short spans. A common length of span is about 30 feet; a common alternation in span is from 30 to 60 feet.

Trestles and viaducts are used for the same purposes; (1) to cross deep ravines where the cost of filling in with earth would be very great : (2) to cross small streams or watercourses ; (3) to cross swamps ; (4) to form the approaches to bridges, especially when at great elevations, where the expense of earth filling would be very great, or where it is necessary to have a waterway greater than that provided by the bridge proper; and (5) where it is either expensive or difficult to obtain filling material, and the time required to complete a permanent work would be too great. In the latter case timber trestles are used to save first cost and loss of time.

Timber trestles are of two kinds, namely, framed trestles and pile trestles.

Pile trestles are constructed by driving either 3 or 4 piles in a row across the line of the road, called a bent, at intervals along the line of from 10 to 15 feet. These piles are cut off to the same level in each row or bent, and to the proper grade from bent to bent. Caps 12 × 12 inches square are placed over each row and drift-bolted to the piles—sometimes they are connected by mortise and tenon; diagonal or X bracing are then spiked or bolted to the piles in each bent, one brace on each side, these crossing each other. These braces are usually planks from 2 to 3 inches thick, and from 9 to 12 inches wide.

When four piles are used, all are driven vertically, or the two outside ones are driven on a batter and the two inner ones vertically. The distance from centre to centre of outside piles is from 11 to 12 feet, and between the two inside piles centre to centre 4½ to 5 feet, as seen on the left half of Fig. 304; when the outside piles are on a batter, they are so set and driven that their upper extremities will be near to the tops of inner piles, as seen in the right half of Fig. 304, while having usually a batter of 3 inches to each vertical foot.

The inner piles are the same distance apart as given above, and the clear interval between the tops of the inner piles and the corresponding batter-piles is from 1 to 1½ feet. The length of caps required in the two cases being 14 and 12 feet respectively.

When three piles are used in a bent, the one is driven in the centre, and the tops of the outside piles are about 3½ feet distance on each side, centre to centre, requiring a cap about 10 feet long. The outside piles may be driven either vertically or on a batter.

There can be no doubt that in either case the batter-piles give a stiffer and stronger trestle, and are capable of bearing a heavier load with the same number of piles than when all are driven vertically.

When the piles are driven to any great depth into the soil the height of a pile trestle above the ground cannot be very great, with any ordinary length of pile that is not over from 20 to 25 feet above the surface.

When over 10 feet high some form of longitudinal bracing should be used. Plank extending diagonally from the top of one bent to the bottom of the next gives a good longitudinal system of bracing.

Two lines of stringers are placed over the cap, each line consisting of two pieces, arranged so as to break joints over the caps.

Each piece of each stringer should be the length of two spans, and vary in cross-section from 6 × 14 inches for 12½ span from bent to bent, or 8 × 16 inches for a 16-foot span. For very heavy engines Fig. 304 shows on the left a half-end view of a trestle-bent with all

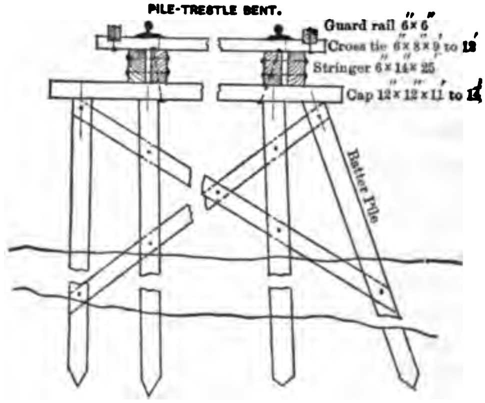
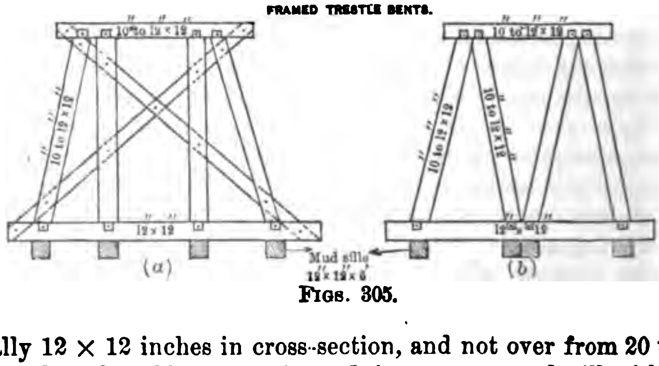


FIG. 304.

piles vertical, and on the right with outside piles on a batter. The caps, stringers, ties, rails, and guard-rails are also shown.

Trestles with Framed Bents.—For a single-story trestle there are two general types shown in Figs. 305 (*a*), (*b*).

Fig. 305(*a*) shows two vertical and two batter posts. These are



FIGS. 305.

usually 12 × 12 inches in cross-section, and not over from 20 to 25 feet in length. These are framed into a cap and sill with the mortise-and-tenon joint, sometimes with square-abutting surfaces, and held in place by iron straps.

Fig. 305(*b*) shows a type of trestle with all inclined posts. This is unquestionably a stronger and stiffer trestle with equal sizes of timbers. The posts of this trestle are often made of 10 × 12 inch timber. The stringers, ties, and guard-rails are the same as shown in Fig. 304 for the pile trestles.

For two or more stories the timbers of the lower are placed in the prolongation of those for the single-story trestles, which, as shown in Fig. 305, will be the uppermost. Additional vertical or inclined members are inserted in the lower stories, and the dimensions are increased in very high trestles to 14 × 14 inches in the bottom one.

Framed trestles are usually supported on mudsills, which are timbers 12 × 12 inches by 4 or 5 feet long, partly imbedded in the soil. Sometimes long sills are placed first, and then the mudsills on these. Occasionally rough masonry pedestals are used, which are far better. When the soil is very soft or swampy four piles are driven and cut off, at or a little below the surface, and upon these the sills of the framed trestles are placed and bolted to the piles.

859. In Fig. 306 is shown a side view of a trestle which is the same for either the framed or pile trestle. This is given mainly to illustrate practically the process of calculating the dimensions of the stringers. No attempt is usually made to proportion the posts to the load that they have to bear, and there is always a large excess of strength in the 10 × 12 or 12 × 12 inch timbers. The same may be said of the caps, sills, and ties.

Nor is there any special effort, as a rule, to proportion the stringers to the loads which they have to bear.

But now that such heavy engines are being introduced, it is well to see whether the standard dimensions of the members give a sufficiently large margin of safety, especially considering the fact that it is not unusual to allow the timbers of a trestle to lose from 25 to 50 per cent of their areas by actual rot before repairing or renewing.

About the heaviest load that can come upon a bent of 12½ feet span will be 75,000 pounds; or assuming that each of the four pairs of drivers of a heavy type of engine have 30,000 pounds, then the bents of four posts will carry 120,000 pounds, or 30,000 pounds to each post, which, if 12 × 12 inches in section, gives 144 square inches, the intensity of pressure per square inch will be 210 pounds nearly; or if only two posts bear the entire load, which is doubtless the case very often, the unit pressure would be about 420 pounds. There will then be ample strength in the posts as com-

monly used. But let us see how it will be with the stringers. Assuming a rather heavy consolidation engine with four pairs of drivers, each pair loaded with 24,000 pounds, spaced as the wheels of this locomotive are, namely, on a base of 14¾ feet, it is evident that only three pairs of drivers can be on any one span at a time. As we can only place three pairs of drivers on the span of 12½ feet centres of bents, or the clear span of 11½ feet, the load can be applied in several different ways, two of which are shown in Fig. 306. On the right-hand half, or Span No. 2, the rear three pairs are placed, with one pair in the centre of the span, and the others symmetrically placed with respect to the centre. And on the left-hand half, or Span No. 1, the front three pairs are placed in such

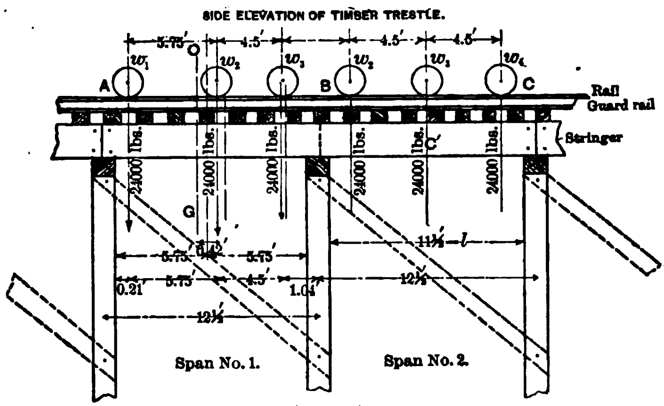


FIG. 306.

position as will give maximum bending moment, the reason for which will be explained later. The principle will simply be stated in this place, namely, *with a system of isolated roads or wheel-concentrations, that position of the load will give maximum bending moment in which the centre of the span bisects the distance between the point of greatest bending moment and the centre of gravity of the system of loads on the span.*

For Span No. 2 the centre of gravity of the three loads is at the centre of wheel w_3, that is, at the centre of the span. Each reaction, then, $R_1 = R_2 = 36{,}000$ pounds. The greatest bending moment is at the centre C, and is

$$M_0 = 36{,}000 \times 5.75 - 24{,}000 \times 4.5 = 99{,}000 \text{ foot-pounds.}$$

For Span No. 1 we must first find the vertical line through the centre of gravity of all the loads by the principle for finding the centre of parallel forces.

Taking moments about the centre of wheel w_1,

$$(w_1 + w_2 + w_3)x_0 = w_2 \times 5.75 + w_3 \times 10.25 = 384,000;$$

hence

$$x_0 = 384,000 \div 72,000 = 5.33 \text{ feet};$$

that is, the centre of gravity of the system is 5.33 feet to the right of w_1, or 0.42 foot in front of w_2; and according to the principle above stated, this distance must be bisected by the centre of the span; that is, wheel w_2 must be 0.21 foot to the right of the centre C, and the line through the centre of gravity of the system 0.21 feet to the left of the centre, as OG.

The maximum bending moment will be under wheel w_2. The positions of the wheels with respect to the centre of the span C and its ends A and B are shown on the drawing. Then, for the reaction R_1 at A,

Taking moments with respect to an axis at B, then

$$R_1 \times 11.5 - w_1 \times 11.29 - w_2 \times 5.54 - w_3 \times 1.04 = 0.$$

$R_1 = 37,294$ pounds and $R_2 = 72,000 - 37,294 = 34,706$ pounds.

Bending moment at centre C

$$= M_0 = R_1 \times 5.75 - w_1 \times 5.54 = 81,480 \text{ foot-pounds.}$$

It is evident, then, for this length of span and wheel concentrations, that the maximum bending moment is when the engine occupies the position shown in Span No. 2. As in this case the load is symmetrically situated with respect to the centre of the span, the centre bending moment is

$$M_0 = w_2 \times 1.25 + \tfrac{1}{2}w_2 \times 5.75 = 99,000 \text{ foot-pounds,}$$

the same as already found by the general method.

Equating this in inch-pounds, that is, $99,000 \times 12 = 1,188,000$ inch-pounds, to the moment of resistance of beams,

$$mWl = 1,188,000 = \frac{fI}{y} = \frac{1}{6}fbd^2,$$

for beams of solid rectangular section; and assuming $f = 1000$, $d = 14$ inches, we find the breadth b in inches. But since this

load is carried by two stringers, each composed of two pieces, the bending moment on each piece will be $\frac{1,188,000}{4} = 297,000$ inch-pounds, hence $\frac{1}{6} \times 1000 \times b \times 196 = 297,000$ and $b = 9.1$ inches. If, however, we allow a modulus of rupture $f = 1500$ pounds, other quantities remaining the same, then $b = 6.1$ inches. This latter is in keeping with the standard dimensions, and gives ample strength.

There would then be two stringers, both composed of two string-pieces, each 25 feet long and 6.1×14 inches in cross-section. For longer spans similar calculations can be made; recollecting that economy of material, as well as stiffness, requires a certain depth, which increases in a well-established ratio with the length.

To find the equivalent uniformly distributed load that will give the same centre bending moment, we have

$$\tfrac{1}{8}(wl)l = \tfrac{1}{8}wl^2 = 297,000; \quad \therefore\ w = 124.764 \text{ pounds per linear inch,}$$

or 1497.2 pounds per linear foot for each string-piece; and for the four pieces, 5989 pounds per linear foot.

Under this load the beam may not deflect more than $\frac{1}{2}$ inch, which is a ratio of deflection to length of $\frac{v_1}{l} = \frac{\frac{1}{2}}{11.5 \times 12} = \frac{1}{276}$ of its length. Then for the ratio of depth to length (see equation (227)), $\frac{d}{l} = \frac{n''f}{4Em'}\frac{l}{v_1}$, in which, for beams of uniform cross-section and supported at both ends while uniformly loaded, $n'' = \frac{5}{12}$; and $m' = \frac{1}{6}$, and for timber $f = 1500$ and $E = 1,000,000$. Then $\frac{d}{l} = \frac{1}{11.6}$ or $d = \frac{138}{11.6} = 11.9$ inches, whereas the beam under consideration is 14 inches deep. If, however, with the same load the length of clear span is increased to say 14 feet = 168 inches, $d = \frac{168}{11.6} = 14.6$ inches.

This is a large deflection under the working load, $\frac{v_1}{l} = \frac{1}{400}$ to $\frac{1}{1000}$ being the usual proportion. With $\frac{1}{400}$, substituting, we find

$$\frac{d}{l} = \frac{1}{8}; \quad \therefore\ d = \frac{138}{8} = 17.25 \text{ inches,}$$

that the deflection may not be more than one third of an inch.

When trestle spans are over 15 or 16 feet in length they are usually trussed in some manner, the effect of which is to divide the original span into two or three segments or bays; consequently the timbers required for the stringers have about the same dimensions as above given, while the proper dimensions of the braces are to be determined by principles hereafter to be explained.

When the spans are long, it is difficult to secure stringers of sufficient length to cover two spans. In this case it becomes necessary to have joints over every cap, the stringers having only the length of one span. The bearing would then be less than 6 inches for each stringer. In such cases bolsters, that is, pieces of timber about 8 or 10 inches deep and 4 or 5 feet long, are placed perpendicularly to the caps and bolted to them, and upon these the stringers rest, by which means a good bearing is obtained.

The caps and sills can be made of single pieces or of two pieces each, bolted together.

860. The greatest shear will be at the points of support. To find the greatest possible shear several trials may be necessary, the maximum shear being equal to the reaction R_1 or R_2. It is seen above that R_1 is greater for the load in the position shown in Span No. 1 of Fig. 306, where $R_1 = 37{,}294$ pounds, whereas on Span No. 2 $R_1 = R_2 = 36{,}000$ pounds. But suppose we move the loads to the left in Span No. 1 until w_1 is just to the left of A, then $R_1 = w_1 + \frac{w_2 \times 5.75 + w_3 \times 1.25}{11.5} = 38{,}609$ pounds. If in Span No. 2 we move the loads until w_2 is just to the right of B, we would find $R_2 = w_2 + \frac{w_3 \times 7.0 + w_4 \times 2.5}{11.5} = 43{,}827$ pounds, which, then, is the condition for maximum shear. As the bending moment is zero at the points of support, it is only necessary to provide material at the ends sufficient to resist the shearing. Taking 400 pounds per square inch as a safe resistance to shearing, then, the maximum shear on each string-piece being $\frac{43827}{4} = 10{,}957$ pounds, there is only required $\frac{10957}{400} = 27$ square inches of material; and each string-piece being 6 inches wide, the depth of the stringer need not be over $\frac{27}{6} = 4.5$ inches. Though this is taken advantage of in floors, wharves, and platforms generally, it is not in trestle-work, the full or near full centre depth being retained throughout; consequently there is always a great excess of strength at the ends of stringers. As the bending moment increases to its maximum near the centre of the span, so the shearing force diminishes, and

becomes zero at the point of maximum bending. A solid rectangular beam of sufficient strength to resist bending will also have sufficient strength to resist the shear.

860a. It is evident that the strength increases in a more rapid ratio with increase of depth than with increase of breadth of rectangular beams. In fact, doubling the breadth only doubles the strength, doubling the depth quadruples the strength. Beams, then, should be made, for economy of material, deep and thin; but if too thin, the beams are wanting in lateral stiffness, and will give way by bending sideways. The rectangular beams are generally, in thickness, from one eighth the depth up to a thickness equal to the depth. They should never have a thickness less than one eighth the depth. For trestles and bridges the usual thickness is from one half to one third the depth.

The following figures, 306½ (a), (b), (c), show the beams of solid rectangular cross-section with the same area of cross-section, $bd =$

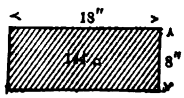


FIG. (a)

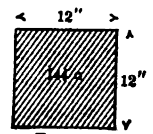


FIG. (b)

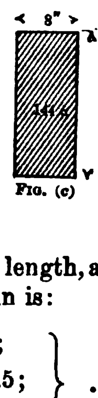


FIG. (c)

FIGS. 306½.

144 square inches. With the same material, same length, and same load, the relative strength to resist transverse strain is:

$$\left.\begin{array}{l} \text{Fig. } (a),\ bd^2 = 18 \times (8)^2 \ = 1152 = 1; \\ \text{Fig. } (b),\ bd^2 = 12 \times (12)^2 = 1728 = 1.5; \\ \text{Fig. } (c),\ bd^2 = \ 8 \times (18)^2 = 2592 = 2.25. \end{array}\right\} \quad . \quad . (456)$$

860b. For a round log beam the moment of inertia of a solid circle $= \dfrac{\pi d^4}{64} = \dfrac{\pi r^4}{4}$, and $b = d$.

$$\therefore\ M_0 = mWl = \frac{f\pi r^4}{4r} = \frac{f\pi r^3}{4} = nfbd^2 = nfd^3;$$

assuming a uniformly distributed load and $n = \dfrac{1}{10.2}$, nearly,

$$\tfrac{1}{8}(wl)l = 0.7854fr^3 = \frac{1}{10.2}fd^3. \quad . \quad . \quad . \quad . \quad (457)$$

Then $wl = 2250$ lbs., or $12\frac{1}{2}$ lbs. per inch of length, and $l = 15$ ft. $= 180$ inches; $\frac{1}{8}(2250) \times 180 = 0.7854 \times 1000 \times r^3$.

$\therefore r = \sqrt[3]{64.458} = 4.01$ inches or $\frac{1}{8} \times 2250 \times 180 = \frac{1}{10.2} \times 1000 \times d^3$.

Hence $d = \sqrt[3]{516.375} = 8.02$ inches in diameter.

860c. As was stated, timber beams are usually of solid rectangular cross-section, as seen in Figs. $306\frac{1}{2}$. For such beams

$$mWl = nfbd^2; \quad \therefore W = \frac{nfbd^2}{ml}; \quad n = \frac{1}{6};$$

and for beams with single weight at the centre $m = \frac{1}{4}$. Hence the centre safe load

$$W = \frac{4}{6}\frac{fbd^2}{l} = \frac{2}{3}\frac{fAd}{l}. \quad . \quad . \quad . \quad . \quad . \quad (458)$$

For a uniformly distributed load, $mwl = \frac{1}{8}Wl$ or $\frac{1}{8}wl^2$; $\frac{1}{8}Wl = nfbd^2$.

$$\therefore W = \frac{8}{6}\frac{fd^2b}{l} = \frac{4}{3}\frac{fAd}{l}. \quad . \quad . \quad . \quad . \quad . \quad (459)$$

It is sometimes convenient, and it is often seen in practical books, such as Trautwine, that l is given in feet while b and d are in inches. Placing $12l$ for l in eqs. (458) and (459), they become, respectively,

$$W = \frac{fAd}{18l} \text{ for a single load at the centre}; \quad . \quad . \quad (460)$$

$$W = 2\frac{fAd}{18l} \text{ for a uniformly distributed load.} \quad . \quad (461)$$

In all of the above equations A is the area of cross-section. Solving equations (458) and (459) with respect to f, we have,

$$\text{From equation (458),} \quad f = \frac{3Wl}{2bd^2} = \frac{3Wl}{2Ad}; \quad . \quad . \quad (462)$$

$$\text{From equation (459),} \quad f = \frac{3Wl}{4bd^2} = \frac{3Wl}{4AD}. \quad . \quad . \quad (463)$$

If, now, we make $f' = \frac{f}{18}$, and substitute in equations (460) and (461) we find, from equation

$$f' = \frac{f}{18} = \frac{Wl}{bd^2} = \frac{Wl}{Ad}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (464)$$

and from equation

$$f' = \frac{f}{18} = \frac{Wl}{2bd^2} = \frac{Wl}{2Ad}. \quad . \quad . \quad . \quad . \quad . \quad (465)$$

When, then, any given beam whose dimensions $b, d,$ and l are known is loaded with a weight that is just sufficient to break it, we find the value of f, from equations (462) and (463), which is called the modulus of rupture when all dimensions are in inches, and for timber this has been found to be from 10,000 to 15,000 lbs. per square inch; and from equations (464) and (465) we find $f' = 600$ to 800 lbs. This is the value of the modulus of rupture when l is in feet and b and d are in inches. Trautwine's tables are calculated upon this supposition.

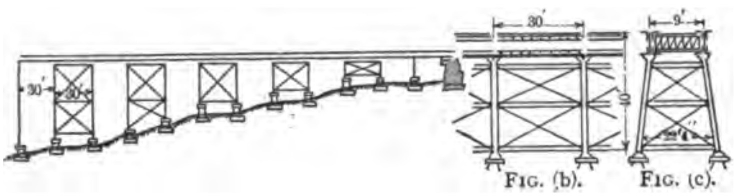
A failure to understand this question fully often leads to confusion and error in results with young engineers, as some tables, such as Rankine's and those of other authors, give the modulus of rupture for timber beams as 10,000 lbs. per square inch, and Trautwine and others give 600 lbs. l is in inches in the first case, and in feet in the second. The results are, however, the same in either case.

With a clear understanding of the above examples on timber beams, there should be no difficulty in determining the dimensions to carry safely any load, or the load that any given beam will carry safely.

861. *Iron Viaduct.*—As an example of this type of structure the writer has selected the viaduct approach to a bridge constructed over the Ohio River, under his superintendence, by the Keystone Bridge Company.

There were 32 spans, each 30 feet long, and 46 spans, each 30 feet long, on the north and south sides of the river respectively, a total length of 2340 feet. The heights of the bents varied from 20 to 45 feet. The highest single-story bent was 29 feet $3\frac{11}{16}$ inches. When the total height did not exceed this figure, intermediate strut braces were used near the top and bottom, and diagonal rods in both vertical and horizontal planes. Usually with heights over 25 feet intermediate struts and ties were used, dividing the bents into two

stories, the upper story about 20 feet high, and the lower story varying, according to the total height. Figs. 307 (a), (b), and (c) show, respectively, a general side view, a side view of two bents and girder span between, and an end view of one bent. The brace system, as shown, was used only in every alternate span. In the formulæ used I = moment of inertia of section ; a is a constant; f = ultimate resistance to crushing in pounds per square inch; S = metal area in the cross-section in square inches; l = length of column in inches or feet; ρ = radius of gyration of the section in the same unit as l; d = least side it rectangular in section, and the diameter if circular.



FIGS 307.

All columns were composed of two 10-in. channels, 35 lbs. per foot, 10.5 sq. in. metal area; the unsupported length of each column, $l = 20$ ft. $= 240$ in. Two columns to each bent, batter 4 in. to a vertical foot; that is, 2 in. to each column. The transverse struts between columns, two 5-in. channels, 6.5 lbs.; vertical and diagonal rods in lower section 1 in. diameter, and also in upper section. Longitudinal horizontal struts, two 6-in. channels, 8.5 lbs.; diagonal rods $1\frac{3}{16}$ in. diameter. Horizontal and diagonal rods 1 in. diameter. For the columns we will use the formula for one end fixed and one pin end.

$$P = \frac{fS}{1 + 1.8a\frac{l^2}{\rho^2}}; \quad f = 39{,}000 \text{ lbs.}; \quad S = 21 \text{ sq. in.};$$

$$l = 20 \text{ ft.} = 240 \text{ in.}; \quad 1.8a = \tfrac{1}{17000}.$$

For latticed columns we have

$$I = \frac{(b_1+t)d^3 - b_1(d-2t_2)^3}{6} = \frac{(2\tfrac{1}{8} + \tfrac{3}{8})10^3 - 2\tfrac{1}{8}(10 - \tfrac{3}{4})^3}{6} = 137.03;$$

$$S = 21 \text{ sq. in.}; \quad \rho^2 = \frac{137.03}{21} = 6.52; \quad \rho = 2.55, \quad \frac{l}{\rho} = 94.12;$$

$$P = \frac{39000 \times 21}{1 + \tfrac{1}{17000}(94.12)^2} = \frac{819000 \times 17000}{17000 + 8858.57} = 538{,}823.5 \text{ lbs.},$$

or 25,659 lbs. per square inch of metal. This, with a factor of safety of 5, gives 107,765 as the safe load per column. The strain-sheet loads, as given by the Keystone Bridge Co., were 93,800 lbs. for the top sections and 112,200 lbs. for the bottom sections of the columns. Taking the dead load at 800 lbs. per lineal foot, and the live load at 4600 lbs., or a total load of $5400 \times 30 = 162,000$ lbs., divided between the two columns, gives actual load on each column 81,000 lbs. from the weight of the structure and train-load. If, instead of latticed channels, those with cover-plates had been used, I and ρ should have been determined from the proper value for I. In other respects the solution of the problem would have been the same. The lattice pieces are short strips, about 2 in. $\times$ $\frac{1}{4}$ to $\frac{3}{8}$ in., riveted to the flanges and placed diagonally, the distance apart of the rivets in the same flange varying according to the size of the channels.

862. Assume a Phœnix column exterior diameter $9\frac{3}{8}$ in., interior diameter 9 in.; metal thickness $\frac{3}{8}$ in.; flanges double thickness, equal to $\frac{3}{4}$ in. and 2 in. long—a total metal area of 17.0 sq. in. In equations for moment of inertia of a Phœnix column we find

$$I = \frac{\pi(r^4 - r_1^4)}{4} + 2bl\left(r + \frac{l}{2}\right)^2; \quad r = 4.7; \quad r_1 = 4.5;$$
$$\pi = 3.1416; \quad b = \tfrac{3}{4}''; \quad l = 2 \text{ in.}$$

Substituting,

$$I = 94.37; \quad A = 17.0; \quad \rho^2 = 5.55; \quad \rho = 2.36.$$

$$P = \frac{fS}{1 + a\frac{l^2}{\rho^2}}; \quad f = 42,000; \quad S = 17.0; \quad a = \tfrac{1}{36000}; \quad l = 20 \text{ ft.} = 240 \text{ in.};$$

$$\frac{l}{\rho} = \frac{240}{2.36} = 101.7; \quad \frac{l^2}{\rho^2} = 10,342.5.$$

Substituting, $P = 476,000$ lbs., or 28,000 lbs. per sq. in.—about 2340 lbs. per sq. in. more than the latticed channel section. The total safe load per column $\frac{1}{5}$ of $476,000 = 95,200$ lbs. With the same value for $\frac{l}{\rho}$ as in the first case, 94.12, we would have found $P = 626,300$ lbs. and safe load $= 125,260$, with the same sectional area of metal. This corresponds with the principle that the farther the metal is from the neutral axis the stronger the column will be.

The above examples will be sufficient to show the practical application of the formulæ. At the Susquehanna bridge, B. & O. Rd., Havre de Grace, about 2500 lineal feet of iron viaduct was used, the height being about 60 ft. All spans were 30 ft. long, and the columns entirely of the Phœnix type, in the equation for which d^2 is used instead of ρ^2. The work is simplified by the fact that we do not have to find the moment of inertia or the radius of gyration, as the least dimension of the column is given, which is represented by d, and, in addition, the formula is in better shape to determine the value of d, i.e., the diameter of a Phœnix column, or the depth of channels in the square column to bear a given load. This is done by solving the equation $P = \dfrac{fS}{1 + a\dfrac{l^2}{d^2}}$ with respect to d, and we get $d^2 = \dfrac{Pal^2}{fs - P}$, from which, in any given case, d can be found, P representing either the breaking load or working load, as the case may be.

Closed columns are stronger than open ones, but we have seen the objection to these. The material should be as far as possible from the neutral axis, so as to increase the radius of gyration, ρ. No unsupported column should be more than about 30 to 40 times its least diameter. Although solid columns and cast-iron columns may have their strength increased by swelling or increasing their diameter at or near the middle of the length, as is done usually in columns of wood and cast iron, it does not seem to have that result in segmental wrought columns.

It simplifies the calculation, in determining the crushing resistance of any column, to omit the area S. Then $P = p =$ the ultimate proof or working strength per square inch of area.

863. A familiar example of a pier composed of cast-iron columns is found in the piers of the Crumlin viaduct. In all such cases the columns are cast into lengths or sections, with sockets or flanges to facilitate the connection of the different sections, which are placed one on top of another, and fastened by bolts through the flanges. The flanges are generally on the outside when above ground and on the inside when below ground, and of sufficient diameter to secure the bolts in position without too much difficulty. At the bottom they are fastened by bolts to a broad base, which generally rests on a masonry pier or pedestal, and a capital with a broad top is fastened at the upper end of the column,

or the top and bottom piece is cast with a projecting cylinder which fits in the end of the column. The ends of all the sections should be planed or turned so as to rest on each other with a full and uniform bearing, and at each junction the several columns should be braced by strut and tie braces, the latter in both vertical and horizontal planes. Lugs or projections should be cast on the columns so as to admit of connection with the braces. The strongest form of column with a given amount of material is the hollow cylinder, and for this reason cast-iron columns are usually of this form. Cast iron is but little used in bridge construction, either for piers or for any part of the superstructure. It is, however, extensively used for columns of buildings and for beams of floors where not likely to be subjected to moving loads causing shocks and vibrations, and where no sudden or extensive change of temperature is likely to occur. If not too long the columns can be cast in one section; otherwise in several sections, connected as mentioned above. In either case they may be cast with the same exterior diameter from end to end, with the same or gradually decreasing metal thickness, or the diameter may gradually decrease and the metal thickness remain the same. The length, however, in no case should exceed from 15 to 25 times the diameter. The following figures, 308 (*a*), (*b*), represent the general design and arrangement of the columns. Fig. (*a*) is a horizontal section near the bottom of the pier; Fig. (*b*) a general elevation. This pier consists of 14 cast-iron columns, 12 inches exterior diameter, and with metal thickness from 1 inch in the bottom section to ¾ inch in the top section. The central columns are vertical; the other columns have a rake or batter, so that while the enclosed space at the bottom measures 60 × 27 feet, the area at the top is reduced to 30 × 18 feet.

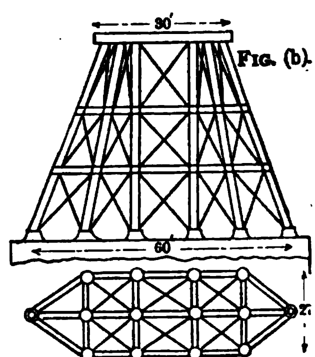


FIG. (b).
FIG. (a).
FIGS. 308.

The horizontal strut braces are cast-iron I-beams 12 inches deep and 5 inches breadth of flange. The diagonal tension braces in vertical planes are flat bars 4 inches × ¾ inch, and the horizontal diagonal ties are rods 2 inches in diameter. The piers are from 180 to 200 feet high, carrying spans 150 feet long. The sections of the columns are 17 feet long. Each column stands upon a base 3 feet

square, which rests upon and is bolted to masonry piers. (See Rankine, Trautwine, and others).

864. In calculating the area of metal required in such columns special formulæ are used. These formulæ are, as explained in paragraphs, for long, hollow cylinders,

$$S = \frac{P(1 + a\frac{l^2}{h^2})}{c} \quad \text{or} \quad P = \frac{Sc}{1 + a\frac{l^2}{h^2}}; \quad . \quad . \quad (466)$$

and

$$P = \frac{Sc}{1 + a\frac{l^2}{r^2}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (467)$$

in which $P =$ crushing load in pounds; $a =$ a constant determined experimentally; $S =$ metal area in square inches; $l =$ length in inches; $h =$ diameter in inches if round, and the least side if square or rectangular in cross-sections; $\rho =$ radius of gyration; $c =$ safe resistance to crushing in pounds; a varies with the material of which the column is made, and upon the bearings of its ends. Three cases may arise: 1. When the ends of the columns or sections have broad, flat capitals, and bases turned or planed. 2. When the columns or sections do not have true or square ends, or have one end in this condition; or when one or both ends are hinged or rest upon pin-bearings. 3. Or when both ends are rough or have hinged or pin bearings. In the first case the columns are said to have fixed ends. For cast iron, $a = \frac{1}{800}$, and when the ends are hinged, or rest against pins; $a = \frac{1}{400}$. For the present purpose, in the second of the above formulæ $a = \frac{1}{3000}$, and $\rho^2 = \frac{d^2}{8}$. The determination of the value of the radius of gyration ρ, and the values of a for the several forms of columns, have been fully discussed in another article.

In the piers above described each of the 14 columns would have to carry approximately 75,000 lbs., including the weight of the rolling load, weight of the bridge, its own weight, and part of the braces. Then $h = 12$ in.

$$\therefore h^2 = 144 \text{ in.}; \quad l = 17 \text{ ft.} = 204 \text{ in.}; \quad l^2 = 41,616.$$

For cast iron,

$c = 80{,}000$ lbs., or safe value $= \frac{1}{5} \times 80{,}000 = 16{,}000$ lbs., $a = \frac{1}{400}$.

Substituting in eq. (466),

$$S = \frac{75000\left(1 + \frac{1}{400} \times \frac{41616}{144}\right)}{16000} = 8 \text{ sq. in.}$$

Substituting in eq. (467),

$$S = \frac{75000\left(1 + \frac{1}{3000} \times \frac{41616}{18}\right)}{16000} = 8\tfrac{5}{8} \text{ sq. in.}$$

The actual metal area in these columns at bottom is $34\frac{1}{2}$ sq. in., and as we have already allowed a factor of safety of 5, the ultimate factor is about 20.

If in columns 12 in. in diameter only 9 sq. in. of metal is taken, the metal thickness would be entirely too small. Therefore in such cases it has to be arbitrarily increased to $\frac{1}{12}$ or $\frac{1}{8}$ of the diameter. We could then assume a thickness of metal of $\frac{3}{4}$ in. or 1 in. In the latter case, $S = 3d$ nearly. Substituting and solving the equation with respect to d, we could find the required diameter of the column.

This would be a rather long and tedious calculation, as d appears in ρ and h in the two formulæ, requiring the solution of an equation of the second degree. But practically for columns from 15 to 20 feet long the diameters would rarely be less than from 6 to 12 inches, and a metal thickness equal to $\frac{1}{12}$ of the external diameter would give a safe column under ordinary circumstances. To find the breaking load of a column 17 ft. long, 12 in. external diameter, metal thickness 1 in., we have,

$$P = \frac{cS}{1 + a\frac{l^2}{r_2}}, \quad c = 80000 \text{ lbs.}, \quad s = 34.5 \text{ sq. in.}, \quad l = 204 \text{ in.},$$

$$\rho^2 = \frac{d^2}{8} = \frac{144}{8} = 18.$$

hence

$$P = \frac{80000 \times 34.5}{1 + \frac{1}{3000} \times \frac{41616}{18}} = 1560000 \text{ lbs.},$$

or 45,216 lbs. per sq. in. = 22.6 tons per sq. in.; and by a similar process for a 6-in. cylinder, metal thickness $\frac{1}{2}$ in.,

$$P = \frac{80000 \times 8.6}{1 + \frac{1}{3000} \times \frac{41616}{4.5}} = 168517 \text{ lbs.},$$

or 19,590 lbs. per sq. in. = 9.8 tons per sq. in., and with a factor of safety of 6, the safe loads per sq. in. would be 3.8 tons and 1.6 tons per sq. in. respectively. The above explains sufficiently the determination of the breaking loads of any given cylindrical column, and also the necessary area to bear any given load. For a hollow square cross-section, $\rho^2 = \frac{(\text{one side in inches})^2}{6}$, other values as above. For the hollow square with thick sides,

$$\square^h,\ h^2 \div 6; \quad \text{for the cross, } \overset{a}{\underset{b}{+}} h;\ \frac{\overline{ab}^2}{24} = \frac{h^2}{24};$$

$$\text{for angle-iron, } \overset{a}{\underset{b}{\text{L}}}{}_c,\ \frac{\overline{ab}^2 + \overline{bc}^2}{12(\overline{ab}^2 + \overline{bc}^2)}.$$

S in every case is the actual metal area of the cross-section in square inches. In cast-iron none but the circular cross-section, as used in the above examples, are employed as a rule. Instead of $a = \frac{1}{3000}$, as used above, Trautwine makes $a = \frac{1}{3200}$, which would give a little greater value for P.

CAST-IRON BEAMS.

865. The usual forms of cross-section for cast-iron beams are the trough or U shape, the tee T or double tee **I**, and these are only used when not exposed to severe blows and changes of temperature. They should be cast without sharp angles, which are lines of weakness, and there should be no sudden changes in thickness of metal in adjoining portions. Such changes should be gradual. As cast iron has a considerably greater coefficient of resistance to crushing than it has to tearing, the main portion of the metal should be so placed that it will be at that surface of the beam under tension; that is, the upper surface, if a cantilever or projecting beam, and at the lower surface, if supported at both ends.

866. Find the centre-breaking load of a cast-iron tee or single-flanged beam, as shown in Fig. 309.

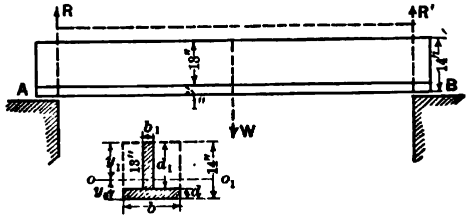


FIG. 309.

Clear length of span = 15 ft.; Total depth of beam = 14 in.; Web, 13 in. × 1 in. = 13 sq. in; Flange, $6\frac{1}{2}$ in. × 1in. = $6\frac{1}{2}$ sq. in.

The general formula $M_0 = mWl = \frac{fI}{y} m = \frac{1}{4}$.

First find the position of the neutral axis *oo*, in Art. XXXIII.

$$y_0 = \frac{\frac{bd^2}{2} + b_1d_1(d + \frac{1}{2}d_1)}{bd + b_1d_1},$$

$$b = 6.5; \quad d = 1, \text{ inches,}$$
$$b_1 = 1; \quad d_1 = 13 \quad ''$$

hence

$$y_0 = \frac{\frac{6.5 \times 1}{2} + 13(1+6.5)}{6.5 + 13} = 5.07 \text{ in.}$$

from bottom of beam, and $y_1 = 14 - 5.07 = 8.93$ in., from the top of the beam.

Assuming the unit resistance to tearing at 20,000 lbs. and to crushing at 80,000 lbs. per square inch, then, since $\frac{20000}{5.07}$ is less than $\frac{80000}{8.93}$, we make $\frac{f}{y} = \frac{20000}{5.07} = 3925$. And in same Art. XXXIII, equation (142), we find

$$I = \frac{1}{3}[b_1y_1^3 + by_0^3 - (b - b_1)(y_0 - d)^3];$$
$$b_1 = 1; \quad y_1 = 8.93; \quad b = 6.5; \quad y_0 = 5.07; \quad d = 1;$$
$$I = \frac{1}{3}[(8.93)^3 + 6.5(5.07)^3 - 5.5(4.07)^3] = 417.13.$$

Hence $mWl = \frac{1}{4}\,Wl = \frac{fI}{y} = 3925 \times 417.13 = 1{,}637{,}135$ in.-lbs., and $l = 15$ ft. $= 180$ in.; $\therefore\ W = 36{,}381$ lbs. $= 18.19$ tons of 2000 lbs., or 16.25 tons of 2240 lbs. The above includes the weight of the beam considered as uniformly distributed; $(13 + 6\frac{1}{2}) \times 180 = 3510$ cu. in. There are about 4 cu. in. in a pound, hence the beam weighs $\frac{3510}{4} = 877.5$ lbs., or 0.44 tons, and the equivalent centre load $= 0.22$ tons. The external breaking load, then, 17.97 tons, or 16.03 tons of 2240 lbs. The actual breaking weight, as given by Trautwine, is 12.5 tons, determined by experiment. Trautwine always uses the ton of 2240 lbs. In this work it will always be taken at 2000 lbs., unless otherwise specially noted. Using a factor of safety of 5, the safe load would be 3.6 tons, and if the load is uniformly distributed, safe load $= 7.2$ tons.

Unless the cast iron is of excellent quality its allowed resistance to tearing should not be more than 18,000 lbs., instead of 20,000, as used in the above example. On this basis the centre-breaking load would only be 16.23 tons.

867. Find the centre breaking load of a double-flanged cast-iron beam with the following dimensions (see Fig. 310): Length of

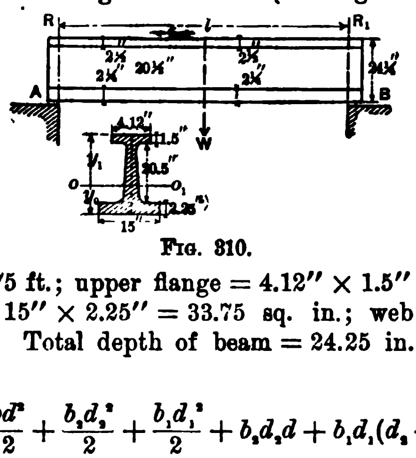


FIG. 310.

span $= l = 30.75$ ft.; upper flange $= 4.12'' \times 1.5'' = 6.18$ sq. in.; lower flange $= 15'' \times 2.25'' = 33.75$ sq. in.; web $= 20.5'' \times 1.5 = 30.75$ sq. in. Total depth of beam $= 24.25$ in. Referring to Art. XXXIII,

$$y_0 = \frac{\dfrac{bd^2}{2} + \dfrac{b_2d_2^2}{2} + \dfrac{b_1d_1^2}{2} + b_2d_2d + b_1d_1(d_2 + d)}{bd + b_2d_2 + b_1d_1}.$$

$b = 15''$; $d = 2.25''$; $b_2 = 1.5''$; $d_2 = 20.5''$; $b_1 = 4.12''$; $d_1 = 1.5''$.

Substituting, we find $y_0 = 8.0''$ and $y_1 = 24.25 - 8.0 = 16.25$ in.

$$I = \tfrac{1}{3}[by_0^3 + b_1y_1^3 - (b - b_2)(y_0 - d)^3 - (b_1 - b_2)(y_1 - d_1)^3].$$

Substituting above values and reducing, we find

$$I = 4771.95; \quad \text{since } \frac{20000}{8} < \frac{80000}{16.25}, \quad \frac{f}{y} = \frac{20000}{8} = 2500;$$

$$M_0 = \frac{fI}{y} = 2500 \times 4771.95 = 11{,}929{,}875 \text{ inch-pounds};$$

$$M_0 = mWl = \tfrac{1}{4} W \times (30.75 \times 12) = 11{,}929{,}875.$$

Hence $W = 129{,}321$ lbs. or 64.66 tons of 2000 lbs. or 57.8 tons of 2240 lbs. The total area of cross-section is 70.68 sq. in.; $70.68 \times 369 = 26{,}081$ cu. in.; 4 cu. in. to the pound gives 6520 lbs., or 2.26, equivalent centre weight = 1.13 tons; external load $64.66 - 1.13 = 63.53$ tons of 2000 lbs., or $57.8 - 1.13 = 56.67$ tons of 2240 lbs. Trautwine gives actual breaking weight 58.0 tons of 2240 lbs. The safe load in the above case $= \frac{63.53}{5} = 12.71$ tons. For a uniformly distributed load we have for the breaking load $63.53 \times 2 = 127.06$ tons, and safe load 25.41 tons. With the aid of a table of squares and cubes of numbers the above calculations can be made with ease and rapidity.

868. Trautwine gives the following empirical rule, known as Hodgkinson's rule:

Multiply area of bottom flange by the total depth in inches, and this product by the constant 2.166, and divide by the clear span in feet. This rule gives for the first case a single-flanged cast-iron beam. $W = \frac{6.5 \times 14 \times 2.166}{15} = 13.2$ tons of 2240 lbs. as compared with 16.03 tons by the general formula. Applying this rule to the beam supported at both ends, double-flanged cast-iron beam $W = \frac{15 \times 2.25 \times 24.75 \times 2.166}{30.75} = 58.5$ tons as compared with 57.8 tons. As these are the centre breaking weights, one half of the weight of the beam should be deducted. The uniformly distributed breaking load would be $13.2 \times 2 = 26.4$ tons and $58.5 \times 2 = 117.0$ tons, but the entire weight of the beam should be deducted in this case.

Knowing the load to be supported and the depth of the beam, we can find, from the above rule, the area of the bottom flange, and then the area of the top flange is taken as equal to one sixth of that of the bottom flange, as the resistance to crushing of

cast iron is about six times that of its resistance to tearing, and the web has the thickness of each flange where it joins them, changing gradually between the two. By an exactly similar process the strength of any form of cast-iron beam can be found, or a beam designed to carry any load.

869. The more common problem is the determination of the proper dimensions and form of beam to sustain safely the given load with the least amount of material. In this the given load should be multiplied by the factor of safety, which is from 4 to 6. This gives the value of W in the preceding formulæ. The clear length is known. The total depth is some fraction of the length varying from $\frac{1}{8}$ to $\frac{1}{18}$ of the clear span. The depth should be as great as practical considerations will allow so as to reduce the amount of material and also to reduce the deflection to a minimum. The allowable deflection may vary from $\frac{1}{260}$ to $\frac{1}{1200}$ of the span. Ordinarily $\frac{1}{40}$ of an inch for each foot of length or $\frac{1}{480}$ of the clear length will answer for beams of not very great length.

DEFLECTION OF CAST-IRON BEAMS.

870. For the single-flanged beam, Fig. 309: The value of the deflection for beams supported at both ends and loaded with a single weight at the centre,

$$v_1 = \frac{1}{EI}\frac{Wl^3}{48} = \frac{n''fl^2}{4Ey_0}. \quad . \quad . \quad . \quad . \quad . \quad (467)$$

Using the first form we have $W = 36{,}381$ lbs., $I = 417.13$, $E = 12{,}000{,}000$, $l = 15$ ft. $= 180$ in. Substituting, we find the value of v_1, all dimensions in inches. It will, however, be easier to use l_1 in feet $= 12l$, and eq. (1) becomes

$$v_1 = \frac{1}{EI}\frac{W\left(\frac{l_1}{12}\right)^3}{48} = \frac{36\,Wl_1^3}{EI}, \quad \text{safe load} = \frac{1}{5}W = \frac{36381}{5};$$

hence

$$v_1 = \frac{36 \times \frac{1}{5}(36381) \times (15)^3}{12000000 \times 417.13} = 0.177 \text{ inch},$$

greatest deflection under working load.

The proof load is from one half to one third the ultimate or breaking load, making $W = \frac{36381}{2}$, $v_1 = 0.442$ inch; and for the same beam under a uniformly distributed load equal in amount to W, take five eighths of the above deflection, that is, under safe load $v_1 = \frac{5}{8} \times 0.177 = 0.11$ in., and under proof load $v_1 = \frac{5}{8} \times 0.442 = 0.278$ in. Using the second part of eq. (1), $v_1 = \frac{n''fl_1^2}{4Ey_0}$, $n'' = \frac{1}{3}$, $f = \frac{20000}{5}$, $l = 180$ in., $E = 12{,}000{,}000$, $y_0 = 5.07$; and substituting, we find $v_1 = 0.177$, same as above.

For the double-flanged cast-iron beams (Fig. 310), centre-breaking load $W = 129{,}321$ lbs.; safe or working load $= 25{,}864$ lbs.; $l = 30.75$ ft.; $I = 4771.95$; $v_1 = \frac{1}{EI}\frac{Wl^3}{48} = \frac{36\,Wl_1^2}{EI} = \frac{n''fl_1^2}{4Ey_0}$.

$$\therefore\ v_1 = \frac{36 \times 25864 \times 29076}{12000000 \times 4771.95} = 0.473 \text{ in.},$$

or

$$v_1 = \frac{n''fl_1^2}{4Ey_0} = \frac{\frac{1}{3} \times 40000 \times 136161}{4 \times 120000000 \times 8} = 0.473 \text{ in.},$$

deflection under working or safe centre load. If the same load is uniformly distributed, $v_1 = \frac{5}{8} \times 0.473 = 0.296$ in. For proof load equal to two fifths breaking load $v_1 = 0.946$ in., and for proof load uniformly distributed $v_1 = \frac{5}{8} \times 0.946 = 0.591$ in.

For the same beams fixed at one end and loaded at the other, $M_0 = Wl = 1{,}637{,}135$ in.-lbs. Hence

$$W = 9095; \quad v_1 = \frac{1}{3}\frac{Wl^3}{EI} = \frac{n''fl^2}{Ey_0} = 0.71 \text{ in.}$$

for single-flanged cast-iron beam, making $W = \frac{9095}{5}$; $I = 417.13$; $E = 12000000$; $f = \frac{20000}{5}$; $y_0 = 5.07$; $l = 180$ in.; $n'' = 1/3$.

For a uniformly distributed load of the same amount

$$v_1 = \frac{1}{8}\frac{Wl^3}{EI} = \frac{n''fl^2}{Ey}.$$

All quantities as above, except n'', $= \frac{1}{8}$; then $v_1 = 0.265$ in. By making $W = \frac{2}{3} \times 9095$ we find the deflection just double the above under proof load; as the load is uniformly distributed, $n'' = \frac{1}{4}$. But m, which appears in the value of f, is $= \frac{1}{2}$; the coefficient then is $n''m = \frac{1}{8}$.

For the double-flanged cast-iron beam, if fixed at one end, we would find W, the breaking load, $= 32{,}330$ lbs.; safe load $= 6466$ lbs.; $l = 30.75 = 369.0$ in.; $v_1 = 1.89$ in. for a single weight at the end.

The last two cases are merely given for purposes of comparison, as projecting beams of such lengths as 30.75 ft. and only 2 ft. deep would never be used; even 15 ft. would be unusual. We see, for instance, that a beam, fixed at one end and loaded at the other, 30.75 ft. long has a deflection of 1.89 in. under a working load of 6466 lbs. If the load was made one half of 25,864, which is the centre safe load when the beam is supported at both ends, then the deflection, assuming the load to be under the proof load, would be $\frac{12932}{6466} \times 1.89 = 3.78$, which is exactly 8 times the deflection of the beam supported at both ends: $8 \times 0.473 = 3.78$ in. Which is equivalent to saying that if a single load at the end of a beam fixed at one end produces a certain deflection, it will require 16 times that same load to produce an equal deflection when the beam is supported at both ends and loaded in the centre. Similar comparisons can be made for other conditions of loading and supporting beams, with the same length and load. Taking, then, the deflection of a beam fixed at one end and loaded at the other as unity, we have: Fixed at one end, load W at the other, 1; uniformly loaded, $\frac{3}{8}$. Supported at both ends: load at centre, $\frac{1}{16}$; uniformly loaded, $\frac{5}{128}$. Which simply means that if under a given load the calculated deflection of a beam fixed at one end and loaded at the other is 2 in., then the same beam uniformly loaded will deflect $2 \times \frac{3}{8} = \frac{3}{4}$ in.; and if same beam is supported at both ends and loaded with the same weight at its centre the deflection will be $2 \times \frac{1}{16} = \frac{1}{8}$ in., and if uniformly loaded it will be $2 \times \frac{5}{128} = 0.078$ in.

871. The deflection of solid beams is found in an exactly similar manner, only being simpler in solution. Assuming the trestle stringer, as solid beams are seldom made of any material other than timber, each string-piece having a uniformly distributed load of 21,600 lbs., length of span $= 15$ ft. $= 180$ in.; $E = 800{,}000$ lbs.;

$b = 7.6$ in.; $d = 16$ in.; $I = \frac{1}{12}bd^3$; $y_0 = r = \frac{16}{2} = 8$ in.; $f = 1500$ lbs.; $l^3 = 5{,}832{,}000$; $l^2 = 32{,}400$; we have, after substituting,

$$v_1 = \frac{5}{8}\frac{1}{48}\frac{Wl^3}{EI} = \frac{n''f_1l^2}{4Ey_0};$$

and also recollecting that $n'' = \frac{5}{12}$, then there results $v_1 = 0.79$ in. Other conditions of loading and supporting timber beams can be similarly found by using the proper value of v_1, and for other materials by giving proper values to W_1, f_1, E, and I.

872. To find the ratio of depth to length in order that a timber beam shall not deflect more than $\frac{1}{480}$ of its length, that is, $\frac{1}{40}$ of an inch per foot of length, which for a beam 15 feet long, is not more than $\frac{15}{40}$ of an inch $= 0.37$ of an inch. Using $\frac{d}{l} = \frac{n''f_0}{4Em'}\frac{l}{v_1}$, $n'' = \frac{5}{12}$; $f_0 = 1000$; $E = 800{,}000$; $m' = \frac{1}{2}$; $\frac{l}{v_1} = 480$.

Then

$$\frac{d}{l} = \frac{5}{12} \times \frac{1000 \times 480}{\frac{1}{2} \times 800000 \times 4} = \frac{1}{8}; \quad \therefore\ d = \frac{l}{8} = \frac{180}{8} = 22.5 \text{ in.}$$

If, however, we apply the above formulæ, allowing a safe working strain of 1500 lbs., thereby increasing the safe load to 21,600 lbs., as used in the preceding paragraph, a much deeper beam would be required. The reason that $\frac{1}{480}$ of the length was adopted was because this amount of deflection is so little that it will not crack plaster on ceilings. Under other circumstances a smaller value for $\frac{l}{v_1}$ could be used. If this were reduced to $\frac{1}{300}$ of its length, or 0.6 in., the required depth would be $\frac{d}{l} = \frac{1}{12.8} = 14.06$ in. for the depth of the beam. From $\frac{1}{8}$ to $\frac{1}{12}$ of the length is probably the usual and best practice, giving an average of $\frac{1}{10}$ for ordinary purposes. The above calculations are for beams supported at both ends and uniformly loaded.

873. As timber beams are often used under walls of houses, it is often required to know what projection beyond the wall is allowable, using a timber of square cross-section. This is a case of a beam fixed at one end and acted upon by pressures which are the resistances of the foundation-beds upon which the timber

rests. In this case it is required to know the value of l in terms of the depth, and we have $\frac{l}{d} = \frac{Em'}{n''f_0}\frac{v_1}{l}$. Making $\frac{v_1}{l} = \frac{1}{480}$, and other quantities as before, we have $\frac{l}{d} = \frac{800000 \times \frac{1}{2}}{\frac{1}{8}1000 \times 480}$. $\frac{l}{d} = 2.5$, if $d = 12$ in., and $l = 30$ in. On either side of the wall, if it is 2 ft. thick, the actual base could be spread over an area at least 7 ft. wide, or if a maximum strain of 1000 lbs. is allowed, and $\frac{v_1}{l} = \frac{1}{300}$, which would be within amply safe limits, $\frac{l}{d} = 4$, or $l = 4d$; if d is 12 in. and $l = 4$ ft., the spread of base would then be 10 ft. wide. It is evident that by using several layers of smaller timbers crossing each other so as to keep the projections within a limit of 2 ft. a spread of base can be secured safely to any desired extent.

874. Before determining the ratio of depth to length for cast-iron beams, it will be better to determine the proper proportions of the flanges and web so that they shall have equal strength above and below the neutral axis, as it is evident that the strength of a beam is that of its weakest part; and if any other part is stronger, it necessarily involves a waste of material, and consequent useless cost, in addition to making the dead weight more than necessary. For these reasons iron and steel beams are seldom, if ever, of a solid rectangular section; the metal is so placed that each unit of area performs its full duty as far as practicable. Few materials have the same resistance to crushing as they have to tearing; as the resistance to bending consists in resistance to crushing of the fibres at and near the extreme upper and tearing of the lower layers, or of tearing of the upper and crushing of the lower layers, respectively, according as the beam is supported at both ends or only fixed at one end, the web usually being assumed only to resist the shearing force and to properly connect the flanges together. In any given beam, then, the material, to resist the bending action, is concentrated in the upper and lower flanges, in the inverse ratio of their resistances to crushing and to tearing per square inch. The web, whether composed of solid plates or open work, is then proportioned so as to connect rigidly the flanges and bear the shearing strain. In ordinary beams practical requirements in manufacture or construction will always provide a sufficiency of material in the web to carry safely its portion of the strain, and as in beams under

transverse strain the intensity of the internal stress is assumed to vary uniformly from zero at the neutral axis to its maximum at the upper and lower surfaces, it is evident that the following relations between the intensities and their respective distances from the neutral axis must exist:

Let y_0 = the distance of the extreme fibre from the neutral axis on the extended side;
y_1 = the distance of the extreme fibre from the neutral axis on the compressed side;
$y_0 + y_1 = d$ = total depth of beam between extreme fibres, or, as sometimes taken, between centres of gravity of the flanges, and sometimes in built beams between centres of rivet-holes in the two flanges;
f_1 = modulus of resistance to crushing per square inch;
f_0 = modulus of resistance to tearing per square inch.

Then

$$f_0 + f_1 : f_0 : f_1 :: d : y_0 : y_1. \quad . \quad . \quad . \quad . \quad (468)$$

Also, let A_1 = the area of the compressed flange;
A_2 = the area of the web;
A_3 = the area of the extended flange.

As the only horizontal stresses are the compressive on one side and tensile on the other, of the neutral axis, the conditions of equilibrium require that the total compressive stresses must be equal to the total tensile stresses, and that they must act in opposite directions; in other words, their algebraic sum must be equal to zero. Taking first the single-flanged cast-iron beam (see Fig. 311), that portion above the neutral axis oo_1, in beams supported at both ends, and below when fixed at one end, will be under compression; f_1 is the greatest proof or working strain in compression per square inch, its mean intensity is $\frac{f_1}{2}$; and as the thickness of the flange is small as compared to the total depth of the beam, we can without sensible error assume the web to extend to the extreme lower surface. The total compression uniformly distributed over the area of the web is, then, $A_2\frac{f_1}{2}$. Similarly, the

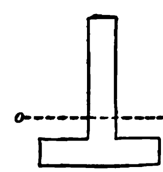


FIG. 311.

mean intensity of the tensile stress distributed uniformly of the same area $= A_2 \frac{f_0}{2}$, and the resultant stress $A_2 \frac{f_1}{2} - A_2 \frac{f_0}{2} = \frac{A_2(f_1 - f_0)}{2}$. This difference is compressive, as f_1 for cast iron is greater than f_0. The total tensile stress in the bottom flange $= A_3 f_0$. Hence for equilibrium $A_3 f_0 = \frac{A_2(f_1 - f_0)}{2}$. Hence

$$A_3 = \frac{f_1 - f_0}{2f_0} A_2, \quad \ldots \ldots \quad (469)$$

which is the required relation between the lower flange and the web.

The relation in eq. (468), if we assume the resistance to crushing at 80,000 lbs. and the resistance to tearing at 20,000 lbs. per sq. in. is 100,000 : 20,000 : 80,000 :: $f_0 + f_1 : f_0 : f_1$, or 5 : 1 : 4 :: $f_0 + f_1 : f_0 : f_1$. Substituting in eq. (469), $A_3 = \frac{4-1}{2} A_2 = 1\frac{1}{2} A_2$; or, assuming $f_0 = 16{,}000$ and $f_1 = 80{,}000$,

$$A_3 = \frac{5-1}{2} A_2 = 2A_2. \quad \ldots \ldots \quad (470)$$

For the double-flanged cast-iron beam a similar course of reasoning gives, as the resultant compression on the web, $= \frac{A_2(f_1 - f_0)}{2}$, compression on upper flange $= A_1 f_1$; and tension on lower flange $= A_3 f_0$. Hence, for equilibrium,

$$A_3 f_0 - \left(A_1 f_1 + \frac{A_2(f_1 - f_0)}{2}\right) = 0; \quad \therefore A_3 = \frac{f_1}{f_0} A_1 + \frac{f_1 - f_0}{2f_0} A_2. \quad (471)$$

Substituting the above values or ratios of f_0 and f_1, we have

$$\left.\begin{aligned} A_3 &= \frac{4}{1} A_1 + \frac{4-1}{2} A_2 = 4A_1 + 1\tfrac{1}{2} A_2; \\ A_3 &= \frac{5}{1} A_1 + \frac{5-1}{2} A_2 = 5A_1 + 2A_2. \end{aligned}\right\} \quad . \; . \quad (472)$$

If, as is usually the case, the web is not to be considered as resisting the bending action, we must include A_2 in A_1. Then

$$A_3 = 5\tfrac{1}{2} A_1 \text{ or } 7A_1. \quad \ldots \quad (473)$$

In words, the extended flange A_2 must be from $1\frac{1}{2}$ to 2 times the area of the web in single-flanged beams, and from $5\frac{1}{2}$ to 7 times the area of the compressed flange in double-flanged beams. The web is then made the same thickness as the flanges at top and bottom, gradually changing between these limits. It must be remembered, however, in thus proportioning the areas of the flanges, that thick castings, that is, when they are from 2 to 3 in. thick, are relatively weaker in proportion than thinner ones, and smaller values of f_0 and f_1 are to be used in finding the breaking load when beams have very thick flanges.

It is evident that in the two cases of cast-iron beams whose breaking loads were determined (Figs. 309 and 310) the single-flanged beam is not well proportioned according to the above requirement (that is, $A_2 = 1\frac{1}{2}A_3$), as in that beam $A_2 = 6.5$ in. and $A_3 = 13$ sq. in.; but if it means that, practically, that portion of the web above the neutral surface is A_3, and the portion below is a part of the lower flange, then $A_3 = (14 - 5.07) \times 1 = 8.97 \times 1 = 8.97$ sq. in. and the lower part $= 13 - 8.97 = 4.03$, and $4.03 + 6.5 = 10.53$ sq. in., and $A_2 = 1.17A_3$.

The beam with two flanges, Fig. 310, gives $A_2 = 33.75$ sq. in. and $A_1 = 6.18$ sq. ins., and we find $A_2 = 5.46A_1$, which agrees fairly well with the previous value.

875. If the beams are not well proportioned, we find the value of the ratio of depth to length of span, for beams fixed at one end,

$$\frac{d}{l} = \frac{n''f_0 \text{ (or } f_1)}{Em'} \frac{l}{v_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (474)$$

and for beams supported at both ends,

$$\frac{d}{l} = \frac{n''f_0 \text{ (or } f_1)}{4Em'} \frac{l}{v_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (475)$$

using whichever is the smaller value, f_0 or f_1, according to the material of the beam. But as beams should be properly proportioned, we will convert the above equations into forms applicable only to equal strength above and below the neutral axis.

From the relation $f_0 + f_1 : f_0 : f_1 :: d : y_0 : y_1$,

$$\frac{f_0}{f_0 + f_1} = \frac{y_0}{d} \quad \text{or} \quad \frac{f_1}{f_0 + f_1} = \frac{y_1}{d}. \quad \text{But} \quad \frac{y_0}{d} = m'; \quad \therefore f_0 = m'(f_0 + f_1).$$

Substituting in equations (474) and (475), there results:

For a beam fixed at one end,

$$\frac{d}{l} = \frac{n''(f_0 + f_1)}{E} \frac{l}{v_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (476)$$

for a beam supported at both ends,

$$\frac{d}{l} = \frac{n''(f_0 + f_1)}{4E} \frac{l}{v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (477)$$

The factor n'' depends on the manner of supporting and loading the beam, $\frac{f_0}{E}$ or $\frac{f_1}{E}$ being the proof strain or working strain of the material, $\frac{l}{v_1}$ the ratio of length of span to the allowed deflection, and $\frac{d}{l}$ the ratio of the length to the depth, so that the deflection may not be exceeded.

In using the above equations—and, in fact, all of the equations involving n'' and f_0 or f_1—it must be borne in mind that if we simply desire to limit the values of f_0 and f_1 to a certain number of pounds per square inch we are not concerned as to what may be the value of the actual load or of the bending moment, both of which quantities will be different for each mode of supporting and loading the beam. We have, then, only to substitute the determined or assumed values of the several quantities. But if we desire to determine the deflection caused by any given load on beams fixed at one end or supported at both ends, the same load being used in each case and the same load either as a single or uniformly distributed load, it then becomes necessary to assume that the value of m, the constant in the bending moment mWl, is contained in f_0 or f_1, as the maximum strain per square inch is different for the same load when applied at one end of a beam and fixed at one end or when uniformly distributed, and also when supported at both ends and loaded at the centre or uniformly loaded. Bearing the above principle in mind, it is evident that these forms of equations are of great convenience and simplicity, as they do not require the actual load to be known until the depth of the beam has been determined. Then, substituting the same value of f in $mWl = \frac{fI}{r}$, we find the load which can be placed on the beam.

876. We will now apply equation (477) to the two cast-iron beams in Figs. 309 and 310, assuming that they are properly proportioned. In the first the beam is a single-flange cast-iron beam of an

 cross-section, loaded in the centre and supported at both ends. $n'' = \frac{1}{3}$; f_0 varies from 16,000 to 20,000 lbs.; f_1 varies from 80,000 to 100,000 lbs. Taking average values and allowing a factor of safety of 5, $f_0 = \frac{1}{5} \times 18{,}000 = 3600$ lbs.; $f_1 = \frac{1}{5} \times 90{,}000 = 18{,}000$ lbs. E varies from 16,000,000 to 20,000,000; average, 16,000,000 lbs. per square inch. Assume $v_1 = \frac{1}{600} l$; $\therefore \frac{l}{v_1} = 600$. Then

$$\frac{d}{l} = \frac{n''(f_0 + f_1)}{4E} \frac{l}{v_1} = \frac{1}{12} \frac{(3600 + 18000)}{16000000} \frac{600}{1} = \frac{108}{1600} = \frac{1}{14.8};$$

$$\therefore d = \frac{180}{14.8} = 12.16 \text{ inches } (12\tfrac{1}{8} \text{ inches, nearly}).$$

If this same beam is uniformly loaded so as to produce the same strains as above, $n'' = \frac{5}{12}$, and

$$\frac{d}{l} = \frac{5}{48} \frac{21600 \times 600}{16000000} = \frac{1}{11.85}; \quad \therefore d = \frac{180}{11.85} = 15.2 \ (= 15\tfrac{3}{16}) \text{ inches,}$$

as the proper depth. This at first sight seems inconsistent, requiring a deeper beam when uniformly loaded than when loaded at the centre; but, as explained in the preceding paragraph, to produce the same strains in the uniformly loaded beam requires twice the load, as compared with a single centre load, to give the same values of f_0 and f_1. If the same load is spread uniformly over the beam, we should reduce $(f_0 + f_1)$ to $\frac{1}{2}(f_0 + f_1)$; then $\frac{d}{l} = \frac{1}{23.7}$ or $d = 7.6$ inches for the depth of the uniformly loaded beam, as seen by the fact that $7.6 = \frac{5}{8} \times 12.16$, since the deflection under a uniform load is only five eighths of that caused by the same load concentrated at the centre of the beam. The actual total depth in this case was 14 inches.

For the double-flanged cast-iron beam, where the length of the span is 30.75 feet, single centre load,

$$\frac{d}{l} = \frac{1}{12} \frac{21600 \times 600}{16000000} = \frac{1}{148}; \quad \therefore d = \frac{30.75 \times 12}{14.8} = 24.93 \text{ inches;}$$

and for the same load uniformly distributed, $\frac{5}{8} \times 24.93 = 15.58$ inches. Or using

$$\frac{d}{l} = \frac{5}{48} \frac{10800 \times 600}{16000000} = \frac{1}{23.66}; \quad \therefore d = 15.58 \text{ inches},$$

making $n'' = \frac{1}{3}$ in the first case and $\frac{5}{12}$ in the second, and halving $(f_0 + f_1) = \frac{21600}{2} = 10{,}800$ lbs.

If, however, the strains are to remain the same, which is equivalent to doubling the amount of the load, then

$$\frac{d}{l} = \frac{5}{48} \frac{21600 \times 600}{16000000} = \frac{1}{11.85}; \quad \therefore d = 31.39 \text{ inches}.$$

877. For a projecting beam, as in foundations where the base is spread by beams which are built in concrete,—which is not an unusual mode of increasing the area of bearing surface under high and heavy buildings,—in this case the beam is fixed at one end, and the resistances acting upward are the equivalent of a uniformly distributed load. In such cases, assuming $\frac{l}{v_1} = 400$, $n'' = \frac{1}{4} f_0 + f_1 = 21{,}600$, and substituting in eq. (476), we have

$$\frac{d}{l} = \frac{n''(f_0 + f_1)}{E} \frac{l}{v_1} = \frac{1}{4} \frac{21600 \times 400}{16000000} = \frac{54}{400};$$

$$\therefore l = \tfrac{400}{54} d, \quad \text{if } d = 15 \text{ in.,} \quad l = 111.11'' = 9.28 \text{ feet}.$$

In such cases, however, it would be better to use beams of less depth and length with shorter projections, and obtain the desired spread by the use of several courses or layers crossing each other at right angles. Old iron rails, thus used in three or four courses, will answer every purpose, and are less costly than cast or rolled beams. For full discussion and application of this principle, see the writer's work on Foundations. Cantilever beams are also largely used for supporting projecting roofs, platforms, balconies, etc.

In the following paragraphs wrought-iron and steel rolled and built beams will be discussed. Very satisfactory and extended tables are given of the strength of such beams in many forms of cross-section, in compact little volumes published by the iron and steel companies. The writer is particularly indebted for much

useful information to the work issued by Cooper, Hewitt & Co., of the New Jersey Steel and Iron Co., Trenton, N. J., and gladly recommends it to young engineers.

WROUGHT-IRON AND STEEL ROLLED BEAMS.

878. The difference between the resistance per square inch to crushing and to tearing is not nearly so great in wrought iron as it is in cast iron, the usual values being 40,000 and 50,000 lbs., respectively. The resistances are more uniform, and can be much better relied upon. The more common form of solid wrought-iron beams is usually known as the rolled I-beam, which is a double-flanged beam with equal flanges. Taking what may be called the extreme differences in the values of f_0 and f_1, viz., $f_0 = 60{,}000$, $f_1 = 36{,}000$, lbs., then $f_0 + f_1 : f_0 : f_1 :: d : y_0 : y_1$, and $8 : 5 : 3 :: d : y_0 : y_1$. As the resistance in this material to tearing is greater than its resistance to crushing, f_0 is greater than f_1, and the compressed flange must have a greater area than the extended flange. Hence the relation between the flange and web will be, for a single-flanged beam,

$$A_1 = \frac{f_0 - f_1}{2f_1} A_2 = \frac{5-3}{2 \times 3} A_2 = \tfrac{1}{3} A_2; \quad . \quad . \quad . \quad (478)$$

and for the double-flanged beam,

$$A_1 = \frac{f_0}{f_1} A_2 + \frac{f_0 - f_1}{2f_1} A_2; \quad A_1 = \frac{5}{3} A_2 + \frac{5-3}{6} A_2 = \frac{5}{3} A_2 + \tfrac{1}{3} A_2. \quad (479)$$

These are exactly similar forms as found in cast-iron beams, except that f_0 in this case being the greater is placed first, and the compressed flange A_1 is found, instead of the extended flange. In this material also the flanges are assumed to resist the bending entirely, and the web the shearing strain. A_3 should then be included in A_2, and eq. (479) becomes

$$A_1 = 2A_2 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (480)$$

879. Single-flange wrought-iron beams are not often used, and although theoretically the compressed flange should be from $1\frac{1}{2}$ to 2 times the area of the extended flange, the usual practice is to give the two flanges the same area, or nearly so. Although nearly every iron and steel company has its own designs and standard cross-sections and dimensions, there is probably no material difference.

The rolled beams vary in depth from $1\frac{1}{4}$ in. to 20 in. and in width of flange from $1\frac{1}{2}$ in., thickness of flange $\frac{1}{8}$ in., with web thickness of $\frac{1}{8}$ in., to width of flange $6\frac{3}{4}$ in., thickness $1\frac{3}{4}$ in., and web thickness $\frac{3}{4}$ inch. These weigh from 1.75 lbs. to 100 lbs. per linear foot. The web is of some uniform thickness, and connected with the flanges by curved surfaces. The flanges decrease in thickness from their junction with the web to their outer edges, sometimes by curved, sometimes by plane, surfaces. The thickness at the outer edges varies from $\frac{1}{8}$ to $\frac{5}{8}$ ins. Although beams can be rolled of any form and dimensions required, it will be generally found advisable and economical to use when practicable the nearest sizes of the standard beams, rather than insist upon having a beam differing in form or dimensions by almost imperceptible changes. The smaller-sized beams are usually made of steel, the larger sizes of either steel or iron.

Assume a rolled beam, Fig. 312, of the New Jersey Steel and Iron Co., Trenton, N. J., with the following dimensions: Breadth b of flanges $= 5\frac{3}{4}$ in., thickness at web $= 1\frac{5}{8}$ in., at outside edge $\frac{19}{32}$ in. Thickness of web $\frac{19}{32}$ in. (0.6 in. nearly). To find the centre breaking load, we have $M_0 = mWl = \frac{fI}{y}$. $m = \frac{1}{4}$; $y = \frac{1}{2}d = 7.5$ in.; $l = 20$ ft. $= 240$ in.; $f_1 = 36{,}000$ lbs.; moment of inertia $I = \frac{1}{12}(bd^3 - b_1d_1^3)$; $A = bd - b_1d_1$. Using average values, $b = 5.75$; $d = 15$; $b_1 = 5.75 - 0.6 = 5.15$; $d_1 = 15 - 2[\frac{1}{2}(\frac{19}{32} + \frac{13}{8})] = 15 - 2.22 = 12.78$ inches. Hence $I = 721.37$; $A = 20.43$ sq. in.

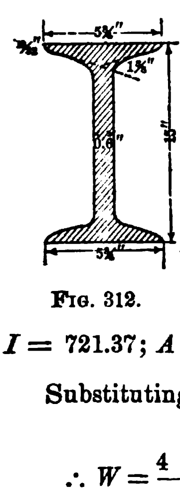


Fig. 312.

Substituting in $M_0 = \frac{1}{4}Wl = \frac{fI}{y} = \frac{36000 \times 721.37}{7.5}$,

$$\therefore W = \frac{4 \times 36000 \times 721.37}{240 \times 7.5} = 57{,}710 \text{ lbs.} = 28.85 \text{ tons}$$

for centre breaking load, and safe load $= \frac{1}{5} \times 28.85 = 5.77$ tons, and for uniformly distributed load $= 2 \times 5.77 = 11.54$ tons. This only allows a working strain of $\frac{36000}{5} = 7200$ lbs. per sq. in., which is sufficiently small. The weight of wrought iron is equal to 480 lbs. per cu. ft. Hence, for a prism of one square foot base and one yard long the weight would be $480 \times 3 = 1440$ lbs., and as the area of its base is 144 sq. in., the weight of any bar or solid beam

is ten times its area in square inches, per yard of its length. The beam Fig. 312 containing 20.43 sq. in., its weight per yard will be $20.43 \times 10 = 204.3$ lbs. per yard, or 68.1 lbs. per foot of length. The total weight of the beam is then $68.1 \times 20 = 1362$ lbs. = 0.68 ton. The above breaking weight $W = 28.85$ tons includes the weight of the beam; but as the weight is uniformly distributed the equivalent centre load would be $\frac{1}{2} \times 0.68 = 0.34$ ton. Hence the external breaking load $= 28.85 - 0.34 = 28.51$ tons. In case of the uniformly distributed load, $W = 57.70$ tons, we should deduct the entire weight of the beam. Hence, $57.70 - 0.68 = 57.02$ tons. The centre safe load being $\frac{57710}{5} = 11{,}542$ lbs., each reaction would be 5771 lbs., which is also the greatest shearing force; and as the depth of the web is 12.78 in. and its thickness 0.6 in., its area is $12.78 \times 0.6 = 7.67$ sq. in., or a strain of only $\frac{5771}{7.67} = 752$ lbs. per sq. in. for the single centre load, or $752 \times 2 = 1504$ lbs. in case of the uniformly distributed load. An allowance of 4000 lbs. would be amply small. We see then, as before stated, that the thickness of the web required by considerations of a practical nature will usually provide sufficient area of web to resist the shearing action.

880. For a steel beam of the same dimensions as the iron beam Fig. 312 the ultimate value of f would be about 75,000 lbs., and safe working strain 15,000 lbs., or about double that of iron. Hence breaking load, including weight of beam, $W = 28.85 \times 2 = 57.70$ tons, or a safe load of 11.54 tons for a single centre load, or 23.08 tons for a uniformly distributed load.

A steel beam that would have about the same strength as the iron beam Fig. 312 would have about the following dimensions:

Breadth of flanges, 5.75 inches; thickness at web 0.95 and at outer edge 0.55 inch; thickness of web, 0.45 inch; depth of beam, 15 inches; metal area, 12 square inches; weight per foot of length, 50 pounds.

DEFLECTION.

881. To find the deflection of the beam Fig. 312 under its safe or working load, at centre, we have $v_1 = \frac{36 Wl^3}{EI}$, l being in feet, $W = 11542$ lbs., $E = 28{,}000{,}000$ lbs., and $I = 721.37$. Substituting, $v_1 =$ 0.165 inch and for the same load uniformly distributed $v_1 = \frac{5}{8} \times 0.165 = 0.103$ inch. But if we use the safe uniformly distributed

load equal to $2w$, the deflection would be $\frac{5}{8} \times 2v_1 = \frac{5}{8} \times 0.33 =$ 0.206 inch.

Or, using the form $v_1 = \frac{n''f_0l^2}{4Ey_0}$, $n'' = \frac{1}{3}$ for the single centre load, $f_0 = 7200$, $l = 20$ ft. $= 240$ inches, $y_0 = \frac{1}{2}d = 7.5$, and substituting, $v_1 = 0.163$ inch.; and for a uniformly distributed load $= 2w$, with $n'' = \frac{5}{12}$, other values as before, then $v_1 = 0.204$ inch. The load $W = 11,542$ is a small fraction too large, otherwise the results by the two equations would give exactly the same results.

For ordinary purposes a safe value of f is from 10,000 to 12,000 lbs.; and substituting in

$$\frac{d}{l} = \frac{n''f}{4m'E}\frac{l}{v_1}, \text{ making } \frac{l}{v_1} = 1000,\ n'' = \tfrac{1}{3}, m' = \tfrac{1}{2}, f = 10,000;$$

$$\therefore \frac{d}{l} = \frac{1}{3}\,\frac{10000 \times 1000}{4 \times \frac{1}{2} \times 28000000} = \frac{10}{168};$$

$$\therefore d = \frac{240 \times 10}{168} = 14.28 \text{ inches depth of beam.}$$

Or if uniformly loaded so as to produce a strain of 10,000 lbs., requiring twice the load in the above case, $\frac{d}{l} = \frac{n''f}{4m'E}\frac{l}{v^1}$, $n'' = \frac{5}{12}$, other values as before, and $d = 17.86$ inches; or for one half of this load we have $\frac{1}{2} \times 17.86 = \frac{5}{8} \times 14.28 = 8.93$ inches for depth of beam. Had we used the expression for beams of equal strength, $\frac{d}{l} = \frac{n''(f_0 + f_1)}{4E}\frac{l}{v_1}$; making $f_0 = 15,000$ and $f = 10,000$, other values as before, $d =$ 14.88 inches, instead of 14.28 inches. This results from the close approximation of equal-flanged wrought-iron and steel beams to the properly proportioned beams giving equal strength above and below the neutral axis.

The above will apply to any form of rolled beams. Such beams are used for the cross-beams or girders of large houses, floor-beams of iron trussed bridges, as well as the longitudinal girders of short-span bridges, from 10 to 25 feet, one, two, or three such girders being placed under each rail and bolted together with packing-blocks between, those under the two rails being connected by struts and diagonal tension-rods, the lateral struts being small channels

or angle-irons latticed. In all of the above examples the clear length of span l is given; the total length includes the bearing on the points of support, which will vary from 6 to 9 inches at each end. For a span of 20 feet the total length of the beam will be from 21 to 21.5 feet, often as much as 22 feet.

The above discussion of beams has been made without reference to their stability against lateral deflection, the assumption being made that proper width of flange has been given to resist lateral deflection under the allowed loads and strains. But beams are often if not generally braced laterally to a greater or less extent, for whatever purpose used, which would materially increase the strength of any given beam. Especially is this the case in the usual fire-proof floors for buildings, which consist of a series of beams, usually I-beams, spaced from 4 to 5 feet apart; between these beams small arches, of common brick, or blocks, or voussoirs of hollow fire-clay brick or tiling, are constructed, over which is placed a thin layer of cement concrete or mortar, or other suitable flooring material, on which the ordinary flooring-plank may be placed; experiment has shown that a section of this hollow arch, 18 inches long or wide, 8 inches thick, and 5 feet span, will carry without breaking over 9000 pounds, allowing 300 pounds per square foot (double the usual load). The greatest load that could come upon this section of arch would be $300 \times 1.5 \times 5 = 2250$ pounds, which allows a wide margin of safety.

Such arches give almost absolute rigidity laterally to floor-beams, and as under such conditions the beams are not subjected to any severe shocks or great vibrations, the actual strength of beams will be very much greater than those allowed in the preceding examples. The following table illustrates both the relative and actual safe loads on a few beams of ordinary dimensions, according as they are supported or unsupported laterally.

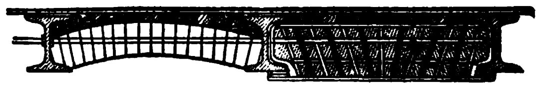


FIG. 312a.

Fig. 312a gives a view of the two forms of fire-proof flooring, of the common and fire-clay hollow blocks, in which the beams have good lateral support. Then the safe, uniformly distributed loads on the beams, in tons of 2000 pounds, will be:

									Supported, tons.	Unsupported, tons.
For	heavy	20	inch	iron	beams,	span	15	feet,	43.40	36.5
"	"	20	"	"	"	"	25	"	25.5	16.3
"	"	15	"	"	"	"	15	"	24.4	19.4
"	"	15	"	"	"	"	25	"	14.1	8.0
"	"	12½	"	"	"	"	15	"	16.6	13.0
"	"	12½	"	"	"	"	25	"	9.5	5.2
"	"	9	"	"	"	"	20	"	4.7	2.5
"	"	8	"	"	"	"	20	"	3.9	2.2
"	"	6	"	"	"	"	15	"	2.4	1.4

For very extended and useful tables, see Cooper, Hewitt & Co.'s book.

Some actual experiments on rolled beams gave the following results:

A heavy 9-inch beam, 14.93 feet span, actual centre load of 32,000 pounds, and failed with this load; safe load = 6400 pounds; actual deflection under safe load, 0.16 inch; elastic limit, 22,000 pounds.

A light 15-inch beam, load uniformly distributed, actual load 90,000 pounds, deflected 2.7 inches, but did not break; calculated safe load, 25,000 pounds; deflection under safe load, 0.36 inch; span, 21.0 feet. The heavy 15-inch beam gives for calculated safe load about 34,000 pounds, with a deflection of 0.42 inch.

A 6-inch light beam, 12 feet span, loaded at centre, failed with 11,000 pounds; elastic limit, 7000 pounds; safe load, 2600 pounds; actual deflection, 0.3 inch. Another 6-inch beam, light, failed with 17,000 pounds; elastic limit, 11,000; safe load, 2624 pounds; deflection under safe load, 0.15 inch; span, 15 feet. On the heavy 6-inch beam, the calculated load when supported laterally, safe load as given in table, was about 2430 pounds for equivalent centre load and when unsupported 1400 pounds. From these as well as from other experiments it is evident that the formulæ with the values used in practice for E, f, and W, give results well under reasonable proof-loads, to say nothing of actual breaking-loads.

WROUGHT-IRON AND STEEL BUILT BEAMS.

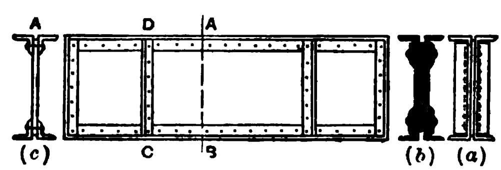
882. Such beams are more commonly used as the floor-beams of large trussed bridges, or the longitudinal girders of iron viaducts with spans from 30 to 60 feet long. If used for any of the purposes

for which cast iron or rolled beams are used, the same general principlės would apply. Therefore in this article built beams will be treated solely as forming the above-mentioned parts of bridges and viaducts.

Wrought iron and steel are rolled in many forms, such as the I-beams, ; channels, ; angle-irons, ; zees, ; tees, . Any of these forms can be used singly, either as beams or columns; but except the I-beams, and perhaps the zee form, both columns and beams are built of two or more of these forms, connected by plates or lattice strips. Built beams are also composed of plates connected by angle-irons, to which they are riveted; and to this form of built beams this article will specially apply. It is an easy matter to properly proportion the parts of this form of beam to the strains upon them by varying the areas of the plates and angles.

BUILT BEAMS.

883. Wrought-iron beams, as for columns, are built up of plates, channels, angles, zees, etc., commonly for the girders of iron viaducts, floor-beams, stringers, etc. They are composed of plates and angles, as shown in the following diagrams. (Figure 313, side view; figure *a*, end view) of a girder of 30-feet span, used on the approaches of the Point Pleasant bridge mentioned above.



FIGS. 813.

Figure 813, side view; figure (*a*), end view; figure (*b*), sectional view, *DC*; figure (*c*), sectional view, *AB*.

The dimensions of the parts of this girder are as follows:

Length, 30 feet; depth, 36 inches. Top flange, one plate 12 × $\frac{7}{16}$ inches, 5.25 square inches. Two angle-irons $3\frac{1}{2}$ × 5 inches, 10.2 lbs. per foot each, 6.12 square inches. Web plate, 36 × $\frac{3}{8}$ inches, 13.5 square inches. Bottom flange, two angle-irons, $3\frac{1}{2}$ × 5 inches, 16.6

lbs., 9.96 square inches. Three angle-irons near the ends, as seen in diagram figure (313d). The extreme end, two angle-irons 3×4 inches, 8.3 lbs.; the other two, two angle-irons $3 \times 2\frac{1}{2}$ inches, 4.4 lbs., as stiffeners for the web. We will now determine the bending moment, shearing stress, and required dimension of the parts of the beam to bear safely the load. The dead load consists of cross-ties 8×10 inches $\times$ 12 feet, spaced 1 foot centres; guard-rails, 6×8 inches inside of rail and 8×12 inches on the outside; two rails each 60 lbs. per linear yard, and the weight of the girders.

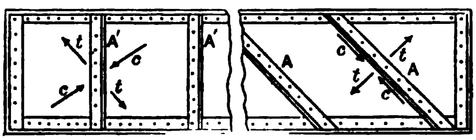


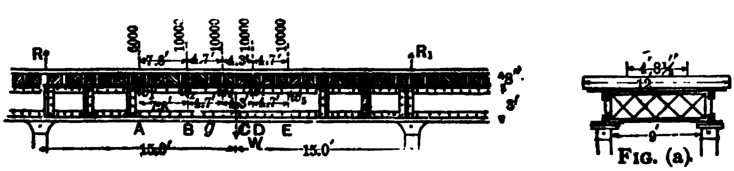
FIG. 313 (d).

Each cross-tie is 80 feet B.M., inside guard-rail 4, outside guard-rail 8, feet B.M. per foot of length. The two rails 40 lbs. per foot, the two girders 340 lbs. per foot. The partial weights of angles, etc. are given in pounds per foot.

Then we have the total dead load per foot of trestle:

Ties and guard-rails, 92 ft. B.M. @ 4 lbs.,	= 368 lbs.		
2 rails, 60 lbs. per yard each, per foot 2×20,	= 40 "		
2 girders, each 170 lbs. per foot,	= 340 "	—	748 lbs.
The dead load on each girder, $\frac{748}{2} \times 30$,	=		11,220 "

which being uniformly distributed, the bending moment at the centre of the span will be $mWl = \frac{1}{8} \times 11{,}220 \times 30 = 42{,}075$ ft.-lbs. As an illustration of the action of several isolated loads on a girder we will suppose the trestle to be loaded with a locomotive, with the weights on each wheel of engine and tender as shown in the following diagram, Fig. 314, which shows also the distances between the



FIGS. 314.

wheel centres, and so placed on the girder as to produce maximum bending moment.

It is seen that we can only get on a girder 30 feet long the front truck and the four driving-wheels of the engine. We will now show the method of determining the position of this load or loads to produce a maximum bending, and the point of greatest bending moment. In any system of isolated loads on a beam supported at both ends it can be shown, by a perfectly simple demonstration, that for the maximum bending the distance between the centre of gravity or point of action of the resultant of all of the loads and the point of greatest bending is bisected by the centre of the span, or, in other words, if the position of the resultant is found at a certain distance on one side of the centre of the span the point of greatest bending will be found at the same distance on the other side of the span; and also that the following relation must exist:

$$\frac{l'}{l} = \frac{R}{w_1 + w_2 + w_3 + \dots w_n}; \quad . \quad . \quad . \quad . \quad (481)$$

in which l' is the distance between the point of greatest bending and the left-hand reaction R, the load supposed to be moving from right to left; l the length of the span, $w_1 + w_2 + w_3 + \dots w_n$ the sum of the loads on the span. The first step will then be to find the centre of gravity of all the loads. To do this we use the well-known principle, that the sum of the moments of the parts of a body taken with respect to any axis perpendicular to the plane of the forces is equal to the moment of the weight of the whole body with respect to that same axis. Having, then, a series of loads or weights, we multiply each load by its perpendicular distance from the assumed axis, and divide the sum of these moments by the sum of all the weights; the quotient will be the unknown distance of the resultant from the same axis. We will now apply this principle to the case under consideration. As the position of the axis is entirely arbitrary, we will take it at the centre of the front load.

7.8′ 4.7′ : 4.3′ 4.7′

w_1 7.8′ w_2 4.7′ w_3 4.3′ w_4 4.7′ w_5

We then have $w_1 \times o + w_2 \times 7.8 + w_3 \times 12.5 + w_4 \times 16.8 + w_5 \times 21.5 = 586,000$ ft.-lbs.; $w_1 = 6000$, $w_2 = w_3 = w_4 = w_5 = 10,000$, lbs.; total load $= 46,000$ lbs. Hence $x_0 = \frac{586000}{46000} = 12.74$ ft. is the distance of the resultant from the centre of the first load A; and as the length AC between the loads w_1 and w_2 is

12.5 ft., the centre of gravity of all the loads is 12.74 − 12.50 = 0.24 ft. to the right of w_3, Fig. 314. Therefore we must move the load w_3 until it is 0.12 ft. to the left of the centre of the span, so that g (the resultant will be 0.12 ft. to the right of the centre c) will then be the point of greatest bending, and the engine will be so placed as to produce the maximum bending; the same method would apply with any length of span and system of loading. Other and heavier engines are now used in which the load on a pair of front trucks is 16,000 lbs., five pairs of drivers 25,000 lbs. each, four pairs of tender-wheels 20,000 lbs. each, the load borne by each girder being one half of the above amounts. With different distances between wheel centres and from the above principles the position of the load for maximum bending can be readily found. By inspection of the relative magnitude of the loads and their distances apart we can determine approximately the proper position of the engine. Having fixed the position of the resultant load, we can now find the value of the reaction R from eq. (481),

$$\frac{l'}{l} = \frac{R}{w_1 + w_2 + w_3 + \dots w_n}.$$

$l' = 14.88$, $l = 30$, $w_1 +$ etc. $= 46,000$ lbs.; hence $R = 22,816$ lbs. Or we can now find the reaction at R of each load separately and take their sum as follows: Load w_5 at E will now be 6.12 ft. from reaction R_1 on right (see Fig. 314), w_4 at D 10.82 ft., w_3 at g 15.12 ft., w_2 at B 19.82 ft., and w_1 at A 27.62 from R_1. Hence reaction at R on left will be

$$\frac{10000(6.12+10.82+15.12+19.82)}{30}+\frac{6000\times 27.62}{30}=22817.33 \text{ lbs.}=R.$$

Error, 1.33 lbs., arising from lever-arms being carried to only two places of decimals. Having now the reaction at R, take moments with respect to g. $22,816 \times 14.88 - 6000 \times 12.5 - 10,000 \times 4.7 =$ 217,502 ft.-lbs. for the resultant moment at g of the reaction R and the loads w_1 and w_2 between R and g due to the live load, to which add the moment due to the dead load, already found, 217,502 + 42,075 = 259,557 ft.-lbs. This moment must be resisted by the compression P in the upper flange and the tension T in the lower flange. These stresses are supposed to act at the centres of the rivet-holes in the upper and lower flanges, which will be less than the total depth of the beam by about 3 inches; hence distance between

centres of rivet-holes $36 - 3 = 33$ inches. Taking moments about an axis at the centre of lower rivets, we have $P \times$ (33 inches) 2.75 ft. $= 259{,}557$; $\therefore P = 94{,}392$ lbs., allowing a working strain of 7000 lbs. per square inch in compression. The upper flange, including cover-plates and angles, should have an area $= \frac{94392}{7000} = 13.47$ sq. in. The actual area in the girder is 11.37 sq. in. On this basis there is not metal area enough. The thickness of the angles in the girder is $\frac{3}{8}$ in. If we increase the thickness to $\frac{9}{16}$ in., each angle-iron would contain 4.47 sq. in., the two 8.94, and if we add area of cover-plate 5.12, we would have $8.94 + 5.12 = 14.06$ sq. in.,—a little greater than necessary. A unit strain of 7000 lbs. gives a factor of safety of about 6. A factor of 5 would in general be ample, which would give 8000 lbs. for safe unit strain, $\frac{94392}{8000} = 11.8$ sq. in.,—a little in excess of the actual area, but near enough. If we allow 10,000 lbs. per sq. in. in tension, which gives a factor of safety of 5, we have area required for bottom flange $\frac{94392}{10000} = 9.44$ sq. in.; but as two rivet-holes are punched in this flange $\frac{7}{8}$ in. in diameter,—we may say 1 in.,—and the thickness of this flange being $\frac{5}{8}$ in., we destroy $\frac{10}{8}$ sq. in. of material. Gross area of flange should then be $9.44 + \frac{10}{8} = 10.69$ sq. in. The actual area is 9.96 sq. in.; so we may conclude that the girder will bear safely the loads used. If larger areas are required so as to reduce the unit strains and give larger factors of safety, the angle-irons should be heavier. Angle-irons are made of certain standard weight per foot of length. For different lengths of angle legs the cross-section in square inches is $\frac{1}{10}$ of the weight per yard. A $5 \times 3\frac{1}{2}$ in. angle-iron $\frac{5}{8}$ in. thick weighs 49.2 lbs. per yard of length. Its area is 4.92 sq. in. Angle-irons in girders always occur in pairs, one on each side of the web, and are riveted together through the web. In girders under 30 ft. span the angle-irons constitute the flange. A cover-plate is not necessary for the bottom flange, but is used over the top flange angle-irons. Angle-irons are given in pounds per foot in this example.

The web is supposed to resist the shearing action of the load alone, the flanges the bending action. The area required for the web in square inches is generally expressed in symbols, $A = \frac{S}{f}$, in which A is area in square inches, S shearing stress at any point, and f safe resistance to shearing in pounds per square inch, which varies from 5000 to 7500 lbs. As the web is generally made of uniform thickness from end to end, we need consider only the greatest shearing stress, which is at the points of support and

equal to the reaction. In the present case the reaction from the dead load is $\frac{11220}{2} = 5610$ lbs. From the live load it is 22,816 lbs., or $S = 22{,}816 + 5610 = 28{,}426$ lbs. Hence $A = \frac{28426}{5000} = 5.7$ sq. in., the unsupported depth of web is 33 in., and the thickness $\frac{5.7}{33} = 0.17$ in., or about $\frac{3}{16}$ inch. The web is seldom made less than $\frac{1}{4}$ in. or more than $\frac{5}{8}$ in. The actual thickness in beam (Fig. 313) is $\frac{3}{8}$ in., or double the thickness required. But owing to the small ratio of $\frac{3}{8}$ in. to 33 in., the web may buckle or bend under the load, and for this reason angle-irons in pairs, which are called "stiffeners," are riveted to the web near the ends. There is no need of them near the centre of the span, as the shearing force is small and vanishes altogether at the point of greatest bending. A pair is placed at each end, and at intervals equal to the depth of the girder near the end. The beam (Fig. 313(d)) shows three angles near each end. Fillers are used where necessary between the web and angle-irons. If the total shear at any point divided by the area of the web is less than from 3000 to 4000 lbs., no stiffeners will be required. We have seen that the shearing force at the ends is 28,486 lbs. This shear gradually decreases by the weight of the dead load per unit of length, or 374 lbs. per foot of length. It will be on the side of safety to suppose it to be constant until the load w_1 at A is reached. Then deduct the entire dead load to that point, equal to $374 \times 2.38 = 890.12$ lbs., and also $w_1 = 6000$. Hence, shearing force from R to $A = 28{,}426$ lbs.; from A to B, $28{,}426 - 6890.12 = 21{,}535.88$; B to g, $21{,}538.88 - 2917.12 - 10{,}000 = 8621.76$; and at g, the point of greatest bending, it becomes 0. These results are a little in excess between the loads, as the shearing force due to the dead load is not deducted until the end of one of the spaces is reached. This shows the method of the determination of the shearing stress from point to point, and as at no point $\frac{S}{\text{area}}$ is over 3000 lbs., theoretically no stiffeners would be required, but they are always used near the ends. Another object in determining the shear at several points is to determine the number and spacing or distance of the rivets apart, connecting the angle-irons with the web, as it is through the rivets that the shearing is transmitted to the web when the load rests on top of the flange. Under this assumption it is evident that the number of rivets required in any given section of the beam would decrease according to the distribution of the load, being constant in the case of a single centre load, gradually and uniformly decreasing, in case of a uniformly distributed load, from the ends

to the centre; and the number required in any given section would be found by dividing the shearing force by the resistance to shearing of the rivets, or the resistance to crushing by the rivets of the web-plate upon which they rest. It is usual to allow 7500 lbs. per square inch as the safe resistance of a rivet in single shear, that is, when they connect only two layers or plates; but in case of heavy moving loads, in order to provide against shock and vibrations a deduction of from 20 to 25 per cent is made, reducing the 7500 to 5625 lbs., which will be used in this case. For resistance to crushing of the web 12,000 lbs. is allowed, or, deducting 25 per cent, 9000 lbs. per square inch of bearing, the bearing surface being equal to the diameter of the rivet multiplied by the thickness of the plate. As in this case the rivet must be sheared on two surfaces, one on each side of web, or is in double shear, the resistance of each rivet will be 11,250 × area of rivet × 0.6 = 6750 lbs., as the area of a $\frac{7}{8}$-in. rivet is 0.6 sq. in. The bearing of a $\frac{7}{8}$-in. rivet on a plate $\frac{3}{8}$ in. thick is $\frac{7}{8} \times \frac{3}{8} = \frac{1}{3}$ in., nearly. Hence, $\frac{1}{3}$ of 9000 = 3000 lbs., safe resistance. This being less than resistance of rivets to shearing, will be used. $\frac{R}{3000} = \frac{28426}{3000} = 9$ rivets. Then in 33 inches of length $\frac{33}{9} = 3.7$ in. between rivet centres. For the next space, $\frac{21536}{3000} = 7$ rivets; $\frac{33}{7} = 4.7$ in. spaces between B and g (Fig. 314); $\frac{8628}{3000} = 3$ rivets; $\frac{33}{3} = 11$ in. spaces. The usual practice, however, would be to space the rivets not more than 3 in. centres at the end sections, and not more than 6 in. at the centre sections, gradually increasing between these limits at the intermediate sections. But it will be shown in the next example that the rivets should be spaced on other principles, and that under some conditions of loading there should be as many rivets in the centre sections as in the end sections.

Where single cover-plates of a uniform thickness, as that in the top flange (Fig. 313), are used, it becomes necessary to determine the number of rivets to connect the cover-plate with the angle-irons. That portion of the flange stress borne by the cover-plate must be transmitted through these rivets, and, as seen in the preceding case, the number of rivets will be determined by the resistance of the plate to crushing at the rivet-bearings. Using $\frac{3}{4}$-in. rivets, and the cover-plate $\frac{7}{16}$ in., the area of bearing equals $\frac{3}{4} \times \frac{7}{16} = 2\frac{1}{64}$ sq. in., and the allowed safe resistance to crushing at each rivet $= \frac{21}{64} \times 9000 = 2953$ lbs. The cover-plate contains $12 \times \frac{7}{16} =$ 5.25 sq. in. Hence $5.25 \times 8000 = 42{,}000$ lbs., the greatest safe resistance to crushing of the plate. Hence $\frac{42000}{2953} = 14.6$ rivets. Using

the nearest even number greater, 16 rivets, 8 on each side, distributed over one half the span, 15 ft., $\frac{15}{8} = 1.88$ ft. intervals. This interval between rivet centres is evidently too great, as the plate acts as a column between the rivets, the length of which is very great as compared with the thickness of the plate. This unsupported length should not exceed 15 times the thickness, or $\frac{7}{16} \times 15 = 6.5$ in. Rivets should then be spaced at this interval on each side of the plate for its entire length. No note is taken here of the decrease in the flange stress towards the ends. In practice the rivets would be spaced at about 6 in. intervals, but at and near the extreme ends for the space of two or three rivets they are placed about 3 in. centre. This matter will be alluded to again in a subsequent example.

In regard to the deflection of built beams there is a want of accurate data, but experiment seems to show that it is much greater than that of solid beams, the coefficient of elasticity being reduced from 29,000,000 to 12,000,000 lbs.; and the determination of the moment of inertia is complicated by the number, size, and variety of shapes into which its parts appear. An approximation can be obtained by supposing the beam to be in the condition of a rolled beam whose depth is the distance between centres of the rivet-holes in the web and the upper and lower flanges solid rectangles, the breadth the same as in the built beam, and the thickness necessary to produce the areas of the actual flanges. The moment of inertia can then be determined, which, together with the value of E above, being substituted in the formula $v_1 = \frac{36\,Wl^3}{EI}$, will give an approximate value for the deflection. In the case of a series of isolated loads, it will be simpler to reduce the actual load to the equivalent uniform load; that is, a load uniformly distributed that would produce the same flange strain at the centre of the span as that caused by the actual loads. This is easily done. As seen above, the moment of the flange strain is $P \times 2.75 = 259{,}557$ ft.-lbs., the centre bending moment for a uniformly distributed load is $\frac{1}{8}(wl)l$; hence $\frac{1}{8}(wl)l = 259{,}557$ lbs. $\therefore wl = 69{,}215.2$ lbs.; $w = 2307.2$ lbs. per foot of span. W in the above formula $= \frac{5}{8}wl = 43{,}259.5$ lbs. The area of the upper flange is 11.37 in., lower flange 9.96; but including rivet-heads, fillers, etc., we can consider each flange as 12 in. area = plate 12×1 in.

In Art. XXXIII, $I = \frac{bd^3 - b_1d_1^3}{12}$, in which $b = 12$ in., $d = 34$, $b_1 = 12 - \frac{5}{8} = 11.72$, $d_1 = 32$ in.

$$I = \frac{12 \times (34)^3 - 11.72 \times (32)^3}{12} = 7300.00;$$

$A = 36$ sq. in., $\rho^2 = 203$, $\rho = 14.25$, $l = 30$.

$$v_1 = \frac{36 \times 43259 \times 27000}{12000000 \times 7300} = 0.48 \text{ in.},$$

as the greatest deflection. A practical rule given by Cooper, Hewitt & Co. for similar beams under safe strain, about the same as used above, is to take the square of the span in feet and divide by 70 times the depth in inches: $\dfrac{l^2}{70 \times d} = \dfrac{900}{70 \times 33} = 0.39$ in. $= v_1$. A great deal of valuable information put in purely practical shapes is contained in these small books of useful information published by the New Jersey Steel and Iron Co. Assuming the deflection to be not more than $\frac{1}{700}$ of the span, we have $\dfrac{d}{l} = \dfrac{n''(f_0 + f_1)}{4E}\dfrac{l}{v_1}$. $n'' = \dfrac{5}{12}$, $f_0 = 10{,}000$, $f_1 = 8000$, $\dfrac{l}{v_1} = 700$, $E = 12{,}000{,}000$. Hence, substituting,

$$\frac{d}{l} = \frac{5}{48} \times \frac{18000 \times 700}{12000000} = \frac{1}{9.14}; \quad \therefore d = \frac{30 \times 12}{9.14} = 39.5 \text{ in.}$$

for the proper depth of the beam. The actual depth is 36 in. This gives as close an approximation to the actual practice as could be expected. As a usual rule, the depth varies in this class of beams from $\frac{1}{8}$ to $\frac{1}{12}$ the span. True economy requires as great a depth as practicable within certain limits. In the girder above considered there was a single cover-plate of uniform thickness in the top flange, but none in the bottom flange; in case of very long girders or those carrying very heavy loads cover-plates are used on both flanges, and often several thicknesses are used, increasing in number towards the centre of the span so as to proportion the flange area to the varying stress upon its different parts. This case will now be considered in the following paragraph.

SHEARING STRESS IN WEB-PLATES.

883*a*. It was seen in discussing plate beams and girders that the web was assumed to bear the entire shearing, that the lines of greatest shear in the web are vertical and horizontal, and that a

pair of shears on two planes at right angles to each other and also to a plane parallel to the shears are accompanied by two direct and normal stresses, one compressive and the other tensile, whose directions are perpendicular to each other, and make angles of 45 degrees with the horizontal line; therefore, at any point in the web it may be considered as subjected to a compressive and tensile strain at right angles to each other. The intensity of the shear in any vertical section is equal to the load supported between it and the centre of the beam, or equal to the reaction diminished by the loads between the end and the section, and this divided by the area of the section. This, then, will be the intensity of the compression or tension acting at any point. As the web is generally very thin as compared to its height between the flanges, it is liable to buckle or become corrugated along lines perpendicular to the compressive stress. The tensile stresses tend to relieve this condition, but it is found necessary, or at any rate advisable, to stiffen the web, which is generally done by riveting pairs of angle-irons, one on each side of the web, to the web-plate. It is evident from the above that such angle-irons or stiffeners should be placed in the lines of compressive stress, that is, sloping outwards and downwards from the centre of the beam towards the ends at angles of 45° to the horizontal, as seen in Fig. 313(d), on the right, at AA. The usual practice for ordinary-sized beams, as a matter of convenience, is to place the stiffeners vertically, as seen at A', A'. The arrow-heads c, c are lines of compressive stress, t, t are lines of tensile stress. Such stiffeners are only needed near the ends of the beams, where the shearing force is the greatest, while it decreases towards the centre sections, becoming zero at or near the centre of the beam. If the intensity of the shear is less than 4000 pounds no stiffener is needed, and where needed its area is found in square inches by dividing the shear at that section by 4000.

ART. L.

FRAMED STRUCTURES—(*Continued.*)

884. In the preceding article the principles developed and the equations obtained in the general discussions of the effect of loads upon beams, and the strength or resistance of beams necessary to maintain equilibrium, were applied to proportioning and designing

beams, when acted upon by the loads used in practice. Such beams, as forming component parts, properly belong to framed structures, whether solid or built of many parts riveted or bolted together; in this latter case the beams themselves come under the head of frames, regardless of their connection with other parts of a larger or more complicated structure.

The general relations and equations, so far as the action of external loads are concerned, are the same whether applied to beams or trusses. But the distribution of the stresses and the proportioning and arranging the members of a truss or bridge to effectively resist these stresses admit of a great variety in designs, and general methods of determining the stress. In this article the general discussion of stresses in trusses will be given, mainly without reference to the actual distribution of loads found in practice.

STRESSES IN ROOF AND BRIDGE TRUSSES.

885. These stresses can be determined either by equating the bending moment to the moment of the internal stresses in all the members cut or separated by the ideal cutting-plane, provided only three acting members, in which the stresses are unknown, are cut by the plane. Or we may determine the stresses by applying the three equations of equilibrium to each and every joint of the truss.

In the latter case, however, we have only two independent equations, as the lines of action of the forces or stresses meet in one point; there must not exist, then, more than two unknown forces or stresses. These equations are, $\Sigma V = 0$ and $\Sigma H = 0$, the requirement being only that the force polygon shall close.

Either method may be applied to any truss for whatever purpose constructed, with the limitations above given. It is often convenient, however, to combine the two methods. The chord stresses may be determined by the method of sections and moments and the web stresses by the force polygon, combining the algebraic with the graphical when deemed expedient; or the algebraic may be used in both cases; or, finally, the graphical method may be applied in each.

It is to be assumed that the reactions have been determined by the methods already explained, and consequently all of the external forces are known.

886. Assuming the form of roof-truss shown in Fig. 315: Let

the truss be cut by a plane dd', which cuts three members, namely, FC, CD, and AB. Consider, then, the portion on the left of the section dd', as shown in figure (a). Some of the forces, W_1 and R_1, are external, and the others are internal stresses, t, t', and c; the latter are unknown. We can apply the three equations of equilibrium, $\Sigma V = 0$, $\Sigma H = 0$, $\Sigma M = 0$, and find these unknown stresses. We may not know what direction to give these stresses, but if either result is negative, we know that for that stress we have assumed the wrong direction of action, and it must be reversed. Or we use three moment equations; this is the method now to be followed.

If we take the axis of moments at C, the moments of the stresses c and t' will be zero as they pass through this axis. We have then only three acting moments, the algebraic sum of which is zero. These moments are, respectively, $+ R_1 \times AE$, $- W_1 \times FG$ and $- t \times CE$. Hence

$$R_1 \times AE - W_1 \times FG - t \times CE = \Sigma M = 0, \quad . \quad (482)$$

From which we find t. If this has a positive value, we have assumed the right direction, and as it acts outward on the piece AE, it is tension. If we prolong the piece CD to intersection with AE at o, and take the axis of moments at that point, the moments of t and t' will be zero, and the three acting moments are $R_1 \times Ao$, $W_1 \times oo_1$, and $C \times oo'$. Hence

$$R_1 \times Ao - W_1 \times oo_1 - c \times oo' = 0. \quad . \quad . \quad . \quad (483)$$

from which c can be found, which is the stress in the member CF. If positive, we have assumed the proper direction of its action, and as it acts inwards on the member it will be compression. Then taking the axis of moments at A, the moments of R_1 and t and c are zero, and we have two acting moments, $W' \times Ao_1$, and $t_1 \times Ao_2$. Hence

$$W_1 \times Ao_1 - t' \times Ao_2 = 0, \quad . \quad . \quad . \quad . \quad . \quad (484)$$

from which t' can be found. It is evident that $W_1 \times Ao_1$ is positive, as it tends to produce right-handed rotation around A, and as there are only two acting moments, $t' \times Ao_2$ must be negative; consequently t' must act outwards from D, producing tension in the piece DC. We have then assumed the proper direction for this

stress. In many cases we can determine the proper direction for the stresses by simple inspection, but not always.

If we take a section d_2d_3, Fig. 315, we cut three members, FA,

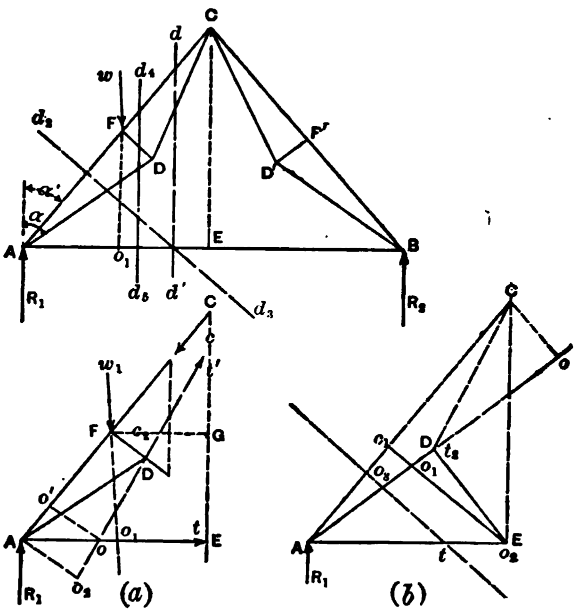
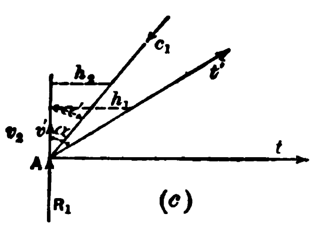


FIG. 315.

DA, and AB; the stresses in these and the reaction R_1 are only forces acting. R_1 and t, the tension in AB, now are known, and it is only necessary to find the stresses in FA and DA. Taking moments about C, Fig. (b), we have



α = Angle between DA and the vertical.
α' = " " FA " " "

$$R_1 \times AE - t \times CE - t_2 \times Co = 0 \text{ [see } (b)], \quad . \quad (485)$$

from which we find t_2. If it is positive, it acts outward from A on the piece AD, as indicated, and shows tension in AD.

Taking moments about E, that of t is zero, and we have

$$R_1 \times AE + t_2 \times o_1o_2 - c_1 \times o_3o_2 = 0, \quad . \quad . \quad . \quad (486)$$

from which c_1 is found. As the first two moments are positive, the third must be negative; that is, the stress c_1 must act inwards A on FA causing compression [see (b)].

We have thus determined the kind and magnitude of the stresses in all members except FD. If, then, we take a section d_4d_4, Fig. 315, we cut CF, FD, AD, and AE; of these all the stresses have been found, except that in FD. The external forces acting on the portion to the left of d_4d_4 are R_1 and W_1. Taking moments about A, the moment of the stress in AD (t_2) is zero, that of R_1 is zero, and that in AE (t) is zero; hence the only acting moments are $W_1 \times Ao_1$ and $c_2 \times AF$, Fig. (a). Hence

$$W_1 \times Ao_1 - c_2 \times AF = 0, \quad . \quad . \quad . \quad . \quad . \quad (487)$$

from which c_2 is found; and in order to give a negative moment it must act inwards on FD towards F, producing compression. As the stresses on the other half of the truss are the same for corresponding members in the left half, all stresses have been determined. This method is simple, and perfectly general in its application to any form of truss loaded in any manner, when all of the external forces are known, provided we can get sections cutting only three members in which the stresses are unknown, which can be done in any truss having a single system of web bracing. It is not, however, generally used to determine the stresses in all of the members, owing to the labor required in obtaining the lengths of so many lever-arms.

It is commonly applied only to those members whose lever-arms are either known from the construction of the truss, or can be readily found, such as the chord stresses in bridge-trusses, which are the depths of the trusses, taking moments for the upper-chord stress about the panel points in the lower chord, and *vice versa;* and in the roof-truss just discussed, Fig. 315, the stress in AB, whose lever-arm about C is CE, the rise of the roof, and that in FD, with lever-arm AF, the half-length of the rafter CA. The stresses in other members can be more readily found by the force polygon applied to each joint.

This latter method will now be applied to finding the stresses in the members of the same roof-truss, Fig. 315. Since at the joint A we have four forces acting, namely, the reaction R_1, the stress t in AB, the stress t_2 in AD, and the stress c_1 in AF, and these, three of which are unknown, meet in one point, A, while we have only two

independent equations, viz., $\Sigma v = 0$, $\Sigma H = 0$, and the same condition of three or more unknown stresses at every other joint or panel point, the determination of the stresses will be impossible unless one of the stresses can be determined independently by the method of moments. As in the preceding case, we can find the stress t in the member AB by moments. This leaves only two unknown stresses at A, namely, those in AF and AD. Drawing the diagram (c), Fig. 315, for the joint A, the stresses being represented by c_1, t_2, and t, decomposing c_1 and t_2 into their vertical and horizontal components, we have $v' = t_2 \cos \alpha$, $h_1 = t_2 \sin \alpha$, $h_2 = c_1 \sin \alpha'$, and $v_2 = c_1 \cos \alpha'$. Then

$$\left.\begin{array}{l} \Sigma V = R_1 + v' - v_2 = R_1 + t_2 \cos \alpha - c_1 \cos \alpha' = 0; \\ \text{and for } \Sigma H, \\ \Sigma H = t + t_2 \sin \alpha - c_1 \sin \alpha' = 0. \quad . \quad . \quad . \quad . \quad . \end{array}\right\} \quad (488)$$

And since α and α' are known, between these two equations we can find, by ordinary algebraic methods, the values of c_1 and t_2. If either of these values are found to be negative, we have assumed the wrong direction for their action. As they stand in diagram (c) and (a), they indicate compression in AF and tension in AD and AB.

At the joint F we have the external force W_1 and the stresses in the three members AF, FC, and FD. W_1, and c_1, the stress in AF, are known; and c and c_2, the stresses in FC and FD, unknown. Placing ΣV and ΣH, each equal to zero, we have two equations, from which c and c_2 can be found. The horizontal components of the stresses at F are found in the same manner as at A. For the joint C we have the stresses in FC, CD, CF'', and CD', and any load at C. Those in FC and CF'' are usually equal, as are also those in CD and CD'. There is therefore only one unknown stress, namely, that in CD or CD', which can readily be found from either $\Sigma V = 0$ or $\Sigma H = 0$.

As the purpose was only to show the principles in the preceding example, only the single load W_1 at F is considered. In actual trusses there would be loads at A, C, F, F_1, and B, all of which would have to enter both into the equations of moments, as well as in those of vertical components. And further, the weight of the structure itself has not been considered. The exact distribution of both dead and live loads will be considered in another paragraph.

These principles are applicable to any truss, whether for

bridges or roof, when the distribution and magnitudes of the external forces are known or determined.

SIMPLE TRUSSED BEAMS.

887. The following types of trussed beams are in common use.

Trussed beams, properly considered, subject the beams to combined bending and direct stress, which is an undesirable condition, leading to ambiguity as to the magnitude and distribution of the stresses, and in many cases to great danger; for, however carefully designed and constructed, undue stresses may be thrown on certain members, and unless a large margin of safety is provided, the structure may collapse under the load.

888. The practice of trussing timber beams with iron rods is especially objectionable. The part of the load necessarily borne by the beam will depend upon the comparative stiffness of the beam and the trussing, which will vary with the permanent elongation of the tie-rods, and also to their temporary contraction or expansion due to changes in temperature. The nuts work loose under constant vibration, and unless attention is given to the structure the component parts cannot be kept to their bearings. Some or all of these considerations also apply, though in a much less degree, when all members are of timber.

The trussing can be placed either above or below, called overtrussing or undertrussing. For any given beam this does not change the magnitude of the stresses. In either case the magnitude of the stresses in different members remain the same under the same conditions of loading, but those members which are under direct compression in the one case will be under direct tension in the other.

889. By applying the principles developed in the discussion of continuous beams, we find the stresses developed in the several members of an undertrussed beam with one intermediate support or triangular truss (Fig. 316), and one with two intermediate supports or trapezoidal truss (Fig. 317), these beams being under a bending stress due to the loads on them, and a direct stress resulting from the tensions developed in the tie-rods due to that portion of the load transmitted through them to the points of support A and B.

A simple inspection of the drawings clearly shows the difficul-

ties of maintaining any exact amount of stress, as indicated in the preceding paragraph. This solution, though more difficult, is given first.

890. *Combined Direct and Bending Stress, Triangular Truss.*—A common application of the preceding principles is to be found in the simple trussed beam used for short spans. Fig. 316 represents a single beam, strengthened by iron rods passing under a

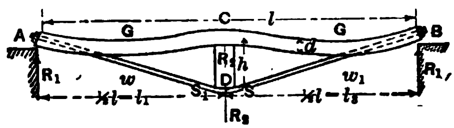


FIG. 316.

short strut or block placed under the centre of a beam of the span l. The points A, B, and C are assumed to be on the same horizontal line. The effect of this is to divide the beam into two equal spans of $\frac{1}{2}l$ each $= AC = AB$, each half being in the condition of a beam fixed at one end C and supported at the other end A and B, respectively. Assuming each section loaded with a single weight at the centre, we have

$$\begin{aligned} &\text{Reaction and shear } B \text{ or } A &&= \tfrac{5}{16}W; \\ &\text{Shear } S \text{ at } C \text{ from span } AC &&= \tfrac{11}{16}W; \\ &\text{Shear } S_1 \text{ at } C \text{ from span } CB &&= \tfrac{11}{16}W; \\ &\text{Reaction at } C = S + S_1 = (\tfrac{11}{16} + \tfrac{11}{16})W &&= \tfrac{22}{16}W. \end{aligned}$$

$R_1 + R_2 + R = (\frac{5}{16} + \frac{5}{16} + \frac{22}{16})W = 2W$, as it should be.

For bending moment at A and B, $M = 0$; for bending moment at C, $M_0' = \frac{3}{16}Wl_1$; for bending moment at G, $M_2 = \frac{5}{32}Wl_2$. (See Art. XXXVI, par. 360.)

From which we see that the bending moment over the point of support C is greater than at the centre of either AC or CB. If AB is then a beam of rectangular cross-section, its dimensions must be determined from the equation $\frac{3}{16}wl = \frac{1}{6}f_1bd^2$.

For the same beam uniformly loaded:

$$\begin{aligned} &\text{Reaction and shear at } A \text{ or } B = R = R_1 &&= \tfrac{3}{8}wl_1; \\ &\text{Shear } S \text{ at } C \text{ from span } AC &&= \tfrac{5}{8}wl_2 (= l_1); \\ &\text{Shear } S_1 \text{ at } C \text{ from span } CB &&= \tfrac{5}{8}wl_1 (= l_2); \\ &\text{Reaction at } C = S + S_1 = (\tfrac{5}{8} + \tfrac{5}{8})wl_1 = \tfrac{10}{8}wl_1 &&= \tfrac{5}{8}wl (l_1 = \tfrac{1}{2}l). \end{aligned}$$

Hence

$$(\tfrac{5}{8} + \tfrac{5}{8} + \tfrac{3}{8} + \tfrac{3}{8})wl_1 = 2wl_1 = wl.$$

For bending moment C, $m_0' = \frac{1}{8}wl_1^2 = \frac{1}{32}wl$; for bending moment G, $m_2 = \frac{9}{128}wl_1^2 = \frac{9}{512}wl^2$. (See Art. XXXVI, par. 361.) Hence

$$\frac{1}{32}wl^2 = \frac{1}{6}f_1bd^2; \quad \therefore \; dh^2 = \frac{6wl^2}{32f}. \quad . \quad . \quad . \quad . \quad (489)$$

Assuming $l = 25$ ft. $= 300$ in.; $w = 480$ lbs. per ft. $= 40$ lbs. per inch; $f = 1000$ lbs.; $d = 12$ in. $\therefore \; b = 4.68$ in.

Length of tie-rod $AD = DB = \sqrt{(\frac{1}{2}l)^2 + h^2}$.

Since centre reaction with uniform load $= \frac{5}{8}wl$, the shear on either side $= \frac{5}{16}wl$; hence

$$\text{Tension in } AD \text{ or } DB = \frac{5}{16}\frac{wl}{h}\sqrt{(\tfrac{1}{2}l)^2 + h^2}; \quad . \quad . \quad (490)$$

making $h = 5$ ft., tension in AD or $DB = 10{,}095$ lbs. at 8000 lbs. per sq. in. $= \frac{10095}{8000} = 1.26$ sq. in., or 1 rod $1\frac{5}{16}$ in. diameter.

In Fig. 316 it is evident that the beam AB is not only subjected to a unit strain of $f_1 = \frac{6wl^2}{32bd^2}$, but has also a distinct compressive strain equal to the horizontal component of the stress in the rods AD or DB, which is $= H = \frac{5}{16}wl \times \frac{\frac{1}{2}l}{h} = \frac{5}{32}\frac{wl^2}{h}$. The unit strain due to this direct compression $= \frac{H}{\text{area of beam}} = \frac{H}{bd} = \frac{5}{32}\frac{wl^2}{bdh} = f_2$; and since the ultimate unit strain should not exceed

$$f = 1000 = f_1 + f_2 = \frac{6wl^2}{32bd^2} + \frac{5}{32}\frac{wl^2}{bdh}, \quad . \quad . \quad (491)$$

Assuming values as before,

$$1000 = \frac{6 \times 40 \times (300)^2}{32 \times b \times 144} + \frac{5}{32} \cdot \frac{40 \times (300)^2}{b \times 12 \times 60},$$

or, more simply,

$$1000 = \frac{6wl^2h + 5wl^2d}{32bd^2h}. \quad \therefore \; b = 9.38 \text{ in.}$$

as the breadth of beam to resist both direct and bending stresses.

Compression on strut $cd = \frac{5}{8}wl$.

891. *Trapezoidal Truss.*—Again, timber beams have sometimes two vertical supports, as shown in Fig. 317. In this case the entire length of beam AB is divided into three sections or panels, AD, DE, and BE, which may be equal or unequal; assumed equal in this case and each represented by l_1. AD and BE are beams fixed at one end and supported at the other; the middle panel DE is a

beam fixed at both ends. Referring to the tabulation of bending moments and shears in Art. 36, paragraphs 360, 361, 362 and 363,

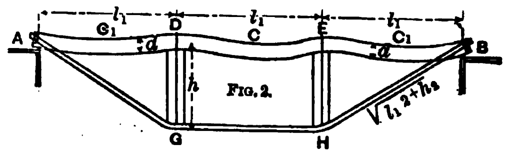


FIG. 317.

we easily write the following, remembering that in this case quantities will be expressed in terms of the length of each panel or span l_1, and not in terms, as in the preceding example, of the entire length of span AB.

First, for a single load at the centre, C, of each span.

Reaction and shear at A or $B = R = R_1 = \frac{5}{16}W$;
Shear S at D or E from span $AD = \frac{11}{16}W$.
Shear S_1 at D or E from span $DE = \frac{1}{2}W$;
Reaction at D or $E = (\frac{11}{16} + \frac{1}{2})W = \frac{19}{16}W$;
Sum of reactions $= 2(\frac{5}{16} + \frac{19}{16})W = \frac{48}{16} = 3W$;
Max. bending moment at D or E from span $AD = M_0' = \frac{3}{16}Wl_1$;
Max. bending moment at D or E from span $DE = M_0' = \frac{1}{8}Wl_1$;
Max. bending moment at C of middle span $= \frac{1}{8}wl_1$.

These are not the conditions of loading, as was seen in the general discussion, to produce equal bending moments at D and E from the adjacent spans.

If the span is uniformly loaded from end to end:

Reaction and shear at A and B $= \frac{3}{8}wl_1$;
Shear S at D from $AD = \frac{5}{8}wl_1$;
Shear S_1 at D from $DE = \frac{1}{2}wl_1$;
Reaction at D or $E = \frac{5}{8}wl_1 + \frac{1}{2}wl_1$ $= \frac{9}{8}wl_1$.
Sum of reactions $= 2(\frac{3}{8} + \frac{9}{8})wl_1$ $= 3wl_1$.
Max. bending at D or E from AD $= \frac{1}{8}wl_1^2$;
Max. bending at D or E from DE $= \frac{1}{12}wl_1^2$.

In such cases it is necessary to either proportion the lengths of the spans or the position and magnitude or intensity of the loading to produce the desired equality, or to proportion and dimension the parts for that position and amount of load giving maximum results. Having found the reactions at D, E, the compression on the verticals will be either $\frac{19}{16}w$ or $\frac{9}{8}wl_1$, according as the spans are loaded with single or uniform loads. In this truss the stress in the inclined

rods AG and HB will be due to the entire reaction at D or E, and not to the half of it, as in the preceding example. Hence

$$\text{Tension in } AG = \text{tension in } HB = \frac{19}{16}w \quad\text{or}\quad \frac{9}{8}wl_1\left(\frac{\sqrt{l_1^2 + h^2}}{h}\right); \quad (492)$$

and for combined bending and direct stress at D or E,

$$\tfrac{1}{12}wl_1^2 = \tfrac{1}{6}f_1bd^2 \quad\text{and}\quad \tfrac{9}{8}wl_1 \times \frac{\frac{1}{2}l}{h} = f_2bd,$$

hence,

$$f = f_1 + f_2 = \frac{1}{2}\frac{wl_1^2}{bd^2} + \frac{9}{8}\frac{wl_1^2}{bdh}, \quad . \quad . \quad . \quad . \quad (493)$$

taking shears and moments from values given for middle span uniformly loaded, and similarly for the centre C.

Such combinations of timber beams and iron rods are decidedly objectionable. If timber beams have to be trussed, it is better to use two inclined struts supporting the beam at the centre C (Fig. 316), or at two points, as D and E (Fig. 317), or to reverse the trussing and place it above the beam. These conditions will be fully explained in the following examples.

892. It is usual in the case of the trussed beam, Fig. 316, to consider the span as made up of two separate beams, AC and BC, hinged or jointed at C; then to consider each beam as bending only under the load upon it. If the load is uniformly distributed, then for the segment $AC = l_1 = \frac{1}{2}l$ we have $\frac{1}{8}wl_1^2 = \frac{1}{6}fbd^2$, from which we readily find b or d by assuming one of them, according to the principles developed in paragraphs 858 to 861.

Then, for the tension in the rods AD and BD and the strut CD, it is assumed that one half the load on AC and one half the load on CB, that is, one half the load on the entire span, $= \frac{1}{2}wl$, for a uniformly distributed load reacts at C, which is the compression on CD. Then $\frac{1}{4}wl$ is transmitted through AD and $\frac{1}{4}wl$ through BD, producing tension in these members equal to

$$\tfrac{1}{4}wl \times \frac{AD}{CD} = \tfrac{1}{4}wl\frac{\sqrt{(\frac{1}{2}l)^2 + h^2}}{h}. \quad . \quad . \quad . \quad . \quad (494)$$

893. In case of the beam, Fig. 317, the entire length is divided into three spans, the length l_1 of each $= \frac{1}{3}l$; and at each intermediate point of support, D and E, there is supposed to rest

$\frac{1}{2}wl_1 + \frac{1}{2}wl_1 = wl_1 = \frac{1}{3}wl$, which gives the compression in the struts DG and EH. This amount of load is transmitted through AG and HB, producing a tension on each

$$= \tfrac{1}{3}wl \times \frac{AG}{DG} = \tfrac{1}{3}wl\frac{\sqrt{(\frac{1}{3}l)^2 + h^2}}{h}. \quad . \quad . \quad . \qquad (495)$$

And for the stress in GH, which is also a tension, we have tension in

$$GH = \tfrac{1}{3}wl\frac{GH}{h} = \frac{1}{9}\frac{wl^2}{h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (496)$$

These stresses (equations (494), (495), and (496)) are found by the principle of the triangles of forces. In Fig. 316 there are three forces acting at the point D, namely, the load $\frac{1}{2}wl$ and the stresses in the tie-rods AD and DB, which can be represented by the three sides of a triangle, respectively parallel to the three forces. And in Fig. 317, at each point G and H, there are three forces, the vertical, $\frac{1}{3}wl$, and the stresses in the members AG and GH, which can be represented by a right-angled triangle.

From which we see that when a load acts at any panel point, and is balanced by any two members of a truss meeting at that point, the stress developed in each member is the load transmitted by that member multiplied by the length of the member and divided by the depth of the truss. If the two members have equal angles of inclination with the vertical, then one half the entire load is transmitted by each; if of unequal inclinations, the loads transmitted by each are inversely as the sines of the angles. If one member is inclined and the other horizontal, the entire load is transmitted by the inclined member, and the stress in the horizontal member is equal to the horizontal component of the stress in the inclined member. These principles are of universal application.

894. While there may be no objection to undertrussing iron beams, as shown in Figs. 316 and 317,—and with the usual factors of safety the dimensions as determined by either of the described methods will give a safe truss,—it will be better with timber beams, if undertrussing is resorted to, to use the means shown in the following figures, 318 (a) and (b):

For short spans, two inclined struts meeting at centre (see figure (a)): As before, AC and BC are treated as separate beams; acted upon by either isolated or uniformly distributed loads. For

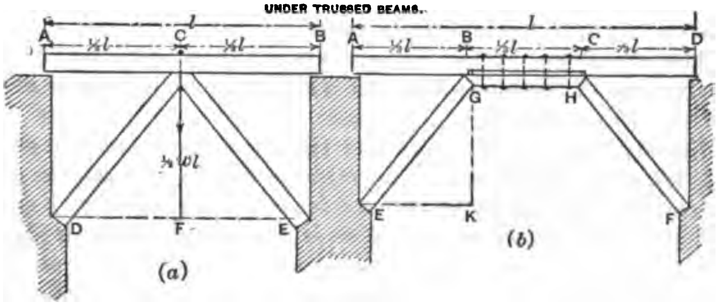
uniform loads, $\frac{1}{2}wl$ is assumed as acting at C. Each strut CD and CE transmits $\frac{1}{4}wl$, and the compression in either is

$$\tfrac{1}{4}wl \times \frac{CD}{CF}. \qquad (497)$$

These struts cause an outward thrust on the supports at D and E equal to

$$\tfrac{1}{4}wl \times \frac{AC}{CF}. \qquad (498)$$

UNDER TRUSSED BEAMS.



FIGS. 318.

In figure (b) there are two intermediate points of support—at B and C. The beam is divided into three panels, each $= \frac{1}{3}l$, which is the load acting at the points B and C. This entire load is transmitted by the struts; hence the compression on each is

$$\tfrac{1}{3}wl \times \frac{GE}{GK}, \qquad (499)$$

and the thrust at G and E

$$= \tfrac{1}{3}wl \times \frac{AB}{GK}. \qquad (500)$$

The equal thrust at G and H, $= \frac{1}{3}wl \times \frac{AB}{GK}$, is taken up by the straining-piece GH.

These are the usual methods of trussing small cheap highway bridges, and also for stringers of trestle-bents when the length of span is over 16 feet. For all bents except the end bents struts abut against them from both sides. There is no tendency, therefore, to overturn the bents, as the horizontal components of the stress in the struts are equal and directly opposed.

For the end bents the struts abut against horizontal pieces,

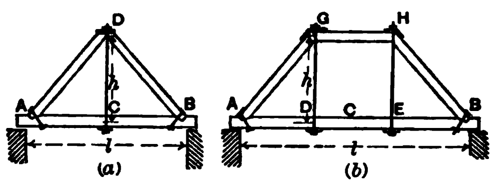
which connect two or three bents together, thereby resisting the overturning tendency.

895. It is in some respects better to overtruss timber beams. Such trusses may be used for roofs or bridges up to lengths of 40 to 45 feet.

These types of trusses are shown in Figs. 319 (*a*) and (*b*). The magnitudes of the stresses in the different members and the equations for them are identical with those found in paragraphs 890, 891, and 892, Figs. 316 and 317, under same conditions of loading and length of span and depth of truss, this latter being usually from one tenth to one eighth the span.

The characters of the stresses are, however, reversed: those members under tension in Figs. 316 and 317 are under compression in Figs. 319 (*a*) and (*b*).

Namely, *AD*, *DB*, *AG*, *GH*, and *HB* are compression members; *DC*, *AB*, *GD*, *HE* are tension members, whether the load consists of isolated or uniformly distributed loads. The loads may be applied along the bottom chord *AB*, as is usually the case in bridges, or it may be applied along the members *AD*, *DB* in (*a*), or *AG*, *GH*, and *HB* in (*b*), as would be the case for roof-trusses. In



FIGS. 319.

either case the apex loads at *D* or *G* and *H* are first found, and when found the manner of finding the stresses is the same for roof or bridge trusses.

The principle of continuity can be applied as explained in paragraphs 890 and 891, or, as is usually done, the method explained in paragraph 892 may be adopted. In this latter case, although no calculation is made for the direct tension in *AB*, a rough allowance is made for it by calculating the dimensions of the segments *AC*, *CB*, *AD*, *DE*, and *EB* as separate beams simply supported at the ends and loaded in any manner. This is on the supposition that there are one or more isolated loads or a uniformly distributed load on *AB*.

The construction may be, however, and often is, such that,

not considering the weight of the beam itself, the external loads are concentrated at the points C, D, and E. In this case the beam is not considered as acted upon by bending at all, but is simply under a direct stress of tension or compression, according as it is overtrussed, in Figs. 319 (*a*) and (*b*), or undertrussed, as in Figs. 316 and 317; this direct stress being simply the horizontal component of the stress in the inclined member AD or AG.

In Fig. 319 (*b*) it is to be noted that the truss may or will be distorted unless the load is symmetrically placed with respect to the centre of the span, that is, if a greater load is at D than at E, or *vice versa*, as the central portion $DGHE$ is a rectangle, whereas the triangular frame is the only one that cannot be distorted without changing the lengths of its members. To provide for this contingency diagonal braces DH and GE must be inserted. These may be both tension or they may be compression members. They are called counterbraces. Approximately this stress will be one third of the excess of the load at D over that at E (or *vice versa*) multiplied by the length of the diagonal and divided by the depth of the truss. The methods of determining when counterbraces will be needed and the stress upon them will be discussed later.

The foregoing trusses can be used either for highway or railway bridges.

896. *The Howe and Pratt Trusses.*—The favorite types of bridges in the United States are the Howe and Pratt trusses.

The Howe truss is built entirely of wood, except the vertical-tension rods, which are iron. It is not used for very long spans. It is used either for highway or railway bridges, especially in those sections of the country where timber is abundant.

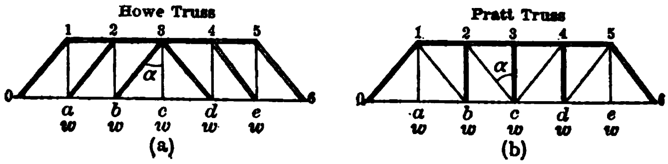
It may be stated here that the top chords of all bridges, except swing and cantilever bridges, are always under compression, and the bottom chords under tension. The classification of bridge-trusses is based upon the material used in the construction, upon the arrangement of the web members, the kind of stress developed, and upon the manner of connecting the parts together.

The Howe truss is distinctly a timber-built truss in which the verticals are under tension and the diagonals under compression. These members are so formed, proportioned, and connected that the kind of stress must always be that for which they are intended.

It differs from the Long truss only in substituting iron rods for the timber verticals framed into and between the chord-timbers.

Whether this is an improvement or not when used for highway bridges may be considered doubtful. The iron verticals have to be continually screwed up to keep all parts to their bearings.

In Fig. 320(a) is shown one of the trusses of a through Howe truss-bridge. The compression members are indicated by double lines, namely, the top chord and all inclined web members. The tension members are the bottom chord and all vertical members. When the intensity and distribution of the load is known, the



FIGS. 320.

stresses are readily determined. For a uniformly distributed load each panel of the bottom or loaded chord $0a$, ab, etc., is treated as a beam under transverse load, whose length is a panel length of the truss combined with a direct tension equal to the sum of the horizontal component of the compression in the diagonal in the same panel and all other panels to the end of the truss. This method of loading, though often used, leads to ambiguity. When used for railway bridges especially the construction should be such that both chords may be only subjected to direct stress. This condition will now be considered. The load at any panel point, a, b, c, etc., will be the load directly acting at that point, a panel load for uniform distribution, and any load transmitted to it from any other panel point. Assuming, then, a load w acting at every panel point, it is evident that the load at c is carried to 3, causing a tension in $3c = w$. At 3 the load divides (by the principle of the lever), $\frac{1}{2}w$ being transmitted by $3d$ and the other $\frac{1}{2}w$ by $3b$. We now have $1\frac{1}{2}w$ at b, which is carried to 2 by $2b$, causing tension in $2b$, thence by $2a$ to a, giving $2\frac{1}{2}w$ at a, which is carried to 1 by $a1$, causing tension on $1a$, thence by 1, 0 to 0, the point of support, and similarly for the other half of the truss.

The stresses in the diagonals are all compressive.

In $3b$ the compression is $\frac{1}{2}w\frac{3b}{3c} = \frac{1}{2}w \sec \alpha$;

" $2a$ " " " $1\frac{1}{2}w\frac{2a}{2b} = 1\frac{1}{2}w \sec \alpha$;

" $1, 0$ " " " $2\frac{1}{2}w\frac{0, 1}{1a} = 2\frac{1}{2}w \sec \alpha$.

The stresses in the chord-panels increase from end to centre of span, and for any panel is the sum of the horizontal components in all the diagonals from end to centre.

$$\text{Tension } 0a = \text{compression in } 1,2 = \qquad (2\tfrac{1}{2}w)\frac{0a}{1a} = 2\tfrac{1}{2}w \tan\alpha;$$

$$\text{"} \quad ab = \quad \text{"} \quad \text{" } 2,3 = \quad (2\tfrac{1}{2} + 1\tfrac{1}{2})w\frac{ba}{2b} = \ 4w \tan\alpha;$$

$$\text{"} \quad bc = \text{tension in} \qquad cd = (2\tfrac{1}{2} = 1\tfrac{1}{2} + \tfrac{1}{2})w\frac{bc}{3c} = 4\tfrac{1}{2}w \tan\alpha,$$

and the same for corresponding members in the other half of the truss.

For unsymmetrical loads, that is, when there is an excess of load on one half the truss over that on the other, counterbraces have to be introduced in certain panels near the centre of the span. These slope in the opposite direction to those shown in Figs. 320 (*a*) and (*b*). These will be explained later.

If the top chord be extended, and supported at its ends by verticals, the load can then run on the top chord, and the structure is then called a deck-bridge. The kind of stress remains the same in each member, and for a uniformly distributed load will be the same in amount, except for the centre vertical, which will not carry any portion of the load resting at the panel point 3. It carries only one panel weight of the lower chord.

897. *The Pratt Truss.*—But little explanation is needed after what has been said above for the Howe truss. The vertical $3c$ carries nothing but a panel weight of the upper chord. The load w at c divides at once, $\frac{1}{2}w$ passing up $2c$ by tension, down $2b$ by compression. At b, then, there is $1\frac{1}{2}w$, which passes up $b1$ by tension and down 0, 1 by compression. The load w at a passes up $1a$ by tension and down 0, 1 by compression. Then

$$\text{Compression in } 2b = \ \tfrac{1}{2}w;$$

$$\text{tension in } 2c \quad = \ \tfrac{1}{2}w\frac{2c}{2b} = \ \tfrac{1}{2}w \sec\alpha;$$

$$\text{"} \quad \text{" } 1b \quad = 1\tfrac{1}{2}w\frac{1b}{1a} = 1\tfrac{1}{2}w \sec\alpha;$$

$$\text{"} \quad \text{" } 1a = w;$$

$$\text{compression in } 0,1 = \ 2\tfrac{1}{2}w\frac{1,0}{1a} = 2\tfrac{1}{2}w \sec\alpha.$$

For chord stresses:

$$\text{Tension } 0a \text{ or } ab = 2\tfrac{1}{2}w\frac{0a}{1a} = 2\tfrac{1}{2}w \tan \alpha;$$

$$\text{" } \quad bc = \text{compression in } 1, 2 = (2\tfrac{1}{2} + 1\tfrac{1}{2})w\frac{bc}{2b} = 4w \tan \alpha;$$

$$\text{Compression in } 2, 3, \text{ or } 3, 4 = (4 + \tfrac{1}{2})w\frac{2, 3}{3c} = 4\tfrac{1}{2}w \tan \alpha.$$

A similar construction, as already described, will convert this truss into a deck-bridge. The only change under a uniformly distributed load will be to increase the compression in $3c$ by w.

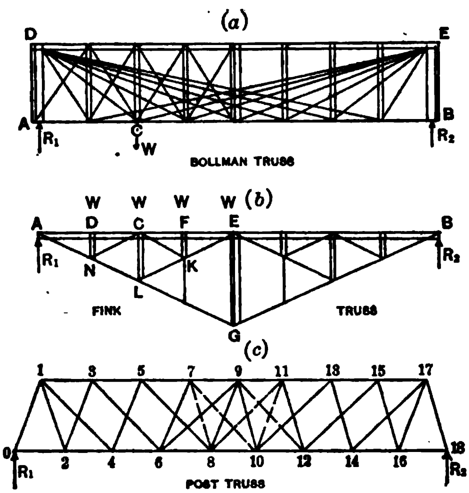
These types of trusses will be discussed in greater detail in another article.

898. *Other Types of Bridge Trusses.*—The following diagrams show some of the best-known types of bridge-trusses, many of which, though found on the older roads, are rarely if ever used at the present time. The methods of determining the stresses are also briefly discussed. They can be used for either highway or railway bridges; and when the magnitudes and distribution of the loads are determined, the methods of finding the stresses are the same for both.

899. *Bollman Truss.*—One of the earliest and simplest forms of truss is known as the Bollman truss. This truss is shown in Fig. 321 (a). The loads are supposed to be concentrated at the panel points, and each load is carried directly to the abutments by two tension rods or bars of unequal length; these bars are not affected by the loads at any other panel points. The top chord, however, acts as a straining-beam for each pair of tension-bars, and receives a compressive strain from each pair; the total compression is simply their continuous sum, and is constant from end to end. There is no strain on the bottom chord. It is therefore only a kind of multiple suspension system. Verticals and short diagonals in the panels are introduced simply to support the upper chord, and transmit its weight to the lower-chord panel points. The bottom chord gives steadiness to the verticals, and transmits any load upon it to the panel points at the feet of the verticals.

Assuming a load w acting at the panel point C: then by the principle of the lever, as C is $\frac{1}{4}$ of the span or 2 panels from A, and

$\frac{3}{4}$ the span or 6 panels from B, the reaction $R_1 = \frac{3}{4}w$ and $R_2 = \frac{1}{4}w$. We then, by the principle of the triangle of forces, have a triangle ACD, whose sides are respectively parallel to the three forces at D, namely, the upward reaction at $D = \frac{3}{4}w$, which is equal to the stress in AD, and compression as it acts inwards on the post towards A; the stress in $DC = \frac{3}{4}w \sec \alpha = \frac{3}{4}w\frac{DC}{AD}$, which acts outward on DC from D, as the direction of the stresses must be continuous in



FIGS. 321.

one direction around the triangle or force polygon; and the stress $DE = \frac{3}{4}w \tan \alpha = \frac{3}{4}w\frac{AC}{AD}$, which act inwards towards D on the piece DE, and hence it is compression. The stresses are then simply the vertical force multiplied by the length of the sides of the triangle parallel to the pieces and divided by the length of the vertical representing the load or reaction; and the kind of stress is determined by the inward or outward action of the forces following in one direction around the triangle, that of one of the three forces being known. Any triangle whose sides are parallel to the members could have been taken, as sec α or tan α, or their equivalent ratios of lengths of sides, would be constant. The triangle ACD is merely taken for convenience. Similarly, at the point E we can take the

triangle ECB. EB, representing the stress in $EB = \frac{1}{4}w$, acting inwards towards E is compression. Then tension in

$$EG = \tfrac{1}{4}w \sec \alpha' = \tfrac{1}{4}w\frac{EC}{EB.};$$

$$\text{Compression in } DE = \tfrac{1}{4}w \tan \alpha' = \tfrac{1}{4}w\frac{BC}{EB}.$$

It is evident that $\frac{3}{4}w\frac{AC}{AD} = \frac{1}{4}w\frac{BC}{EB}$, as these are the only horizontal components, which is proved also by $BC = 3AC$, the one being 6 and the other 2 of the equal panels. The use of the number of panels, instead of absolute lengths, very much facilitates the computation of reactions and stresses. Although the Bollman truss is rarely if ever built at the present time, the method of computing stresses is fully entered into, as it is equally applicable to any form of truss, at any particular joint or panel point, when the load suspended at the point or transmitted to it is known, and will not again be explained.

900. *The Fink Truss.*—This truss is a modification of and improvement on the Bollman truss. It was used for a number of years, and many Fink bridges are found on old roads, but, like the Bollman truss, it is rarely if ever built now. This truss is shown on Fig. 321 (*b*). The calculation of the stresses is a little more complicated, as the load at one panel point may be transmitted to some other panel point, and a portion of every load travels first to the centre of the span and thence to its abutment. The transmission of these loads will be better understood by reference to the figure.

A load w at D passes down to N, causing compression in $DN = w$; at N it divides $\frac{1}{2}w$, passing directly through AN, producing tension; the other $\frac{1}{2}w$ passes to C, causing tension on $NC = \frac{1}{2}w\frac{NC}{DN}$. This is total tension on NC. From C it passes to L through LC, causing compression in $LC = \frac{1}{2}w$; at L, $\frac{1}{2} \times \frac{1}{2}w = \frac{1}{4}w$ passes through LA to A, causing tension in LA. The other $\frac{1}{4}w$ passes to E through LE, causing tension; thence down EG, causing compression. At G one half, that is, $\frac{1}{8}w$, passes through AG to A, causing tension; the other $\frac{1}{8}$ passes through GB to B. The load at F passes to K, thence $\frac{1}{2}w$, to E, down EG, thence $\frac{1}{4}$ to A and $\frac{1}{4}$ to B. The other $\frac{1}{2}w$, passes to C through CK, down CL; $\frac{1}{4}$ through AL to A, the other $\frac{1}{4}w$

to E; through EL, thence to G; thence $\frac{1}{8}w$ to A through AG, and $\frac{1}{8}w$ through GB to B.

We have then, finally, the following loads or reactions:

$$\text{At } A,\ \overset{\text{Load at } D.}{(\tfrac{1}{2}w+\tfrac{1}{4}w+\tfrac{1}{8}w)}+\overset{\text{Load at } C.}{(\tfrac{1}{2}w'+\tfrac{1}{4}w')}+\overset{\text{Load at } F.}{(\tfrac{1}{4}w_2+\tfrac{1}{4}w_2+\tfrac{1}{8}w_2)}+\overset{\text{Load at } E.}{(\tfrac{1}{2}w_3)}$$
$$=\tfrac{22}{8}w = 2\tfrac{3}{4}w;\ w = w_1 = w_2 = w_3.$$

To prove this the resultant load $4w$ acts half-way between C and F, that is, $5\frac{1}{2}$ panels from B. The reaction R_1 at A is, then,

$$R_1 = 4w \times \frac{5\frac{1}{2}\text{ panels}}{8\text{ panels}} = \frac{22}{8}w_1;$$

from which the stresses in AC and AN can be found as in the Bollman truss. By noting that AN forms a part of the three trusses ANC, ALE, and AGB; that NL forms a part of the two trusses ALE and AGB; and that LG forms a part of only one truss, AGB,—called, respectively, the tertiary, the secondary, and the primary trusses,— the tension on AN is greater than on NL and that on NL is greater that that on LG. Since each segment, two panels in length, of the top chord, namely AC, CE, etc., forms a part of all three trusses, the compression in each is the same when the truss is fully loaded, which is the condition for maximum stress. As this truss is not now used, it will be useless to discuss further. The truss used in Fig. 321 (*b*) is for a deck bridge, the rolling load being on the top chord. It can be constructed as a through bridge with the load on the bottom chord. The Bollman truss could be used as a deck bridge; in the figure it is shown as a through bridge.

In the two trusses the compression members are indicated by double lines, the tension members by single lines. The compression members were usually made of cast iron, tension members of wrought iron.

901. *The Post Truss.*—The Post truss is shown in Fig. 321 (*c*). This truss differs from other forms in having the web struts inclined instead of vertical, with a horizontal reach of one-half panel, and web ties inclined, with a horizontal reach of a panel and a half. As in the other forms of trusses, the maximum chord stresses occur with a full uniform load, in which case the web members meeting at the panel points 7, 9, and 11 do not act, and the chord stresses are found as in any other truss, being simply the horizontal components of the stresses in the web members added together from

the end to the panel under consideration: for instance, in the panel 1, 3 the sum of the horizontal components in the members 0, 1; 1, 2; 1, 4; and in 3, 5 the sum of those in 1, 0; 1, 2; 1, 4; 3, 2; and 3, 6; and so on to the centre panels 7, 9 and 9, 11. For the web stresses, as it is impossible to separate the systems, since they are connected at the centre, the best that can be done is to assume one system to be, 0, 1; 1, 4; 4, 5; 5, 8; 8, 11; 11, 12; 12, 15; 15, 16; 16, 17; 17, 18; and the other, 0, 1; 1, 2; 2, 3; 3, 6, etc.; and then find the stresses for each system separately, as in any other truss; the counters for the first system become 8, 9, 12.

"At the time that this truss was invented, it was considered important to avoid anything like indirect strains. A bridge should be so designed that the strain could be transmitted to each member directly on the line of the axis of that member, viz., the Post bridge, being based on theoretical considerations, all connections top and bottom were made on pins; top chord pieces abutted square; floor-beams were hung from panel points by double-looped hangers passing over chord-pins and around pins in the floor-beams at right angles to the chord pins. It was admirably designed to relieve the structure of all strains tending to deform a member, but the stiffness of the connections was sacrificed, and it would now be considered a very loose structure." This truss is no longer used.

902. *The Baltimore Truss.*—This truss is shown in Fig. 322, and a modified form of it, known as the Petit truss, is very generally used for long spans. The stresses are easily determined: the panel lengths are short, while the diagonals have an economical inclination.

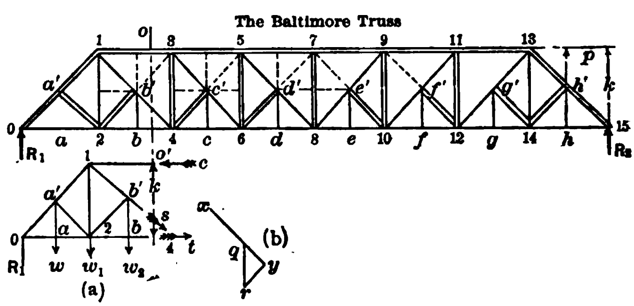


FIG. 322.

The chord stresses are best found by moments or sections. Passing the cutting-plane oo', intersecting the upper chord panel 1, 3, the diagonal 1, 4, and the bottom chord panel b, 4, we find

the compression in panel 1, 3 by taking moments about the lower chord panel point 4. The stresses are c, t, and s [see diagram (a)]; the external forces are w, w_1, and w_2; and the reaction R_1, which is readily found by the principle of the lever, and for maximum chord stresses the truss should be fully loaded. Taking moments about 4, the intersection of s and t, the moments of s and t are zero. Then

$$R_1 \times 4p - w \times 3p - w_1 \times 2p - w \times p - c \times k = 0, \quad . \quad . \quad . \quad (501)$$

in which p is the panel length $= 0a = a2 = 2b$, etc., and k is the depth of the truss $= 1, 2$; $3, 4$, etc. From eq. (501), c, the stress in 1, 3, is found, and if this is positive, as it should be, it acts inwards towards 1, and hence it is a compressive force. Taking moments about the panel point 1, the intersection of s and c, the moments of s and c are zero, and we have, since the moment of w_1 is zero also,

$$R_1 \times 2p - w \times p + w_2 \times p - t \times k = 0, \quad . \quad . \quad (502)$$

from which we find the stress t, which should be positive ahd act outwards from b, indicating tension in b, 4. In the same manner, taking a cutting-plane at any other point, we can find the chord stresses in the panel cut. It is evident that the tension in 2, b is equal to that in b, 4, as they are only separated by a vertical member b, b', which cannot change the horizontal stress.

Web Stresses.—The stress or tension in any subvertical aa', bb' cc', etc., is equal to the weight or load suspended from its lower extremity, the maximum value of which is a panel weight of structure plus a panel weight of live load $= (w + w')p$, in which w is the weight of the structure and w' of the moving load per foot of length, and p the panel length in feet. It is evident that, in whatever manner we may look at it, this load acting at the point b' must be divided equally between the two directions $2b'$ and $1b'$. The polygon of forces, diagram (b), however, clearly shows this, for at the point b' we have the external force $(w + w')p$; the stresses in $2b'$, $1b'$, and $b'3$. Then in diagram (b) draw qr vertically to represent the load $(w + w')p$, from r draw ry parallel to b'2 in Fig. 322, and through q draw the line yx parallel to 1, 4, and from x back to q. The length qx is not as yet known, being dependent upon the loads transmitted from other panel points through the tension member 1, 4, which is immaterial for present purposes, but represents the stress on the corresponding member of the Pratt truss under the same condition of loading as already explained; since 1, 4 and $b'2$ have equal inclinations, the

point y is on a horizontal line bisecting qr, and consequently the vertical component of the stress ry in $b'2$ is one half of qr, the load on bb'; the other is found in 1, 4. Then compressive stress $b'2$ is represented by the line ry, whose vertical component is $= \frac{1}{2}(w + w')p$; and the same for the members $c'4$, $d'6$, $a'2$, and the corresponding members on the other side of the centre, namely, $e'10$, $f'12$, $g'14$, and $h'14$.

For $0a'$, $b'4$, $c'6$, $d'8$, $e'7$, $f'9$, the vertical components of the stresses are the maximum positive shears in the panels to which these members belong. These maximum positive shears occur when the head of the rolling load extends from the right abutment to the panel points a, 4, 6, 8, e, and f, respectively, for the above members.

The stresses in the verticals 3, 4; 5, 6; 7, 8 are equal to the vertical components in $3c'$, $5d'$, and $7e'$, respectively, plus whatever load is assumed to rest at their upper extremities, and are compressive stresses.

The end vertical 1, 2 is in tension, the stress being the vertical component in $a'2$ + vertical component in $b'2$ + load at 2 $= 2(w + w')p$. And the same for the vertical 13, 14.

The vertical component of the stress is $1b'$ = the shear in the panel $2b$ − the vertical component $b'2$. This will be a maximum when the rolling load reaches the panel point b. For a load at b adds to the reaction on the left, that is, the positive shear, in the panel $2b$, $\frac{13}{16}w'p$, while it only adds to the negative component in $b'2$, $\frac{1}{2}w'p$, an increase of $\frac{13}{16}w'p - \frac{8}{16}w'p = \frac{5}{16}w'p$; therefore more is gained than lost by loading the point b. The shear in $1b'$ diminished by $\frac{1}{2}(w + w')\ p$ gives the vertical component of the maximum stress in $1b'$; and similarly for $3c'$ and $5d'$, and for corresponding members on the other side of the centre.

The shears or the resultant vertical components being thus found, the stresses are then multiplied by the length of the member and divided by the vertical reach, or its equivalent, the secant of the angle of inclination to the vertical, as in any other truss or frame.

For the member $a'1$ the resultant vertical component of the maximum stress is = shear in $a'1$ + the vertical component in $a'2$; for although loading the point a decreases the positive shear in the panel $a2$ by $\frac{1}{16}w'p$, it increases the vertical component in $a'2$ by $\frac{1}{2}w'p$, thus increasing the resultant vertical component in $a'1$ by $(\frac{1}{2} - \frac{1}{16})w'p = \frac{7}{16}w'p$. Hence for maximum stress in $a'1$ the bridge should be fully loaded.

Under certain conditions of loading the members $c'4$, $d'6$, $e'10$, and $f'12$ are under tension, for when the rolling load extends from the right to the panel point 10, only loading the shorter segment of the bridge, the vertical component of the stress in $e'10$ = positive shear (or reaction at the left) + the vertical component in $e'9$; the vertical component $e'9$ is one half the load at e, but loading e decreases the tension in $e10$ arising from the loads up to and including 10 by an amount equal to the reaction of this load on the right, which is $\frac{9}{16}w'p$, or a resultant decrease in the tension in $e'10$ of $(\frac{9}{16} - \frac{8}{16})w'p = \frac{1}{16}w'p$. In other words, we lose more than we gain by loading the point e, and for maximum tensile stress in $e'10$ the load should only extend to the panel point 10, the member $8e'$ not acting under this condition of loading, and the maximum tensile stress in $e'10$ is the positive shear in $e'10$ multiplied by the secant of the angle of inclination. The stress in $f'12$ is found in the same manner. It may happen that the shear in $f'12$ + vertical component in $f'11$ may be negative, in which case there can never be any tension in $f'12$. And in the case of $g'14$, the third panel from the end, the load extending from 15 to g, the result will be negative; the member $g'14$ can never be under tension, and there is no need of a member being used between the points 11 and g'. This corresponds to the conditions necessary for a counter-brace in these panels, as discussed in the Pratt truss.

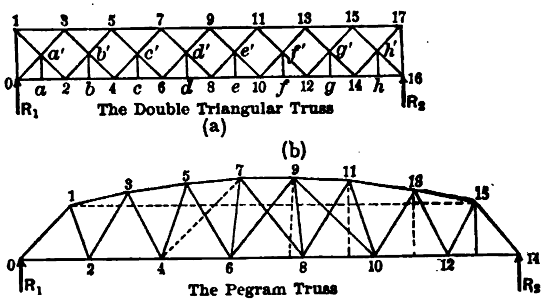
903. The Petit truss is very similar to the Baltimore truss, the only difference being in having a curved or inclined upper chord, which is a more economical design for long spans. The stresses are found in a similar manner. The chord stresses are best found by moments, as in the preceding case. The bottom-chord stresses are found as before, the depths of the truss at the several panel points being the lever-arms for these stresses.

The moments of the stresses in the top-chord panels are taken about the lower-chord panel points; but the lever-arms, being the perpendicular distances between the chord segments and the lower-chord panel points, have to be calculated or scaled from a large drawing, as explained in the Whipple or double-intersection truss.

The stresses in the web members are found as in the Baltimore truss.

In these long-span trusses the subverticals bb', cc', etc., are extended to intersection with the top chords; also, horizontal members are inserted between the intersections of the diagonals and the vertical posts. These are shown by dotted lines in left half of

truss. They form no part of the truss proper. They are merely intended to support and stiffen the upper-chord panels and the long vertical posts.



FIGS. 323.

904. The double triangular truss is shown in Fig. 323 (*a*). It has a double system of triangular bracing. The Kentucky and Indiana Cantilever Bridge, and the recently constructed Memphis Bridge, are of this form. For short spans the subverticals are omitted, and it has riveted connections. For long span it has pin connections. Leaving the subverticals *aa'*, *bb'*, etc., out of consideration, the truss can be divided into two separate systems. Certain panels of the top chord are common to both systems; and the maximum stress in those portions is found by adding together the stress due each system, taken separately. As the maximum stress occurs under full load, the web members 9, 6 and 9, 10 would not act; and on the left of the centre point the systems are 8, 7; 7, 4; 4, 3; 3, 0 and 6, 5; 5, 2; 2, 1 and 1, 0, respectively, and corresponding portions on the other half. The web stresses are found as in the simple triangular truss or Warren girder, taking the systems separately and independently. There is no material difference in the case shown in Fig. 323 (*a*), except as modified by the introduction of the subverticals, the stresses on these being simply the load acting at their lower extremities. This load is supposed to be transferred to the main panel points 0, 2, 4, 6, etc., and combined with any other load found at those points. In other words, whatever stress in this transference is caused in the horizontal or inclined members of the truss must be added to the stresses in those members of the truss caused by the existing loading. For instance, the load at *c* is supposed to be transferred to the main panel points 4 and 6 by the little triangular truss 4*c*'6, causing a compression in

$4c'$ and $c'6$ and a tension in 4, 6. And as the maximum tension in 4, 6 occurs when all points are loaded, this tension due to the small truss must be included in that found by the usual method of moments or otherwise. The compression in the upper-chord panels is found by considering the entire load as acting and concentrated at the main panel points.

The dead-load web stresses are found in similar ways, treating the systems independently, but adding the stresses thus found in those members forming a part of both systems.

For maximum live-load stress in any member the load must extend over the longer segment and, reaching to the vertical, pass through its lower extremity. For instance, the maximum tension in $3b'$, the main panel points at 4, 8, 12, are to be fully loaded; but as a part of the loads acting directly at b and c are transferred to 4, at d and e to 8, and at f and g to 12, all of these points must be loaded. But for maximum tension in $b'4$ the point b should not be loaded, for the half of this load transferred to 4 tends to produce compression in $b'4$, which would reduce the tension in $b'4$ caused by other loads; whereas on the portion $3b'$ it causes tension, that is, the same kind of stress as is produced by other loads, and for this member the effect is the same as if it acted directly at 4. For maximum compression in $5b'$, all points up to and including c must be loaded, since the load at c causes tension in $5c'$, and consequently compression in $5b'$; and for maximum compression on the portion $b'2$ the load must extend to b, as the load at b tends to produce compression in $b'2$ of the same kind as caused by other loads, and tension in $b'5$, which is of opposite kind to that caused by other loads to the right.

905. The Pegram truss, Fig. 323 (b), is an economical and good design. There are the same number of panels in each chord. The panels of the lower chord are longer than those of the upper chord, but those in the separate chords are of the same length, except that, for convenience of manufacture, as the splices in the top chord are towards the ends of the truss from the pin points, the centre panel, measured from centre to centre of pins, is a fraction shorter than the other panels, in order to permit of the chord sections between the splices being of the same length. The upper-chord panel points are in the arc of a circle, the chord of which, 1, 15, is about one and one-third to one and one-half panel lengths shorter than the bottom chord or span. The versed sine of this arc may be so taken that the lengths of all the posts may be nearly

equal, or they may decrease in length towards the ends, where the shear is the greatest. In a deck bridge the upper chord is made straight and the lower chord curved.

The chord stresses can be found by moments, as explained in Whipple truss with curved upper chord; as also, the web stresses may be calculated in the same way. But it will generally be found more convenient by taking the difference between the horizontal components of the chord stresses in the same panel, which will be the horizontal component of the stress in the diagonal, from which the stress itself can be determined. This is fully explained in the discussion of the Whipple truss. The live-load stresses in the chords and in the diagonals 0, 1 and 1, 2 are a maximum under full load, and are therefore found as for dead-load stresses. For maximum live-load stresses in 2, 3 and 4, 3, all joints must be loaded from the right up to and including 4. For 4, 5 and 5, 6 the load should extend to 6.

The graphical method, either alone or combined with the method of moments, can be applied to the determination of the stresses in any of the foregoing trusses. These methods in detail are found fully discussed in such works as those of Dubois, Johnson, and Burr.

FRAMED TRUSSES, CONSIDERED WITH REFERENCE TO ACTUAL LOADS OCCURRING IN PRACTICE.

ROOF TRUSSES.

906. *Dead Load.*—The fixed load supported by a roof-truss consists of (1) the weight of the truss itself; (2) the covering, including the covering proper, such as slate, tin, or shingles; the sheeting, rafters, and purlins; and, (3) when it exists, the weight of ceilings or floors suspended from the truss. The weight of the covering and that of ceilings and floors can be directly computed.

The total weight of the covering will vary from 5 to 30 pounds per square foot of roof surface. The weight of the truss can only be calculated from some empirical formulæ. It has been found approximately equal to $\frac{1}{24}$ the length of the span in feet multiplied by the area of the roof surface included between any two trusses. If, then, $l =$ the span, b the distances between the trusses, in feet, and the area equal to bl square feet, the weight of one truss in pounds $= \frac{1}{24}bl^2$.

If, however, after this assumed weight the actual truss differs materially, a recalculation should be made.

907. *Live Load.*—The moving or variable load consists of the wind, snow, and floor loads. The maximum wind pressure is estimated at from 30 to 56 pounds per square foot of surface normal to its directions.

Experiments have shown that the pressure per square foot on large surfaces is less than that on small surfaces; and from these experiments it would seem safe to assume a maximum pressure of 45 pounds per square foot upon small surfaces, and 30 pounds on large ones. The more recent empirical formulæ for a pressure per square foot of surface is $p = 0.004v^2$. v being the velocity in miles per hour, with 30 pounds per square foot, $v = 86\frac{1}{2}$ miles nearly, which is an unusually high velocity, and would be called a hurricane.

The longitudinal component of the pressure of the wind upon a roof is zero for smooth surfaces, and nearly so for any ordinary surface. The normal component is usually calculated from Hutton's formula,

$$p' = p \sin \alpha^{1.84 \cos \alpha - 1},$$

in which p' = normal component; p = pressure per square foot on a vertical surface; α = angle of inclination of the roof with the horizontal. Assuming $p = 30$ pounds, and α different values from 5° to 60°, the following values of p' result.

TABLE LXXIII.

$\alpha = 5°$,	$p' = 3.9$	pounds.	$\alpha = 35°$,	$p' = 22.6$	pounds.
$\alpha = 10°$,	$p' = 7.2$	"	$\alpha = 40°$,	$p' = 25.1$	"
$\alpha = 15°$,	$p' = 10.5$	"	$\alpha = 45°$,	$p' = 27.1$	"
$\alpha = 20°$,	$p' = 13.7$	"	$\alpha = 50°$,	$p' = 28.6$	"
$\alpha = 25°$,	$p' = 16.9$	"	$\alpha = 55°$,	$p' = 29.7$	"
$\alpha = 30°$,	$p' = 19.9$	"	$\alpha = 60°$,	$p' = 30$	"

From an inclination of 60° to that of a vertical surface the value of $p' = 30$ pounds, as near as may be.

The snow load is taken at from 10 to 30 pounds per square foot of horizontal projection of surface, the weight of a cubic foot of snow being from 5 to 12 pounds, according to its dryness. When α is from 45 to 60°, depending on the smoothness of the roof covering, the snow load need not be considered. The floor loads will vary from

50 to 150 or more pounds per square foot. A full allowance should be made in the calculation.

The weight of the covering, sheeting, snow, and wind loads are transferred to the trusses by means of purlins. It is better to place the purlins at those points on the rafters directly supported by struts, called joints; otherwise the rafters will have to be dimensioned to bear both direct compression and bending.

In most roof-trusses the panel lengths are equal, and commonly in the same plane on each side of the centre; the purlins are usually placed at the joints or panel points. In such case the panel loads are equal; and are concentrated at the joints. It is also usual to assume the weight of the truss as concentrated in equal portions at the same point, at the points of support one half of a panel weight is assumed to rest. The above includes all weights and loads, namely, those of the structure, snow, and wind. If the roof is arched or steeper along some portions than along others, then both snow load and wind pressure will vary from panel to panel. In such cases the joint or apex loads will be unequal, but still equal to the half sum of the adjacent panel loads. When the purlins rest on the rafters or chords of the truss at points between the joints the portion of the load resting on them at these points is assumed to be transferred to the adjacent joints in the inverse ratio of the distances of these points from the joints.

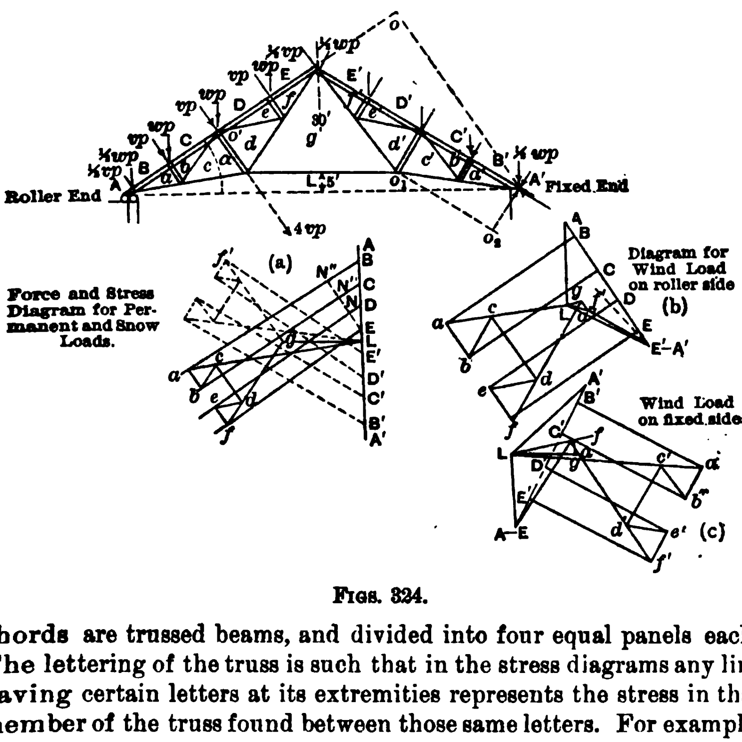
The reactions at the points of support are both vertical, arising from the weight of the structure, and snow and floor loads, if these latter exist. The direction of the reactions, arising from wind-pressure, will depend upon the manner of supporting the truss. If both ends are fixed, the wind reactions will be parallel to the direction of the resultant wind load. If one end is free to move, as when resting on rollers, the reaction at the roller end will be vertical, and that at the fixed end will have to be determined from the conditions existing. If one end is fixed and the other rests on a smooth, sliding plate, there will generally be a horizontal component at this end, which will be equal to the vertical component multiplied by the coefficient of friction, which may be taken as equal to one-third.

As roof-trusses are not subjected to many changes in the position or distribution of loads, in order to produce maximum stresses in its members, the graphical method of determining stresses is especially suitable, as only one or two stress diagrams are necessary.

908. Referring to Articles XXI and XXII, where are given the methods of finding stresses by the graphical method, and recollect-

ing the diagrams are merely built up of a series of triangles and polygons placed together along lines which are common to two or more of them, and that in determining the kind of stress in the various members the polygon of stress for each joint must be considered separately, so that the direction of action of stress represented by the common line may be different when taken as an independent part of each stress polygon, the following diagrams will be readily understood:

In Fig. 324(a) is shown a form of truss known as the French Truss. It is a common form, and considered an economical design for spans not exceeding 150 feet in length. The two rafters or upper



FIGS. 324.

chords are trussed beams, and divided into four equal panels each. The lettering of the truss is such that in the stress diagrams any line having certain letters at its extremities represents the stress in that member of the truss found between those same letters. For example, the stress in the upper-chord panel between the letters B and a in Fig. 324 is represented by the line Ba in the diagram Fig. 324(a), the member between f and g by fg, the load at the centre strut between C and D by CD in the line of loads, and so on. Considering first the weight of the structure itself, and calling wp the

panel weight, at each of the joints a load $= wp$ is concentrated, except that at A and A' only $\frac{1}{2}wp$ acts directly. These loads are indicated by the vertical arrows at the joints, and are the only external forces considered in diagram Fig. 324 (a). Then draw the vertical line $AA' = 8wp$, total dead load on the truss, and divide it so that to any given scale, say 5000 pounds per inch, AB and $A'B'$ will each equal $\frac{1}{2}wp$, and the other divisions $= wp$; then with the centre point the force polygon for loads is AA' acting downwards $= 8wp$, $A'L$ upwards $= 4wp$, and LA upwards $= 4wp$, the reactions at A' and A, respectively. Then for the joint A we have the upward reaction $= AL$; the load $= \frac{1}{2}wp = AB$ acting downward, both known; and the unknown stresses in Ba and aL. Then from B draw a line parallel to the piece Ba, and from L a line parallel to the piece La, forming the closed polygon LA, AB, Ba, and aL, or $LABaL$. Ba acts towards the left, that is, inward on the rafter towards A, hence compression; while aL must act towards the right or outwards from A on the piece aL, hence tension. For the joint at the first strut between B and C we have the load pw, the stress just determined in the member Ba, and the unknown stresses in the strut ab and the panel bC. In diagram (a) the known terms are $BC = pw$ and Ba the stress in Ba; from a and C draw the lines ab and Cb parallel to the members Cb and ab. They will represent the unknown stresses, the force polygon being $BabCB$. As Ba has been found to be a compressive stress, it must act inwards towards the joint now under consideration, that towards the right from a to B,—the opposite direction from that given to it when the diagram for joint A was under consideration; BC acts downward; Cb to the left or inwards towards the joint, hence compression in Cb; ba acts upwards towards the left, that is, inwards on the strut ab towards the joint, hence this member is under compression. For the joint at the foot of this strut between ac we have the four stresses in ab, aL, cb, and cL; the first two are known. From b draw the line bc parallel to the member bc intersecting the known stress line aL in c; the polygon for this joint is $LabcL$. Since the members aL and cL are in the same straight line, as ab must act inwards towards the joint between a and c on the member ab, then bc acts upwards to the right, indicating tension on bc; cL acts outwards to the right, or tension on cL. This brings us to the joint, at the main strut cd, between C and D. Here it is seen that we have three unknown stresses, namely, that in cd, de, and De; whereas the method now being followed permits

of only two unknown stresses at any one joint; we must therefore be able to find one of these unknown quantities by some independent method.

Passing on to the joint between the panels D and E we would still have three unknown stresses; but it is evident that the only stress that can come upon the member ed arises from the stress developed in ef by the load wp, which is equal to the normal component of that load. Or, decomposing DE, diagram (a), into components parallel and perpendicular to the rafter, DN and NE are these components, respectively. It is further evident that the compression in the chord De is greater than that in Ef by the parallel component ND, that in Cb greater than that in De by the same amount, and that in Ba greater than that in Cb by the same. Therefore if we prolong EN as indicated to intersection with Ba at N'', the increments of stress, in the successive panels towards A over that in the upper-chord panel Ef, are, respectively, ND, $N'C$, and $N''B$; and the stress lines to the left of $ENN'N''$ must be of the same length, i.e., $aN'' = bN' = eN = fE$. If, then, through the points D and E of line of loads we draw De and Ef parallel to the members De and Ef of the truss, and prolong ab of diagram (a) to intersection with them, at e and f respectively, De and Ef will represent the stresses in the chord panels De and Ef, and the intercepted line ef will represent the stress in the strut ef. Having now found the stress in chord panel De, there remains only two unknown stresses at the joint between C and D, namely, that in cd and de. Then draw from c a line parallel to the strut cd, and from e a line parallel to the member de, intersecting at d in diagram (a); then the stress polygon $CbcdeDC$, in which cd is the compression in strut cd, and ed is the tension in de. For the joint between c and d the stress polygon is $LcdgL$ and for the joint between e and f is $efgde$. The kind of stress in the several members can be determined as explained fully above. The full lines in diagram give the stresses in all of the members in the left half of the truss. As the truss is symmetrically loaded, corresponding members in the other half have the same kind and magnitude of stress in them. As a check, however, the diagram for the other half should be drawn; this is indicated by the dotted lines, the closing of the last line corresponding to fg; that is, $f'g$ at the point g checks the whole work, or the work can be checked by moments without drawing the second half of the diagram. Compression members are indicated by double lines, tension by single lines, in Fig. 324.

The same diagram exactly can be used for the stresses due to the snow load, the scale being changed to suit the panel weight of snow; or if the panel weight of snow is, say, three fourths of that due to weight of structure, simply take three fourths of the stresses scaled off from diagram (*a*): they will be the stresses due to the snow load, and will be of the same kind as those for dead load.

909. *Wind Stresses.*—As the reaction at the roller end is vertical whether the wind blows from the right or left, the stress diagrams will not be the same in the two cases; consequently there must be one stress diagram for the wind blowing from one side, and a second diagram when blowing from the other. Diagram Fig. 324 (*b*) is for the wind blowing from the left, or against the truss on the roller-end side. The normal component of the pressure for different wind velocities is given in table, paragraph 907. Calling this per panel vp, we have, acting normally to the top chord at each of the panel points between B and C, C and D, and D, E, a pressure vp; and at A and the apex between E and E', $\frac{1}{2}vp$. The diagram (*b*) is then readily constructed. Draw AE' to represent $4vp$ parallel to the normal wind pressure, and divide this line so that $AB = \frac{1}{2}vp$, $= EE'$, and $BC = CD = DE = vp$, acting from A towards E'. Then, since the reaction at A, the roller end, is vertical, we can readily find this reaction from $\Sigma m = 0$, with axis of moments at A':

$$\Sigma m = R \times L - 4vp \times oo' = 0, \quad . \quad . \quad . \quad . \quad (503)$$

from which the reaction R at A is found. Lay off this vertically from $A = AL$. Then LA' will be the wind reaction at A', the fixed end of the structure, ALA' being the force polygon in diagram (*b*). Then for the joint A the stress polygon is $LABaL$. As LA acts upward, AB downward, Ba to the left, or inwards on the member Ba, it will be compression; and as aL acts to the right or outwards on the member aL, it is tension. For the joint between B and C the stress polygon is $BabC$; the stress polygon for joint ab is $LabcL$; for DC, $CbcdeDC$. For wind pressure there is no trouble at the joint between CD, since the pressure at that point, vp, as well as that transmitted to it through the members bc and $de = \frac{1}{2}vp + \frac{1}{2}vp$, or a total of $2vp$, acts normally to the chord, or in the direction of the strut dc. The stress on this strut is known, and equal to $2vp$; and there are then only two unknown stresses, namely, those in DE and de, and there is no difficulty in drawing the stress polygon $CbcdeDC$. For joint between D and E, stress

polygon is $DefED$; for joint ef, stress polygon $defgd$. From which it will be seen that by giving proper direction to the stresses in the several stress polygons, the stresses in all members of the left half of the truss are of the same kind as those caused by dead and snow loads, the resultant stresses being the sum of all three stresses; except that in the horizontal tension member gL, when only acted upon by dead and snow loads, the wind load tends to produce compression, as seen from the stress polygon for the joint between c and d, which is $cdgLc$, since cd acts downwards, dg to the right, gL to the left, inwards on the piece Lg towards the joint, that is, gL indicates compression. As in the case considered there is no direct action of the wind on the right half of the truss, the entire effect of the wind pressure must be transferred to it at the apex joint EE', or possibly through Lg also. The stress polygon for the joint EE' is $gfEE'f'g$, from which it will be seen that the stresses in the members $E'f'$ and $f'g$ are compressive; and for the joint $d'c'$ the stress polygon is the little triangle gLf' (or a'). As all of these act inwards towards the joint gd', gL and Lc' are compressive, and the same as in gf' and La'. For the joint A', the stress polygon is La' (or f') $A'L$, $A'L$ being the reaction, and others stresses. From the above it is seen that when the wind blows from the left, as indicated, the stress on the right side rafter or chord is compressive, and represented by the stress line A' (or E') f', and is constant from end to end, and of the same kind as those caused by dead and snow loads; but that the tendency is to cause compression on the members gf', gd', $c'L$ and aL, and gL, all of which are under tension due to dead and snow loads. The actual stress then existing is that due to the difference between the compressive and tensile stresses. This difference will usually be of the same kind as the dead-load stresses, unless the truss has a great rise as compared with the length of span. In this case the wind stresses would be greater than the dead-load stress, and these members may be at times under compression, and should be counterbraced so that they may act as both struts and ties. There is no wind stress on the members $f'e$, $e'd'$, $d'c'$, $c'b'$, or $b'a'$.

When the wind pressure acts on the right or fixed-end side the stress diagram is as given in Fig. 324 (c). The reaction at the roller end A is vertical, and found from $\Sigma m = 0$, as before. The lever-arm of $4pv$ is now, however, o_1o_2, instead of oo'. As in eq. (503),

$$\Sigma m = R \times L - 4vp \times o_1o_2 = 0, \quad . \quad . \quad . \quad (504)$$

from which the vertical reaction at A is $R = LA$ (Fig. 324 (c)). $A'E$ is the line of loads; and from A or E lay off $AL = R$, vertically; LA' is the closing line. The force polygon is then $A'LA$, and LA' is the reaction at A' in magnitude and direction. The diagram (c) is then constructed similarly to diagram (b). This can readily be constructed, and the magnitude and kind of stresses determined as described above.

The kind of stress that will result cannot be predetermined. The direction of action with respect to the joint under consideration must be followed out for each stress polygon taken separately, commencing with those that are known, and following continuously around the polygon. All stress lines acting on the member considered inwards towards the joints indicate compression, and those acting outward indicate tension. Scaling off all the stress lines, reducing them to pounds or tons according to value of the unit used in laying off the load lines, adding all of the same kind or sign to determine maximum stress, taking the differences where the kind of stresses from some loads is different from others, and assuming compression as plus (+) and tension as minus (−), if the difference is the same sign as that due to the dead load, the member will always have the same kind of stress. If, however, it is of the opposite sign to the dead-load stress, the member must be counterbraced, that is, designed and connected to bear both tension and compression.

910. In the above example, assume a length of span $AA' = L = 97.5$ feet; rise $= 35$ feet, or 30 feet above lower tension-rod gL: then the length of one side of the roof $= \sqrt{48.75^2 + 35^2} = 60.0$ feet; panel length $= 15$ feet; angle of inclination of side $=$ arc-sin $\frac{35}{60} = 35° = \alpha$; the distance centre to centre of trusses $= 20$ feet; weight of truss itself $= \frac{1}{24}bl^2 = 7922$ pounds; weight of roof covering 20 pounds per square foot, and per truss $60 \times 2 \times 20 \times 20 = 48{,}000$ pounds. Then total dead load $= 48{,}000 + 7922 = 55{,}922$; panel weight $= wp = \frac{55922}{8} = 6990$ pounds. These are the joint loads represented by $BC = CD$, etc.; $AB = A'B' = 3495$ pounds.

Taking the snow load at 20 pounds per square foot of projected area, equal to span × distances between trusses, total snow load $= 39{,}000$ pounds $= 97.5 \times 20 \times 20$, and per panel $= \frac{39000}{8} = 4875$ pounds. Diagram Fig. 324 (a) can be used for this loading by making $BC = CD$, etc., $= 4875$ pounds, or, what is the same thing, multiplying dead-load stresses by $\frac{4875}{6990} = 0.697$, nearly.

From the table, paragraph 907, the normal component of the

pressure on a surface sloping at 35° is 22.6 pounds; and as this can act only on one side of the roof at a time, total wind pressure $= 22.6 \times 60 \times 20 = 27,120$ pounds, and per panel $= vp = \frac{27120}{4} =$ 6780 pounds. This is represented, in diagrams (*b*) and (*c*), by $BC = CD = B'C' = C'D'$ and $AB = EE' = A'B' = EE' = 3390$ pounds. The values of the stress lines are determined by the same scale.

911. Another common form of roof-truss is shown in Fig. 325. All inclined members are under compression; all verticals and horizontal rods are under tension. This truss is often constructed with wooden compression members and iron tension members. The same may be said of the French truss, just discussed. The force and stress diagrams for the truss Fig. 325 can be readily constructed, following the principles laid down in the preceding discussion. The stresses are easily calculated from the joint diagrams, as there are only three members meeting at any joint. The

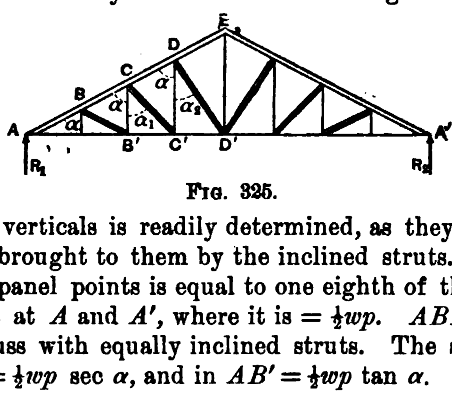


FIG. 325.

stress on the verticals is readily determined, as they simply transmit the load brought to them by the inclined struts. The load at each of the panel points is equal to one eighth of the entire load $= wp$, except at A and A', where it is $= \frac{1}{2}wp$. ABB' is a simple triangular truss with equally inclined struts. The stress, then, in AB or $BB' = \frac{1}{2}wp \sec \alpha$, and in $AB' = \frac{1}{2}wp \tan \alpha$. For the truss ACC' the apex load at $C = wp + \frac{1}{2}wp$; $\frac{1}{2}wp$ passing from B to B' and from B' to C. The stress in $AC = \frac{1}{3} \times 1\frac{1}{2}wp \sec \alpha$; in $CC' = \frac{2}{3} \times 1\frac{1}{2}wp \sec \alpha'$; and in the chord $AC' = \frac{1}{3} \times 1\frac{1}{2}wp \tan \alpha$. For the truss ADD', apex load at $D = wp + \frac{2}{3} \times 1\frac{1}{2}wp = 2wp$. Stress in $AD = \frac{1}{4} \times 2wp \sec \alpha$; in $DD' = \frac{2}{4} \times 2wp \sec \alpha_2$; and in chord $AD' = \frac{1}{4} \times 2wp \tan \alpha$. For the truss AEA', apex load $= \frac{1}{2}wp + \frac{3}{4} \times 2wp = 2wp$ from left half and $2wp$ from right half, with top chords at equal inclination. Stress in $AE = EA' = 2wp \sec \alpha$, and in chord $AA' = 2wp \tan \alpha$.

AB forms a part of all four trusses, *BC* of the three larger, *CD* of the two larger, and *DE* only of the largest or primary truss; and similarly for AB', $B'C'$, and $C'D'$.

Hence, summing up,

$$\left.\begin{array}{l}
\text{Compression in } DE = 2wp \sec \alpha; \\
\quad\text{“}\qquad\text{“}\ \ CD = 2wp \sec \alpha + \tfrac{1}{2}wp \sec \alpha; \\
\quad\text{“}\qquad\text{“}\ \ BC = 2\tfrac{1}{2}wp \sec \alpha + \tfrac{1}{2}wp \sec \alpha; \\
\quad\text{“}\qquad\text{“}\ \ AB = 3wp \sec \alpha + \tfrac{1}{2}wp \sec \alpha; \\
\text{Tension in } CB' = \tfrac{1}{2}wp;\ \text{in } DC' = wp;\ \text{and in } ED' = (1\tfrac{1}{2} \\
\quad + 1\tfrac{1}{2})wp = 3wp; \\
\text{Compression in } BB' = \tfrac{1}{2}wp \sec \alpha;\ \text{in } CC' = wp \sec \alpha; \\
\quad \text{and in } DD' = 1\tfrac{1}{2}wp \sec \alpha_2.
\end{array}\right\} \qquad (505)$$

$$\left.\begin{array}{l}
\text{Tension in } C'D' = 2wp \tan \alpha; \\
\quad\text{“}\qquad\text{“}\ \ B'C' = 2wp \tan \alpha + \tfrac{1}{2}wp \tan \alpha; \\
\quad\text{“}\qquad\text{“}\ \ AB' = 2\tfrac{1}{2}wp \tan \alpha + \tfrac{1}{2}wp \tan \alpha.
\end{array}\right\} \qquad (506)$$

Stresses from snow loads are found by substituting the proper numerical value for *wp*, as already explained in paragraph 910, in the above equations. For wind stresses it would be better to apply the stress diagrams or use the method of moments, as the kinds of stress in any member can be more readily determined.

Often the Crescent truss or some form of arch truss is used for long spans. The arch truss is usually hinged at crown and springing. For train-shed roofs the arch often extends to the floor, the horizontal components at the springing being resisted by means of a tie-rod placed beneath the floor.

If the arch has no hinges, or has but the two hinges at the abutment, the stresses depend upon distortion as well as upon the static load.

For the full discussion of roof-trusses of these types see such works as Dubois and Johnson, as it would be beyond the scope of this volume.

Trusses of this kind will, however, be briefly discussed in another connection later.

BRIDGE TRUSSES.

912. The general definition of a frame has been given in Article XXII, paragraph 208. In this article a framed structure is to be considered as composed of straight members, connected or joined at their extremities in such a manner as to act sensibly as a rigid body.

This applies equally to: (1) Bridge-trusses, either for railway or highway bridges; (2) Roof-trusses.

The principles and their applications are, however, the same,

whatever may be the character of the structure or for whatever purpose constructed.

The determination of the magnitude and character of the stresses will in general be perfectly simple, even in the most complicated frames, when the direction, magnitude, and points of application of the loads or external forces are assumed or known. The first and important questions are, What are the loads, and in what manner should they be distributed on a structure in order that they may produce the maximum stresses on the members of the truss? The determination of these questions is very difficult and in many respects unsatisfactory, and as a consequence many different assumptions and suppositions have been made. These differ considerably in the practice of the several bridge companies and in the requirements of engineers.

In either case the loads are divided into two kinds: (1) Dead Loads and (2) Live Loads.

913. *Dead Load.*—The dead load is the weight of the structure itself. Evidently this must be assumed, or calculated by some empirical formula deduced from the actual weights of structures that have been erected.

This can now be done with such a close degree of approximation that a second calculation will not in general be necessary. This load is commonly considered as uniformly distributed over the entire structure or bridge, and is commonly expressed in pounds or tons per foot of length.

914. *Live Load.*—The live or moving load consists of trains, vehicles, people, snow, and the force of the wind, or, in short, of any temporary and moving load or force that may come upon a structure, or not in any wise connected with or forming a part of the structure itself.

The purpose for which the frames or trusses are constructed determines the special kind of live load which it will have to support.

It will therefore be necessary to discuss and determine both the dead and live loads by considering the form and purpose of the structure.

RAILWAY BRIDGES.

915. A railway bridge or span consists of two parallel and vertical or of two inclined trusses or frames, connected by a system of horizontal braces called wind or lateral braces; and a floor system, the purpose of whichis to support directly the moving load and transmit this load to the trusses which support it and carry the

load, both dead and live, to the piers, abutments, or supports. This floor system consists of floor-beams, stringers, ties, guard-rails, iron rails, etc. Each truss is a frame composed of two chords, either parallel or inclined to each other, and at vertical distances apart, determined by questions of strength, stiffness, length of span, convenience and economy in design, manufacture, and erection.

These chords are connected by a system of braces, called the web system, which taken together constitute the truss. The truss as a whole is designed to act as a beam, while each of its component parts or members is subjected only to a longitudinal stress, that is, a stress either of direct compression or tension. With long panels, and where the live load is distributed along the chords, these are under bending stress, in addition to the direct longitudinal stress.

The kind of stress, whether tension or compression, borne by each member of a truss will depend upon its position in the truss, and upon the manner of loading and supporting the truss.

As a rule, the top chords of bridge-trusses are under compression, the bottom chords under tension, the exceptions being the cantilever trusses and the trusses of swing-bridges, under certain conditions. The diagrams illustrating all of the more common types of bridge-trusses have been given, only one or two of which are now in common use. It will be seen from these designs that the web members may be designed, connected, and placed with respect to the load, so that any one may act either as a strut or tie, with the sole limitation that they must be alternately in tension or compression; and each will always be permanently under tension or compression, except in the Warren or Zigzag girder, in which some of the web members will be under tension at one time and compression at another. This is called a counterbraced system.

To avoid this alternation of stress in the same member, counterbraces are introduced in certain panels, in which case the other web members, called main braces, are always under the same kind of stress in any particular truss.

Until quite recently all spans over 60 to 70 feet in length were composed of trusses, while spans under those lengths were either trussed, or composed of solid-built beams. This limit has now, however, been extended to lengths of 100 or 120 feet or more; very recently a plate girder of about 180 feet in length has been built. The bending moments and shears due to live or moving loads will now be considered.

BENDING MOMENTS IN A BEAM.

916. In the preceding application of the balance of moments the structure has been taken as whole, and only the question of the equilibrium of the externally applied forces been considered. The equations of equilibrium were only used to find the magnitude and direction and points of application of the unknown forces from those that were known, which we have seen is always possible when there are only three unknown forces.

Having now seen the manner of determining the magnitudes and disposition of the external forces on a beam or truss in equilibrium, we come to consider the effect of these forces upon the beam, and the manner in which they are or should be resisted.

Under this division of the subject we apply the equations of equilibrium to only a portion of the beam, conceived to be separated from the other portion of the beam by an ideal section. Equilibrium must then be produced by the supposed application of certain external forces, which are equal to actual internal stresses developed by the resistances of the material to strains. The moment of the external forces, taken with respect to the neutral axis of the ideal surface, whichever portion of the beam may be considered, is called the bending moment at that section. And whatever may be the distribution of the external forces, the sum of the moments of all of these forces on one side of the section must be equal to that on the other, but of opposite sign.

We usually call a bending moment positive when it tends to produce convexity downwards or causes a tensile stress in the lower fibres. The positive bending moment has then the same sign as the moment of the resultant force on the left of the section, and negative when the reverse.

The bending moment at any section C, in a beam supported at both ends, is always positive.

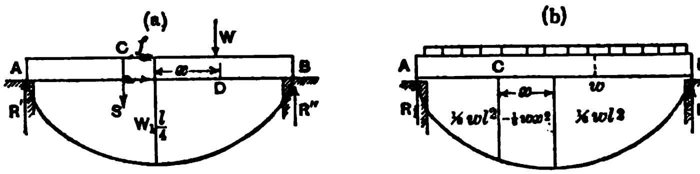
The effect of placing any additional load on a beam supported at both ends is simply to increase the abutment reactions, and consequently the positive bending moment at any section. For a uniform load the maximum bending moment occurs then at any section when the load covers the beam from end to end. From Art. XXIII we see that the bending moment at any point distant x from the centre of the uniformly loaded beam, Fig. 326 (b), is

$$M = \tfrac{1}{8}wl^2 - \tfrac{1}{2}wx^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (507)$$

or, under another form,

$$M = \tfrac{1}{2}w\left(\frac{l^2}{4} - x^2\right) = \tfrac{1}{2}w\left(\frac{l}{2} - x\right)\left(\frac{l}{2} \times x\right). \quad . \quad . \quad (508)$$

In words, the bending moment at any point is one half the load per unit of length (one foot or one inch), multiplied by the product of



FIGS. 326.

the segments (in feet or inches) into which the beam is divided. Eq. (508) is that of a parabola, the middle ordinate of which is $\frac{1}{8}wl^2$, x being zero at the middle point.

For a single isolated load W_1 (see Fig. 326 (a)) the reaction on the right is $R'' = W\dfrac{\frac{l}{2} + x}{l}$, and the bending moment at any point is the greatest when the load is at that point; and hence at D

$$M = W_1\left(\frac{\frac{l}{2} + x}{l}\right)\left(\frac{l}{2} - x\right) = W_1\left(\frac{l}{4} - \frac{x^2}{l}\right). \quad . \quad . \quad (509)$$

This will have its maximum when $x = 0$; that is, at the centre of the beam, where $M_0 = W\dfrac{l}{4}$. Eq. (509) is that of a parabola whose maximum ordinate is $W\dfrac{l}{4}$.

917. The determination of the point of maximum bending when there are any number of isolated loads, either with or without a uniform load on a portion of the beam or span, will be fully discussed in another paragraph.

As an illustration of the method of determining the point of maximum bending and the curve of maximum moments, take any two equal loads capable of moving along a beam, but at the same time maintaining a fixed distance apart represented by a, as shown

in Fig. 327. Let W_1 and W_2 be the loads, l the length of the span, x the distance of the left-hand load from the centre of the beam, and $x - a$ the distance of the right-hand load from the centre. For the bending moment under the left-hand load, W_1, the resultant moment will be that due to both loads. The left reaction due to the load W_2 is $R_1 = W_2 \dfrac{\frac{l}{2} + x - a}{l}$. Its moment under W_1 is then

$$M' = W_2\left(\frac{\frac{l}{2}+x-a}{l}\right)\left(\frac{l}{2} - x\right) = \frac{W_2}{l}\left(\frac{l^2}{4} - x^2 - \frac{al}{2} + ax\right). \quad (510)$$

This equation is that of a parabola having a maximum ordinate, at $x = \frac{a}{2}$, thus equal to

$$M_2 = \left(\frac{l^2}{4} - \frac{al}{2} + \frac{a^2}{4}\right)\frac{W^2}{l} = \frac{W^2}{l}\left(\frac{l}{2} - \frac{a}{2}\right)^2. \quad . \quad . \quad . \quad (511)$$

This is the curve in Fig. 327, $Ab'B'$, with maximum ordinate at b' distant from the centre G of the beam $\frac{a}{2}$.

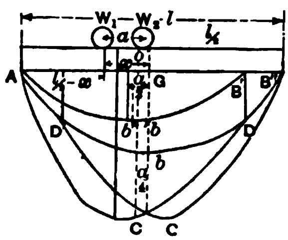


Fig. 327.

The curve of moments for the load W_1 is the parabola AbB as the loads move over the beam (see eq. (509)) with maximum ordinate at the centre of the beam.

The ordinates of these two curves added together will give the curve representing the sum of the moments, namely, $ACDB$. The equation of the portion ACD is found by adding eqs. (509) and (510), recollecting that the loads are equal, $W_1 = W_2 = W$. Hence total moment

$$M = \frac{W}{l}\left(\frac{l^2}{2} - 2x^2 - \frac{al}{2} + ax\right). \quad . \quad . \quad . \quad . \quad (512)$$

For a maximum, $\frac{dM}{dx} = 0$. Hence $x = \frac{a}{4}$.

Changing the origin to this point by making $x = x' + \frac{a}{4}$ in eq. (512),

$$M = \frac{W}{2l}\left[\left(l - \frac{a}{2}\right)^2 - x'^2\right]. \quad . \quad . \quad . \quad . \quad (513)$$

This is the equation of a parabola with axis vertical passing through the origin. The maximum ordinate is, for $x' = 0$, $\frac{W}{2l}\left(l - \frac{a}{2}\right)^2 = oC$ in the figure. For moment under the right-hand load the curve is $AD'C'B$, and is symmetrical to $ACDB$. The greatest moments for the left half occur under the left load, and for the right half under the right load. The maximum moment in the beam is found at a distance from the centre equal to one fourth the distance between the loads, in which case the loads straddle the centre in such a manner that the left load is $\frac{1}{4}a$ to the left of the centre, and the right load is $\frac{3}{4}a$ to the right of the centre, or, as more commonly expressed, the position of the loads for maximum bending is such that the distance between the centre of gravity of the loads and the point of maximum bending is bisected by the centre of the span. This will be better understood later.

918. These bending moments at any section of the beam must be balanced by the moment of the internal stresses indicated by the arrows acting on the ideal section in Fig. 326 (a), and represented by the general equation $M_0 = \frac{fI}{y}$ (see paragraph 317); and for equilibrium the bending moment of the external forces must be equal to the moment of resistance, $M = M_0$.

SHEAR IN BEAMS.

919. Referring to Fig. 326 (a), the algebraic sum of the forces acting on either side of the ideal section is called the shear on that section. For convenience, when the direction of the resultant force on the left portion of the beam tends to move it upwards on the right portion, the shear is called positive. The sign of the resultant force is evidently positive on one side and negative on the other side of the section. This shear must be balanced by the internal stress in the section. This is called the shearing stress, and is indicated by the downward force S in the figure.

The shear at any point C (Fig. 326 (b)) in the beam is equal to the reaction on the left minus the load between A and C.

$$S = R_1 - wx = w\left(\frac{l}{2} - x\right), \quad . \quad . \quad . \quad . \quad . \quad (514)$$

for a uniform load of the intensity w. This is the equation of a straight line having a maximum positive ordinate $= w\frac{l}{2}$ when $x = 0$, and maximum negative ordinate, $= -w\frac{l}{2}$, when $x = l$, and $S = 0$ when $x = \frac{l}{2}$, the origin being taken at A, the left-hand end of the beam. This condition is shown in Fig. 328 (a). The condition for maximum bending moments from a moving load is that the entire span is loaded, and is therefore the same as that due to the weight of the structure when a uniform moving load is taken, so that w in the formulæ for bending moments can be taken to include both dead and live loads. Not so when isolated live loads are used.

When we come to consider the shears, it is essential to separate the live load and the dead load. Shears due to the dead load are to be determined by the principles of the preceding paragraph, and illustrated in Fig. 328 (a).

The shears due to the live load are now to be considered alone. The two are to be subsequently combined.

In Fig. 328 (b), let a uniform live load extend from B to the section C, covering the longer segment. This is evidently the

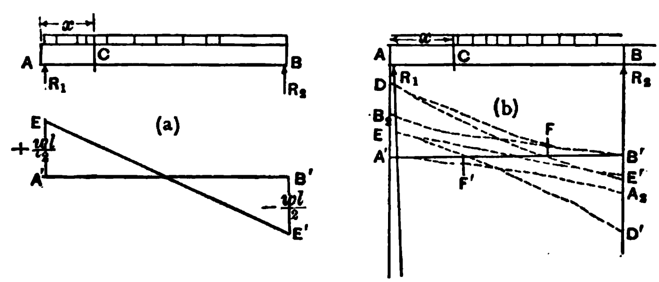


FIG. 328.

position of moving load for maximum positive shear at C, since adding loads on the right of the section increases the left reaction

R_1, and consequently the positive shear. While placing a load to the left of the section, although it adds to the left reaction a certain portion of the load, the entire load must be taken from the reaction to find the shear at C. Therefore, for every load placed on the left, more is lost than gained, thereby reducing the positive shear. The moving load must reach the section, but not pass it on the left.

With the load, then, in the position as shown in Fig. 328 (*b*), the reaction R_1 and shear are equal. Then

$$R_1 = S = w\left(\frac{l-x}{2}\right)\left(\frac{l-x}{l}\right) = \frac{w}{2l}(l-x)^2. \quad . \quad . \quad (515)$$

This is the equation of a parabola $B'B_2$ with vertical axis $B'B$, for $x = 0$, and $S = \frac{wl}{2} = AB_2$, maximum positive shear. For the maximum negative shear, the left portion AC, must be fully loaded, and no load on BC. Then the shear

$$S = -R_2 = -wx\frac{x}{2l} = w\frac{x^2}{2l}, \quad . \quad . \quad . \quad . \quad (516)$$

which is the equation of the parabola $A'A_2$.

If the straight line EE', figure (*a*), represents the dead load shears, the maximum positive shears will be represented by the curve DFE', whose ordinates are the sum of the ordinates of the straight line EE' and of the parabola B_2B'. This line crosses $A'B'$ at F, the dead-load negative shears to the right of this point being greater than the live-load positive shears. From F to B' positive shear cannot occur.

Combining the ordinates of dead-load shear line EE' with those of the parabola for negative shears $A'A_2$ due to live load, and we find the curve $EF''D'$ for final shear line. This line crosses $A'B'$ at F'', giving maximum negative shears between F'' and B'; no negative shears being possible between F'' and A'.

Between F and F'' both kinds of shear are possible. It is only necessary to find the maximum positive shears, since the negative shears are equal and symmetrically disposed.

For a single isolated load W_1 the maximum positive shear at C occurs when the load is just to the right of the section, and is

$$R_1 = S = W_1\left(\frac{l-x}{l}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (517)$$

which is the equation of a straight line $B'B_2$, Fig. 328(b); and when $x = 0$, the ordinate $A'B_2 = W_1$.

The maximum negative shear occurs when the load is just to the left of C, and is

$$S_1 = -R_2 = -W_1\frac{x}{l},$$

and is represented by the straight line $A'A_2$, Fig. 328(b).

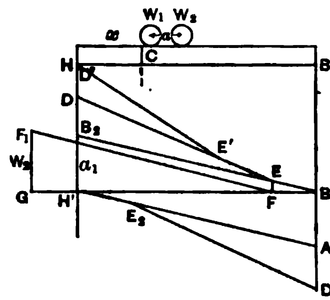


FIG. 329.

920. With two equal loads W_1 and W_2 maintaining a constant distance a apart, Fig. 329, the maximum positive shear occurs when W_1 is just to the left of the section C. For the load W_1 the shear line is the same, $B'B_2$. The shear line for the load W_2 is found by obtaining the left-hand reaction, viz.,

$$R_1 = S_2 = W_2\frac{l-(x+a)}{l}. \quad . \quad . \quad . \quad . \quad (518)$$

This becomes zero for $x = l - a$, that is, for the point F, and would obtain its maximum for $x = -a$, that is, at a point G to the left of A', where it becomes $S_2 = W_2$, and the shear line is F_1F. That portion to the left of H' is not possible, but can be used to construct the shear line. Adding the corresponding ordinates of the lines F_1F and $B'B_2$, we find the total shear line DEB'. If there were three loads, W_1 would have the same position, also W_2, and it would only be necessary to find the reaction for the third load, W_3. Construct the shear line for W_3 as for W_2, combine the ordinates giving the total shear line $D'E'EB'$, the curve approaching a parabola as the number of loads increased, as the condition would approach that of a uniform load. The total negative shears are found in the same way as shown below the line $H'B'$ in $A'E_2D'$, the total shear line for two loads.

$\Sigma H = 0$, or that equation of equilibrium taking in consideration horizontal components, has not been used, as all external forces have been taken as acting vertically, the usual condition for beams and trusses.

ANY NUMBER OF MOVING ISOLATED LOADS.

921. With any system of isolated loads on a bridge the reactions due to each load are found, and the sum of these is the final

reaction sought. Assuming any number of loads, w_1, w_2, w_3, etc., to w_n, to be on a truss, the known distances between them, a, b, c, d,

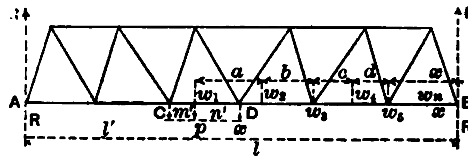


FIG. 330.

etc., and the distance of the rear load w_n from the nearest point of support $= x$, as in Fig. 330. The reaction R due to all of the loads

$$= \frac{w_1(a+b+c+d+\text{etc.},\ldots+x)}{l} + \frac{w_2(b+c+d+\text{etc.},\ldots+x)}{l}$$

$$+ \frac{w_3(c+d+\text{etc.},\ldots+x)}{l} + \text{etc.}, + \frac{w_n x}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (519)$$

As any number of loads, depending on the values of a, b, c, d, etc., may rest between C and D, and as these loads are supposed to be supported at the panel points C and D, if m', m'', m''', and n', n'', n''', etc., are their respective distances from C and D, the portion of these loads resting at C will be $\frac{w_1 n'}{p}$, $\frac{w_2 n''}{p}$, $\frac{w_3 n'''}{p}$, etc. Then R will be the shear at any point between A and C, and for any point between C and D the shear will be

$$= S = R - \left(\frac{w_1 n'}{p} + \frac{w_2 n''}{p} + \text{etc.}\right). \quad . \quad . \quad . \quad (520)$$

If the loads advance towards the left by a very small distance Δx, the new reaction

$$R' = R + (w_1 + w_2 + w_3 + \text{etc.},\ldots + w_n)\frac{\Delta x}{l}. \quad . \quad (521)$$

The new shear will be

$$S_1 = R' - \left(\frac{w_1 n'}{p} + \frac{w_2 n''}{p}\ldots\right) - (w_1 + w_2 + \text{etc.})\frac{\Delta x}{p};$$

hence

$$S_1 = R - \left(\frac{w_1 n'}{p} + \frac{w_2 n''}{p} + \ldots\right) + (w_1 + w_2 + w_3 + \text{etc.},\ldots + w_n)\frac{\Delta x}{l}$$
$$- (w_1 + w_2 + \text{etc.},)\,\frac{\Delta x}{p};$$

$$S_1 = S + (w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n)\frac{\Delta x}{l} - (w_1 + w_2 + \ldots)\frac{\Delta x}{p}; \quad (522)$$

$$S_1 - S = \frac{\Delta x}{l}\left[(w_1 + w_2 + w_3 + \text{etc.}, \ldots + w_n) - (w_1 + w_2 + \ldots)\frac{l}{p}\right]. \quad (523)$$

If this difference is positive just before reaching zero, S_1 will be a maximum. Placing it ($S_1 - S$) equal to zero, and making $\frac{l}{p} = n$ the number of panels, we have

$$n(w_1 + w_2 + \ldots) = w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n. \quad (524)$$

The shear in any panel will be a maximum when n times the load it contains is equal to, or as nearly so as possible, the entire moving load on the bridge. It makes no difference whether the loads on the panel CD are w_1 or w_1 and w_2, or whether the loads w_1 and w_2, etc., have passed the point C, and w_3 and w_4 are resting on that panel, as it can be shown by exactly a similar process that in this case $n(w_3 + w_4 \ldots) = w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n$, or, in other words, it is immaterial where the head of the moving load may be; but the value of the shear will then be the reaction diminished by the sum of the entire loads that have passed to the left of the point C, and also that portion of the loads on the panel in question supported at the point C; in symbols,

$$S = R - (w_1 + w_2 + \text{etc.}) - \left(\frac{w_3 n'}{p} + \frac{w_4 n''}{p} + \text{etc.}\right). \quad (525)$$

The relation in eq. (524), $n(w_1 + w_2 \ldots) = w_1 + w_2 + w_3 + \text{etc.}, w_n$, is then general. As n is a whole number, $\dfrac{w_1 + w_2 + w_3 + \text{etc.}, w_n}{w_1 + w_2 \ldots}$ must also be a whole number. This will rarely be the case; but by assuming that one of the weights rests at a panel point, we can suppose that such a part of it acts on the two panels adjacent as will make the above ratio a whole number; and as n', n'', n''', etc., do not appear in equation (524), we can always take one of the loads as resting on the panel point at the rear end of the panel in question. The value of the shear can be put in the following form: In eq. (520)

$$S = R - \left(\frac{w_1 n'}{p} + \frac{w_2 n''}{p} + \frac{w_3 n'''}{p} \ldots\right),$$

in which, substituting the value of R from equation (519), and expressing n', n'', n''' in terms of a, b, c, etc., which is easily

done by reference to the following diagram, Fig. 330 (*a*), taken from Fig. 330, $n' = a + b + y$, $n'' = b + y$, $n''' = y$, y being the distance in the diagram between the wheel concentrations w_3 and w_4; but as these may be any two of the wheels, we may call them w_{n_1-1} and w_{n_1}. Making the substitutions,

$$S = \frac{1}{l}[(w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \text{etc.}, \ldots$$
$$+ (w_1 + w_2 + w_3) + \text{etc.}, w_n)x] - 1/p[(w_1a + (w_1 + w_2)b$$
$$+ \text{etc.}, \ldots (w_1 + w_2 + \text{etc.}, \ldots + w_{n1-1})y]; \quad . \quad . \quad . \quad . \quad . \quad (526)$$

w_{n1-1} being the rear load in the panel in question and w_{n1} the load that is placed at the rear extremity of the panel.

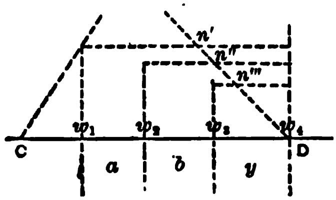


FIG. 330 (*a*).

922. The position of moving load for the greatest bending moment is found by a similar course of reasoning as in determining the maximum shear. Assuming n_1 loads in front of the point D, Fig. 330, and y_1 the distance of the load w_{n_1} in front of the point D, Fig. 330, about which moments will be taken, the moment at D

$$= M = R \times AD - [w_1(a + b + c + \text{etc.}, + y_1)$$
$$+ w_2(b + c + \text{etc.}, \ldots + y_1) + w_3(c + y_1) + \text{etc.}, \ldots + w_{n_1})y_1].$$

Make $AD = l_1$, and substituting value of R from equation (519), we have

$$M = \frac{l_1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \text{etc.}, \ldots$$
$$+ (w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n)x] - w_1a - (w_1 + w_2)b$$
$$-(w_1 + w_2 + w_3)c - \text{etc.}, \ldots -(w_1 + w_2 + w_3 + \text{etc.}, \ldots w_{n_1})y_1. \quad (527)$$

If the whole load moves forward by the small quantity Δx, the new moment

$$M_1 = M + \frac{l_1}{l}(w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n)\Delta x$$
$$- (w_1 + w_2 + w_3 + \text{etc.}, \ldots w_{n_1})\Delta x;$$

$$M_1 - M = \Delta x\left[\frac{l_1}{l}(w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n)\Delta x\right.$$
$$\left. - (w_1 + w_2 + w_3 + \text{etc.}, \ldots w_{n_1})\right].$$

$M_1 - M$ must be either positive or zero for a maximum. Hence

$$\frac{l_1}{l} = \frac{w_1 + w_2 + w_3 + \text{etc.}, \ldots + w_{n_1}}{w + w_2 + w_3 + \text{etc.}, \ldots + w_n}. \quad . \quad . \quad . \quad (528)$$

This relation will rarely exist unless w_{n_1} is supposed to rest at a panel point such as D; and such a portion of it as may be necessary to produce the above equality is supposed to rest in front of D. Again, for any condition of the loading the greatest bending moment will occur at that point or section in a beam or truss where the shear is zero. If the shear is zero at D or any other panel point, then R must be equal to the sum of the loads $w_1 + w_2 + w_3 + \text{etc.}, \ldots + w_{n_1}$, and equation (528) then becomes

$$\frac{l'}{l} = \frac{R}{w_1 + w_2 + w_3 + \text{etc.}, \ldots w_n}; \quad . \quad . \quad . \quad (529)$$

or the centre of gravity of the entire load is at the same distance from one end of the truss as the point of maximum bending is from the other end, which is equivalent to saying that the centre of the span is at the middle of the distance between the centre of gravity of the given system of loads and the point of greatest bending. If the centre of gravity of the wheel concentrations is found by the method explained in paragraphs 858 and 883 to be one foot to the right of the centre of the span, the point of greatest bending will be one foot to the left of the centre. The application of this principle will be repeatedly made in the following pages.

923. As the above conditions of a maximum assume that w_{n_1} rests at a panel point where the maximum bending exists, y_1 in eq. (527) becomes zero, and hence

$$M = \frac{l_1}{l}[w_1 a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \text{etc.}, \ldots$$
$$+ (w_1 + w_2 + w_3 + \text{etc.}, \ldots + w_n)x] - w_1 a - (w_1 + w_2)b$$
$$-(w_1 + w_2 + w_3)c, \text{ etc.}, \ldots (w_1 + w_2 + w_3 + \text{ etc.}, \ldots w_{n_1 - 1})y; \quad (530)$$

y, as before, being the distance between the loads $w_{n_1 - 1}$ and w_{n_1}.

All of the above equations apply to any single system of web members in which there is one vertical and one inclined member in each panel, such as the Howe or Pratt trusses, since the panel points are in the same vertical line and the moments are the same for both chords about any panel point and, in fact, for all members except the chord stresses in the loaded chord, when the web is composed of all inclined members. In this case the panel points

in either chord are opposite the centre of the panels in the other chord. The general formulæ applicable to any single system of web bracing will be found when we come to discuss the Warren or Triangular Truss. It will readily be seen by examining equations (526) and (530) that both the shears and bending moments are expressed in terms of the weights multiplied by the respective distances between them. They differ only in form from the usual expressions for shears and moments. The only unknown factors are the values of x and y. The value of x is the distance from the last or rear load and the nearest point of support B on the right; and the value of y is the distance between the wheel concentrations w_{n1-1} and w_{n_1}, as w_{n_1} is usually at a panel point; y is simply equal to the distance between two of the wheel centres, and is therefore known. x is easily found as soon as the position of the front wheel is known. All loads are supposed to come in from the right and move across the span towards the left; the terms front and rear are then understood.

CONCENTRATED LOADS AND EQUIVALENT UNIFORM LOADS.

924. Having seen the general method of determining the stresses in the several members of a truss, both in amount and kind, under any condition of loading, we will now consider the various assumptions and suppositions made in determining the amount and distribution necessary to give the maximum stress of a particular kind in each and every member of a railway truss.

AMOUNT AND DISTRIBUTION OF THE DEAD LOAD.

925. When the moving load rests on the bottom chords and, consequently, passes between the two trusses, the structure is called a *through bridge*. When it rests on the top chords, the entire structure being beneath it, the structure is called a deck bridge.

926. Deck bridges or spans, although possessing many advantages, will only be discussed in a general way in this volume, and then mainly in respect to these advantages. The essential principles of design and construction are the same, and as it is not intended to discuss exhaustively the subject of bridge construction, but only to explain clearly the principles and their applications so far as they may seem necessary to be understood by every engineer, many volumes having been written on this subject; and to these the special student in the design and construction of bridges is referred.

DEAD LOADS, OR THE WEIGHTS OF STRUCTURES.

927. The weight of a bridge, although essential to be known in calculating the stresses in its members, can only be definitely determined after the structure has been finally designed. The probable weight, therefore, has to be assumed, or approximately determined by means of some empirical formula based upon the actual weights of existing bridges.

The dead load is considered as uniformly distributed over the entire span, and commonly as concentrated at the panel points of the loaded chord.

Clearly a portion of the panel weight of dead load should be taken as acting at the panel points of the unloaded chord. In practice this division of the load varies according to the kind of truss used, but will not vary far from two thirds of the panel weight at the lower-chord panel points and one third at the upper-chord panel points of the truss.

After the amount and distribution of the load are determined, the stresses can be found by any of the methods already explained or by those developed in the following pages, that one being adopted which will give the required degree of accuracy with the least labor. But not that one which requires the least labor, as is often done, regardless of the accuracy of the results.

WEIGHTS OF HIGHWAY BRIDGES.

928. The weight of a highway bridge may be greater or less than that of a railway bridge of the same length of span. The floor system of a highway bridge varies from 20 to 25 pounds per square foot of surface, and from 300 to 500 pounds per linear foot, according to the width of roadway and method of construction.

The following formulæ give approximately the weight (see Modern Framed Structures):

For trusses and lateral system per foot of length,

$$W = 2l + 50, \qquad (531)$$

to which must be added the weight (300 to 500 lbs.) of the floor system.

The live load on a highway bridge is taken as anywhere from 50 to 120 pounds per square foot of floor surface, or from 800 to 2400 pounds per linear foot of span. For specially heavy loads, such as road engines, it is necessary or advisable to employ the method of

wheel concentrations to determine the dimensions of floor-beams, floor-beam hangers, etc. Otherwise in highway bridges both dead and live loads are commonly considered as uniformly distributed, whether covering the entire span, as when determining maximum chord stresses, or over a part of the span for maximum stresses in web members.

WEIGHTS OF RAILWAY BRIDGES.

929. Several formulæ have been devised for calculating the weight of any proposed iron or steel bridge.

Mr. Rankine deduces a formula expressing the proportion of weight of structure to load. This formula requires the determination of the corresponding quantities connected with a bridge of a similar design already in existence, which is not always convenient to obtain; and if it were badly proportioned a corresponding defect in the proposed design would result. This formula is therefore of little practical value.

Pegram's formulæ for weight of iron and steel structures are as follows:

$S=$ length of span, centre to centre of end pins, in feet;
$W=$ total or shipping weight of iron or steel, in pounds.

For iron bridges under 200 feet span,

$$W=\left(75+\frac{S}{a}\right)S\sqrt{S}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (533)$$

in which $a=$	4.5 for class D.	Uniform train load 3600 pounds per foot; two decapod engines and train; equivalent concentrated load 30,000 pounds.
$a=$	7.0 " " T.	Pennsylvania Ry.; uniform load 2900 pounds per foot, equivalent concentrated load 25,000 pounds.
$a=$	9.0 " " C.	N. Y., L. E. & W. Ry.; two 80-ton consolidation engines, followed by uniform load of 2240 pounds per foot.
$a=$	12.0 " M.	Cincinnati Southern Ry.; two 69-ton mogul engines; uniform train load 1820 pounds per foot.

For iron bridges over 200 feet span,

$$W = \left(5 + \frac{S}{b}\right)S^2, \quad . \quad . \quad . \quad . \quad . \quad (534)$$

in which $b = 100$ for class C, $b = 80$ for class T, $b = 68$ for class D.

For steel bridges over 300 feet span,

$$w = cS^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (535)$$

in which $c = 6$ for class C, 6.7 for class T, and 7.1 (about) for class D. The decapod engine in the above formulæ, class D, has 135,000 lbs. on five pairs of drivers, on a wheel-base of 18 feet. The Erie Railway has adopted a still heavier decapod rolling load = 161,000 lbs. on the drivers.

For deck spans add 10 per cent.; for double-track spans, 90 per cent. The equivalent coucentrated loads of 30,000 lbs. and 25,000 lbs. are supposed to be placed at those points on the uniform load which will produce maximum stress in the member under consideration; i.e., it is conceived to be a single concentrated load rolling on the uniform load.

Such formulæ with the aid of experience will give the proper weight of the structure sufficiently near the actual; and if in any case the assumed weight falls short of the actual weight, a second calculation, with a weight somewhat greater than the first calculated weight, must be made.

Referring to the five methods of loading described in the next paragraph, the fifth is only used for highway bridges. For the second the equivalent uniform load is often used, and may be that which will produce the same stresses in all members as the actual concentrated loads, if this were possible, but it is not. The equivalent uniform load that will produce the same chord stresses at the centre of the span or at some other point, say the point one fourth or one third the length of the span from one end, will give practically the same chord stresses throughout as the concentrated loads, but the web stresses will be appreciably smaller. This method would then require a different equivalent load for chord stresses and for web stresses.

The single rolling concentrated load would seem to provide partly for this difficulty.

Other engineers place the concentrated load in the centre of the span, and others again at other points. None of these methods,

however, give as satisfactory or reliable results as the actual wheel concentrations, and seem to have mainly the purpose of saving labor where a great many calculations and estimates have to be made, and are therefore useful and convenient for the bridge companies, but are of little value to the ordinary bridge engineer, as they are all based upon and derived from the actual wheel concentrations.

The following formulæ are given by Burr and Johnson:

If l = the length of span in feet, w = dead load in pounds per foot of length, then, approximately,

$$w = 5l + 350 + 400 \text{ pounds.} \quad . \quad . \quad . \quad . \quad (536)$$

$5l + 350$ pounds includes the weight of the two trusses, the lateral braces for both bottom and top chord, the floor-beams, stringers, etc., or all of the steel and iron in the bridge, and 400 pounds, which includes iron rails, guard-rails, cross-ties, spikes, and bolts. This latter is capable of being calculated for any particular dimensions of timber ties, guard-rails, iron rails, etc., to be used. The above formula is for the weight of a single-track through-bridge of iron and steel, pin connected.

The formula for the Howe truss bridge is

$$w = 6.5l + 275 + 400 \text{ pounds.} \quad . \quad . \quad . \quad (537)$$

For a double-track bridge add ninety per cent. For the load on each truss take one half of the above values of w.

AMOUNT AND DISTRIBUTION OF LIVE LOADS.

930. The almost universally accepted live load for railway bridges is two of the heaviest engines in use, coupled together and followed by a train load of 3000 pounds per linear foot uniformly distributed. The weights of the engines are taken as concentrated at the wheel-bearings. This is called the wheel-concentration system It is about equivalent to a uniformly distributed load of 4000 pounds per foot under engines and tenders, that is, for a distance of about 103 feet, and 3000 pounds per foot on the remaining portion of the length of span. While this is not the exact condition of loading, it is accepted, being as near to it as is practicable, and gives strains at least equal to or in excess of those which could occur under any but unusual conditions.

The labor, however, required in calculating the stresses has led to the substitution of several other systems, which are claimed as giving the same maximum stresses, or as conforming better to the actual conditions, some, however, only being adopted for the sole purpose of saving labor. These are as follows:

(1) Two concentrated excess loads, placed 50 feet apart, and at the head of the uniform train load for shears or maximum stresses in the web members, and in the middle of the uniform load for bending moments or stresses in the chords.

(2) A uniformly distributed excess load over the length of two engines and their tenders, that is, about 100 feet in length, supposed to rest on a uniformly distributed load at its front end, and used for calculating both shears and bending moments.

(3) A single concentrated excess load; which simply means that the span is to be covered in part, or wholly, with a uniformly distributed load, according as shears or bending moments are to be determined; and the single load is to be placed at that point on the uniform load that will cause the maximum stress, which would be at the panel point in question for both chord and web stresses, namely, at the head of the uniform load for web stresses, and moved from panel point to panel point for chord stresses, the uniform load extending over the entire span for these latter stresses.

(4) The use of a given uniform load in determining both chord and web stresses, but in the middle of the train. The uniform load that would cover a given span space is supported to be concentrated on two or four axles. This is somewhat like the general method, the actual wheel concentrations being at the middle of the train instead of at the head.

(5) The use of the so-called equivalent uniform load.

All of these systems are or have been used; but the more conservative engineers still require the general or wheel-concentration method, from which the five substitute methods are derived, and to which it is intended to approximate. It may be therefore regarded as standard.

931. *Method of Equivalent Uniform Loads.*— An equivalent uniform load is one that will produce the same maximum stresses in all the members as is caused by the actual wheel concentrations. No single equivalent uniform load can be found that will do this. Therefore that load which comes nearest to doing so is what is commonly called the Equivalent Uniform Load.

It is not uncommon to consider that the equivalent uniform load

is that which will produce the same bending moment and chord stress at the centre of the span. Calling this load per unit of length w, we know that for a uniformly distributed load the bending moment at the centre of the span is a maximum, and

$$M_0 = \tfrac{1}{8}wl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 537(a)$$

It is found, however, that a closer approximation is to find the bending moment for the uniform load at the quarter point, that is, half-way between the centre and the end. Calling this M', we have

$$M_0' = \tfrac{1}{8}wl^2 - \tfrac{1}{32}wl^2 = \tfrac{3}{32}wl^2, \quad . \quad . \quad . \quad . \quad 537(b)$$

and in the two cases,

$$w = \frac{8M_0}{l^2}, \quad \text{and} \quad w = \frac{32M_0'}{3l^2}; \quad . \quad . \quad . \quad . \quad (538)$$

which give the equivalent uniform loads per foot of length under the respective suppositions, provided we have already determined the bending moments at the centre and at the quarter points when the span is loaded with the actual wheel loads, as these moments must be equal to M_0 and M_0' respectively. Having found w, we can find the moment at any joint, say the nth, when we know the number of panels N and the length of each panel $\frac{l}{N}$. Since the load on each panel is $w\frac{l}{N}$, the total load $= \frac{wl(N-1)}{N}$; each reaction $= \frac{wl(N-1)}{N \times 2}$; its distance from the nth joint is $\frac{nl}{N}$, and its moment $= \frac{wl(N-1)}{2N}\frac{nl}{N}$; the number of loaded joints between the end of the span and the nth joint $= n - 1$; number of panels $= (n-1)\frac{l}{N}$; load $= w(n-1)\frac{l}{N}$; the distance of the resultant from the nth joint is $\frac{1}{2}n\frac{l}{N}$; its moment is

$$w(n-1)\frac{l}{N} \times \tfrac{1}{2}n\frac{l}{N} = w\frac{l^2}{2N^2}(n-1)n.$$

Hence resultant moment at the nth joint, the end joint being zero,

$$M = \frac{wl(N-1)}{2N} \times \frac{nl}{N} - w\frac{l^2}{2N^2}(n-1)n = w\frac{l^2}{2N^2}(N-n)n. \quad (539)$$

This is only a general application of eq. (538) (see also par. 916 to 918) to finding the bending moment at any point, using panel loads and panel lengths instead of loads per foot and actual lengths from the end of the span.

932. When the maximum shears and web stresses are to be calculated with the equivalent uniform load, it is necessary to assume the head of the load to cover the longer segment, fully loading all joints on one side of the panel in question, and all joints on the other side entirely unloaded, as in simple beams.

Under these conditions and assumptions the bending moments and chord stresses will agree closely with the results obtained by the actual wheel loads, w being found from eq. (538) (see also pars. 919 and 920); as will also the web stresses, provided "*the panel length is not less than one eighth* of the span. When there are more than eight panels, and when the engine loading is much greater than the train loading, the shears are always too small." (See "Modern Framed Structures," by J. B. Johnson.)

The expression for the maximum shear in any panel with the equivalent uniform load is, using same notation as above, as follows:

$$S_e = \frac{wl}{2N^2}(N-n)(N-n+1). \quad . \quad . \quad . \quad . \quad (540)$$

Mr. Johnson draws in substance the following conclusions:

(1) That the uniform load with a single moving concentrated load gives better results than the equivalent uniform load. The results for chord stresses are the same by the two systems.

(2) That the equivalent uniform load gives very much too small results for the web system when there are more than eight panels.

(3) That it is safer and better to compute the stresses and to fix the dimensions of both truss and plate-girder simple-span bridges by employing the actual wheel loads.

(4) That for engines very much heavier than the train loads the actual wheel loads should be employed for plate-girders, stringers, floor-beams, and hip verticals.

These conclusions are based on comparisons with Cooper's class

"Extra Heavy A" loading, which takes two locomotives coupled together, having on each wheel of the front truck 8000 pounds, on each wheel of the four pairs of drivers 15,000 pounds, and on each wheel of the tender 9000 lbs., followed by a uniform load of 1500 pounds per foot.

ART. LI.

APPLICATION OF THE PRECEDING PRINCIPLES TO THE DESIGNING OF SOME OF THE SIMPLE TIMBER TRUSSES.

933. TIMBER frames, so-called, may be either entirely built of timber, or the connections, such as bolts, pins, nails, straps, and fish-plates, may be and usually are made of iron. When, however, the main tension-rods are of iron, as in the ordinary triangular, trapezoidal, and Howe truss bridges, the trusses are properly called combined iron and timber structures; but these will be included under the head of timber trusses, as this is the common construction for both highway and railway bridges.

The following drawings show the usual connections for timber tension verticals with top and bottom chords of the all-timber trusses. These apply alike to the triangular, trapezoidal, Long, and Burr trusses. The Burr truss is the Long truss, stiffened by an arch rib of timber springing from the abutments and bolted to the members of the simple truss at all intersections. This truss has been extensively used for long spans in bridges for highways.

The combination of the truss and arched rib leads to ambiguity in regard to the distribution of loads and stresses on the two systems; and although it is possible that one or the other may have to carry the entire load, each necessarily stiffens the other, thereby providing a greater capacity for resisting the effects of the load; and to this extent at least it is safe to assume that under all conditions and circumstances each system will carry a certain portion of the load.

In Figs. 331 are shown the usual connections between the chord and web members.

In all timber joints, except those for lengthening struts, and to some extent in these, there is necessarily a sacrifice of the areas and strength of the members connected, due to cutting daps, shoulders, bolt-holes, etc. In the figures the effective areas are indicated at *a*.

In figure (*a*) are shown an end view and side view of common forms of joints for connecting a timber vertical, when composed of a single stick, with the top and bottom chords and inclined web members. Figure (*d*) shows the same; the long, inclined rods are intended to give lateral stability to the truss. Figures (*b*) and (*c*)

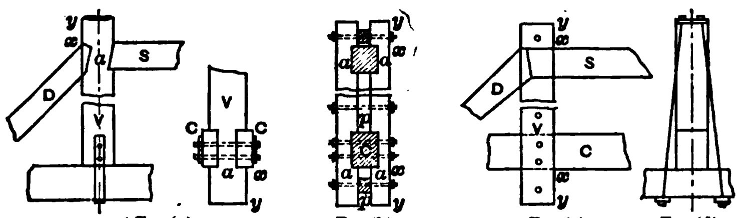


FIG. (a). FIG. (b). FIG. (c). FIG. (d).

FIGS. 331.

are shown, respectively, as section and side view of these connections, when the verticals are composed of two pieces, the chords passing between and shouldered into them. Timber joints for connecting two struts, a strut and a tie, and two ties, will be illustrated in the trusses given.

TRIANGULAR TRUSS FOR RAILWAY AND HIGHWAY BRIDGES.

934. The clear width between the trusses will be 16 feet; the height of the truss at the centre will be 7.0 feet. A floor-beam will rest at the centre of the span. Longitudinal stringers will rest with one end on the end supports and the other on the floor-beam. A solid floor of 3-inch oak plank will be spiked to the stringers (see Fig. 332). The moving load per square foot varies from 70 to 100 pounds, and will be taken in this example at 100 pounds per square foot. The greatest strain on all the main members will occur when the span is entirely loaded. We will suppose that five eighths of the load rests upon the centre floor-beam. Clear length of span 30 feet. The total load will then be $30' \times 100$ lbs. $\times$ 16 ft. $= 48{,}000$ pounds, and the dead load 645 pounds per foot of length$=30 \times 645 = 19{,}350$ pounds. Hence total load $= 48{,}000 + 19{,}350 = 67{,}350$ pounds; load on centre vertical $CD = \frac{5}{8} \times 67{,}350 = 42{,}095$ pounds, divided between the two side trusses; load on each$=21{,}048$ pounds; area required for iron vertical $\frac{21048}{10000} = 2.1$ square inches; one rod $1\frac{5}{8}$ inch diameter, or two rods, each $1\frac{3}{16}$ inch diameter. In the tri-

angle ACD, $CD = 7.0$ feet, $DA = 15$ feet, $CA = 16.5$ feet. Compression $AC = \frac{21048}{2} \times \frac{16.5}{7} = 24{,}807$ pounds. As these members should not exceed a certain ratio of length to least side, $\frac{l}{d} = 20$ as a rule. Substituting this in the formula for long columns, $P = \frac{5000}{1 + \frac{1}{250}\frac{l^2}{d^2}}$, we have $P = 1923$ pounds as the ultimate resistance to crushing per square inch, and allowing a factor of safety of 6, the safe unit of pressure will be about 320 pounds; hence area required for AC and $BC = \frac{24807}{320} = 77.5$ square inches. We could use, then, one piece 8×10, one piece 9×9, or two pieces 4×10 inches, packed. For the tension on the tie-beam AB we have $\frac{21048}{2} \times \frac{15}{7} = 22{,}551$ pounds. For safe tensile strength 1000 pounds

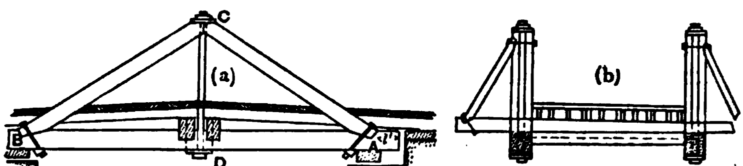


FIG. 332.

per square inch is allowed. This requires $\frac{22551}{1000} = 22.5$ square inches for effective area; but in fitting timber structures an allowance must be made for boring holes, cutting shoulders, loss of strength when two or more pieces are to be connected to transmit tensile stress, etc.; and a certain depth is required to prevent sagging or to give the requisite stiffness when the unsupported length is as much as 10 to 15 feet. We will then assume the depth to be 10 inches, and the breadth, for convenience of construction, should be at least as wide as the strut. The bottom chord, then, should be one piece 9×10 in. $\times$ 33 feet, or two pieces $4\frac{1}{2} \times 10$ inches. If single-length pieces cannot be secured, three pieces $4\frac{1}{2} \times 10$ inches should be used, to allow for loss of strength in connecting timbers to bear a tensile strain. The floor-beam is under a bending action due to 42,095 pounds, uniformly distributed over its length, 16 feet. $mWl = nfbd^2$, $m = \frac{1}{8}$, $W = 42{,}095$, $l = 16 \times 12 = 192$ inches, $n = \frac{1}{6}$, $f = 1500$ pounds, $d = 16$ inches, hence $b = 21$ inches; then floor-beam, two pieces $10\frac{1}{2} \times 16$ inches. As the joists are spaced $1\frac{1}{4}$ feet apart and are 15 feet long, each joist would carry $1\frac{1}{4} \times 15 \times 100 =$

2250 pounds, assuming $d = 12$ inches and using the above formula, $b = 1.82$ inches, as the breadth of each longitudinal joist. The above calculations have been made on the basis of a closely packed crowd, and one of unusual weight. The extreme smaller limit allowed by some engineers is only one half of the above, or 50 pounds per square foot, a fair average being 75 pounds. These loads would give materially smaller dimensions. If such small loads are used it would be better to proportion the floor-beams by using the heaviest loads that could come upon a pair of wheels, which might reach 5 tons, or 5000 pounds on each wheel. Although highway bridges are made for a double roadway, it will rarely happen—and there is no necessity of its ever happening—that two such heavy loads would meet at the same time. Assuming, then, 5000 pounds on each wheel, and the wheels 6 feet apart, the maximum bending moment would be $5000 \times 5 \times 12 = \frac{1}{6} \times 1000 \times b \times (14)^2$. $\therefore$ $b = 9.2$ inches, or a floor-beam 10×14 inches for a roadway 16 feet wide; and should be used if it is greater than that required for the assumed load per square foot uniformly distributed, and one and one-half times the above dimensions if it is intended to allow the heavy loads, or, more exactly, $10{,}000 \times 7 - 5000 \times 6 = \frac{1}{6} \times \frac{1000 \times b \times (14)^2}{12}$. $\therefore$ $b = 15.0$ in.; floor-beam, one piece 15×14 inches, or two pieces $7\frac{1}{2} \times 14$ inches, packed, assuming two heavy wagons to pass directly in the centre of the bridge at the same time. When two pieces for any one member are used they should be bolted together, with packing-blocks between. Large wrought washers should be used at the bottom and top of the vertical rods. The end of the inclined strut should rest against a shoulder cut in the tie and about one third to one half the depth of the strut, and, in addition, a bolt should pass through the strut and tie obliquely. The shoulder should be far enough from the end of the tie, so as not to shear off by the pressure, which is the horizontal component of the stress in the strut—which in this case is 22,551 pounds. The resistance to shearing along the grain is, for oak, from 752 to 2300 pounds per square inch, and for pine, from 200 to 600 pounds per square inch, by different authorities. Assuming 600 pounds, and for safety 150 lbs., $l = \frac{H}{150b}$, l = distance of shoulder from end of tie-beam in inches, H = horizontal thrust in pounds, b = breadth of tie in inches, $H = 22{,}551$ pounds, $b = 9.0$ inches; $\therefore$ $l = 16.7$ inches. Generally 2 feet should be allowed. Below is the bill of material.

4 inclined struts,	9×9 in.	×16.5 ft. =	446.0 ft. B.M.
2 pieces for bottom chord,	9×10 "	×33.0 " =	496.0 " "
2 " " floor-beam,	$10\frac{1}{2}$×16 "	×17.5 " =	490.0 " "
28 " " joists,	2×12 "	×18.0 " =	1008.0 " "
2 " " guard-rails,	6×8 "	×30.0 " =	240.0 " "
1 piece for lateral stays,	6×10 "	×22.0 " =	110.0 " "
2 pieces for lateral struts,	6×6 "	× 9.0 " =	54.0 " "
30 pieces for flooring plank,	3×12 "	×17.0 " =	1530.0 " "
2 " " wall-plates,	6×12 "	×17.0 " =	204.0 " "
			4578.0 " "

Allowing 4 lbs. per ft. B.M. for oak, we have 4578×4	=18,312.0 lbs.
2 vertical tension rods $1\frac{5}{8}$ in.×8.5 ft.	= 119.0 "
4 bolts at ends of struts 1 in.×2.0 ft.	= 22.0 "
2 wrought washers 1 in.×6 in. sq.	= 20.0 "
Spikes for guard-rails, floors, etc.	= 150.0 "
Total weight of bridge	=18,623.0 "

Weight per foot of length $= \frac{18623}{30} = 621$ pounds; assumed weight, 645 pounds, which is on the safe side. The entire structure is assumed to be of oak or southern pine. By using light pine timber—which weighs from 3 to $3\frac{1}{2}$ pounds per foot B.M.—for all parts except the flooring plank, and this of oak 2 inches thick, the dead weight per foot would not exceed 400 pounds. The cost of such a bridge should not exceed $160.

The required number of joists would only be twenty-two pieces, but it is well to double three or four of the joists near the centre, as well as near the outside, of the roadway to support the heavy concentrated loads. Truss members of smaller dimensions than the above are often used, but there is no economy in so doing. Diagonal struts and rods should also be used between the bottom chords for lateral cross-bracing. Simple plank, spiked to the bottom chords diagonally, is frequently used for this purpose in ordinary highway bridges. Spans of this length can be undertrussed, as shown in Fig. 318 (*a*) or as in (*b*). Such struts are liable to be swept away by the accumulation of drift or ice, acted upon by a strong current, in times of flood.

The flooring plank is commonly only 2 inches thick; sometimes two courses of plank are used—the lower course only from 1 to $1\frac{1}{2}$ inches thick, laid diagonally: this increases materially the lateral stiffness of the structure.

935. *Timber Triangular Truss for Railway Bridges.*—Assuming a locomotive of the type shown in Fig. 342 (*a*), we can get on a 30-foot clear span the front truck, 15,000 pound; the 4 pairs of drivers, 96,000 pounds, and 1 pair of wheels of the tender, 15,000 pounds; or 120,000 pounds in all.

With equal weights on the two sides of the centre of the span, assuming dead load at 800 pounds per foot, or 24,000 pounds in all, and taking $\frac{5}{8}$ $(120,000 + 24,000) = 93,750$ as the load on floor-beam, then on each vertical rod 46,875 pounds, each vertical tension-rod CD $\left(\frac{46875}{8000}\right) = 5.86$ square inches (Fig. 332), compression on inclined struts CA and $CB = \frac{46875}{2} \times \frac{16.5}{7} = 50,312$ pounds, requiring $\frac{50312}{320} = 158$ square inches; and for the bottom chord

$$\frac{46875}{2} \times \frac{15}{7} \times \frac{1}{1000} = 50.2 \text{ square inches.}$$

The floor-beam load is 93,750 on two points 4′ 8½″ apart, and weight of beam is 700 pounds; each reaction $= 46,875 + 350 = 47,225$ pounds. With floor-beam 18 inches deep the required breadth $= 15$ inches.

For the struts CA and CB, 1 piece 12×13.2 inches, or 2 pieces 9×10 inches. The area of 50.2 square inches for the bottom chord is too small, for the reasons stated in the preceding example; and, moreover, as the dimensions of the floor system are so large, a different arrangement of the floor-beams is advisable, which in turn will affect the dimensions of the bottom chord, as well as those of

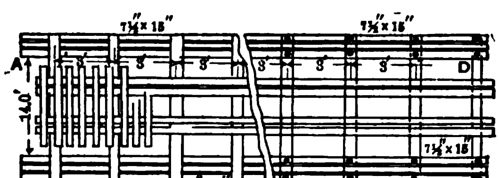


Fig. 332 (*a*).

the stringers. It is better, then, to use a series of floor-beams, either resting upon or suspended from the bottom chord. Suspending the floor-beams by means of bolts has the advantage of economizing in head-room, but it does not give as steady or as rigid an arrange-

ment. Sometimes the floor-beams are placed very close together, in which case the rails can be spiked directly to them, but unless very close the rail will be too heavily strained. A better arrangement is to place the floor-beams 3 feet centres and place stringers upon them. The clear distance between the trusses is 14 feet. Fig. 332 (*a*) shows the two positions of the floor-beams. The chord will then be in the condition of beams supported at both ends, 15 feet long, and loaded at intervals of 3 feet. It will be safe to suppose that the four drivers rest on the half span 15 feet, and that this weight of 96,000 lbs. is equally distributed over the 5 floor-beams by the stringers, or on each floor-beam 19,200 lbs., and resting on each chord at the ends of the floor-beam 9600 lbs. The reaction at A or D will be $\frac{48000}{2}$ lbs., and moment at centre of AD, Fig. 332, will be $\frac{48000}{2} \times \bar{7}.5 - 9600 \times 6 - 9600 \times 3 = 93{,}600$ ft.-lbs.; hence $93{,}600 \times 12 = \frac{1}{6}\, 1500 \times b \times (15)^2$. $b = 20$ inches, $d = 15$, $bh = 300$ square inches, to resist the bending moment, to which must be added 50.2 square inches to resist the direct strain, making 350.2 square inches, or 3 pieces each $7\frac{1}{2} \times 15.6$ inches. For the floor-beams, each may have a pair of driving-wheels to support 24,000 lbs., or 12,000 lbs. resting on each wheel at a distance of $7.0 - 2.5 = 4.5$ feet from end of floor-beam. Greatest moment, $12{,}000 \times 4.5 \times 12 = 648{,}000$ in.-lbs., which would require a 10×16 in. beam, in which case a single string-piece 8×9 inches for the rail to rest upon would answer; but this gives a very large floor-beam, and a form of stringer rarely used. It will therefore be found both convenient and economical to use a stringer of sufficient dimensions to distribute the load of 12,000 lbs. over three or four floor-beams. If distributed over three floor-beams, the above moment would be $\frac{648000}{2} = 324{,}000$ in.-lbs. $= \frac{1}{6}\, 1500 \times b + (14)^2$. $b = 6.6$ inches; the floor-beams then are $14 \times 6\frac{1}{2}$ inches. With an unsupported length of stringer of 6 feet, the greatest moment would be $\frac{12000}{2} \times 3 \times 12 = \frac{1}{6}\, 1500b = (12)^2$. $\therefore b = 9$ inches. The stringers would then be under each rail 1 piece 9×12 inches, or two pieces $4\frac{1}{2} \times 12$ inches. The cross-ties in this case need not be over 6×4 inches.

936. The following is the bill of material for a single-track railroad bridge 30 feet clear span, 10 feet high. The calculation to be made on this height. Triangular truss. Clear distance between trusses, 14 feet.

4 vertical truss rods 5.86 sq. in., each $1\frac{7}{8}$ inches diameter; 4

end struts each 11 × 12 inches × 18.0 feet, or 12 pieces each 4½ × 10 inches; 2 bottom chords each 3 pieces 7½ × 15 inches × 33 ft.; 10 floor-beams, each 1 piece 6½ × 14 inches × 17.5 ft.; 2 pieces for rest of truss brace 8 × 8 inches × 8 ft.; 4 string-pieces each 4½ × 12 inches × 33 ft.; 30 cross-ties 4 × 6 inches × 8 ft.; 4 guard-rails 6 × 8 inches × 33 ft.; 2 pieces for lateral truss braces 6 × 6 inches × 12 ft.; 4 bolts for ends of inclined struts, 1¼ inches diameter 2 ft. 9 inches long; 20 bolts for suspending floor-beams from bottom chords, 1½ inches diameter, 2 ft. 9 inches long; 20 bolts for fastening stringers to floor-beams ¾ inch diameter, 2 ft. 5 inches long; 60 spikes for cross-ties ½ inch × 10 inches; 60 bolts for guard-rails, ½ inch diameter, 12 inches long. Wrought washers from ⅛ to 1 inch thick, and from 2 to 6 inches square, according to size of bolt. For cross-bracing between chords, 3 pieces 6 × 6 inches × 14 ft., and 4 rods 1 inch diameter and 23 ft. long.

From the above the number of feet B.M. of timber can be found, and also weight of iron, as in preceding examples. The cost of the structure depends upon the cost of the material, price of labor, etc. Allowing $30 per 1000 feet B.M. for the timber and 6 cents a pound for the iron will give a close approximation to the cost of the completed structure.

937. In the preceding examples we have entered into much detail in order to show, first, that the dimensions of the parts as determined by the formula are necessarily greatly modified by the practical requirements of construction, and how and for what reasons these changes are made; and, second, to show that practical considerations require a proper adjustment in the number and arrangements of the parts so as to avoid heavy, cumbersome members in some portions of the structure and too light and flexible members in other portions, so that all of the parts may be proportioned, not only to improve the appearance of the structure, but also that they may be conveniently framed and bear fair or proper proportion of the loads. These matters are purely within the province of the practical man and cannot, from their nature, be provided for in theoretical dimensions or be embodied in formulæ. Experience and experiment can alone guide in these matters. This is equally true of small and large bridges, but especially so of the latter. The omission of a seemingly unimportant plate, bolt, or rivet may endanger the safety of an entire structure, no matter how carefully and fully all of the parts have been proportioned by a long and thorough calculation; and, on the other hand, unnec-

essarily heavy members and increased number and size of bolts, plates, and rivets may be found necessary theoretically which are not necessary in fact, owing to the failure of theory to fully provide for the proper correlation in number or positions of the parts in respect to their mutual support and relief of each other. And it cannot be too often stated or forcibly impressed upon young engineers especially, that any effort to build bridges by purely theoretical considerations and formulæ will almost inevitably end in disastrous failure. The theory will fail, and so will the structure.

938. Taking now a span of 42 ft. clear width between trusses for a highway bridge and 14 ft. for a railway bridge, divided into three equal panels of 14 ft. each. The left half shows details for a highway bridge and the right for a railway bridge, and assuming weight of truss at 600 lbs. per linear foot (see Fig. 333). For the highway bridge, live load 80 lbs. per sq. ft., depth of truss 10 ft., total live load $= 16 \times 42 \times 80 = 53{,}760$ lbs.; for the dead load $600 \times 42 = 25{,}200$ lbs., aggregate $= 78{,}960$ lbs. On each of the two trusses $= 39{,}480$ and on each vertical $= 39{,}480 \times 0.367 = 14{,}489$ lbs. $\frac{14489}{8000} = 1.81$ sq. in., or 2 rods $1\frac{1}{8}$ in. diameter, for CD and C_1D_1, Fig. 333. This load is transmitted to the points C

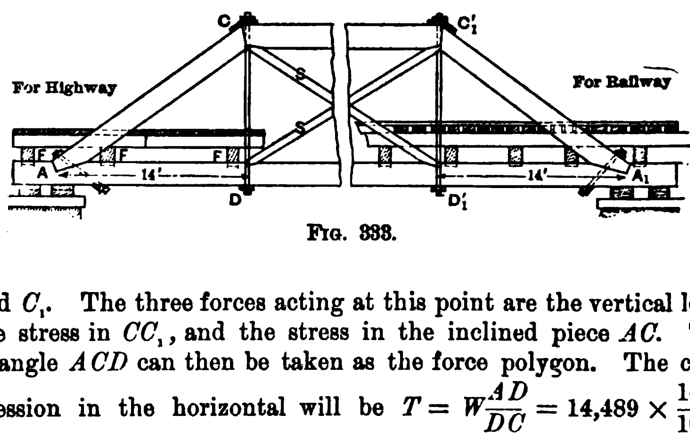


FIG. 333.

and C_1. The three forces acting at this point are the vertical load, the stress in CC_1, and the stress in the inclined piece AC. The triangle ACD can then be taken as the force polygon. The compression in the horizontal will be $T = W\frac{AD}{DC} = 14{,}489 \times \frac{14}{10} = 20{,}285$ lbs. The compression in $AC = C = W\frac{AC}{DC} = 14{,}489 \times \frac{17.2}{10} = 24{,}921$ lbs. But clearly the dimensions obtained from the above stresses would be smaller than would be justified in practice, and they would not be less than those used in the preceding triangular

truss, for the reason there given. Usually one third of 39,480 is taken instead of 0.367 of 39,480 as the load on the verticals.

The counterbraces CD' and $C'D$ are necessary to prevent distortion when there is an excess of load on one side of the centre over that on the other. In a highway bridge, if we assume that 10,000 pounds is supported at D by the two trusses, then 5000 will be carried by one truss at D. Of this 5000 pounds two thirds would be carried by AC to A, and only one third to A' by the counter CD'.

The stress on $CD' = \frac{1}{3} \times 5000 \times \frac{17.2}{10} = 2867$ pounds; or, allowing only 250 pounds per square inch, there would be required 11.5 square inches, or one piece 3×4 inches, and the same for $C'D$. Other dimensions can be fixed by proceeding in the same manner as in the triangular truss.

939. Considering the right-hand half of Fig. 333 as the general design for a railway bridge, a uniformly distributed load might be used if we only knew what it should be, but we would have to use the wheel concentrations in order to determine it unless we happened to have a table of equivalent loads handy.

We can apply eq. (524), namely, $n(w_1 + w_2 \text{ etc.}) = w_1 + w_2 + w_3 + \text{etc.}, w_n$. Assuming that the front truck has passed D, we have 28 feet from D to A'. The four pairs of drivers cover $14\frac{3}{4}$ feet, leaving $13\frac{1}{4}$ feet, which will leave room for the front wheels of the tender, covering 11.92 feet (see Fig. 342 (*a*)), the rear wheels being to the right of A'. As there are three panels, $n = 3$. $3 \times 15{,}000 = 45{,}000 < 4 \times 24{,}000 + 2 \times 15{,}000 = 126{,}000$ pounds.

Moving the load to the left so that the front pair of drivers has passed the point D sufficiently far to bring another pair of wheels of the tender on the bridge, and substituting, we have $3(15{,}000 + 24{,}000) = 117{,}000 < 141{,}000$ total load, which still does not satisfy the condition of a maximum; but by placing w_3 at D we can consider 8000 pounds as acting to the left and 16,000 pounds to the right of D: then $3(15{,}000 + 24{,}000 + 8000) = 141{,}000$, which does satisfy the condition for maximum position of loads. Then, from equa. (519),

$$R_1 = (w_1 \times 39.83 + w_2 \times 31.75 + w_3 \times 26.0 + w_4 \times 21.5 + w_5 \times 17.0 + w_6 \times 9.92 + w_7 \times 5.09 + w_8 \times 0.26) \div 42 = 74{,}680 \text{ pounds};$$

and the shear at $D = 74{,}680 - (w_1 + w_2) = 35{,}680$ pounds. Or, as will be shown later, the maximum floor-beam load will be when

the two adjacent panels are (the more nearly) equally loaded, which will be when, as above, w_3 is at D; then the reaction at D will be equal to $(w_1 \times 0.17 + w_2 \times 8.25 + w_3 + w_4 \times 9.5 + w_5 \times 5.0) \div 14 = 40{,}890$ pounds. As this is the larger, we will use it. Then on each vertical tension-rod 20,445 pounds, and compression on $CC' = 20{,}445 \times \frac{14}{10} = 28{,}623$ pounds $=$ tension on AA'. Compression on struts AC or $A'C' = 20{,}445 \times \frac{17.5}{10} = 35{,}778$ pounds. With a limiting ratio of $\frac{l}{d} = 20$, the ultimate resistance to crushing per square inch is $p = \frac{5000}{1 + \frac{1}{250} \times 400} = 1923$, and 320 lbs. with factor of safety 6 pounds. Area of strut $CC' = 90$ square inches, or 1 piece 9×10 inches; and for AC and $A'C'$, 1 piece, 10×11.2 inches.

For bottom chord, floor-beams, stringers, etc., the same considerations should determine their arrangements and dimensions as in the triangular truss.

With the condition of loading just employed, find the reaction at D'. The difference between this and that found at D will give excess load at D, from which the stress on the counters can be found as already explained. Assuming the load to come in from the left, with the first or second driver at D, also find the reactions at D and D', and compare this with the one above mentioned, adopting the greater of the two in finding stress on counterbraces. The subject of counterbraces will be fully discussed in other paragraphs.

940. Triangular and trapezoidal trusses, when made of iron, are usually of the deck kind, that is, Figs. 332 and 333 turned upside down, in which case for the same length of spans and loads the stresses would be the same in amount but of opposite kind to those found in the foregoing examples. The tension members are usually iron eye-bars, the compression members are usually channels, and the connections are made with pins, or all the connections may be riveted. The truss Fig. 333 can be converted into a deck-bridge as it stands by extending the top chord and supporting its ends over the abutments. In this case the stresses will be of the same kind as already found.

HOWE TRUSS BRIDGE.

941. The distinctive features of the Howe truss are tension verticals made of iron; all other members, top and bottom chords, and

diagonal web members are of wood. The diagonals are in compression.

This truss has been extensively used in this country for both highway and railway bridges for spans up to 200 feet in length, especially in those sections of the country abounding in timber. The framing and connection of the members are simple, and it can be framed and erected by any intelligent and skilful carpenter. For highway bridges timber angle-blocks are commonly used.

As the diagonals are in compression, it is usual to make the panels considerably shorter than in the Pratt truss, in which they are equal or nearly equal to the depth of the truss. The struts are therefore more nearly vertical, and in better positions to bear a compressive stress. In the following skeleton truss the length of span is taken at 114 feet, depth of truss 16 feet.

It is common to adjust the panel length so that the angle which the inclined members make with the vertical may be about 30°; the tan of 30° = 0.577. This would give a panel length = 16 × 0.577 = 9.23 ft. As the span is 114 ft., there will be 12 panels of 9.5 ft.

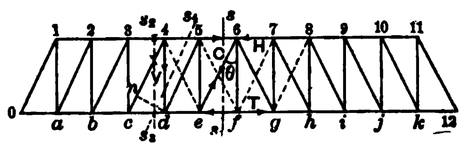


FIG. 334.

each = 114.0 ft. Fig. 334 shows this arrangement. It is only necessary to consider one half the span, as the corresponding members in the other half have the same maximum stresses. Length of span 114.0 ft., panel length = 9.5 ft., depth of truss = 16 ft., clear width between trusses = 12 ft., length of diagonal = 18.6 ft., sec. 0.1, $a = \frac{18.6}{16} = 1.16$, tan 0.1, $a = \frac{9.5}{16} = 0.594$. Total weight of bridge = 700 lbs. per foot, and for each of the two trusses 350 lbs. per lineal foot. Of this 80 lbs. × 9.5 = 760 lbs. per panel will be taken as concentrated at the upper-chord panel points, and 270 × 9.5 = 2565 lbs. at each lower-chord panel point. Total dead load per panel = 3325 lbs., live load per foot for each truss = $\frac{1000}{2}$, and per panel = 4750 lbs., or a total of 8075.

The shears for chord stress, rolling load extending over entire span, are:

In $0a$, 1, 2	$= 5\frac{1}{2}$ (8075)................	44,413 lbs.
" ab, 2, 3	$= (5\frac{1}{2} + 4\frac{1}{2}) \times 8075$	80,750 "
" bc, 3, 4	$= (10 + 3\frac{1}{2}) \times 8075$.......	109,013 "
" cd, 4, 5	$= (13\frac{1}{2} + 2\frac{1}{2}) \times 8075$.......	129,200 "
" de, 5, 6	$= (16 + 1\frac{1}{2}) \times 8075$.......	141,313 "
" ef, fg	$= (17\frac{1}{2} + \frac{1}{2}) \times 8075$.......	145,350 "

The chord stresses = shears $\times$ tan of inclination = 0.594 = 0.6, nearly.

Compression 1, 2 = tension $0a$ = 44,413 $\times$ 0.6 = 26,647 lbs.
" 2, 3 = " ab = 80,750 $\times$ 0.6 = 48,450 "

Compression 3, 4 = tension bc = 109,013 $\times$ 0.60 = 65,407 lbs.
" 4, 5 = " cd = 129,200 $\times$ 0.60 = 77,520 "
" 5, 6 = " de = 141,313 $\times$ 0.60 = 84,787 "
" ef = 145,350 $\times$ 0.60 = 87,210 "

Find now the shears in the vertical and inclined members. 1st. Due to the dead load:

Shear in $6f$ =	2,565	=	2,565 lbs.
" " $6e$ =	$\frac{1}{2}$(2,565) + 380	=	1,663 "
" " $5e$ =	1,663 + 2,565	=	4,228 "
" " $5d$ =	4,228 + 760	=	4,988 "
" " $4d$ =	4,988 + 2,565	=	7,553 "
" " $4c$ =	7,553 + 760	=	8,313 "
" " $3c$ =	8,313 + 2,565	=	10,878 "
" " $3b$ =	10,878 + 760	=	11,638 "
" " $2b$ =	11,638 + 2,565	=	14,203 "
" " $2a$ =	14,203 + 760	=	14,963 "
" " $1a$ =	14,963 + 2,565	=	17,528 "
" " 01 =	17,528 + 760	=	18,288 "

2d. Due to rolling load:

Load at g shear in $7g$	$= R = 5 \times 4750 \times \frac{3}{12}$	= 5,938 lbs.
" " g " " $7f$	$= R = 5 \times 4750 \times \frac{3}{12}$	= 5,938 "
" " f " " $6f$	$= R = 6 \times 4750 \times \frac{3.5}{12}$	= 8,313 "
" " f " " $6e$	$= R = 6 \times 4750 \times \frac{3.5}{12}$	= 8,313 "
" " e " " $5e$	$= R = 7 \times 4750 \times \frac{4}{12}$	= 11,083 "
" " e " " $5d$	$= R = 7 \times 4750 \times \frac{4}{12}$	= 11,083 "

Load at d shear in $4d = R = 8 + 4750 + \frac{4.5}{12} = 14{,}250$ lbs.
" " d " " $4c = R = 8 + 4750 + \frac{4.5}{12} = 14{,}250$ "
" " c " " $3c = R = 9 + 4750 + \frac{5}{12} = 17{,}813$ "
" " c " " $3b = R = 9 + 4750 + \frac{5}{12} = 17{,}813$ "
" " b " " $2b = R = 10 + 4750 + \frac{5.5}{12} = 21{,}771$ "
" " b " " $2a = R = 10 + 4750 + \frac{5.5}{12} = 21{,}771$ "
" " a " " $a1 = R = 11 + 4750 + \frac{6}{12} = 26{,}125$ "
" " a " " $01 = R = 11 + 4750 + \frac{6}{12} = 26{,}125$ "

Stress in $6f$ = tension $= (2{,}565 + 8{,}313) = 10{,}878 = 10{,}878$ lbs.
" " $6e$ = compression $= (1{,}663 + 8{,}313) \times 1.16 = 11{,}572$ "
" " $5e$ = tension $= (4{,}228 + 11{,}083) = 15{,}311$ "
" " $5d$ = compression $= (4{,}988 + 11{,}083) \times 1.16 = 18{,}642$ "
" " $4d$ = tension $= (7{,}553 + 14{,}250) = 21{,}803$ "
" " $4c$ = compression $= (8{,}313 + 14{,}250) \times 1.16 = 26{,}173$ "
" " $3c$ = tension $= (10{,}878 + 17{,}813) = 28{,}691$ "
" " $3b$ = compression $= (11{,}638 + 17{,}813) \times 1.16 = 34{,}163$ "
" " $2b$ = tension $= (14{,}203 + 21{,}771) - = 35{,}974$ "
" " $2a$ = compression $= (14{,}963 + 21{,}771) \times 1.16 = 42{,}611$ "
" " $1a$ = tension $= (17{,}528 + 26{,}125) - = 43{,}653$ "
" " 01 = compression $= (18{,}288 + 26{,}125) \times 1.16 = 51{,}519$ "

On the counterbrace $7f$ or $5f$ the shear from the live load is 5938 lbs.; the shear on the main brace $6e$ or $6g$ due to the dead load is 1663 lbs. These counterbraces will then be needed, and the stress on them will be $(5938 - 1663) \times 1.16 = 4959$ lbs. If the live load is at h, the shear in $8g = R = 4 \times 4750 \times \frac{2.5}{12} = 3958$ lbs. in $8g$ or $4e$. The dead-load shear in $5d$ is 4988 lbs., which being the greatest, these counterbraces are not required; but as the above shears are so near equal, small braces are generally used, but no other panel towards either end will need them. The dimensions can now be determined: Maximum compression on top chord 87,210 lbs.; $\frac{87210}{500} = 174.4$ sq. in. 4 pieces 4×11 inches of varying lengths so as to break joints. Ends cut square so as to abut true against each other. The cross-section uniform from end to end.

Each end strut 01, $\frac{51519}{300} = 172$ sq. in., 3 pieces 5.25×11 in.
Diagonal strut $2a$, $\frac{42611}{300} = 142$ " 3 " 4.4×11 "
" " $3b$, $\frac{34163}{300} = 114$ " 2 " 5.25×11 "
" " $4c$, $\frac{26173}{300} = 88$ " 2 " 4.0×11 "
" " $5d$, $\frac{18642}{300} = 62$ " 2 " 4.0×8 "

Counterbrace	$4e$,		1 pieces	4.0	×	8 "
"	$6e$, $\frac{11572}{300} = 38.6$ sq. in.,		2 "	4.0	×	8 "
"	$5f$,		1 "	4.0	×	8 "
Vertical tension-rod	$1a$, $\frac{43653}{10000} = 4.36$	sq. in.,	2 rods	$1\frac{1}{16}$		in. diam.
" " "	$2b$, $\frac{35874}{10000} = 3.6$	"	2 "	$1\frac{1}{2}$		"
" " "	$3c$, $\frac{28691}{10000} = 2.9$	"	2 "	$1\frac{3}{8}$		"
" " "	$4d$, $\frac{21803}{10000} = 2.2$	"	2 "	$1\frac{3}{16}$		"
" " "	$5e$, $\frac{15311}{10000} = 1.54$	"	2 "	1		"
" " "	$6f$, $\frac{10878}{10000} = 1.1$	"	2 "	$\frac{7}{8}$		"

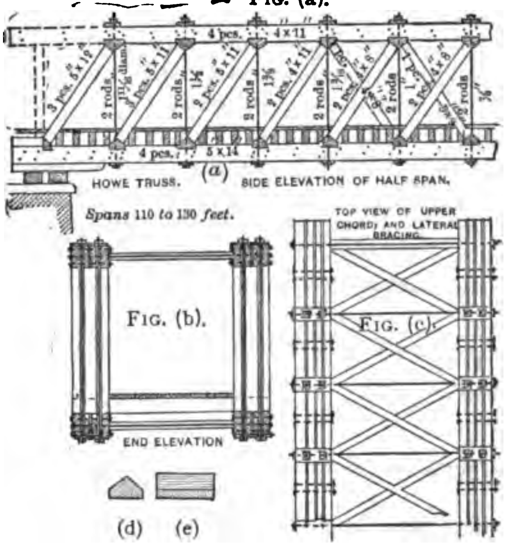
The determination of the dimensions of the bottom chord will depend upon the design of the floor system. If floor-beams rest at intervals on the chord between the panel points, it will have to sustain not only direct tension, but also a bending strain; and it must be proportioned to resist both. If, however, floor-beams rest only at the panel points, and longitudinal joists are used, it will have to resist only the direct strain. It is safe to allow 1000 lbs. per square inch in tensile strain, but owing to the difficulty of connecting timber so as to transmit tension, the necessary cutting and framing causes a waste of material from one third to one half; we will therefore only allow a unit strain of 500 lbs. per square inch, and the bottom chord will then be the same as that in the case of the top chord, viz., 4 pieces 4 × 11 in., to resist direct stress.

To determine the dimensions of the floor-beam, the load may be considered as practically uniformly distributed over its length, and equal to a panel weight of the floor system and live load. The floor system will weigh about 20 to 25 lbs. per square foot × 12 = 9.5 × 25 = 2850 lbs.; the live load 1000 × 9.5 = 9500, or a total of 12,350 lbs.; centre moment $\frac{1}{8} \times 12{,}350 \times 12 \times 12 = 222{,}300$ in.-lbs. $= \frac{1}{6} \times 1000 \times b \times 144 \quad \therefore b = 9.26$, or 2 floor-beams each 6 × 12 inches at each panel point. These can be placed so close to the panel point that the bending action on the chord will be small, and the floor-beams, 6 × 12 inches, will be ample. At each panel point hard-wood angle-blocks will be required (Figs. 335 (*d*) and (*e*)) of these forms. The top chord-pieces must be bolted together with about $\frac{3}{4}$-inch screw bolts, 4 bolts at every joint, and 2 bolts at the centre of each panel. Packing-blocks are to be used between the chord-pieces, about $1\frac{1}{2}$-inch thick, at all bolts.

In the bottom chord one of the joints, either scarfed or with fish-plates, for lengthening ties must be used, and additional bolts between the joints as for the top chord. The diagonals simply rest

against the angle-blocks. The vertical rods pass between the chord-pieces and through wrought-plate iron washers 1 inch thick, 4 inches wide, and of a length equal to the breadth of the top chord. The screw end of the rod should be upset, so that the actual area of the rod may be its effective area. Cast washers are commonly used for

FIG. (a).



FIGS. 335.

the chord-bolts. A bolster-piece should be bolted to the bottom of the bottom chord at the ends, 6 inches × breadth of chord and 5 or 6 feet long. This rests directly on a timber wall-plate composed of 2 pieces 10 or 12 inches × 12 inches, and extending under both chords, the wall-plate resting under masonry.

Sometimes the iron verticals are omitted and timber tension verticals used, called king-posts, the diagonals resting against a shoulder cut on the vertical, and the chord-pieces let into daps cut in the posts 2 or more feet from the ends and fastened by bolts.

For spans 150 feet or more, arch-ribs are used in connection with the regular truss. The ribs composed of two pieces 6 × 14 inches, one on either side of the truss and bolted to it, spring from skew-backs built in the masonry some distance below the ends of the bottom chord, and rising at the centre above the middle of the depth of the trusses. This construction is called the Burr Bridge. In

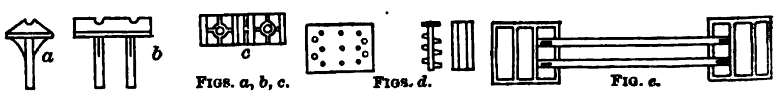
such a combination the condition of the strains is doubtful, and the only safe plan is to design the one or the other to carry the entire load, the truss stiffening the arch-rib or *vice versa,* and supporting such a proportion of the load as may rest upon it, when the combined structure assumes a condition of equilibrium. For a long-span double-track bridge three trusses may be used, giving two entirely separated roadways. In highway bridges the dead weight of the structure is so great that complications as to the distribution of the live load between the trusses, when one roadway is loaded and the other not loaded, need not exist. The distribution in railway bridges, however, is a matter of importance. The reader is referred to Rankine's "Civil Engineering" for a discussion of this subject. The Howe Truss bridge is used to a great extent. The bill of material for the above bridge can be easily calculated from the above determined stresses. If any member is very long the allowable unit strain must be determined by the equation

$$P = \frac{5000}{1 + \frac{1}{250}\frac{l^2}{d^2}} = \text{the ultimate crushing load in lbs. per sq. in.,}$$

$$\left(\frac{l}{d} \text{ must not exceed about 30 to 40,}\right)$$

or by the straight-line formulæ, equations (130), (131), and (132), par. 313.

The details given in figures 335 (*a*), (*b*), (*c*), (*d*), and (*e*), and 336 (*a*), could be used also for railway bridges; but in this case the angle-blocks should be of iron, and of the following designs (Figs. 335½ *a*, *b*, and *c*). For the bottom chord the timbers are connected



FIGS. *a*, *b*, *c*. FIGS. *d*. FIG. *e*.

FIGS. 335½.

by iron plates with projecting teats or ribs instead of wooden keys or fish-plates, and iron bars connect these across the joint, as seen in figures *d* and *e*. These bars have sometimes hook-shaped ends, which encircle bolts or pins. With such modifications the design of a railway bridge is exactly similar to the above, and with a uniformly distributed load the method of determining the stresses is identical. The resultant stresses are very much greater, and, consequently, the dimensions of the various members. But, as before mentioned, for railway bridges it is not consistent with the best

practice to consider the rolling load as uniform, and the stresses should be determined from the actual wheel weights or wheel concentrations of the heavy types of engines now in use. (See Fig. 342 (*a*).)

942. It is always advisable, when stresses are calculated as in the foregoing examples, to check the results obtained by successive additions. For the end diagonal 0, 1, Fig. 334, the shear must be one half the load on the truss, no matter how it may be distributed; hence the total load $= 114 \times 1000 = 114{,}000$ pounds, and on each truss 57,000 pounds due to live load. Total dead load $= 114 \times 700$

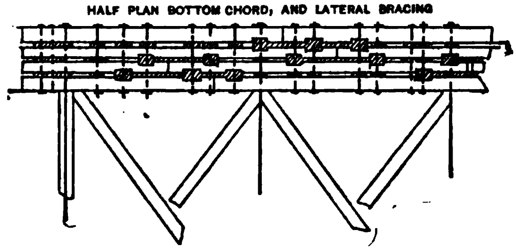


FIG. 336 (*a*).

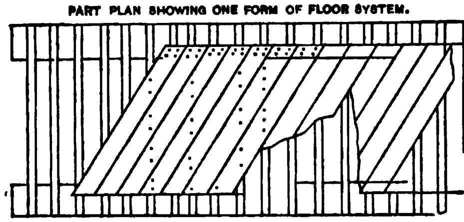


FIG. 336 (*b*).

$=79{,}800$ pounds, and on each truss 39,900 pounds. The aggregate, $57{,}000+39{,}900=96{,}900$ pounds. Each reaction is, then, 48,225 lbs. But it is evident that one-half a panel weight rests directly at the end panel point $0 = 4037$ pounds; hence the load transmitted through 0, 1 or its shear is equal to $48{,}225-4037=44{,}188$ lbs.. to which must be added the weight of 0, 1 itself, taken at one third an upper-chord panel load of the dead load $= 250$ pounds, nearly; hence shear in $0, 1=44{,}188+250 = 44{,}438$, and compression in 0, 1 $= 44{,}438 \times 1.16 = 51{,}548$ pounds; whereas the actual compression found was 51,519 pounds—a difference of only a few pounds. This

is a useless refinement, and a check agreeing within a few hundred pounds would answer every purpose; but it is introduced here merely to show that the tabulations of the shears and stresses were accurately made for the assumed distribution of the load, as it could hardly be possible for errors to occur in any part of the table, which was formed by a series of additions commencing at the centre of the span and extending to the end of the truss, making each term depend upon those preceding it, if the last term is correct.

The chord stresses are checked by taking moments with respect to any section, generally cutting the centre panel. As this involves the reaction at the points of support, we simply use, in case of uniformly distributed loads, the general expression $mWl = \frac{1}{8}(wl)l = T \times d_1$; wl = entire load on each truss; l = length of span in feet; T = tension in fg or ef, centre panels; and d = depth of of truss in feet. In the present case $wl = 96,900$ pounds; $l = 114$ feet; $d = 16$ feet; then

$$\tfrac{1}{8} \times 96,000 \times 114 = T \times 16. \quad \text{Hence } T = 86,302 \text{ pounds.}$$

Referring to table for horizontal stresses, stress on $ef = 87,210$ pounds.

This apparent discrepancy is easily accounted for by observing that the multiplier—that is, the tangent of the angle of inclination θ in Fig. 334—is taken at 0.6, whereas its actual value is 0.594, which multiplied by the shear gives $145,350 \times 0.594 = 86,338$ pounds, only 36 pounds greater than the value of T above. The multiplier is only $= 0.59375$ exactly, which would make the agreement exact. The tangent of the angle of inclination should generally be carried to three places of decimals. Making it a little greater than the actual value errs on the side of safety, and saves some labor, and for this reason it was made 0.6 in the above example. This proves the accuracy of the table for chord stresses. It will be observed that compression in any panel in the upper chord is equal to tension in the lower-chord panel included between the same pair of parallel diagonals. In the above calculation the section is supposed to be taken as shown in Fig. 334, SS_1, cutting upper-chord panel 5.6, main diagonal $6e$, and bottom chord ef, cutting only three acting members, as the diagonal $5f$ being a counterbrace does not act when the load covers the entire span; and by taking the axis of moments at the panel point 6, the moments of the stresses become $H \times 0 + C \times 0 - T \times f.6 = T \times 16$. By taking the section at

any other point, as S_2S_2, and taking moments about an axis at point 4, the moment of the reaction at $0 = 48{,}450 \times 38' = 1{,}841{,}100$ foot-pounds, and the moment of the load on $0d = 32{,}300 \times 19' = 613{,}700$ foot-pounds; hence $T \times 16 = 1{,}841{,}100 - 613{,}700 = 1{,}227{,}400 \quad \therefore \; T = 76{,}713$ pounds tension on cd, a little less than in table, for the same reason as given above.

The greatest stress on $c4$ is found when the rolling load reaches the point d, its reaction r at $0 = 14{,}250$ pounds, and taking moments about the point d (Fig. 334), as the moving load does not extend over the entire span, it becomes necessary to find the reactions due to the dead load and live load separately. The reaction due to the live load being $r = 14{,}250$, and that due to the dead load on each truss being one fourth of the total load $= 114 \times 700 \times \frac{1}{4} = r = 19{,}950$ pounds $\quad \therefore \; R = 19{,}950 + 14{,}250 = 34{,}200$ pounds. The dead load on the truss between 0 and $d = \frac{700}{2} \times 38 = 13{,}300$ pounds $= w$. The remaining forces are the stress in $3.4 = H$, in $c.4 = C$, and $c.d = T$. As moments are taken with respect to d, the moment of T is zero. Lever-arm of $H = 16$ feet; the lever of $c.4$ is $nd = 4.d \sin \theta = 16 \times 0.5 = 8.0'$. Then the equilibrium of the moments will be expressed by

$$R \times y - w \times y_1 - H \times 16 - C \times 8 = 0.$$

in which all the terms are unknown. But with the section s_2s_2 at any point all of the terms can be found directly except C; hence C can be found. H being the compression in the top chord panels when the load extends over more than one half the span, is not the maximum chord stress as found, but must be computed independently for each position of the moving load. With the load at d, the reaction $r = 14{,}250$ lbs. is the shear due to the live load between 0 and d. Hence in each panel the direct stress, or rather increment of stress, is $14{,}250 \times 0.6 = 8550$ lbs., which is the horizontal component of the constant compression in 0, 1; $2a$; $3b$, due to the shear r; the accumulated stress in $3, 4 = 8550 + 8550 + 8550 = 25{,}650$ lbs.; the accumulated dead-load stress in 3, 4 is $13.5 \times 3325 \times 0.6 = 26{,}933$ lbs., which is the continued sum of the horizontal components of the compressions in 0.1; $2a$; $3b$, due to dead load; $\therefore$ compression $H = 25{,}650 + 26{,}933 = 52{,}583$ lbs. In the present case $y = 0d = 38$ ft.; $y_1 = bd = 19$ ft. Substituting in equation (1),

$$34{,}200 \times 38 - 13{,}300 \times 19 - 52{,}583 \times 16 - C \times 8 = 0.$$

Hence, Compression in $4c = C = 25,697$ lbs.;
" " " $= 26,173$ "

In table the apparent error arises, as in the former cases, from W being about 200 lbs. less than above, or 13,100 instead of 13,300, which gives $C = 26,172$ lbs.

If we take a section s_3s_4, cutting the upper chord 4, 5, lower chord cd, and the vertical $4d$, the forces would be the reaction R; the load on $0d$, compression 4, 5; tension $4d$ and tension cd for equilibrium $- R \times y + w \times y_1 - H \times 16 + V \times y_2 = 0$.

$T \times 0 =$ zero, as the axis will be taken to intersect the bottom chord. Taking axis at the point 0, $y = 0$; $\therefore R \times y = 0$, $y_2 = 0d = 38$ ft.; $\therefore V = 21,190$ lbs., in table 21,803 lbs. With loads extending over only a part or even the whole of the spans, checking the tabulated stresses in diagonals and verticals by the method of moments, as seen above, is somewhat tedious. The first method of checking these stresses is simpler and more satisfactory (see par. 942. The above, however, is introduced to show the application of the method of moments.

IRON-TRUSSED BRIDGES.

EYE-BARS AND PIN-CONNECTIONS.

943. In the ordinary truss bridges the tension members, whether in the bottom chord or web system, are usually made of flat bars

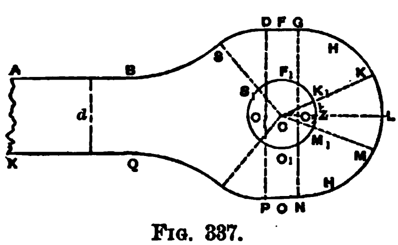


FIG. 337.

with enlarged heads in sets of two or more, which are connected through holes or eyes in the heads by a single large pin. The greatest care is necessary in designing and forming these heads.

It is found by experiment that if d is the depth of the bar, its thickness t should be between $\frac{1}{4}$ and $\frac{1}{8}$ d, and the diameter of the pin-hole $D = 0.75$ to $0.8\ d$. The several sections of the head on radial

planes from the centre of the pin-hole should have such areas that when the eye-bar is tested to failure, the fracture should be as likely to take place in the body of the bar as on any part of the head. The diameter of the pin should be equal to the diameter of the hole; a variation of from $\frac{1}{50}$ to $\frac{1}{100}$ of an inch is, however, considered excellent workmanship.

When the eyebar is strained the pin bears on a surface of the wall of the hole of greater or less extent near z, and that portion of the bar is under direct compression and extension. Those portions about KC, FC, and SC and corresponding portions on the other half of the head are under direct tension and bending, at some points, H, where there is a point of contraflexure, and the stress in the direction of the circumferences changes from compression to tension. Such is a general statement of the nature and direction of the stresses developed in the head. Many experiments lead to the following proportions of the various sections through the centre of the pinhole: Let R = radius of the pin; $FF_1 = 00_1 = 0.66d_1$; $GKLMN$ is a semi-circumference described from the point C_2 (so adjusted that $ZL = 0.87d$) with a radius $= R + 0.66d$; SD is a portion of the same curve described from C_1 as a centre ($CC_1 = CC_2$); SB is any curve of a long radius, so as not to reduce the metal area near SC too rapidly; $ABQX$ is a part of the body of the bar. To simplify this form of head the circumference, or that portion from F to D through L, is described from C as a centre with a radius $= R + 0.8$ (or $0.9)d$, C being the centre of the pinhole, the portions FSD and QO formed as before. The head should not be welded to the body of the bar. The thickness of the head is sometimes made greater than that of the body of the bar. If the simpler form of head is used $FF_1 = 00_1$ should be $0.8d$. The only objection to increasing the thickness of the head is the increase in the bending action on the pin, as this bending action is very great under any circumstances.

944. The diameter of the pin is found from

$$M = \frac{fI}{y} = \frac{f\pi d^3}{32} = \frac{fAd}{8};$$

hence

$$d^3 = \frac{32M}{\pi f} = \frac{32M}{3.1416f}, \quad \therefore d = 2.17\sqrt[3]{\frac{M}{f}}, \quad . \quad . \quad (541)$$

f being the modulus of rupture. The bending moment M depends

upon the magnitude and direction of the stresses upon the members connected by the pin.

Pins, like rivets, are subjected to direct shearing and bending action, and moreover their diameters are determined by the bearing resistance of the walls of the hole in contact with them.

There will always be sufficient resistance to shearing when either of the other requirements are satisfied.

But its diameter to resist shearing in any case can be found. If f is the coefficient of resistance to tearing and f_1 to shearing, then $f_1 = \frac{4}{5}f$ or $0.80f$ (nearly). If A and A_1 are the areas of the bars and pin respectively, then in single shear $fA = f_1A_1$ or $A_1 = \frac{fA}{f_1} = \frac{6}{5}A$. Knowing, then, the area of the bar acting on the pin on either side, the area of the pin must be 1.2 times greater; or if n bars act in one direction and $n + 1$ in the other, then there would be $2n$ sections to be sheared, or total area sheared $= 2nA_1 = 2n\frac{\pi d^2}{4} = 1.5708nd^2 = \frac{6}{5}A$; A being the total area under direct tension of the bars.

$$\therefore\ d = \sqrt{\frac{6A}{5 \times 1.5708n}} \quad \text{or} \quad d = \sqrt{\frac{A}{1.3093n}}. \quad . \quad . \quad (542)$$

Eq. (542) is of small moment so far as pins are concerned, but is useful in case of rivet diameters. For the proper diameter to give sufficient bearing surface on the plates or bars the area of bearing surface is assumed to equal the diameter of rivet or pin multiplied by the thickness of the bars or plates $= dt$; or if f_1 be the coefficient of bearing resistance, then total resistance

$$R = f_1dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (543)$$

$f_1 = 1.4f$ for wrought iron and $1.25f$ for steel in thin plates, and for thick plates in wrought iron $1.25f$.

Taking f for wrought iron at 40,000 lbs., and for steel plates when thin 75,000 lbs. and when thick 60,000 lbs. per sq. in., knowing R and f, and assuming either d or t, the other can be found. The above values for f are ultimate values; the working values should be not more than from $\frac{1}{4}$ to $\frac{1}{6}$ of the ultimate resistances. With thicker bars and plates d would be correspondingly small. But increase in thickness of bars increases greatly the

bending action. Thick bars are then to be avoided. In addition thick bars have a smaller ultimate resistance than thinner ones.

945. We will now find the bending moment M in eq. (541). The eye-bar and pin-connection joint usually occur in trussed bridges, both in the top chord and bottom chord. Fig. 338 (a) shows a side view of an ordinary joint in the bottom chord of trussed bridges. Fig. 338 (b) shows horizontal projection of the same.

There are several bars in the adjacent panels forming the ten-

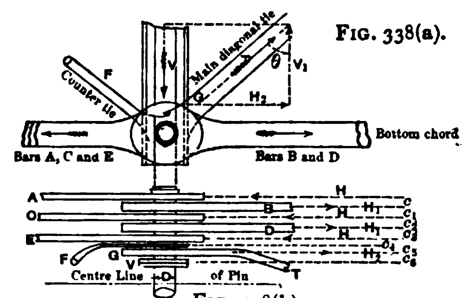


FIG. 338(a).

FIG. 338(b).

sion-bars of the bottom chord, and inclined or diagonal members ordinarily composed of two main bars on one side and of two counter-rods on the other. These counters do not act under maximum conditions of stress, and will not be considered as under strain, but being in position they affect the lengths of the lever-arms. Also, there is a vertical member or strut, either resting directly at the centre of the pin, or at points on either side and equally distant from the centre. It is evident, since all members are symmetrically situated on either side of the pin centre, that it is only necessary to consider one half of the pin, which can then be considered as a beam fixed at centre of the pin or at the bottom of the vertical, the maximum bending moment in either case being the same. Sometimes it can be seen by inspection that the maximum bending moment is at the foot of the pin-plate V, or centre of the pin. But often, if not generally, it will be necessary to find the bending moment at several points. Let H, H, H be the horizontal pull on the bottom-chord bars in the panel to the left of the pin, and H_1, H_1 that on the right; H_2 the horizontal component of the tension in the diagonal $G = T \sin \theta$, and V_1 its vertical component $= T \cos \theta = V$, the compression on the vertical. Then the pulls to the right

will be H_1, H_1, and $T \sin \theta$; those to the left, H, H, and H. The vertical stresses are tension $= T \cos \theta$; and V, compression in the vertical. Equilibrium requires $T \cos \theta = V_1 = V$, and $H + H + H = H_1 + H_1 + H_2(T \sin \theta)$. If c, c_1, c_2, c_3, etc., be the distances from centre to centre of the thickness of the bars, Fig. 338(b), and assuming that the centre of pressure between any eye-bar and the pin is at the centre of thickness of the eye-bar, which is approximately true, we can either find by the principle of the centre of parallel forces the position of the resultant of all of the horizontal forces or the resultant couple, and thence the bending moment at any point of the pin, or we can take the algebraic sum of the moments of the tension on each bar with respect to any axis, and the same for the vertical stresses. It will probably be better to find the resultants by the first method if the point of greatest bending is known, otherwise use the second method. By this latter method we have the following moments:

$$\left.\begin{array}{lll}
\text{About } B = M & = & Hc; \\
\text{“} \quad C = \text{“} & = & H(c + c_1) - H_1c_1; \\
\text{“} \quad D = \text{“} & = & H(c + c_1 + 2c_2) - H_1(c_1 + c_2); \\
\text{“} \quad E = \text{“} & = & H(c + c_1 + 2c_2 + 2c_3) \\
 & & \quad - H_1(c_1 + c_2 + 2c_3); \\
\text{“} \quad F = \text{“} & = & H(c + c_1 + 2c_2 + 2c_3 + 3c_4) \\
 & & \quad - H_1(c_1 + c_2 + 2c_3 + 2c_4); \\
\text{“} \quad G = \text{“} & = & H(c + c_1 + 2c_2 + 2c_3 + 3c_4 + 3c_5) \\
 & & \quad - H_1(c_1 + c_2 + 2c_3 + 2c_4 + 2c_5); \\
\text{“} \quad V = M & = & H(c + c_1 + 2c_2 + 2c_3 + 3c_4 + 3c_5 + 3c_6) \\
 & & \quad - H_1(c_1 + c_2 + 2c_3 + 2c_4 + 2c_5 + 2c_6) \\
 & & \quad - H_2c_6.
\end{array}\right\} \quad (544)$$

The values of c, c_1, c_2, etc., are the half sums of the thicknesses of the adjacent bars, increased by the necessary clearance between them, which may vary from $\frac{1}{16}$ to $\frac{1}{8}$ inch. In the above the total tension on each of the bars on one side of the panel point is supposed to be equal, and equal to H. If they vary, the first term in the above equations (543) would consist of two or more separate expressions. For moments about V we would have, calling H, H', H'' the pulls,

$$\text{moment about } V = H(c + c_1 + c_2 + c_3 + c_4 + c_5 + c_6) + H'(c_2 + c_3 + c_4 + c_5 + c_6) + H''(c_4 + c_5 + c_6).$$

And similarly for the second term:

$$\text{Moment about } V = -H_1(c_1 + c_2 + c_3 + c_4 + c_5 + c_6) + H_1'(c_3 + c_4 + c_5 + c_6).$$

$$\text{Moment about } V = -H_2c_6$$

for the third term; and the sum of these terms would be the resultant moment.

The above forces being in a horizontal plane, their moments are taken about a vertical axis. The only vertical forces are the compression in V and the vertical component of the tension in G; these are equal and in opposite direction, and hence form a couple whose moment is about V, Fig. 338(b), $T\cos\theta\ (= V_1 = V) \times c_6 = M_2$, and is about a horizontal axis; hence the resultant moment in eq. (1)

$$= M = \sqrt{M_1^2 + M_2^2}. \quad \ldots \ldots \quad (545)$$

By the first method, referring to Fig. 338(b), we have three bars, A, C, E, under tension on the left. Their resultant would be $H + H' + H''$ if unequal, or $3H$ if equal; and two bars on the right, $H_1 + H_1'$, if equal; and the horizontal component H_2, the resultant of these, $= H_1 + H_1'$, or $2H_1 + H_2 = R$ also. Hence the resultant moment $M_1 = R \times l$, l being the distance between the two resultants, the moment of the vertical forces remaining as before, M_2 and

$$M = \sqrt{M_1^2 + M_2^2}, \quad \ldots \ldots \quad (545a)$$

as before.

If the other half of the pin was considered, the bending moments at corresponding points would have been the same.

It is sometimes convenient to find the bending moments graphically. In this case the stress in the diagonal G is not decomposed into its components.

As was seen in Art. XXII, par. 203, a moment can be represented by a straight line in position, magnitude, and direction, the line being drawn perpendicular to the plane of the couple in such a manner that looking along the line towards the plane of the couple the moment of the couple shall be right-handed. In par. 945, Figs. 338 (b) and (a) represent the relative position of the several members connected at a lower-chord joint of a Pratt truss, and situated at the right of the centre of the span. The several stresses acting as in-

dicated by the arrows, as in this case, the resultant tension of all of the horizontal forces acts to the left, and taking moments about the foot of the vertical V, a vertical line will represent the moment. But in order that the moment may appear right-handed, it will be necessary to view it from below; then drawing a vertical line to represent to any scale the magnitude of the moment, and acting upwards, it will represent the moment. The moment of the tension in G must be taken about an axis perpendicular to the plane of the couple, and this must be viewed from above to appear right-handed about an axis at V; hence drawing a line perpendicular to G, and to same scale as the vertical line, it will represent the moment of the stress in G. The moment of the vertical stress V is zero; hence the closing line of the triangle BC will represent the resultant moment to the same scale.

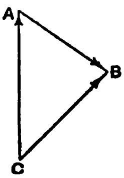


FIG. 339.

In figure 339, CA represents moment of horizontal forces, AB moment of stress in diagonal, and CB resultant moment.

946. For joints in the upper chords of trusses the principle is the same. The pin there passes through the webs of the channels or sides of the box beams forming the chords, the verticals and diagonals being connected to the pin between the sides of the chord. Moments are then found by considering the pin as a beam supported at both ends and loaded at intermediate points. Fig. 340 shows this condition. The greatest bending moment would be anywhere between the inside pieces a and b. In this case it will not be necessary to find the reactions at the supports of the pin, as it is symmetrically loaded with equal forces on each side of the centre. If the pin plates v are outside, the diagonals G on the inside, then the moments of the horizontal forces will be $T\sin\theta(c + c_1) = M_1$ and of the vertical forces $T\cos\theta(c + c_1) - v(= T\cos\theta)c = T\cos\theta c_1 = M_2$; $\therefore M = \sqrt{M_1^2 + M_2^2}$, as before. A diagram in every way similar to 339 will represent this case graphically. A horizontal line would represent the moment of the stress on the vertical, an inclined line perpendicular to the diagonal would represent its moment, and the closing line BC the resultant moment.

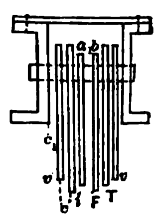


FIG. 340.

Often at the upper-chord joints, as in the Warren or triangular truss, where two main diagonal members meet and also a vertical suspending-rod, and always where the end posts rest against the top chord, there will be two inclined members, one under compres-

sion and one under tension, with the vertical suspending-rods or bars. In these cases the moment polygon will have four sides, constructed as follows: Fig. 341 shows a skeleton sketch of the

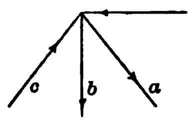


FIG. 341.

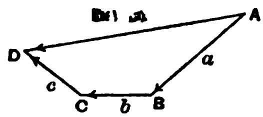


FIG. 341(b).

joint and Fig. 341 (b) the moment polygon. The construction of the polygon is as follows: The moment of the force *a*, Fig. 341, to appear right-handed must be viewed from the right, hence *AB*, Fig. 341(*b*), must act towards the left; similarly the line *BC*, representing the moment of *b*, and also the moment of the force *c*, must be viewed from below, and the force lines *CD*, acts upward. Then *AD* will be the resultant moment, and must act in a direction opposed to the other moments. If equilbrium is sought it must act in the same direction. The moment of the horizontal chord force is zero. The analytical solution is similar to that fully discussed in the preceding cases. Upper-chord pins generally have their greatest stress with the web members, that is, when the load extends over the longer segment; the lower-chord pins, when the load extends over the entire span.

947. In this article the rolling loads will be taken as the weights on the wheels—called concentrated wheel weights—of two

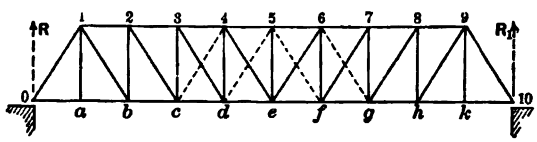


FIG. 342.

Type of Locomotive.

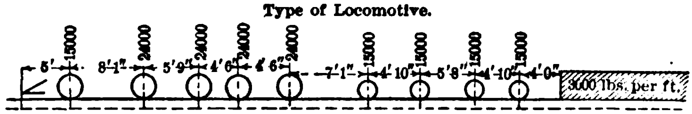


FIG. 342(*a*).

heavy locomotives coupled together, followed by a uniformly distributed load. The formulæ used will be found in equations (526) and (530).

Fig. 342 represents a through all iron or steel Pratt truss, verticals in compression, diagonals in tension. Length of span =

200 feet centre to centre of end pins; panel lengths $0a$, ab, cd, etc., $=20$ feet; number of panels$=10$; depth of truss$=30$ feet between pin centres. From eq. (536) $w = 5l + 350 + 400 = 1750$ pounds. Assuming the weight of the two trusses and lateral bracing per foot of length at five times the length of span, expressed in pounds, then

Weight of two trusses and lateral bracing	$= 200 \times 5 =$	1000 lbs.
" " stringers and floor-beams per linear foot	$=$	350 "
" " cross-ties, guard-rails, rails, spikes, etc., per linear foot	$=$	400 "
Total dead weight per foot of length of truss	$=$	1750 "

Weight on each truss per linear foot $= 875$ pounds. Assuming 275 pounds as acting on upper chord, we have, for each upper-panel point, $w_2 = 275 \times 20 = 5500$ pounds $=$ one panel weight of dead load.

As illustrated in the above diagram, there are 96,000 pounds concentrated in the space of 14′ 9″, 60,000 pounds on 15′ 4″, and 171,000 pounds over 54′ 3″. It is now generally specified that the rolling load shall consist of two such locomotives coupled together and followed by a uniform load of 3000 pounds per foot. It is further evident that it is erroneous to suppose that the entire dead load acts directly at one chord panel point; and although we cannot in advance determine exactly how much load should be allowed for the two chords, we can at least approximate it. The use of the wheel concentrations is really no more complicated than that of a uniformly distributed load. In either case it requires the summation of the reactions or shears of each individual load or part of a load. It requires a little more labor, perhaps, but no different principles.

Lower-chord fixed load, 600 pounds per lineal foot, or 12,000 pounds per panel, $= w_1$; total panel weight $= 17{,}500$ lbs. $= w_1 + w_2$; length of diagonals $= 36.05$ feet; multiplier of diagonal stresses $=$ sec angle $0, 1, a = \frac{36.05}{30} = 1.2$; and for horizontal stresses $=$ tan angle $0, 1, a = \frac{20}{30} = 0.66\frac{2}{3}$,—the rolling load to consist of two consolidation locomotives of the type shown in Fig. 342 (a), coupled together, followed by a uniform load of 3000 pounds per lineal foot.

Recollecting that the stresses in the verticals are equal to the shears, and in the diagonals are the shears multiplied by 1.2, we write easily these stresses, due to the dead load:

Compression in $5e$ =		w_2 =	5,500 lbs.
Tension " $4e$ =	$\left\{\begin{array}{c}\frac{1}{2}(w_1+w_2)\times 1.2 = \\ \frac{1}{2}(12{,}000+5500)\times 1.2\end{array}\right\}$	=	10,500 "
Compression " $4d$ =	$8750 + 5500$	=	14,250 "
Tension " $3d$ =	$(14{,}250 + 12{,}000) \times 1.2$	=	31,500 "
Compression " $3c$ =	$26{,}250 + 5500$	=	31,750 "
Tension " $2c$ =	$(31{,}750 + 12{,}000) \times 1.2$	=	52,500 "
Compression " $2b$ =	$43{,}750 + 5500$	=	49,250 "
Tension " $1b$ =	$(49{,}250 + 12{,}000) \times 1.2$	=	73,500 "
" " $1a$ =	$12{,}000$	=	12,000 "
Compression in 01 =	$(61{,}250 + 12{,}000 + 5500) \times 1.2$	=	94,500 "

To prove the correctness of the above, the stress in 01 must be equal to one half the entire load on the bridge multiplied by 1.2, or $17{,}500 \times 4\frac{1}{2} \times 1.2 = 94{,}500$ pounds, the same as in the table.

The chord stress in any panel is the sum of all of the shears between the end of the truss and the panel in question multiplied by $0.66\frac{2}{3}$. Using the above shears, we write the chord stresses $61{,}250+12{,}000+5500=78{,}750$; $78{,}750+49{,}250+12{,}000=140{,}000$; etc.

Chord stress	$0a$, ab, tension	$78{,}750 \times \frac{2}{3}$ =	52,500	lbs.
" "	1, 2, compression	$140{,}000 \times \frac{2}{3}$ =	93,333	"
" "	bc, tension	$140{,}000 \times \frac{2}{3}$ =	93,333	"
" "	2, 3, compression	$183{,}750 \times \frac{2}{3}$ =	122,500	"
" "	cd, tension	$183{,}750 \times \frac{2}{3}$ =	122,500	"
" "	3, 4, compression	$210{,}000 \times \frac{2}{3}$ =	140,000	"
" "	de, tension	$210{,}000 \times \frac{2}{3}$ =	140,000	"
" "	4, 5, compression	$218{,}750 \times \frac{2}{3}$ =	145,833	"

To prove this series, the shear in 01 is $4\frac{1}{2}$ panels weight; in $1b$ $3\frac{1}{2}$, in $2c$ $2\frac{1}{2}$, in $3d$ $1\frac{1}{2}$, in $4e$ $\frac{1}{2}$ panel weight; hence $4\frac{1}{2} + 3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2} = 12\frac{1}{2} \times 17{,}500 \times \frac{2}{3} = 145{,}833$; and similarly for any other panel. The load is not uniformly distributed, but concentrated at panel points.

948. To find the shears for the rolling load and the bending moments, construct the following table for use in connection with equations (526) and (530). Column 1 is the number of the wheel from the head of the moving load. Column 2 contains the continued summation of the wheel concentrations and the distances between them; column 3 the product of the numbers in column 2; and column 4 contains the sum of the numbers in column 3 from

LOCOMOTIVE TABLE LXXV.

1.	2.	3.	4.
w_1 1	15,000 × 8.08	121,200	121,200
w_2 2	39,000 × 5.75	224,250	345,450
w_3 3	63,000 × 4.50	283,500	628,950
w_4 4	87,000 × 4.50	391,500	1,020,450
w_5 5	111,000 × 7.08	785,880	1,806,330
w_6 6	126,000 × 4.83	608,580	2,414,910
w_7 7	141,000 × 5.67	799,470	3,214,380
w_8 8	156,000 × 4.88	753,480	3,967,860
w_9 9	171,000 × 9.00	1,539,000	5,506,860
w_{10} 10	186,000 × 8.08	1,502,880	7,009,740
w_{11} 11	210,000 × 5.75	1,207,500	8,217,240
w_{12} 12	234,000 × 4.50	1,053,000	9,270,240
w_{13} 13	258,000 × 4.50	1,161,000	10,431,240
w_{14} 14	282,000 × 7.08	1,996,560	12,427,800
w_{15} 15	297,000 × 4.88	1,434,510	13,862,310
w_{16} 16	312,000 × 5.67	1,769,040	15,631,350
w_{17} 17	327,000 × 4.83	1,579,410	17,210,760
w_{18} 18	342,000 × 4.00	1,368,000	18,578,760

the top down to, and including the number in, the same horizontal line with the number selected in column 4, *and is the value* of that part of the positive parenthesis, $w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c$ + etc., in equation (526), which is as follows:

$$S = \frac{1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots (w_1 + w_2 + w_3 + \ldots w_n)x] - \frac{1}{p}[w_1a + (w_1 + w_2)b + \ldots (w_1 + w_2 + \ldots w_{n1-1})y]. \quad (546)$$

949. The value of x is easily determined. Let the load come in from the right and move towards the left, which is the condition always assumed unless specifically stated otherwise. If, then, the head of the load w_1 rests at k, we find from column 2 of the table $8.08 + 5.75 + 4.50 = 18.33$ feet. Hence wheel w_4 will be on the bridge. As the panel length is 20 feet, and as x is the distance of the rear or last load from the right-hand end of the bridge at point 10, we have $20 - 18.33 = 1.67$ feet $= x$. Similarly, if the head of the train rests at h_1, then $8.08 + 5.75 + 4.50 + 7.08 + 4.83 = 34.74$ feet; two panels = 40 feet from h to 10; hence $40 - 34.74 = 5.26$ feet $= x$, and so on. These calculations will be facilitated by forming a table, giving the distances of each wheel from the

head of the train, which can be compared directly with the length of span covered by the rolling load as follows:

TABLE LXXVI.

Distance from front load w_1 =

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
0	8.08′	13.83′	18.33′	22.83′	29.91′	34.74′	40.41′	45.24′

Second engine:

w_{10}	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	w_{16}	w_{17}	w_{18}
54.24′	62.32′	68.07′	72.57′	77.07′	84.15′	88.98′	94.65′	99.48′

Head of Uniform Load.
103.48′

From which table, when w_1 or any other load is at a panel point, we can find at once how many wheel concentrations are on the bridge, and the distance of the last load from the right-hand point of support 10. For instance, if the head of the train is at f, from f to 10 = 4 panels = 80 feet. The next less number is 77.07 feet, under wheel w_{14}, which is the last load on the bridge, and 80 − 77.07 = 2.93 feet = x. Again, if wheel w_3 is at h, from h to 10 = 40 feet; wheel w_3 is 13.83 from w_1; then 45.24 − 13.83 = 31.41 feet, and 54.24 − 13.83 = 40.41; hence the last wheel on the bridge is w_9 over 45.24 feet. If the head of the load is at c, from c to 10 = 7 panels = 140 feet. As the front end of the uniform load is 103.48 feet from the head of the train, there will be 140 − 103.48 = 36.52 feet of uniform load of 3000 pounds per foot on the bridge. This load acting through its centre of gravity will be w_n of eq. (546). w_{n-1} being the last wheel of the second tender, the distance then between w_{n-1} and w_n will be $(x + 4)$, x being the half length of the uniform load, equal in the above case to $\frac{36.52}{2} = 18.26$ feet, and then distance between w_{n-1} and $w_n = 18.26 + 4 = 22.26$ feet; which will be the last of the series of distances a, b, c, d, etc. We are now prepared to determine the maximum shears. The condition for maximum shear is shown by eq. (524):

$$n(w_1 + w_2 + \text{etc.}) = w_1 + w_2 + w_3 + \ldots w_n. \quad (547)$$

n = number of panels = 10 in this case, and w_n being the last load on the bridge. For maximum shear this condition must be fulfilled as nearly as possible.

950. Let, then, the rolling load come in from the right, and w_7

rest at k, distant from $10 = 1$ panel. We find over 18.33 the wheel w_4; then $x = 20 - 18.33 = 1.67$ feet. From Table LXXV, opposite w_4 in column 2, we find total weight on bridge $= 87,000$ pounds. If now the load w_1 has just passed k, w_1 will be in the panel kh, and $nw_1 = 10 \times 15,000 = 150,000 > 87,000$ by substituting in eq. (547). Therefore this position of the load does not produce maximum shear in panel hk or in main brace $h9$; but if we suppose that w_1 rests at panel point k, and that 8700 pounds of this load rests in front or to the left of k, then

$$n \times 8700 = 87,000 = w_1 + w_2 + w_3 + \dots w_n,$$

the total load on the bridge; which satisfies eq. (547), and is the position for maximum shear or tension in $8k$, or maximum compression in $h9$. The brace $8k$ is not used; as will be seen later, it is not needed. If used it would be a counterbrace.

If the load extends to the panel point h, distance $h10 = 40$ feet. In Table LXXVI we find w_7 over 34.74 feet; then $x = 40 - 34.74 = 5.26$ feet. Opposite w_7 in Table LXXV, column 2, total load on bridge $= 141,000$ pounds, and $nw_1 = 10 \times 150,000 > 141,000$, placing w_1 at h and 14,100 pounds acting to the left of h. Hence

$$10 \times 14,100 = 141,000 = w_1 + w_2 + w_3 + \dots w_n,$$

in which $w_n = w_7$. This gives position of load for maximum shear and tension in $h7$, which will be another counterbrace if needed. If we place w_2 at h (as with w_1 at h wheel w_8 is so near the end of the bridge, as in Table LXXVI we find it over the distance 40.41, or $40 - 40.41 = -0.41$ foot, to right of point 10), then $x = 40 + 8.08 = 48.08 - 45.24 = 2.84$ feet, and $w_9 = w_n$ over 45.24 is the last load on the bridge. w_1 is in panel gh. Hence $10 \times 15,000 = 150,000 < 171,000$, opposite w_9, Table LXXV, but by assuming only 2100 pounds of w_2 to act to left of h,

$$10(15,000 + 2100) = 171,000 = w_1 + w_2 + w_3 + \dots w_n.$$

Both of these positions of the load should be tested by substitution in eq. (546) to see which gives the greatest shear. With w_2 at h, from Table LXXV we find that

$$5.75 + 4.50 + 4.50 + 7.08 + 4.83 + 5.67 + 4.83 = 37.16.$$

Hence $x = 40 - 37.16 = 2.84$ feet, same as found by Table LXXVI.

If we place w_1 at g, $g10 = 3$ panels $= 60$ feet, $w_{10} = w_n$, $x = 60 - 54.24 = 4.76$ feet, and as $10 \times 150{,}000 < 186{,}000$, it is clear that this does not satisfy eq. (547) within a small limit of error. Hence we must place w_2 at g. Then

$$x = 60 + 8.08 = 68.08 - 68.07 = 0.01 \text{ foot.}$$

Hence $w_{11} = w_n$ and

$$10(15{,}000 + 8400) = 234{,}000 = w_1 + w_2 + w_3 + w_n(- w_{11}).$$

By assuming that 8400 pounds of w_1 acts to left g, w_2 at g will give maximum in $g6$, a counterbrace, and will be needed, as will be seen later. If we place w_3 at g, $w_{11} = w_n$, $10(15{,}000 + 24{,}000) = 390{,}000 > 258{,}000$ by too great a difference.

w_2 at f. $f.10 = 80$ feet. $x = 80 + 8.08 - 84.15 = 3.93$ feet; $w_{15} = w_n$; $10 \times 15{,}000 = 150{,}000 < 297{,}000$, $10(15{,}000 + 14{,}700) = 297{,}000$, which satisfies conditions for a maximum shear on $f5$, a counterbrace, and will certainly be needed. Hence it will be best to try w_3 at f. $f10 = 80$ feet. $x = 8.08 + 5.75 + 80 = 93.83$, $93.83 - 88.98 = 4.85$; $w_{16} = w_n$; $10(15{,}000 + 24{,}000) = 390{,}000 > 312{,}000$, which is evidently very far from satisfying the required condition. w_2 at f comes nearer the condition for a maximum than any other loading possible.

w_2 at e. $e10 = 100$ feet; $x_1 = 8.08 + 100 = 108.08$, $108.08 - 103.48 = 4.60$ feet $=$ length of uniform load on the bridge. Hence $x = \dfrac{4.80}{2} = 2.30$ feet, the distance of the centre of gravity of the uniform load, which is w_n, to the point 10. Wheel $w_{18} = w_{n-1}$; distance between w_{n-1} and $w_n = x + 4 = 6.30$ feet, which is the multiplier for the last load in column 1, Table LXXV (instead of 4), it being the last term of the series of distances a, b, c, d, etc. Total uniform load on the bridge, $4.60 \times 3000 = 138{,}000$ pounds. Total load, $w_{18} + 13{,}800 = 342{,}000 + 13{,}800 = 355{,}800$ pounds; $10(15{,}000 + 20{,}580) = 355{,}800 = w_1 + w_2 + w_3 +$ etc. $- w_n$. w_2 at e gives then maximum shear and tension in $e4$, which is a main brace. Both engines, tenders, and part of the uniform load are on the bridge, and as we have seen the condition for a maximum shear is independent of the exact position of the load resting on the panel under consideration, hence it may be that it is not necessary to place any one of the wheels at a panel point, as we can increase or decrease the entire load on the bridge, moving the

whole load forward or backwards, still keeping the same loads on the panel in front of the panel point e, or any other point $n(w_1 + w_2 + \text{etc.})$ remaining constant, and the exact equality in equation (547) thereby established.

With w_2 at d: $d10 = 6$ panels $= 120$ feet; $x_1 = 8.08 + 5.75 + 120 = 133.83$, and $133.83 - 103.48 = 30.35$ feet of uniform load on the bridge. Hence $x = \frac{30.35}{2} = 15.18$ feet, $w_{18} = w_{n-1}$, and $30.35 \times 3000 = 91,050$ pounds of uniform load $= w_n$; distance between w_n and $w_{n-1} = x + 4 = 19.18$ feet, last multiplier. Total load on bridge $= 342,000 + 91,050 = 433,050$ pounds; $10(15,000 + 24,000) = 390,000$ pounds $< 433,050$; but $10(15,000 + 24,000 + 4305) = 433,050 = w_1 + w_2 + w_3 + \ldots w_n$; w_2 at d gives maximum shear on $d3$, another main brace.

w_2 at c: $c10 = 140$ feet; $x_1 = 8.08 + 5.75 + 140 = 153.83$, and $153.83 - 103.48 = 50.35$ feet, length of uniform load; $x = \frac{50.35}{2} = 25.18'$ $w_{18} = w_{n-1}$; and $50.35 \times 3000 = 151,050$ pounds $= w_n =$ total uniform load. Entire moving load on bridge $= 342,000 + 151,050 = 493,050$ pounds; $10(15,000 + 24,000) = 390,000 < 493,050$; but $10(15,000 + 24,000 + 10,305) = 493,050 = w_1 + w_2 + w_3 + \ldots w_n$. This will be maximum position for shear. Distance between w_{n-1} and $w_n = 29.18$ feet.

w_2 at b: $b10 = 160$ feet; $x_1 = 160 + 13.83 - 103.48 = 70.35$ feet uniform load; $x = \frac{70.35}{2} = 35.18$ feet; $w_{18} = w_{n-1}$; $w_n = 70.35 \times 3000 = 211,050$ pounds. Total load $= 342,000 + 211,050 = 553,050$ pounds $= 10(15,000 + 24,000 + 16,305)$. Distance between w_{n-1} and $w_n = x + 4 = 39.18$ feet, which is condition for maximum shear and tension in $1b$.

w_2 at a: $a10 = 180$ feet; $x_1 = 8.08 + 5.75 + 180 - 103.48 = 90.35$ feet of uniform load; $x = 45.18$, $w_{18} = w_{n-1}$, $w_n = 90.35 \times 3000 = 271,050$ pounds. Total load $= 613,050$ pounds $= 10(15,000 + 24,000 + 22,305)$, condition for maximum shear and compression in 01. Distance between w_{n-1} and $w_n = 49.18$ feet.

951. Recollecting that when the uniform load is on the bridge w_n is this load concentrated at its centre of gravity, x its distance from the nearest reaction, and that the distance between w_{n-1} and w_n is $= x + 4$, we can then proceed to find the shears due to the rolling load in any web member, as we have already found position of load on the bridge for maximum shear. Substituting in eq. (526),

$$\left.\begin{aligned} S = \frac{1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots \\ + (w_1 + w_2 + w_3 + \ldots + w_n)x] \\ - \frac{1}{p}[w_1a + (w_1 + w_2)b + \ldots + (w_1 + w_2 + \ldots w_{n_1-1})y]. \end{aligned}\right\} \quad (548)$$

Since until the load has passed the point g, or so long as the front load w_1 rests at a panel point for maximum shear, there are no terms in front of the panel point and the negative part of the value of S becomes zero. Hence

$$S = \frac{1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots + (w_1+w_2+\ldots+w_n)x]. \quad . \quad . \quad (549)$$

With w_1 at k (see paragraph 950): $x = 1.67$, $w_n = w_4$; then $w_1 + w_2 + w_3 + \ldots w_n(= w_4) = 87{,}000$ pounds, in Table LXXV, column 2, opposite wheel w_4. The first part of the value of S is

$$w_1a+(w_1 + w_2)b + (w_1 +w_2+w_3)c+\ldots(w_1+w_2+w_3 + \ldots w_{n-1})q,$$

q being the distance between w_{n-1} and w_n, or, in this case, between w_3 and w_4, $= 4.50$ (Table LXXV); and as the term simply expresses the summation of the moments up to and including $w_{n-1} = w_3$, the value of this term is found opposite $w_{n-1} = w_3$ in Table LXXV, column 4, $= 628{,}950$ foot-pounds $= w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c +$ etc. $l = 200$ feet. Substituting in eq. (549),

$$S = \frac{1}{200}[628{,}950 + 87{,}000 \times 1.67] \div 2.$$

As the table is constructed for the entire load carried by both trusses, then shear in

$$8k = \frac{1}{200}\left(\frac{628950}{2} + \frac{87000}{2} \times 1.67\right) = 1936 \text{ pounds.} \quad . \quad (1)$$

Table LXXV is calculated by using the weights on a pair of drivers and tender-wheels, 24,000 pounds, 15,000 pounds, which should be divided between two trusses. For shears and stresses in each truss, $\frac{15000}{2}$, $\frac{24000}{2}$, etc., should be used, or the shears should be divided by 2 in all cases.

w_1 at h: $x = 5.26$; $w_n = w_7$; $w_1 + w_2 + w_3 \ldots w_n = 141{,}000$ pounds (see column 2, Table LXXV, opposite w_7); $w_{n-1} = w_6$. Hence opposite w_6, column 4, Table LXXV, $w_1a + (w_1 + w_2) + (w_1 + w_2 + w_3)c = 2{,}414{,}910$ foot-pounds.

$$\text{Shear in } 7h = \frac{1}{200}\left(\frac{2414910}{2} + \frac{141000}{2} \times 5.26\right) = 7892 \text{ pounds.} \quad (2)$$

If w_2 be placed at h, $x = 2.84$, $w_n = w_9$, and $w_1 = w_{n_1-1}$; $w_1 + w_2 + w_3 +$ etc., $+ w_n = 171{,}000$ pounds (see column 2, Table LXXV, opposite w_9); $w_{n-1} = w_8$. Then $w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c +$ etc., $= 3{,}967{,}860$ (column 4, Table LXXV opposite w_8), and as w_1 is in front of the point h, the negative term in eq. (546),

$$-\frac{1}{p}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + \ldots w_{n_1-1})y],$$

becomes

$$-\frac{1}{p}(w_{n_1-1})y = -\frac{1}{p}(w_1a) = \frac{1}{p}15{,}000 \times 8.08 = \frac{1}{p}121{,}200 .$$

(See column 4, Table LXXV, opposite its wheel, w_1.) Then substituting in eq. (546), $p = 20$,

$$S = \frac{1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots + (w_1 + w_2 + w_3 + \ldots w_n)x]$$
$$-\frac{1}{p}[w_1a + (w_1 + w_2)b + \ldots + (w_1 + w_2 + w_3 + \ldots w_{n-1})y];$$
$$S = \frac{1}{200}\left[\frac{3967860}{2} + \frac{171000}{2} \times 2.84\right] - \frac{1}{20} \times \frac{121200}{2} = 8109 \text{ lbs.} \quad (3)$$

As this result is a little greater than with w_1 at h (see (2)), the maximum shear is secured with w_2 at h (see (3)). Although 8109 − 7892 = 217 pounds is small, it shows the importance, in all cases of doubt, as to the maximum position of the load, of trying several positions of the load to find the actual maximum shear.

For maximum shear in $g6$: w_2 at g; $w_n = w_{12}$; $x = 0.01$; $w_{n-1} = w_{11}$; $w_{n_1-1} = w_1$; $a = y = 8.08$. First positive term in column 4, Table LXXV, opposite w_{11}. Second term in column 2, Table

LXXV, opposite w_{12}. Negative term, column 4, Table LXXV, opposite $w_1 = 121{,}200$; $p = 20$. Substituting, eq. (546),

$$S=\frac{1}{200}\left[\frac{8217240}{2}+\frac{234000}{2}\times 0.01\right]-\frac{1}{20}\cdot\frac{121200}{2}=17{,}524 \text{ lbs.} \quad (4)$$

For maximum shear in $f5$: $x = 3.93$; w_1 at f; $w_n = w_{16}$; $w_{n-1} = w_{14}$; $w_{n_1-1} = w_1$; $a = y = 8.08$. First positive term opposite w_{14} in column 4 = 12,427,800; second positive term opposite w_{16} in column 2 = 297,000; negative term as above.

$$\text{Shear in } f5 = S = \frac{1}{200}\left[\frac{12427800}{2}+\frac{297000}{2}\times 3.93\right] - \frac{1}{20}\cdot\frac{121200}{2} = 30{,}963 \text{ pounds.} \quad (5)$$

For maximum shear in $4e$: $x = 2.30$; w_1 at e. As now the rolling load is on the bridge, the total load is $w_1+w_2+w_3+\ldots+w_{n-1}+w_n$, in which $w_{n-1} = w_{18}$ and w_n = total uniform load = $4.60 \times 3000 = 13{,}800$ lbs. Hence total load = 13,800 + 342,000 (from column 2, Table LXXV, opposite w_{18}); but since the distance between w_{n-1} and $w_n = x + 4$, the last term, column 2, Table LXXV, should be $342{,}000 \times (x + 4)$. Then the first positive term in the value of S would be $= 17{,}210{,}760 + 342{,}000 \times (x + 4) = 17{,}210{,}760 + 342{,}000 \times 4 + 342{,}000 \times x = 18{,}578{,}760 + 342{,}000 \times 2.30 (= x) = w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots$. The second positive term $= (342{,}000 + 13{,}800) \times x = (w_1 + w_2 + \ldots w_{n-1} + w_n)x$; then $w_1a + (w_1 + w_2)b + \ldots + (w_1 + w_2 + w_3 + \ldots w_{n-1})q + w_1 + w_2 + \ldots w_{n-1} + w_n)x = 18{,}578{,}760 + 342{,}000 \times 2.30 + (342{,}000 + 13{,}800) \times 2.30 = 18{,}578{,}760 + (2 \times 342{,}000 + 13{,}800)2.3$. Hence when the uniform load comes on the bridge take the last number in column 4, Table LXXV, for the first term, and add to this for the second part of the positive twice the sum of all the engine loads found in column 2 opposite w_{18} plus the total uniform load, multiplied by the distance of the centre of gravity of the uniform load from the nearest reaction (which is $= x$). Then substituting in the value of S the negative term the same as before, hence with w_1 at e

$$\text{Shear in } 4e = \frac{1}{200}\left[\frac{18578760}{2}+\frac{(2\times 342000)+13800}{2}\times 2.30\right] - \frac{1}{20}\cdot\frac{121200}{2} = 47{,}434 \text{ pounds.} \quad (6)$$

This is the first main brace, and the maximum stress or tension is found by multiplying 47,434 $\times$ sec $\theta(=1.2)$. (See following table.)

With w_1 at d: $x = 15.18$; uniform load $= w_n = 91{,}050$ pounds; $w_{n-1} = w_{18}$; $w_{n_1-1} = w_2$, $w_{n_1} = w_3$. Apply the preceding rule for positive terms, and noting that w_1 and w_2 are both in front or to the left of d, the negative term

$$-\frac{1}{p}[w_1a + (w_1 + w_2)b + \ldots + (w_1 + w_2 + \ldots + w_{n_1-1})y]$$

becomes

$$= -\frac{1}{p}[w_1a + (w_1 + w_2)b] = -\frac{1}{20}\frac{345450}{2},$$

found in column 4, Table LXXV, opposite $w_{n_1-1} = w_2$. We then have

$$\text{Shear in } 3d = \frac{1}{200}\left[\frac{18578760}{2} + \frac{(2 \times 342000) + 91050}{2} \times 15.18\right]$$
$$- \frac{1}{20}\cdot\frac{345450}{2} = 67{,}124 \text{ lbs.}, \quad \ldots\ldots\ldots \quad (7)$$

another main brace.

With w_1 at c: $x = 25.18$ feet; $w_{n-1} = w_{18}$; $w_{n_1-1} = w_2$; $w_{n_1} = w_3$; uniform load $w_n = 151{,}050$ pounds. Hence

$$\text{Shear in } 2c = \frac{1}{200}\left[\frac{18578760}{2} + \frac{(2 \times 342000) + 151050}{2} \times 25.18\right]$$
$$- \frac{1}{20}\cdot\frac{345450}{2} = 90{,}377 \text{ lbs.} \quad \ldots\ldots\ldots \quad (8)$$

With w_1 at b: $x = 35.18$; $w_{n-1} = w_{18}$; $w_{n_1-1} = w_2$; $w_{n_1} = w_3$; uniform load $w_n = 211{,}050$. Hence

$$\text{Shear in } 1b = \frac{1}{200}\left[\frac{18578760}{2} + \frac{(2 \times 342000) + 211050}{2} \times 35.18\right]$$
$$- \frac{1}{20}\cdot\frac{345450}{2} = 116{,}531 \text{ lbs.} \quad \ldots\ldots\ldots \quad (9)$$

All of the above shears produce compression in the verticals and tension in the diagonals.

w_1 at a: $x = 45.18$; $w_{n-1} = w_{18}$; $w_{n_1-1} = w_2$; $w_{n_1} = w_3$; uniform load $w_n = 271{,}050$ pounds. Hence

$$\text{Shear in } 01 = \frac{1}{200}\left[\frac{18578760}{2} + \frac{(2 \times 342000) + 271050}{2} \times 45.18\right]$$
$$- \frac{1}{20} \cdot \frac{345450}{2} = 145{,}684 \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

01 is under compression.

All the quantities substituted in the values of S are given in paragraphs 948, 949, and 950. The above shears in the diagonals are also the compressive stresses in the verticals that meet the diagonals at their upper extremities, due to the rolling load.

In paragraph 947 the stresses on the diagonals and verticals have already been found, due to the dead load or weight of the structure. These, then, combined by adding algebraically to the stresses found for the moving load, will give the total or ultimate stresses. It will have been noticed that the shears in any panel before the head of the moving load has reached the centre have been mentioned as if existing in the structure. They are not shown in the end panels, but are shown by dotted lines in the centre panels. These shears produce or tend to produce tension in the diagonals mentioned, or, what is the same thing, compres-pression in the main diagonals in the same panel, shown by full lines, which are under tension from the dead load. As both kinds of stresses cannot exist in the same diagonal, the resultant stress will be the difference. This is the principle upon which stresses in counterbraces are determined. The above shears give the stresses in the diagonals when multiplied by 1.2, the ratio of the length of the diagonal to its vertical reach.

Stresses due to Rolling Load.				Dead Load. (See par. 947.)	Total.
Tension in	$8k$ =	1,936 × 1.2 =	2,323	73,500	
" "	$7h$ =	7,892 × 1.2 =	9,470	52,500	
" "	$6g$ =	17,524 × 1.2 =	21,029	31,500	
" "	$5f$ =	30,963 × 1.2 =	37,156	10,500	26,656
Compression in	$5e$	.... =	30,963	5,500	36,463
Tension "	$4e$ =	47,434 × 1.2 =	56,921	10,500	67,421
Compression "	$4d$	.... =	47,434	14,250	61,684
Tension "	$3d$ =	67,124 × 1.2 =	80,549	31,500	112,049
Compression "	$3c$	.... =	67,124	31,750	98,874
Tension "	$2c$ =	90,377 × 1.2 =	108,452	52,500	160,952
Compression "	$2b$	.... =	90,377	49,250	139,627
Tension "	$1b$ =	116,531 × 1.2 =	139,837	73,500	213,337
" "	$1a$	.... =	41,530*	12,000	53,530
Compression "	01 =	145,684 × 1.2 =	174,821	94,500	269,321

* Found on next page.

The tension on vertical $1a$ due to rolling load is found as in case of the greatest floor-beam reaction, that is, it is greatest when the loads on oa and ab are equal, or most nearly so, as in the following diagram. Placing wheel w_4 at a, there is 63,000 pounds on oa

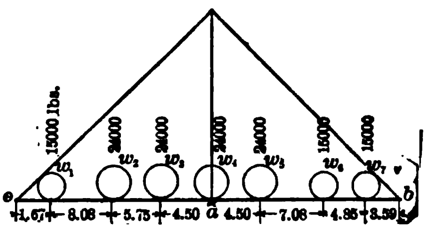


FIG. 343.

and 54,000 on ab, the equality being satisfied by dividing w_4 at a. For one truss the loads are to be divided by 2; then, for the reaction of all the loads at a,

$$r = \frac{24000}{2} + \frac{15000 \times 1.67 + 24000(9.75 + 15.50 + 15.52)}{2 \times 20}$$
$$+ \frac{15000(8.42 + 3.59)}{2 \times 20} = 41,530 \text{ lbs.}$$

The dead-load stress in $1a = 12,000$ pounds; $\therefore$ total tension $1a = 53,530$ pounds, as above. The main diagonals get maximum stress when more than one half the span is loaded. The object of placing the rolling load at the points k, h, g, and f is to determine whether a counterbrace is needed, and the stress on it. The counterbrace is not needed if the dead stress on the main brace in any panel is equal to or greater than the stress in the same panel from the live load covering the shorter segment. Evidently $8k$ and $6g$ and $7h$ are not needed, but $6g$ had better be introduced, simply as a matter of precaution. $5f$, on the contrary, is necessary, as the stress is the difference between that due to the dead and live loads, and the latter is the greatest, the difference being 26,656 pounds. These members are shown by the dotted lines in Fig. 342, paragraph 947, $5f$ and $5d$.

952. To determine the chord stresses we use equation (530), in connection with the table of locomotive loads and moments.

From equation (530),

$$\left.\begin{aligned} M = \frac{l_1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots \\ + (w_1 + w_2 + w_3 + \ldots w_n)x] - w_1a - (w_1 + w_2)b \\ -(w_1+w_2+w_3)c \ldots -(w_1+w_2+w_3+\ldots+w_{n_1-1})y; \end{aligned}\right\} \quad (550)$$

w_{n_1-1} being the loads in front of the panel point at which moments are taken and y the distance between the wheel concentrations w_{n_1-1} and w_{n_1}. The values of x and y are to be found for each panel. The position of the moving load for maximum bending moment is expressed by the relation given, equation (528), as follows: $\frac{l_1}{l} = \frac{w_1 + w_2 + w_3 + \ldots + w_{n_1}}{w_1 + w_2 + w_3 + \ldots + w_n}$, l_1 being the distance to the rear extremity of the panel in question from the left-hand support. When the load comes in from the right, the rear extremity of a panel is that end nearest the centre of the span. Instead of expressing $\frac{l_1}{l}$ in feet, it saves labor to express them in the number of panels. In this example $l = 10$; l_1 is 1, 2, 3, etc., panels. For panel stress in 4, 5; upper chord $l_1 = 5$; $l = 10$. Placing $w_{14} = w_{n_1}$ at e, we have $w_1 + w_2 + w_3 + \ldots + w_{n_1} = 282{,}000$ pounds, from column 2 of Table LXXV, opposite wheel w_{14}; the length of the uniform load on the bridge will be $= 100 - (7.08 + 4.83 + 5.67 + 4.83 + 4.00) = 73.59$ feet; hence $3000 \times 73.59 = 220{,}770$ pounds; total load on the bridge $= w_1 + w_2 + w_3 + \ldots + w_n = 342{,}000 + 220{,}770 = 562{,}770$ pounds. Substituting in eq. (528), $\frac{5}{10} = \frac{1}{2} = \frac{282000}{562770} = \frac{1}{2}$, nearly, which fulfils the required condition for maximum bending action, $x = \frac{73.59}{2} = 36.8$ feet. y is the distance between wheel concentrations $w_{13} = w_{n_1-1}$ and $w_{14} = w_{n_1} = 4.50$ feet. Then, substituting in eq. (550), first term $= 18{,}578{,}760$, column 4 of Table LXXV, second term $= 2 \times 342{,}000 + 220{,}770$. The negative term is found in column 4, opposite $w_{n_1-1} = w_{13} = 10{,}431{,}240$. Hence, dividing by 2,

$$M = \frac{5}{10}\left[\frac{18578760}{2} + \left(\frac{2 \times 342000}{2} + \frac{220770}{2}\right) \times 36.8\right]$$

$$- \frac{10431240}{2} = 7{,}752{,}862 \text{ foot-pounds.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For upper 3, 4: $l_1 = 4$, $l = 10$; $w_{13} = w_{n_1}$ at d; then $w_1 + w_2 + w_3 + \dots + w_{n_1} = 258,000$. Length of uniform load $= 120 - (4.50 + 7.08 + 4.83 + 5.67 + 4.83 + 4.00) = 120 - 30.91 = 89.09$ feet (d being 120 feet from point 10 or R_1). Uniform load $= 3000 \times 89.09 = 267,270$ pounds; $w_1 + w_2 + w_3 + \dots w_n = 342,000 + 267,270 = 609,270$ pounds. $\frac{4}{10} = \frac{258000}{609270} = \frac{4}{10}$, nearly, by eq. (528). Hence w_{13} at d gives maximum moment panel 3, 4.

$$\therefore M = \frac{4}{10}\left[\frac{18578760}{2} + \left(\frac{2 \times 342000}{2} + \frac{267270}{2}\right) \times 44.55\right]$$

$$- \frac{9270240}{2} = 7,556,447 \text{ foot-pounds.} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For upper chord 2, 3: $l_1 = 3$; $l = 10$; $w_{10} = w_{n_1}$ at c. Then $w_1 = w_2 + w_3 + \dots + w_{n_1} = 186,000$ pounds; length of uniform load $= 140 - 49.24 = 90.76$; $x = 45.38$ feet. Uniform load $= 3000 \times 90.76 = 272,280$ pounds; $w_1 + w_2 + w_3 + \dots + w_n = 342,000 + 272,280 = 614,280$ pounds. Hence

$$M = \frac{3}{10}\left[\frac{18578760}{2} + \left(\frac{2 \times 342000}{2} + \frac{272280}{2}\right) \times 45.38\right]$$

$$- \frac{5506860}{2} = 6,542,782 \text{ foot-pounds.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Upper chord 1, 2: $l_1 = 2$; $l = 10$; $w_6 = w_{n_1}$ at b; $w_1 + w_2 + w_3 + \dots + w_{n_1} = 126,000$. Length of uniform load $160 - 73.57 = 86.43$; $x = 43.22$ feet. Uniform load $3000 \times 86.43 = 259,290$ pounds; $w_1 + w_2 + w_3 + \dots w_n = 342,000 + 259,290 = 601,290$ pounds. Hence

$$M = \frac{2}{10}\left[\frac{18578760}{2} + \left(\frac{2 \times 342000}{2} + \frac{259290}{2}\right) \times 43.22\right]$$

$$\frac{1806330}{2} = 5,031,610 \text{ foot-pounds.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We can now find the chord stresses. In lower chord oa, ab, hk, $k10$, multiply the shear in 01 by $\frac{0a}{1a} = \frac{20}{30} = \frac{2}{3}$.

Tension due to rolling load $= 145,984 \times \frac{2}{3} = 97,123$ pounds.

For the stresses in the other panels divide the moments just found by the depth of the truss $= 30$ feet.

Compression, 4, 5; 5, 6 $= \frac{7752861}{30} = 258{,}428.70$ lbs.
" 3, 4; 6, 7 $= \frac{7556447}{30} = 251{,}881.60$ "
Tension, *de*, *ef* $= \frac{7556447}{30} = 251{,}881.60$ "
Compression, 2, 3; 7, 8 $= \frac{6542782}{30} = 218{,}092.73$ "
Tension, *cd*, *fg* $= \frac{6542782}{30} = 218{,}092.73$ "
Compression, 1, 2; 8, 9 $= \frac{5031610}{30} = 167{,}720.33$ "
Tension, *bc*, *gh* $= \frac{5031610}{30} = 167{,}720.33$ "

To the above add the stresses due to the dead load (see par. 947):

Max. compression,	4, 5; 5, 6	$= 258{,}429 + 145{,}833 = 404{,}262$	lbs.
" "	3, 4; 6, 7	$= 251{,}882 + 140{,}000 = 391{,}882$	"
" ttension,	*de*, *ef*	$= 251{,}882 + 140{,}000 = 391{,}882$	"
" compression,	2, 3; 7, 8	$= 218{,}093 + 122{,}500 = 340{,}593$	"
" tension,	*cd*, *fg*	$= 218{,}093 + 122{,}500 = 340{,}593$	"
" compression,	1, 2; 8, 9	$= 167{,}720 + 93{,}333 = 261{,}053$	"
" tension,	*bc*, *gh*	$= 167{,}720 + 93{,}333 = 261{,}053$	"
" "	*oa*, *ab*	$= 97{,}123 + 52{,}500 = 149{,}623$	"

953. This completes the stresses, except for the lateral or wind bracing, the object of which is to so connect the two trusses that they may act together in resisting the bending action of the wind on the trusses, as well as on the exposed surface of a train on the bridge, and to prevent the bodily overturning of one truss. This construction then gives two horizontal frames or trusses acted upon by a supposed horizontal force, distributed to a greater or less extent uniformly over an area, for plate-girders or enclosed trusses, equal to the length of the span multiplied by the depth of the truss. In this case the centre bending moment would be the same as that of a horizontal beam resting on two points of support and uniformly loaded, viz., $m = \frac{wl^2}{8}$, in which w is the pressure per lineal inch of each girder, equal to the product of the wind pressure per square inch by the depth of the truss in inches, the horizontal diagonal bracing resisting the horizontal shear, so to speak, of the wind pressure; and on these assumptions the stresses in the various members are determined in the same manner as the members of the vertical trusses acted on by vertical loads. The trusses of railway bridges seldom present solid surfaces exposed to the wind, and the actual surfaces of the chords and web members are a small

percentage of the area of the corresponding truss surface. The actual pressure of the wind in any given locality is uncertain. The area of the actual exposed surface is made up of relatively small parts, at different angles, which not opposing a solid front to the wind, cut and divide it up, confusing, mixing, as it were, the original force of the wind; consequently the magnitude, the distribution, and the effect of the wind on such a structure is uncertain and exceedingly variable. This has led to many theories and suppositions, upon which the determination of the dimensions of the wind bracing is based. About the highest wind pressure observed is from 90 to 100 pounds per square foot. Almost every engineer gives a different requirement for the wind pressure, and apparently in an entirely arbitrary manner, such as 50 pounds per square foot on the structure alone, and 30 pounds per square foot on train and structure surface combined, or 300 pounds per foot per length of train considered as a moving load; or, again, to allow 56 pounds for a portion of the depth of the truss equal to the distance between the rail and the top of the train, and the same pressure on the actual metal surface exposed to the wind of one truss above and below the train surface allowed above, and 30 pounds on the same surface of the leeward truss; and so on almost indefinitely. The specifications for the Susquehanna River bridge, B. & O. Ry., Havre de Grace, was a horizontal wind pressure of 50 pounds per square foot on all exposed surfaces, and on a moving train averaging 10 square feet per linear foot of track. The simplest and easiest of application is contained in the specifications of the Phœnix Bridge Company, substantially as follows: The dimensions of the lateral bracing between the chords shall be determined by assuming a dead-load pressure of 150 pounds per lineal foot of span, and for the chord carrying the moving load an additional pressure of 300 pounds per lineal foot of track, treated as a live or moving load. The working unit stresses on tension members is 15,000 pounds per square inch, and on compression members 9000 pounds. Upon these suppositions the stresses in and dimensions of the various members can be found in the same manner as has been fully explained for the ordinary vertical truss members. The horizontal lateral bracing has no weight, except that of its own, to carry, which can be neglected. It must be remembered, however, that the stress in the chords must be considered as a part of the stress only, and should be combined with the stress in the chords due to their forming a part of the vertical trusses, and their dimen-

sions fixed accordingly. Any further refinement on a subject where the acting forces are so uncertain would seem to be useless, an allowed pressure per lineal foot, with a large factor of safety, being as safe, and decreasing the uncertainty of the actual pressure used arising from actual train and truss surfaces. It may be stated that a pressure of about 30 pounds per square foot will derail a loaded passenger train, and of 56 pounds per square foot a loaded freight train. Such pressures are unusual, as the velocity would be from 80 to 100 miles per hour, causing a hurricane. A wind called a storm does not produce a pressure over from 12 to 20 pounds per square foot. Stresses due to wind pressure will be computed in another paragraph.

954. In determining the dimensions of the members of this bridge, the general specifications of the Phœnix Bridge Company will be used. The clear width between the trusses will be 14 feet. The allowed tensile stresses will be as follows per square inch:

For lower-chord eye-bars	10,000	pounds.
" main diagonal eye-bars near the end of the span	10,000	"
" counterbraces	7,333	"
" vertical near end of span	7,333	"
" other tension diagonals to be graded between	7,333 and 10,000	"
" plate-hangers on floor-beams (net section)	8,000	"
" bottom flanges of floor-beams and stringers (net section)	8,000	"
" lateral braces	12,000	"

For compressive stresses, upper chord and end posts, per square inch,

$$p = \frac{\frac{1}{4}(39000)}{1 + \frac{1}{50000}\frac{l^2}{\rho^2}} \text{ for flat ends,} \quad p = \frac{\frac{1}{4}(39000)}{1 + \frac{1}{30000}\frac{l^2}{\rho^2}} \text{ for pin ends.}$$

For posts at centre of span reduce the value of p by 20 per cent; for all other intermediate posts the unit stresses to be graded between the end and centre values. But for lateral compression braces the values of p may be increased 20 per cent.

For top flanges of stringers and floor-beams the unit stress must not exceed 7000 pounds per square inch. The unit pressure between pins and pin-holes and rivets and rivet-holes, 12,000 pounds per square inch. In stringers and floor-beams, and their connec-

tions with each other and the trusses, this value shall be reduced 25 per cent. The greatest unit-shearing resistance in rivets and pins 7500 pounds. In stringers and floor-beams and their connections reduce the above 25 per cent. The greatest unit resistance to bending of iron pins = 15,000 pounds, and the centres of pressure to be taken at the centre of the thickness of the bars and plates connected by the pins. No cast iron whatever shall be used in any part of the structure. All parts shall be accessible for inspection and painting. The unsupported width of any plate in compression shall not exceed 30 times its thickness. The pitch of the rivets in compression members shall not exceed 16 times the thickness of the thinnest plate through which the rivets pass. An initial stress of 5000 pounds per square inch shall be added to that produced by the vertical load in all adjustable tension members. For present purposes, the above specifications will be sufficient. The width of the upper chord will depend to a great extent upon the thickness of the eye-bars and verticals of the web system. The eye-bars should be made as thin as practicable, as this reduces the bending moment on the pins. It is also advisable to keep the thickness as uniform as possible, but variations in certain limits are allowable and usual.

955. With the allowable unit strains the following dimensions are found by dividing the total stress by the unit strains. For bottom-chord panels:

					Inches.
Panels *de*, *ef*,	tension	$= \frac{391882}{10000} = 39.19$	sq. in.	= 6 bars	$5\frac{1}{2} \times 1\frac{3}{16}$
" *cd*, *fg*,	"	$= \frac{340593}{10000} = 34.06$	" "	= 6 "	$5\frac{1}{2} \times 1\frac{1}{32}$
" *bc*, *gh*,	"	$= \frac{261058}{10000} = 26.11$	" "	= 4 "	$5\frac{1}{2} \times 1\frac{3}{16}$
Panels *oa*, *ab*, 10*k*, *hk*,	"	$= \frac{140491}{10000} = 14.05$	" "	= 2 "	$5\frac{1}{2} \times 1\frac{9}{32}$

The unit stresses in the main diagonals are found by dividing 10,000 − 7333 = 2667 by 4, the number of braces between the centre and the end of the span, which gives a rate of decrease of about 670 pounds. We find the stresses in diagonals and verticals in par. 951. All diagonals are under tension, except the end diagonal, 01, which is under compression.

Main braces,	1*b*, 9*h*	$= \frac{213337}{10000} = 21.34$	sq. in.	= 2 bars	$6\frac{1}{2} \times 1\frac{5}{8}$ inches.
" "	2*c*, 8*g*	$= \frac{160952}{9330} = 17.25$	" "	= 2 "	$6\frac{1}{2} \times 1\frac{5}{16}$ "
" "	3*d*, 7*f*	$= \frac{112042}{8660} = 12.94$	" "	= 2 "	$6\frac{1}{2} \times 1$ "
" "	4*e*, *e*6	$= \frac{67421}{7994} = 8.44$	" "	= 2 "	$5 \times \frac{7}{8}$ "
For end vertical,	1*a*, 9*k*	$= \frac{58530}{7820} = 7.30$	" "	= 2 "	$4 \times \frac{15}{16}$ "
Counterbraces,	5*d*, 5*f*	$= \frac{28656}{7880} = 3.64$	" "	= 2 rods	$1\frac{1}{4}$ in. diam.

COMPRESSION MEMBERS.

956. As no strut or column should have its least side less than one fortieth of its length, which in this case will be $\frac{30 \times 12}{40} = 9$ inches, we will assume the columns to be composed of two 10-inch channels placed 10 inches apart and latticed, with pin-plates riveted to the upper and lower ends of the vertical struts. These must be considered as columns with pin ends.

The moment of inertia of the cross-section will depend upon the metal thickness of the channels, which is really what we wish to know. It becomes necessary to assume a trial thickness. The lightest standard 10-inch channel weighs 48 pounds per yard; the web thickness is $\frac{5}{16}$ inch, with an average flange thickness of about $\frac{1}{4}$ inch. These values substituted in equation (150) for moments of inertia for latticed columns, we find the radius of gyration ρ; when neutral axis is taken parallel to the channels $\rho = 3.7$ inches, and when perpendicular to the web of the channels $\rho = 5.6$ inches. As the length of unsupported column in planes parallel to the greater value of ρ is 30 feet = 360 inches, $\frac{l}{\rho} = \frac{360}{5.6} = 64.3$ inches, but is supported, in direction of planes parallel to smaller value of ρ, by cross or lateral braces at about 22 feet above its lower extremity. $\frac{l}{\rho} = \frac{264}{3.7} = 71.4$ inches. Using the greater of these two values, and substituting in equation, $p \doteq \frac{\frac{1}{5}(39000)}{1 + \frac{1}{30000}\frac{l^2}{\rho^2}}$, we find the safe resistance to crushing per square inch for the end strut,

$$p = \frac{7800 \times 30000}{30000 + (71.4)^2} = 6667 \text{ pounds}$$

for the end post; and deducting from this 20 per cent, we have 5334 for the centre strut or column, and $(6667 - 5334) \div 3 = 440$ pounds nearly for the rate of decrease in unit strains on the posts from end to centre of span. Then on end vertical column $01 = 6667$ pounds, on $2b = 6227$, on $3c = 5787$, on $4d = 5347$, and on $5e = 5347$.

Dimensions of vertical struts will be as follows:

				Lbs. per yd.
Compression on 2*b*, 8*h*	$= \frac{139627}{6227}$	= 22.42 sq. in.	= two 10-in. channels,	115
" " 3*c*, 7*g*	$= \frac{98874}{5787}$	= 17.10 " "	= " " "	86
" " 4*d*, 6*f*	$= \frac{61684}{5347}$	= 11.54 " "	= " " "	60
" " 5*e*	$= \frac{36463}{5347}$	= 6.82 " "	= " " "	48

For 5*e*, two 10-inch channels 35 pounds per yard would answer, but the 48-pound 10-inch channel is about the lightest made. The excess is on the side of safety. The object in reducing the safe unit stresses on the web members towards the centre of the span is to provide against the injurious effects of the stresses being applied suddenly and removed suddenly on account of these members being under only a feeble permanent strain from the weight of the structure, consequently the strain is more in the nature of a suddenly applied load or shock. But the web members towards the ends are under a large permanent strain. The initial stress in the counterbraces increases the compression in 5*e*, but this is fully provided for by the excess of area in the 48-pound channels. The channels of all of these vertical struts are latticed with $2\frac{1}{4} \times \frac{5}{16}$ inch pieces placed at an angle of about 60° with the axis of the strut.

957. The thickness of the pin-plates connecting the verticals with the pin must be determined. This, however, requires the diameter of the pin to be known; the one or the other must therefore be assumed. Experience alone is the only guide, but with this the proper diameter of the pin can be closely approximated, but may require two or more recalculations. Assuming a pin $4\frac{15}{16}$ diameter, each inch in length of the pin will give a bearing resistance of $4\frac{15}{16} \times 12{,}000 = 59{,}256$ pounds. The floor-beams may either be connected with the pin by hangers, or they may be fastened to the posts by plates and rivets. This latter connection is assumed in this case. Consequently the stress transmitted to the pin by the posts is equal to the compression on the strut increased by a bottom-chord panel weight, which in this case has been found for the vertical $1a = 53{,}530$ pounds. Hence for post 2*b* bearing stress on pin $139{,}627 + 53{,}560 = 193{,}187$ pounds; hence thickness of the bearing plates, one on each side of the centre of the pin:

For post 2*b*	$= 193{,}187$	$\div 59{,}250 = 3.26$ in.,	2 plates	each	$1\frac{9}{16}$ in.
" " 3*c*	$= 98{,}874 + 53{,}530$	$\div 59{,}250 = 2.57$ "	2 "	"	$1\frac{1}{4}$ "
" " 4*d*	$= 61{,}684 + 53{,}530$	$\div 59{,}250 = 1.94$ "	2 "	"	1 "
" " 5*e*	$= (36{,}463 + 53{,}530 + 8333)$	$\div 59{,}250 = 1.66$ "	2 "	"	$\frac{7}{8}$ "

The initial stress in each of the two counterbrace bars $5f$ is 5000 pounds. The vertical component (of this total stress 10,000) = 8333 is added to the pressure on the post.

958. For the joint a floor-beam plate-hangers are used at the centre of the pin. The following diagram (Fig. 344) shows the arrangement of the members, with their stresses, on one half of the pin. T and T_1 are the stresses on the chord bars oa and ab, respectively; the stresses on these are equal for joint a, and each equal to $\frac{140490}{2} = 70,245$ pounds; $v =$ the vertical stress on one of the bars $1a = \frac{53530}{2} = 26,765$ pounds; the stress on the floor-beam hanger

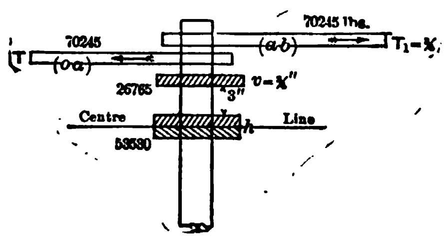


FIG. 344.

$h = 53,530$ pounds. Taking moments with respect to centre line of pin: The moment of $h = 0$; moment of v in a vertical plane = $26,765 \times 3 = 80,295$ inch-pounds. As T and T_1 are equal and act in the opposite direction, the resultant moment is simply equal to either one multiplied by the distance between the centres of their thickness $= 70,245 \times (\frac{41}{64} + \frac{1}{8} + \frac{41}{64}) = 70,245 \times 1.41 = 99,045$ inch-pounds = the moment in a horizontal plane; hence resultant moment $= \sqrt{(99,045)^2 + (80,295)^2} = 127,543$ inch-pounds, and diameter of pin $D = 1/11.3 \sqrt[3]{127,543} = 4.45$ inches $= 4\frac{15}{32}$ inches. For the point b neither the chord bars of the adjacent panels nor the web members receive their maximum strain under the same condition of moving loads or at the same time. But it will be assumed that the chord bars in adjacent panels do receive maximum tension at the same time, and are of the amounts given. It is then easy to determine the magnitude of the tension that must exist in the diagonals and verticals meeting the chord bars at any panel point or joint under consideration.

For convenience of reference the "strain-sheet" for one half of the span is introduced. (See Fig. 346.)

959. To find the diameter of the pin at joint b: Equilibrium at

that joint requires that the algebraic sum of the horizontal components, and also of the vertical components, of all forces acting at that point shall be zero. Hence the horizontal component of the stress in $1b$ existing at the same time must be equal to the increment of the chord stresses or the difference between the tension in ab and bc, or $261{,}053 - 140{,}490 = 120{,}563$ pounds, and for each of the two bars $1b = 60{,}282$ lbs. $= H$; then the vertical component will be

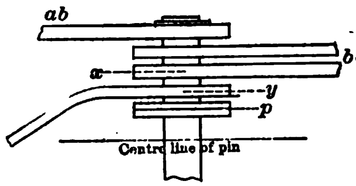


FIG. 345.

$$60{,}282 \times \frac{1a}{ab}(= 1\tfrac{1}{2}) = 90{,}423 \text{ lbs.} = v.$$

These components will be used in finding the pin diameter. The arrangement of the bars and plates on one side of the centre of the pin is shown in Fig. 345. Thickness, $1b = 1\frac{3}{4}''$; $ab = 1\frac{9}{32}''$; tension $ab = 70{,}245$ pounds; two bars $bc = 1\frac{3}{16}''$ each; tension $=$ 65,264 pounds; each pin-plate $p = 1\frac{9}{16}''$.

As we can only determine the point of greatest bending moment by trial, try first the point x. There will only be two mo-

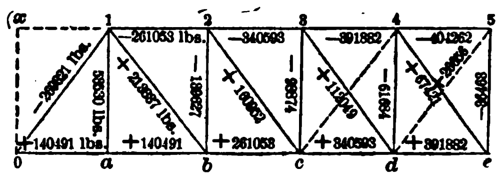


FIG. 346.—Strain-sheet of Iron Pratt Truss. Diagonals in Tension, Verticals in Compression.

ments, that of the stress in ab and one of the bars bc, as the pin is in the condition of a beam fixed at one end and loaded at intermediate points. Hence moments about $x = 70{,}245(\frac{19}{32} + \frac{1}{8} + 1\frac{3}{16} + \frac{1}{8} + \frac{41}{64}) - 65{,}264(\frac{19}{32} + \frac{1}{8} + \frac{19}{32}) = 102{,}069$ inch-pounds, an eighth of an inch being allowed as clearance between each bar and the centre of pressure or stress at the centre of the thickness of the bars. Moments about $y = 70{,}245 \times 4.272'' - 130{,}528 \times 2.25''$ $= 6419$ inch-pounds, both bars bc acting in this case, taking mo-

ments about the centre of pin-plate p. The acting moments will now be due to the stress in ab and the horizontal component of the stress in $1b$, both in the same direction, and the stresses in the two bars bc acting in the opposite direction. Hence we have $70,245 \times 6.053 + 60,282 \times 1.781 - 130,528 \times 4.031 = 6397$ inch-pounds, or the same as about the axis y. We also have a vertical moment about p, which is the vertical component of the stress in $1b \times 1.781 = 90,423 \times 1.781'' = 161,043$; hence the resultant moment $= \sqrt{(6419)^2 + (161,043)^2} = 161,171$ in.-lbs.; hence $D = \frac{1}{11 \cdot 3} \sqrt[3]{161,171} = 4.82'' = 4\frac{13}{16}''$, as the required pin diameter. For the joint c, horizontal component of stress in $2c = 3 \times 56,766 - 2 \times 65,264 = 39,770$; vertical component $= 39,770 \times \dfrac{1a}{ab} = 59,655$ pounds; stress itself $= 59,655 \times 1.2 = 71,586$ pounds, under assumed chord stresses.

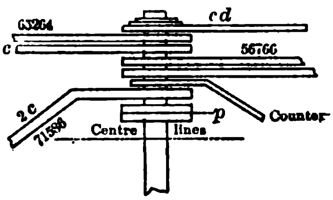


FIG. 347.

Trial moments should be taken about several points, but they will only be taken about the point p, the centre of the pin-plate.

The moment of each stress could be found as in the preceding case, but it is simpler to find the resultant of the opposed and parallel stresses. The resultant of the two stresses bc is parallel and in the same line with the two outside bars cd. It is therefore $130,528 - 113,532 = 16,996$ pounds. Its moment is $16,996(\frac{1}{16} + 1\frac{3}{16} + \frac{1}{8} + 1\frac{1}{32} + \frac{1}{8} + 1\frac{1}{32} + \frac{1}{8} + 1 + \frac{1}{8} + 1\frac{5}{16} + \frac{1}{8} + \frac{5}{8}) = 116,848$ inch-pounds. The pin-plates are $1\frac{1}{4}$ inch, or each $= \frac{5}{8}$ inch; the counter is 1 inch thick. The remaining moments are $= -56,766(\frac{33}{64} + \frac{1}{8} + 1 + 1\frac{5}{16} + \frac{1}{8} + \frac{5}{8}) + 39,770(\frac{31}{32} + \frac{1}{8} + \frac{5}{8}) = -161,380$; $\therefore$ resultant moment in a horizontal plane $= -161,380 + 116,848 = -44,532$ inch-pounds. The only acting moment in a vertical plane is the vertical component of the stress in the diagonal $2c = 59,655 \times (\frac{31}{32} + \frac{1}{8} + \frac{5}{8}) = 83,890$ inch-pounds. The resultant moment $= \sqrt{(44,532)^2 + (83,890)^2} = 94,977$ inch-pounds; hence diameter of pin $D = \frac{1}{11 \cdot 3} \sqrt[3]{94,977} = 4.03 = 4\frac{1}{32}$ inches.

The above calculations are long and tedious; and instead of decomposing the stress in the diagonals into vertical and horizontal components, the moment of the stress in the diagonals can be used direct. The stress on each of the diagonal bars was found to be 71,586 pounds; its moment with respect to p is $71,586 \times (\frac{31}{32} +$

$\frac{1}{8} + \frac{5}{8}$) = 100,700 inch-pounds; and the moment of the chord stresses is 116,848 − 56,766 × $\frac{245}{64}$ = −100,459 inch-pounds. Then draw *ab* vertically to represent the moment of the chord stresses = 100,459 inch-pounds, and *ac* normal to 2*c*, to represent the moment, 100,700 inch-pounds; the closing side of the triangle *bc* will represent the resultant moment = 94,977 inch-pounds. In the same manner the resultant moments on the pins at *d* and *e* can be found either graphically or analytically. But it is necessary to see whether the bearing resistance of the smallest pin above found against the end of the eye-bar is sufficient. Taking the panel *de*, each bottom-chord bar has a stress of $\frac{391882}{6}$ = 65,314 pounds. The bearing resistance of a 4.32 pin against the end of a bar $1\frac{3}{16}$ inch thick is 4.32 × $1\frac{3}{16}$ × 12,000 = 62,208 pounds, which is less than what is required. We must either increase the diameter of the pin or the thickness of the bar at and near the head, which is sometimes done. If we use a 5-inch pin, the bearing resistance is 5 × $1\frac{3}{16}$ × 12,000 = 71,250. In the same way, for the pin at *b*, each bar carries 70,245 pounds. The bearing resistance is $4\frac{15}{16}$ × $1\frac{9}{32}$ × 12,000 = 77,064 pounds, which is sufficient. This examination should be made at each joint. For the panel points of both chords it would seem wise not to use any pin of less diameter than those determined by one or the other of the above considerations—whichever requires the largest pin. Nothing will be gained generally by making the pins too small, and, in addition, it is economical not to vary the diameters of the pins to any extent. When the dimensions of the parts of the top chord are determined, it will be necessary to see the pin diameter required. Less diameter will generally be sufficient, except at the end panel point of the top chord.

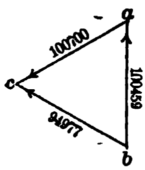


FIG. 347½.

960. For the dimensions of the top chord, taking the centre panel 4, 5, we know that about $\frac{432662}{8000}$ = 54 square inches will be required. The width will be taken at 21 inches and the depth at 18 inches. The following parts will about make the required area of cross-section: One cover-plate 21 × $\frac{9}{16}''$ = 11.81 square inches; two angles, *AB*, 3×3″, 20 pounds per yard = 4.0 square inches; two angles, *CD*, 5×3″, 55 pounds per yard, 11 square inches; two web-plates 18 × $\frac{13}{16}''$ = 29.2 square inches. Total, 11.81 + 4.0 + 11.0 + 29.2 = 56.01

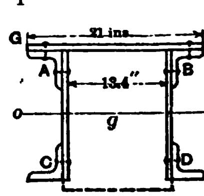


FIG. 348.

square inches. To determine the value of the radius of gyration, first find the position of the centre of gravity, taking moments about the horizontal centre line of the cover-plate GH. The centre of gravity of the 3×3 inch angles is found either by balancing a model on a knife-edge or by calculation. In this case the metal thickness is $\frac{3}{8}$ inch, nearly, and its centre of gravity is 0.95 inch from its base; and for the 5×3 inch angles $\frac{3}{4}$ inch thick, is 0.83 inch from the base of the 5-inch leg.

Moments about GH, 3×3 in. angles	$= 4 \times 1.23$	$=$	4.92 in.-lbs.
" " GH, 5×3 " "	$= 11.0\times 17.47$	$=$	192.17 "
" " GH, two web-plates	29.2×9.28	$=$	270.98 "
			468.07 in.-lbs.

Hence $468.07 \div 56 = 8.36$ inches, as the distance from the centre line of the cover-plate to the centre of gravity of the entire cross-section g; and $8.36 - .28 = 8.08$ inches, the distance from the lower surface of the cover-plate to g. The moment of inertia of the cross-section can then be found by substitution in a formula, or by the general principle that the moment of inertia of a surface composed of symmetrically arranged rectangles is the sum of all the areas of the rectangles by the square of the distance from their centres of gravity to the neutral axis of the entire cross-section, increased by the sum of the moments of inertia of the several parts with respect to an axis through their own centres of gravity, and parallel to the principal axis oo' in Fig. 348. We then have,

For the cover-plate,	$11.81 \times \overline{8.36}^2 + \frac{21}{12} \times (\frac{9}{16})^3 =$	825.71
" " 3×3 in. angles,	$4.00 \times \overline{7.13}^2 + 2 \times 1.8 =$	206.96
" " 5×3 " "	$11.0 \times \overline{9.09}^2 + 2 \times 3.46 =$	915.85
" " web-plates,	$29.2(0.92)^2 + 29.2(\frac{\overline{18}^2}{12})$	813.22
Total moment of inertia,		2761.74

Square of the radius of gyration $\rho^2 = \dfrac{2761.74}{56} = 49.32$; hence $\rho = \sqrt{49.32} = 7.02$ inches, and $\dfrac{l}{\rho} = \dfrac{20 \times 12}{7.02} = 34.19$. We can now find the allowable stress per square inch on chord members. For flat ends

$$p = \frac{7800}{1 + \frac{1}{50000}\frac{l^2}{\rho^2}} = 7622 \text{ pounds}.$$

for pin ends

$$p = \frac{7800}{1 + \frac{1}{30000}\frac{l^2}{\rho^2}} = 7508 \text{ pounds.}$$

The upper chord will be flat-end columns, except for the end panels, which is one end fixed and one pin end, which is the mean between the above, $\frac{7622 + 7508}{2} = 7565$ pounds per square inch.

Upper chord panel 1, 2, $\frac{261053}{7565}$ = 34.51 sq. in.

One cover-plate 21 × $\frac{7}{16}$ in. = 9.20 " "
Two 3 × 3 in. angles, 20 lbs. per yard, = 4.00 " "
Two 5 × 3 in. angles, 40 lbs. per yard, = 8.00 " "
Two web-plates 18 × $\frac{3}{8}$ in. = 13.50 " "
—— 34.7 sq. in.

Upper chord panel 2, 3, $\frac{340583}{7622}$ = 44.68 sq. in.

One cover-plate 21 × $\frac{7}{16}$ in. = 9.20 " "
Two 3 × 3 in. angles, 20 pounds, = 4.00 " "
Two 5 × 3 in. angles, 50 pounds, = 10.00 " "
Two web-plates 21.0 × $\frac{17}{32}$ in. = 22.30 " "
—— 45.50 sq. in.

Upper chord panel 3, 4, $\frac{391882}{7622}$ = 51.41 sq. in.

One cover-plate 21 × $\frac{9}{16}$ in. = 11.31 " "
Two 3 × 3 in. angles, 20 pounds, = 4.00 " "
Two 5 × 3 in. angles, 55 pounds, = 11.00 " "
Two web-plates 21 × $\frac{10}{16}$ in. = 26.24 " "
—— 52.55 sq. in.

Upper chord panel 4, 5, $\frac{404262}{7622}$ = 53.04 sq. in.

One cover-plate 21 × $\frac{9}{16}$ in. = 11.31 " "
Two 3 × 3 in. angles, 20 pounds, = 4.00 " "
Two 5 × 3 in. angles, 55 pounds, = 11.00 " "
Two web-plates 21 × $\frac{21}{32}$ in. = 27.56 " "
—— 53.87 sq. in.

This last is a little smaller than the trial area, 56.06 square inches, but near enough. The end strut 01 will be taken as 18 inches in depth, same as the upper chord, and the radius of gyration will be nearly the same, $\rho = 7.02$. The length of the strut

is $35.17 \times 12 = 422.04$ inches; $\frac{l}{\rho} = \frac{422.04}{7.02} = 60.12$; as it has both pin ends, $p = \frac{7800}{1 + \frac{1}{30000}\frac{l^2}{\rho^2}} = 6961$ pounds; area required for 01 $\frac{269321}{6961} = 38.70$ square inches.

1 cover-plate, $21 \times \frac{9}{16}$ in.	=	11.31 sq. in.
Two 5×3 in. angles, 45 lbs.	=	9.00 " "
Two 3×3 in. angles, 25 lbs.	=	5.00 " "
Two web-plates, $18 \times \frac{3}{8}$ in.	=	13.50 " "
		38.81 sq. in.

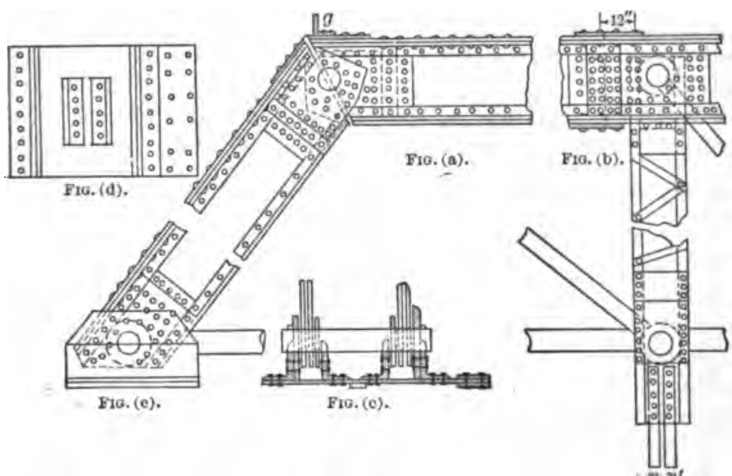
The relative dimensions of the cover-plates, angles, and web should be so adjusted that the centre of gravity of the cross-section shall not deviate far from the centre of the depth of the beam. This provides space above the pins for the heads of the web members. The centre of the pins, but for the bending action of the weight of the beam, should pass through the centre of gravity of the cross-section. The chord should be deep so as to reduce the bending. It is sometimes made only 16 inches for a 200-foot span, but 18 inches is to be preferred.

The upper extremity of 01 does not bear against the chord; both bear directly on a pin. The bearing resistance of a 5-inch pin on a web member $\frac{3}{8}$ inch thick is $5 \times \frac{3}{8} \times 12{,}000 = 22{,}500$ pounds; whereas the stress in the end strut is on each web $\frac{269321}{2} = 134{,}661$ pounds, and on the web of the chord panel 1, 2, $\frac{261053}{2} = 130{,}527$ pounds. The thickness of the bearing surface to resist this latter stress would be $5 \times t \times 12{,}000 = 130{,}527$, or $t = 2.17$ inches. By riveting plates to both the inside and outside of the web of the end post and chord the required thickness can be secured. It would be better, however, to increase somewhat the diameter of the pin. A pin $5\frac{1}{2}$ inches diameter would require a plate thickness of $5\frac{1}{2} \times t \times 12{,}000 = 130{,}527$, or $t = 1.98$ inches, say 2 inches, thick. The following plates should then be riveted to the web of the chord: A $\frac{3}{4}$-inch plate on the outside, and two plates, respectively $\frac{1}{2}$ and $\frac{3}{8}$ inches on the inside, $\frac{3}{4} + \frac{3}{8} + \frac{1}{2} + \frac{3}{8} = 2$ inches. Using $\frac{3}{4}$-inch rivets, the number required is found as follows with a $5\frac{1}{2}$-inch pin: The bearing resistance of the reinforcing plates will be respectively $5\frac{1}{2} \times \frac{3}{4} \times 12{,}000 = 49{,}500$ pounds; $5\frac{1}{2} \times \frac{1}{2} \times 12{,}000 = 33{,}000$ pounds; $5\frac{1}{2} \times \frac{3}{8} \times 12{,}000 = 24{,}750$

pounds. The bearing resistance of $\frac{3}{4}$-inch rivet will be respectively $\frac{3}{4} \times \frac{3}{4} \times 12{,}000 = 6750$ pounds, $\frac{3}{4} \times \frac{1}{2} \times 12{,}000 = 4500$ pounds, and $\frac{3}{4} \times \frac{3}{8} \times 12{,}000 = 3375$ pounds. This requires for the several plates $\frac{49500}{6750} = 7.3$, $\frac{33000}{4500} = 7.4$, $\frac{24750}{3375} = 7.4$, in all 23 rivets. The shearing strength of the $\frac{3}{4}$ rivets is $0.44 \times 7500 = 3300$ pounds per rivet. This resistance will generally be found in excess when the bearing resistance is secured. The same thickness of plates is generally used at the end of the inclined strut 01, but arranged somewhat differently. One of the inside thickening plates of the chord and one of the inside plates of the post project beyond their respective ends, and are so cut that they reach into the ends of the chord and post, overlapping each other. These are called "jaw-plates," through which the pin passes. This arrangement holds the ends of the chord and post in their proper position and prevents any lateral slipping. The other thickening-plates are cut square with the ends of the chord and post respectively, and simply bear against the pin on surfaces cut to fit it, a fraction less than a semicircle, as the ends of the chord and post are separated by about $\frac{3}{8}$ inch. The exact thickness and position of the thickening plates vary with the taste of the designer or the convenience of construction and erection. The lower extremity of the end post also rests against a pin. The number and thickness of the plates at this point depends on the vertical component of the stress in the end strut, and with the same size pin as at the top will be practically the same. The required thickness on each side of the centre for a $5\frac{1}{2}$-inch pin is $5\frac{1}{2} \times t \times 12{,}000 = \dfrac{269321}{2 \times 1.2}$ (see Figs. 346 and 349). $\therefore t = 1.6$ or $\frac{5}{8} + \frac{5}{8} + \frac{3}{8} = 1.62$ inches. The pin rests upon vertical plates riveted to a base composed of one or more thick plates of considerable area so as to distribute the pressure over a large surface. This construction is called the pedestal. The end lateral braces of the lower chord are connected with the base-plate. The thickness of the vertical plates must be sufficient to bear the vertical pressure. These plates may be collected together on the outside or inside of the web of the end post, or partly on both sides. This latter distributes the pressure to better advantage over the base-plate, the proper space being left between the two to permit easy entrance of the sides of the post, and also the bottom-chord bars of the end panel. The exact arrangement of these parts is also a matter of taste or convenience. The following figures 349 (*a*), (*b*), (*c*), (*d*), show these details at the upper and lower ends of the end post and at the

end of the upper chord. Also details of an intermediate post at upper and lower extremity for connection with top and bottom chord and floor-beam.

Figure (*a*) shows connections at the upper extremity of the end post with the chord. The jaw-plate riveted to the end post is shown in full lines on the outside; all inside plates are shown by the dotted lines. Figure (*e*) shows lower end of inclined post and side elevation of pedestal. Figure (*c*) shows vertical cross-section through axis of lower pin and perpendicular to the plane of the truss, with the vertical ribs for the pin bearing; the sum of their



FIGS. 349.

thicknesses must be equal to the bearing resistance of the pin, and spaced so as to admit entrance of the end of the strut and chord bars. Figure (*d*) shows plan of pedestal and extended plate for lateral connections. Figure (*b*) shows the connection between the posts and chords. The number and thickness of the pin-plates at the top depends upon the stress on the posts, and must be determined for each post. At the lower ends the channels extend downward below the pin; to these angles are riveted, and to these plates riveted to the floor-beams at their ends are connected. This connection is greatly preferred to the floor-beam hangers. About 12 inches to the left of the upper pin the connection for the ends of two segments is shown, cover-plates being used on top, bottom, and

both sides. The number and arrangement of the rivets are shown at each point of junction. A bent cover-plate is also placed over the joint between the end post and the upper chord. Many plates, angles, and general details for connection of the lateral or wind bracing are not shown, but the above diagrams give a general idea of the arrangement and dimensions of the main details, all of which can be varied to a greater or less extent according to the ideas of the designer or the peculiar conditions of the case. It is evident that a smaller pin at the bottom than at upper extremity of the post could be used, but it would require an increased number of thickening-plates and ribs in the pedestal, and require additional cost and labor in the shops, resulting from another variation in the size of the pins. We will therefore use $5\frac{1}{2}$-inch pins for the end post and posts $2b$ and $3c$, and 5-inch pins for the posts $4d$ and $5e$. To determine the thickness of the pin-plates for the vertical $2b$, the pressure on the strut at the top is $\frac{139628}{2} = 69{,}814$ pounds on each side of the post. Thickness of pin-plates is then $t = \frac{69814}{5\frac{1}{2} \times 12{,}000}$ $= 1.06$ inches, one plate $\frac{3}{8}$ inch outside and one plate $\frac{5}{8}$ inch inside; one $\frac{3}{4}$-inch rivet $= 0.44$ square inch area. Hence shearing resistance of one rivet $7500 \times 0.44 = 3300$ pounds, and the number of rivets $\frac{69814}{3300} = 21$ rivets. The bearing resistance of a $5\frac{1}{2}$ pin on a $\frac{5}{8}$-inch plate is $5\frac{1}{2} \times \frac{5}{8} \times 12{,}000 = 41{,}250$ pounds. The bearing resistance of a $\frac{3}{4}$ rivet on a $\frac{3}{8}$ plate is $\frac{3}{4} \times \frac{3}{8} \times 12{,}000 = 3375$. Hence the number of rivets required to fasten the $\frac{3}{8}$-inch plate to the $\frac{5}{8}$-inch plate will be $\frac{41250}{3375} = 12$ rivets, and the bearing resistance of 21 rivets connecting the $\frac{3}{8}$ plate with the flanges of the channel is $21 \times 3375 = 70{,}785$ pounds, which is somewhat greater than the pressure on one side of the post, 69,814 pounds. Therefore the above arrangement and thickness of the pin-plates are satisfactory. Two $\frac{1}{2}$-inch plates might have been used, but the thinner the outside plate consistent with strength the better, as less clear width between the webs of the chord, and as consequence a smaller pin, will be required.

961. The bearing on the lower-chord pin at the lower extremity of the verticals is equal to the pressure on the post increased by the lower-chord panel weight due to both dead and moving loads. This latter will be taken equal to the tension in $1a = 53{,}530$ pounds. Hence for the post $2b$ the pressure on each pin-plate will be 69,814 $+ \frac{53530}{2} = 96{,}579$ pounds; hence thickness of pin-plate $= \frac{96579}{5\frac{1}{2} \times 12000}$

$= 1.46$ inches $=$ three plates, $\frac{1}{2}$ inch $= 1.5$ inches; the rivets required $= \frac{96579}{3300} = 30$. The bearing resistance of these rivets $= 30 \times 3375 = 101,250$ pounds, which is in excess of the total pressure. The rivets near the pin-hole must be countersunk, so that the eye-bars of $1b$ brace may lie close to the post. This must be done whenever plates or bars have to be placed close together. The pin-plates and rivets for the other posts are determined in exactly the same manner, the actual results varying as the pressure on the posts vary, decreasing towards the centre of the span. Where counters connect with the top of the posts the vertical component of the initial strain of 5000 pounds $= \frac{5000}{1.2} = 4166$ pounds. For post $4d$ as two counter-rods connect with it, the pressure used should be $69,841 + 8332 = 78,173$ pounds, as was stated. These counterbraces, $4c$, were not necessary, but are usually inserted. Post $5e$ has, however, four counter-rods connected at its upper extremity, $5d$ and $5f$, which are necessary; and the pressure to be used in calculating the thickness of the top pin-plates $= 36,463 + 16,664 = 53,127$ pounds, and for the bottom pin-plates $= 53,127 + 53,530 = 106,657$ pounds, from which the thickness of the pin-plates and the number of rivets can be calculated. The above pressures must be divided by 2 for each side of the centre of the pins.

962. To determine whether the web of the upper chord needs thickening-plates near the pins, find the horizontal component of the stress on the diagonal meeting at that point. Taking, for instance, the panel point 3, the stress on one of the bars $3d = \frac{112049}{2}$ $= 56,025$ pounds; the horizontal component of this stress is $=$ $56,025 \times \frac{cd}{3d}\left(= \frac{20}{36}\right) = 31,125$ pounds. The web-plate of the upper chord 3, 4 is $\frac{10}{16}$ inch thick; the bearing resistance of a 5-inch pin is $5 \times \frac{10}{16} \times 12,000 = 37,500$ pounds. Therefore no plate is needed at that point, and evidently none can be required between this point and the centre of the span. At the joint 2 we have the horizontal component of the stress in $2c = \frac{20}{36} \times \frac{160952}{2} = 44,709$ pounds for each bar. The bearing resistance of a $5\frac{1}{2}$-inch pin on web-plate 2, 3, $\frac{17}{32}$ inch thick, $= 5\frac{1}{2} \times \frac{17}{32} \times 12,000 = 35,040$ pounds. Hence $44,709 - 35,040 = 9769$ pounds, which must be carried by a

thickening-plate; $t = \dfrac{9679}{5\frac{1}{2} \times 12000} = 0.15$ inch, or one plate $\frac{1}{4}$ inch thick riveted to each web-plate. The shearing resistance of $\frac{3}{4}$ rivets $= 3300$ pounds. Hence $\dfrac{9769}{3300} = 3$ rivets. The bearing resistance $= \frac{3}{4} \times \frac{1}{4} \times 12{,}000 = 2250$ pounds. Hence $\dfrac{9769}{2250} = 4.3$ rivets; say, six rivets. At the joints between two segments of the upper chord, which should always be near a panel point, where the bending action is small, the main duty of the cover-plate is to hold the ends together. If the ends are not cut true they must have the bearing resistance on the rivets on either side of the joint equal to the stress in the panel where the joint is made. As shown in figure 349 (*b*), paragraph 960, the joint is in panel 1, 2, where the stress is 261,053 pounds, or 130,527 on each side. Using, then, on the sides a $\frac{5}{8}$ and $\frac{3}{8}$ inch plate as in the figure, and on top and bottom $\frac{3}{8}$-inch plates, we have for the $\frac{5}{8}$-inch plate, twelve rivets, bearing resistance $5625 \times 12 = 67{,}500$ pounds; $\frac{3}{8}$-inch plates, eight rivets, $3375 \times 8 = 27{,}000$ pounds; and in the top and bottom $\frac{3}{8}$-inch plates, twelve rivets in the two, $3375 \times 12 = 40{,}500$ pounds. Total bearing resistance 135,000 pounds. The total shearing resistance of thirty-two rivets in single shear would be $3300 \times 32 = 105{,}600$; but twelve of these would be in double shear, which would add 39,600 pounds to the above, or a total shearing resistance of 144,200 pounds. It is easy by lengthening the plates to increase the number of rivets to any desired extent to insure perfect safety. The under side of the bottom chord and end posts are latticed. The usual lattice-bars are $4 \times \frac{3}{8}$ inches, two rivets at each end. At the ends and near the joints in the chord segments broad plates are used. For the intermediate posts, which are latticed on both sides, $2 \times \frac{5}{16}$ inches will do. The lattice-bars make an angle about 60° with the axis of the member.

FLOOR-BEAMS AND STRINGERS.

963. In the preceding example the determination of the stresses in and dimensions of the several members of the Pratt truss and the loads assumed having been fully discussed, the floor systems will now be briefly considered. The length of the floor-beams will vary with the width from centre to centre of the trusses, and the mode of supporting the ends.

In iron and steel bridges of the present day they are rigidly connected with the posts by means of plates and rivets. Floor-beams are suspended by hangers in small bridges and highway bridges.

The length of the stringers depends upon the length of the panels. They may rest on top of the floor-beams, or they may be riveted to the web-plates of the floor-beams.

The dimensions of floor-beams and stringers discussed in this and the following paragraphs are taken without reference to any particular bridge.

The rails of the track rest upon cross-ties supported by the stringers. These cross-ties act as floor-beams, as the stringers are not placed directly under the rails. Guard-rails are bolted to the ties.

The flanges are supposed to resist the whole bending action of the loads, and the web to resist the shears. The following data are assumed:

The dimensions of the floor-beam are as follows: Length, 20 feet; depth, 36 inches; between rivet-holes, 33 inches. The loads will be the reactions of stringers, 25 feet long, riveted to the web of the floor-beams at A, D, and B, and placed 6-foot centres, carrying a consolidation locomotive with the concentrated wheel-weights and distance apart of wheels as shown in Fig. 351.

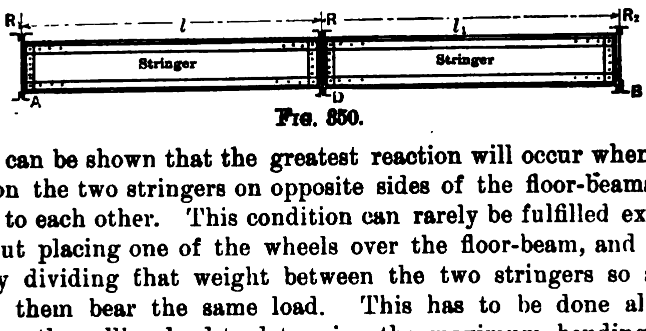


FIG. 350.

It can be shown that the greatest reaction will occur when the load on the two stringers on opposite sides of the floor-beams are equal to each other. This condition can rarely be fulfilled exactly without placing one of the wheels over the floor-beam, and arbitrarily dividing that weight between the two stringers so as to make them bear the same load. This has to be done also in locating the rolling load to determine the maximum bending moment and maximum shear on girders or stringers. Therefore in Fig. 350 we place the third driver D over the floor-beam in question, and by supposing 9000 pounds of this weight to rest on the stringer to the right and 1500 on the one to the left, we find that we have on the left the front truck 15,000 + 2 drivers 48,000 + 15,000 from D = 78,000 pounds; and on the right 1 driver 24,000

+ 3 tender-wheels 45,000 + 9000 from $D = 78,000$ pounds. This location of the load has to be made by trial, but can be closely approximated by inspection. Now, having the position of the loads on the stringers so as to produce the maximum reaction at R, we find this reaction as follows: Multiply each load by its distance from the other end of the stringers at R_1 and R_2, and divide by the length of the stringer. The sum of all these quantities will be the desired reaction. If the spans are unequal, let l and l_1 be their respective lengths; w_1, w_2, w_3 the loads on l, and w_5, w_6, w_7, w_8 the loads on l_1; x_1, x_2, x_3, etc., their respective distances from R_1 and R_2. Then the reaction at R will be

$$R = \frac{w_1x_1 + w_2x_2 + w_3x_3}{l} + \frac{w_5x_5 + w_6x_6 + w_7x_7 + w_8x_8}{l_1} + w_4. \quad (551)$$

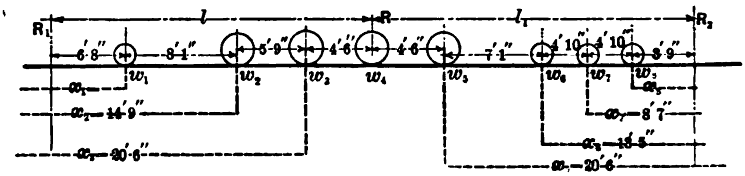


Fig. 351.

Note.—Figs. 350 and 351 are to be considered together. Fig. 350 shows the connections between stringers and floor-beams.

w_1, w_6, w_7, and $w_8 = 15,000$ pounds; w_2, w_3, w_4, and $w_5 = 24,000$ pounds. Assuming $l = l_1 = 25$ feet, and substituting in eq. (551), $R = 96,960$ pounds, the entire load on the two spans $= 156,000$ pounds. If this load were uniformly distributed over the entire length and the two spans supposed to be continuous, then R would be $= \frac{5}{8} \times 156,000 = 97,506$. Specifications generally call for the load on the bridge to consist of two consolidation locomotives followed by a uniform load of either 2240 or 3000 pounds per lineal foot. The latter load on the two spans l and l_1 would be $3000 \times 50 = 150,000$ pounds; the reaction R from this load would then be $= \frac{5}{8} \times 150,000 = 93,750$ pounds. So we may conclude that the above value of $R = 96,960$ pounds will be the maximum reaction. It is generally intended that the prescribed uniform load will approximate to the equivalent uniform load, or that which will give the same centre bending moment as the actual wheel concentrations will give. The above loading is to be divided between the two stringers; hence R for one line of stringers $= 48,375$ pounds, which

is supposed to act at A and B (Fig. 352). As this is the reaction due to the rolling load, we must add that due to weight of stringers. This has to be assumed in the first calculation, and if it does not agree with the actual weight thus found, a new weight must be taken and the calculation made again. To calculate the weight of stringer now under consideration would require a new distribution of the load in order to find maximum bending moment and shearing stress. As the stringer under consideration carries a heavy load, we will take it at, say, 200 pounds per foot, and assuming weight of cross-ties, guard-rails, iron rails, etc., at 200 pounds, we can safely say that the total dead load will be 400 pounds per lineal foot of stringer. This is one half of the weight of the cross-ties, guard-rails, iron rails, spikes, etc., per foot plus the weight of the stringer per foot. We would then have the reaction due to the dead load, $400 \times 25 = 10{,}000$ pounds, which added to the reaction found above for the live load, gives $48{,}375 + 10{,}000 = 58{,}375$ pounds, as the final load transmitted to the floor-beams at A and B by each stringer. We are now prepared to find the proper dimensions of the floor-beam. The weight of this will have to be assumed, as we have taken heavy moving loads, giving a large reaction. We will take a floor-beam 36 inches deep, 33 inches between rivet-hole centres, weighing 200 pounds per foot of length; hence total weight $200 \times 20 = 4000$ pounds. The bending moments (see Fig. 352) at points half way between end of beam and A, where the stringer rests, at A, and at the centre, will be as follows: The moment at the centre will be the same as at A and B. That due to the dead load or weight of beam will vary at each point, and will give the following combined moments:

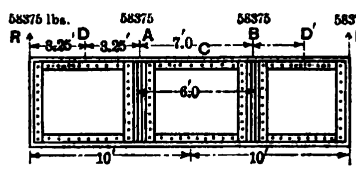


FIG 352.

At D, live load $58{,}375 \times 3.25 +$ dead load $2000 \times 3.25 - 200 \times 3.25 \times \frac{3.25}{2} = 195{,}162.5$; flange strain, $P_s = \frac{195162.5}{2.75} = 70{,}968$ pounds. (Eq. (2).)

At A, live load $58{,}375 \times 6.5 +$ dead load $2000 \times 6.5 - 200 \times 6.5 \times \frac{6.5}{2} = 388{,}212.5$; flange strain, $P_1 = \frac{388212.5}{2.75} = 141{,}167$ pounds. (Eq. (3).)

At C, live load $58{,}375 \times 6.5 +$ dead load $2000 \times 10 - 200 \times 10$

$\times \frac{10}{2} = 389{,}437.5$; flange strain, $P = \frac{389437.5}{2.75} = 141{,}614$ pounds. (Eq. (4).)

If we only allow 7000 pounds per square inch in compression, the flange area at the three points will be, respectively,

at $D = 10.14$ sq. in.; at $A = 20.17$ sq. in.; at $C = 20.23$ sq. in.;

and allowing 8000 for the lower or tension flange,

at $D = 8.86$ sq. in.; at $A = 17.65$; and at $C = 17.8$ sq. in.

The following flanges will satisfy the above requirements:

Top flanges, two 5-in. × 4-in. angles, each 55 lbs. per yd.,	11 sq. in.
One cover-plate, ¾ in. thick × 12 in. or two ⅜ in. × 12 in.	9 " "
	20 " "
Bottom flange, two 5-in. × 4-in. angles, 55 lbs. per yd.,	11 sq. in.
" " one cover-plate ¾ in. or two plates ⅜ in. × 12,	9 " "
	20 " "

This allows in the bottom flange for loss in punching holes only 2 inches of metal, whereas the actual loss is 3.8 square inches. This would doubtless be safe, as we have only allowed 8000 pounds per square inch resistance; and with 9000 pounds, which gives a safety factor of 5, the required area would be 15.74 square inches; or if we had increased the total depth of beam from 36 to 40, there would have been required 16.22 square inches. This latter would be better, as then the required area of the top flange would be reduced to 18.54 square inches, and in addition the stiffness of the beam would be greatly increased.

To determine the thickness of the web of the floor-beam, find the reaction at either end. This is equal to the weight of one half of the floor-beam added to the stringer reaction, $2000 + 58{,}375 = 60{,}375$ pounds; allowing 5000 pounds per square inch, $\frac{60375}{5000} = 12.07$ square inches; and thickness, $\frac{12.07}{33} = 0.37$ in. $= \frac{3}{8}$ inch. With such heavy loads it would be as well to reinforce the web where the stringers rest by riveting plates to the web of the floor-beam. Stiffeners should also be introduced between the ends and the stringers, and heavy angles at the ends of the floor-beam.

We can now proceed to determine the number of rivets, based

on the supposition that the rivets transmit the stresses to the flanges. As we have already seen that the flange stresses increase from the end towards the centre, viz., from the end of the stringer to D, it is (eq. (2)) 70,968 pounds; the resistance of the web to crushing at the rivet bearing is $\frac{7}{8} \times \frac{3}{8} \times 9000$ pounds (as in preceding example) = 3000 pounds. Hence number of rivets between R and $D = \frac{70968}{3000} = 24$, distributed over 3.25 feet; hence $\frac{39}{24} = 1.6$ inches rivet centres. In the next section we have only to provide for the increase of flange strain, or $141,167 - 70,968 = 70,199$ pounds. $\frac{70199}{3000} = 23$ rivets; $\frac{39}{23} = 1.7$ inches, distance apart of rivets; and in the section from D to C, $141,614 - 141,167 = 447$ pounds, for which one rivet would be required. It is plain that in the end sections the rivets are too close together and in the middle sections they are too far apart, and if we space them $2\frac{1}{2}$ inches apart for seven feet from each end and 6 inches the balance of the length the conditions of safety will be fulfilled. If we had made the web $\frac{5}{8}$ inch thick, in which the safe bearing of the rivet would be $\frac{7}{8} \times \frac{5}{8} \times 9000 = 5000$ pounds, and $\frac{70968}{5000} = 14$ rivets, spaced $\frac{39}{14} = 2.8$ inches in the first two sections from the end, that is, from R to A, in either event the rivets between A and B should not be more than from 6 to 9 inches apart, in order to avoid too great a length of small columns under compression. The same spacing of the rivets will exist in the bottom flange. For the rivets in the cover-plate we proceed as in the preceding example. It is better to use one thick plate than two thinner ones. Using, then, the $\frac{3}{4}$-inch plate and $\frac{3}{4}$-inch rivets, we have, for the bearing power of each rivet, $\frac{3}{4} \times \frac{3}{4} \times 9000 = 5062$ pounds; $\frac{3}{4}$-inch cover-plate 12 inches broad would contain $\frac{3}{4} \times 12 = 9$ square inches. Total resistance to crushing would be $9 \times 7000 = 63,000$ pounds; hence $\frac{63000}{5062} = 12.5$ rivets, or say 14 rivets, seven on each side, to be distributed over 10 feet, or $\frac{120}{7} = 17$ inches rivet centres. But the plate being only $\frac{3}{4}$ inch thick, for reasons above stated the rivets should not be over $\frac{3}{4} \times 10 = 7.5$ inches. This would, no doubt, be sufficient, but as a fact in practice they are placed at about 3-inch centres near the end, and increasing to 6 inches near the centre. The only difference in the bottom cover-plate is that 8000 pounds is allowed, and the effective area is $12 - 1.5$ (for rivet-holes) $= 10.5$ in. $\times \frac{3}{4} = 7.9$ sq. in. $\times 8000 = 63,200$ pounds, which gives, as above, 14 rivets. But to make the parts act together as one whole, the rivets in the bottom flange are seldom spaced more than from 6 to 8 inches apart, and one half of this distance for about three rivets from the end.

In the floor-beam under consideration the stringer is riveted to the web of the floor-beam; therefore it remains to determine the number of rivets required. At each end of the stringer angle-irons must be riveted of sufficient cross-section to carry the reaction at the end. The reaction in this case, as will be shown in the next paragraph, for the assumed rolling load is 34,190 pounds; one half the weight of the stringer is 5000 pounds; total reaction at end of stringer 39,190 pounds. Allowing 4000 pounds per square inch, $\frac{39190}{4000} = 9.79$ square inches; two 5 in. $\times$ 4 inch angles, 55 pounds per yard, give 11 square inches. These are to be riveted to the ends of the stringers and should extend the full depth between the flange angles top and bottom, fillers being placed between them and the web of the floor-beam of the thickness of the heavier flange-angles, as they both strengthen the web and are necessary for a full and close fit of end angles. Intermediate stiffeners, 4×4 inch, 35 pounds, will also be riveted to the web of the floor-beam. If the ends of these cannot be easily bent to fit the flange angles, fillers must also be used between them and the web.

Now to determine the number of rivets necessary to transfer the locomotive reaction at the stringer ends, we have, as found in equation (551), Figs. 350 and 351, $R = 48,375 +$ dead load $10,000 = 58,375$ pounds, and allowing 3000 pounds for resistance of web at rivet bearing, $\frac{58375}{3000} = 20$ rivets, or ten rivets in each end angle-iron, as seen in Fig. 352. For the end angles of the floor-beam we have, first, the reaction above, and, second, the one half weight of the floor-beam $= 58,375 + 200 \times 10 = 60,375$ pounds, or metal area $\frac{60375}{4000} = 15.09$ square inches. Two 6×4 inch angles, 75 pounds per yard, are sufficient. The number of rivets required for these would be $\frac{60375}{3000} = 20.1$ rivets, as seen in Fig. 352. Intermediate stiffeners, two 4×4 inch, 35 pounds.

The bill of material for floor-beam as calculated is as follows:

1 web-plate $36 \times \frac{3}{8}$ in. $\times$ 20 ft., 15 pounds per square foot	900 lbs.
2 upper-flange angles, each 5×4 in. $\times$ 20 ft., 55 pounds per yard....	733 "
1 cover-plate $12 \times \frac{3}{4}$ in. $\times$ 20 ft., 30 pounds per square foot..........	600 "
2 bottom-flange angles, each 5×4 in. $\times$ 20 ft., 55 pounds per yard ..	733 "
1 " " cover-plate $12 \times \frac{3}{4}$ in. $\times$ 20 ft., 30 lbs. per square foot	600 "
4 end angle-irons, each 6×4 in. $\times$ 8 ft., 75 pounds per yard.........	300 "
4 intermediate angles 4×4 in. $\times$ 3 ft., 35 pounds per yard..........	140 "
4 filling-plates, ends, $6 \times \frac{5}{8}$ in. $\times 2\frac{1}{4}$ ft., 25 pounds per square foot ...	120 "
8 " $4 \times \frac{5}{8}$ in. $\times 2\frac{1}{4}$ ft., 25 pounds per square foot.........	154 "
2 heads for $\frac{3}{4}$-in. rivets weigh $\frac{1}{4}$ lb., $\frac{7}{8}$-in. rivets $\frac{3}{8}$ lb.; say	200 "
Total weight of floor-beam....................................	4480 "

This is $\frac{4480}{20} = 224$ pounds per foot, which is 24 pounds per foot more than was assumed. Strictly speaking, a recalculation should be made, with a weight of, say, 230 pounds per foot of length for the beam. All parts in the above bill of iron, except in the web-plate, are very heavy per yard of length, as about the heaviest type of engine was used, and a long floor-beam for a single track, carrying long stringers. For instance, in the strain-sheet for the floor-beams of a bridge 415′ 11⅝″ long, length of floor-beam 20 feet, carrying stringers 24′ 11¼″ long, designed by the Keystone Bridge Company, the bill of material for the floor-beam is given as follows:

2 upper-flange angles 3½ × 5 in. × 20 ft., 20.3 pounds per foot.......	812 lbs.
2 bottom-flange angles 3½ × 5 in. × 20 ft., 18.4 pounds per foot......	736 "
Web 36 × $\frac{7}{16}$ in. × 20 ft., 18 pounds per square foot...............	1080 "
(No cover-plates were used; and no allowance for stiffeners, end angles, fillers, rivets, etc., appears. These, except the cover-plates, will be added of the ordinary dimensions.)	
4 end angles 5×4 in., 36 pounds per yard, each 3 feet long.........	144 "
8 stiffening angles 3 × 2½ in. × 3 ft., 16 pounds per yard............	128 "
4 fillers 3 × ¾ in. × 2¼ ft., 30 pounds per square foot................	70 "
8 " 5 × ¾ in. × 2¼ ft., 30 " " " "	240 "
Allow for rivets...	150 "
Total weight of a floor-beam..................................	3360 "

Weight per foot of length $\frac{3360}{20} = 168$ pounds. If in the above two upper-flange angles 5 × 4 inches, 36 pounds per yard, and a cover-plate to correspond had been used, the same strength could have been secured. The beam is strong enough for the type of engine used in Fig. 353 (*b*), followed by a train load of 2240 pounds per foot of length of stringers. This and the other type of engine used in Fig. 352, followed by a train load of 3000 pounds per foot, may be taken as the extremes of light and heavy rolling loads. Where cover-plates can be dispensed with it is to be preferred, especially in the bottom flange, as it saves labor of punching holes, loss of material resulting thereby, and is simpler and more solid,—all of which considerations are of some moment. Cover-plates are not necessary when angle-irons not over ¾ inch thick will answer. Angle-irons are generally spoken of by their weight per yard or per foot of length. Their area in square inches is equal to one tenth of their weight per yard. For any definite length of legs angle-irons vary in weight between certain limits. For instance, a 5 × 3½ inch angle varies from 30.5 to 58.1 pounds per yard, and

contains in cross-section from 3.05 to 5.81 square inches. Angles are formed of almost any size from $\frac{3}{4} \times \frac{3}{4}$ inch, weighing from 1.72 to 2.46 pounds per yard, from $\frac{1}{8}$ to $\frac{3}{16}$ inch in thickness, to angles 6×6 inches, weighing from 57.5 to 97.3 pounds per yard, and from $\frac{1}{2}$ to $\frac{7}{8}$ inch thick; and between these limits almost any combinations of lengths of arms, weight per yard, thickness and area of cross-section can be obtained. As mentioned elsewhere, it is better to use those dimensions and weights that are made by the iron and steel companies rather than insisting upon unusual thicknesses and weights.

964. To close this part of the subject we will determine the dimensions of the stringers resting on the floor-beam considered, and the locomotive weights used in the last paragraph, principally to show how the reactions at the end of the stringers were found, and the principles upon which the length and thickness of the cover-plates are determined when two or more layers are required.

The stringer Fig. 350 will be taken at 25 ft. long, 2.5 deep, weighing 200 lbs. per lineal foot. The type of engine Fig. 351, paragraph 963, will be used. Flange stresses will be found at points 3, 6, and 9 ft. from the end and at the centre. The position of the rolling load for maximum bending moment, by the principles established in paragraph 883, will be as shown in Fig. 353 annexed, and the flange stresses at the above points will be as follows: It will be

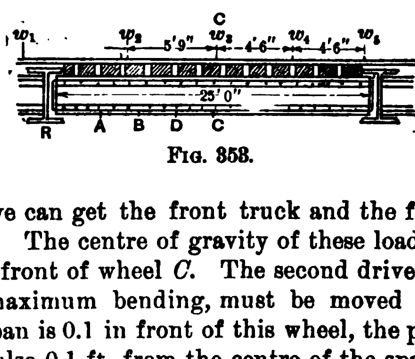


FIG. 353.

observed that we can get the front truck and the four drivers in a length of 25 ft. The centre of gravity of these loads will be found to be 0.2 ft. in front of wheel C. The second driver from the end, the load for maximum bending, must be moved then until the centre of the span is 0.1 in front of this wheel, the point of greatest bending being also 0.1 ft. from the centre of the span on the other side. For this position of the load we have

$$R = \frac{24000(18.15 + 12.40 + 7.90 + 3.40)}{25} = 40{,}176 \text{ lbs.},$$

divided between two stringers·

$$R = \frac{40176}{2} = 20{,}088 \text{ lbs.}$$

Reaction due to dead load, $400 \times 25 \div 2 = 5000$ lbs.

Hence final moments M_4 between R and A, M_3 between A and B, M_2 between B and D, and M_1 between D and C, and the values of the flange stresses P_4, P_3, P_2, P_1 in the same sections, are:

$$M_4 = 20{,}088 \times 3 + 5000 \times 3.0 - 400 \times 3 \times \tfrac{3}{2} = 73{,}464 = P_4 \times 2.25 \text{ feet};$$
$$\therefore\ P_4 = 36{,}655 \text{ lbs.}$$

$$M_3 = 20{,}088 \times 6 + 5000 \times 6.0 - 400 \times 6 \times \tfrac{6}{2} = 143{,}328 = P_3 \times 2.25 \text{ feet};$$
$$\therefore\ P_3 = 63{,}700 \text{ lbs.}$$

$$M_2 = 20{,}088 \times 9 + 5000 \times 9.0 - \tfrac{24000}{2} \times 2.15 - 400 \times 9 \times \tfrac{9}{2} = 183{,}792 = P_2 \times 2.25 \text{ feet}, P_2 = 81{,}685 \text{ lbs.}$$

$$M_1 = 20{,}088 \times 12.5 + 5000 \times 12.5 - \tfrac{24000}{2} \times 5.65 - 400 \times 12.5 \times \tfrac{12.5}{2} = 217{,}050 = P_1 \times 2.25 \text{ feet } P_1 = 96{,}467.$$

Required upper flange area at $C = \frac{96467}{7000} = 13.78$ sq. in.;
" lower " " " $C = \frac{96467}{8000} = 12.08$ sq. in.

Upper flange, 2 $\angle s$ 5 × 4 in., 47 lbs. per yard; area, 9.4 sq. in.
1 cover-plate, 12 × $\frac{3}{8}$ .... 4.5 " "

13.9 sq. in.

Bottom flange, 2 $\angle s$ 5×4 in., 62 lbs. per yard, area = 12.4 " "

Greatest reaction at end of stringer, 39,190 lbs.; $\frac{39190}{4000} = 9.8$ sq. in.; web, $27 \times \frac{3}{8} = 10.12$ sq. in.; end angles, $\frac{39190}{4000} = 9.8$ sq. in.; 2 $\angle s$ 5 × 4 in., 55 lbs. = 11 sq. in.; 4 fillers, 4 × $\frac{11}{16}$ in.; 8 $\angle s$ 4 × 3 in., 20.9 lbs. No fillers needed, as they can be bent at ends to fit.

The principle upon which the greatest shear in any section or panel can be obtained is seen in equa. (524)—that for the greatest shear the load that it contains multiplied by the total number of panels in the span must be equal to or nearly equal to the entire load on the span; in symbols, $n(w_1 + w_2 + \ldots) = (w_1 + w_2 + w_3 + \ldots w_n$. This condition will be nearer fulfilled by placing the front driver at or near the point of support at R (see Fig. 353). We then have the four drivers and one wheel of the tender on the

stringer. Hence

$$R_1 = \frac{24000(19.25 + 14.75 + 10.25)}{2 \times 25} + \frac{24000}{2} + \frac{15000 \times 3.17}{2 \times 25}$$
$$= 34{,}190 \text{ lbs.},$$

to which add 5000 lbs. for dead load. $R = R_1 + 5000 = 39{,}190$ lbs., as already used on page 1100. The panels or sections in this case may be from $2\frac{1}{2}$ to 4 ft. long. As the load rests on top of the stringer, the greatest load that can rest on any of the 3-foot sections into which we have divided the stringer will be $\frac{24000}{2} = 12{,}000$ lbs. The flange strain, as seen above, varies from section to section. The increments are as follows:

Flange strain P_4	betw. R and A	$= 36{,}655$ lbs.	$= 36{,}655$ lbs.
" " $P_3 - P_4$	" A " B	$= 63{,}700 - 36{,}655$	$= 27{,}045$ "
" " $P_2 - P_3$	" B " D	$= 80{,}352 - 63{,}700$	$= 16{,}652$ "
" " $P_1 - P_2$	" D " C	$= 96{,}467 - 80{,}352$	$= 16{,}115$ "

The flange strain is horizontal, and distributed over the section; the load is vertical, and may be taken as uniformly distributed over the same distance, which is generally taken equal to the depth of the beam, which would be 2.5 ft.; but as this is somewhat arbitrary, the distance has been taken in this example at 3 ft. The shear on the rivets between these points will be the resultant of these forces.

$$\left.\begin{array}{llll} s_4 \text{ betw. } R \text{ and } A = \sqrt{(36655)^2 + (12000)^2} = 38{,}569 \text{ lbs.;} \\ s_3 \quad \text{"} \quad A \quad \text{"} \quad B = \sqrt{(27045)^2 + (12000)^2} = 29{,}589 \quad \text{"} \\ s_2 \quad \text{"} \quad B \quad \text{"} \quad D = \sqrt{(16652)^2 + (12000)^2} = 20{,}525 \quad \text{"} \\ s_1 \quad \text{"} \quad D \quad \text{"} \quad C = \sqrt{(16115)^2 + (12000)^2} = 20{,}092 \quad \text{"} \end{array}\right\} \quad (1)$$

These amounts divided by 3000 will give the number of rivets in pounds.

In the following example a Howe truss deck-span will be used, having an odd number of panels.

965. The rolling load will consist of two consolidation engines,

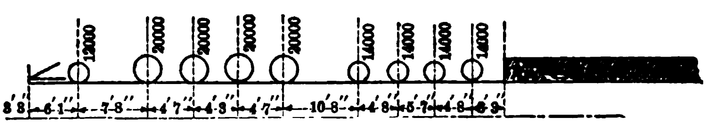


FIG. 358 (b).

followed by a uniform load of 2240 pounds per linear foot. The above diagram shows the weights on each pair of wheels, with the distances between them. In this example the equivalent uniform load will be determined and used; hence it will only be necessary to find the bending moment at the centre of the span from the actual wheel concentrations.

The following is a skeleton sketch of the truss (see Fig. 354): Length of span = 185.0 ft.; 9 panels, each 20.55 ft.; depth of

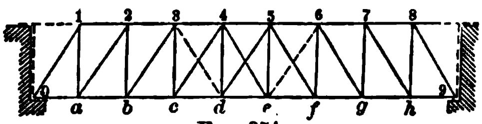


FIG. 354.

truss = 27 ft. It will be found that when $w_{1_1} = w_{n_1}$ is just to the left of the point d that the condition for a maximum bending is obtained. w_{1_1} will then be w_{n_1-1}. There will be about 67 ft. of rolling uniform load on the bridge = 2240 × 67 = 150,080 pounds. Only one half of this, as well as the wheel concentrations given above, will be carried by each of the two trusses: $w_1 + w_2 + w_3 + \ldots + w_{n_1} = 200{,}000$ lbs.; $w_1 + w_2 + w_3 + \ldots + w_n = 296{,}000 + 150{,}080 = 446{,}080$. The locomotive table used in this example will be found in the following paragraph. For the centre panels $l_1 = 4$, and as $l = 9$, using panels instead of distances, the condition of loading for a maximum (see eq. (528)) will be fulfilled very nearly, viz.,

$$\frac{l_1}{l} = \frac{w_1 + w_2 + w_3 + \ldots + w_{n_1}}{w_1 + w_2 + w_3 + \ldots + w_n} = \frac{4}{9} = \frac{200000}{446080} = \frac{4}{8.92},$$

nearly correct. Small changes made by moving the load a short distance either forward or backward might satisfy exactly the above relation. This could only be found by repeated trials, and that giving the greatest bending moment should be adopted. The above is near enough for the present purpose. Substituting in eq. (530), paragraph 923, as was done in finding moments eq. (1) and following, x in this case is $\frac{67}{2} = 33.5$, w_n in the second term is the uniform load = 150,080 pounds, and using the locomotive table mentioned,

$$M = \frac{4}{9}\left[\frac{16336338}{2} + \left(\frac{2 \times 296000}{2} + \frac{150080}{2}\right) \times 33.5\right] - \frac{7808639}{2}$$
$$= 5{,}250{,}481 \text{ ft.-lbs.}$$

We can now find the equivalent uniform load per linear foot, $\frac{1}{8}wl^2 = \frac{1}{8}w \times (185)^2 = 5,250,481$; hence $w = 1228$ lbs., the equivalent uniform load for one truss. We have now the following data: Span $= 185$ ft.; panel length $= 20.55$ ft.; depth of truss, 27 ft.; number of panels $= 9$; dead load, 760 lbs. per linear foot; live load, 1228 lbs. per linear ft. If we were to use upper and lower chord panel weights, as in previous cases, the upper-chord fixed load would be taken at 525 and lower-chord at 235 lbs. per foot; but following the method usually adopted in case of uniform loads, the entire load is assumed as concentrated in the upper or loaded chord. Dead-load panel weight $w = 15,618$ lbs.; live-load panel weight $= 1228 \times 20.55 = 25,236$ lbs. $= w_1$; the length of the diagonal $= 33.93$ ft.; sec of angle $01a = \frac{33.93}{27} = 1.257$; tan angle $01a = \frac{20.55}{27} = 0.761$. As the greatest chord stresses occur when the entire space is covered with the rolling load, the formula for horizontal stresses remains the same: $H_n = \frac{(w + w_1)l}{k}, \frac{n(N - n)}{2N}$. (See Rankine's C. E.; see also paragraph 931.) $(w + w_1) = 40,854$ lbs., $l = 185$, $N = 9$, $k = 27$, $n = 1, 2, 3, 4$ successively.

Tension in $0a$ = comp. in $1, 2 = \frac{40,854 \times 185}{27} \frac{(9 - 1)}{18} = 124,411$ lbs.

"	" ab =	"	" $2, 3$ =	$15,551.4 \times 2(9 - 2)$	$= 217,720$ "
"	" bc =	"	" $3, 4$ =	$15,551.4 \times 3(9 - 3)$	$= 279,925$ "
"	" cd =	"	" $4, 5$ =	$15,551.4 \times 4(9 - 4)$	$= 311,028$ "
"	" de and ef				$= 311,028$ "

The calculation is simplified by observing that $\frac{40854 \times 185}{27 \times 18}$ is constant and equal to 15,551.4; the only variable faction is $n(N-n)$. As there is no centre vertical, it is evident that when the span is entirely covered by the rolling load the diagonals $4e$ and $5d$ cannot be under stress at all, as the load on one side of the centre tends to produce in one of them the same kind and amount of stress that the load on the other side of the centre tends to produce in the other; therefore neither of them are under stress. In other words, they are both counterbraces, and the tension in the panel *de* is therefore the same as in the panel *cd*.

To Determine the Stresses in the Verticals and Diagonals.—As the diagonals are compression members and the load is on the top chord, and the verticals tension members, the vertical $4d$ receives its entire stress from the moving load, and this will be a maximum

when the rolling load reaches the point 5; should it extend to 4 the load would be transmitted to $4e$, and a stress would be developed in $4e$. This would act counter to the stress in $5d$ due to the load at 5, and thereby reduce the amount of load that would otherwise reach the point d. Assuming the rolling load to come in from the right and reach the point 6, the rolling-load shear in $5f$ would be $3 \times 25{,}236 \times 2 \div 9 = 16{,}824$ lbs.; dead-load shear in $5f$ would be one panel weight $= 15{,}618$ lbs. As these shears would tend to produce stresses of opposite kind in $5f$, the resultant would be the difference $= 16{,}824 - 15{,}618 = 1206$ lbs.; and as that due to the live load is the greatest, a counterbrace would be needed, shown by the dotted line $6e$; the stress on it would be $1206 \times 1.257 = 1516$ pounds. No other counterbrace will be needed between this one and the end of the span. When the rolling load reaches 5, the shear in $5d$ will be $4 \times 25{,}236 \times 2\frac{1}{2} \div 9 = 28{,}040$ pounds; and as there is no dead-load shear on $5d$, the shear will be transmitted to $4d$; hence tension on $4d = 28{,}040$ pounds. The stress in $5d$ is $28{,}040 \times 1.257 = 35{,}236$ pounds compression, and the same on $4e$ when the load comes in from the left; $3d$, with same compression as $6e = 1516$ pounds, is a counterbrace. When the load reaches the point 4, or passes the centre, both dead and live load produce the same kind of stress in $4d$. The general formula for shears in the diagonals remains the same as if the load was on the bottom chord; but the stresses in the verticals will be diminished by one panel weight $(w + w_1) = 40{,}854$. This requires some little trouble. Making the substitutions, then, in the equation for shears $V_n = w\left(\frac{N+1}{2} - n\right) + w_1\frac{(N-n)(N-n+1)}{2N}$ (see Rankine's C. E., also paragraph 932), we can write the following stresses:

$$\text{Comp. in } 4c = 15{,}618\left(\frac{9+1}{2} - 4\right) + 25{,}236\frac{(9-4)(9-4+1)}{18} \times 1.257$$
$$= 57{,}678 \times 1.257 = -\ 72{,}501 \text{ lbs.}$$

Stress in $3c = +\ 57{,}678$ lbs. tension.

$$\text{Comp. in } 3b = 15{,}618\left(\frac{9+1}{2} - 3\right) + 25{,}236\frac{(9-3)(9-3+1)}{18} \times 1.257$$
$$= 90{,}120 \times 1.257 = -\ 113{,}281 \text{ lbs.}$$

Tension in $2b = +\ 90{,}120$ lbs.

$$\text{Comp. in } 2a = 15{,}618\left(\frac{9+1}{2} - 2\right) + 25{,}236\frac{(9-2)(9-2+1)}{18} \times 1.257$$
$$= 125{,}366 \times 1.257 = -\ 157{,}585 \text{ lbs.}$$

Tension in $1a = +\ 125{,}366$ lbs.

$$\text{Comp. in } 01 = 15{,}618\left(\frac{9+1}{2} - 1\right) + 25{,}236\frac{(9-1)(9-1+1)}{18} \times 1.257$$

$$= 163{,}416 \times 1.257 = -\ 205{,}415 \text{ lbs.}$$

This can be checked as follows: Take any vertical, as $1a$; this receives from the dead load 3 panel weights and from the live load $\frac{7 \times 4}{9}$ panel weights; it gets nothing from either load at the point 1. Therefore the stress in $1a$ is $15{,}618 \times 3 + 25{,}236 \times \frac{28}{9} = 125{,}366$ pounds.

01 receives half the load on the entire bridge. $(15{,}618 + 25{,}236) \times 4 = 163{,}416$ lbs.; the stress $= 163{,}416 \times 1.257 = 205{,}414$.

From the foregoing cases it is evident that the method of uniformly distributed loads does not give accurate results, and could not be relied on for very long spans. Approximate results are obtained with comparatively little labor, but even then the question of the equivalent uniform load has to be settled, as every different span and system of loading would have a different equivalent uniform load that only gives a correct result at one point in the span, and it is necessary to find the maximum chord stress at the centre from the actual loading in practice in order to deduce the equivalent uniform load. Concentrating the dead load entirely in one chord, as has been seen, makes another material error. With good tables and a little practice in handling the general formulæ for the wheel concentrations, the additional labor required is unimportant,

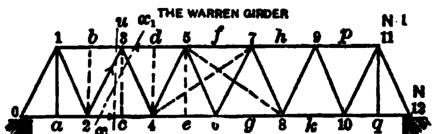


Fig. 355.

and more than counterbalanced by the satisfaction of at least securing results that are in some degree of accord with the actual practical conditions. The writer believes that he has fully explained the principles applicable to a single system of vertical and diagonal bracing, whether compression verticals or compression diagonals, or the reverse. An example will be given of a truss in which all of the web members are inclined; this is commonly known as the

Warren or Zigzag Girder. The skeleton sketch above, Fig. 355, shows this design.

966. The design Fig. 355 shows a series of verticals by dotted lines. These are not essential members of the truss, and are only used when in long spans the panel lengths, such as *bd* or 3, 5, are of such a length that they would deflect under their own weight or any load upon them, and when used they would only carry one panel weight of the length 2*c* or 3*d*. They will not be considered at all in this example. The upper chord is under compression, lower chord under tension. In the web members the end diagonals and all members parallel to them on the same side of the centre are compression members, the others tension members. In other words, under maximum stress all web members inclining upwards to the centre are compression members, as 2, 3, and those inclining downwards towards the centre are tension members, as 3, 4. Therefore the stresses on the two web members meeting at a point are of the opposite kind. In this type of bridge, under some conditions of the moving load, as will be seen later, some of the members that are under compression from the dead load may be thrown into a state of tension by the rolling load, and they must therefore be formed and connected to resist tension and compression both. They are then said to be counterbraced. If this is not desired, members such as 5, 8 and 7, 4 must be introduced. These are called counterbraces. Some builders prefer them, as they do not necessitate any member to pass from a state of compression to one of tension, which may affect injuriously the strength of the material, and it is more convenient to form and connect a member so that it will always be under the same kind of strain. Assuming the following dimensions and other data—

Length of span $= 120$ ft.; sec angle $01a$ or $a12 = \dfrac{22.36}{20} = 1.12$;

" " panel $= 20$ ft.; tang angle $01a$ or $a12 = \dfrac{10}{20} = 0.50$;

" " diagonal $= 22.36$ ft.;

Depth of truss $= 20.0$ ft.

Assuming that the weight of the two trusses with the lateral bracing is 500 pounds per linear foot or 250 pounds for one truss, one half concentrated at each chord, the weight of the cross-ties,

guard-rails, rails, etc., 200 pounds per foot for each truss, and the stringers, etc., 125 pounds for each truss per foot—

Upper-chord panel weight 125×20	$= 2500$ lbs.
Lower-chord panel weight $(125 + 200 + 125) \times 20$	$= 9000$ lbs.
	11,500 lbs.

The dead-load stresses in the web members are as follows:

Tension in 5, 6	$=$	$\frac{1}{2}(9000) \times 1.12,$	$= 5040$ lbs.
Compression in 5, 4	$=$	$[\frac{1}{2}(9000) + 2500] \times 1.12$	$= 7840$ lbs.
Tension in 4, 3	$=$	$(7000 + 9000) \times 1.12$	$= 17,920$ lbs.
Compression in 3, 2	$=$	$(16,000 + 2500) \times 1.12$	$= 20,720$ lbs.
Tension in 1, 2	$=$	$(18,500 + 9000) \times 1.12$	$= 30,800$ lbs.
Compression in 0, 1	$=$	$(27,500 + 2500) \times 1.12$	$= 33,600$ lbs.

The above can be checked, as it is evident that the shear in 0, 1 will be three panels upper and two and one half panels lower-chord load, $2500 \times 3 + 9000 \times 2\frac{1}{2} = 30,000$ pounds, as the shear in any member is the weight between it and the centre under a uniform load, and $30,000 \times 1.12 = 33,600$ pounds stress in 0, 1.

Recollecting that the chord stresses in any panel are the continued sum of the horizontal components of the stresses in the diagonals between it and the end of the span, or the shears by the tangent of the angle of inclination ($= 0.50$), the chord stresses are easily written.

Lower chord 0, 2:	Tension	$= + 30,000$	$\times 0.5 = + 15,000$ lbs.
Upper chord 1, 3:	Compression	$= - (30,000 + 27,500)$	$\times 0.5 = - 28,750$ lbs.
Lower chord 2, 4:	Tension	$= + (57,500 + 18,500)$	$\times 0.5 = + 38,000$ lbs.
Upper chord 3, 5:	Compression	$= - (76,000 + 16,000)$	$\times 0.5 = - 46,000$ lbs.
Lower chord 4, 6:	Tension	$= + (92,000 + 7,000)$	$\times 0.5 = + 49,500$ lbs.
Upper chord 5, 7:	Compression	$= - (99,000 + 4,509)$	$\times 0.5 = - 51,750$ lbs.

This can be checked as follows:

Dead load per linear foot $= 11500/20 = 575$ pounds;

Centre bend. mom. $= \frac{1}{8}w'l^2 = \frac{1}{8} \times 575 \times (120)^2 = 1,035,000$ ft.-lbs.

Upper-chord stress 5, 7 $= 1035000/20 = - 51,750$ pounds.

To find the web and chord stresses due to the rolling load, which will consist of two engines and tenders followed by a uniform load of 2240 pounds per lineal foot, construct the following table of moments, using two consolidation engines coupled together, as

shown in Fig. 353(*b*), paragraph 965. Column 1 gives wheel number; column 2, the continued sum of the weights on a pair of wheels multiplied by the distances between the centres of gravity of the wheels; column 3, the numerical products of numbers in column 2; column 4, the continued sum of the numbers in column 3. The locomotives here used would be considered as comparatively of a light weight. Those used in Table LXXV were about the heaviest type of engine now in general use. There are, however, some types still heavier. But whatever the weight or construction of the locomotive used, a similar table can be readily constructed.

TABLE LXXVII.

1.	2.	3.	4.
w_1	12,000 × 7.666	92,000	92,000
w_2	32,000 × 4.583	146,660	238,660
w_3	52,000 × 4.25	221,000	459,660
w_4	72,000 × 4.583	329,980	789,640
w_5	92,000 × 10.666	981,280	1,770,920
w_6	106,000 × 4.666	494,600	2,265,520
w_7	120,000 × 5.583	669,960	2,935,480
w_8	134,000 × 4.666	625,250	3,560,730
w_9	148,000 × 9.333	1,381,290	4,942,000
w_{10}	160,000 × 7.666	1,226,560	6,168,580
w_{11}	180,000 × 4.583	824,940	6,993,520
w_{12}	200,000 × 4.25	850,000	7,843,520
w_{13}	220,000 × 4.583	1,008,260	8,851,780
w_{14}	240,000 × 10.666	2,559,840	11,411,620
w_{15}	254,000 × 4.666	1,185,170	12,596,790
w_{16}	268,000 × 5.583	1,496,250	14,093,040
w_{17}	282,000 × 4.666	1,315,820	15,408,860
w_{18}	296,000 × 3.25	962,000	16,370,860

It will aid in the use of the above table to take the continued sum of the distances between the wheels which will give the distance of any wheel from the front one w_1, also the length of the moving uniform load on the bridge at any time. (A similar table would be useful in connection with other examples.)

TABLE LXXVIII.

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
0	7.666	12.249	16.449	21.082	31.748	36.414	41.997	45.663
w_{10}	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	w_{16}	w_{17}	w_{18}
55.996	63.662	68.245	72.495	77.078	87.744	92.410	97.993	102.659,

102.659 + 3.25 = 105.909 feet to the front end of the uniform load.

When w_1 is $120 - 105.909 = 14.091$ feet from one end of this span, w_2 will be $20.0 - 14.091 - 7.666 = -1.757$ feet to the right of the panel point 2, and the uniform load will not yet be on the bridge; but if the load moves forward until w_2 rests at the panel point 2, which is in accord with the requirement that one of the loads shall be at a panel point, there will be 1.757 feet of uniform load on the bridge, as the head of the uniform load is $105,909 - 7.666 = 98,243$ feet from point 2, and the distance from 2 to the end of the span is five panels = 100 feet, $100.0 - 98,243 = 1.757$, and $x = 0.879$, and similarly for any other position of the moving load. If w_3 is at 2, $1.757 + 4.583 = 6.34$ feet of uniform load, and $x = 3.17$; w_4 at 2, $6.34 + 4.25 = 10.59$ feet of uniform load, $x = 5,3$. w_1 will then be only $120 - (105.909 + 10.59) = 3.501$ feet from the end of the bridge; and if w_5 is at 2, w_1 will have passed off the bridge. But after the uniform load comes on the bridge it may happen that it will not be necessary to place a wheel at a panel point to fulfil the condition for a maximum shear. With any wheel at any point it is easy to take at once the number of loads on the bridge, and to determine the length of the uniform load. To illustrate. Place w_3 at panel point 6; the distance from this point to N is three panels = 60 feet. Add to this the number under wheel w_3, $60 + 12.249 = 72.249$ feet. This is the distance from w_1 to the rear end of the span at point 12. The next less number to this is found under wheel w_{12} (which will be the last load on the span $= w_n$) $= 68.245$, and $72.249 - 68.245 = 4.004$ feet $= x$. With w_3 at 4, $80 + 12.249 = 92.249$. Next less number is 87.744 under wheel w_{15}: $92.249 - 87.744 = 4.505 = x$. w_3 at 2, $100 + 12.249 = 112.249$ feet. As this exceeds all of the distances given, the uniform load has entered the bridge. $112.249 - 105.909 = 6.34$ feet is the length of the uniform load on the bridge, which is now w_u, and $\frac{6.34}{2} = 3.17 = x$. In the above $w_{n_1-1} = w_2$. The last result, 6.34 feet, is the same as that found by another process above.

This tabulation can be applied to any length of span or length of panel and system of wheel concentrations. The above results can be checked by placing the wheels at the points indicated and summing up the resulting distances between the wheels and the end of the span. (This method was used in the preceding examples, it is tedious.) Let the load now come in from the right and rest successively at the panel points 10, 8, etc. The condition of maximum shear to the left of these points is $n(w_1 + w_2 \ldots) =$

$w_1 + w_2 + w_3 + \ldots w_n$, with w_2 at 10; $w_1 = w_{n_1-1}$; $w_n = w_4$; and as there are six panels, $n = 6$. Hence $6 \times 12{,}000 = 72{,}000$ pounds; $w_1 + w_2 + w_3 + \ldots w_n = 92{,}000$ pounds; but by assuming that $3333\frac{1}{3}$ pounds of w_2 acts to the left of point 10,—$6(12{,}000 + 3333\frac{1}{3}) = 92{,}000$. Hence w_2 at 10 gives maximum shear. Substituting in eq. (548),

$$S = \frac{1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \ldots + (w_1 + w_2 + \ldots w_n)x] - \frac{1}{p}[w_1a + (w_1 + w_2)b + \ldots + (w_1 + w_2 + \ldots w_{n_1-1})y].$$

For maximum shear in 9, 10; 8, 9: $l = 120$; $p = 20$; $x = 20 + 7.666 - 21.082 = 6.584$.

Shear in 9, 10; 8, 9:

$$= \tfrac{1}{120}[\tfrac{789640}{2} + \tfrac{92000}{2} \times 6.584] - \tfrac{1}{20}\cdot\tfrac{92000}{2} = 3514 \text{ lbs.} \quad (1)$$

The first part of the positive term runs to $w_{n-1} = w_4$, as $w_n = w_5$, and is found in column 4 of table, opposite w_4; the second part, involving x, is found in column 2, opposite w_5. The negative term contains only $w_1 = w_{n_1-1}$; $y = a = 7.666$, as $w_2 = w_{n_1}$ is at the panel point, and is found in column 4, opposite wheel w_1.

For maximum shear in 8, 7; 7, 6: $w_2 = w_{n_1}$ at 8; $w_1 = w_{n_1-1}$; $x = 40 + 7.666 - 46.663 = 1.003$. Hence $w_n = 9$; $w_{n-1} = 8$; $6(12{,}000 + 12{,}666\frac{2}{3}) = w_1 + w_2 + w_3 + \ldots w_n = 148{,}000$ pounds.

Shear in 8, 7; 7, 6

$$= \tfrac{1}{120}[\tfrac{3560730}{2} + \tfrac{148000}{2} \times 1.003] - \tfrac{1}{20}\cdot\tfrac{92000}{2} = 13{,}155 \text{ lbs.} \quad (2)$$

Maximum shear in 6, 5; 5, 4: $w_3 = w_{n_1}$ at 6; $w_2 = w_{n-1}$; $x = 60 + 12.249 - 68.245 = 4.004$ feet. Hence $w_n = w_{12}$; $w_{n-1} = w_{11}$; $6(12{,}000 + 20{,}000 + 1333\frac{1}{3}) = w_1 + w_2 + w_3 + \ldots w_n = 200{,}000$ pounds.

Shear in 6, 5; 5, 4

$$= \tfrac{1}{120}[\tfrac{6993620}{2} + \tfrac{200000}{2} \times 4.004] - \tfrac{1}{20}\cdot\tfrac{238660}{2} = 26{,}510 \text{ lbs.} \quad (3)$$

Maximum shear in 4, 3; 3, 2: $w_3 = w_{n_1}$ at 4; $w_2 = w_{n_1-1}$; $x = 80 + 12.249 - 87.744 = 4.505$ feet. Hence $w_{15} = w_n$; $w_{14} = w_{n-1}$; $6(32{,}000 + 10{,}333\frac{1}{3}) = 254{,}000 = w_1 + w_2 + \ldots w_n$.

Shear in 4, 3; 2, 2

$$= \tfrac{1}{120}\left[\tfrac{11411620}{2} + \tfrac{264000}{2} \times 4.505\right] - \tfrac{1}{20} \cdot \tfrac{238660}{2} = 46{,}356 \text{ lbs.} \quad (4)$$

For maximum shear in 1, 2; 0, 1: w_3 at $2 = w_{n_1}$; $w_{n_1-1} = w_2$; $x_1 = 100 + 12.249 - 105.909 = 6.340$ feet. Hence there will be 6.34 feet of uniform load on the bridge $= 2240 \times 6.34 = 14{,}202$ pounds; $6(12{,}000 + 20{,}000 + 20{,}000) = 312{,}000$; but $w_1 + w_2 + w_3 + \ldots + w_{n-1} + w_n) = 296{,}000 + 14{,}202 = 310{,}202$.

If we place w_4 at 3, $x_1 = 100 + 16.449 - 105.909 = 10.54$ feet. Hence $2240 \times 10.54 = 23{,}610$ pounds; $6(52{,}000 + 1268\tfrac{1}{2}) = 296{,}000 + 23{,}610 = 319{,}610$. We will therefore take w_n at 3 for maximum shear in 1, 2; 0, 1:

$$x = \frac{10.54}{2} = 5.27 \text{ feet}; \quad w_n = 23{,}610; \quad w_{n-1} = w_{18}; \quad w_{n_1} = w_4; \quad w_{n_1-1} = w_3.$$

Shear in 1, 2; 0, 1

$$= \frac{1}{120}\left[\frac{16370860}{2} + \frac{2 \times 296000 + 23610}{2} \times 5.27\right] - \frac{1}{20} \cdot \frac{459660}{2} = 70{,}108 \text{ lbs.} \quad (2)$$

In this last case the uniform load is on the bridge, and, following the rule explained in the preceding example, we write for the first part of the positive term the sum of all the moments found in column 4 opposite wheel w_{18}, and for the second part twice the sum of all the wheel concentrations found in column 2 opposite wheel w_{18}, increased by the uniform load, this sum being multiplied by x and the whole then divided by 2. So long, then, as w_4 rests at a panel point, the first quantity in the above expression, $\tfrac{16370860}{2}$ would remain constant in the second term, the uniform load would increase, and also the value of x. The negative term would also be constant. If it became necessary to place $w_5 = w_{n_1}$ at a panel point, the negative term would become $-\tfrac{1}{20} \cdot \tfrac{189640}{2}$, and be constant until $w_6 = w_{n_1}$. It is usual to give the load as carried by the two trusses. The table is calculated on this basis; therefore for stresses on members of one of the trusses all quantities are divided by 2.

CHORD STRESSES.

It was stated in par. 923 that eq. (530) was not applicable to the chord stresses of the loaded chord when all of the web members were inclined, but did apply to the unloaded chord, the upper chord in this case. The reason is easily understood. Eq. (530) gives the bending moment about a panel point in the loaded chord; and when the web is composed of a single system of one diagonal and one vertical meeting at a point, the lever-arm and the resultant load are the same for any panel point, whether in the top or bottom chord, and the stress, which is equal to the moment divided by the depth of the truss, is necessarily the same in the upper and lower panels included between two adjacent diagonals. But when the web members are all inclined the panel points in the two chords are not in the same vertical line, and there are no two panels which have the same stress, as a different pair of diagonals meet at every point in both chords; therefore the position of the rolling load for maximum chord stress is different for each panel. Examining the skeleton sketch (Fig. 355) to determine the stress in panel 3, 5, we take moments about the lower-chord panel point 4; this reduces the moments of the internal stresses or resistances of the members of the bridge to the moment of the upper-chord panel 3, 5, the lever-arms of the stresses in the bottom chords and web members being zero, as they pass through the axis. Equating this moment of resistance to the external bending moment, there results one equation with one unknown quantity. Any other position of the axis would result in more unknown quantities than the equations obtained, hence unknown quantities are indeterminate. We cannot, then, with the axis at 4 determine the stress in the bottom chord, or with the axis at any other point in a vertical line through 4. The lever-arms for the top and bottom chord cannot be the same; but if we take the axis through point 5, the only acting moment would be the stress in the bottom-chord panel 4, 6. Therefore, to make eq. (530) applicable to the determination of the stresses in the bottom-chord panels, the length of the lever-arm, of the reaction, l_1 must be either increased or decreased by a certain part of a panel length, which varies with the inclination of the diagonals. In this design it is one half of a panel in either case. As the negative term contains the sum of the moments of the loads between the reaction and the panel point of the lower chord, it must be either increased or diminished by the

sum of the moments of the loads in any panel, such as 4, 6, acting at the panel point of the lower chord. Also it will be seen that y_1, which in eq. (530) is the distance from the wheel w_{n-1} to w_{n_1}, must now be the horizontal distance from w_{n_1} to the new axis at 5 instead of at 4. Writing eq. (530),

$$\begin{aligned} M = \frac{l_1}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \dots \\ (w_1 + w_2 + w_3 + \dots w_n)x] \\ - w_1a - (w_1 + w_2)b - (w_1 + w_2 + w_3)c - \dots \\ - (w_1 + w_2 + w_3 + \dots w_{n_1-1})y_1, \qquad (552) \end{aligned}$$

which is still applicable to the unloaded chord.

Let s = any fraction of a panel length, $4e = \frac{1}{2}p$ in this case, which is constant for the same span, R_2 the resultant of the loads on any panel, such as 4, 6, and z its distance from 6; p the panel length; w_{n_1} either resting at or in front of a panel point 4. We have $l_2 = l_1 + s$, the distance from the end of the span to the axis of moments, and $y_2 = y_1 + s$. The reaction of the loads on the panel 4, 6 at 4 $= \frac{zR_2}{p}$; its moment with respect to $e = \frac{zR_2}{p} \times s$ (see Figs. 356). We can then write the following:

$$\begin{aligned} M = \frac{l_2}{l}[w_1a + (w_1 + w_2)b + (w_1 + w_2 + w_3)c + \dots \\ + (w_1 + w_2 + w_3 + \dots + w_n)x] - w_1a - (w_1 + w_2)b \\ -(w + w_2 + w_3)c - \dots - (w_1 + w_2 + w_3 + \dots + w_{n_1})y_2 \\ - \frac{zR_2s}{p}; \qquad (553) \end{aligned}$$

noting that x, y, and z, are variables, and moving the load forward by the small quantity $\Delta x = \Delta y = \Delta z$. The conditions for a maximum moment will be

$$\frac{l_2}{l}[w_1 + w_2 + w_3 + \dots + w_n] \\ - w_1 + w_2 + w_3 + \dots + w_{n_1}) - \frac{SR_2}{p} = 0;$$

$$\frac{l_2}{l} = \frac{w_1 + w_2 + w_3 + \dots + w_{n_1} + \frac{SR_2}{p}}{w_1 + w_2 + w_3 + \dots + w_n} \qquad (554)$$

These last equations, (553) and (554), are applicable to any single system of bracing, equations (528) and (530) being only

applicable to the unloaded chord where the web members are all inclined. Using then equations (528) and (530) for the upper chord panels:

For upper chord 1, 3, $w_4 = w_{n_1}$ at 2; $x_1 = 100 + 16.449 - 105.909 = 10.54$ feet of uniform load on bridge; $l_1 = 1$; $l = 6$.

$$\frac{l_1}{l} = \frac{1}{6} = \frac{w_1 + w_2 + w_3 + \ldots w_{n_1}}{w_1 + w_2 + w_3 + \ldots w_n} = \frac{72000}{296000 + 10.54 \times 2240} = \frac{1}{4.44};$$

but assuming that 18,731 lbs. of w_4 acts to the right of panel point 2, $\frac{72000 - 18731}{319610} = \frac{l_1}{l} = \frac{1}{6}$. By placing $w_3 = w_n$ at 2, $\frac{l_1}{l} = \frac{1}{6} = \frac{52000}{296000 + 6.34 \times 2240} = \frac{1}{6}$, nearly. We will try both of these positions, substituting in

$$\begin{aligned} M = \frac{l_1}{l} & [w_1 a + (w_1 + w_2) b + (w_1 + w_2 + w_3) c + \ldots \\ & + w_1 + w_2 + w_3 + \ldots + w_n x] \\ & - w_1 a - (w_1 + w_2) b - (w_1 + w_2 + w_3) c - \ldots \\ & - (w_1 + w_2 + w_3 + \ldots + w_{n_1 - 1} 1) y_1. \end{aligned}$$

With $w_4 = w_{n_1}$ at 2; $w_{n_1 - 1} = w_3$; $x_1 = 10.54$, and total uniform load $= 10.54 \times 2240 = 23{,}610$ lbs.; $x = 5.27$.

$$M = \frac{1}{6}\left[\frac{16370860}{2} + \frac{2 \times 296000 + 23610}{2} \times 5.27\right] - \frac{459650}{2} = 1{,}404{,}764 \text{ foot-pounds.} \quad (1)$$

With $w_3 = w_{n_1}$ at 2, $w_{n_1 - 1} = w_2$; $x_1 = 100 + 12{,}249 - 105{,}909 = 6.34$, and total uniform load $= 6.34 \times 2240 = 14{,}202$ lbs.; $x = \frac{6.34}{2} = 3.17$.

$$M = \frac{1}{6}\left[\frac{16370860}{2} + \frac{2 \times 296000 + 14202}{2} \times 3.17\right] - \frac{238660}{2} = 1{,}405{,}047 \text{ foot-pounds.} \quad (1a)$$

Practically there is no difference between the bending moment whether w_4 or w_3 is taken at the point 2, but as the latter is a little greater it will be used. As before stated, in all cases of doubt two

or more positions of the load should be tested in order to determine which is the greatest.

Upper chord 3, 5: $w_6 = w_{n_1}$ at 4; $x_1 = 80 + 31.748 - 105,909 = 5.839$ ft.; uniform load $= 5.839 \times 2240 = 13079$ lbs.; $x = 2.92$; $l_1 = 2$; $l = 6$; $w_{n_1-1} = 5$; $w_n = 13.079$ lbs.; $\frac{l_1}{l} = \frac{1}{3} = \frac{106000}{300079} = \frac{1}{3}$, nearly.

$$M = \left[\frac{116370860}{3\quad 2} + \frac{2 \times 296000 + 13079}{2} \times 2.92\right] - \frac{1770920}{2} \Big\} \quad [= 2,139,358 \text{ ft.-lbs.} \qquad (2)$$

For upper chord 5, 7: $w_{10} = w_{n_1}$ at 6; $x_1 = 60 + 55.996 - 105,909 = 10.087$ ft.; uniform load $= 22,995$ lbs.; $x = 5.04$; $l_1 = 3$; $l = 6$; $\frac{l_1}{l} = \frac{1}{2} = \frac{160000}{318995} = \frac{1}{2}$, nearly, which is condition for maximum:

$$M = \frac{1}{2}\left[\frac{16370860}{2} = \frac{2 \times 296000 + 22995}{2} \times 5.04\right] - \frac{4942000}{2} \Big\} \quad [= 2,396,609 \text{ ft.-lbs.} \qquad (3)$$

It may sometimes happen that, when the condition for a maximum bending moment is fulfilled, the front wheel w_1 may have passed off the bridge to the left hand. We must then subtract the weight on that wheel, or wheels, multiplied by its distance from the near reaction + the length of the span, from the positive term, and add to the negative term its weight multiplied by its distance from the near reaction + the distance of the near reaction from the panel point under consideration. For example, if the position of the load for maximum stress in upper chord 5, 7 had been taken when $w_{11} = w_{n_1}$ was at point 6, the front wheel w_1 would be 63.662 ft. (see Table LXXVIII for wheel distances) to the left of 6, or $63.662 - 60 = 3.662$ ft. to the left of the left-hand reaction, or that much off the bridge; then $60 + 63.662 - 105.909 = 17.753$ ft. of uniform load on the bridge $= 39,767$ lbs.; total load on the bridge $= (296,000 + 39,767) - 12,000 = 323,767$; or, from the value of M, equation (3), deduct $\frac{12000}{2}(3.662 + 120.0)$ from the positive term, and add $\frac{12000}{2}(3.662 + 60)$ to the negative term. Then, substituting in equation (530), we have, $x = 8.88$; $l_1 = 3$; $l = 6$,

$$\left.\begin{aligned} M = \tfrac{1}{2}\Big[\frac{16370860}{2} + \frac{2 \times 296000 + 39767}{2} \times 8.88 - \frac{12000}{2} \times 123.662\Big] \\ - \frac{6168580}{2} + \frac{12000}{2} \times 63.662 = 2{,}439{,}399 \text{ ft.-lbs.} \end{aligned}\right\}, \qquad (3a)$$

being greater than the value of M in equation (3) by 42,790 ft.-lbs., again showing the importance, where great accuracy is required, and when the uniform load is on the bridge, of trying several positions of the moving load to find the ultimate maximum. It is true that the difference in the stress is only $\frac{42790}{20} = 2140$ lbs., which would be no serious error.

For tension in the lower-chord panels, use equations (553) and (554). But for panel 0, 2 the tension is simply the horizontal component of the stress in 0, 1, which is 70,108 lbs. $\times$ 0.5 (see equa. 2 for shears, page 1114) = 35.054 lbs., due to rolling load.

For lower chord-panel 2, 4 use eq. (554) to find maximum position of load. If $w_2 = w_{n_1}$ is placed at point 2, then w_5 will be 1.167 feet to the left of c, which is in the same vertical plane with upper-chord panel point No. 3, about which moments are to be taken (see Fig. 356 (a)), which is a panel taken from Fig. 355, in which l_2

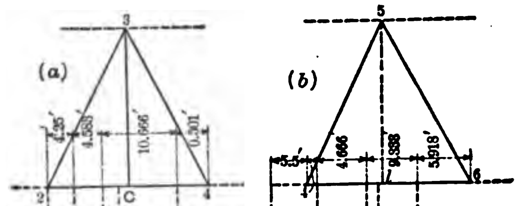


FIG. 356.

$= 1.5$ and $l = 6$ panels. The resultant of the loads $w_4 + w_5 + w_6 = R_2$ is found to be 10.1 feet from point 4, or 0.1 foot to left of C. $R_2 = 54{,}000$ pounds; $\dfrac{s}{p} = \dfrac{1}{2}$ is the distance $2c$; $w_1 + w_2 + w_3$ $(= w_{n_1}) + \dfrac{sR_2}{p} = 52{,}000 + \dfrac{54{,}000}{2} = 79{,}000$ pounds. Then 100 $+ 12.249 - 105.909 = x_1 = 6.34$ feet, and uniform load on the bridge $= 2240 \times 6.34 = 14.202$ pounds; $x = 3.17$. Total load $= w_1 + w_2$ $+ w_3 + \ldots + w_{n-1} + w_n$; ($w_n = 14{,}202$ pounds :) $(w_{n-1} = w_{18}) =$

296,000 + 14,202 = 310,202 pounds. Then the condition for maximum moment is $\frac{l_2}{l} = \frac{1.5}{6} = \frac{79000}{310000} = \frac{1}{4}$ nearly; hence this position of the load gives maximum bending in 2, 4. Then substituting in eq. (553), maximum bending in 2, 4 is

$$M = \frac{1}{4}\left[\frac{16370860}{2} + \frac{2 \times 296000 + 14202}{2} \times 3.17\right] - \frac{238660}{2}$$
$$- \frac{52000}{2} \times 10 - \frac{54000}{2} \times \frac{10.1}{2} = 1{,}742{,}884 \text{ foot-pounds.} \quad (5)$$

The first term is found in col. 4, Table LXXVII, opposite w_{18}; the second is the sum of the loads in col. 2 opposite wheel w_{18} + the uniform load multiplied by x. The first negative term is the sum of the products $w_1a+(w_1+w_2)b$, etc., to w_{n_1-1}, and is found in col. 4 opposite $w_{n_1-1}=w_2$. The second negative term is $(w_1+w_1+w_2+\ldots+w_{n_1})y_2$, where $w_{n_1}=w_3$, $y_2=10=2c$; and the third negative term is $\frac{zsR_2}{p}$, in which $s/p = \frac{1}{2}$; $R_2 = 54{,}000$, $z = 10.1$,—all terms divided by 2.

For maximum bending in lower-chord panel 4, 6, axis of moments at 5 in same vertical line with e (see Fig. 356 (b)), $l_2 = 2.5$ and $l = 6$ panels. w_7 at 4; then w_8 will be 0.249 foot to right of e. Then $80 + 36.414 - 105.909 = 10{,}505$ feet $= x_1$ of uniform load on the bridge = 23,532 pounds; hence total load = 296,000 + 23,532 = 319,532 pounds. Sum of loads to $w_7 = w_{n_1} = 120{,}000$, and

$$\frac{sR}{p} = \frac{1}{2}(w_8 + w_9 + w_{10}) = \frac{14000 + 14000 + 12000}{2} = 20000 \text{ lbs.};$$
$$\frac{l_2}{l} = \frac{2.5}{6} = \frac{5}{12} = \frac{140000}{319532} = \frac{5}{11.4}.$$

As in the term $\frac{sR_2}{2}$ the factor z, which determines the position of the resultant, does not enter, the position of the resultant in panel 4,6 is immaterial. If, then, we move the entire load on the bridge to the left, provided w_8 does not reach the point 4 (see Fig. 356 (b)), we will increase the total load on the bridge, increasing the denominator of the above fraction without affecting the enumerator. If, then, the load moves $5\frac{1}{2}$ feet to the left, w_8 will still be to the right

of 4 by $5.583 - 5.5 = 0.083$ foot, and we bring $2240 \times 5.5 = 12{,}320$ pounds more of uniform load on the bridge. Then

$$\frac{l_2}{l} = \frac{5}{12} = \frac{140000}{331852} = \frac{5}{11.85}$$

very nearly correct.

A trial might be made with w_8 at 4; but when a load passes a panel point, sudden and relatively great changes take place, and with the condition of maximum bending nearly satisfied, and as this subject has been so fully discussed, we will assume this position of the load as giving maximum bending. As w_7 is now 5.5 feet to left of 4, $y_2 = 5.5 + 4e = 15.5$ feet, $R_2 = 40{,}000$ pounds $= w_8 + w_9 + w_{10}$, it will be 5.83 feet to right of w_8 or 1.64 feet to right of w_9 $= 9.333 + 5.918 - 1.164 = 14.09$ feet from $6 = z$. For this position of the load w_8, being 0.83 foot to right of 4, $x_1 = 80 + 41.997 - 0.83 - 105.909 = 16.005$ feet of uniform load on the bridge $= 2240 \times 16.005 = 35{,}851$ pounds.

$$x = \frac{16.005}{2} = 8.0 \text{ ft.}; \quad \frac{szR}{p} = \frac{40000}{2} \times \frac{14.09.}{2}$$

Substituting in eq. (553),

$$M = \frac{5}{12}\left[\frac{16370860}{2} + \frac{2 \times 296000 + 35851}{2} \times 8\right] - \frac{2265520}{2}$$

$$- \frac{120000}{2} \times 15.5 - \frac{40000}{2} \times \frac{14.09}{2} = 2{,}253{,}354 \text{ foot-pounds}, \quad (6)$$

for maximum bending moment in 4, 6.

The corresponding panels have the same shear and bending moment on the other side of the centre of the span.

To determine absolutely that position of the rolling load which produces maximum bending moment in any panel requires several trials; but it may be said in general that the load must always extend well over the entire span. This enables the position of the load to be determined by inspection, approximately, and but little change from the first position will be required, as was seen in finding the moment in panel 4, 6 (see 6). It is not always necessary that any wheel should be placed exactly at a panel point, when the uniform load is over a part of the span.

FINAL MAXIMUM STRESSES. WEB MEMBERS.

966½. We can now write the stresses in the web members, which will be the sum of the shears due to both loads, multiplied by 1.12.

			Dead Load.	Live Load.		Lbs.
Compression	0, 1;	11, 12 =	(30,000 +	70,108)	× 1.12 =	− 112,121
"	3, 2;	9, 10 =	(18,500 +	46,356)	× 1.12 =	− 72,639
"	4, 5;	7, 8 =	(7,000 +	26,510)	× 1.12 =	− 37,531
Tension	1, 2;	10, 11 =	(27,500 +	70,108)	× 1.12 =	+ 109,311
"	3, 4;	8, 9 =	(16,000 +	46,356)	× 1.12 =	+ 70,839
"	5, 6;	7, 6 =	(4,500 +	26,510)	× 1.12 =	+ 34,731

The above are the maximum stresses with the moving load so placed as to give the same kind of stress in each member as that caused by the dead load.

When the rolling load only extends over the shorter segment, that is, before it passes the centre of the span, it produces a stress of the opposite kind to that caused by the dead load. When the rolling load comes in from the right and reaches the point 10 it tends to produce a tension in 9, 10, which is the shear. See (1), page 1113, 3514 × 1.12 = 3936 bls. The dead-load stress (compression) in 9, 10 (see page 1110, compression in 3, 2) = 20,720 lbs. As these different stresses cannot exist at the same time, the resultant stress is 20,720 − 3936 = 16,784 lbs. compression, and is a permanent stress and the minimum that can ever exist. This member need not then be counterbraced, and consequently 8, 9 need not be counterbraced, as in this case these braces enter by pairs.

If now the rolling reaches the point 8, the dead-load shear on 8, 7 is compression and equal to 7840 lbs.; the live-load shear is 13.155 lbs. and is tension. This latter being the greater of the two, the member 8, 7 will change from compression to tension when the rolling load is entering the bridge, and it must therefore be counterbraced, so as to carry a compressive strain of 37,531 lbs. and a tensile strain of (13.155—7000) × 1.12 = 6894 lbs. To bear the compression this member must have a cross-section of the form required for columns or struts; and to bear the tensile stress its ends must be fixed or connected by pins, rivets, or otherwise to the chords; or with the strut in the form of channels, flanged or boxed beams, these may simply be proportioned and con-

nected to bear the compression, and an iron rod or bar can be laid along or in the column and connected and proportioned to bear the tension. This is probably better than the first arrangement, as the member then does not have to change from compression to tension, or *vice versa*. The same remarks apply to 7, 6; 5, 6; and 5, 4. Either of the above arrangements may be called counterbraced members. or finally these members may be formed and connected simply to carry one kind of stress of maximum value from both loads, and the counterbraces 5, 8 and 7, 4 may be introduced to prevent the alteration in the kind of strain.' Then when the rolling load reaches point 8, as 8, 7 is not connected to receive tension, the shear in 5, 8, which is equal to the reaction on the left, or nearly so, must be carried by 5, 8. This shear (equa. (2), page 1113) = 13,154, and tension in 5, 8 or 4, 7 (when load comes in from the left)

$$= 13{,}155 \times \frac{5,8}{5e} = 13{,}155 \times 1.8 = 23{,}679 \text{ lbs.}$$

The member 7, 6 may be called upon to bear a compression = (13,155 − 4500) × 1.12 = 9694 lbs.

FINAL CHORD STRESSES.

These are found by dividing the moments in equations (pages 1118–19) (1*a*), (2), (3*a*), and (pages 1120–21), equations (5) and (6) (by the depth of the truss), and to these results add chord stresses in same panels due to dead load, page 1110.

$$\text{Upper chord} \left\{ \begin{matrix} 9.11 \\ 1.3 \end{matrix} \right\} \text{comp'n} = \frac{1405047}{20} + 28{,}750 = -99{,}002 \text{ lbs.}$$

$$\text{“} \quad \text{“} \left\{ \begin{matrix} 7.9 \\ 3.5 \end{matrix} \right\} \text{“} = \frac{2139338}{20} + 46{,}000 = -152{,}967 \text{“}$$

$$\text{“} \quad \text{“} \quad 5.7 \quad \text{“} = \frac{2396609}{20} + 51{,}750 = -171{,}580 \text{“}$$

$$\text{Lower} \quad \text{“} \left\{ \begin{matrix} 10.12 \\ 0.\ 2 \end{matrix} \right\} \text{tension} = 35{,}054 + 15{,}000 = +\ 50{,}054 \text{“}$$

$$\text{“} \quad \text{“} \left\{ \begin{matrix} 8.10 \\ 2.4 \end{matrix} \right\} \text{“} = \frac{1742884}{20} + 38{,}000 = +125{,}144 \text{“}$$

$$\text{“} \quad \text{“} \left\{ \begin{matrix} 6.8 \\ 4.6 \end{matrix} \right\} \text{“} = \frac{2253354}{20} + 49{,}500 = +162{,}168 \text{“}$$

This style of bridge does not seem to be as extensively used as it was formerly.

It is frequently built partly of wood and partly of iron; compression members of wood, tension members of iron. Iron rods run along those struts that need to be counterbraced. By allowing the proper unit strains for wood or iron the dimensions are easily calculated, as in the preceding examples.

The details of connections are not materially different from other types of bridges, except in case of those members which alternate from tension to compression and the reverse.

In very long spans requiring a considerable depth of truss, the panel lengths 0, 2, 1, 3, etc., would be too great. In that event $1a$, $3c$, $5e$, and, in deck bridges, $b2$, $d4$, etc., are introduced, according to the position of the loaded chord. These verticals are not, however, main members, that is, they do not receive any transmitted load from any other portion of the truss, simply carrying by compression or tension a single-panel ($0a$, ab, $2c$, etc.) load of dead and live load; but, when used, every panel point of both chords is loaded with the moving or live load. Hence the web members do not enter by pairs; that is, in the case that has been discussed the shear arising from the moving load on 3, 4 and 3, 2 is the same; but when the verticals are used, brace 3, 2 would carry not only one upper-chord panel load of the dead load, but one panel load of the live load resting at c more than 3, 4, and should be proportioned according. The panels in this case, however, are only one half the length, $2c = \frac{1}{2}(2, 4)$, etc.; otherwise both the amount and kind of stress would be found as in the case already fully discussed.

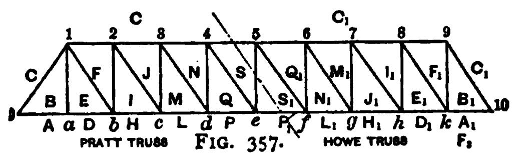
GRAPHICAL METHOD OF DETERMINING STRESSES IN TRUSSES.

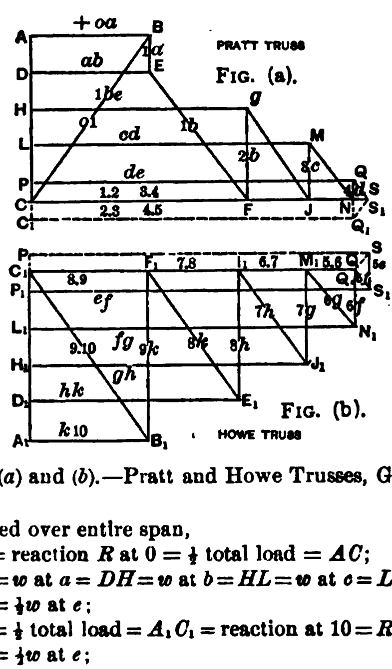
967. The principles underlying the graphical method of determining stresses were fully discussed in Art. XXII, and applied to roof-trusses in Art. L, paragraphs 908, 909, 910.

In this paragraph these principles will be applied to several types of bridge-trusses. This method is simple and easy of application, and is sufficiently accurate for any ordinary purpose. It is oftener better to combine the method of sections or moments, and in some cases necessary. It is frequently adopted, and is especially advantageous in complicated structures and those bridge-trusses having curved upper or lower chords.

While for the more simple roof-trusses and such bridge-trusses as the Howe, Pratt, and Warren there may be no special advantage over the methods already fully discussed, the following applications

to these trusses will serve to impress the principles and methods, so that there will be no difficulty in extending them to the more complicated trusses. The diagrams when drawn with lines of different colors show clearly to the eye those members under different kinds of stress. In order to prevent possible errors, some of the stresses should always be checked by moments.





FIGS. 357 (a) and (b).—Pratt and Howe Trusses, Graphical Method.

Uniformly loaded over entire span,

$CA =$ reaction R at $0 = \frac{1}{2}$ total load $= AC$;

$AD = w$ at $a = DH = w$ at $b = HL = w$ at $c = LP = w$ at d;

$PC = \frac{1}{2}w$ at e;

$C_1A_1 = \frac{1}{2}$ total load $= A_1C_1 =$ reaction at $10 = R_1$;

$C_1P_1 = \frac{1}{2}w$ at e;

$P_1L_1 = w$ at $f = L_1H_1 = w$ at $g = H_1d_1 = w$ at $h = D_1A_1 = w$ at k.

It is never necessary to deal with but one half the span, as stresses in correspondingly placed members of the other half of the truss are the same, and only one half is used in the trusses and diagrams here discussed, the closing lines for the other half being indicated by dotted lines in the stress diagrams. Fig. 357 shows a

half Pratt truss to the left of e and a half Howe truss to the right of e. In actual practice, however, to insure accuracy and to avoid errors, it is better to construct the stress diagrams for the whole span, as the exact correspondence in length and relative positions of the lines in opposite directions prevents errors of direction and accurate scaling of the several lines, as one error must be carried entirely through the diagram.

968. The accompanying diagrams, Fig. 357 and (*a*), apply to one half of the Pratt truss. The left half of Fig. 357 is a skeleton diagram of truss, assumed to be loaded at every panel point, a, b, c, etc. There will then be four full loads w on each half of the truss and one load of $\frac{1}{2}w$ at e, the centre of the span. Lay off AC, figure (*a*), downwards equal to $4\frac{1}{2}w$. This must be balanced by the vertical reaction upwards $R = CA = AC$. This closes the force polygon. All lines in figure (*a*) lettered with the capitals A, B, C, etc., are the stresses on the members included between the same capital letters. Corresponding lines are also indicated by the small letters or numbers, as in Fig. 357; for instance, taking at random the line or member $1b$ or EF, this is also marked $1b$ or EF, figure (*a*); and similarly CF or 1, 2 is marked CF or 1, 2 in stress diagram, figure (*a*). Either of these systems of notation can be used. It will be noticed that C refers to the end-post, and also to all of the upper-chord panels. The lower-chord panels have, however, a separate letter for each. This double system of notation is followed in all of the diagrams and will not be further explained.

Having, then, figure (*a*) laid off with the external force polygon $ACCA$, then going to joint 0, there are only three forces acting: the reaction AC upward, the horizontal stress in oa, and the inclined stress in 01. Then in figure (*a*) drawing AB and CB from the extremities of AC and parallel respectively to oa and 01, Fig. 357, we have the first stress polygon ABC, figure (*a*). CB is the stress in 01, and AB is the stress in $0a$; as CA acts upwards, and as equilibrium requires the other stresses to act continuously around the force polygon, therefore AB acts to the right from A towards B, that is, outwards or away from 0. It therefore produces tension in $0a$, and is marked + in diagram. Also stress BC must act downwards to the left, from B towards A, acting inwards towards 0 means compression in 01, marked − in diagram. Going to the panel point 1, we have four forces: w at a, stresses in 01, $1b$, and 1, 2. We already have BC = stress in 01; and as this is compression, it must now be supposed to act inwards towards 1, i.e., from C

towards B, figure (a). Drawing a vertical line from B to $E = w$ downwards, an indefinite horizontal line from C parallel to 1, 2, and an inclined line EF parallel to $1b$, their intersection forms the closed polygon with lines acting from C to B, B to E, E to F, F to C, these lines representing the stresses in the members to which they are parallel. EF acting outwards from 1 on $1b$ gives tension; FC acting to the left inwards towards 1 is compression in 1, 2; BE is tension in $1a$, acting downwards and outwards from 1; DE is tension in ab = tension $0a$, which is evident, since $1a$ is vertical and can have no horizontal component. The actual polygon for this point is from A to B, B to E, E to D, D to C, and from C to A, equilibrium requiring a downward force DC (= loads to the right of a), together with BE to balance CA. Now at the point b we have ab and FE already determined, and w at b to find $b2$ and bc. As FE is tension, it must act outwards and upwards on $1b$ from b. The stress polygon for this point is then F to E to D to H, and then the horizontal and vertical lines HG and FG respectively give the stresses in bc and $2b$. bc, acting outwards from b to the right, is tension in bc; and Fg, acting downwards and inwards on $2b$ towards b, is compression in $2b$. The remainder of the diagram is but a repetition of the above, at 2 and c, 3 and d, 4 and e. This results in making the stress on $5e = ss_1 = 0$. It has, however, to carry one upper-chord panel weight at 5. If the line of loads AC is laid off to represent the sum of the dead and live loads, this diagram gives the maximum chord stresses, but not maximum vertical and diagonal stresses. Figure (a) then is only useful so far as the horizontal lines are concerned, if there is a rolling load to come on the structure. It will be noticed that all of the upper-chord panel stresses are found in the same horizontal line CS: $1,2 = CF$; $2,3 = CJ$; $3,4 = CN$; and $4,5 = CS$.

HOWE TRUSS.

969. Taking now the half-span Howe truss, Fig. 357 and (b), it will be noticed that the horizontal and diagonal stresses are the same in magnitude but opposite in kind. The bottom chord stresses are all found in one horizontal line $C'Q = CS$, of Fig. (a) and that the verticals are all one panel greater and are tension instead of compression, all acting away from the panel point instead of towards it, and that the end vertical $9k$, corresponding to $1a$, has a tension of $3\frac{1}{2}$ panel loads more than $1a$, since it now is

a main member and has transmitted through it the panel loads between the centre of the span and the vertical itself, whereas $1a$ has to carry but a single panel load. Further, it is noted that the centre vertical $5e$ carries a panel load ss_1, Fig. (b), where ss_1, Fig. (a), is zero.

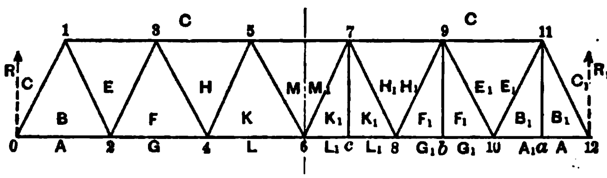
The stress diagram itself is constructed precisely as in the preceding case. $C_1A_1B_1$ corresponds to the stresses in 9,10; $k10$ and reaction $R_1 = \frac{1}{2}$ total load on span; for joint 9 is B_1 to C_1 to F_1 to B_1, and as C_1B_1 is compression, it must act upwards on 10,9 towards 9; then C_1F_1 acts to the right inwards towards 9, hence is compression; and F_1B_1 acts downwards and outwards on $1a$ from 9, hence is tension, and similarly for any other panel point. The stress on a member having been once found as a compression, its direction then must always act inwards on the member towards the panel point considered. It must then be conceived as changing its direction according as the panel point under consideration is at its top or bottom extremity.

WARREN GIRDER.

970. *Graphical Method.*—Uniform load over entire span. Even-numbered joints loaded only. In Fig. (a), AC acting downward equals half total load; CA acting upward equals reaction R at 0. $AG = w$ at 2; $GL = w$ at 4; $LC = \frac{1}{2}w$ at 6; $LL_1 = w$ at 6. Figs. 358 and (a) refer to a Warren girder only loaded at the even-numbered joints. Fig. 358, left half, shows skeleton diagram when loaded only at lower chord joints.

For the joint 0 only three forces exist, the reaction R and the stresses in 01 and 02. The force polygon ABC, Fig. (a), gives reaction AC upwards, AB stress in 02 to the right, and acting from 0 is tension, BC downwards towards 0 on 01 is compression. For joint 1: CB upwards towards 1, compression; BE downwards on 1,2, outwards from 1, is tension; and CE, inwards to the left, on 1,3, towards 1, compression. For joint 2: stress in 1,2 and 02, BE and AB respectively, are known; and also load $w = AG$. Since we have found stress AB in 1,2 to be tension, it must act towards the left and outwards from 2, then follow from E to B upwards from B to A to the left, downwards from A to G, then draw from GF horizontal line parallel to 2,4, meeting a line from E parallel to 2,3, in F, then FG to the right and outwards, on 2,4, from 2 is tension in 2,4; and FE downwards on 2,3, towards 2, is com-

pression. The stress polygon for this joint is *EBAGFE*, and for joint 3 it is *CEFHC*, and so on through the diagram. Observe that the diagonals meeting in the upper or unloaded chord enter by pairs, the stress being the same in amount but opposite in kind, one compression sloping upwards to the centre, the other tension downwards towards the centre, from the ends as 2, 3 and 3, 4 respectively.



FIGS. 358.—Warren Girder.—Graphical Method.

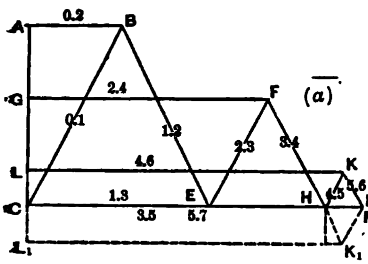


FIG. 358 (*a*).

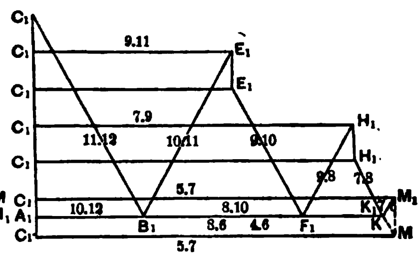


FIG. 358 (*b*).

Fig. (*c*) Rolling load, joint 2 only loaded.

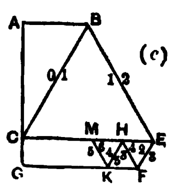


FIG. 358 (*c*).

WARREN GIRDER LOADED AT ALL JOINTS.

Uniform load over entire span. All joints loaded. $C_1A_1 = \frac{1}{2}$ total load; A_1C_1 = reaction R_1 at 12; $C_1C_1 = w$ at 11; $E_1E_1 = w$ at 10; $C_1C_1 = w$ at 9; $H_1H_1 = w$ at 8; $C_1C_1 = w$ at 7; $mm_1 = w$ at 6.

This case is shown in Figs. 358 and (*b*). There will be five and a half panel loads on each half of the truss, whole loads at *a*, 10, *b*,

8, c, and one half panel load at centre panel joint 6. Laying this downward in Fig. 358 (b) from C_1 to A_1, the reaction upwards $= R_1$ from A_1 to C_1. Then for joint 12 the stress polygon is A_1C_1 upwards, C_1B_1 downwards on 11, 12 towards 12 compression, and B_1A_1 towards left outwards from 12 on a12 is tension. For joint 11 stresses in 11, 12 $= B_1C_1$, upwards towards 11; C_1C_1 load at a downwards from 11, tension a1; then draw the horizontal line C_1E_1 parallel to 9, 11, meeting B_1E_1 parallel to 10, 11; the stress polygon for the joint 11 is the crossed quadrilateral B_1 to C_1 to C_1 to E_1 to B_1, or $B_1C_1C_1E_1B_1$, as B_1C_1 is upwards, C_1C_1 downwards. C_1E_1 must act towards the right on 9, 11 inwards towards 11; hence compression E_1B_1 must act downwards on 10, 11 outwards from 11, hence tension. For panel point 10, the stress polygon is $A_1B_1E_1E_1F_1A_1$. At joint 9 the forces are w at b (E_1E_1), compression in 9, 11 (E_1C_1), compression in 9, 10 (E_1F_1) known, and stress in 7, 9 and 9, 8 unknown. Since stress in 9, 10 is compression, it must act upwards towards 9, then from F_1 to E_1, E_1 to C_1, C_1 to C_1; then draw C_1H_1 parallel to 7, 9, meeting F_1H_1 parallel to 9, 8 at H_1; then C_1H_1 acting towards the right on 7, 9 inwards towards 9 is compression, and H_1F_1 acting downwards on 9, 8 outwards from 9 is tension. A similar construction at other points completes the stress diagram. Starting at the centre M_1M, it is observed that the shear in each diagonal is one panel greater for each successive diagonal towards the end; and in addition, although the panel weights are only half of those in the first cases, yet a larger portion of the entire load is required to be carried by the members of the truss. The extension and rapidity of the application is evident, and by it the amount and kind of stress on each member is determined with great rapidity, as has been readily seen in all of the examples.

ROLLING LOADS.

Also diagrams for rolling loads can be constructed for the above trusses, so as to obtain the maximum stresses in diagonals and verticals due to rolling loads, in their proper positions on the trusses. These must be added to the dead-load stresses found by diagrams similar to figures (a) and (b), the panel loads being simply the dead-load weight instead of both dead and live load over the entire span. The value of stresses in pounds or tons would be less, but otherwise there would be no change in the diagrams. In that case the diagonal and vertical lines would be the maximum stresses due to the dead weight.

In figure (c) suppose a single load to rest at point 2, Fig. 358, we have the stress diagram for this condition, AG acting downwards would be the load at 2. We would then have to calculate the reactions by the principle of the lever. There being 6 panels, R would be $\frac{5}{6}w$ and R_1 $\frac{1}{6}w_1$; then laying off upwards $\frac{1}{6}w$ to C and $\frac{5}{6}w$ to A, the external force polygon is completed; then on AC acting upwards draw AB and CB, these would be the stresses on 0,1 and 0,2, compression and tension respectively; then on BC, for the joint 1 draw BE and EC, tension on 1,2, and compression on 1,3 respectively. For the joint 2, we have EB, BA, AG, known and acting as indicated by the order of the letters $EBAG$; then drawing GF and EF parallel to 2, 4 and 2, 3, respectively, meeting in F_1, GF is tension in 2, 4, as it acts outwards from 2, and FE is tension on 2, 3, as it acts upwards and outwards from 2; then for joint 3 we have CE, EF, known; HF, and CH, the stresses on 3, 4 and 3, 5; and the polygon of forces then in order would be $CEFHC$. HF acting upwards towards 3 would be compression in 3, 4. The diagonal stresses would now remain constant, as seen at 3, 4 = 2, 3 etc.; but here we note that the load at 2 produces tension on 2, 3 and an equal compression on 3, 4, whereas, referring to figure (a), we had compression on 2, 3 and tension on 3, 4, due to uniform load, consequently the rolling load produces a contrary stress on 3, 4 and 2, 3 to that caused by the dead load. If then 2, 3, figure (a), is greater than (EF), or 2, 3, figure b, that member will always be compressive; if otherwise, then it would change from compression to tension when the load comes on the bridge, the resulting tension being difference between 2, 3, figure (a) and 2, 3, figure (c). In other words it should be counterbraced, as fully explained in preceding articles. It is evident, for such simple trusses, that no labor is saved by resorting to the graphical method for rolling loads, as we have to calculate the reactions anyhow, and the reaction is the shear in 3, 4 or 2, 3, causing a stress different from the dead load; but the dead load shear in 3, 4 and 2, 3 is 1½ panel loads; that is, the load between 4 and the centre of the span, which can be made mentally; and, moreover, a different diagram must be constructed for every new position or number of moving loads on the span. Until the rolling load passes the centre it tends to reverse all of the dead-load stresses in the members on the same side of the centre; after it extends over more than one half of the span it causes the same kind of stress in the web members as the dead-load stresses and the ultimate maximum is the sum of the two. This subject has, however, already been fully discussed. A similar dia-

gram could be constructed with two or more loads on the span, but the above sufficiently illustrates the application of the principles.

GRAPHICAL METHOD : THE WHIPPLE OR LINVILLE TRUSS.

971. The Whipple truss is a modification of the Pratt truss. It is in fact composed of two Pratt trusses having common chords, but two independent sets of diagonals and verticals. When both chords are horizontal the truss can be separated into two Pratt trusses, and the stresses calculated as already explained; the stress on any chord panel being the sum of those obtained from the two separate trusses. This form of truss then presents no difficulties and involves no special principles which have not been already discussed.

In long spans, however, the upper chord is usually curved, the curve being approximately that of a parabola. (See Figure 359.)

It is evident in this case that the truss cannot be separated into independent trusses, owing to the inclination of the upper chord the two systems are mutually interdependent, since the members in one system induce stresses in the other. In such a truss there will necessarily be some ambiguity in determining the stresses, and it becomes necessary to make certain assumptions which may or may not be true. If, however, the assumptions err on the side of safety no harm can result. It is therefore impossible to intersect the structure in any direction by a plane that will not cut more than three members. The stresses can then only

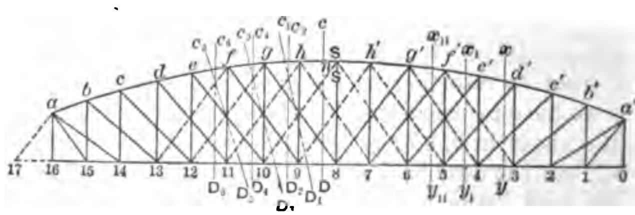


FIG. 359.

be determined by making certain assumptions, which cannot involve any danger. The following assumptions are made :

1. If a uniform load covers the entire truss, the counterbraces are not needed. These are $13f$ and the other dotted lines parallel to it on the same side of the centre of the span, and $3f'$ and those members parallel to it on the same side of the centre.

2. That the point at which the reaction R_1, diminished by the loads at the panel points, is zero ($R - w + w_1 + w_2 +$ etc. $= 0$), divides the load into two parts, and that each part travels to the nearest abutment by the most direct route.

3. At the point where $R - w + w_1 + w_2 +$ etc. $= R - \Sigma w = 0$, only those diagonals that slope upwards and away from that point are supposed to act, and this whether they are main or counter-braces. Assume for any condition of loading that makes $R - \Sigma w = 0$ at panel point 13, for instance, all verticals act, but only the diagonal $13f_1$ and those sloping upwards towards the right to R_1 including both $a'2_1$ and $a'1_1$ and on the left of 13_1 only those sloping upwards to the left, $b13$, $a14$, $a15$, and similarly for any other point.

With these assumptions it will be always practicable to find some point through which a plane can be passed, only intersecting three members of the truss, the stresses in these three being found either by moments or graphically. The position of the plane can then be shifted so as either to intersect only three members, or if intersecting four members, one of which has already been intersected by another plane and the stress determined, the remaining stresses can then be found. The method by moments requires much labor, especially if the various lever-arms are calculated. By scaling the lengths of the lever-arms on a large drawing the labor is materially reduced and, with care, the results sufficiently accurate. But for structures of this kind the graphical method greatly simplifies the determinations of the stresses.

The stresses in a few of the members will be determined by moments to illustrate the principle, which applied to all of the members in succession will determine the nature and amount of stress in each.

Length of span $= 200$ ft.	Height at centre $= 35$ ft.
" " panel $= 12.5$ ft.	" " ends $= 15$ "
Number of panels $= 16$.	Radius of upper chord $= 260$ ft.

Upper-chord panel weight $= 3.125$ tons $= w_2$. } Dead load.
Lower-chord panel weight $= 5.000$ tons $= w_1$. }
Uniform rolling load per panel $= 13.0$ tons $= w$.
Four panels rolling load at the front, $= 17.0$ tons each, $= w'$.
Engine excess per panel $= w' - w = 4$ tons $= e$.

Find first the stresses due to the dead load.

$R = R_1$, the reactions at the ends of the truss.
$R = 7.5(w_2 + w_1) = 7.5 \times 8.125 = 60.94$ tons.

As the counterbraces do not act under the dead load, a plane CD to the left of the centre vertical cuts only three members, hs, $g8$, and 9,8. The lever-arm of the stress H in hs is the line $n8$ perpendicular to it; its length can be found by calculation or by scale. It, however, differs so slightly from $s8$ that it will be taken equal to it. Hence we have

$$7.5(w_2 + w_1) \times 100 - 7(w_2 + w_1) \times 50 - H \times 35 = 0.$$

$H = 92.9$ tons. The stress in 9, 8 and $g8$ can now be found by the construction of the force polygon

$$R - \Sigma w = 7.5(w_2 + w_1) - 7(w_2 + w_1) = 4.062 \text{ tons.}$$

Laying a line parallel to $hs = H$ to a scale of 20 tons to the inch, $\frac{92.9}{20} = 4.65$ in., and a vertical $= \frac{4.062}{20} = 0.203$ in., and completing the polygon by lines parallel respectively to 9,8 and $g8$, these lines will represent the stresses, and the arrow-heads their directions (Fig. 359 (a)). We have AB upwards $= 0.203$ in. $=$ 4.062, $AD = 4.65$ in. $= 92.9$ tons compression in hs; BC acting outwards to the right along 9,8 is tension $= 4.575$ in. $= 91.5$ tons; and CD acting downwards or outwards in $g8$ is tension $= 0.125$ in. $= 2.5$ tons. This is the only panel on the left of the centre of the span in which a plane can be taken that will not cut more than three members, except the end panel, where we know that 16,15 is not under stress; but this point should be proved rather than assumed. But a cutting-plane C_1D_1 cuts gh, $8g$, $h9$ and 9,8. The stresses in two of these members having been determined, the remaining two can be found. Assuming the length and direction BC and CD, diagram figure (a), drawing then from the extremities B and D lines, respectively parallel to gh and $h9$, DF and BF closing the polygon. DF, acting to the left or inwards on gh, is compression $= 4.666$ in. $= 93.32$ tons. The line FB acting upwards is the upward force to produce equilibrium. This must be found in the stress in $h9$, and the resultant of the external forces $R - \Sigma w =$ $7.5(w_2 + w_1) - 7w_1 - 6w_2 = 7.19$ tons. As only six upper-chord panel loads are between the end of the span and the cutting plane, and seven lower-chord panel loads, then $FB - 7.19$ tons

must be the stress in $h9$, and acting upwards will be tension in $h9$; $FB - 7.19 = 0.45 \times 20 - 7.19 = 1.81$ tons. In the language of of graphics, we first draw FB upwards and then, to close the polygon, $R - \Sigma w$ downwards; the difference, acting in the direction of the greater, would be the stress in the vertical member cut. Under the dead load it is to be noted that $h9$ is under a tensile stress of 1.81 tons. $s8$ will also be under a small tensile stress. The ultimate stress is

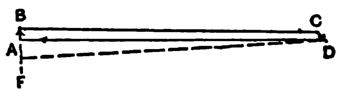


FIG. 359 (*a*).

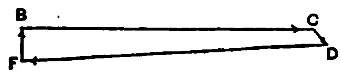


FIG. 359 (*b*).

to be found by adding algebraically the stresses due to the live load in these members, which will vary both in kind and amount. As the load passes from panel point to panel point the greatest of these results must be selected and added algebraically to the dead-load stress; this will give a maximum tensile and compressive stress. These members must then be counterbraced, that is, formed and connected to act both with tension and compression. It may be noted that diagrams Fig. 359 (*a*) and (*b*) have two lines in common, BC and CD. Fig. 359 (*b*) could have been formed on (*a*) by simply drawing two lines DF and FB as indicated by the dotted lines in Fig. 359 (*a*); as it avoids drawing over and over again the same lines to scale, labor and time are both saved. In this consists the graphical method—building one diagram on another along common lines. For accuracy the scale of the drawing should be large —not less than 10 tons to the inch. The diagram Fig. 359 (*c*) will be constructed to a scale of 15 tons to the inch. A skeleton diagram of the truss should be drawn carefully on the same paper so as to secure perfect accuracy in the parallelisms of the lines. In diagram Fig. 359 (*c*) *a* and *b* are shown by the dotted lines enclosed by the letters $BCDF$. Take a cutting-plane C_2D_2 cutting gh, $g8$, $f9$, and 10,9. Stresses in gh and $g8$ are known, the other two can be found; $R - \Sigma w = FG$; $7.5(w_2 + w_1) - 6(w_2 + w_1) =$ 12.19 tons; $FG = \frac{12.19}{15} = 0.813$ in., drawing lines from G and c parallel respectively to 10,9 and $f9$ of Fig. 359, intersecting at c_1. Then Gc_1, tension in 10,9, $= 5.95$ in. $\times 15 = 89.25$ tons; and cc_1, tension in $f9$, $= 0.259$ in. $\times 15 = 3.88$ tons; the arrow-heads indicating the direction and kind of stress. Plane c_2d_2 cutting fg, $f9$, $g10$, 10,9—fg, $g10$ unknown—draw cG_1 and GG_1 respectively parallel to fg and $g10$; $R - \Sigma w = 60.94 - 6w_1 - 5w_2 = 15.315$ tons;

$GG_1 = 0.921$ in. $\times$ 15 = 13.815 tons acting upwards, hence stress in $g10 = 15.315 - 13.815 = 1.5$ tons acting downwards = compression. Compression in $fg = cG_1 = 6.12$ in. $\times$ 15 = 91.8 tons.

Cutting-plane C_4D_4 intersects fg, $f9$, $e10$, 10,11; unknown, $e10$ and 10,11. $R - \Sigma w = 60.94$ tons$-5(w_2 + w_1) = 20.315$ tons; lay off $G_1H = \frac{20.315}{15} = 1.354$ in. From H and c_1 draw Hc_2 and c_1c_2 parallel respectively to 10,11 and $e10$, intersecting in c_2. Hc_2 tension in 10,11 = 5.58 in. $\times$ 15 = 83.7 tons; c_1c_2 tension in $e10$ = 0.54 in. $\times$ 15 = 8.1 tons.

Plane C_5D_5 cuts ef, $e10$, $f11$, 10,11; unknown, ef, $f11$; c_1H_1, parallel, to $ef = 6.00 \times 15 = 90.0$ tons, compression. $R - \Sigma w = 60.94 - 5w_1 - 4w_2 = 23.44$ tons; $HH_1 = 1.47$ in. $\times$ 15 = 22.05; compression in $f11 = 23.44 - 22.05 = 1.39$ tons, acting downwards. Similar cutting-planes, cutting each time four members, two of which are unknown, and lines drawn parallel to these will form each time a four-sided polygon, with the known stresses. $R - \Sigma w$ is laid off upwards in finding bottom-chord and diagonal stresses, and downward in finding top-chord and vertical stresses, and noting that diagonal and chord stresses are shown on diagram c by corresponding and parallel lines, but that the vertical stresses are the differences between the vertical lines closing the four-sided polygon formed by the lines representing the stresses in the upper and lower chord and the diagonal cut by the same plane which cuts the vertical and the value of $R - \Sigma w$ for that plane. $R - \Sigma w$ in these cases is laid off downwards: if this is the greatest, the difference acts downwards and indicates compression; if the smallest, the difference acts upwards and indicates tension. Bearing this in mind, the completion of the diagram is simply a repetition of what has gone before, and only results will be given for the remaining members. The lines on the diagram show the members of the truss to which they correspond; the arrow-heads indicate the direction and consequently the kind of stress. $R - \Sigma w$ = 28.44 tons = $H_1I = 1.9$ in.; $Ic_3 = 5.2$ in. $\times$ 15 = 78.0 tons tension in 12,11; c_2c_3 = tension $d11 = 0.57$ in. $\times$ 15 = 8.6 tons. $R - \Sigma w = 31.595$ tons; compression $e12 = 31.595 - II_1 = 31.595 - 1.7 \times 15 = 6.095$ tons; compression $de = 5.75 \times 15 = 86.25$ tons. $R - \Sigma w = 36.565$ tons = $I_1K = 2.438$ in.; Kc_4 = tension 13,12, = 4.4 $\times$ 15 = 66.0 tons; c_3c_4 = tension $c12 = 1.1 \times 15 =$ 16.5 tons. $R - \Sigma w = 39.69$ tons; c_3K_1 = compression in $cd =$ 5.375 $\times$ 15 = 80.625 tons; compression in $d13 = 39.69 - 2.15 \times$

15 = 7.44 tons. $R - \Sigma w = 44.69$ tons $= K_1M = 2.979$; $c_6M =$ tension in 14,13 $= 3.314 \times 1.5 = 49.7$ tons; $c_4c_6 =$ tension $b13$ $= 1.4 \times 15 = 21.0$ tons. $R - \Sigma w = 47.815$ tons; $c_4M_1 =$ compression $bc = 4.64 \times 15 = 69.6$ tons; compression $c14 = 47.815$ $- (MM)2.312 \times 15 = 13.135$ tons. $R - \Sigma w = 52.815$ tons $=$ $m_1N = 3.521$ in.; $Nc_6 = 1.267 \times 15 = 21.0$ tons, tension on 14,15; $c_6c_5 =$ tension $a14 = 2.3 \times 15 = 34.5$ tons. $R - \Sigma w = 60.94 -$ $w_1 = 55.94$ tons. $NN_1 = 2.438$ in. $\times 15 = 36.57$ tons; compres-

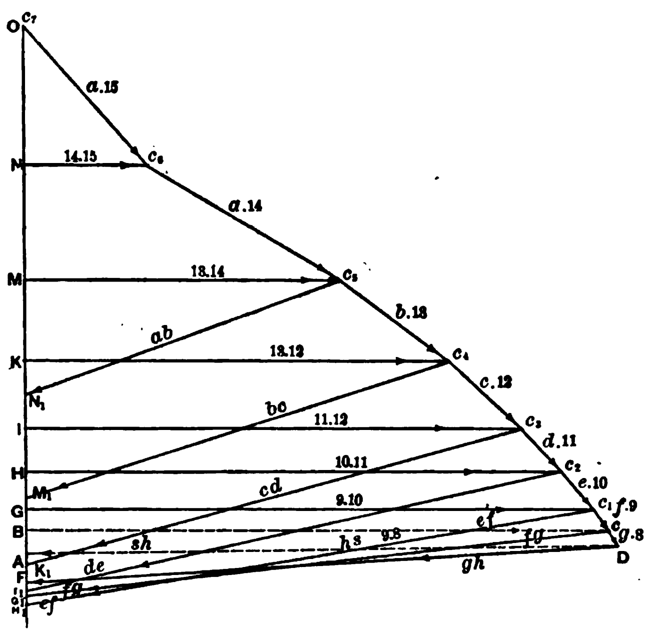


FIG. 859 (c).

sion $b15 = 55.94 - 36.57 = 19.37$ tons; c_6N_1 compression $ab =$ $3.513 \times 15 = 52.7$ tons.

Plane cutting ab, $a14$, $a15$, 16,15: $R - \Sigma w = 60.94 - 3.125 =$ $N,0 = 57.815$ tons $= 3.854$ in.; $c_6c_7 =$ tension $a15 = 2.12 \times 15$ $= 31.8$ tons. Stress in 16,15 = zero. This is evident, as the end member is vertical. Had it been inclined outward, there would be tension on 16,15 and 16,17 equal to the horizontal component of the stress in $17a$, these members are shown by dotted lines in 16,17 and $a17$. Stress on $a16$ = compression $= 60.94 + 3.125 = 64.065$

tons, as w rests at a. If $a17$ had been used instead of $a16$ as end strut, the compression on $a17 = 64.065 \times \frac{a17}{a16}$.

972. Owing to possible confusion of lines, errors in direction or scaling of lines in diagram Fig. 359 (c), checks should be made as the diagram is constructed. Upper- and lower-chord stresses can be readily checked by moments, as their lever-arms can be calculated or scaled from the drawing of the truss. The stress in the verticals are found in the vertical line OH_1 of the diagram, and is the vertical closing line of the four-sided polygon, formed by lines parallel to the upper- and lower-chord panels and diagonal and vertical intersected by the cutting-plane, diminished by $R - \Sigma w$ for that position of the plane and supposed to act downwards; the sign of this difference indicates the direction of the stress as shown in the preceding discussion. The difference in the lengths of the verticals can be calculated from the equation of the curve of the upper chord, or the lengths can be scaled direct from a large drawing. The lengths of the lever-arms of the stresses in the chord panels can also be determined by scaling or by calculation. To illustrate the method of checking the stresses by moments, or finding the stresses by the method of moments, we will assume a plane $c_{11}D_{11}$, one of the planes used in constructing diagram No. c. Cutting bc, $b13$, $c14$, and 14, 13, draw the following diagram, Fig. 359 (b'). Exact scaling is not here used and is not necessary. The upper-chord lines are taken as either the chord or tangent lines of the actual curves of the chord panels; the lettering is the same as in Fig. 359. The lengths of all of the members of the truss in feet are known. As the cutting-planes intersect four members, one of them must be known as to kind and amount of stress. Let this be the tension in $b13$ any one of them except the stress in the vertical $c14$ can be readily found, commencing at the centre of the span as in diagram Fig. 359 (c). Required to find stresses in bc, 14, 13 and $c14$. As the moments for each must be taken about an axis at the intersection of the other two, the lever-arm of the stress in bc is $n14 = c14 \cos n14c$; angle $n14c = cbm$, which can be found, as we know cm, cb, and bm; also angle m in the triangle cbm. Calling $n14c = \theta$, compression in $bc \times c14 \cos \theta =$ moment of external forces $= R \times 16, 14 - (w' + w_2 + w_1) \times 14{,}15$, from which stress in bc can be found. Tension in $14{,}13 \times c14 = R \times 16{,}14 - (w' + w_2 + w_1) \times 14{,}15$; from which tension in bottom chord 14,13 is found. Stress in $c14 \times O14 = - R \times O16 +$

$(w' + w_2 + w_1) \times O15 + (w' + w_1) \times O14$. We must find $O14$ and $O15$, which being substituted in the equation, we can find the stress in $c14$. From Fig. 359 (b'), $O14 : c14 :: bm : cm$. $bm = 14,15$; $cm = c14 - b15$; $\therefore O14 = \dfrac{c14 \times 14,15}{c14 - b15}$; $O15 = O14 - 15,14$. If the stress in chord panel 14,13 is known, the lever-arm of the stress in the diagonal $b13$ is $cn_1 = bc \times \sin cbn$, which is a known quantity. $\therefore$ stress in $b13 \times bc \sin cbn_1 = R \times 16,14 - (w' + w_2 + w_1) \times 15,14$. If the stress in $c14$ is known, the lever-arm of the stress

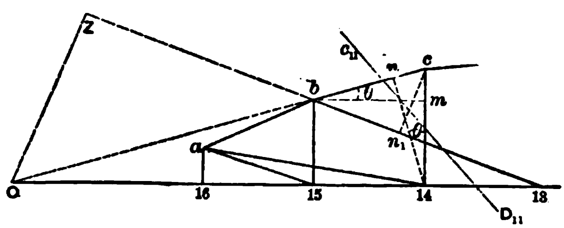


FIG 359 (b').

in $b13$ would be $ZO = O13 \times \sin O13Z$. Other combinations, depending upon which one of these stresses is known, will readily suggest themselves, as the lever-arms will be different for the unknown stresses in each case. Assuming that the stress in the diagonals is known, the following rules will give the lever arms of the stresses in any upper- or lower-chord panel and any vertical, without drawing separate diagrams for each.

1st. The lever-arm of any upper-chord panel is equal to the length of the vertical multiplied by the cosine of the angle made by the chord panel in question with a horizontal line.

2d. The lever-arm of the stress in any lower-chord panel is equal to the length of the vertical.

3d. The lever-arm of the stress in the vertical is equal to the product of the length of one bottom-chord panel by the length of the vertical itself, divided by the difference between the vertical in question and the next one on the side of the axis of moments (as O).

These rules apply to any case where the cutting-plane is placed as in the above case with respect to the members cut, and the diagonal stress is known. Other conditions will give different rules. The moments of the external forces R and Σw vary from panel to panel, of course.

972½. Whether the stresses above determined by moments will be tension or compression will depend upon the relative value of the moments of R and Σw and the direction of these moments. The moment of the stress will have the sign of the smaller of the two. If, as is usually the case, the portion of the truss taken is less than one half of the span, and on the left of the cutting-plane and centre of the span, the moment of R will be the greater. It may, however, be negative or positive: negative if the axis is on the left, as at O, and positive if the axis is to the right, as at c or 14. In this latter case, as R is positive and greater than Σw, the moment of the stress in bc is negative, and then the stress must act inwards on c_1b, Fig. 359 (d), and consequently must be compression. The moment of the stress in 14,13 must also be negative with respect to axis at c, and the stress must act outwards from point 14, causing tension. Also, moment of stress in b 13 with respect to axis at c must be negative, and the stress acting outwards and downwards from n_1b causes tension at b13. If, however, the axis is at O, moment of R is negative, moment of stress in b13 positive, still requiring the stress to act outwards from n_1b. Also, in this case, the moment in c14 being positive, the stress must act downwards on c14, causing compression. The stresses found by this method should agree practically with those found graphically in diagram 359 (c).

Also the web stresses can be checked by recollecting that the vertical component of the stress in any diagonal must be equal to the algebraic sum of the stress in the vertical which meets it at its lower extremity and the load which hangs from the same point. The algebraic sum is taken, since in some of the verticals the stress is sometimes tensile; with parallel and horizontal chords it would always be the numerical sum: which is likewise true with all but a few verticals in trusses with curved upper chords. These checks should be used at several points to prevent errors arising and accumulating.

ROLLING-LOAD STRESSES.

973. To determine the stresses due to the rolling load assume the load to come in from the right and rest successively at different panel points (Fig. 359), when the load reaches the point 3, the foot of the counter-brace $f'3$, which is the first one needed; then, according to the third assumption, only the diagonals $f'3$ and those (nearly) parallel to it on the left, all of the way to R,

the left-hand reaction will act, and all verticals except $e'4$, $d'3$, and $c'2$ will act, and on the right only the diagonals $b'3$, $a'2$, and $a'1$, and the verticals $b'1$ and $a'0$. (See Fig. 359.) A plane xy cuts $e'd'$, $f'3$, and 4,3. There will be three panels of uniform load and excess loads $= 3(w + e)$, the resultant acting at 2; hence reaction R on the left $= 3 \times 17.0 \times \frac{2}{16} = 6.375$ tons $= R$. $R - \Sigma w = 0$ at 3, or head of moving load. Taking moments about 3, stress in $e'd' \times n3 = 6.375 \times 162.5'$. $n3$ by scale $= 26$ ft.; compression $e'd' = 39.46$ tons. Diagram 359 (b_1) gives the force polygon, in which CD parallel to $f'3 = 1.269$ in. $\times 15 = 19.0$ tons acting downwards, is tension. The letters indicate the corresponding members of the truss, the arrow-heads the direction of the stress. Until the rolling load covers the entire span the chord stresses are of no value, except in so far as they may be necessary to determine the stresses in the diagonals and verticals; x_1y_1 cuts $f'e'$, $f'3$, $g'4$, and 5,4; $f'3$ is known; $f'e'$ can be found by moments; rolling load at 4, $R - \Sigma w = 0$ at 4. Lever-arm of $e'f' = 30$ ft. by scale, hence compression in $e'f' \times 30 = 4(w + e) \times \frac{2.5}{16} \times 150' = 10.625 \times 150'$; $\therefore$ compression in $e'f' = 53.15$ tons; $R = 10.625$ tons, and $f'3$ is not supposed to act with this position of moving load, $g'4$; tension $= 1.6 \times 15 = 24$ tons. (See Fig. 359 (b_2).)

If the plane cuts $g'f'$, $g'4$, $h'5$, 6, 5, rolling load at 5, then $g'4$ does not act. Reaction $R = 5w \times \frac{3}{16} + 4e \times \frac{3\frac{1}{2}}{16} = 15.69$ tons. Lever-arm of $g'f' = 32.5$; compression in $g'f' = 15.69 \times 137.5 \div 32.5 = 66.4$. Diagram Fig. 359 ($b_3$) gives stress in $h'5$ = tension $= 1.9$ in. $\times 15 = 28.5$ tons. Up to the point 5, $R - \Sigma w$ has become zero at the head of the train, and the above diagrams give the stresses in the counters. When the load reaches the point 6, $R = 6w \times \frac{3\frac{1}{2}}{16} + 4e \times \frac{4\frac{1}{2}}{16} = 21.56$ tons. As a panel weight at the head of the train is $w + e = 17$ tons, $R - 17 = 4.56$ tons; it is seen then that $R - \Sigma w$ becomes zero at the point 5, and according to the second assumption the acting diagonals on the left will be $5h'$ and those sloping upwards and outwards or in the same direction as $h'5$, and on the right $5d'$ and those sloping the same way. Taking a plane cutting $g'f'$, $h'5$, and 6,5, and taking moments with respect to axis at 5, compression in $g'f' = R \times 137.5' - (w + e) \times 12.5 \div 32.5 = 84.38$ tons. When $R - \Sigma w$ is zero at the head of the train, the only external force is the reaction; but when it is zero at some point in the

rear, there will be one or more panel loads on the same side of the cutting-plane as R, and the moment of these loads must be deducted from the moment of the reaction. In the above case there was but 1 panel load $(w + e)$, and its lever-arm was 1 panel $= 12.5$ feet. The moment of the stress in $g'f'$ is equal to the difference; where there are more loads, as 2,3, or $4(w + e)$, the moment of the resultant must be deducted, and for the same reason $(w + e)$, $2(w + e)$, $3(m + e)$ must be deducted from R for the resultant vertical forces. For the above position of the load and cutting plane we can then construct the following diagram, 359 (d). Draw a vertical line O_1O, and at some point draw a line parallel to $g'f' = \frac{84.38}{20} = 4.219$ inches. A scale of 20 tons to the inch will be used for convenience; for accuracy it should not exceed 10 tons. $R - w + e = 4.56$ tons, equal to AB; the remaining lines parallel to $h'5$ and 6,5, marked with the same letters, are the stresses giving tension in $h'5 = 0.9$ inch $\times\ 20 = 18.0$ tons. Now cut $h'g'$, $h'5$, $g'6$, and 6,5; draw $g'h'$ parallel to same line in truss meeting the vertical at c: it is compression in $h'g'$. $BC - AB =$ stress in $g'6 = 0.19 \times 20 = 3.8$ tons; this acting upwards is tension, as $AB = 4.56$ tons acts downwards and BC upwards. The next plane cuts $h'g'$, $h'5$, $s6$, and 7,6. This position of the plane leaves $R = 21.56$ tons $= 1.07$ inches as the only external load. Lay this off from C to D.h' 5 is known, and $g'h'$ cuts the vertical at C. Then drawing lines parallel to 7,6 and $s6$ from D and the upper extremity of $h'5$, intersecting in F_1, FF_1 is the stress in $s6 = 0.75$ inch $\times 20 = 15$ tons, which, acting downwards, is tension. Next cut sh', $s6$, $h'7$ and 7,6; as $s6$ and 7,6 are known, draw from F a line parallel to $sh' = FG =$ compression in sh'; this line cuts the vertical in G. GD acts upwards and R downwards from D to C; therefore $DC - GD = 1.07 - 0.56 =$ 0.51 inch $\times\ 20 = 10.2$ tons, which must act downwards, as R is the greater and laid off downwards. GC is then the compression in $h'7$. The vertical component in $s6$ would evidently be the compression in $s8$ if the upper chord was horizontal; but this is diminished by the upward components in the two panels sh and sh' at the centre; therefore if we draw from F_1, the upper extremity of $s6$, a line parallel to the panel sh on the left of s, as F_1G_1, G_1G, will be the compression in $s8$, as it must act downward, and equal 8.8 tons. The vertical components of sh and sh' are found in DG_1 and GB. Let the rolling load rest at 7. Reaction $R = 7w \times \frac{4}{16} + 4e \times \frac{5\frac{1}{2}}{16} = 28.25$ tons; as two panel loads would be 34 tons, $R - \Sigma w = 0$

at panel point 6. Therefore $s6$ and those diagonals on the left sloping the same way will act, as also $6e'$ and those sloping the same way on the right. Take a plane cutting $h'g'$, $s6$, and 7,6. Lever-arm of $h'g'$ with respect to axis at 6 = 34.0 feet. Compression in $g'h' = R \times 125 - (w + e) \times 12.5 \div 34 = 97.61$ tons. $R - w + e = 11.25$ tons. Proceeding as before, we construct diagram

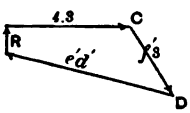


FIG. 359 (b_1).

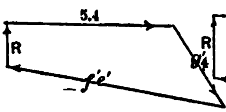


FIG. 359 (b_2).

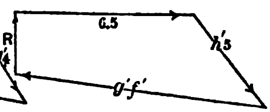


FIG. 359 (b_3).

359 (e). Draw $AB = \frac{11.25}{20} = 0.562$ inch, also $AF_0 = \frac{97.61}{20} = 4.88$ inches parallel to $g'h'$; $F_0F = 1.05 \times 20 = 21$ tons = tension, $s6$. Next a plane cutting sh', $s6$, $h'7$, and 7,6; draw F'_0C parallel to sh'; then $AC = 0.75 - 0.562 = 0.188 \times 20 = 3.760$ tons = tension on $h'7$, and acting upwards. Cut sh', $s6$, $h7$, and 8,7; lay off $CD = R = \frac{28.25}{20}$, and draw FF_1 and DF_1. FF_1 = stress in diagonal $h7$ = 0 8 inch $\times$ 20 = 16.0 tons tension. Then cut the left-hand panel, hs, $h7$, $s8$, and 7,8. Draw FG parallel to hs on the left of the cen-

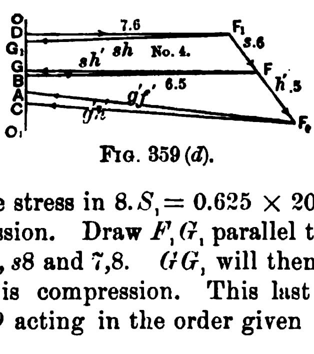


FIG. 359 (d).

tre. GC will be the stress in $8.S_1 = 0.625 \times 20 = 12.5$ tons, acting downwards, compression. Draw F_1G_1 parallel to gh on the left; the plane cutting gh, $h9$, $s8$ and 7,8. GG_1 will then be the stress on $h9$ acting downwards is compression. This last stress polygon is $DF_1F_1G_1GG_1GCCD$ acting in the order given as shown by arrowheads.

Let the rolling load reach the point 8, which is the centre of the span. $R = 8w \times \frac{4\frac{1}{2}}{16} + 4E \times \frac{6\frac{1}{2}}{16} = 35.75$ tons, as this is greater than 34.0 tons two panel loads. $R - \Sigma w = 0$ at the point 6, and $2(w + E)$ will be in front of the point 6. The first cutting-plane cuts $h'g'$, $s6$, and 7, 6, the same diagonals acting as in the last case. Lever-arm of $h'g'$ = 34.0 ft. with respect to axis at 6; hence compression in $g'h' = R \times 125 - 2(w + E) \times 1\frac{1}{2} \times 12.5 \div 34.0$

$= 112.68$, and $R - 2(w + E) = 1.75$ tons. Diagram [illegible] this position of the load is constructed in the same [illegible] before. First draw a line parallel to $g'h'$ meeting a [illegible] at A; then a distance upward $= 1.75 \div 20 = 0.09$ in.; $F_1A = \dfrac{112.68}{20} = 5.634$ in.; and then lines parallel to $s6$, and [illegible] lines to scale are the stresses. $F_0F =$ tension in $s6 =$ [illegible] $= 10.0$ tons. Next plane cuts sh', $s6$, $h'7$, and 7.6. Tension [illegible] $= BC - AB = 6.26 - 1.75 = 4.51$ tons $=$ tension. Next plane cuts sh', $s6$, $h7$, and $8,7$. $CD = 35.75 - w + E = 18.75$ tons; [illegible] draw FF_1 and DF_1. Tension $h7 = 0.75 \times 20 = 15.0$ tons; [illegible] hs, $s8$, $h7$, $7,8$. $GC = CD - GD = 18.75 - 15.0 = 3.75$ tons, acting downwards, is compression on $s8$. Now cut sh, $g8$, $h7$, and [illegible] Lay off $GG_1 = \dfrac{35.75}{20} = 1.788$ in. $F_1F_2 =$ tension on $g8 = 1.2 \times 20 = 24$ tons. Cut $gh, g8, h9, 9, 8$. From F_1 draw F_1H, stress in [illegible] and $HG = 0.5 \times 50 = 10.0$ tons, acting downward, is compression in $h9$. Then cut $f, g, g10, h9, 9, 8$. Draw F_2H_1 parallel to fg. HH_1 acting downward is compression on $g10 = 0.85 \times 20 = 17.0$ tons.

As the rolling advances, an exactly similar diagram is constructed for each point—a convenient order for the cutting planes is: First, a vertical plane in the panel immediately in front of the panel point at which $R - \Sigma w$ is zero. This, under the assumption as to acting web members, cuts only three members, one upper and one lower chord panel and one diagonal, for which a four-sided stress polygon is constructed, the vertical side being $R -$ (0, 1, 2, or 3) $(w + E)$, as the case may be. The next, an oblique plane cutting the same lower-chord panel and diagonal; but the vertical and upper-chord panel is in advance of the first plane: from this is found the stress in the vertical, which is equal to the difference between the first vertical line and that found between the horizontal and the line parallel to the last upper-chord panel cut. The difference will act downwards or upwards, as the first vertical is greater or less than the second: if less, downwards; if greater, upwards.

The third plane is vertical, cutting the same upper-chord panel and diagonal as before, but a bottom-chord panel and diagonal in advance; a single four-sided polygon constructed on the other two stresses enables us to determine the two unknown stresses. The vertical is either R, or R reduced by one panel less load than before.

The fourth plane is an oblique plane cutting a new upper chord

and a new vertical, but the same lower chord and diagonal as in the third plane; a similar diagram to that formed for the second plane gives the vertical and upper-chord panel stress. The fifth plane cuts a new upper chord, a new vertical, and the same diagonal and lower chord as in plane 4, plane 5 being oblique and between and parallel (nearly) to the acting diagonals. The above order of planes will never cut more than two acting members and sometimes only one.

After the rolling load reaches the centre and passing covers more than half of the span, it is not necessary to determine so many

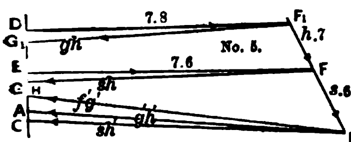


FIG. 359 (e).

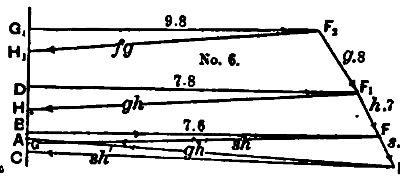


FIG. 359 (f).

diagonals and verticals for each position of the head of the moving load, as the maximum stresses in diagonals and verticals will generally be found at the head of the moving load; they will, however, enter into the diagram. But for the diagonals and verticals near the centre the stresses in all must be determined for each position of the load, as they may be called upon to act either as tension or compression members alternately, and must be counterbraced. Where more than one stress is found for the same member, the greatest of each kind must be selected in proportioning the dimensions and connections of the member, the remaining being ignored. The necessary counterbraces on one side are all determined before the load reaches the centre of the span; corresponding ones must be introduced on the other side for the load coming in the opposite direction. The chord stresses, as already mentioned, are of no value in these diagrams, except as they are necessary parts of the diagrams. As acting members these stresses are not maxima, until the entire span is covered. It will not be necessary to give other diagrams, but one more will be given for a point selected at random (Fig. 359 (g)); all others will be similarly constructed. Selecting the point 13 for the head of the moving load, $R = 13w \times \frac{7}{16} - 4e \times \frac{11\frac{1}{2}}{16} = 85.44$ tons, as $5 \times w + 4e = 81.0$ tons; $R - \Sigma w = 0$ at the 6th panel point to the rear, which is the point 8;

therefore the acting diagonals will be $8g$ and those sloping the same way on the left, and $8g'$ and those sloping the same way on the right. These are all main braces, no counters supposed to be acting.

1st plane cutting hs, $g8$, and 9,8. The lever-arm of hs is 34.95 ft. with respect to 8. The external forces are R_1, acting upwards; its lever-arm is 8 panels = 100 ft., and $4(w + e)$ acting downwards; lever-arm = $3\frac{1}{2}$ panels = 43.75 ft., and one w, acting downwards; lever-arm = 1 panel = 12.5 ft.; hence compression in $hs = [85.44 \times 100 - 4(w + e) \times 43.75 - w \times 12.5] \div 34.95 = 154.69$ tons. To avoid such long lines, the scale for this diagram Fig. 359 (g) will be 30 tons to the inch; this would be too small a scale for accuracy.

Draw a vertical line OO_1; then draw $F_0A = \dfrac{154.69}{30} = 5.156$ in. and parallel to sh; from A upwards draw a line $AB = 85.44 - 81.0 \doteq 4.44$ tons $= \dfrac{4.44}{30} \doteq 0.148$ in. Then, in the closing lines F_0F and BF, F_0F is tension in $g8 = 0.125$ in. $\times 30 = 3.75$ tons, $BF =$ tension in 9.8.

2d plane gh, $g8$, $h9$, and 9,8. Draw F_0c parallel to gh; then $AC =$ tension on $h9 = 0.25 \times 30 = 7.5$ tons.

3d plane cuts gh, $g8$, $f9$ and 10,9. Lay off CG upwards $= R - 4(w + e) = 17.44$ tons $= 0.581$ inches, and draw GF_1 and FF_1 parallel respectively to 10,9 and $f9$. $FF_1 =$ tension in $f9 = 0.25$ inch $\times 30 = 7.5$ tons.

4th plane, fg, $f9$, $g10$, and 10,9. Draw FD parallel to fg. $CD =$ tension on $g10 = 0.25 \times 30 = 7.5$ tons.

5th plane cuts fg, $f9$, $e10$, 11,10. Lay off $DG_1 = R - 3(w + e) = 34.44$ tons $= 1.15$ inches, and draw G_1F_2 and F_1F_2. $F_1F_2 = 0.375 \times 30 = 11.250$ tons, tension in $e10$.

6th plane, ef, $e10$, $f11$, 10,11. HD acting downwards = compression on $f11$, = 1.9 tons.

7th plane cuts ef, $e10$, $d11$, and 12,11. $HH_1 = R - 2(w + e) = 51.44$ tons $= 1.715$ inches. Draw H_1F_3 and F_2F_3 parallel respectively to 12.11 and $d11$. $F_2F_3 =$ tension on $d11 = 0.85$ inch $\times 30 = 25.50$ tons.

8th plane cuts de, $e12$, $d11$, 12,11. Draw F_3K parallel to de. KD acting downwards $= 0.065$ inch $\times 30 = 1.9$ tons, compression.

9th plane cuts de, $d11$, $c12$, and 13,12. Lay off $KK_1 = R - w + e = 68.44$ tons $= 2.281$ inches, and draw K_1F_4 and F_3F_4 parallel respectively to 13,12 and $c12$. $F_3F_4 =$ tension in $c12 = 0.85$ inch $\times 30 = 25.5$ tons.

10th plane cuts *cd*, *d*13, *c*12, 13,12. Draw F_3M parallel to *cd*. MK = compression on *d*13 = 0.6 inch × 30 = 18 tons.

The 11th plane cuts *cd*, *c*12, *b*13, and 14,13. Lay off $MM_1 = R = \frac{85.44}{30} = 2.85$ inches, and draw M_1F_5 and F_4F_5. $F_4F'_5$ = tension on *b*13 = 1.85 inches × 30 = 55.30 tons.

The 12th cutting-plane cuts *bc*, *c*14, *b*13, 14,13. Draw F_4N parallel to *bc*, MN = 0.5 inch × 30 = 15.0 tons; compression *c*14.

The 13th plane cuts *bc*, *b*13, *a*14, and 15,14. Lay off $NN_1 = R$

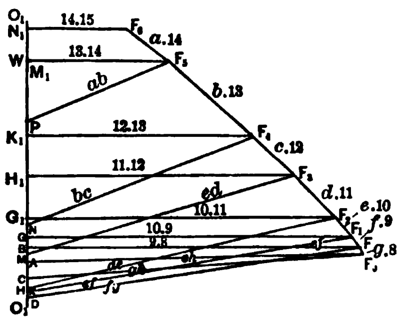


FIG. 359 (*g*).

− 2.85 inches. Draw M_1F_6 and F_5F_6. F_5F_6 − tension on *a*14 = 0.89 × 30 − 26.7 tons. Finally, cut *ab*, *a*14, *b*15, and 15,14. Draw F_6P parallel to *ab*. NP_1 acting downwards = compression on *b*15 = 1.562 inches × 30 = 46.86 tons. The only stresses of importance in the last diagram are the compression in *b*15 and the tension in *b*13. These are their greatest, and, it has been seen, the maximum stress in the main braces occurs at the head of the train, after the rolling load passes the centre, as was the case in trusses with parallel chords. The stresses, however, in many other members enter into the diagram, it is well to note the kind and amount of stress in the verticals near the centre of the span, as they need counterbracing, and it is important to know the maximum stress of each kind in them, which can only be done by obtaining the stress from the load at every point; the stresses being tabulated, the greatest can be selected. When the rolling load reaches the point 15, $R - \Sigma w = 0$ at the same point 8. A diagram similar in every respect to Fig. 359 (*g*) for the head of the load at 13 will be constructed, but in this case the chord stresses will be the greatest possible, and the lines in it parallel to the chord members must be scaled and reduced to tons. These added to the dead-load stresses

in the same chord members give the total stress. These can and should be checked by moments to avoid possible errors in direction and length of the lines in the diagrams. The method of moments has been fully explained and illustrated by examples.

There can be no doubt that the stresses, determined under the assumptions made, are not the actual stresses existing in the completed structure, but they are the greatest that can possibly exist in any member, as all the members omitted in any case will, when introduced, acts counter to the others, and will either relieve some or all of them. This of course results in the use of more material than actually required if the problem was determinate without the use of assumptions, and it may in general be stated that where ambiguities in the stresses exist there cannot result economy in material.

If the curved upper chord of the above-discussed truss be extended to intersection with the bottom chord, we have the type of truss known as the Bow-string girder; the same general methods are used for determining the stresses, and similar ambiguities exist. The bow-string girder is rarely, if ever, used for anything but highway bridges.

ART. LII.

THE CANTILEVER.

974. In trusses already discussed there are two points of support, and whatever may be the condition of loading, the reactions or supporting forces or resistances are determined by the principle of the lever. The reactions may be equal or unequal: the sum of the two must always be equal to the total load on the truss, and they are supposed to act vertically upward, while loads act downward. If the resultant of the loads be found and its point of application also, the reactions will be inversely proportional to the two segments into which the beam or truss is divided by the line of action of the resultant. If, then, the resultant be supposed to be separated into two parts, respectively equal to the reactions, there will be formed two couples. The forces of the couples are equal to the reactions, and their lever-arms are the segments of the span. One couple will be right-handed and the other left-handed, and their moments will be equal. If, then, we support the beam or truss at the centre, remove the end supports, and conceive the reactions to act downwards and the resultant upwards, without changing their lines of action or their magnitudes, we have the cantilever

proper. Two couples will still exist; their directions will be changed, however, and their moments will still be equal. The direction of the bending action is changed, and tends to make the upper chord convex and to be lengthened, and the lower chord concave and to be shortened, thereby producing tension in the upper chord and compression in the lower chord. The web members can be arranged and connected so as either to act as tension or compression members. Of two members meeting at a point the one must be a compression and the other a tension member. The two arms of the cantilever may be equal or unequal in length, as also the weights of the two arms, provided the moments of the two couples remain equal. If, as in the case of railway bridges, one arm carries a moving load and the other does not, this latter must be anchored or weighted down by a force the moment of which with respect to the point of support shall be equal to the difference between the moments of the loaded and unloaded arm about the same axis. Except for the members connecting the two arms over the point of support, each arm may be considered as fixed at the point of support. The following (Fig. 360) illustrates the last condition. The stresses are calculated as follows:

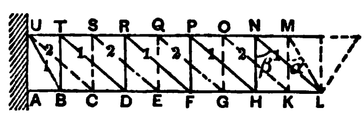


FIG. 360.

The diagonals in the above overhanging truss are tension members, the verticals compression members. By turning the truss upside down, making AL the top chord, or reversing the diagonals so as to slope upwards towards the free end, the kind of stress would be changed. All of the web members could be inclined, alternating as tension and compression members. The above design shows the common form used in what are called cantilever bridges, and is also one arm of a drawbridge when open. It may have either a single or double system of web members. The latter is used, but each system is supposed to be independent; the stresses due to each are first determined and the results added together. Either the upper or lower chord may be curved: if a through bridge, the upper chord; if a deck bridge, the lower chord. In very long spans it is now usual to build out from the two adjacent piers two projecting trusses towards each other, then to build between these an ordinary truss, its ends resting upon the ends of the cantilever arms, the greatest reaction at these ends being the end loads of the arms. Assuming the lengths of the panel to be 10 feet $= KL = GH = SR$, etc.;

depth of truss = 20 feet; diagonals ML and $UB = 22.36$ feet; diagonals NL, QK, etc., = 28.28 feet; tan $KML = 0.5$; sec $KML = 1.118$; tan $HNL = 1.0$; sec $HNL = 1.41$.

Upper-chord panel weight $= w$ ⎫ due to dead load.
Lower- " " " $= w_1$ ⎭
Load transmitted to or the reaction at $L = R$.
Rolling loads at K, H, G, F, etc., $= w'$.

Assuming that NL receives the entire load at L, extreme length of span $AL = 90$ feet, angle $KML = \alpha$, $HNL = \beta$, and rolling load is supposed to extend entirely over AL. The stresses are then easily written for the vertical and diagonal members. For the first system we have stress in $NL = R \sec \alpha$, tension; stress in $NH = R + w$, compression.

Tension in	$PH = (R + w + w_1 + w') \times \sec \beta$	$= 31.02$	tons.
Compress. in	$PF = R + 2w + w_1 + w'$	$= 25.00$	"
Tension in	$RF = [R + 2(w + w_1 + w')] \times \sec \beta$	$= 47.94$	"
Compress. in	$RD = R + 3w + 2w_1 + 2w'$	$= 37.00$	"
Tension in	$TD = [R + 3(w + w_1 + w')] \times \sec \beta$	$= 64.86$	"
Compress. in	$TB = R + 4w + 3w_1 + 3w'$	$= 49.00$	"
Tension in	$UB = [R + 4(w + w_1 + w')] \times \sec \alpha$	$= 64.84$	"

For the second system:

Compression in	$MK = w$	$= 3.00$	tons.
Tension in	$OK = (w + w_1 + w') \times \sec \beta$	$= 16.92$	"
Compression in	$OG = 2w + w_1 + w'$	$= 15.00$	"
Tension in	$QG = 2(w + w_1 + w') \times \sec \beta$	$= 33.84$	"
Compression in	$QE = 3w + 2w_1 + 2w'$	$= 27.00$	"
Tension in	$SE = 3(w + w_1 + w') \times \sec \beta$	$= 50.76$	"
Compression in	$SC = 4w + 3w_1 + 3w'$	$= 39.00$	"
Tension in	$UC = 4(w + w_1 + w') \times \sec \beta$	$= 67.68$	"

The numerical results are obtained from the following: $R = 10.0$ tons, $w = 3.00$ tons, $w_1 = 4.0$ tons, $w' = 5.0$ tons, $\sec \beta = 1.41$, $\sec \alpha = 1.118$.

For the chord stresses, if ML carries no load, NM is simply a beam 10 feet long carrying only its own weight. Recollecting that the stresses in the chord panels are the sums of the horizontal components of the stresses in the diagonals from the free end up to and including the diagonal in the panel under consideration, and that the components are the shears in the diagonals multiplied by the tangent of the angle of inclination, the shears being the sum

of the loads from the free end up to and including the point at the foot of the diagonal if in tension, and to the foot of the vertical passing through its upper extremity if in compression, we write the following general expressions, and also the numerical results with the data above given, and with $\tan \beta = 1$ and $\tan \alpha = 0.5$. As in the panels included between any two adjacent diagonals, the stress in the upper and lower chords is equal.

Tension in		Compression in				
ON	$=$	KL	$= R \times \tan \beta$	$=$	10.0	tons.
PO	$=$	HK	$= (R + w + w_1 + w') \tan \beta$	$=$	22.0	"
QP	$=$	GH	$= 2(R + w + w_1 + w') \tan \beta$	$=$	44.0	"
RQ	$=$	FG	$= [2R + 4(w + w_1 + w')] \times \tan \beta$	$=$	68.0	"
SR	$=$	EF	$= [3R + 6(w + w_1 + w')] \times \tan \beta$	$=$	102.0	"
TS	$=$	DE	$= [3R + 9(w + w_1 + w')] \times \tan \beta$	$=$	138.0	"
UT	$=$	CD	$= [4R + 12(w + w_1 + w')] \times \tan \beta$	$=$	184.0	"
		BC	$= [4R + 16(w + w_1 + w')] \times \tan \beta$	$=$	232.0	"

$$\text{Compression in } AB = [4R + 16(w + w_1 + w')] \times \tan \beta + [R + 4(w + w_1 + w')] \times \tan \alpha = 261 \text{ tons.}$$

975. The above results can be checked by moments, which will be applied to the bottom chord which contains the largest number of panels; and as the two systems are considered separately, and the axis of moments for any section must be the point of intersection of the diagonals and the upper chord, the axis of moments will be different for the two systems. Taking a vertical plane cutting PO, PH, QG, and FG, the axis of moments for the first system, marked 1, in Fig. 360 will be at P, and for the second system, marked 2, will be at Q; and similarly for any other position of the cutting-plane.

$$\text{Compression in } FG = [R \times 40 + (w + w_1 + w') \times 20 + 2(w + w_1 + w')] \times 30 \div 20 = 68.0 \text{ tons.}$$

Section through RQ and DE, axis of moments R and S:
Compression in $DE = (10 \times 60 + 24 \times 30 + 36 \times 40) \div 20 = 138.0$ tons.

Section through SR and DC, axis of moments T and S:
Compression in $DC = (10 \times 80 + 36 \times 40 + 36 \times 40) \div 20 = 184.0$ tons.

Section through BC and TS, axis of moments T and U:
Compression in $BC = (10 \times 80 + 36 \times 40 + 36 \times 50) \div 20 = 232.0$ tons.

Section through UT and AB, axis of moments at U:
Compression in $AB = (10 \times 90 + 48 \times 40 + 48 \times 50) \div 20 = 261.0$ tons.

These results agree exactly with the stresses obtained by the general formulæ above, which proves their accuracy and also the accuracy for the general values of the stresses in diagonals and verticals, as the expressions for the chord stresses are obtained from and are dependent upon them. The stresses in the diagonals and verticals could also be checked by moments, as no error could exist except in the numerical results; with the checks above given it is not necessary. If a section be taken parallel to the diagonals, cutting only vertical and horizontal members, such as *SR, RD, QE*, and *EF*, the only horizontal forces are the stresses in *SR* and *EF*. The conditions of equilibrium require that they should be equal and in opposite directions. This proves what was before assumed. With curved chords it would be necessary to use methods similar to those explained in the preceding example. Otherwise no difficulties exist. Even when this truss is used for a deck bridge, only the end diagonal is made a compression member, as shown by the dotted lines in Fig. 360; the other diagonals would remain as tension members.

CANTILEVER BRIDGES.

976. Cantilever bridges consist, then, of two overhanging trusses, of a greater or less length determined by theoretical and practical considerations, and a smaller span resting upon or suspended between their free ends. The loads thus transmitted are the values of the *R* used in the last formulæ. The projecting arms of the cantilever are balanced by similar arms extending in the opposite direction, which are connected with parts of another span, or anchored to piers or abutments in order to counterbalance the rolling load. For spans over 550 or more feet in length this type of bridge is probably usual, and some engineers use it even with very much shorter spans. Through-bridges require temporary supports, which consist of a frame in the form of trestlework for the ironwork of the bridge before its parts are fully connected so as to be self-supporting. This temporary frame is called the "false-work," and is removed after the span is connected and swung, as it is termed. It constitutes a considerable item in the cost of erecting bridges, and consequently is avoided as much as possible in crossing deep rivers or ravines, from the difficulty of securing safe foundations, and the cost resulting from this or the great heights required. It sometimes happens that it is impracticable to get intermediate supports, or the requirements of

navigation prohibit their use. In any of the above cases cantilever bridges may, or are required to, be used, as they can be erected without false-work, the overhanging trusses being self-supporting, as they are constructed over the pier and extend in opposite directions. These arms also serve to support the weights of the suspended trusses until they can be connected at the centre of the spans.

Cantilever and swing bridges will be briefly discussed in the following paragraphs. A few special problems will be given in the following application of the equations of equilibrium.

HINGED OR FREE-ENDED STRUCTURES—APPLICATION OF THE EQUATIONS OF EQUILIBRIUM.

977. The conditions of equilibrium or balance of structures and the equations expressing these conditions were fully discussed in Arts. XLIX and L. In applying these to structures we can consider (1) the structure as a whole balanced under the externally applied forces, or loads and reactions; (2) the equilibrium of the system of forces and stresses applied at each joint; (3) any portion of the structure separated from the other part or parts and balanced under the externally applied forces, and the stresses in the members cut or severed by the ideal plane or planes of division, the stresses in the members being replaced by equal external forces.

Since we can always obtain three equations of equilibrium, the external forces can always be found when not more than three are unknown. These equations are:

$$\left.\begin{array}{lll} \text{Algebraic sum of horizontal component,} & & \Sigma H = 0; \\ \quad\text{"} \quad\quad \text{"} \quad \text{" vertical} & \text{"} & \Sigma V = 0; \\ \quad\text{"} \quad\quad \text{"} \quad \text{" all moments,} & & \Sigma M = 0. \end{array}\right\} \quad . \quad (555)$$

If, then, we take a bridge truss, as a whole, acted upon by vertical forces or loads, in which case the truss is simply equivalent to a beam loaded normally to its length or longitudinal axis, the equation for bending moments, eqs. (508), (509), for a beam supported at both ends and uniformly loaded, is that of a segment of a parabola, the double ordinate of which is the length of the beam, and its vertex vertical below the centre of the beam, the vertical ordinate of the curve at any point representing the bending moment at the corresponding point in the beam. This is repre-

sented by the diagram Fig. 361 (a). In a trussed beam, or simply a truss, the vertical loads are assumed to be concentrated at the panel points. The equilibrium polygon will no longer be the line cd and the parabola ced, but the latter will be changed into a series of straight lines connecting the extremities of the vertical ordinates, the angles of this polygon being on the parabola as shown in Fig. 361 (a), the load being symmetrically distributed with respect to the centre. In this case it is evident that equations (555) are satisfied. As all loads are vertical by the principle of the lever, the reactions are not only vertical, but equal to each other, and together equal to the total load; hence $\Sigma V = 0$. Whether the truss rests on rollers at one end or not, the horizontal thrusts on the supports will be zero at both A and B, and $\Sigma H = 0$. If moments be taken about the point B, the moment of R'' is zero, the moment of R'

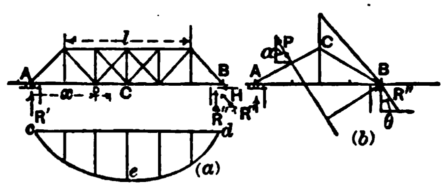


FIG. 361.

is $R'l$, and the sum of the moments of the loads on truss will be equal to the moment of the resultant load, Σw, which, acting at the centre point C, will be $\Sigma w \times \frac{1}{2}l$, and $\Sigma M = R'l - \Sigma w \times \frac{1}{2}l = 0$, since

$$R' = \tfrac{1}{2}\Sigma w = R''. \quad . \quad . \quad . \quad . \quad . \quad . \quad (556)$$

If the load is not symmetrically situated with respect to the centre of the truss, $\Sigma H =$ zero for the same reason as before, and $\Sigma V = R' + R'' - \Sigma w = 0$ from the principle of the lever, and the values of R' and R'' are found from the same principle, but will be unequal; or assuming that Σw acts at the point D distant x from A, and taking moments with respect to A, that of R' $= 0$ as before, and $\Sigma M = R''l - \Sigma wx = 0$, since by the principle of the lever $R'' = \Sigma w \frac{x}{l}$. Then $R' = \Sigma w - R''$, or, taking moments about the point B, $\Sigma M = R'l - \Sigma w(l-x) = 0$, since $R' = \Sigma w \frac{(l-x)}{l}$.

978. As an illustration of the action of oblique forces on a frame or truss, take the roof-truss, Fig. 361(b). The above principles

and equations are applied in exactly the same manner so far as the weight of the truss itself and any vertical forces or loads are concerned, but to make the solution general we will combine these loads with the action of the wind, the direction of which is taken as normal to the roof, calling P the normal wind pressure, and Σw the weight of the roof and its load acting through the centre or apex C, and calling forces acting upwards and to the right, and moments tending to produce rotation in the direction of the motion of the hands of a watch, positive, and when tending in the opposite directions negative.

If the end A of the truss rests on rollers moving without friction, the reaction R' at that end must be vertical, as there can be no horizontal thrust under that condition. At the end B or fixed end of the truss, however, the reaction R'' at that end will make some angle θ with the vertical. Its horizontal component will be R'' sin θ, and its vertical component will be R'' cos θ. Decomposing the wind pressure into vertical and horizontal components, respectively equal P cos α and P sin α, then

$$\left.\begin{array}{l} \Sigma H = P \sin \alpha - R'' \sin \theta = 0, \text{ and} \\ \Sigma V = R' + R'' \cos \theta - P \cos \alpha - \Sigma w = 0; \\ \text{and taking moments about } B, \\ \Sigma M = R'l - \Sigma w \times \tfrac{1}{2}l - Py = 0. \end{array}\right\} \quad (557)$$

From ΣM, $R' = \frac{\frac{1}{2}\Sigma wl + Py}{l}$; from ΣH, R'' sin $\theta = P$ sin α. Substituting value of R' in ΣV, $R'' \cos \theta = P \cos \alpha + \Sigma w - \frac{\frac{1}{2}\Sigma wl + Py}{l}$; then

$$\frac{R'' \sin \theta}{R'' \cos \theta} = \tan \theta = \cot \alpha + \frac{\Sigma w}{P \sin \alpha} - \frac{\frac{1}{2}\Sigma wl + Py}{lP \sin \alpha}. \quad (558)$$

From eq. (558) the angle θ can be found, which fixes the direction of R'', and substituted in ΣH determines R''. We therefore have R' and R'' in magnitude, direction, and points of application.

If a structure is composed of two or more trusses connected by hinges or pins about which they are free to turn, each portion between any two neighboring pins can be considered, so far as equilibrium is concerned, as an independent truss acted upon by

its own weight, and by any loads resting upon it or transferred to it from the adjacent trusses. To each part thus considered the above principles and equations can be applied, that is, the three equations (555) of equilibrium. Taking the curved truss or arch, Fig. 362, hinged at A, B, and C: there are two separate framed structures, AC and BC. At the point C we have a simple case of action and reaction, that is, equal, parallel, and directly opposed forces; and this is true whether the externally applied loads on the two portions are equal or unequal. The general arrangement of the forces or their vertical and horizontal components at the points A, B, and C is shown by the arrows, the double-headed arrows at C indicating the equal and opposite action of the forces upon the two halves.

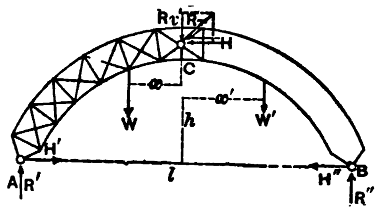


FIG. 362.

It is only necessary, however, to consider one half the whole structure, though where the external loads or forces are unequal on the two parts or not symmetrically distributed, both sides should be considered. In writing the equations of equilibrium we may not always be able to give the proper sign to some of the terms; and if we do not, the sign of the numerical result should indicate this.

Taking AC, the left-hand half of the truss, we write at once the three equations of equilibrium. There are only two horizontal and three vertical forces or components. Hence

$$\Sigma H = H' - H = 0; \quad \Sigma v = R' - v' - W = 0. \quad . \quad (559)$$

Moments about C,

$$\Sigma M = R' \times \tfrac{1}{2}l - Wx - H'h = 0; \quad . \quad . \quad . \quad (560)$$

and for BC, the right-hand half of the truss, and noting that the directions of the components at C must be reversed, then

$$\Sigma H = H - H'' = 0; \quad \Sigma v = R'' + v' - W' = 0; \quad . \quad (561)$$

and taking moments again about C, thereby reversing the signs of the lever-arms or moments of W' and R'' and H'',

$$\Sigma M = - R'' \times \tfrac{1}{2}l + W'x' + H''h = 0. \quad . \quad . \quad (562)$$

Or, if we choose, we can find the vertical components by considering the structure as a whole:

$$\Sigma v = R' + R'' - W - W' = 0. \quad . \quad . \quad . \quad . \qquad (563)$$

ΣM about A or B is zero; hence

$$\Sigma M = -W\times(-\tfrac{1}{2}l+x) - W'\times(-\tfrac{1}{2}l - x') + R''\times(-l) = 0; \quad (564)$$

from which R'' can be found, and substituted in eq. (563) gives R'. Or we may find R' from the ΣM about B:

$$\Sigma M = R'\times l - W(\tfrac{1}{2}l + x) - W'(\tfrac{1}{2}l - x') = 0.. \quad . \quad (565)$$

Let $l = 60$ feet, $x = 7.5$ feet, $x' = 15$ feet, $h = 30$ feet, $W = 3000$ pounds, $W' = 2000$ pounds. Then from eqs. (560) and (562) we have, since $H' = H''$, $R' \times \frac{1}{2}l - Wx = R'' \times \frac{1}{2}l - W'x'$; and from eqs. (559) and (561), since $v' = v'$, $+ R' - W = -R'' + W'$; and finding R' by combining these two equations we find $R' = 2375$ pounds, after substituting values of W, W', x, x', and l; then substituting R' in eq. (559), $v' = R' - W = -625$. The negative sign merely indicates that the wrong direction was given to v' in eq. (559), that is, v' should have been assumed as acting upwards, and the equation should have been written $\Sigma v = R' + v' - W = 0$, and eq (561) $\Sigma v = R'' - v' - W' = 0$; $\therefore R'' = W + v' = 2625$ pounds. As a check, taking the structure as a whole, and substituting in eq. (565), we obtain R' directly: $R' \times 60 - 3000 \times 37.5 - 2000 \times 15 = 0$; $\therefore R' = 2375$ as before, and from eq. (563) $R'' = 2625$, which substituted satisfies eq. (564), or R'' could have been found from this equation. Then from eq. (562) $H'' = \dfrac{R'' \times \frac{1}{2}l - W'x'}{h} = \dfrac{48750}{30} = 1625$ pounds $= H' = H$, or $H' = 1625$ pounds direct from eq. (560), which checks the accuracy of all the foregoing calculations.

979. In Fig. 364 is shown a structure composed of three trusses, AGB, $BEFC$, and CKD, hinged at the points A, G, B, C, K, D.

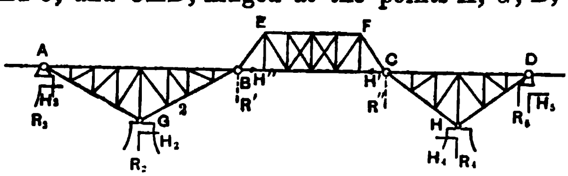


FIG. 363.

These frames or trusses can then be considered as independent trusses except in so far as any load or loads resting on the middle

truss are transferred to the other trusses at the points B and C, as determined by the principle of the lever. If, then, Σw is the load on this truss, and it acts at a point distant x from B, we have

$$R' = \frac{\Sigma w(l - x)}{l} \quad \text{and} \quad R'' = \frac{\Sigma wx}{l}. \quad . \quad . \quad . \qquad (565\tfrac{1}{2})$$

Then a portion of the load $\Sigma w = R'$ is to be taken as acting vertically downward on the truss AGB at its end B, and as this truss must now be balanced under all of the external forces acting on it,

$$R_3 + R_2 - \Sigma w' - R' = 0;$$

and taking moments about B, (566)

$$\Sigma M = R_3 l' + R_2 \times \tfrac{1}{2}l' - \Sigma w'(\tfrac{1}{2}l' - x) - R_1 \times 0 = 0.$$

Since R' has been found, we have two equations and two unknown quantities, namely, R_2 and R_3, the reactions at G and A, respectively. Combining these equations, we find

$$R_3 = -\frac{2\Sigma w'x + R'l'}{l'}, \quad \text{and} \quad R_2 = \frac{2R'l' + \Sigma w'l' + 2\Sigma w'x}{l'}. \quad (567)$$

Or, taking moments about A,

$$\Sigma M = -R' \times (-l') - \Sigma w'[-(\tfrac{1}{2}l' + x) + R_2 \times (-\tfrac{1}{2}l') = 0;$$

hence

$$R_2 = \frac{2R'l' + \Sigma w'l' + 2\Sigma w'x}{l'},$$

as above.

The negative sign in the value of R_3 merely indicates that the reaction at A is negative, or acts downward.

An exactly similar set of equations can be written out for the truss CHD. It is to be noted that $\Sigma w'$ simply means the sum of the loads acting directly on the truss under consideration, and does not include the load transferred from any other truss, as given by the values of R' and R'' in equation ($565\frac{1}{2}$) above, R' being taken as an independent load acting at the end B and $\Sigma w'$ as the resultant of the loads on the truss acting through their common centre of gravity. R_3 and its moment $R_3 l'$ were purposely taken with the wrong sign, in order to call attention to the significance of the signs in determining the direction of action of the forces. It is,

however, immaterial whether a negative sign shall indicate an upward or a downward force; but in all cases in this volume it indicates a downward action, and the positive sign an upward one.

Assuming that each of the panel points of the central truss is loaded with 4000 pounds, then the reactions $R' = R'' = 10,000$ pounds; also that the truss AGB is loaded at each of five panel points with a load $w = 10,000$, so that $\Sigma w' = 50,000$ pounds, and the resultant acts 10 feet to the right of the point G, and the span $AB = l' = 80$ feet. Substituting these values in equation (567), we find the reactions $R_2 = 82,500$ pounds, and positive; $R_1 = 22,500$, and negative. These values, together with that of R', substituted in equations (566), satisfy them:

$$R_1 + R_2 - \Sigma w' - R' = -22,500 + 82,500 - 50,000 - 10,000 = 0.$$

From the above example we see that the end A of the span must be weighted with 22,500 pounds, or it must be anchored down to the abutment, in order to maintain the structure in equilibrium. In general the direction of the reactions can be determined by inspection; but it is better to write out fully the equations of equilibrium and determine the values and directions of action of the unknown forces from them.

CANTILEVER BRIDGES.

980. A typical cantilever bridge truss is shown in Fig. 364. It may be conceived to be composed of three interdependent parts, namely, (1) the anchorage arm AB, (2) the cantilever arm BC, (3) the suspended span CD; on the right of D are corresponding cantilever and anchorage arms, similar to BC and AB, respectively. In the design selected it is only necessary to consider the three parts mentioned, as the structure will be discussed from D to the left up to A. Similar or identical conditions exist if the parts from C to the right are considered.

The suspended span $CKLD$ is under precisely similar condition as any non-continuous, through, or deck span, supported at C and D, or rather at K and L, by the suspending rods or bars CK and DL, respectively. In Fig. 364 it is simply a deck Pratt truss. Its top chord is in compression under all circumstances, and maximum stresses exist when fully loaded. The same remark applies to the bottom chord, except that it is under tension. The maximum web stresses occur when the head of the moving load reaches the upper extremity of the vertical, passing through the lower extremity of the diagonal in question, and the upper extremity of the

vertical in question the load covering the longer segment, and maximum stresses in end posts or diagonals occur when the entire truss KL is loaded. These stresses are determined as already fully explained in other paragraphs. The cantilever arm BC has its maximum chord and web stresses when the greatest possible reaction is at C, arising from loads on the suspended span CD, that is, when CD is fully loaded with the live load; and at the same time the live load extends from C on the cantilever arm up to the upper extremity of compression diagonals, or to the verticals passing through their lower extremities for tension diagonals, and to the upper extremities of verticals for maximum compression in them.

The anchorage arm AB has to be considered from two conditions of loading: (1) When there is no downward reaction at A, in which case loads upon BC or CD cause no stress in any of its members, only serving to fix the end of the truss along the line BG. It is then in the condition of any cantilever or projecting truss, and stresses in all members will be greatest when loaded, as fully explained in paragraph 974 and in Fig. 360, and will be found as there shown. (2) When the downward reaction is the greatest possible at A, this downward reaction is due entirely to loads on the portions BC and CD, and will be a maximum when there is no live load on AB, as any live load on AB tends to cause an upward reaction at A, the effect of which is to reduce the downward reaction due to loads on other portions of the structure. In this connection it is to be noted that this downward reaction causes or tends to cause tension in the upper chord AB and compression in the lower chord $AF \ldots D_0 \ldots G$, whereas the weight of the truss AB and any load upon it causes or tends to cause compression in the upper chord and tension in the lower chord, provided they are of sufficient magnitude to bring the truss to a bearing at A on its abutment. In this latter case the span AB becomes a simple truss supported at both ends, and the maximum stresses will be found as has been fully explained. Therefore stresses under both suppositions must be determined. Where they are of the same kind in any member, the sum of the two must be taken as an ultimate maximum; where they are of opposite kind, the member must be proportioned to bear either.

The greatest shears will evidently be affected by the magnitude of the reaction R. If, as in Fig. 364, a downward shear, which produces compression in AF_0 and $K'G$, and tension in F_0B', B_0D', D_0E', and E_0K', is called a main shear, any condition of loading which increases the downward reaction will increase the main shears, and consequently the stresses in the members mentioned; the maxi-

mum downward reaction is the maximum bending moment at B (which occurs when there is no moving load on AB, and BC and CD are fully loaded) divided by a, the length of span AB. The maximum main shear in any panel due to the load on AB is found when the live load centres at A and moves towards B, it being considered a non-continuous truss. To find, then, maximum main shears loads should be placed on BC and CD in positions which give maximum downward reaction R, and maintained at this while the moving load enters at A and advances to B. AB is treated as a non-continuous span. The maximum moving-load stresses thus found combined with the fixed-load stresses will give the resultant maximum web stresses.

It may happen, owing to the length of the anchorage arm, that the maximum downward pull R will be less than the upper reaction at A due to the live load and dead load, respectively. In such a case it will not be necessary to anchor the end of the span AB to the abutment at A, as the reaction will always be upward.

The stress in counters, such as $B'D_6$ if in tension, or $B'F_0$ in compression, or AF_0 in tension, will be found by assuming the moving load to advance from the pier at B to the extremity A of the anchorage arm, the span AB being a simple non-continuous truss, and no moving load on the cantilever arm or suspended span. Whether the posts or vertical members will receive their greatest stresses as main or web members can only be determined by trial. Those nearest the extremity of the anchorage arm will probably receive their greatest stress as counters, those nearest the main pier as main web members.

The foregoing gives a general explanation of methods of determining the stresses when the loads are distributed to give them maximum values. The following mathematical solution furnishes general formulæ applicable to all types of cantilever trusses, while establishing the truth of the statements made above:

Let a = length of anchorage arm AB;
c = length of cantilever arm BC;
l = length of suspended span CD;
x_1 = distance from D to centre of gravity of loads on l;
x = distance of centre of gravity of loads on cantilever arm BC from D;
x_2 = distance of centre of gravity of loads to left of U from U;
x_3 = distance of centre of gravity of loads to right of U from U;
d = distance between U and C;
p = panel length VU.

The values of x_1, x, x_2, and x_3 are found by the application of

the principle for centre of parallel forces (paragraph 204). $\Sigma' w_1$ is the summation $w_1 + w_2 + w_3 +$ etc., that is, all of the loads on the span BC. Σw_0 is the sum of the loads on CD.

The reaction R at A is determined by a simple application of the principle of moments. That portion of Σw_0 which is transmitted to C is, by the principle of the lever, $= R_2 = \Sigma w_0 \frac{x_1}{l}$.

Taking moments about B, that of R_1 is zero and the acting forces are R_2, and the load Σw_1 on BC, whose lever-arm, with respect to an axis at B, is $= l + c - x$. The lever-arm of R_2 is c, and that of R is a; hence

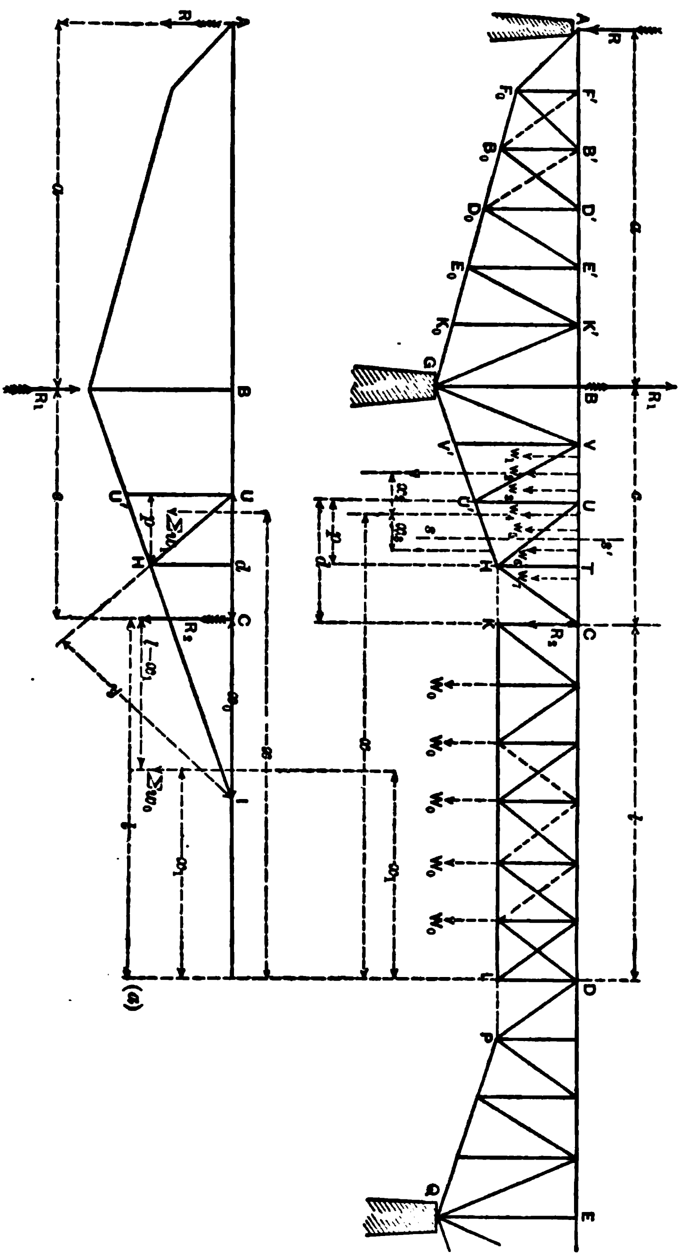
$$\left.\begin{aligned} R \times a &= R_2 \times c + \Sigma w_1(l + c - x); \\ \therefore\ R &= \frac{c}{a} \cdot \frac{x_1}{l} \Sigma w_0 + \left(\frac{l + c - x}{a}\right) \Sigma w_1; \\ R_1 &= R + R_2 + \Sigma w_1 = \Sigma w_0 \left(\frac{c}{a} + 1\right) \frac{x_1}{l} + \Sigma w_1 \left(\frac{l + c - x}{a} + 1\right); \\ R_2 &= \Sigma w_0 \frac{x_1}{l}. \end{aligned}\right\} \quad (568)$$

These are the reactions for the given conditions of loading, as seen in Fig. 364. We will first find the stress in the inclined web member UH of the cantilever arm due to this loading. Pass an ideal section SS', cutting the chords UT and $U'H$ and the diagonal UH. Considering only that portion of the truss to the left of the section and the loads and reactions corresponding, these will be the reaction R at A, the reaction R_1 at B, the loads w_1, w_2, and w_3; and, as usual, the loads in the panel cut are considered as concentrated at the adjacent panel points. By the principle of the lever, that portion of the loads w_4, w_5, and w_6 supported at U will be

$$w_4 + w_5 + w_6 - (w_4 + w_5 + w_6)\frac{x_3}{p} = (w_4 + w_5 + w_6)\left(1 - \frac{x_3}{p}\right).$$

In order to eliminate the moments of two of the unknown stresses, namely, that in UT and $U'H$, prolong the chords to intersection at I in Fig. 364 (a), and take moments with respect to an axis at I. The only stress whose moment is not zero is that in the inclined member UH. Prolong UH until a normal y can be drawn to it from the axis at I. Then acting moments are as follows ($CI = x_0$):

$$\text{Tension in } UH \times y = R_1(c + x_0) - R(a + c + x_0)$$
$$- (w_1 + w_2 + w_3)(x_2 + d + x_0) - \left[(w_4 + w_5 + w_6)\left(1 - \frac{x_3}{p}\right)\right] d + x_0).$$



ELEVATION OF CANTILEVER BRIDGE.

DIAGRAM FOR MOMENTS AND STRESSES.

FIG. 364.

The load w_7 to the right of TH is not considered when the cutting-plane is between U and T.

Substituting values of R, R_1, and R_2 from equations (568), cancelling similar terms with opposite signs, collecting and combining those terms having the same coefficients, we can write:

$$\left.\begin{aligned}\text{Tension in } UH = S = \frac{1}{y}\Big[&\frac{x_0x_1}{l}\Sigma w_0 + (x - l + x_0)\Sigma w_1\Big] \\ &- (w_1 + w_2 + w_3 + w_4 + w_5 + w_6)(d + x_0) \\ &- (w_1 + w_2 + w_3)x_2 + (w_4 + w_5 + w_6)(d + x_0)\frac{x_3}{p}.\end{aligned}\right\} \quad (568a)$$

In order that S may be a maximum or minimum, ΔS must be zero when x, x_1, x_2, and x_3 each vary by the same amount, all loads being moved forward by the small distance Δx. The variation of x_3 will have an opposite sign to that of x, x_1, and x_2.

$$S' = S + \frac{1}{y}\Big[\frac{\Delta x x_0}{l}\Sigma w_0 + \Delta x \Sigma w_1 - (w_1 + w_2 + w_3 + w_4 + w_5 + w_6)0$$
$$- (w_1 + w_2 + w_3)\Delta x - (w_4 + w_5 + w_6)(d + x_0)\frac{\Delta x}{p}.$$

This equation assumes the following form, when $S' - S = \Delta S = 0$:

$$w_1 + w_2 + w_3 + (w_4 + w_5 + w_6)\frac{d + x_0}{p} = \frac{x_0}{l}\Sigma w_0 + \Sigma w_1.$$

Since $\Sigma w_1 = w_1 + w_2 + w_3 + w_4 + w_5 + w_6$, there results

$$(w_4 + w_5 + w_6)\frac{d + x_0}{p} = \frac{x_0}{l}\Sigma w_0 + w_4 + w_5 + w_6. \quad (568b)$$

If there should be several maxima indicated by this last equation, the greatest must be determined by trial; that is, several conditions of loading may satisfy the conditional equation (568*b*). We see also from this equation that the loads to the left of the panel under consideration, namely, w_1, w_2, and w_3, do not enter into the conditions for a maximum, as explained in the general discussion. As the equation (568*b*) of condition involves the distance d between c and U with a uniformly distributed load, it must extend from C to U.

Having fixed the proper position of loads by equation (568*b*), the corresponding values of x, x_1, x_2, and x_3 can be determined by the principle for finding the centre of parallel forces; and proper substitutions made in equation (568*a*) will give the maximum value of shear S.

GREATEST CHORD STRESSES IN THE CANTILEVER ARM.

To find the greatest chord stresses in the panel $UTHU'$: For tension in the upper-chord panel UT the centre of moments will be at H, the intersection of UH and $U'H$, and the lever-arm of the tension in UT is $TH = n$. For the compression in $U'H$ the centre of moments will be at U, and the lever-arm will be normal from U let fall on $U'H = n'$. This can be determined by scale or by calculation. Taking the same ideal cutting-plane SS' and U as the centre of moments, the acting forces to the left of the section are the reactions R and R_1 and the loads w_1, w_2, and w_3, and the stress in $U'H = C_0$.

The moments with respect to U are as follows:

$$C_0 \times n' = R_1(c-d) - R(a+c-d) - (w_1 + w_2 + w_3)x_2.$$

Substituting values of R, R_1, and R_2 from equations (568) and reducing,

$$C_0 \times n' = \left[\Sigma w_0\left(\frac{c}{a}+1\right)\frac{x_1}{l} + \Sigma w_1\left(\frac{l+c-x}{a}+1\right)\right] \times (c-d)$$

$$-\left(\frac{c}{a}\frac{x_1}{l}\Sigma w_0 + \frac{l+c-x}{a}\Sigma w_1\right) \times (a+c-d) - (w_1 + w_2 + w_3)x_2$$

$$-\Sigma w_0\frac{dx_1}{l} - \Sigma w_1(l+c-x) + \Sigma w_1(c-d) - (w_1 + w_2 + w_3)x_2;$$

$$C_0 = \frac{1}{n'}\left[\Sigma w_1 x - \Sigma w_1(l+d) - \frac{dx_1}{l}\Sigma w_0 - (w_1 + w_2 + w_3)x_2. \qquad (568c)\right.$$

Allowing the loads to advance by the small amount Δx, then

$$C_0' = C_0 + \frac{1}{n'}\left[\Sigma w_1 \Delta x - \frac{d\Delta x}{l}\Sigma w_0 - (w_1 + w_2 + w_3)\Delta x\right].$$

For a maximum, $C_0' - C_0 = \Delta C_0 = 0$, and

$$\Sigma w_1 - \frac{d}{l}\Sigma w_0 - (w_1 + w_2 + w_3) = 0;$$

$$\Sigma w_1 - (w_1 + w_2 + w_3) = w_4 + w_5 + w_6;$$

$$\therefore \frac{d}{l}\Sigma w_0 = w_4 + w_5 + w_6. \quad . \quad . \quad . \quad . \qquad (568d)$$

This is the equation of condition that must be satisfied for position of loads producing maximum chord stresses. Several positions of loads may satisfy eq. (568d). The greatest must be determined by trial, then the corresponding values of x, x_1, x_2, and x_3; and the values w_0, w_1, etc., substituted in eq. (568c), give the maximum value of C_0'.

It will be noted that eq. (568d) is independent of w_1, w_2, and w_3, the loads between U and the main pier G; also, x_0 does not appear. It then applies to trusses with parallel chords as well as those with inclined chords. It should be noted that eq. (568b), of condition for maximum web stresses, does involve x_0; therefore for parallel chords x_0 is infinitely great, and the terms not involving x_0 may be omitted as of insignificant values. Eq. (568b) then becomes

$$(w_4 + w_5 + w_6)\frac{x_0}{p} = \frac{x_0}{l}\Sigma w_0;$$

and hence

$$w_4 + w_5 + w_6 = \frac{p}{l}\Sigma w_0, \quad . \quad . \quad . \quad . \quad (568e)$$

which is the equation of condition to be satisfied for position of loads giving maximum web stresses, and may be stated as follows: The position of loads for maximum web stresses in cantilever arms with parallel chords must be such that the sum of the loads on the cantilever arm between its extremity and the upper extremity of the diagonal in question is equal to entire load on the suspended span multiplied by a panel length of the cantilever arm and divided by the length of the suspended span.

CANTILEVER TRUSSES UNDER UNIFORMLY DISTRIBUTED LOADS.

In the case of uniform loads all the preceding general formulæ apply, but they can be placed in simpler forms.

R_2 will have its maximum value when the uniform load covers the entire span l; hence $\Sigma w_0 = wl$ and $x_1 = \frac{l}{2}$. The load extends from C to $u = wd$, and $x = l + \frac{1}{2}d$. There is no load to the left of U, and $x_2 = 0$. The load covers the panel UT, and $x_3 = \frac{1}{2}p$.

Making these substitutions in eq. (568), for reactions,

$$\left.\begin{aligned} R &= wl\,.\,\frac{c}{a}\,.\,\tfrac{1}{2} + wd\left(\frac{2c-d}{2a}\right) \\ &= w\frac{c}{a}\left(\tfrac{1}{2}l + d - \frac{d^2}{2c}\right); \\ R_2 &= w\frac{l}{2}; \quad R_1 = R + R_2 + wd; \\ R_1 &= w\frac{l}{2}\left(\frac{c}{a} + 1\right) + wd\left(\frac{c}{a} - \frac{d}{2a} + 1\right). \end{aligned}\right\} \quad . \quad . \quad . \quad (568f)$$

Eq. (568a) for maximum stress in UH becomes

$$S = \frac{w}{2y}[x_0(l+d) + (d-p)(d+x_0)]. \quad . \quad . \quad (568g)$$

For parallel chords terms not involving x_6 and with + or − signs can be omitted, while x_0 as a multiplier can be expressed in terms of y, and may be taken as $= y \sec \alpha$, α being the angle between the diagonal and a vertical line. Then eq. (568g) becomes

$$S = w\left(\frac{l}{2} + d - \frac{p}{2}\right) \sec \alpha. \quad . \quad . \quad . \quad . \quad (568h)$$

These equations apply to any panel in the cantilever arm by giving proper values to x, x_1, etc., d and x_6.

When all of the web members are inclined, as is the case in the Warren girder, it is necessary to provide for the condition that the panel points in upper and lower chord are not in the same vertical line, and changes similar to those made in paragraph 966 must be made in the formulæ.

ERECTION STRESSES.

In the preceding discussion only stresses arising from the weight of the structure and the loads upon it have been considered. But as cantilever bridges are constructed without falsework, the method of erection adopted is to build out, from each of the main piers G and Q, each cantilever arm and the adjacent half of the suspended span. It is evident, then, that the cantilever arms themselves will be subjected to the same kinds of stresses in each member as when completed, that is, the upper chords will be under tension and the lower chord under compression. The web members will be stressed according to the design of the truss, and no reversion in the kind of stress will occur in any member. The suspended span during process of erection will have stresses in all of its members of different kinds from those developed in the completed structure, as until it is connected at its centre each half will be a projecting or cantilever truss, and will have to sustain tension in top chord and compression in bottom chord. It will also be noted that the members LP and HK will be under compression during erection. These erection stresses are sometimes very great, and ample provision must be made to sustain them. Proper arrangements must be made for expansion and contraction at each end of the suspended span. For this purpose folding wedges or wedges and rollers removed after erection, in connection with oblong pin-holes, are used.

WIND STRESSES.

Wind stresses on cantilever structures should be computed with great care. The stresses in the chords and web members are to be

calculated in precisely a similar manner as for the fixed and live vertical loads on the structure. There will in general be found a horizontal reaction at A, the amount depending on the relative proportions of anchorage and cantilever arms and suspended span. This reaction must be provided for at A by such a connection between the anchor arm and the masonry as will at the same time permit of longitudinal contraction and expansion.

ECONOMICAL LENGTHS OF SPANS AND ARMS.

The suspended span may be advantageously taken equal to from 0.5 to 0.55 of the entire length between main piers G and Q, and the cantilever arm BC or DE equal to 0 25 of the same.

It has been found in the best practice that the anchorage arm AB should be 1.67 to 2 times the length of the cantilever arm BC, with the suspended span twice or a little more than the latter. Making $L =$ the full length of the entire structure $= 2a + 2c + l$, then, approximately,

$$a = 0.226L; \quad c = 0.113L; \quad l = 0.32L.$$

The foregoing discussion of cantilever structures follows the method given by Mr. Burr in his work on "Stresses in Bridge and Roof Trusses." The brief and general discussion of the types shown in Figs. 365 and 366 are on the lines laid down in Modern Framed Structures. For thorough discussions of cantilever structures reference should be made to the above works.

981. *Niagara River Cantilever Bridge.*—This bridge consists of two double cantilevers AB and CD, and the suspended span $CBD'E$. Each of the cantilevers has two points of support, namely, the steel piers and the abutments at A and D. Where greater length is required, other suspended spans could have one end of each suspended from A and D, respectively, the other ends of which could rest upon abutments. In this case the weights of these spans must balance the excess load upon the main span $FD'EG$. If, as in the Niagara Bridge, Fig. 365, the ends of the structure are at A and D, the equilibrium or balance is secured by anchoring these extreme ends to the abutments; or the weights of the cantilever arms AF and GD must be sufficiently great to balance the weight of the main span and its loads under all conditions.

Another arrangement of spans for a cantilever bridge is shown in Fig. 366, which is the design of the Kentucky and Indiana Bridge. In this there is a centre through-span resting on two

points of support, from each end of which project the cantilever arms, to the ends of which are suspended spans, the other ends of which are suspended from other cantilever arms. *CD*, the central through-span; *BC* and *DE*, the cantilever arms; *AB* and *EF*, the

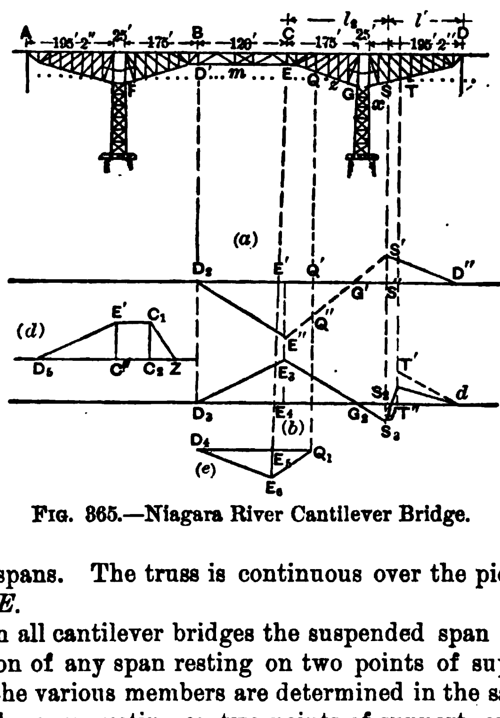


FIG. 365.—Niagara River Cantilever Bridge.

suspended spans. The truss is continuous over the piers *C* and *D* from *B* to *E*.

981½. In all cantilever bridges the suspended span is simply in the condition of any span resting on two points of support. The stresses in the various members are determined in the same manner as any similar span resting on two points of support, and from any

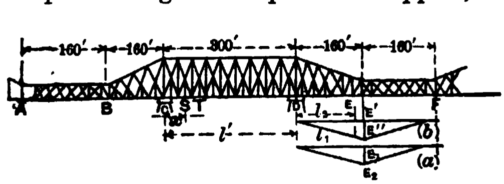


FIG. 366.

condition of loading the reactions are the resistances at the ends of the cantilever arms due to the loads on the suspended span. These reactions act at the ends of the cantilever arms, and must be combined with the loads, if any, on the cantilever arms in order to

determine the stresses in the members of those arms; and, as has been mentioned, proper loads or anchorage must be provided on the balancing arms of the cantilevers AF and GD, Fig. 365, and on the through-span CD, in Fig. 366.

In both forms of cantilever bridges shown in Figs. 365 and 366 there are but two supports for the cantilevers proper; hence the reactions can be readily found.

In Fig. 365 the bridge is hinged at the points B and C; consequently it is divided into three independent portions, namely, AB, BC, and CD.

The stresses in the members of the span BC are determined from the loads on it, including its own weight, with the loads in position to produce the maximum stresses.

Then this span must be loaded so as to give maximum reactions at B or C. This load, then, constitutes the end load on the cantilever arm DF or EG, which is then to be combined with the total load on and the weight of the arm itself. These arms are then in the condition of a cantilever fixed or balanced at one end, that is, at the points F and G. (See Fig. 360.)

The reactions at the points of supports A and F or G and D, are found by the simple application of the principle of the lever. These reactions and the loads being known, the shears and bending moments in any panel or at any section are easily determined by the application of the principles and equations already found.

In the form of truss shown in Fig. 366 the reactions at C and D are to be found by the application of the principles and equations established in the discussion of the continuous girder over two points of support. When the reactions are found the shears and bending moments at any section are found as already explained.

982. Cantilever bridges being used mainly for long spans, the double systems of web members are commonly used for the sake of economical construction.

Such systems do not admit of the use of the wheel-concentration method of loading, and some of the methods of equivalent uniform load, or uniform load with one or two excess or concentrated loads, are adopted; and it is usual to consider each system of web bracing to be independent, and the loads are assumed to be carried by the system to which it belongs. (See Fig. 360.)

The use of cantilever bridges is confined mainly to those situations or conditions which require very long spans, or where the difficulties and cost of erecting false-work upon which to build the

bridge are very great or, it may be, impracticable from several causes, such as over deep channels with rapid currents, and where the false-work would be a serious obstruction to navigation.

Having given the equations for maximum stresses and those for determing the positions of the moving loads required to produce them, paragraph 980, the writer will limit this discussion to finding the reactions at the points of support for a given condition of loading. These being found, the stresses resulting can be easily found by the application of the general principles for the determination of stresses in beams or trusses fixed at one end or supported at both ends.

983. *Niagara Cantilever Bridge* (see Fig. 365).—Since in this bridge there are no diagonals in the panels over the towers F and G, the trusses are free to turn at those points as if supported on a single pin, and as the suspended span is simply hung from the ends of the cantilever arms, we may consider the entire structure as composed of three independent trusses on each side of the bridge, namely, AB, BC, and CD. Each of the trusses, then, considered as a whole, must be independently balanced under the loads acting upon it.

Dead Loads.—The weight of the centre span BC being uniformly distributed, the stresses in the several members are to be found in the same manner as for any truss simply supported at both ends and uniformly loaded. Each reaction at B and C will be equal to one half the weight of the truss, or entire dead load, or one half of the dead load is transmitted and supported at each of the points B and C, and constitutes a single load acting on the cantilever lever-arms at these points. We then have two equal and similarly loaded trusses AB and CD, acted upon by one half of the weight of the centre span at their ends B and C, and a dead load equal to its own weight, assumed to be concentrated at and divided equally or otherwise, according to the design of the structure, amongst the several panel points. The stresses, then, in the two arms AF and FB are found as for the cantilever truss (Fig. 360), having one end fixed and the other free. The condition of equilibrium requires that the moments of the loads on the two arms with respect to the centre point F must be equal. And since a concentrated load exists at the point B on the arm FB, this must be balanced by an equal or an equivalent load on the arm AF. This may be accomplished by making AF similar in every respect to FB, and anchoring the end A to the abutment, or by a longer arm,

or a shorter arm, having the proper weight and distribution of the weight in order to develop the proper moment to bring about a balance at F.

The same requirements and conditions exist for the arms CG and GD.

Rolling Load.—For maximum positive moments on the arm GD this arm should be fully loaded with the rolling load.

The maximum positive moment in the arm CG and maximum negative moment in GD will occur when the portions of the trusses from B to G are fully loaded.

Let single rolling load w come in from the right and rest at D, then the moment at any section S is zero, not considering the dead load. If, then, we draw a horizontal line D_2D'', as shown in Fig. 365 (a), this will be the line of zero moments. Project the point D on D_2. As this load advances towards G the moment at S will be positive and increasing, reaching a maximum when the load is over S. The reaction at G will then be $w\frac{l' - x}{l'}$, and this multiplied by the distance from G to $S = x$ will be the bending moment at G; hence $M = w\frac{x(l' - x)}{l'}$. Lay off this moment under S as represented by $S'S''$, and join S' and D'; then the ordinate of the line $S'D'$ will represent the moment at S for the load at corresponding points in the truss directly above—the reaction at G must be determined for each position of the load. After the load passes S the moments again decrease until it rests above the centre of the tower at G, when the moment becomes zero at S, as shown by the moment-line crossing the zero line at G'. As the load passes G it causes a negative or upward moment at S, which becomes a negative maximum when the load reaches the point C, the end of the cantilever arm GE. The value of this moment is found by multiplying the reaction at D by its distance from S. The reaction is evidently found from the proportion $R : w$ (at C) as $l_2 : l'$. $\therefore R = w\frac{l_2}{l'}$; its lever-arm $l' - x$. Hence moment $= w\frac{l_2(l' - x)}{l'}$, and negative. Laying off this moment below the zero-line and represented by $E'E''$, and connecting E'' and S', any ordinate of this line will represent the moment at S with the load at points on the truss vertically above the ordinates; for example, the ordinate $Q'Q''$ represents the moment S, when the load is on the truss above Q. As the load passes to the left of C and rests on the suspended

span, its moment at S is still negative, but decreasing, as only a part of it is supported at C. When it reaches B it is carried entirely by the truss AB; its moment at S becomes zero, as indicated at D_2. Connect D_2 with E''. The ordinate of D_2E'' is the moment at S when the load is at a point on the suspended truss BC vertically above the ordinate on the truss. The line $S'E''$ is a straight line, since $\frac{S'S''}{E'E''} = \frac{x}{l_2}$.

It is evident from the diagram, Fig. 365 (a), that, if the entire arm GD is loaded, the area $G'S'D''S''G'$ will represent the maximum positive or downward moment S, and that when the suspended span BC and the arm CG are both loaded throughout, the maximum negative moment at S occurs, and is represented by the area $D_2G'E''D_2$. Therefore, to find the maximum positive moment at any section S in the arm GD, the entire arm GD must be loaded with the rolling load; but no load must be to the left of G. For maximum negative moment at any point S in GD the distance BG must be loaded throughout, but no load must be on the arm GD.

The diagram Fig. 365 (b) represents graphically the shears produced in the panel ST in the same manner as diagram (a) represents moments.

When the rolling load enters at D the shear is zero. As it passes on it will be the increasing reaction at G, until it reaches the panel point T to the right of S. When it passes T a part of w is supported at S, and when a point is reached at which the reaction at G is equal to the portion of the load transferred to S, the shear becomes zero, crossing the zero line dD_3; the shear then becomes negative, and has its maximum value when the load reaches S, after which the shear is equal to the reaction at D, and decreasing to zero, when the load reaches G. When the load passes G to the left it causes a negative reaction at D and a positive shear at S, increasing to a maximum when the load reaches the end of the arm C, and represented by E_3E_4, figure (b). Its value is $w\frac{l_2}{l}$. From that point on to B it decreases, and becomes zero when the load reaches B, as seen at D_3.

Therefore, for the maximum positive shear in any panel ST, the arm GD should be loaded from D to T, and also the entire distance from B to G, and is represented by the two areas $DT''y$ and $G_2E_3D_3G_2$. The maximum negative shear is represented by the area yS_3G_2, the load extending from some point between S and T to G.

For moments on the arm EG at any point Q it is evident that only that portion of the truss between B and Q should be loaded. With the load at B the moment at Q is zero. As the load advances towards C an increasing negative moment is developed at Q, due to that portion of w transferred to C, becoming a maximum when at C. As the load passes to the right of C, its moment at Q decreases, becoming zero when it reaches the point vertically above Q, a load between Q and G produces no bending at Q, while a load to the right of G would cause a moment in the opposite direction. The maximum moment, then, will be when the load extends from B to Q, and is represented by the area $D_4Q_1E_4$, Fig. 365 (c).

The positive shear in the panel Qz of one system of web bracing commences with the load at B, being zero for that point, and increasing to a maximum when the load reaches C. It then remains constant until the load reaches Q, after which it decreases as the load approaches z, and when at z the shear at Q again becomes zero, the diagram Fig. 365 (d) representing maximum positive shear at Q, its area being proportional to the sum of the reactions at C of all the loads on BC, plus the sum of the loads on EQ, plus the reactions at Q of the loads on Qz.

The several diagrams, Figs. 365 (a), (b), (c), and (d), are equally applicable to the design of the cantilever bridge shown in Fig. 366, if only that portion of the truss from A to D be considered, the central span CD of Fig. 366 corresponding with the shore arm GD of the truss in Fig. 365, the points S and T occupying the same relative position in the two cases. In Fig. 365 there is no suspended span at the end D, whereas in Fig. 366 there is a second cantilever arm DE and the suspended span EF. Therefore a load coming in from the right and passing F produces both negative shears and moments at any point S, both reaching a maximum when the load reaches E. The diagrams for these are shown in Figs. 366 (a) and (b), respectively. With the load at F, both moment and shear at S are zero. They gradually increase until the load reaches E, where they have maximum negative values. The shear in the panel ST is equal to the negative reaction at $C = W\frac{l_1}{l'}$, and the moment at G is $W\frac{xl_1}{l'}$, represented by the middle ordinates respectively, E_1E_2 and $E'E''$. As the load passes E these decrease and become zero when the load reaches D. For the other moments and shears exactly similar diagrams are to be constructed. For

maximum negative moments in CD both of the spans AC and DF should be loaded.

The discussion of these cantilever bridges is only entered into far enough to show the general effects of loads on different parts of the structure considered as a whole; for general formulæ see paragraph 980. For economy the suspended span should be made about four tenths of the total opening; but the longer this span compared to the total opening, the less will be the deflection. The maximum vertical deflection of the hinge C (Fig. 365) under the test load was about 9 inches. The corresponding deflection of a bridge with short cantilever arms and a long suspended span was only $3\frac{1}{2}$ inches.

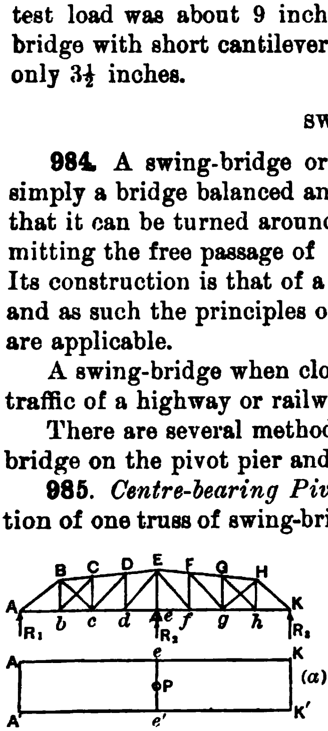
SWING-BRIDGES.

984. A swing-bridge or, as often miscalled, a drawbridge, is simply a bridge balanced and supported in such a manner on a pier that it can be turned around to the axis of a stream, thereby permitting the free passage of river craft on one or both sides of it. Its construction is that of a continuous girder of two or more spans, and as such the principles of continuous girders as fully discussed are applicable.

A swing-bridge when closed forms a continuous support for the traffic of a highway or railway across a stream.

There are several methods of balancing and supporting a swing-bridge on the pivot pier and on the piers at its ends when closed.

985. *Centre-bearing Pivot.*—In Fig. 367 is shown a side elevation of one truss of swing-bridge with three points of support at A, e, and K, and in Fig. 367 (a) a plan of the bridge. If the ends of the truss A and K do not touch or rest on the end piers or supports when only loaded with its own weight, there will be no end reactions, the truss being balanced over the centre, and the truss is simply in the condition of two cantilever arms balancing each other. If a moving load comes on one arm, Ae for example, this arm is deflected until a reaction is developed; the unloaded arm eK lifts, still having no reaction. If the live load covers both arms there may or may not be any end reactions. In the latter case there are, as before, two balanced cantilever arms. The bridge is then called a tipper.



FIGS. 367.

In the following discussion it will be assumed that when the bridge is closed the ends are raised by their supports.

By means of the principles and equations given, the moments, and consequently the reactions, for any condition of loading can be found. The two spans Ae and Ke are assumed equal, and the loads are taken as concentrated at the panel points b, c, d, etc.

Since the continuous girder is simply supported at the ends A and K, the moments at these points are zero.

The full discussion of the Theorem of Three Moments will not be given in this volume. For this the reader is referred to such works as Dubois, Burr, Johnson, and to Bender on "Continuous Bridges." (See Van Nostrand's Science Series, No. 26.)

From the principles and equations for continuous girders already discussed we learned (1) that continuity of girders developed moments in a vertical plane over the points of support, these moments being represented by couples whose lever-arm is the depth of the truss and whose equal and opposite forces are the tension and compression in the upper and lower chords respectively; (2) that the effect of these moments was to change the shears and bending moments resulting from any condition of loading from those caused by the same loading on any span, considered simply as a beam supported at its two ends; (3) that reactions must not be taken for shears at the piers, except at the extreme left- and right-hand piers of a series; the shears are the reactions from each span taken separately, whereas the reactions in intermediate piers are the sum of the shears over that pier arising from the two adjacent spans; (4) that while the shears or partial reactions from the loads on any span are not the same as for a truss simply supported at the ends, the sum of these shears is the same, and equal the total load on the span considered independently and separately, provided the supports are rigid, as is usually assumed; (5) that in a series of any number of spans the end spans are taken as in the condition of beams supported at one end and fixed at the other. Therefore the bending moments at the two extreme ends, where the beam or truss simply rests on its support, are zero. (6) Under this latter supposition it is always possible to obtain as many equations as there are unknown moments. Therefore the moments, shears, and reactions can always be found, no matter how many spans there may be.

986. To determine the moments, shears, and reactions at the piers of a continuous truss the Theorem of Three Moments is used,

—an equation of equilibrium expressing the relations between the moments at the piers and the loads of any three consecutive piers and spans between them, taken in order, as 1, 2, 3; 2, 3, 4; 3, 4, 5; etc. Or calling the middle pier of the three the rth, the one on the left will be the $r-1$th and the one on the right the $r+1$th pier. The left span will be l_{r-1} and the right span l_r. The moments at the piers will be, respectively, M_{r-1}, M_r, and M_{r+1}.

Then, for a uniformly distributed load on the first span of the intensity W_{r-1} and on the second of W_r, we write

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -\tfrac{1}{4}(W_{r-1}l^3_{r-1} + W_rl_r^3). \quad (568)$$

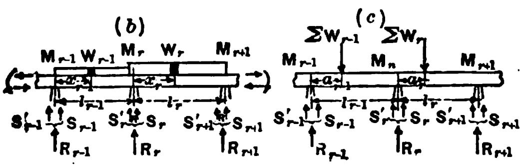
And for any system of concentrated loads on the two spans represented by ΣW_{r-1} and ΣW_r, respectively, when the distance of the centre of gravity of the loads on the first span is distant from the left of the three piers by $a = kl_{r-1}$ or $k = \dfrac{a}{l_{r-1}}$, we write

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r \\ = -\Sigma W_{r-1}l_{r-1}^2(k - k^3) - \Sigma W_rl_r^2(2k - 3k^2 + k^3). \quad (569)$$

The conditions of loading as shown in Fig. 368 (b) are expressed by equation (568), and those in Fig. 368 (c) by equation (569).

It is to be noted that at each pier there are two partial reactions, one from the load on each of the contiguous spans. These are the shears immediately to the left and right of the piers. The final or total reaction in each of the piers is the sum of the two shears at that pier.

Since the loads on the trusses of bridges are usually concen-



FIGS. 368.

trated at the panel points, equation (569) is the one that will be used in the following discussion. And as in determining the stresses in individual members it is necessary to consider the effect of each panel concentration separately, and then to combine these

separate results to obtain maximum stresses, we will consider only the single load W_{r-1}. Then $\Sigma W_r = 0$; $\Sigma W_{r-1} = W'_{r-1}$.

Referring now to Fig. 369, which shows one of the two trusses of a swing-bridge having a centre-bearing pivot consisting of two spans simply supported at the two ends, and $r = 2$, $M_{r-1} = M_1 = 0$, $M_{r+1} = M_3 = 0$. Then equation (569) becomes

$$2M_2(l_1 + l_2) = -W_1 l_1^2 (k - k^3);$$

$$\therefore M_2 = -\frac{W_1 l_1^2}{2(l_1 + l_2)}(k - k^3). \quad . \quad . \quad . \quad (570)$$

This gives the bending moment over the centre or pivot pier e.

To find now the reaction R_1 at A, conceive the truss to be cut by a vertical plane at Ee, removing the right-hand span and taking moments with respect to an axis in the cutting-plane. The acting moments are M_2, that of R_1 and that of the load W distant from $A = a$; $l_1 = Ae$. This condition is shown in Fig. 369, the moment of R_2 being zero. Then

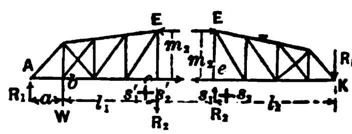


FIG. 369.

$$M_2 = R_1 l_1 - W(l_1 - a); \;\therefore\; R_1 = \frac{W(l_1 - a) + M_2}{l_1}.$$

Substituting the value of M_2 from equation (570), we have

$$R_1 = -\frac{W_1 l_1}{2(l_1 + l_2)}(k - k^3) + W_1 - W_1 \frac{a}{l_1}.$$

And making $\frac{a}{l_1} = k$ and $l_1 = \frac{l_2}{n}$ or $l_2 = nl_1$,

$$R_1 = \frac{W_1}{2(1+n)}[2(1+n) - k(1 + 2 + 2n) + k^3]. \quad (571)$$

Taking, then, the right-hand half: the acting moments are M_2, and that of R_3 with respect to an axis at e. Hence $M_2 = R_3 l_2$, as there is no load on l_2.

$$\therefore R_3 = \frac{M_2}{l_2} = -\frac{W_1 l_1^2}{2(l_1 + l_2)l_2}(k - k^3);$$

$$R_3 = -\frac{W_1 l_1^2}{2(l_1 + nl_1)nl_1}(k - k^3) = -\frac{W_1}{2(1+n)n}(k - k^3). \quad (572)$$

And since ΣV, or the sum of all vertical components, must be zero, $\Sigma V = R_1 + R_2 + R_3 - W_1 = 0$. $\therefore R_2 = W_1 - R_1 - R_3$. Substituting values of R_1 and R_3,

$$R_2 = W_1 - \frac{W_1}{2(1+n)}[2(1+n) - k(1+2+2n) - k^3] + \frac{W_1}{2(1+n)n}(k - k^3);$$

$$R_2 = \frac{W_1}{2n}\left[\frac{2n(1+n) - n[2(1+n) - k(1+2+2n) + k^3] + (k - k^3)}{1+n}\right]. \quad (573)$$

Reducing, $R_2 = \frac{W_1}{2n}(2nk + k - k^3) = \frac{W_1}{2n}[k(1+2n) - k^3]$.

For loads in the span l_2 make $l_1 = nl_2$, and $a = kl_2$ = distance of the load from the right-hand support K. Then R_1 becomes R_3 and R_3 becomes R_1.

The above equations (571–3) are all that are necessary to find the reactions for any continuous girder over three supports.

Where the two spans are equal, $l_1 = l_2 = l$, and $n = 1$. These equations then become

$$\left.\begin{aligned} R_1 &= \frac{W_1}{4}(4 - 5k + k^3); \quad R_3 = -\frac{W'}{4}(k - k^3) \\ \text{and} \qquad & \\ R_2 &= \frac{W_1}{2}(3k - k^3). \end{aligned}\right\} \quad (574)$$

Having found the reactions, the stresses are found as in any truss. The live load can be taken as acting at one panel point, at two, or any number of points. Eqs. (574) gives, then, the reaction due to each load by simply changing W_1 and k to correspond with the magnitude of the load and its position with respect to the point of support, A or K, in Fig. 369. The sum of these partial reactions will be final reactions for the loading existing at any time. A moving load may enter the bridge at one end A and cover a portion or the whole of the first span, and pass on to the second span, and finally out entirely at the other end K; every position may or does produce a different stress in the several members, either in kind or in magnitude, or in both. The point then is to determine that position of the moving load which will give maximum stresses in the members.

987.—*Maximum Stresses.*—Live load is assumed to be on one arm only for maximum tension in lower chord and maximum compression in upper chord; also for maximum web stresses from end towards centre.

From equations (574) we find the left-hand reaction (Fig. 370), $R_1 = \frac{W_1}{4}(4 - 5k + k^3)$, which is positive for all values of k from 0 to 1. The moment, then, at any section C to the left of W_1 is simply $M = R_1 x$, and positive. For any point C' to the right of W_1 we have the moment

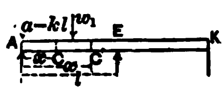


FIG. 370.

$$M = R_1 x_1 - W_1(x_1 - kl) = W_1 k\left(l - x_1 \frac{5 - k^2}{4}\right).$$

This moment becomes zero when $l = x_1 \frac{5 - k^2}{4}$, and if x_0 is that value of x_1 for which $M = 0$,

$$\frac{x_0}{l} = \frac{4}{5 - k^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (575)$$

To the left of this point of zero moment the moment is positive, and beyond or to the right it is negative. When $k = 0$, $\frac{x_0}{l} = \frac{4}{5} = 0.8$; when $k = 1$, $\frac{x_0}{l} = 1$. Hence all loads in the first span give positive moments at all points to the left of the point where $\frac{x}{l} = 0.8$.

As loads in the second span would give negative values for R_1, and consequently negative moments in the first span, no loads should exist on the second span when we are seeking maximum positive moments, or maximum compression in the upper chord and maximum tension in the lower chord; but the first span should be fully loaded for these maximum stresses at any point to the left of $\frac{x}{l} = 0.8$. For the remaining 0.2 of the span as many different loadings are required as there are centres of moments, which are panel points—usually not more than one. The point over the centre support is not used, as the greatest positive moment at that point is zero. The loading taken must be such that the point of zero moments is on the right of the centre of moments taken. For all

practical purposes, however, one position of the uniform live load is sufficient for maximum positive moments giving maximum compression in upper chord and tension in lower chord, and that is when the first span is fully loaded. The total reaction R_1 being then found, the stresses in the members are readily computed.

988. *Web Stresses.*—To find the maximum positive shear or maximum tension in those diagonals inclining downwards towards the right.

The positive shear in any panel is equal to the reaction R_1 minus the loads between R_1 and the panel in question. As any load on the second span causes a negative reaction on the left at A, thereby introducing a negative shear, there should be no loads on the second span. And as any load between R_1 and the panel under consideration increases the positive reaction by only a portion of the load itself, whereas the entire load must be deducted to obtain the shear, no loads should exist between A and the panel under consideration, but all points to the right of this panel should be loaded. For maximum shear in the panel bc, for instance, only the panel points c and d should be loaded with the live load. Having fixed upon the position of the loads for maximum positive shears, the reactions are found, and consequently the shears and the stresses in the web member in the ordinary way. The condition of loading giving the maximum stresses in the diagonals gives also the maximum stresses of the opposite kind in the verticals, meeting the respective diagonals at their upper extremities.

In the truss represented in Fig. 369, when closed the truss will be a continuous girder over three supports, if the ends A and K are raised, for dead loads, consisting of two equal spans, and also for live loads so long as the end reactions are positive. If the ends are not latched down, there can be no downward or negative reaction. But if the ends are not sufficiently raised, a live load on the first span may cause the second span to rise until the right end is lifted from its support. This frequently happens, and when this condition is found, the first span should be considered as an independent span for live-load stresses; and the dead-load stresses are the same as for the bridge open, or not bearing on the end supports, being considered as two balanced cantilever arms, as fully discussed, after omitting the external loads.

989. However, to compensate for want of proper adjustments, and for the effects of unequal chord temperatures on the deflection at the ends, two assumptions are made in calculating the dead-load

stresses: (1) The ends just touching with no positive reactions; and (2) the end reactions equal to those of a continuous girder over three supports.

Practically the end reactions will vary between the limits of these assumptions. The ends should always be raised so that the end reaction shall be at least a mean between those resulting from the two conditions assumed. The ends should therefore be raised sufficiently to take out at least one half the deflection of the ends arising from the weight of the structure, the bridge open or spans swinging. This would still leave the ends of the arms below the true level of the chord by one half of the deflection due to dead loads. This can be diminished by shortening the upper chord—usually in the centre panel, or panels near the centre—just enough to raise the ends to the true level of the lower chord.

If the ends are left free to hammer, under extreme variations of temperature, the ends may be thrown out of line, possibly causing derailment of a train.

The simplest form of end lift for a swing-bridge is one in which wheels are attached to the ends of arms which rest on top of plates bolted to the masonry. These plates are bevelled each way from the crown, thereby facilitating both the opening and closing of the bridge. Some kind of automatic latch is necessary to stop the bridge at its proper position, or the wheels may be fastened to the supports, and the ends of the bridge provided with bevelled plates.

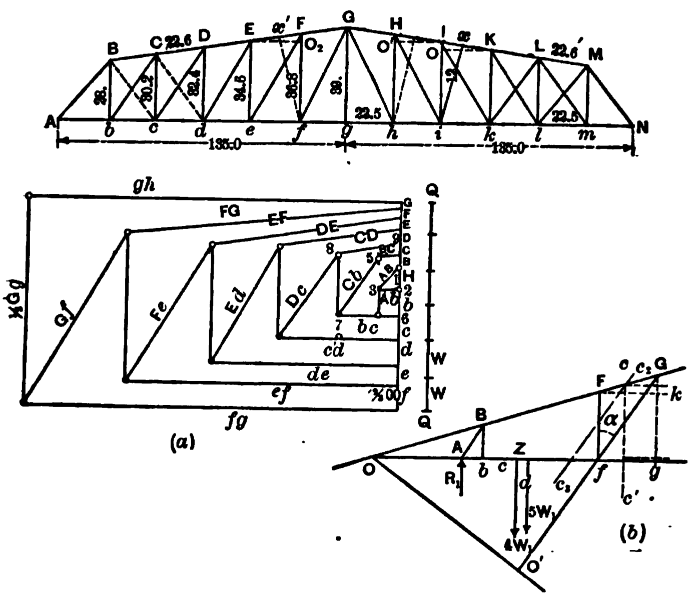
There are many other devices for raising the ends of swing-bridges, such as wedges, wheels forced into bearing after closing and latching the bridge by means of gearing, the screw-jack or direct lift, etc.

The entire dead load in a centre-bearing swing-bridge is carried on a vertical pin or pivot when swinging or when not resting on the end supports. If raised at the end on bearing-plates when closed, wedges are inserted at *ee'*, Fig. 367 (*a*), thereby relieving the pivot from all live load except the panel load at *ee'*.

990. For all lengths up to 100 feet end to end plate-girder swing-bridges should be used. For lengths from 100 to 160 feet the riveted-truss design is preferable. The advantage of this latter design is that the trusses can be made in two halves of the full length of each span, and then coupled up over the pivot pier by means of eye-bars with pin connections. As the only stress in the top chord over this centre support is tensile, and as at this point the bottom chord is under compression, a butt-joint can be

made at the meeting of the bottom flanges or chords with only such splicing as is used for a compression-joint. Both of these types of swing-bridges are usually pin-bearing: when closed the girders rest on side supports, which relieve the pivot from any but one panel weight of live load. About four wheels are used under the riveted-truss swing-bridge, simply to balance the bridge when swinging, but not intended to relieve the pivot of any load. These balance-wheels are necessary under any form of centre-bearing bridge, whether of girder or truss design. They are placed and connected to roll around on a circular track on top of the pier.

The limitation of length for plate girders to 100 feet is simply necessitated by the difficulty of transporting longer girders in one length. Plate girders 136 feet long have been shipped for short distances. This is, however, dangerous, especially when shipped for long distances over all kinds of roads. Very recently plate girders 180 feet long have been built and erected.



FIGS. 371.

APPLICATION OF THE PRECEDING PRINCIPLES.

991. In the following example we will take the case of a centre-bearing swing-bridge. One of the trusses is shown in Fig. 371.

Length from end to end 270 feet; of each of the two spans 135.0 feet, divided into six panels of 22.5 feet each. End vertical $Bb =$ 28.0 feet, centre vertical $Gg = 39.0$ feet. The increment in length for each successive vertical from end to centre $= 2.2$ feet. The lengths of the verticals are then $Bb = 28$, $Cc = 30.2$, $Dd = 32.4$, $Ee = 34.6$, $Ff = 36.8$, and $Gg = 39.0$ feet. The lengths of the diagonals can be readily calculated or scaled from a large drawing, and are $AB = Bc = 35.9$, $Cb = 37.7 = Cd$, $Dc = 39.5$, $Ed = 41.3$, $Fe = 43.1$, and $Gf = 45.0$. The length of all bottom-chord panels is $= 22.5$, and of all top-chord panels 22.6 feet.

In determining stresses in chord panels by the method of moments it is to be recollected that for bottom-chord stress the centres of moments are the top-chord panel points at which the top-chord panel and diagonal in the same panel intersect, and consequently the lever-arms are the respective verticals passing through the same point. For top-chord panel stresses the centres of moments are the intersection of the bottom chord and the diagonal in the same panel, and the lever-arms for top-chord panel stresses are the respective perpendiculars from the bottom-chord panel points to the upper-chord panel in question. For instance, the centre of moments for the stress in panel HI is $xi = Ii \cos \alpha$, $\alpha =$ angle IKO. Hence $\cos \alpha = \frac{OK}{IK} = \frac{O'I}{HI}$;

$$\therefore\ xi = Ii\frac{O'I}{HI}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (576)$$

and similarly for any other chord panel.

The dead load or weight of the two trusses, track, etc., per foot of length may be taken approximately at $2w = 5l - 50 + 400 = 5 \times 270 - 50 + 400 = 1700$ pounds, and for one truss per foot $=$ 850, and per panel $850 \times 22.5 = 19{,}125$ pounds. The live load will be taken at 3000 pounds uniform load per foot of both trusses, two engine excesses 20,000 pounds each at head of uniform load and placed two panel lengths apart, or for each truss uniform load per foot 1500 pounds or 33,750 per panel, and the two excesses per truss 10,000 pounds each.

Case 1.—The dead-load stresses are to be found firstassuming the bridge swinging free of the supports. The dead load will be assumed as divided between the bottom-chord and top-chord panel points; $\frac{1}{3}$ of 19,125 $= 6375$ at each upper-chord panel point, and $\frac{2}{3}$ of 19,125 $= 12{,}750$ pounds at each of the bottom-chord panel points. The

stresses can be determined algebraically as in the cantilever beam, Fig. 360, or more readily by the graphical method as shown in Fig. 371 (a). Lay off the line of loads QQ', dividing it into equal divisions, each representing W, a panel load to any desired scale. As $\frac{3}{8}W$ acts at the lower-chord panel points, transfer to the parallel line (to QQ') divisions f, e, d, c, bA equal to $\frac{3}{8}W$. These will represent the lower-chord panel loads. Then lay off B, C, D, E, F, and G equal to $\frac{1}{8}W$. These represent the upper-chord panel loads due to weight of truss. Only $\frac{1}{2}$ of $\frac{1}{8}W$ rests at G from the weight of the left-hand span. The diagram is then constructed as follows: Taking the free end of the truss A, the force A and the stresses in AB and Ab are the only acting forces. The little triangle constructed on the force A and numbered 1, 2, 3 in diagram (a) is the stress polygon for the joint A, and as A acts downward, 2,3, the stress on Ab, acts inward on it towards A, hence compression in Ab; 3,1 represents the stress on AB, and acting outwards from A is tension.

Taking the joint B, the acting forces are the stresses in AB, Bb, and BC, and the load $\frac{1}{8}W$. Taking division B in diagram (a), the four-sided figure 1,4,5,3 is the stress polygon. 4,1 acts downward $= \frac{1}{8}W$; 1,3 to the left and outward from B, hence tension on AB, as before found; 3,5 acts upwards, that is, inward on Bb towards B, hence compression in Bb; and 5,4 to the right, that is, outward on BC from B, hence tension on BC. Then for the joint b we have the load $\frac{3}{8}W$; stress in Ab, bc, and Bb. The stress polygon is 2,6,7,5,3,2, showing tension in Ab, bc and bC, and compression in Bb.

For the joint C the stress polygon is 4,9,8,7,5,4, and similarly for other joints. To check the accuracy of this diagram, find the stress in the lower-chord panel fg by moments. For this panel the centre or axis of moments is G; the lever-arm is $Gg = 39.0$ feet. The loads are $\frac{3}{8}W$ at A, with lever-arm $Ag = l = 135$ feet; and the resultant at B and b, C and c, etc., to those at G and $g = 5W$, with lever-arm from middle of de to $g = 2\frac{1}{2}$ panels $= 56.25$ feet. Then, for compression in $fg = T$,

$$T \times 39 = \tfrac{3}{8}W \times 135 + 5W \times 56.25$$

$$= \tfrac{3}{8} \times 19{,}125 \times 135 + 5 \times 19{,}125 \times 56.25.$$

$F = 184{,}300$ pounds, compression. This should agree with the length of the line fg, diagram (a), reduced to pounds, according to the scale used for the line of loads QQ'.

Case 2.—Considering, next, the bridge closed with ends raised, so that it becomes a continuous truss over three supports, and dead load only acting, the loads at A and N resting on the points of support need not be considered. For this condition a stress diagram can be readily constructed or the stresses determined by moments.

For the reaction at A the loads at b, c, d, e, and f give positive reactions, and the loads at h, i, k, l, and m give negative reactions. The difference between these two give the final reaction. Substituting first in equation (574), $R = \frac{W_1}{4}(4 - 5k + k^3)$, we find, after making $W_1 = 19{,}125$ pounds and $k = \frac{a}{l} = \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}$, and $\frac{5}{6}$ successively, for the load at b, c, d, e, and f the sum of the positive reactions:

Load at b, $k=\frac{1}{6}$, $= \frac{W_1}{4}(4-5k+k^3) = \frac{19125}{4}(4-5\times\frac{1}{6}\times(\frac{1}{6})^3) = 15{,}186$

" " c, $k=\frac{1}{3}$ .... $= 11{,}333$

" " d, $k=\frac{1}{2}$ .... $= 7{,}769$

" " e, $k=\frac{2}{3}$ .... $= 4{,}595$

" " f, $k=\frac{5}{6}$ .... $= 1{,}970$

Sum of positive reactions at A 40,853

For negative reactions from loads at h, i, k, l, and m:

Substitute in equation (374) $R_2 = -\frac{W_1}{4}(k - k^3)$;

Load at h, $k = \frac{5}{6} = -\frac{19125}{4}\left(\frac{5}{6} - \left(\frac{5}{6}\right)^3\right)$ .... $= -1208$

" " i, $k = \frac{2}{3}$ .... $= -1771$

" " k, $k = \frac{1}{2}$ .... $= -1793$

" " l, $k = \frac{1}{3}$ .... $= -1417$

" " m, $k = \frac{1}{6}$ .... $= -775$

6,964

Final reaction at A 33,889

Having now the reaction at A, we can find the stresses in the members either graphically or algebraically.

To find by moments the stress in fg: The centre of moments is G; lever-arm of the stress, $Gg = 39$; of the reaction $Ag = 135$; and of the resultant of the five loads at b, c, d, e, $S = \frac{1}{2}Ag = 67.5$. Then stress in $fg = (33{,}889 \times 135 - 5 \times 19{,}125 \times 67.5) \div 39 = 48{,}197$

pounds. To determine the kind of stress, if we conceive a plane to cut the truss just to the left of Gg, intersecting the members FG, fG, and fg, and moments to be taken about G, the stress in fg must have such a direction that its moment has the same sign as the least of the remaining moments, namely, of R_1 and ΣW_1, with respect to G, in the case taken is $R_1 \times 135$, which is positive or right-handed with respect to G; therefore the stress in fg must act towards the left, or inwards towards f. Hence it is compression.

For the stress in FG, centre of moments is at f, its lever-arm is $x'f = Ff \cos \alpha = Ff\frac{EO_2}{EF} = 36.8 \times \frac{22.5}{22.6}$. Then stress in $FG \times x'f = R_1 \times Af - \Sigma W_1 \times \frac{1}{2}Af$. Stress in $FG = (33{,}889 \times 112.5 - 4 \times 19{,}125 \times 56.25) \div 36.8 \times \frac{22.5}{22.6}$. Hence stress in $FG = 18{,}850$ pounds, and as it must have a moment right-handed, as R_1Af is right-handed and less than $\Sigma W \times \frac{1}{2}Af$ with respect to f, it must act on FG outwards from F; that is, it is a tensile stress.

The stress in the web member fG can be found by prolonging FG and fg to intersection at some point on the left; then find the perpendicular from fG prolonged to this point. It will be the lever-arm of the stress in fG. In Fig. 371 (b), FG and fg have been prolonged to intersection at O; the perpendicular OO' is the lever-arm of the stress in fG; OA the lever-arm of R_1, and Od the lever-arm of the five loads from b to f. Intersecting the truss with the vertical plane cc', and taking moments about O, we have

$$\Sigma M = 5W_1 \times Od - R_1 \times OA - \text{stress in } Gf \times OO' = 0. \quad (577)$$

To find Od, OA, and OO', we have

$$Ff : Bb :: Of : Ob \text{ or } Of - bf; \quad \therefore \; Of = \frac{Ff \times bf}{Ff - Bb}.$$

Substituting values, $Of = \frac{36.8 \times 90}{36.8 - 28} = 387.727$, or 387.73 nearly.

Then $OA = 387.73 - 112.5 = 275.23$, and $Od = 275.23 + 67.5 = 342.73$; $OO' = Of \sin(90 - \alpha)$, or $\cos \alpha$, $= Of\frac{22}{22}\frac{.5}{.6} = 336$; and substituting in eq. (577), stress in $Gf = \frac{5W_1 \times 342.73 - R_1 \times 275.23}{336}$

$$= \frac{5 \times 19125 \times 342.73 - 33889 \times 275.23}{336} = 69{,}781 \text{ pounds.}$$

This stress is evidently tension, since its moment must be of the same kind as the least of the other two; that is, of R_1, which is left-handed about O.

As seen, this method is long and troublesome, and it is therefore better to find the stress in the web members either by diagram, or by noting that in any panel the vertical component of the stress in any diagonal is equal to the shear in the panel either increased or diminished by the vertical component of the stress in the inclined chord segment in the same panel.

The stress in FG has been found to be 18,850 pounds; its vertical component is $V = 18{,}850 \times \frac{FO_2}{FG} = 18{,}850 \times \frac{2.2}{22.6} = 1828$ pounds. The shear in the panel $= R_1 - 5W_1 = -95{,}625$; hence the vertical component of the stress in $Gf = 95{,}625 - 1828 = 59{,}908$ pounds, and the stress itself $= 59{,}908 \times \frac{30}{26} = 69{,}125$ pounds tension,—practically the same as found by moments.

The stresses in the verticals could be found by moments, as readily seen from Fig. 371 (b), using the lever-arms Of or Og, or any other distances from O. For instance, changing the position of the cutting-plane to c_2c_3, then $4W_1 \times OZ - R_1 \times OA -$ stress in $Ff \times Of = 0$. From which stress in Ff can be easily determined, which is compression, as it must act upwards towards F on Ff.

If this process is applied to all members, it will be found that BC, CD, DE, EF will be under compression, and, as shown above, FG under tension, for the top chord; and that ab, bc, cd, and de of the bottom chord under tension, while ef and fg, as shown above, are under compression.

Of the web members bC will not act; Ba will be under compression; Bc, cD, dE, eF, fG will be under tension; Cd not acting. Of the verticals Bb will be under tension. All other verticals will be under compression.

The conditions in this Case 2 are that the ends are raised and the truss is in the condition of a continuous girder over three points of support, while in the preceding Case 1 the ends were assumed to be swinging free, or the bridge open. For this condition the stresses are found in Fig. 371 (a), and as shown the entire top chord was under tension, and entire bottom chord under compression. All acting diagonals, which are those sloping upwards towards the centre support, were under tension, and all verticals under compression. Bc and Cd, sloping downwards, do not act.

DETERMINATION OF STRESSES DUE TO LIVE LOADS.

992. Two cases will be considered: (3) when the live load is only on one span, for maximum compression in upper chord and maximum tension in lower chord; also for maximum web stress from the end towards the centre. (4) When the live load is on both arms or spans, for maximum tension in upper chord and maximum compression in lower chord; also for maximum web stresses from the centre towards the end.

Case 3.—It was seen in paragraph 987 that, for a strictly accurate analysis of stresses due to live loads, it would be necessary to find for each condition of loading the point of zero moment, and to obtain maximum stresses only those panel points to the left of this point should be loaded; but that for practical purposes this was not necessary, and that for maximum compression in the upper chord and maximum tension in the lower chord it is only necessary to assume the first span fully loaded. This condition will be assumed. The reactions are to be found by the use of eqs. (574) for R_1, R_2, and R_3 precisely in the same manner as was done in Case 2. The successive values of k are the same; the panel loads or W_1 are now, for $k = \frac{1}{6}$, one panel weight of uniform load and one excess load at $b = 33{,}750 + 10{,}000 = 43{,}750$ pounds; for $k = \frac{2}{6} = 33{,}750$ pounds at c; for $k = \frac{1}{2}$ or $\frac{3}{6} = 43{,}750$ pounds at d; for $k = \frac{4}{6}$ or $\frac{2}{3} = 33{,}750$ pounds at e; and for $k = \frac{5}{6} = 33{,}750$ pounds at f. Substituting these values in $R_1 = \frac{W_1}{4}(4 - 5k + k^3)$: then $R_1 = 31{,}761 + 20{,}003 + 16{,}249 + 8110 + 3477 = 79{,}600$ pounds.

There is no negative reaction at A to deduct from the above, as in Case 2, for dead load, since there is no live load on the second span. But there is a negative reaction at the right-hand support N (see Fig. 371), and unless held down that end will lift if not already sufficiently raised. This value of R_3 can be found by substituting in $R_3 = -\frac{W_1}{4}(k - k^3)$.

$$R_3 = -(2527 + 3126 + 3750 + 2501 + 1368) = -13{,}272 \text{ pounds.}$$

Having now the end reactions, the reaction at the pivot pier can be found from ΣV, that is, vertical forces equal to zero.

Having the reaction R_1 at A, we can now find the stresses in the various members in the same manner as was adopted in Case 2 for

dead loads. It was, however, stated that when the live load on one span lifted the other span from its end support the first span should be considered as an independent span, in which case it is evident that the stress in fg will be zero. It will then be found that all members of the top chord will be in compression, and of the bottom chord in tension. The stress in ef, for instance, is found by moments as follows: The forces are the stresses in ef; the reaction R_1; the uniform load $4W$ at the points b, c, d, and e, and the excess loads at b and d. Then $\Sigma M = R_1 \times Af - 4W \times 2\frac{1}{2}$ panels $-$ excess at $b \times bf -$ excess at $d \times df -$ stress in $ef \times fF = 0$, centre of moments at F. $R_1 = 79{,}600$; $Af = 112.5$ feet; $W = 33{,}750$; $2\frac{1}{2}$ panels $= 56.25$ feet; excess $= 10{,}000$; $bf = 90$; and $df = 45$ feet; $fF = 36.8$ feet. Substituting and reducing, stress in $ef = (8{,}955{,}000 - 8{,}943{,}750) \div 36.8 = 336$ pounds tension.

If, however, the engine is headed towards the right, placing the excess loads at d and f, the reaction R_1 will be 75,082 pounds, only the excess at d will have a moment about F, and stress in $ef = (8{,}446{,}725 - 8{,}043{,}756) \div 36.8 = 10{,}950$ pounds tension; and since ef is under compression for dead loads, the difference if tensile would be maximum stress of this kind which could come upon it, which would be small. In the same manner the maximum compressive stress in FG due to this loading can be determined by taking centre of moments about f. This member is under tensile stress from dead loads; its maximum compression will then be very small under any condition.

With one span only loaded, but few of the web members will be under their maximum stresses of either kind. First, because the shear in any panel being the difference between the reaction R_1 and any load between R_1 and the panel in question, such panel points should not be loaded; and second, because in some of the the web members the maximum stress occurs when both spans are either fully or partially loaded. With the first span Ag fully loaded, the end post AB has its maximum compression, since this is equal maximum reaction R_1 at A multiplied by $AB \div Bb$. Any load in the second span would give a negative reaction at A, thereby diminishing R_1. The maximum compression, then, in $AB = R_1 \times \frac{AB}{Bb} = 79{,}600 \times \frac{35.9}{28} = 102{,}060$ pounds. The reaction due to dead load under Case 2 is 33,889 pounds; then $33{,}889 \times \frac{35.9}{28} = 43{,}450$

pounds, which being also compression, the ultimate maximum compressive stress in $AB = 102{,}060 + 43{,}450 = 145{,}510$ pounds.

The maximum tension in Bb is one panel weight of live load $= 40{,}000$ pounds $+$ bottom-chord panel weight of dead load $= 12{,}750$ pounds $= 52{,}750$ pounds.

For maximum tension in Bc the point b should not be loaded, and the excess loads should be placed at c and e. The reaction R_1 should be found for loads at c, d, e, and f, with excess at c and e. The reaction is the shear in the panel bc. This, minus the vertical component of the stress in BC, will give the vertical component of the stress in Bc, which, multiplied by the length $\frac{Bc}{Bb}$, gives the tension in Bc.

For maximum tension in Cd neither of the panel points b and c should be loaded, only the points d, e, and f, with excess at d and f; for which position of the load a new reaction should be found; then tension in Cd and corresponding compression in Cc. As no other web members have maximum compression when live load is only on a part or the whole of the first span, it is not necessary to consider them for this loading.

Case 4.—The condition of loading considered in this case contemplates two trains on the bridge at the same time, and, in addition, that the ends are insufficiently raised. Assume a load of uniform intensity covering the second span, and a train, entering the first span from the left, headed by engines and advancing until the whole bridge is covered. The maximum negative shears will then occur at the head of the train, giving also maximum web stresses. When the second span is fully loaded with a uniform load of 33,750 pounds per panel, the result will be to raise the end A. Substituting in eq. (574), R_2, now $R_1 = -\frac{W}{4}(k - k^3)$, giving the successive values to $k = \frac{1}{8}, \frac{2}{8}$, etc.; and $W = 33{,}750$ pounds. Then $R_1 = -(1367 + 2500 + 3164 + 3125 + 2148) = -12{,}304$ pounds, which simply means that a load resting at A of 12,304 pounds is necessary to hold the end down. Any load, such as a panel weight at the head of the train $= 43{,}750$ pounds, will be exerted to the above value in holding the end down, and the remainder supported by the end pier directly. The maximum stress, then, of tension in AB will be $12{,}304 \times \frac{AB}{Bb}\left(= \frac{35.9}{28}\right) = 15{,}776$ pounds tension.

The compression in *Bb*, if both chords were horizontal, would simply be the reaction R_1; but since the chord panel BC is sloping upwards towards the right and under tension, it gives an upward component, thereby reducing the compression in *Bb*.

The tension in BC, taking moments about b, is $= -12{,}304 \times \dfrac{22.5}{Bb \cos \alpha} = -12{,}304 \times 22.5 \times \dfrac{22.6}{28 \times 22.5} = -9931$. The vertical component of this is $9931 \times \dfrac{2.2}{22.6} = 975$, and the resultant compression in $Bb = 12{,}304 - 967 = -11{,}337$ pounds.

The same result can be obtained directly, instead of first finding the stress in BC and then its vertical component, by simply multiplying the entire reaction by the length of the vertical less the rise per panel of top chord divided by the length of the vertical: that is, $12{,}304 \times \dfrac{28 - 2.2}{28} = +11{,}337$ pounds, compression in *Bb*, the same as already found, namely, 11,337 pounds.

For maximum tension in diagonal bC, second span fully loaded, head of train at b; hence 43,750 at b. For this load

$$R_1 = \frac{W}{4}(4 - 5k + k^3) = \frac{43750}{4}(4 - \tfrac{5}{6} + (1/6)^3) = +34{,}802 \text{ pounds},$$

reaction at A. The negative reaction at A from load on second span same as before $= -12{,}304$ pounds; hence resultant reaction at $A = +34{,}802 - 12{,}304 = +22{,}498$ pounds. The shear in the panel $bC = 22{,}498 - 43{,}750 = -21{,}252$ pounds, to which we must add the vertical component of the stress in BC from this loading: Vertical component of stress in

$$BC = 22{,}948 \times 22.5 \times \frac{22.6}{28 \times 22.5} \times \frac{2.2}{22.6} = 1767 \text{ pounds};$$

the vertical component of stress in $bC = -(21{,}252 + 1767) = -23{,}019$; the stress itself in $bC = -23{,}019 \times \dfrac{bC}{cC}\left(= \dfrac{35.9}{30.2}\right) = -27{,}695$ pounds tension; and for maximum compression in Cc follow same rule as for *Bb*. Then compresssion in

$$Cc = 27{,}695 \times \frac{30.2 - 2.2}{0.2\ 3} = +25{,}677 \text{ pounds}.$$

For maximum tension in cD and compression in Dd: head of train at c, 43,750 pounds at c, 30,000 at b. Find, then, reaction,

shears, and stress as above. For maximum dE, and compression in Ee: head of train at d, 43,750 pounds at d, 30,000 at c, and 43,750 at b; and proceed in this manner until the train reaches from A to f, the second span being fully loaded in all cases.

For maximum chord stresses, the one position of the load covering both spans is all that is practically necessary to consider. This loading causes stresses of the same kind in all chord members, as was found in the case of the truss swinging freely and under the action of its own weight alone.

Four cases have now been considered: Cases 1 and 2, dead load only acting; Case 1, bridge open or closed without resting on end supports; Case 2, ends raised, the bridge then in the condition of a continuous girder over three supports; and two cases of live loads; Case 3, live load on one span only, in positions for maximum stresses in chords and webs; Case 4, live loads on both arms or spans, in positions for maximum stresses in chords and web.

Case 3 must be combined with Cases 1 and 2 in order to find maximum compressive and tensile stresses, and the same combinations must be made for Case 4. The methods of determining stresses in each of the members has been clearly indicated for each case both in kind and magnitude. For convenience of reference the following table, showing kinds of stress, and ultimate maximum of both kinds is given; — sign tension, + sign compression:

TABLE LXXIX.

Dead Load.				Live Load.			
Case 1.		Case 2.		Case 3.		Case 4.	
Chords.	Web.	Chords.	Web.	Chords.	Web.	Chords.	Web.
$-BC$	$-AB$	$+BC$	$+AB$	$+BC$	$+AB$	$-BC$	$-AB$
$-CD$	$-bC$	$+CD$	not act'g, bC	$+CD$	not max. bC	$-CD$	$-bC$
$-DE$	$-cD$	$+DE$	$-cD$	$+DE$	" cD	$-DE$	$-cD$
$-EF$	$-dE$	$+EF$	$-dE$	$+EF$	" dE	$-EF$	$-dE$
$-FG$	$-ef$	$-FG$	$-eF$	$+FG$	" eF	$-FG$	$-eF$
$+Ab$	$-fG$	$-Ab$	$-fG$	$-Ab$	" fG	$+Ab$	$-fG$
$+bc$	$+Bb$	$-bc$	$-Bc$	$-bc$	$-Bc$	$+bc$	not max. Bc
$+cd$	$+Cc$	$-cd$	$-Bb$	$-cd$	$-Cd$	$+cd$	" Cd
$+de$	$+Dd$	$-de$	$+Cc$	$-de$	$-Bb$	$+de$	$+Bb$
$+ef$	$+Ee$	$+ef$	$+Dd$	$-ef$	$+Cc$	$+ef$	not max. Cc
$+fg$	$+Ff$	$+fg$	$+Ee$	not act'g, fg	not max. Dd	$+fg$	$+Dd$
	$+Gg$		$+Ff$		" Ee		$+Ee$
not acting.			$+Gg$		" Ff		$+Ff$
BC and Cd		not acting,			" Gg		$+Gg$
		bC and Cd					

For ultimate maximum compression in BC, combine Cases 2 and $3 = +BC+BC$ for the two cases. For maximum tension in BC combine Cases 1 and 4, $= -BC, -BC$, after finding numerical values for BC under the two conditions of dead and live load; and similarly for other members. Those members which have both a maximum for tension and compression must be proportioned and connected so as to resist both when called upon. All of the chord members may have to resist both tension and compression, as will also the web members AB and Bb. The web members Cc, Dd, Ee, Ff, and Gg, are under compression from all loads, and should be proportioned by the total stress from these two cases for dead and live load giving the greatest total. Members bC, cD, dE, eF, fG, Bc, and Cd are under tension from all loads, and should be proportioned and connected accordingly.

RIM-BEARING TURNTABLE.

993. In the rim-bearing swing-bridges, instead of the entire dead load when swinging being carried on a vertical pin or pivot as for a centre-bearing, the entire dead load is supported on a circular girder called a drum, which in turning moves upon rollers; or these two conditions may be combined, so as to make the bridge partly rim-bearing and partly centre-bearing. In general, plate-girder swing-bridges are centre-bearing, and truss bridges are rim-bearing, or rim-bearing and centre-bearing combined.

The rim-bearing swing-bridges may have three or four supports. When the ends are raised so that the truss becomes a continuous girder over four supports, the reactions must be determined as for a girder continuous over four points of support for dead load only; the reactions being found, the stresses are found as in the preceding case. But it is difficult to compute the live-load stresses. A common practice is to make the computation of stresses on the assumption that the middle space (the one over the turntable, which is short) is zero, which reduces the truss to the condition of two spans with single centre bearing. The discussion is then the same as already given. Where four supports are used, each truss is usually supported by two links connected to the top of a rigid frame resting on the centre pier, so that no shear can be transmitted across the centre span or panel, causing equal moments at the two centre supports, thereby simplifying the determination of stresses; or, if the two trusses or spans are carried by a single link to a rigid frame,

there will be only three points of support—in which the analysis is the same as for a simple two-span bridge. Many other devices have been suggested, so that the bridge may be made continuous when being opened, and be two simple spans when closed. Dead-load stresses are then computed, as already explained, for the condition of two overhanging or cantilever arms when open; and when closed both dead and live load stresses are computed as for two independent and simple spans.

994. In all cases of swing-bridges a constant moment of inertia of cross-section has been assumed. Although this assumption is not true, it is an error on the safe side, and is the only one practicable in computing stresses.

For complete discussions of swing-bridges in all their conditions of loading and supporting, see Burr, Dubois, and Johnson.

GENERAL REMARKS ON THE SELECTION OF BRIDGE DESIGNS.

995. While the several questions of first cost, cost of maintenance, comparative durability of material used, and, finally, safety in case of accident, such as a derailment of a train, are of great importance, the question of first cost is to a great extent the controlling factor in selecting any particular design—this including not only cost of material and manufacture, but also cost and safety of transportation, and that of erection as determined by the local conditions at the site of the bridge, such as height of structure above water surface, depth of water, rapidity of current, character of material of the bed of the river, etc.; these latter mainly determining, however, the general design as a choice between a simple trussed bridge or a cantilever bridge.

The following remarks are intended to apply principally to such questions as the selection of through or deck bridges considered as a whole, involving the quantities of material in superstructure and in masonry or substructure, the selection of plate or trussed girders, and in the latter riveted or pin connections.

996. Deck-bridges are usually the cheapest in first cost, owing mainly to a decrease in the height and cost of the masonry or other material in the piers. For short spans there may be some saving of iron and steel in the trusses, as these may be placed closer together than in through-spans of the same length, thereby reducing the length of the iron floor-beams, or practically dispensing with the iron floor system by supporting the cross-ties directly on the top chords.

997. When the total length of bridge is great, it becomes an important question to determine whether a few long spans or many short spans would be economical. This question merely resolves itself into a comparison of the total cost of a number of short spans and many piers, or that of long spans and correspondingly reduced number of piers.

The following formula gives a close approximation to the weight of iron in a single-track pin-connected span:

$$W = al^2 + kl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (578)$$

W = total weight of iron in the span; a, a constant = 5; l = length of span; k, a quantity varying with the assumed live load, and constant for any particular live load; al^2, taken together, represents the weight of the trusses, and kl the weight of the floor system. Both a and k really should vary with the live load, but it is usual to make $a = 5$ and $k = 350$. The weight per linear foot of span is

$$w = \frac{W}{l} = 5l + 350. \quad . \quad . \quad . \quad . \quad . \quad . \quad (579)$$

This does not include cross-ties, rails, guard-rails, etc., which for the two trusses is taken at 400 pounds per foot of track. For any given span the total weight of iron can be found from eq. (578), the cost of which can be determined by allowing from 4 to 5 cents per pound.

In spanning a river 1200 feet wide we could use, say, 5 spans 240 feet each, or 3 spans 400 feet each. In the first case there would be required 2 abutments and 4 piers; in the latter, 2 abutments and 2 piers. The weight of iron in the first case $W = (5 \times (240)^2 + 350 \times 240)$ multiplied by the number of spans (5).

$$\therefore \; W = 1{,}860{,}000 \text{ pounds;}$$

and in the second

$$W = (5 \times (400)^2 + 350 \times 400) \times 3 = 2{,}820{,}000 \text{ pounds;}$$

giving a difference in favor of the shorter spans of 960,000 pounds at 5 cents = $48,000; and unless the cost of the two additional piers is greater than this sum the shorter spans would be more economical. Formulæ have been given to make this comparison by a single calculation. These are based on a constant or equal cost for each of the piers, which would rarely be even approximately true, and the results of such computations would be misleading. With sev-

eral borings the cost of foundations and masonry can be calculated to a fair degree of approximation, and with but little labor, by reducing all quantities to a basis of cubic yards, and the iron superstructure calculated by eq. (578), proper comparisons can be made.

998. *Designs for Bridges.*—(1) Plate girders are the simplest in design. Defects due to errors in calculating stresses and in faulty workmanship are to a great extent eliminated; they require little attention, except repainting, after erection, and, finally, are the cheapest for spans under 60 feet.

$$\left.\begin{array}{ll} \text{Weight per linear foot for deck girders,} & w = 9l + 110; \\ \text{" \quad " \quad " \quad " \quad " through "} & w = 8\tfrac{3}{4}l + 300, \end{array}\right\} \quad (579\tfrac{1}{2})$$

for the usual floor systems. For solid iron floors of any of the usual types, trough-shaped sections built of plates and angles, $w = 10l + 600$.

These are the safest and most rigid forms of floor system, but are expensive. The troughs are filled with concrete, usually bituminous, and covered with ballast of broken stone or gravel.

(2) Riveted truss-bridges are cheaper than other forms for spans not exceeding 80 to 100 feet in length. They are often considered stiffer and safer than pin-connected trusses. The secondary stresses developed cause these trusses to be less satisfactory than pin-connected trusses. These are built up of plates and angles.

(3) Pin-connected truss-bridges are usually adopted in this country for spans over from 80 to 100 feet in length. One principal advantage is the freedom from secondary stresses, which permits of a more satisfactory designing and proportioning of the truss-members.

The single-intersection truss is now almost exclusively used in pin-connected truss-bridges. The iron Pratt truss, with tension diagonals and compression verticals, may be taken as the standard truss in this country. It is simple in its details, and has proved more satisfactory than any other type for short spans, though somewhat heavier and more costly than the Warren truss. But for long spans, as modified in the Petit truss, it is lighter and less costly.

The Warren or triangular truss requires less material in its construction than the Pratt truss. It is often used for short spans, especially for deck spans. An objection often raised to the Warren truss is on account of the continual reversal of stresses in some of the web members. The floor-beams have to be suspended from the chord-pins, or special verticals have to be introduced at each panel

point in order to make a rigid riveted connection for the floor-beams. The latter method increases the cost; the former is very objectionable. There are no adjustable members in the truss. For this reason it is sometimes used; it is rarely used for long spans. The Petit truss seems to be the one more commonly used for long spans.

The Whipple truss consists of two simple Pratt trusses combined. It is often called a double-intersection Pratt truss. The stresses are determined by assuming two independent trusses or systems, and combining the resultant stresses in the chords which are common to both systems. For railway bridges this type of truss has been to a great extent abandoned, and the Baltimore or Petit truss is now preferred. The ambiguities and uncertainty of computations of stresses by the ordinary methods in the double-intersection systems has led to the adoption of the other types mentioned. These ambiguities exist to a great extent when, as for long spans, economy requires the curved or inclined chord.

LATERAL TRUSSES FOR WIND PRESSURE.

999. Lateral pressure upon bridge or roof trusses, commonly called wind pressure, is resisted by means of systems of braces connecting the chords, these forming two horizontal trusses. In roofs which are composed of a number of parallel trusses, two pairs, one at each end, are thus connected to resist wind pressure. The remaining and intermediate trusses are simply braced to resist buckling.

The lateral systems may be of the same design or type as the vertical trusses, such as the Pratt, Howe, or Warren truss. As provision must be made to resist the wind pressure from either direction, both sets of diagonals will be required. The floor-beams are used for the transverse members of the lower lateral system in through-bridges, and for the upper lateral system in deck-bridges.

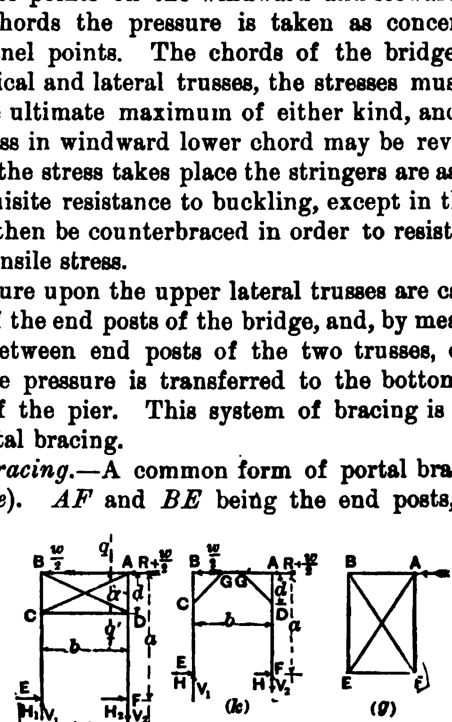
1000. *Amount and Distribution of Pressure.*—Several different assumptions are made as to the amount and distribution of the pressure. For railway bridges from 30 to 50 pounds per square foot is assumed for the unloaded bridge, the surface being that actually exposed to the action of the wind. This is usually taken as the area of the vertical projection of the floor system plus twice the vertical projection of one truss, and about 30 pounds upon bridge and train surface, that portion on the train surface being treated as a moving load. A more exact and definite dis-

tribution is to assume a uniform pressure of 150 pounds per lineal foot on the unloaded chord, and 450 pounds per lineal foot on the loaded chord, of which 300 pounds are considered to be a moving load. It is then only necessary to find the panel-point concentrations, and find the chord and web stress, as already fully explained for a vertical truss loaded in the same manner.

For the unloaded chords the pressure may be divided equally between panel points on the windward and leeward sides, but on the loaded chords the pressure is taken as concentrated at the windward panel points. The chords of the bridge being chords for both vertical and lateral trusses, the stresses must be combined to obtain the ultimate maximum of either kind, and it may occur that the stress in windward lower chord may be reversed. Where a reversal of the stress takes place the stringers are assumed to provide the requisite resistance to buckling, except in the end panels. These must then be counterbraced in order to resist either a compressive or tensile stress.

The pressure upon the upper lateral trusses are carried by them to the tops of the end posts of the bridge, and, by means of a system of bracing between end posts of the two trusses, over a pier or abutment, the pressure is transferred to the bottom of the posts or the top of the pier. This system of bracing is known as the portal or portal bracing.

Portal Bracing.—A common form of portal bracing is shown in Fig. 372 (*e*). *AF* and *BE* being the end posts, the plane of



FIGS. 372.

EBAF may be either vertical or inclined. Let a = length *AF* of end post, d = depth of portal bracing from *A* to *D*. Assuming the wind to be from the right, one half of the pressure on the intermediate panels is transferred by the lateral truss to the point *A*; call this *R*. In addition about one half of a full panel weight is concentrated at each of the points *A* and *B*; call this $\frac{w}{2}$. Then

there is acting at A a pressure $= R + \frac{w}{2}$ and at $B = \frac{w}{2}$. These external forces are balanced by the unknown external forces acting in the plane of the portal at the lower extremities of the end posts. These may be taken as the horizontal and vertical forces H_1, H_2, V_1, V_2.

$$\left.\begin{aligned} \Sigma H = 0 \text{ gives } H_1 + H_2 - \left(R + \frac{w}{2} + \frac{w}{2}\right) = 0, \\ H_1 + H_2 = R + w; \\ \Sigma V = 0 \text{ gives } V_1 = -V_2; \end{aligned}\right\}, \quad (580)$$

and taking moments about E or F,

$$V_1 b = (R + w)a; \quad \therefore V_1 = - V_2 = (R + w)\frac{a}{b}. \quad . \quad (581)$$

Assuming $H_1 = H_2$, then

$$H_1 = H_2 = \frac{R + w}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (581\tfrac{1}{2})$$

Assuming a section qq' cutting the portal bracing, taking centre of moments at D, and noting that with tension diagonals AC will not be in action with wind from the right, only three acting members being intersected, namely, AB, BD, and CD, the only acting moment about D is the stress in AB. The external forces acting on the portion to the right of qq' are $R + \frac{w}{2}$, H_2, and V_2. Moment of $V_2 = 0$. Hence

$$\Sigma M = \text{compression in } AB \times d - \left(R + \frac{w}{2}\right)d - H_2(a - d) = 0;$$

$$\text{Compression in } AB = \frac{\left(R + \frac{w}{2}\right)d + H_2(a - d)}{d}. \quad . \quad (582)$$

Whether $R + \frac{w}{2}$ will be found at A or B depends upon the type of lateral bracing used. With tension diagonals, as in the Pratt truss, the condition assumed above is right; but with compression diagonals, as in Howe truss, the compression AB would be less than that given above by R, as can be readily shown by substituting $\frac{w}{2}d$ for $\left(R + \frac{w}{2}\right)d$ in the above equation.

Taking moments now about B, we find

$$\Sigma M = \text{compression in } CD \times d - H_2 a + V_2 b = 0;$$

$$\text{Compression in } CD = \frac{V_2 b - H_2 a}{d} = \frac{(R + w)a}{2d}. \quad . \quad (583)$$

The only vertical force to the right of qq' is V_2. This, then, is the shear at that section. Hence

$$\text{Tension in } BD = V_2 \sec \alpha = (R + w)\frac{a}{b} \sec \alpha. \quad . \quad (584)$$

The force V_1 produces a uniform compression in the post BE, and the force V_2 acting downwards reduces the compression in the post AF from the dead and live loads on the bridge—only as far up as the point D, however, but not from D to A.

The horizontal forces H_2 and H_1 produce a bending stress upon the posts AF and EB as far up as the points D and C, where it becomes a maximum, and

$$=H_2(a - d) = \frac{R + w}{2}(a - d). \quad . \quad . \quad . \quad (585)$$

The effect of this moment is to increase the compressive stress on the inner side of the leeward post EB. As this post is also under compression from dead and live loads and also from the force V_1, the maximum compression is the sum of the three, and it should be designed to resist this combined stress. The other post is relieved by the tensile stress caused by V_2. But when the wind blows from the left this post AF will have to resist the combined stress. Therefore both posts should be designed to resist the same maximum compression. Similarly, when the wind blows from the left BD is not in action, but AC will have a tensile stress equal to that found for BD.

With inclined end posts the forces, represented by V_1 and V_2 in the preceding case and acting vertically, must be decomposed in two components. If α is the angle of inclination to the vertical, then for V_2 in the foregoing equations we must substitute $V_2 \cos \alpha$. The horizontal component $V_2 \sin \alpha$ acts directly on the lower chords, and increases the tension already existing; whereas $V_1 \cos \alpha$ acts inwards, tending to produce compression in the lower or windward chord, and in any event reducing the tension stress already existing. A reversal of stress often occurs in the end panels of the windward chord. These horizontal components are uniform throughout the length of the bridge.

In small bridges, owing to the want of head-room, simple knee-braces are used, as shown in Fig. 372 (k).

Portals with Fixed End Posts.—In the preceding discussion the end posts have been considered incapable of resisting bending

moments at their lower extremities E and F, and maximum bending moments and stresses in posts and portal have been found as if the lower segment of the post from F to D were a cantilever beam. If, however, the ends F and D are so anchored as to fix these points in position, the effect is to reduce greatly the bending moments and stresses; and as the upper ends may be taken as also fixed, the reactions H and V may be considered as applied at points half-way between E and C or F and D. At these points the moments are zero, and at D and C only one half as much as before determined, as the lever-arm is changed from $a - d$ to $\frac{1}{2}(a - d)$.

Sway-bracing.—In deck-bridges, as shown in Fig. 372 (g), sway-bracing is placed at each panel point. And in through-bridges, when the depth of the trusses is over about 25 feet, this bracing is sometimes designed to carry the wind pressure from one chord to the other at each panel point, in which case but one lateral system is required. This bracing is similar to the portal bracing shown in Fig. 372 (e); the external lateral force is the wind pressure on one panel, the portal and all intermediate sway-bracing being subjected to the same loads; the resulting vertical reactions act as loads, upward or downward as the case may be, upon the main trusses.

In addition to transferring one half the wind pressure, sway-bracing prevents lateral vibration and swaying of the vertical trusses, stiffens the long columns, and is of advantage in the erection.

In deck-bridges and deep through-bridges the sway-bracing maintains the rectangular shape of the cross-section.

In deck-bridges the portal bracing is as shown in Fig. 372 (g). The stresses in AB and FB and EF, and the direct stress in BE, are found as already explained. There is no bending moment in any member, nor is there any tension in AF.

EXPANSION-BEARINGS FOR BRIDGE SUPERSTRUCTURES.

1001. When a structure rests on practically rigid supports, some provisions for its change in length arising from changes in temperature must be made. The three devices for permitting the contraction or expansion of bridge trusses are (1) sliding-plates, (2) rockers, and (3) rollers

The first is the simplest, and consists essentially of a metal plate planed to a smooth surface and set on top of the masonry pier, and

a similar smooth surface connected to the bottom of the end post, bearing and sliding upon the first plate. If these surfaces can be kept lubricated the arrangement acts fairly well. It is still employed under riveted structures of moderate length, though but little used for long spans.

The rocker consists simply of a section of a wheel, the weight resting on the axle and the wheel rolling on its circumference. The great weight and consequent friction on the axle cause more or less sliding, which works the bearing gradually to one edge of the section, until finally the rocker falls over. This device is now rarely employed.

Rollers have always been the favorite device. The top bearing rolls on the rollers and the rollers roll on the bottom bearing. Friction is or can be practically eliminated. There are no sliding surfaces.

The sole problem, then, is to devise a form of rollers which will always roll. The function which an expansion-joint has to perform is to transfer the weight of the superstructure, whose length changes, to a fixed substructure, without exerting on that substructure any other than a purely vertical strain. Horizontal strains, whether arising from changes in dimensions, or irregularities of bearings, should be entirely eliminated.

Formerly a nest of small rollers rested on a flat cast-iron plate, and the planed end of the post, usually of cast iron, rested on the rollers. In this arrangement it was difficult if not impracticable to preserve the parallelism of the rollers, and some of the rollers would be greatly overloaded. This design has been abandoned, the more common practice being to transfer the load through bolsters to the rollers. The end posts of the bridge, bearing on pins which are supported on the bolsters, are free to rock. Formerly both cast-iron plates and cast-iron rollers were used, the rollers being completely enclosed between the plates. In this arrangement it was difficult to keep the rollers clean. An improvement on this design consisted in cutting a series of parallel grooves or dust-traps in the bottom cast-iron plate, which permitted of cleaning the rollers. The later practice has been to use for the bottom or wall plate a thin wrought-iron plate, planed on its upper surface, and stiffened by two angle-irons, which also acted as guides for the rollers. The top plate, which forms the bottom of the bolster, was also fitted with angle-irons. This arrangement completely enclosed the rollers, and prevented proper cleaning. The rollers were seldom over 3 inches in

diameter, and the collection of a little dirt around them caused considerable friction, and in addition the thin bottom plate, when not well bedded on the masonry, would be indented, locking the rollers. It was likewise difficult to maintain the nest of rollers in a rectangular form.

The remedy for the above defects is to use a thicker and stiffer wall-plate, and to increase the diameter of the rollers. The use of large rollers naturally led to the adoption of segmental rollers; that is, the rollers are planed off on the sides, having therefore two plane or flat sides and curved surfaces top and bottom. The rollers could thereby be placed closer together on centres. The main objection to these rollers arises from the danger of tipping over if moved farther than intended. This has happened, and in later designs provisions have been made to prevent it.

A still later development is to support the bottom bearing-plate on a thick wall-plate of cast-iron—the bearing-plates and rollers made of steel. These plates had ribs or guides fitting into grooves in the rollers. These cast-iron wall-plates have been cast with ribs, forming a series of rectangular pockets, which were filled with cement mortar.

Mr. Geo. S. Morison has designed and patented many devices for meeting the objections above mentioned to the use of the small rollers between two plates, the first of which substitutes for the bottom plate upon which the rollers rest a series of T rails riveted to a thin plate; the flanges of these rails are planed off to bevelled surfaces which overlap, bringing the heads close together. This composite plate rests on a large wall-plate casting. The rails were planed off on top. The middle rail projected above the others, forming a guide-rib for the rollers .The dust drops .betweens the rails, and can be removed without trouble.

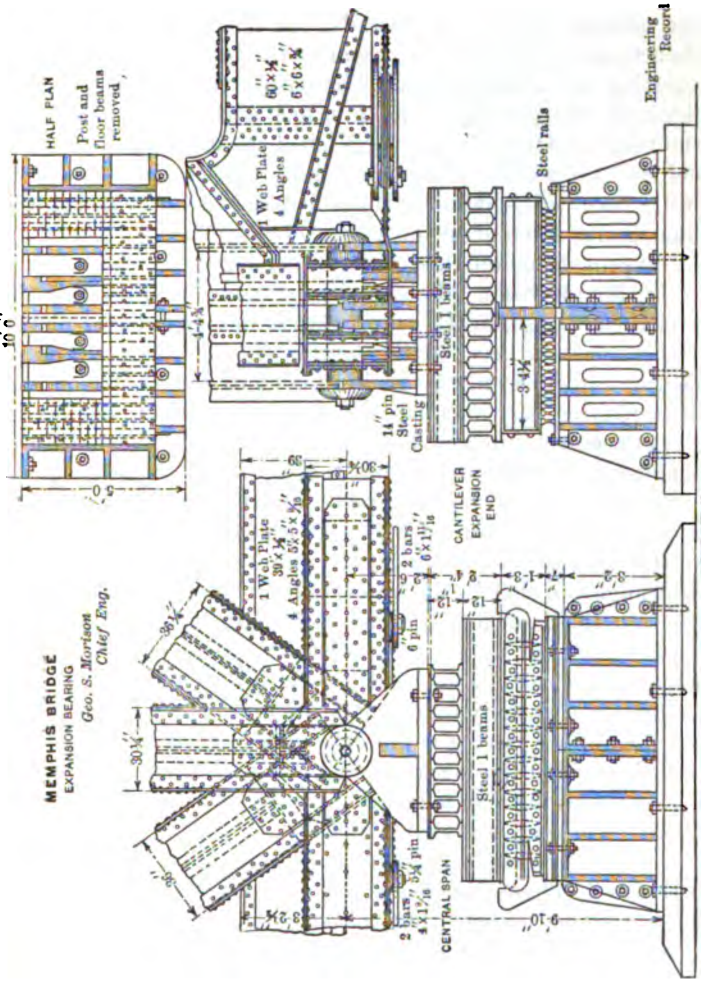
The plate above forms the bottom of the bolster and remains as shown in Fig. 373, in its general construction, consisting essentially of a series of vertical ribs riveted to the plate, the end bearing-pin resting on curved grooves cut in the top of the ribs.

In Fig. 373 are shown side and end elevation of the entire bearing used on the recently constructed bridge over the Mississippi River at Memphis.

The central span of this bridge is 621 feet long, from the ends of which project cantilever arms. This arrangement is similar to the design shown in Fig. 366.

Not considering a change of length due to strains, provision

had to be made for a change of length of about 8 inches, due to changes in temperature. The dead weight carried to each bearing is 2,454,000 pounds, and total due to dead and live loads 4,036,000



FIGS. 373.

pounds. An expansion-bearing was required for a possible movement of 10 inches and a weight of 2000 tons, which should roll freely. It is perhaps the largest expansion-bearing ever built. Large segmental rollers, 15 inches in diameter, made of forged steel, were

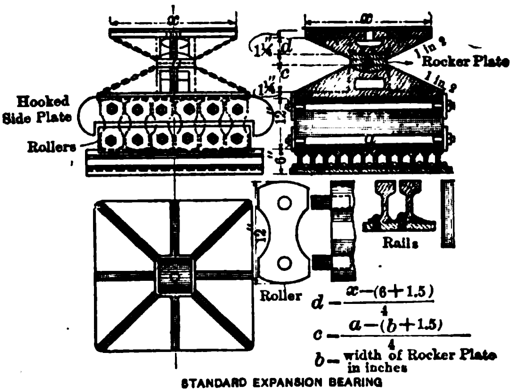
used. The sides were planed out hollow. The rollers were placed 6 inches between centres, and would shut into each other, after the European practice. The entire arrangement is shown in the figures. The wall-plate casting is 10 feet square. The rail-plate on the casting consists of 30 rails. A 4-inch steel plate in the centre acts as a guide-rib. There are 30 rollers, 15 inches in diameter, resting on the rail-plates. The bolster resting on the rollers was built up of steel plates and I beams, with a central guide-rib for the rollers. Over this is a large steel casting, which carries the 14-inch pin, on which the entire weight of the superstructure is thrown. Steel lock-plates for the rollers are also provided as a precaution against a too great movement of the rollers.

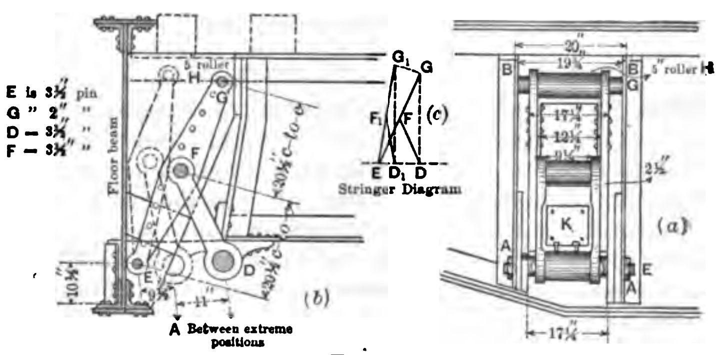
An improvement on this construction has been made and patented by Mr. Morison. This device is shown in Fig. 374. The main change consists in the use of what is termed a rocker-plate for the pin-connection between the bolster and the truss. The pin-connection provided only for rocking in one direction, which might result in making very unequal bearings, owing to a want of parallelism between the pin and the rollers caused by irregularities in workmanship or other disturbing causes. This is clearly seen in the drawings, Fig. 374. The plate bearing on the rollers is a steel casting, with a turned socket in the centre and at the top; above this another steel casting, with a turned socket below, its axis at right angles to that in the lower casting. Between the two and fitting into these sockets is the rocker-plate, which is a polished steel plate with an upper and lower cylindrical surface at right angles to each other. This bearing is placed below the chord line of the truss and entirely independent of any pin-connections in the truss. The upper casting is therefore free to rock in any direction without disturbing the lower casting, which rests upon the rollers.

The same arrangements below the rail-plates are provided as in Fig. 373. These are not shown in Fig. 374.

The rollers are made 12 inches in diameter and placed 6 inches from centres. These dimensions are kept constant for all conditions. It will be noticed that the sides of these rollers are plane surfaces near the top and bottom, and only hollowed near the centre of the depth. The rollers will not, therefore, shut into each other, but will strike before turning over. The space between can still be cleaned with a brush. The details of construction are clearly shown in the drawings. The bolster or the bottom chord is placed on the top plate, to which it is bolted rigidly. "A rocking

motion is possible in any direction, and the bearing may be depended upon to distribute the weight, not only uniformly over the several rollers, but uniformly over the length of each roller. The number of rollers may vary from 3 to 12; length in inches, from 6 to 24; total bearing in inches, from 45 to 720; safe load, at 3000 pounds





FIGS. 374.

One bracket is removed. Expansion-jointss G and D are always on same straight line, although distance between E and D varies. Hence D is always on the circumference of a circle of which EG is the diameter. $EF = GF = DF$. Angle EDG is always 90°. Since GD is vertical, ED is horizontal.

per linear inch, from 135,000 to 2,160,000 pounds; width of rocker-plate, from 4 to 14 inches; depth of lower casting, from 3 to 14 inches.

The American practice has been to make the permissible weight per linear inch of the roller in pounds equal to 500 $\sqrt{d}$, d being the

diameter of the roller. "This rule is incorrect in principle and vicious in its results," as it encourages the use of small instead of large rollers, a 12-inch roller being allowed to carry only twice the load that would be put on a 3-inch roller. Mr. Morrison accepts $500\sqrt{d}$ as correct for a 4-inch roller, i.e., 1000 pounds per linear inch, and afterwards in proportion to the diameter, viz., 250 pounds for a 1-inch roller and 3000 pounds for a 12-inch roller. The above weights are calculated on this basis. (See *Engineering Record*, Dec. 9, 1893.)

An ingenious device is shown for supporting the end of a long line of stringers on a floor-beam, and allowing at the same time for the expansion and contraction, which in this case was as much as 8 inches. In Figs. 374 (*a*) and (*b*), figure (*a*) is an end view projected on the floor-beam, and figure (*b*) is a side view showing the connections. The full lines *EFDFG* show the position of the link-expansion members when in one position and the dotted lines when in another. By this arrangement the points *G* and *D* are always on the same straight line, as indicated in diagram (*c*), and the straight line joining *E* and *D* is horizontal.

The drawings show clearly the construction.

ECONOMICAL DEPTH FOR TRUSSES.

1002. The proper and economical depth of beams and trusses, so far as its theoretical determination is concerned, can be readily calculated, as it corresponds with that which gives the least quantity of material for the same degree of strength. As the depth increases, the smaller is the stress in the flanges or chords, while the shear remains the same for the same length of span and load, the web members are theoretically the same; but practically a limit is soon reached, as the increased length of web members would require increased areas of cross-section or increasing material to give proper stiffness, and various elements of a practical nature forbid excessive depths, and economy of material does not always mean economy of manufacture, construction, and erection. The general practice is to use a depth equal to from one fifth to one tenth of the span, with a fair average in trusses of long spans of from one seventh to one eighth of the span.

It is doubtless a great mistake to determine the dimensions of parts and consequently weight of bridges by the use of a too small rolling load, as there has been a great tendency to increase of the weights of engines—far beyond that for which the large majority

of bridges have been constructed; whereas but a small per cent increase in weight of structure would have provided for a much heavier load, as it has been established that for short spans an increase of rolling load to the extent of about 20 per cent only requires about 3 per cent increase in weight, and only 8 per cent for spans up to 100 feet in length; and these additional weights do not imply an equal increase in cost. From spans of 100 to 200 feet, the increased weight for the ordinary heavy-type locomotive, known as the Pennsylvania Railroad Specifications, the equivalent uniform load being about 3000 pounds per linear foot, to the Decapod rolling load followed by a uniform rolling load of 3600 pounds per linear foot, varies in increase of weight from 8.1 to 13.2 per cent, whereas the average increase in cost will be only about two thirds of this percentage, or increase in cost from 5.4 to 8.8 per cent. In short spans, not over 100 feet, an increase of depth of girders from ½ to 1 foot does not increase the weight more than about 1 per cent; and for spans 180 feet an increase of depth from 26 to 28 feet does not increase the weight over 2 per cent, this depth being from 1/6½ to 1/7 of the span; a span of 520 feet steel, an increase in depth from 50 to 58 feet only increased the weight 3 per cent, the depth being from 1/10.4 to 1/9 of the span.

The weight of a drawbridge, including turn-table, wheels, and hand-turning machinery, will be very nearly the same as for a fixed span of the same total length to carry the same live load.

The evident conclusion from the above is that a good depth, while not adding materially to the weight of the truss or cost of the same, has many advantages, and that, all things considered, there is neither wisdom nor economy in still specifying rolling loads to be used in calculating stresses on bridges far below actual heavy loads now in use on some roads. As the increased cost is but a small per cent, many advantages are gained, greater confidence would exist in the actual safety and permanence of the structure, and, in addition, future contingencies would be amply provided for. We should at least not be far behind the heaviest loads now used.

ECONOMICAL LENGTH OF SPAN.

1003. Many questions have to be considered and settled before the actual question as to what is an economical length of spans of any particular bridge structure. These mainly involve the question of location and site, which has been discussed in a previous volume by the writer, on Foundations and Substructures.

Having then selected the type of bridge—suspension, cantilever, Linville or Whipple, Pratt or Howe truss; and the kind of material to be used—steel, iron, or wood; type of moving load, etc., a thorough knowledge of the nature of the bed of the stream, the depth from the water surface to the rock or other firm material which is to form the foundation-bed is necessary and valuable for many reasons. Also, information as to the nature and extent of floods, whether liable to great gorging of ice or drift, clear height above high water required by law in the interest of navigation, as well as the number, length, and position of channel spans; whether some of the spans can be deck-bridges or all are required to be through-bridges. The latter is now required under the law governing bridge structures over navigable streams. Often a special act of Congress is necessary allowing the construction on any special terms or conditions the particular case may seem to require. All of these considerations and conditions so far modify any general formulæ or equation that nothing seems to be left but to make a series of approximate estimates of actual resulting cost. There need not be many such trial calculations, as certain general considerations enable us to arrive approximately at the proper relation between cost of superstructure on the one hand, and substructure and foundations on the other.

As a general statement it may be said that where the foundations are not costly and substructures of no very great height, economy points to a large number of piers or supports and short spans. On the contrary, where cost of foundations is large, or great height of piers is required above water surface, economy requires few piers and long spans. Again, these questions are greatly modified by actual and relative cost of material and erection of superstructure. and that of the cost and construction of substructure and foundations, which are ever-varying quantities.

Some engineers act upon the assumption that the economical span is that which will cost as much as one pier, which, again, is based upon the assumption that the cost of a span varies as the square of the span—neither of which assumptions is accurate or correct.

In the *Engineering News* of Dec. 14th, 1889, the following expression occurs:

$$p = \frac{s^2 - ab - 400}{b}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (586)$$

p being the cost of one pier, and the conclusion being that the cost of bridge is least when the cost of piers equals the above expression, in which s = length of one span in feet.

CLASS M.	CLASS C.	CLASS T.
Mogul Engine.	Consolidation Engine.	Typical Consolidation Engine.
Uniform Train Load, 1820 lbs. per foot.	Uniform Train Load, 2240 lbs. per foot.	Uniform Train Load, 3000 lbs. per foot.
$a = 2220$	$a = 2180$	$a = 2120$
$b = 3.38$	$b = 3.1$	$b = 2.89$

The above engine weights, or their equivalents, have already been given.

The writer does not propose to criticise or judge of the value of such formulæ, but he knows that as a practical question in many cases the change of position of a pier of only a few feet may enormously increase or decrease the cost of the foundation and pier, whereas but little affecting the cost of a span. A few calculations based upon reliable information of cost of materials, cost of construction and erection, will usually be a more reliable guide.

EXAMPLES OF LONG AND HEAVY BRIDGES.

1004. *Interstate Bridge, Omaha, Nebraska.*—The swing-span of this structure is 520 feet long. It is the largest and, with one exception, the heaviest swing-span in the world. Its weight is 3,000,000 pounds, while that of the Harlem River Bridge is 4,400,000 pounds, their respective lengths being 520 feet and 389 feet. The former carries two tracks for railway and two roadways; the latter is for 4-track railway. The two next in length are the Thames River, Conn., 503 feet for 2-track railway; and the Arthur Kill, Staten Island, N. Y., 500 feet, for 1-track railway. The height is 95 feet at centre, 50 feet at first hips, and 25 feet at the end hips. The two trusses are 30 feet apart, centre to centre. The webs and chords are computed for a live load assumed at 9600 pounds per lineal foot, one arm loaded, or 8000 pounds per lineal foot, both arms loaded. The wind pressure is taken at 600 pounds per lineal foot for the lower lateral system, and 280 pounds for the upper lateral system.

The dead load is assumed at 6100 pounds per lineal foot. The main requirements were as follows: All metal medium steel, except for adjustable members, which are of wrought iron; rivets of soft steel; there are also cast-steel portions. All steel manufactured by the open-hearth process. Rivet steel shall have tensile strength of 57,000 pounds per square inch; elastic limit not less than 30,000 pounds; elongation 25 per cent in 8 inches, and a reduction of area 45 per cent; and must bend double, flat, without sign of fracture on the convex side of the bend.

The medium steel shall have ultimate tensile strength of 64,000 pounds per square inch, and elastic limit at least one half of the tensile strength; the per cent of elongation 1,200,000 pounds divided by the unit tensile strength in pounds, and a reduction of area of 2,400,000 pounds, divided by the same quantity. Some of the more important features of this bridge are shown in Fig. 374½.

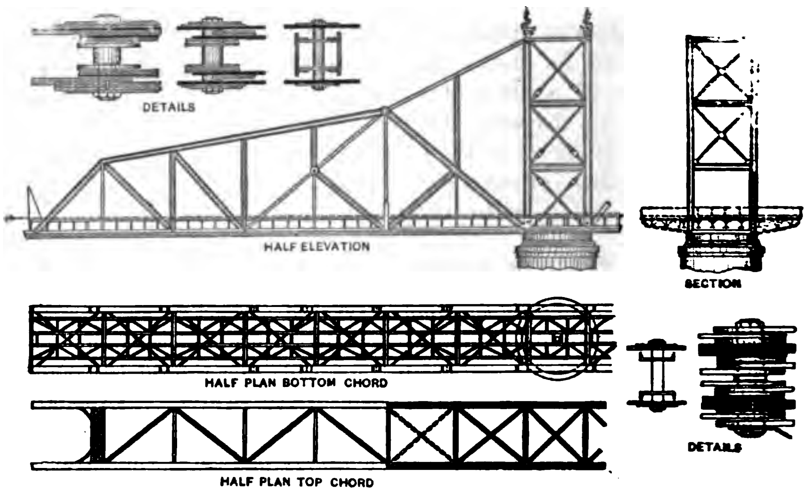


FIG. 374½.

The Vistula River Bridge at Fordan, Germany, is notable both on account of its great length and the extensive use of basic steel in its construction. Its length is 4346 feet. This is made up of five river spans about 328 feet each, and thirteen land spans of about 203 feet each. Of 10,800 tons of metal in the superstructure over one half was produced by the Thomas basic process, and the remainder by the Martin basic process. The cost was about $2,100,000, of which $500,000 went for foundations, $250,000 for masonry, $1,062,500 for superstructure, and the remainder in miscellaneous works.

The lengths of some other European bridges are: Bridge now under construction across the Danube, known as the Czernavoda Bridge, 12,628 feet long; the Tay Bridge, Scotland, 10,496 feet; the Forth Bridge, 7852 feet; the Waal Bridge at Moerdijk, Holland, 4822 feet; the Syzran Bridge, Russia, over the Volga, 4777 feet.

ERECTION OF BRIDGES.

A not unimportant, laborious, and expensive portion of bridge construction concerns the proper means of erecting structures in place. The cost of staging, falsework, and machinery is an important factor in determining the type of structure to be used, the material of which it is to be built, the location of the structure, and and the time and labor required, as well as the ultimate total cost.

The term falsework is applied to any temporary supports used in connection with the erection of structures of any kind, and not included in the general term of plant, which includes derricks, pile-drivers, tackle, boilers, engines, etc., and machinery and tools of all descriptions.

The temporary nature of the structure required is an unfortunate condition that has been a fruitful source of enormous loss of property, time, and labor, and of fearful loss of life and limb. To avoid unnecessary expense the frailest structures are often constructed. Care is not taken to secure a firm and stable foundation. The frames erected on them are often made of defective material and of small dimensions, connected and jointed in a loose and haphazard manner. Often but little bracing is used; or, in other words, all kinds of risks are taken, for the reason that the load is considered as fixed, and the falsework is of a temporary character.

The two recent disasters, resulting unquestionably from weak and poorly constructed falseworks, with their appalling loss of life, should be a warning to engineers and contractors not to take any unnecessary risks in designing and constructing falseworks for the erection of the gigantic structures built at the present time. Space forbids a full discussion of this subject.

Get rid of the idea of building only a temporary structure, and the principles of construction will be the same for falseworks as for more permanent structures.

The falseworks required in the construction of arches and tunnels have been fully discussed and described under the heads of Centres and Tunnel Linings. Commonly due thought and care is given to the design and construction of these structures, as this involves not only the questions of danger, delay, and cost, but also one of absolute necessity in executing the works properly or at all.

The following remarks will therefore only apply to the falseworks required in the erection of bridges.

For all ordinary structures, especially when at no great height

of roadway or length of span, the work can be erected from staging and by the use of derricks, shears, gin-poles, and the like, or some inexpensive framing, such as framed or pile trestles, according to circumstances.

The gin-pole is capable of performing a great deal of heavy lifting, even to very great heights, and offers many conveniences and advantages when properly supported, guyed, and carefully handled, but is both dangerous and risky otherwise. A gin-pole is nothing but a single stick of timber bearing on a broad, solid support at one end, and held in a more or less inclined position by three or more guy ropes attached to its upper and free end. These guys hold the pole in place, and by carefully slackening or tightening on the guys its top can be raised, lowered, or swung horizontally through reasonable distances and angles. The power is applied by steam or hand power working by means of spools, drums, windlasses, ropes, blocks, and other useful tackle. It is simple, inexpensive, easily placed, taken down, and removed to some other place.

For the erection of high iron viaducts with short spans between the bents of iron columns a great many devices simple but crude in design are used. The bents themselves can be connected on the ground and lifted bodily in place when not too long and heavy, or can be built in place story by story. For these purposes simple projecting beams, having their rear ends well lashed to the completed portion of the trestle and rigged with the usual blocks, ropes, etc., or the gin-pole or shears erected on the completed trestle, can be used. Where the work is of sufficient magnitude to justify the expense, platforms or cars can be run on the completed trestle and lashed down to it when in use. This car carries the poles, beams, shears, machinery, etc.

For short spans the girders can be either lifted bodily in place or moved end on from one bent to the next in front, the front end being held up when necessary by any of the means above described. Economy will usually govern the method to be adopted.

Bridge Spans.—Where the spans are not very long and heavy, or very high above the water surface, it is usual to build a pile trestle or a framed trestle resting on the bed of the river directly, or on a crib or pile foundation, depending on the depth of water, rapidity of current, and character and depth of the material forming the bed of the stream. Any trestle design is suitable for this purpose; or the same general design as shown in Fig. 374½, which is intended for a long high span, may be used.

Upon this rests directly the bottom-chord members and such other members as may be convenient to place on it. To support the upper chord during erection, and until all parts are connected up so that the bridge becomes self-supporting, another fixed frame may be erected on the trestle.

This frame consists essentially of a series of uprights, caps, and cross-pieces placed at the panel points. This frame should be well braced both longitudinally and transversely, and upon these frames the top-chord pieces rest. The bottom and top chords are then connected by the web members, thus completing the trusses.

The top lateral bracing and so much of the bottom lateral bracing as may be considered necessary are also connected up. The bridge is then swung, as it is called, by removing blocks, wedges, etc. Other members, including floor-beams and stringers, may or may not be placed before tearing down the falsework.

Long and High Spans.—In Fig. 374½ is shown a vertical elevation of one of the bents for the falsework of the Louisville and Jeffersonville Bridge across the Ohio River. This falsework was intended to support a span 550 feet long, whose lower chord was about 100 feet above the water surface.

Each frame bent is supported on eight piles driven into the bed of the river, and arranged as shown in the drawing. These bents were placed about 25 feet intervals centre to centre. It was designed to have the usual X braces in both transverse and longitudinal directions, also horizontal longitudinal braces from end to end of the falsework. There is some question as to whether these braces were placed when the entire trestle collapsed.

Instead of the fixed frame resting on the trestle for the support of the top chord, as already described, a movable structure called a *traveller*, as shown in Figs. 374½ (*a*) and (*b*), is commonly used. The general construction of this traveller is clearly shown in the drawings. Figure (*a*) shows a side elevation, and figure (*b*) a vertical section or end view.

This moves along on wheels carried on a temporary track, and is used to hold the members of panels until they are so connected that they are supported by other portions of the bridge itself. As shown, the clear width between the side frames is somewhat greater than that of the trusses of the bridge from out to out, and the clear height is greater than that of the highest point of the top chord of the bridge. The traveller can be moved from end to end clear of the trusses.

The members are lifted and suspended by tackle attached to the top frame and beams of the traveller.

This entire structure, with an almost completed iron and steel span 550 feet long, collapsed and fell into the river.

As is usually the case in such disasters, there is a conflict of testimony as to the direct cause of the failure. It is stated on one hand that the upper framework of the trestle was not braced as designed, especially with the diagonal and longitudinal braces. Consequently the bents did not act together as a unit, but gave way in detail, owing to the overloading and careening of one bent, the bents simply falling forward or folding the one on the other. While admitting a want of complete bracing, the bridge company claims that the falsework was sufficiently braced to stand any ordinary pressure from wind, current, and drift, and that the collapse was due to the unusual pressure caused by a cyclone, the pressure from which careened the traveller, which acting with a great leverage, brought an undue load on one side of the falsework; and as a further evidence of this they cite the collapse of a similar and adjoining span, practically completed, and swinging clear of all falsework, on the same day.

The writer does not undertake to decide between these two statements, as the exact facts and conditions will probably never be known.

This accident resulted in the falling of some 4,000,000 (in the two spans) pounds of iron and steel beams, columns, rods, etc., into the river, the total destruction of the traveller and 550 feet of falsework, the killing of some twenty to twenty-five men and the injuring of as many more, and finally a delay of one or two years in the completion of the structure.

There is one point, however, that to the writer seems a sufficient cause for the disaster, namely, the foundation bents, composed of piles all driven vertically, and having an unsupported length, between the lowest brace and the bed of the river, of some 32 feet or more. The fact that such structures have stood and served their purposes does not disprove the assertion; for all such bents are in a precarious condition, and if from any cause the direction of the pressure varies from the vertical, these piles will spring and bend to a dangerous extent; and if there is any tendency for the bents to careen as a whole, an unsupported length of 30 feet can be sprung from 2 to 4 feet from the vertical with no great effort, and in this

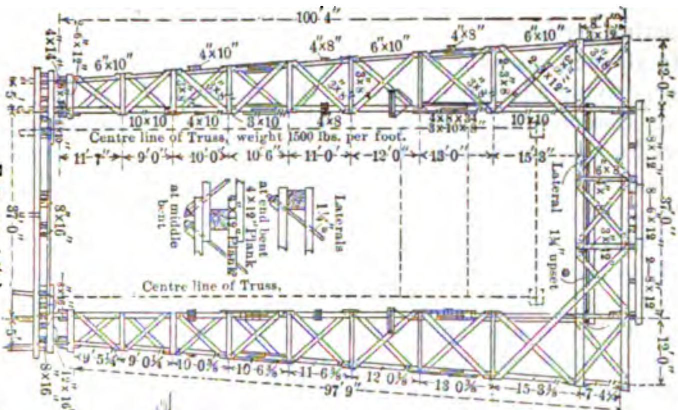


FIG. 374½(b).
END ELEVATION OF TRAVELER

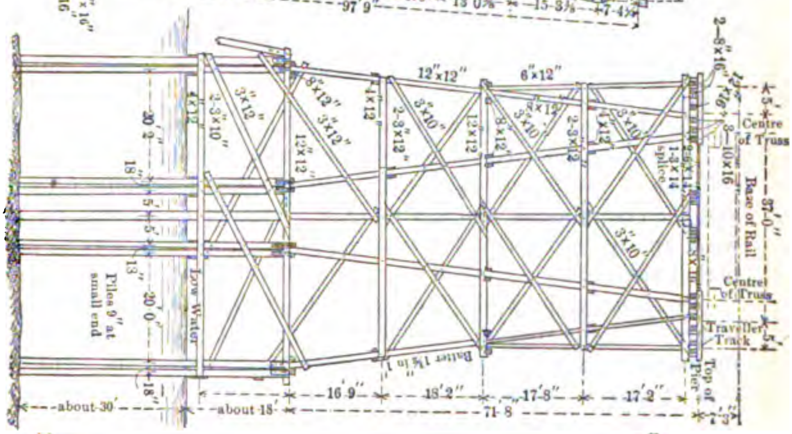


FIG. 374½.
END ELEVATION OF FALSE WORK

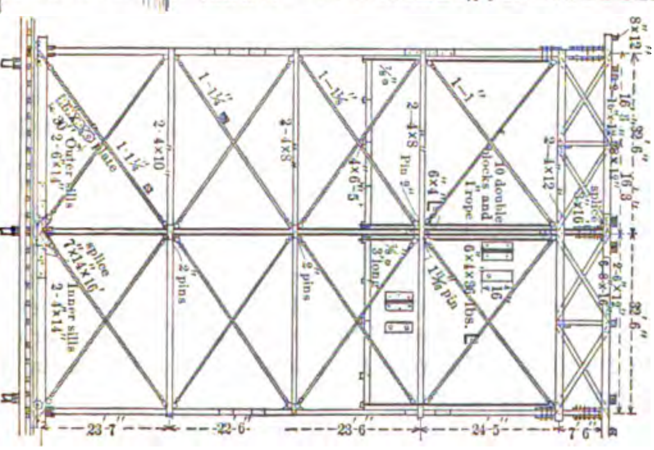


FIG. 374½(a).
SIDE ELEVATION OF TRAVELER

condition they are not capable of sustaining any great load or pressure acting vertically.

In all such cases the outer piles of a bent should be driven on a batter, driving additional piles for this purpose if necessary; or a strong system of X bracing should be bolted to the piles, using divers for this purpose below the water surface. It is, however, an easy matter to do this without divers, bolting the lower end of the brace to the pile, before driving it its full depth, at such a point as will finally be at or near the bed of the river, lashing its upper end loosely to the pile, and finally inclining it at the proper angle, and bolting its upper end to the adjacent or ultimate pile in the same bent. This can also be done in longitudinal planes, bracing the separate bents together unless the bents are very far apart, bents 25 feet apart and in a depth of water of 25 feet deep only requiring a brace 35 feet long. These could readily be stiffened by a transverse brace resting at the point of crossing of the braces, running in opposite directions.

Considering that in two recent accidents no doubt can exist as to the great unsupported lengths of piles being a potent factor in the collapse of the falsework, resulting in the death of some fifty men and the crippling of as many more, it certainly seems that no such weak and defective construction should be allowed merely to save a little expense.

Frequently, to avoid the use of high and long falseworks, or where the construction of falsework is very difficult and costly, or impracticable, it is advisable or necessary to adopt the cantilever type of bridge instead of the ordinary trusses.

COMBINED DIRECT AND BENDING STRESSES.

1005. Any tension or compression member placed in a horizontal position, in addition to the direct stress, is subjected to a transverse strain due to the bending action of its own weight. This condition always exists in the panels of the top and bottom chords of bridges. If, as is often the case, the load is transferred direct to the chord at points between the joints, this bending action may be very large; or if the direct-acting stress be placed towards one side of the axis of member, or acts on an already bent member, there will result a bending moment equal to the product of the load or direct stress by the distance between the line of action of the load and the longitudinal axis of the member.

It is to avoid this bending action that joints are designed to bring the centre-of-gravity lines of all members to a common point, thereby avoiding secondary stresses from bending.

Taking, then, a panel length of the loaded chord subjected to the bending action of loads transferred to it at several points by floor-beams or cross-ties and subjected to direct stress of tension or compression at the same time: the loads, including the weight of the chord section, deflect or bend it. The direct stress is assumed to be uniformly distributed over the area of the cross-section of the member, and its resultant as acting at the centre of gravity of the section. It therefore tends to increase the bending or deflection caused by the weight of the member and any load upon it if it is a compressive stress, and to diminish it if it is a tensile stress.

The maximum admissible fibre stress must be at least equal to (1) the fibre stress caused by the external load and weight of the beam increased by (2) the fibre stress caused by the bending action of the direct stress on the member, increased by (3) the intensity of the direct stress, and the member must be designed and dimensioned to resist all three.

If P is the total direct stress, A the area of the cross-section, v the maximum deflection, then the fibre stress or intensity of direct stress $= f_1 = \frac{P}{A}$; the bending moment due to direct stress $= Pv_1 = M_2$; and if M_1 = bending moment due cross-breaking load, while the moment of resistance to bending $= M_0 = \frac{fI}{y_1}$, the equilibrium requires $M_0 = \frac{fI}{y_1} = M_1 \pm M_2 = M_1 \pm Pv_1$, and the maximum deflection due to any load $= v_1 = \frac{nfl^2}{Ey_1}$ (see eq. (197)). Substituting v_1 in the value of M_0 above, we have $\frac{fI}{y_1} = M_1 \pm \frac{Pnfl^2}{Ey_1}$; hence $f = \dfrac{M_1y_1}{I \pm \frac{Pnl^2}{E}}$, and the maximum admissible fibre stress

$$f_0 = f + f_1 = \frac{M_1y_1}{I \pm \frac{Pnl^2}{E}} + \frac{P}{A}. \quad . \quad . \quad . \quad . \quad (587)$$

This equation is true for all forms of cross-section, and conditions of loading and supporting beams. The value of the factor n has

been given in paragraph 339 and table. For a beam supported at both ends and loaded at the centre $n = \frac{1}{12}$ or $\frac{4}{48}$, and for the same beam uniformly loaded $n = \frac{5}{48}$, noting that $n = \frac{1}{4}n''$ in equa. (477). It is therefore practically the same for any condition of loading met with in practice, and for a sufficiently close approximation may be assumed to be constant, and equal to $\frac{5}{50}$ or $\frac{1}{10}$. Substituting this value in eq. (587), it becomes

$$f_0 = f + f_1 = \frac{M_1 y_1}{I \pm \dfrac{Pl^2}{10E}} \pm \frac{P}{A}. \quad . \quad . \quad . \quad . \qquad (588)$$

When the direct stress is compressive, $-\dfrac{Pl^2}{10E}$ must be used; and when tensile, $+\dfrac{Pl^2}{10E}$.

The above conditions often occur in deck-bridges for railways, where the rolling load is practically distributed uniformly over the top-chord panels, and is not uncommonly the case in through-bridges for highways.

As a practical example, take the case of a railway deck-span for a railway bridge.

Let l = panel length = 20 feet = 240 inches;
" P = direct compressive stress in the member = 404,262 pounds;
" A = area of cross-section = 53.87 square inches;
" y_1 = distance from compressed side to the neutral axis = 9 in.;
" I = moment of inertia of cross-section = 2762;
" w = weight per foot of length per truss = 875 pounds;
" w^1 = moving load per foot of length per truss = 1500 pounds;
" $w + w'$ = 2375 pounds per foot of length, and per inch of length $= w = \frac{2375}{12} = 200$ pounds nearly. This data is taken from paragraphs 947 to 960.

The bending moment from transverse load $= M_1 = \frac{1}{8}wl^2 = \frac{1}{8} \times 200 \times (240)^2 = 1{,}440{,}000$ inch-pounds. Substituting in eq. (588), assuming $E = 28{,}000{,}000$,

$$f_0 = f + f_1 = \frac{1440000 \times 9.0}{2762 - \dfrac{400000 \times 57600}{10 \times 28000000}} + \frac{400000}{53.87} = 12{,}268 \text{ pounds.}$$

This gives the resultant compressive stress on the extreme upper fibres, and is greatly in excess of the safe unit stress allowed. The special top-chord section taken was not designed to carry any external load. Finding M_1 for the weight of the beam itself, it will be found to be about 108,000 inch-pounds, which substituted for 1,440,000 pounds in the above examples gives $f_0 = 7800$ pounds per square inch nearly, which is not excessive. To overcome this moment due to the weight of the chord sections, it is necessary to place the pin far enough below the neutral axis of the cross-section, that the bending moment of the direct stress may be equal and opposite to that of the weight of the chord segment. This position of the pin is also required to allow sufficient clearance for the eye-bar heads beneath the cover-plate.

The above principles have been applied to trussed beams, either of the triangular or trapezoidal types, considering the loaded chord as a continuous beam. It is, however, better to consider the trussing members or braces as bearing all the direct tensile or compressive stress and the beam as simply sustaining the bending section of the cross-bending loads, its length equal to one half the span for the triangular truss and one third the span for the trapezoidal truss. For a full discussion of this subject, see Johnson's Modern Framed Structures.

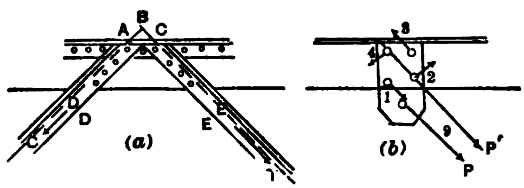
SECONDARY STRESSES.

1006. Secondary stresses are those stresses caused by bending action resulting (1) from the connection of the members at a joint in such a manner that their centre of gravity lines do not meet at a point (2) any member under combined direct and bending stress, and (3) when the connections are such that they are not free to rotate when the live load comes upon the truss, and, as a consequence, the members must spring or deflect as the structure deflects.

Nearly all riveted structures are subjected to secondary stresses, which may be as much as 75 per cent of the primary stresses. Pin-connected structures are to a great extent free from secondary stresses, especially those arising from rigidity at the joints.

In Fig. 375 (*a*) is shown a not uncommon form of riveted truss for lattice girders of no great depth, which is a single-intersection Warren riveted truss. Let the chord be composed of one plate 12 in. $\times \frac{1}{2}$ in., and two 3×4 in. angles, 25 pounds per yard, and each of the diagonals two 3×4 in. angles, 25 pounds per yard, riveted along the 4-inch leg.

If the stress along each diagonal, compression in one and tension in the other, is taken at 40,000 pounds, which may be taken as uniformly distributed over the area of cross-section, and the resultant acting along the gravity lines DB and EB, then, taking moment about the point A, we have $M = T \times AB$. With $AB = 5\frac{1}{2}$ inches and $T = 40,000$ pounds, $M = 220,000$ inch-pounds. All of the mem-



FIGS. 375.

bers meeting at a joint will resist this bending action in proportion to their moments of inertia directly and inversely to their lengths, or, as it may be termed, in proportion to their relative rigidities. It is evident that, as the moments of inertia of the diagonals are small as compared with those of the chords, while the lengths of the diagonals are greater than the panel lengths of the chords, the chord will have to supply the much larger portion of the resistance, and it is commonly assumed that the chord alone resists the bending action from the eccentric position of the members. Then from $M_e = \frac{fI}{y_1}$ we find the fibre stress in the chord due to this moment $f = \frac{M_e y_1}{I}$. This is to be divided between the chord members meeting at the joint. The moment of inertia of this chord section is 137.6, and the distance from the neutral axis to the extreme lower fibre is 8.16 inches. Taking $M_0 = \frac{1}{2} \times 220,000 = 110,000$ inch-pounds, and substituting we have $f = \frac{110,000 \times 8.16}{137.6} = 6531$ pounds, which is tension on one side of the joint and compression on the other, due entirely to the eccentric position of the members; and unless this effect was considered, and the chord member designed only for a direct intensity of stress of about 7000 to 8000 pounds, it will be seen that the actual stress may be nearly twice as great as that for which the structure was designed.

In Fig. 375 (b) is shown a common connection for small lateral rods. In this case the lateral rod is connected by means of a pin to

a plate fastened by four rivets to another member. Let the pull on the rod be 14,000 pounds, on the supposition that each of the rivets is subjected to a shear of 3500 pounds, and each bearing its proper share of the pull.

Let the direction of the pull be parallel to two of the diagonally opposite rivets 2 and 4, and the eccentricity a be 4 inches. Without disturbing the existing conditions of equilibrium we can introduce two equal and directly opposed forces, each equal to P, and in the line joining the centres of rivets 2 and 4, as indicated, which is equivalent to a direct pull P' along this line, and a couple whose moment is Pa. This moment is left-handed, and causes a stress on each of the rivets, as indicated by the arrows. The moment $Pa = 14{,}000 \times 4 = 56{,}000$ inch-pounds, about the centre of gravity of the four rivets. This point from each of the four rivets placed in the corner of a square $4\frac{1}{4}$ inches on a side will be 3 inches, which are the respective lever-arms of the stresses on the rivets, and as these all act in the same direction, we have, for the shearing stress due to the bending moment on each rivet, $S \times 4 \times 3 = 56{,}000$. $\therefore S = 4666\frac{2}{3}$ pounds, whereas the direct shear is 3500 pounds.

The resultant shear on rivet $1 = 3500 + 4666\frac{2}{3} = 8166\frac{2}{3}$ pounds;
" " " " " $3 = 3500 - 4666\frac{2}{3} = -1166\frac{2}{3}$ "
and the resultant shear on rivets 2 and $4 = \sqrt{\overline{3500}^2 + \overline{4666\frac{2}{3}}^2} =$ 5833 pounds;

the latter being the hypothenuse of a right-angled triangle, of which the two shears are the sides.

The shearing stress on rivet 1 is more than twice that which it was designed to carry, and over one and one half times on 2 and 4. The minus sign for shear in rivet 3 merely indicates that the direction of the shear is opposite in direction to that in rivet 1.

SUSPENSION BRIDGES.

1007. Suspension bridges consist essentially of two or more cables, composed of chains, bar links, flat bars connected by means of pins, and, in the more recent constructions, of iron or steel wire, from which a platform of some kind is suspended upon which the moving load is carried. The chains or cables pass over high masonry or steel towers, and their ends are anchored to heavy masonry abutments. The stress on the cables is tensile, having its least mag-

nitude at the lowest point of the cables and its maximum at the towers. Owing to the great height of towers required, and the consequent great bending action even with a small unbalanced horizontal component at their tops, it is of the first importance that the reaction in the tower should be vertical.

This condition will usually exist when there is no frictional resistance to the movement of the cables on the towers, in which case it is immaterial whether the inclinations of the two portions of the cable adjacent to the tower are the same or not, it only being necessary that the horizontal components of their tension shall be equal. There are certain conditions, however, which may require some frictional resistance at the top of the pier. This condition will be briefly discussed a little farther on.

The principles determining the stress at different points in the cables and the form of curve assumed under any condition of loading was discussed under the head of chains and cords.

Conceiving the bridge to be intersected by a vertical plane at the lowest point of the cable where the tension is horizontal, and replacing the stress by the force H, the cable being uniformly loaded with a dead load and live load, w and w' per foot of length, the origin taken at C, and taking moments about any point D, whose co-ordinates are $CE = x$ and $DE = y$, then $Hy = \frac{(w + w')x}{2} \times \frac{1}{2}x$ for one of the two cables, from which

$$x^2 = \frac{4H}{w + w'}y, \quad \ldots \ldots \quad (589)$$

which is the equation of a parabola referred to the vertex.

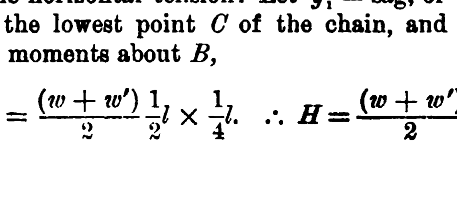


FIG. 376.

To find the horizontal tension: Let y_1 = sag, or height of the tower above the lowest point C of the chain, and l = the span. Then, taking moments about B,

$$Hy_1 = \frac{(w + w')}{2}\frac{1}{2}l \times \frac{1}{4}l. \quad \therefore \; H = \frac{(w + w')}{2}\frac{l^2}{8y_1}. \quad (590)$$

Substituting this value of H in eq. (589), we have the curve of equilibrium for a load uniformly distributed along the horizontal referred to its lowest point, $x^2 = \frac{l^2 y}{4y_1}$.

In such a curve of equilibrium the stress is one of simple tension, and the horizontal component of the tension is constant, since the external forces are all vertical. If we consider the portion of the cable between C and D, which is held in equilibrium by the three forces, tension at $C = H$, tension at $D = T$, and the load between C and D, which can be represented by the right-angled triangle DGK, in which the angle $DGK = \alpha$,

$$DE = T = GK \sec \alpha = H \sec \alpha = H \frac{\sqrt{dx^2 + dy^2}}{dx}; \quad (591)$$

and for the tension in cable at the tower,

$$T_1 = H \sec \alpha' = \sqrt{H^2 + \left(\frac{w+w'}{2} \cdot \frac{l}{2}\right)^2} = \left(\frac{w+w'}{4}\right) l \sqrt{1 + \frac{l^2}{16y_1^2}}, \quad (592)$$

after substituting H from eq. (590).

In the above discussion the cables have been assumed to be uniformly loaded from end to end.

When, however, the cable is not uniformly loaded from end to end, as when a live load covers only a part of the span, the cable will assume another curve of equilibrium for that load, unless resisted in some manner; and for this purpose stiffening trusses are used as shown in Fig. 376. This truss may be attached to the cable itself, or attached to the vertical hangers or suspending bars. The truss is not intended to aid in carrying the load to the supports, but only to maintain a uniform distribution of the load over the cable.

The truss should be made so rigid that its deflection, up or down, at any point under a concentrated load will be very small as compared to the deflection that would take place in the cable if the truss were not used. The load is supposed to rest directly on the truss, and it simply distributes the load evenly and uniformly over the cable, whatever may be the condition of loading. A full discussion of the theory of stiffening trusses will be found in Johnson on Modern Framed Structures.

"It is common to use stay-cables reaching from the tops of the

towers to the bottom of the stiffening truss, out to about one fourth the span from the towers. At this point the stays become tangent to the main cables at the towers. The stays are superfluous members, and when introduced *the distribution* of the load must be determined by the principle that it divides itself amongst the systems in direct proportion to their relative rigidity or inversely as their deflections."

THE DIRECTION AND AMOUNT OF PULL ON ANCHORAGE.

The horizontal component of the pull on the anchorage is equal that at the centre of the span as given in eq. (590), the vertical component is equal to H tan α, α being the angle which the cable at the anchorage makes with the horizontal; and if w_2 is load per foot on the shore side, as indicated to the right of the tower B in Fig. 376, including the weight of the cable itself, which may be the only load; H the horizontal component of the pull on the cable, and constant from anchorage to anchorage, i.e., from O to O', under vertical loads; V the vertical component of the pull on the cable at O, which point is taken as the origin of co-ordinates, and at a horizontal distance l' from the tower, and at a vertical distance of y_1 from the top of the tower B, then, taking moments about B, we have

$$- Hy_1 + Vl' + \frac{wl'^2}{2} = 0; \quad \therefore \quad V = H\frac{y_1}{l'} - \frac{wl'}{2}; \quad . \quad (593)$$

and taking moments about any point D',

$$- Hy + Vx + \frac{w_2x^2}{2} = 0; \quad \therefore \quad V = H\frac{y}{x} - \frac{w_2x}{2}; \quad . \quad (594)$$

from which

$$H\frac{y_1}{l'} - \frac{w_2l'}{2} = H\frac{y}{x} - \frac{w_2x}{2}; \therefore y = \frac{w_2}{2H}x^2 - \left(\frac{w_2l'}{2H} - \frac{y_1}{l'}\right)x, \quad (595)$$

which is the equation of the curve of the cable $BD'O$; and for the tangent of the angle which it makes with the horizontal at any point

$$\frac{dy}{dx} = \frac{w_2}{H}x - \left(\frac{w_2l'}{2H} - \frac{y_1}{l'}\right) = \tan \alpha. \quad . \quad . \quad . \quad (596)$$

For $x = 0$, that is, at the anchorage O,

$$\tan \alpha' = \frac{y_1}{l'} - \frac{w_2l'}{2H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (597)$$

$\frac{y_1}{l'}$ is the tangent of the angle that a straight line from B to O makes with the horizontal. Since w_2l' is or may be very small as compared with $2H$, the negative term in eq. (597) may be neglected, or, in other words, the cable $BD'O$ will deviate but little from a straight line. The vertical pull on the anchorage is

$$V = H \tan \alpha' = \frac{Hy_1}{l'} - \frac{w_2l'}{2}. \quad . \quad . \quad . \quad . \quad (598)$$

To find the angle of inclination of $BD'O$ at the tower, make $x = l'$ in eq. (596); then

$$\tan \alpha' = \frac{w_2l'}{2H} + \frac{y_1}{l'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (599)$$

This angle should be such as to cause a vertical reaction in the tower. To accomplish this it may be necessary to transfer some horizontal components of stress from stays to cables on the saddle, which may be done through their frictional resistance to sliding without any special means of attachment.

Instead of only two cables, as considered in the above discussion, there may be four or more cables, divided into sets of two in the same vertical plane. In each set the cables may be parallel or they may have different sags.

When the suspension cables are made of high-grade steel wire having an ultimate strength of 160,000 pounds per square inch and maximum working load of 40,000 pounds per square inch, there will be great economy in the use of suspension bridges. Greater care is necessary in protecting the small wires used in cables than when larger members are used.

1008. For the construction of highway bridges there are some advantages in using this type of bridge for spans exceeding 300 or 400 feet in length, but for railway bridges it is not to be recommended, except for very long spans, owing to the great cost required to stiffen sufficiently a suspension bridge to carry the heavy train loads.

1009. The East River Suspension Bridge is designed to carry all kinds of traffic, including railway-cars pulled by means of endless wire ropes. In this bridge there are four cables, suspended in three spans. The middle or river span between centres of towers is 1595.5 feet long, and the shore spans between towers and anchor-

age abutments 954.5 feet each. Each cable consists of 19 strands of 332 parallel steel wires, and contains therefore 6308 wires, which have a total ultimate strength of 10,730 tons. Each strand is secured with a 7-inch pin of iron to two anchor-bars $1\frac{1}{2} \times 9$ inches. The wires do not pass around the pins directly, but around a cast-iron shoe, which rests against the pin, and which increases the curve of bending from 7 to 17 inches.

The last link of the anchor chain, to which the strands are attached, is arranged in four tiers. In each link there are alternately 9 and 10 bars, of sizes varying from 3×9 inches for the upper to 3×7 inches for the lower links. The pins also diminish proportionally from 7 to 5 inches in diameter.

About 10 or 12 feet forward of the shoe the two halves of a strand are combined into one, and all strands before leaving the masonry are squeezed into a round cable. The cables, as solid cylinders, emerge from the anchorage masonry 8 feet below grade-line of bridge.

The towers, containing 40,000 cubic yards of masonry each, are built entirely of granite. Their dimensions at high-water line are 140×59 feet, and on top below the cornice 126×43 feet. The top is 272 feet, and the saddles, in which the cables rest, 267.54 feet above high water. The cast-iron saddles, weighing 12 tons each, rest on 43 rollers of $3\frac{1}{2}$ inches diameter, which are movable on a cast-iron bed-plate $4\frac{1}{2}$ inches thick and weighing 11 tons.

The anchorages are built of limestone with granite corners, the dimensions at the base 132×119.33 feet, at ground-line 124×111 feet, and at top 117×104 feet.

Two arches run lengthwise through the masonry, dividing it into three piers, in which the anchor chains are bedded. The total height of the anchorage above high water is at the face 89 feet. It falls towards the rear at $3\frac{1}{4}$ per cent slope.

The deflection in the finished bridge is 124.74 feet, below the saddle-plates.

Length of river span..................	1595.5	feet.
" " each land span, 930 feet......	1860	"
" " Brooklyn approach.........	971	"
" " New York "	1562.5	"
Total length..................	5989	"

'idth of bridge	85 feet.
'umber of cables	4
'iameter of each cable	$15\frac{1}{2}$ inches.
umber of wires in each cable (No. 7 gauge)	6300
ltimate strength of each cable	11,200 tons.
epth of foundation-beds below high water	45 to 78 feet.
otal height of towers above high water	277 "
lear height above high water at 50° F	135 "
rade of roadway	$3\frac{1}{4}$ per 100 feet
'eight of each anchor-plate	23 tons.
otal cost of bridge	$9,000,000

For a full discussion of the construction of this structure see
ın Nostrand's Science Series, No. 32—"Cables of the East River
·idge," by Wilhelm Hildenbrand, C.E.

SUPERSTRUCTURE OF THE TOWER BRIDGE.

1010. A general elevation of this bridge is shown in Fig. 377.
ıe towers consist of four built steel columns, octagonal in section
:h, and $119\frac{1}{4}$ feet in height from the bottom of the bed-plates.
ch of the four columns is built of T's, angles, and plates, and
s an external diameter of $5\frac{3}{4}$ feet. The plates vary in thickness

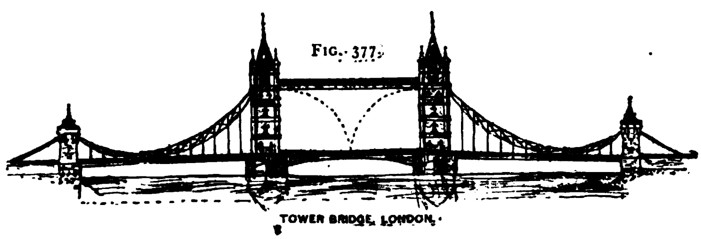


FIG. 377.

m $\frac{7}{8}$ to $\frac{5}{8}$ inch. The columns are braced together by the steel
lers carrying the several landings and by a system of diagonal
cing and wind struts.

There are two elevators in each tower by means of which pas-
gers are conveyed to the several landings and to the overhead
tway between the towers. Staircases are also located in each
er. The steelwork of each tower is covered with a masonry
ng.

The shore towers are similar in construction, but only 39 feet high. The masonry housing is only used for architectural effect.

Suspension Spans.—The two shore spans are constructed on the suspension system. The suspension frames or chains consist of two rigid segments for each, with hinged connections between the towers and with anchorage system at the shore and river towers.

The shorter of the main segments measures horizontally 106½ feet, and the longer 190½ feet. From these chains the roadway is suspended.

The Bascule Span.—This span is 226 feet long between centres of pivots, and is divided into two leaves, hinged at the towers and disconnected at the centre of the span. The counterbalance arms are 50 feet long.

The span is cleared by each of the leaves revolving on a hinge at the towers, the projecting arms revolving upwards towards the towers and the counterbalance arms downward into the bascule chambers of the piers. This general arrangement is shown in Fig. 377.

High-level Footways.—There are two footways reaching from tower to tower near their tops. Each consists of two cantilever arms with a suspended centre truss. The cantilevers are anchored to the towers. The ties which take the pull of the chains of the suspension spans are concealed in the footways. These ties are built up of eight thicknesses of 1-inch steel plates 2 feet deep.

Expansion-bearings on the shore columns of the river towers and the river columns of the shore towers allow for the expansion and contraction of the suspension system.

Siemens-Martin steel was used throughout which had a tensile strength of from 27 to 52 tons per square inch and an elongation of 20 per cent in 8 inches. About 11,000 tons of steel and 1200 tons of ornamental cast-iron work was used.

The lifting or turning of the bascule span, elevators, etc., was effected by means of hydraulic machinery.

The total length of the structure, including the approaches, is one half mile. Total height of towers above foundation-bed, 293 feet. There were used in the work 235,000 cubic feet of granite, 20,000 tons of cement (about 100,000 barrels), 70,000 cubic yards of concrete, 31,000,000 bricks, and 14,000 tons of iron and steel.

The cost was apportioned as follows:

The piers and abutments	$653,700
Northern approach, anchorage, etc.	292,380
Southern " " "	191,915
Hydraulic machinery	426,100
Iron and steel superstructure	1,685,500
Masonry superstructure	745,600
Paving and lighting	151,600
Total cost	$4,146,795

For full particulars of this novel and interesting structure, see *Engineering Record*, Dec. 1893, and *Engineering News*, Jan. 1894.

For a description of the construction of the foundations and nasonry see "Foundations," paragraph 429½, Figs. 195 (*e*) to (*q*), nclusive.

ART. LIII.

SOLID AND BRACED ELASTIC ARCHES.

1011. The elastic theory of the arch has been developed and iscussed by many writers, and many efforts have been made to pply this theory to the designing of ordinary masonry arches omposed of voussoirs or blocks of stone cemented together. The onclusion reached had been that it could not be applied to ma- onry arches; but recently the question seems to have been taken p again, and it is claimed by some, especially by German experi- enters, that the theory is capable of this application even to dinary voussoir arches, but especially to the design of concrete ches, which has of late become so common, whether with concrete one or in combination with wire netting or curved iron beams.

While the writer does not propose to discuss in full this theory d its applications, the interest lately elicited in it seems to demand e exposition of some of the fundamental and simpler principles volved, and to refer the reader to the works of Eddy, Greene, in, and others who have fully developed the graphical method of plying this theory, and to Dubois and Johnson, who have devel- ed the mathematical or analytical treatment of solid and braced ches.

1012. The term *solid arch* is applied to arches having a con- nuous web connecting the flanges; the term *braced arch* is plied to arches in which the flanges are connected by the usual

tension and compression web members. Such arches, when made of iron or timber, are capable of supplying tensile or compressive resistances and resistance to bending and shear. When these are known, the stresses in the chords or flanges and in the bracing or web are found in every respect as if the structure were a simple truss. It would seem quite clear that the theory would be applicable to concrete arches equally with those of iron or timber, except that the modulus of elasticity, coefficients of resistance to crushing and tearing apart are not so well known, defined, or constant as in iron and timber; but within the limits of our knowledge of these coefficients as applied to voussoir arches, the theory would seem to be applicable; but at present few engineers would risk their reputation by relying upon the tensile and adhesive strength of mortar in the joints of masonry arches to give stability to the structure, although it is recognized that these resistances do exist, and that the actual stability is thereby increased. The theory pertaining to solid arches is therefore only applicable to voussoir arches when the actual curve of pressure lies within the middle third of the arch-ring, so that there is no tension at any part of the joint.

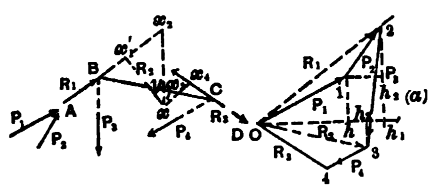
The great strength found in the combined concrete and iron arch may arise from the great initial fibre resistance developed, as was mentioned in discussing concrete arches.

For the clearer understanding of the elastic theory as applied to arches or curved beams, it will be necessary to explain the relations between the equilibrium or funicular polygon and the bending moments and stresses at any section of the arch.

Conditions of Equilibrium of any System of Forces in the same Plane whose Lines of Action do not meet in One Point.—In paragraph 200 it was seen that if any system of forces in the same plane act through the same point and are in equilibrium, then a closed polygon can be drawn whose sides are parallel to the directions of the forces and whose lengths are proportional to the magnitude of the forces; or, conversely, if any system of forces in the same plane are parallel to the sides of a closed polygon, and whose magnitudes are proportional to the lengths of those sides, that such a system of forces will be in equilibrium and their lines of action will meet at one point. This condition only involves the two equations of equilibrium, namely, ΣV or Σ, vertical components equal to zero, and ΣH or Σ, horizontal components equal to zero. This polygon is called the *force polygon.*

If, however, the lines of action of the system of forces do not meet in a common point, three equations of equilibrium must be satisfied, namely, $\Sigma V = 0$, $\Sigma H = 0$, and $\Sigma M = 0$; that is, the algebraic sum of the moments of the forces about an axis perpendicular to the plane of the forces must also be zero.

If, as shown in Fig. 378, there is a system of forces, P_1, P_2, P_3, P_4, acting as shown, we could find the resultant of P_1 and P_2 by



FIGS. 378.

constructing a parallelogram on these forces; then in like manner the diagonal with the next force P_3, and finally this resultant with P_4. The diagonal of this last parallelogram will be the resultant of the entire system. Or, more simply, in Fig. 378 (a) draw the force polygon P_1, P_2, P_3, P_4, and then draw the closing line $O2$ or R_1; this will be the resultant of P_1 and P_2; $O3$ or R_2 is the resultant of R_1 and P_3, or of P_1, P_2, and P_3; and, finally, draw $O4$ or R_3; this will be the resultant of R_2 and P_4, or of all the forces P_1, P_2, P_3, and P_4, if it acts from O towards 4, and the force necessary to produce equilibrium if it acts from 4 to O, that is, continuously around the polygon with the other forces. Then, in Fig. 378, commencing at A, the point of intersection of P_1 and P_2, draw the line AB parallel to R_1 of Fig. 378 (a) to intersection with P_3 at B; then BC parallel to R_2 of (a), intersecting P_4 at C; and then CD parallel to R_3. The broken line $ABCD$ is called the equilibrium polygon; AB, BC, CD, the segments. Then for equilibrium the force polygon must close, and the closing line $O4$ or R_3 must coincide with the last segment of the equilibrium polygon, the first giving equilibrium of translation and the second equilibrium of rotation.

It is further to be noted that each segment of the equilibrium polygon is the line of action of the resultant of all the forces to the left of that segment, namely, P_1 and P_2 to the left of AB; P_1, P_2, and P_3 to the left of BC; etc. Also, it is the line of action of the resultant of all the forces to the right of that segment, if the system of forces is a balanced one.

Drawing the horizontal line Oh' in Fig. 378 (*a*) and the vertical lines $1h$ and $2h'$, it is evident by inspection of the diagram that the ray R_1, which is the resultant of the forces P_1 and P_2, has for its horizontal component the line Oh', which is equal to the sum of the horizontal components of the forces P_1 and P_2, viz., $Oh' = Oh + hh'$, and similarly the vertical component $2h' = h_22 + h_2h'$; or, in words, the sum of the horizontal and the sum of the vertical components of the forces P_1, P_2, to the left of any segment, AB for example, of the equilibrium polygon are equal respectively to the horizontal and vertical components or projections of the ray or resultant force to which the segment is parallel. Similarly the horizontal and vertical projection of the ray R_2 are respectively equal to the sum of the horizontal and vertical components of the three forces P_1, P_2, and P_3 to the left of the segment BC parallel to R_2.

Moments of the Forces.—Taking any point x in Fig. 378 as the centre of moments, then since AB is the line of action of the resultant R_1 of the forces P_1 and P_2, it follows that $R_1 \times xx'$ is equal to the sum of the moments of P_1 and P_2 with respect to x, and $R_2 \times xx_3$ = sum of the moments of P_1, P_2, and P_3 with respect to x, and $R_3 \times xx_4$ = the sum of the moments of P_1, P_2, P_3, and P_4 with respect to x. The algebraic sum is of course meant, taking right-handed moments as positive and left-handed as negative. In words, the sum of the moments of all the forces to the left of any segment of the equilibrium polygon is the product of the force or ray parallel to that segment by the perpendicular distance between the segment and the centre or axis of moments. Again, if in Fig. 378 we draw from the centre of moments x a vertical line xx_2, then comparing the triangles $xx'x_2$ and $O2h'$ in the two diagrams, they are both right-angled triangles, and the angle at x_2 is equal to the angle at 2, since the sides are parallel. The triangles are then similar. Hence $O2$ or $R_1 : Oh' :: xx_2 : xx'$.

$$R_1 \times xx' = Oh' \times xx_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (600)$$

And in the same manner, since the triangles xyx_3 is similar to the triangle $O3h_2$, $O3 : Oh_2 :: xy : xx_3$; hence

$$(O3) \text{ or } R_2 \times xx_3 = Oh_2 \times xy. \quad . \quad . \quad . \quad . \quad (601)$$

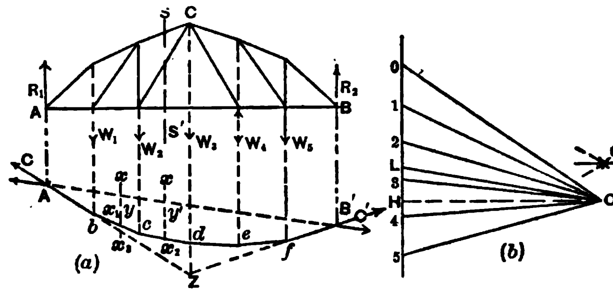
From which we learn the important principle, that the sum of moments of the forces to the left of the segment, or the moments

of resultants $R_1 \times xx'$ and $R_2 \times xx_2$ are respectively equal to the horizontal projections of the rays or forces multiplied by the respective vertical distances from the centre of moments x to the segments of the equilibrium polygon parallel to these rays R_1 and R_2, namely, $Oh' \times xx_2$ and $Oh_2 \times xy$. These principles apply also to the system of forces on the right of any segment if the entire system of forces is a balanced one.

It can be readily determined whether these moments are right-handed or left-handed, since the segments are the respective lines of action of the resultants whose directions are indicated by following the force polygon around. In the triangle $O1,2$, $O1$ or P_1 acts to the right from O to 1, 2,1 or P_2 acts from 1 towards 2; and since R_1 is the resultant it must act in contrary direction, that is, from O towards 2 or from A towards B, producing, therefore, a right-handed moment about x. If R_1 acts from 2 towards O, it would be equal and opposite to P_1 and P_2, or rather their resultant, and produce equilibrium. Since R_1 acts from O towards 2, 2,3 or P_3 from 2 towards 3, then the resultant of P_1, P_2, and $P_3 = O3$; or R_2 must act from O towards 3 or B towards C, also producing a right-handed moment, and so on.

1013. *Parallel Forces.*—If the forces are all parallel, and according to the usual assumption also vertical, the reactions will also be parallel and vertical. The force or load polygon becomes a vertical line, the forces or loads acting downwards and the two reactions acting upward, and together equal to the sum of the loads. The rays or resultant forces parallel to the segments of the equilibrium polygon all radiate from the same point, and terminate in the load line at the respective extremities of the single forces. In this case, instead of each ray having its own and separate horizontal projection a single line will be the common horizontal component or projection of all rays, whatever may be their inclination. Let ACB be any structure loaded with vertical forces W_1 to W_4, equal or unequal, as indicated in Fig. 379. Then in Fig. 379 (b) draw the force polygon 0,5, the load line divided into segments representing the loads, viz., $0,1 = W_1$, $1,2 = W_2$, etc. Then from any point O, called the pole, draw $O0$ and $O5$ to the extremities of the load line. These are evidently the components in these directions of the total load, and if assumed to act in the directions indicated by the arrows C and C' in Fig. 379 (a), will hold in equilibrium the loads W_1 to W_4; or they can be taken to replace a force equal to the resultant of the loads, but acting in the opposite direction, that is,

upward, and therefore holding the loads in equilibrium. As the position of the pole O is entirely arbitrary, we can thus divide the resultant into any two directions,—these components C and C' or $O0$ and $O5$, respectively. Then in diagram (b) draw a series of radiating lines from O to the points 1, 2, 3, and 4, which divide the load line into the separate loads assumed to be unequal in the dia-



FIGS. 379.

grams. We can now construct the equilibrium polygon $A'bcdefB'$, taking any point A' vertically below A and drawing the line C parallel to $O0$, producing it to intersection with the vertical W_1 at b. $A'b$ is the first segment; then from b a line parallel to $O1$ to intersection at c with W_2 prolonged, bc is the second segment; and so on to the point B', fB' being the last segment and parallel to the component $O5$, C representing the direction of action of 0,5 when equilibrium is assumed. We can now divide the components C and C' into vertical components and into components acting along the closing line $A'B'$ simply by drawing in (b) a line OL parallel to $A'B'$. For the joint A' the force or stress polygon is $0OL$, and for B' the stress polygon is $OL5$, OL being the equal and opposite forces acting along the closing line $A'B'$, $0L$ and $L5$ the vertical reactions at A and B, respectively.

Taking the segment $A'b$ in (a), the only vertical force acting to the left of this segment is $R_1 = OL$ in (b), and to the left of the segment bc in (a) the vertical forces are R_1 acting upward and W_1 downward. Their sum (algebraic) is therefore $R_1 - W_1 = L1$ in (b). Therefore when the ray OL has been drawn parallel to the closing line $A'B'$, the sum of the vertical forces (or the sum of the vertical components for inclined forces) acting to the left of any segment is represented by the vertical distance on the line of loads from L to the extremity of the ray parallel to that segment. The sum of the horizontal components is zero.

Moments about any point are found as in the preceding paragraph. The moments of the forces to the left of any centre x would be the vertical distance $x'y$ multiplied by the horizontal projection of the ray to which that segment is parallel, which in diagram (b) is the horizontal line OH, the common projection for all rays with vertical forces, that is, the sum of the moments of the forces R_1 and W_1 acting to the left of the segment $bc = M = OH \times x'y$, and similarly for the sum of the moments of the forces R_1, W_1, and W_2 to the left of cd about $x_1 = M' = OH \times x_2y'$, etc. The line OH is called the pole distance of the force polygon.

For vertical forces, therefore, the sum of all of the moments of the external forces to the left or right of any segment about any axis is equal to the vertical distance intercepted between the closing line and the segment of the equilibrium polygon multiplied by the pole distance of the force polygon. This is true whether the point x or centre of moments is vertically opposite the segment or the segment produced. From Fig. 379 (a) the sum of the moments of R_1 and W_1 to the left of the segment bc is

$$M = x'y \times OH; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (602)$$

and the sum of the moments of the forces R_1, W_1, and W_2 about the same point x,

$$M_2 = x_2y \times OH, \quad . \quad . \quad . \quad . \quad . \quad . \quad (603)$$

x_2y being the vertical distance from the segment cd produced. Consequently

$$M_2 - M = x'x_2 \times OH \quad . \quad . \quad . \quad . \quad . \quad (604)$$

is the moment of the single force about x. In general the moment of any single force about any point is equal to the vertical distance passing through the centre of moments (x) and intercepted between the two segments parallel to the two components of the single force multiplied by the pole distance. This is known as Culmann's Principle.

The above principles are applicable to any structure, and after constructing the force diagram (b) and the equilibrium polygon (a) we at once pass to the moment at any point. The structure may be a beam, or a roof or bridge truss. Other methods have been used in structures of these kinds, and its special application to bent beams and metallic arches for bridge or roof trusses will be alone

given. Ordinarily the scale to which the force polygon is drawn will be different from that to which the scale of the equilibrium polygon is drawn. Hence the numerical value of the pole distances in pounds or tons must be obtained from the scale of the force polygon, and the length of the vertical ordinate in units of distance, feet, or inches must be deduced from its own scale.

The position of the resultant of all or any portion of the separate forces can readily be found.

If the forces are not parallel, the resultant in force polygon will always be represented by the line necessary to close the polygon in magnitude and direction. For parallel forces the resultant is parallel to the forces in direction and equal to their sum in magnitude. If, then, we can find one point on the line of action of the resultant, it will be completely determined. For the entire system of forces, whether inclined or parallel, the directions of the extreme forces are the same as those of the first and last segments of the equilibrium polygon, as C and C' in Fig. 379 (*a*); and as these are the two components of the resultant, their intersection z is one point on the line of action of the resultant. Similarly, for the resultant of the loads W_1, W_2, and W_3, one point on its line of action would be the intersection of the segments $A'b$ and cd to the left and right of the forces.

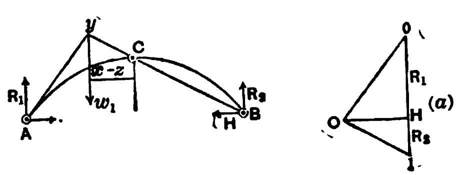
We have already seen that the position of the pole O, or the direction of the closing line, is immaterial. It may be, however, sometimes desirable to have the closing line $A'B'$, Fig. 379 (*a*), parallel to AB, that is, to the axis of a beam or bottom chord of a truss, or, commonly speaking, horizontal. It is only necessary to construct the diagram (*b*), locating the point L as has already been done. Then through L draw a line parallel to the beam or chord, placing the new pole O' anywhere on this line. Draw the rays from O' to the same points on the load-line, and with these construct a new equilibrium polygon. Then the closing line will be parallel to AB. In choosing the pole distance or the position of the pole, all that is required is to give well-defined angles of intersection between adjacent segments of the equilibrium polygon. If we double the distance OH, we will halve the ordinates of the equilibrium polygon, and *vice versa;* so that in any case, so long as the pole is on the same line OH, the product of the pole distance and the ordinate, that is, the moment of any point on any vertical line, will be constant. Again, we can adopt such a scale that the length of the ordinate of the equilibrium polygon gives the moment

direct. Since, under vertical forces, the sum of the moment of all the forces to the left or right of the segment bc about any point in the vertical line xx_1 is $M = x'y \times OH$, and assuming that the scale of the force polygon is 5000 pounds to the inch, then $OH =$ 2 inches, equivalent to 10,000 pounds, and the scale of the equilibrium polygon is 40 feet to the inch; then $x'y = \frac{1}{4}$ inch, equivalent to 10 feet; then $M = 10 \times 10{,}000 = 100{,}000$ foot-pounds.

Then by simply changing the scale of the equilibrium polygon, so that the vertical ordinates may represent $40 \times 10{,}000 = 400{,}000$ pounds to the inch, the length of any ordinate then gives at once the moment at any point along that ordinate. If we want the moment at the section SS', Fig. 379, the ordinate $x_1y' = \frac{3}{8}$ inch; hence the moment is $\frac{3}{8} \times 400{,}000 = 150{,}000$ pounds. The moments at A and B are zero. The equilibrium polygon is, therefore, a moment polygon.

1014. *Arches of Metal.*—Under the head of Arches of Metal or Timber may be classed curved beams of iron or steel and of timber. These may have either open or solid webs, with flanges or chords or curved trusses. With reference to their design and modes of support, arches may be (1) hinged at three points, namely, at the two abutments and at the crown; (2) hinged at the abutments only; (3) fixed rigidly at the abutments and continuous between them.

In paragraph 978 it was shown that a truss or arch hinged at three points could be treated as two independent trusses, and that the reactions and stresses could be found by the ordinary methods. As the truss is free to turn at the hinges or pins, the moments at these points are zero. The equilibrium polygon must then pass through these points. Let Fig. 380 be any curved beam hinged at A, C, and B, and acted upon by any single load w_1. To draw the equilibrium polygon it is only necessary to draw the segment through the points B and C to intersection with the line of action



FIGS. 380.

of the load w_1 at y; then from y draw the other segment to A. The force polygon is given in Fig. 380 (a), in which OH is the

pole distance, $O0$ is the ray parallel to the segment Ay, and $O1$ is the ray parallel to yB; $0H$ is the vertical reaction at A, and $1H$ that at B; OH is the common horizontal component or the horizontal reaction at A and B. These are the components of the reactions $O0$ and $O1$.

Let l' be the half span, r the rise, and $-z$ the horizontal distance between C and the point of application of any load w_1. Then, from the principle of the lever,

$$\left.\begin{aligned} R_1 : w_1 :: l' - z : 2l'; \quad \therefore\ R_1 = w_1\frac{l' - z}{2l'}; \\ R_2 : w' :: l' + z : 2l'; \quad \therefore\ R_2 = w_1\frac{l' + z}{2l'}. \end{aligned}\right\} \quad . \quad (605)$$

Taking the centre of moments at C, and considering the portion of the truss AC, we have

$$R_1 \times l' - H \times r + w_1 \times z = 0; \quad \therefore\ H = \frac{R_1 \times l' + w_1 \times z}{r}. \quad (606)$$

Substituting value of R_1,

$$H = w_1\frac{l'(l' - z)}{2l'r} + \frac{w_1 \times z}{r} = \frac{w_1(l' - z)}{2r}. \quad . \quad . \quad (607)$$

For a load on the portion BC at the same distance from C, R_1 would become R_2, and *vice versa;* and as no loads are on AC,

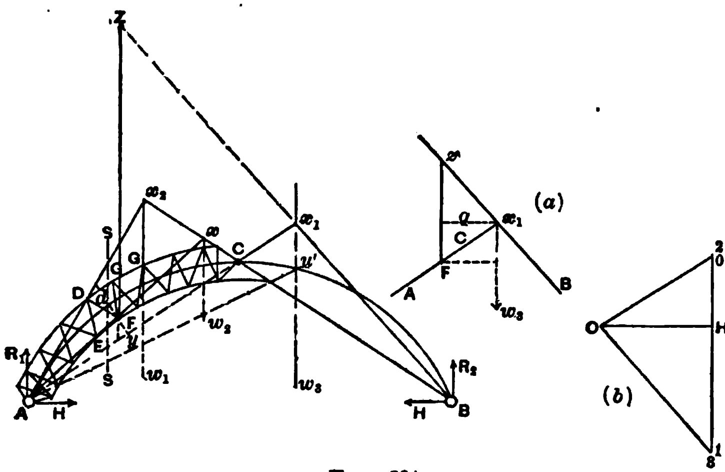
$$R_1 \times l' = H \times r; \quad \therefore\ H = \frac{w_1(l' - z)}{2r}. \quad . \quad . \quad (608)$$

In the above equations distances to the right of C are essentially positive, those to the left negative.

The above calculations are to be made for each separate load. The total vertical reaction $= \Sigma R_1$ and ΣR_2, and horizontal components ΣH. The components of the reactions being found, the stresses can be found by moments or otherwise.

1015. The position of loads for maximum stresses can be readily found, recollecting that the segments x_2A and x_2B (see Fig. 381) are the lines of action of the components of the load w_1, and xA and xB those of the load w_2 when acting towards A and B respectively, and produce equilibrium, or are the lines of action of the reactions when acting towards x_2 and x respectively, these lines passing through the panel points D and F. Considering the panel $DEFG$, the stress in DG has its centre of moments at F. It

is therefore clear that any load acting through x has no moment about F, since, cutting the members DG, DF, and EF with the vertical plane SS', the only acting force is the reaction whose line of action xA passes through F. Therefore no stress can be developed in DE by the load w_2.



Figs. 381.

For loads between x and F the line of action of the reaction will be above F, and, as this force acts towards x, its moment will be right-handed about F, to balance which the stress in DG must act towards the left, or inwards on DG towards D, causing compression. For loads between D and A the only force acting on the right of DG is the reaction at B acting along Bx, evidently causing compression in DG. Therefore, for maximum compression in DG, the load should extend from A to x; and as Ax is the line of zero moment, all loads between x and B produce tension in DG, and, provided that only this portion of the arch is loaded, is the condition for maximum tension in DG.

For stress in EF the centre of moments is D. The line of zero moments for this point is x_2A. For all loads to the left of x_2 the line of actiôn of the reaction is above D, and its moment would be right-handed with respect to D; the moment of the stress in EF then must be left-handed, that is, the stress must act outwards from E, causing tension. For all loads on the right of x_2 the line of action of the reaction passes below D, giving left-handed moments

about D, and therefore the stress in EF must act inwards towards E, causing compression in EF.

If x should be found to the right or below the hinge C, then all loads produce compression in DG, since all reaction lines would be above Ax, the lowest possible reaction line at A having to pass through C.

To find the stress in DG for any load at x_2, the only acting force to the left of the section SS' is the reaction represented in magnitude by the line $O0$ in Fig. 381 (b), and in direction by Ax_2, Fig. 381. Its moment is then $O0 \times DF$, DF being the perpendicular distance from Ax_2 to the centre of moments F. The moment of the compression in DG is the stress multiplied by Fd; hence the stress is

$$DG = \frac{O0 \times DF}{Fd}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (609)$$

Finding the stress in DG for loads at each of the panel points from A to x in the same manner, the sum of all these partial stresses will give the maximum compression in DG.

For any load between x and C draw a line from the intersection of the line of action of the load and BC to A. This line will pass below F, and will be the line of action of the reaction; its magnitude will be found as for $O0$, and its moment will be $O0 \times$ the perpendicular let fall from F. This moment divided by Fd gives the tension in DG for that load. For any load w_3 acting at u' the reaction line at A would be ACx_1, and would have same direction for any and all loads to the right of C; consequently its lever-arm would be the same for all such loads, and equal to Fy' for tensile stress in DG. The magnitude of the reaction would have to be found for each load either analytically or by diagram. The force diagram for the load w_3 is shown in Fig. 381 (b), $O2$ being the reaction at A, and AC its direction. Therefore tensile stress for this load in DG is

$$\text{Tension in } DG = \frac{O2 \times Fy'}{Fd}. \quad . \quad . \quad . \quad . \quad (610)$$

The sum of these partial tensions for each load to the right of x gives maximum tension in DG.

For the stress in the chord segment EF the centre of moments is D; and recollecting that all loads to the left of x_2 cause tension in EF, while all loads from x_2 to B produce compression, the

maximum of each kind of stress is determined in a similar manner to that in DG.

For either system of loads we could construct the equilibrium polygon for that system and find the moment, by Culmann's principle, for a single load, or, by the general method, for the entire system at any point. Applying Culmann's principle for the moment about the point F, prolong Bx_1 to intersection with the vertical through F. So much of this vertical as is included between Bx_1 produced and Ax_1, multiplied by the pole distance OH', will be the moment for w_2 at F. This is readily seen by comparing figures (a) and (b). From (b) the moment of w_2 about F is $w_2 \times q$, and from the similarity of the triangles zx_1F, and $O23$,

$$OH' : 2, 3, \text{ or } w_2 :: q : zF;$$

hence

$$OH' \times zF = w_2 \times q = O2 \times Fy'. \quad . \quad . \quad . \quad (610\tfrac{1}{2})$$

This is really another proof of the Culmann principle. The general method of the use of the equilibrium polygon in finding moments will be explained in next paragraph.

To find the stress in the diagonal DF cut by the section SS': All loads from D to A cause compression in DF, whereas all loads from F to B cause tension. It is only necessary to find the stress for each load from A to D. Their sum will give maximum compression, and the stress for each load from F to B added together will give maximum tension. These stresses are more easily found graphically than by moments. We can find the stresses in the chord segments DG and EF as already explained, taking that condition of loading which causes maximum stresses in the diagonal DF. Knowing the angle of inclination of these chord segments to a vertical, we can find the vertical components of these stresses. The vertical shear in the panel from loads between A and D will be the vertical reactions minus the loads, and for loads from F to B the vertical shear will simply be the vertical reaction at A. The algebraic sum of these components and shears for either system of loads will be the vertical component of the stress in DF, from which the stress follows at once.

In the three-hinged arch the hinges may be placed at any three points. For loads between C and B the vertices of the polygons will all be on the line CA produced; that is, one of the reaction lines will coincide in direction with CA, but the magnitudes of the

reactions will vary with the position of the load. For loads on *AO* the diagram will be reversed; that is, all of the vertices of the polygons will be on a line drawn through *B* and *C*.

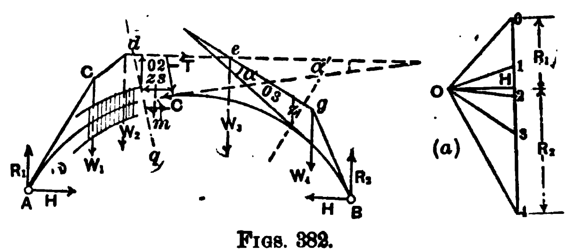
The equilibrium polygon must pass through the three hinges, and is consequently fixed in position, since no bending moment is found at these points.

For any system of loading an equilibrium polygon can be constructed for the entire system. But it will in general be easier to find the effect of a single load at successive points and combine their effects than to treat several or all of the loads together. For single loads the equilibrium polygon is composed of two straight lines, and with three hinges the direction of these lines is at once located, as fully shown in the preceding discussions.

One great advantage of the three-hinged arch is that changes of temperature do not have any straining effect. The crown rises and falls without seriously affecting the conditions of strains in the two halves of the arch. If the crown sinks a little, the value of the horizontal component of the reaction will be slightly increased. The equilibrium polygon will move with the curve of the arch.

1016. *Stresses Determined by means of the Equilibrium Polygon.*—Arches without hinges, or with only two hinges at the abutments, are more difficult to discuss, since we know neither the direction of action nor the magnitude of the reactions. In such cases the reactions depend not only upon the magnitude of the loads, which are usually assumed to be vertical, but also upon the form and material of the arch.

Assuming, for the present, that the directions of the reactions are known, that the arch has four loads acting upon it, namely,

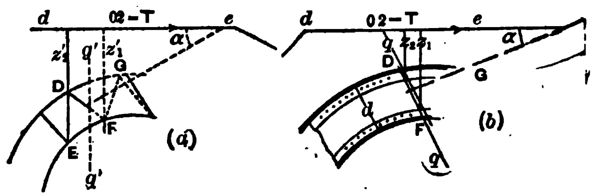


FIGS. 382.

W_1, W_2, W_3, and W_4, as in Fig. 382, then in Fig. 382 (*a*) draw the load line $O4$, and divide it into segments $O1 = W_1$, $1, 2 = W_2$, etc. Then draw the lines OO and $O4$ parallel to the reaction lines *Ac* and *Bg*. From the pole *O* thus found draw the radiating lines

$O1$, $O2$, $O3$. The equilibrium polygon $AcdegB$ is drawn as already explained. Any segment of this polygon being the line of action of the resultant of all the forces to its right or left, we have for the direction of the resultant of OO and W_1 the segment cd, and for its magnitude the parallel, say $O1$. For the forces OO, W_1, and W_2, $O2$ is the magnitude of the resultant, and de its line of action.

Taking a portion of the arch to the left of the section qq', whether an open truss or a plate girder, between the points d and e as shown in Figs. 383 (a) and (b), we can assume that the single force $O2$ acting along the line de will produce the same stresses as actu-



FIGS. 383.

ally caused by the force OO, or its component R_1 and H, and the loads W_1 and W_2. This force $O2$ must then be in equilibrium with the stresses at the section. These stresses may be taken as consisting of a uniformly distributed stress or thrust C acting in the direction of the tangent, a shear S at right angles to C, and a bending moment M. If α is the angle between the stress C and the segment de, then

$$C = T\cos\alpha; \quad S = T\sin\alpha; \quad \text{and} \quad M = OH \times z; \quad (611)$$

in which $T = O2$, OH the pole distance, and z the vertical line between the segment de and the centre of moments.

In a solid beam the fibre stresses result directly from the above equations. Thus the thrust, shear, and bending moment are easily and rapidly found after drawing the equilibrium polygon. For any section of the arch between e and g, T would be represented in magnitude by $O3$ and in direction and position by eg; α' would be the angle of the tangent to the curve at the section and the line eg; z would be the vertical from the arch line to the segment eg. (See right half Fig. 382.)

For a braced arch, as in Fig. 383 (a), and a plate girder (b) cut

by the section qq': Taking moments about F, and dividing by the depth d of the truss, we find at once the stress in

$$DG = \frac{OH \times z_1}{d}, \quad \text{and the stress in } EF = \frac{OH \times z_2}{d}. \quad (612)$$

The shear in the panel $S = T \sin \alpha$, as given above.

In these cases the flanges are assumed to resist the entire direct stress and bending stress, and the web resists the shear. If the chord segments DG and EF are not parallel, then the stress in the brace DF must be found by taking the moment of the resultant T about their point of intersection and dividing by the lever-arm of DF with respect to the same point. In the plate girder the shear is assumed to be uniformly distributed over the sectional area of the web-plate.

1017. *Deflection of a Curved Beam.*—It would be beyond the scope of this work to enter into a full discussion of the deflection of a curved beam, and the vertical and horizontal components of the motion of any point of the beam, the change of position arising from whatever cause. Therefore only a few of the equations and their applications will be given. For a full discussion of these matters the reader is referred to the works of Prof. C. E. Greene, "Trusses and Arches;" Wm. Cain, "Theory of Solid and Braced Elastic Arches;" J. B. Johnson, "Modern Framed Structures;" and A. Jay Dubois, "Strains in Framed Structures."

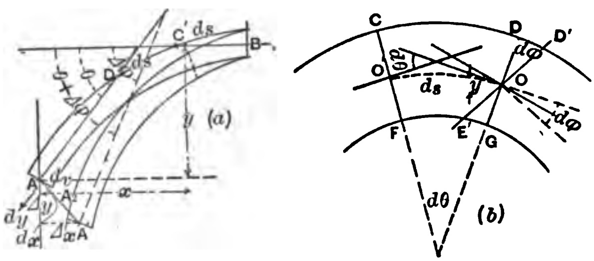
Let M be the moment of the external forces on one side of the section about a point in the neutral axis; I the moment of inertia of the section; E the coefficient of elasticity of the material; ds the original length of the fibre at a distance y from the neutral axis; da an elementary area of the cross-section; $d\theta$ the central angle between the normal sections at the two ends of ds; $\Delta\phi$ the change in angle between the end faces or end tangents due to *bending;* then the angle included between a tangent to the neutral axis of a curved beam when not strained and the tangent at the same point after deflection will be given by the equation

$$\Delta\phi = \int_A^B \frac{Mds}{EI}. \quad \ldots \ldots \quad (613)$$

This value of $\Delta\phi$ is readily found as follows: In Fig. 384 (a) is shown an originally bent beam, AB being any portion of it. As it is desired to find the relative movement of any point A with respect

to B, we may consider the end B as not moving under the deflection. The deflected beam is then shown in $A'B$.

The tangents to the curved beam at the points A and B make the angle θ with each other before deflection, and those at A' and B



Figs. 384.

the angle $\theta + \Delta\phi$ after deflection, the point A moving to A', the components of this motion being Δy and Δx.

If, now, we consider an indefinitely small portion of this beam drawn to a larger scale, as shown in diagram (*b*) included between two normal sections CF and LG. After deflection the face LG assumes the position $D'E'$. For the original length of any fibre whose distance from oo' is y, the inclination of the end tangents is $d\theta$, and the inclination of the tangents at O before and after deflection will be $d\phi$.

The change in the length of ds (either an increment or decrement, according as it is above or below oo') is $yd\phi$, and since the total alteration in length is $yd\phi$, the alteration per unit of length is $\dfrac{yd\phi}{ds}$ = strain. Then if f is the corresponding stress and E the coefficient of elasticity, then

$$E = \frac{\text{stress}}{\text{strain}} = \frac{f}{y\dfrac{d\phi}{ds}}; \quad \text{hence } f = E\frac{yd\phi}{ds}.$$

The stress per elementary area $da = fda$, the elementary moment $fday$, and total moment of resistance for the entire section

$$LG = \int_G^L fday = \int_G^D Ey^2 da\frac{d\phi}{ds}.$$

Equating this to the bending moment at the section of the external forces M, and noting that E is constant for the same

material, and $\frac{d\phi}{ds}$ is also constant for any particular section, and y^2da is the moment of inertia of the section, then

$$M = Ey^2da\frac{d\phi}{ds} = EI\frac{d\phi}{ds}; \quad d\phi = \frac{Mds}{EI}.$$

Since $\Delta\phi$ is the entire change of inclination of the tangents at A and B, then

$$\Delta\phi = \int d\phi = \int_A^B \frac{Mds}{EI}, \quad . \quad . \quad . \quad (613\,(a))$$

which is the same as equa. (613).

If, then, we divide the segment BA of the beam into any number of equal lengths, each equal to ds, and assume that the total change of position AA' is effected by the bending of each ds successively, we can find that portion due to the bending of the first segment $BC'' = ds$, as follows: The bending of BC'' through the angle $d\phi$ causes the point A to deflect through the distance $AA_1 = dv$, turning through the same angle $d\phi$ with a radius r about C''. If x and y are the co-ordinates of C'', dx and dy will be the components of dv. From similar triangles $dy : dv :: x : r$, and $dx : dv :: y : r$; $dy = \frac{xdv}{r}$, and since $dv = rd\phi$, $dy = xd\phi$; $dx = yd\phi$. Substituting, $d\phi = \frac{Mds}{EI}$; and noting that $\Delta y = \int dy$ and $\Delta x = \int dx$, we have

$$\left.\begin{aligned} \Delta y &= \int dy = \int_A^B xd\phi = \int_A^B \frac{Mxds}{EI} \\ \Delta x &= \int dx = \int_A^B yd\phi = \int_A^B \frac{Myds}{EI}. \end{aligned}\right\} \quad . \quad . \quad (614)$$

It is to be observed that in all of the above, owing to $d\phi$ being very small, the arc itself has been used instead of its tangent.

Equations (613) and (614), giving values of $\Delta\phi$, Δy, and Δx, which are unknown quantities, can be used in finding reactions or in locating the position of the resultant at any section only by making certain assumptions.

If, for instance, we assume that the tangents of an arch at the abutments are fixed in direction, then

$$\Delta\phi = \int_A^B \frac{Mds}{EI} = 0; \quad . \quad . \quad . \quad . \quad . \quad (615)$$

and further, if no horizontal or vertical displacement can occur, i.e., for an arch with ends fixed, then also

$$\Delta x = \int_A^B \frac{Myds}{EI} = 0, \quad \text{and} \quad \Delta y = \int_A^B \frac{Mxds}{EI} = 0. \quad . \quad (616)$$

For an arch hinged at the abutments and continuous between them, only the horizontal component of the motion of A is zero; that is,

$$\Delta x = \int_A^B \frac{Myds}{EI} = 0; \quad . \quad . \quad . \quad . \quad . \quad (617)$$

and if the arch is hinged at some intermediate point and fixed in direction at the ends, the horizontal and vertical displacements of the hinged point C' for the part AC' must be the same as for the part BC'. Hence we have

$$\left.\begin{aligned} \Delta x &= \int_{C'}^B \frac{Myds}{EI} = \int_{C'}^A \frac{Myds}{EI}, \\ \text{and} \qquad \Delta y &= -\int_{C'}^B \frac{Mxds}{EI} = -\int_{C'}^A \frac{Mxds}{EI}, \end{aligned}\right\} \quad . \quad . \quad . \quad (618)$$

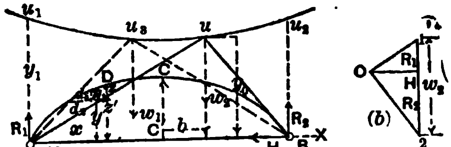
the origin of co-ordinates being taken at C'.

If the arch is fixed in direction at one end, but hinged at the other,

$$\int_A^B \frac{Myds}{EI} = 0, \quad \text{and} \quad \int \frac{Mxds}{EI} = 0, \quad . \quad . \quad (619)$$

the origin being at the hinged end.

(a)



FIGS. 385.

PARABOLIC ARCH WITH TWO HINGES.

1018. We have seen that with three hinges the directions of the reaction lines are readily found, as one of them will always have to

pass through two of the hinges. This is not possible with a two-hinged arch, and the problem is to find the locus of the intersection of the reaction lines of any and all loads. This being found, the reactions in magnitude, direction, and points of application are then found as in the preceding cases. Since the arch is hinged at the abutments we know that the reactions must pass through the hinges, as the bending moments are zero for these points; and this, with

$$\varDelta x = \int_A^B \frac{Myds}{EI} = 0, \quad . \quad . \quad . \quad . \quad . \quad (620)$$

is all that we do know.

If, then, in Fig. 385 we have a two-hinged parabolic arch, with the tollowing data given: The rise $CC' = r$; the half span $AC' = C'B = l'$; the distance of any load w_2 from the centre $= b$, measured positively towards the right; x and y the co-ordinates of any point of the arch with origin at A. Assuming any point u as the intersection of the reaction lines for the load w_2, we can construct the force polygon (b), having OH for the pole distance. The corresponding equilibrium polygon is AuB; z the vertical ordinate from the segment Au to any point D of the arch;—then the moment

$$M = OH \times z = Hz. \quad . \quad . \quad . \quad . \quad . \quad . \quad (621)$$

The coefficient of elasticity being constant, we have, substituting Hz for M in equa. (620),

$$\int_A^B \frac{Myds}{EI} = \frac{H}{E}\int_A^B \frac{zyds}{I} = 0, \quad \text{or} \quad \int_A^B \frac{zyds}{I} = 0. \quad (622)$$

If it is further assumed that the moment of inertia I increases from the crown to the springing-line in the same ratio as the secant of the angle of inclination i of the arch at any point to the horizon, $\frac{I}{\sec i} =$ a constant $= I_0$, the moment of inertia at the crown. From Fig. 385 $dx = ds \cos i = \frac{ds}{\sec i}$. Substituting in equa. (622),

$$\left.\begin{aligned} \int_A^B \frac{zyds}{I} = \int_A^B \frac{zyds}{I_0 \sec i} = \frac{1}{I_0}\int_A^B zydx = 0, \\ \text{or} \qquad \int_A^B zydx = \int_0^{2l'} zydx = 0. \end{aligned}\right\} . \quad (623)$$

From Fig. 385, $z = y - z'$, eq. (623), becomes

$$\int_0^{2l'} y^2 dx - \int_0^{2l'} z'y dx = 0. \quad . \quad . \quad . \quad . \quad . \quad (624)$$

Substituting in this equation the value of $y = \frac{rx}{l'^2}(2l' - x)$, as found from the equation of the parabola with origin at A, and also the value of $z' = y_0 \frac{x}{l' + b}$, as found from the proportion derived from Fig. 385,

$$z' : x :: y_0 : l' + b,$$

in which y_0 is the ordinate of the point u, we will find that the first term of eq. (624) becomes

$$\int_0^{2l'} y^2 dx = \frac{16}{15} r^2 c;$$

and the second term

$$\int_0^{2l'} z'y dx = \frac{ry_0}{6}\left(5l' - \frac{b^2}{l'}\right);$$

hence

$$\frac{16}{15} r^2 c - \frac{ry_0}{6}\left(5l' - \frac{b^2}{l'}\right) = 0.$$

Solving with respect to y_0, we have

$$y_0 = \frac{32l'^2 r}{25l'^2 - 5b^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (625)$$

This equation fixes the position of the point u for the load w_2; uA and uB are then the reaction lines for the load w_2 at a distance b from the centre. Giving any desired number of values to b for the positions of different loads, we can locate the curve u_1uu_2, which is locus of the vertices of equilibrium polygons for single loads.

For the load W_1 prolong its line of action to intersection with this curve; then U_2A and U_2B will be the reaction lines for the load W_1. A force polygon can be constructed to correspond; the values of H and R_1 and R_2 can be found, and also the moment M, at any point of the arch; and similarly for any number of loads.

The position of loads for maximum stresses and the stresses in

the members of the arch can then be determined as in paragraphs 1015, 1016, and 1017.

1019. For a circular arch it will be sufficiently accurate to assume a parabolic arch of the same span and of such a rise that the areas included between the arch and the chord from springing to springing points, which latter is the same length in both cases, shall be equal.

If the rise of a circular segmental rib is not more than one tenth of its span, a parabolic arch of the same span and rise will sensibly coincide with it.

1020. *Temperature Stresses.*—While in a three-hinged arch there are no temperature strains, in an arch with only two hinges the strains caused by a change in temperature may be considerable. If the temperature rises above that for which there is no strain, a horizontal pressure will be exerted against the supports at its ends. This tendency to increase its length is resisted by the abutment reactions, and stresses are developed in the arch. These reactions acting inwards give negative moments at every point, causing tension in the outer flanges. If the temperature falls below that for no strain, the tendency is to shorten the span, which is resisted by the reactions of the abutments acting outwards, and causing positive moments at every point, resulting in compression in the outer flanges.

Since the abutments are considered as fixed or rigid, the length of span remains unchanged, and any alteration in the form of the arch can only manifest itself by a rise or fall of the crown of the arch. Expansion and contraction are in proportion to the change of temperature t in degrees, the coefficient of expansion e of the material, and to the length of span $2l'$. The total change in length can then be taken equal to

$$\pm 2l'et. \qquad (626)$$

The plus or minus value is to be used according as t indicates a rise or fall in temperature.

If $\pm H_t$ represents this horizontal thrust or pull, it will exert on all parts of the arch both a direct stress and a bending action. Since the condition for an invariable length of span is

$$\Delta x = \int_A^B \frac{Myds}{EI} = 0,$$

we must now have

$$\int_0^{2l'} \frac{Myds}{EI} \pm 2l'et = 0;$$

or, since $ds = dx \sec i$, and $I = I_0 \sec i$,

$$\frac{1}{EI_0}\int_0^{2l'} Mydx \pm 2l'et = 0. \quad . \quad . \quad . \quad . \quad (627)$$

M for any point of the rib is $H_t y$, y being any ordinate of the arch; then

$$\frac{H_t}{EI_0}\int_0^{2l'} y^2dx \pm 2l'et = 0. . \quad . \quad . \quad . \quad . \quad (628)$$

It has already been shown that, for a parabolic rib with rise r and half span l', $\int_0^{2l'} y^2dx = \frac{16}{15}r^2l'$. Substituting and solving for H_t,

$$H_t = \pm \frac{15EI_0te}{8r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (629)$$

The maximum bending moment occurs at the middle of the span, where the lever-arm of H_t is r; then

$$\text{Maximum } M = H_tr = \frac{15}{8}\frac{EI_0ter}{r^2} = \frac{15}{8}\frac{EI_0te}{r}. \quad . \quad . \quad (630)$$

The temperature stresses can now be found and must be combined with the maximum stresses found for live and dead loads.

1021. *Parabolic Arch with Fixed Ends.*—In the case of an arched rib with fixed ends we neither know the direction, magnitude, nor point of application of the reactions. Three conditions must be fulfilled in order to find the equilibrium polygon of any load, namely, that

(1) the change of inclination at the abutments $\Delta\phi = \int_A^B \frac{Mds}{EI} = 0$;

(2) the change in the length of span $\Delta x = \int_A^B \frac{Myds}{EI} = 0$;

(3) the abutment deflection $\Delta y = \int_A^B \frac{Mxds}{EI} = 0$.

With these three equations we find the point whose ordinate is y_0, which is the common intersection of any load and its components or reaction lines, and the position of the points of application of the reactions at the abutments, which will be vertically above or below the points A and B, or the ends of the arch, by the distances y_1 and y_2, respectively. The process is similar in every respect to that followed in determining y_0, the ordinate of the vertex of equilibrium polygon for the arch with hinged ends. There are three unknown quantities; the analysis is long and tedious, though involving no especial difficulties. The results only will therefore be given. For any load at a distance b from the centre of the span on the right, y_1 being the vertical ordinate of the end of the equilibrium polygon above the left abutment, that is, the one farthest from the load; y_2 the vertical ordinate to the end of the equilibrium above or below the abutment nearest the load; then

$$y_1 = \frac{2}{15}\frac{l' + 5b}{l' + b}r; \quad y_2 = \frac{2}{15}\frac{l' - 5b}{l' - b}r. \quad . \quad . \quad . \quad (631)$$

These values vary with the position of the load. For instance, as b varies from $+ l'$ to $- l'$, y_1 varies from $+ \frac{6}{15}r$ to $- \infty$, and y_2 varies in the opposite direction. The value of y_0 becomes

$$y_0 = \tfrac{6}{5}r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (632)$$

This is the equation of a straight line parallel to AB, and situated $\frac{6}{5}r$ above AB, or $\frac{1}{5}r$ above the crown of the arch. Therefore one point on the reaction lines is the intersection of the vertical load line, and this straight line for all loads. But y_1 and y_2 require a separate calculation for each load.

1022. Temperature stresses, when the ends are fixed, are found by a similar process, as in the case of the arch with hinged ends. The points of application of the thrust H_t is not, however, at the points A and B, but at a distance above these points, $y_1 = y_2 = \frac{2}{3}r$, the rise being r. This changes the value of M from $M = H_t y$ to $M = H_t(y - y_1)$. These substitutions being made in

$$\frac{1}{EI_0}\int_0^{2l'} Mydx \pm 2l'et = 0$$

gives

$$H_t = \frac{45EI_0te}{4r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (633)$$

The resulting stresses may be computed by moments and shears or by diagram.

As we have seen, the reactions do not pass through the points A and B, but through points above or below the ends. This means that we not only have at each end a horizontal and a vertical component of the reaction, but also a bending moment which varies with the position of the load, and must be of such a magnitude as will keep the ends of the arch fixed in direction.

We further note that, with the same change in temperature, the temperature thrust H_t is six times as great with fixed ends as compared with its value for hinged ends, and the direct stress in the ribs will therefore vary in the same proportion.

The maximum moment at the springing for the rib with fixed ends is four times as great as at the crown with hinged ends, and of the opposite kind, and taken at the two crowns is twice as great.

For a full and exhaustive treatise on this subject, accompanied with numerous examples worked out in the greatest detail for all forms of curves of intrados, and both metal and stone used, the reader is referred to Arches, Part III, "Trusses and Arches," by Charles E. Greene, a most interesting and instructive work throughout.

ART. LIV.

FORMATION AND PERMANENT WAY.

1023. It is convenient to divide a railway or a highway construction into two parts, namely, Formation and Permanent Way. The works which constitute the Formation include earthworks, drains, retaining-walls, masonry piers, foundations, timber and iron trestles, bridges, tunnels, etc., in short, everything necessary to receive the permanent way, the permanent way being those parts of structures upon which the traffic is directly carried. The first division of the subject has been already fully discussed in preceding articles.

Railway permanent way will be taken to include the ballast, ties, and iron or steel rails, or track, as it is commonly called.

1024. *Ballast.*—Earth, cinders, slag, gravel, broken stone and burnt clay are used for ballast. Convenience, cost, and safety determine the material to be used in any particular case.

Earth ballast is nothing more than the ordinary earth used in

the embankments and found in the excavations. The ties are simply laid on the bed, and the earth thrown and compacted around and between the ties. It is of little value unless exceptionally good material is convenient, and unless surfaced in such a manner that it is even with the top of the ties at and near the centre line and then slopes off so as to be even with the bottom of the ties at their ends. In good weather a fairly good track can be thus made. It is employed upon thousands of miles of track in the South and West, and is commonly the only kind of ballast used on new roads.

Cinder and slag ballast are often used where these materials are found convenient and cheap, and make good ballast.

Broken-stone ballast, or a mixture of coarse and fine gravel, is the best material, giving elasticity and good drainage. Almost any stone but the softer varieties of sandstone and disintegrating shales and limestones are suitable for the purpose. Although it is better with either of these materials to lay a bed of stone, gravel, or slag, from 9 to 12 inches in thickness on the earthen bed, and upon this layer place the ties, it is the usual custom to first lay the ties and rails and do the ballasting subsequently with construction trains. In this latter case the ties and track have to be raised and the ballast forced and rammed under them. This method bends the rails and throws the track out of line. But rapidity of construction and saving in cost usually control such matters. In either case the ties should be entirely imbedded in the ballast, which should slope off from the ends of the ties at about 1 to 1.

Knowing the number and dimensions of cross-ties, and depth of ballast beneath them, it is easy to calculate the ballast required per yard or mile of track. Taking cross-ties $6'' \times 8'' \times 9'$, and taking 2640 to the mile, their volume would be 3 cubic feet or one ninth cubic yard to the tie, or 293 cubic yards per mile. Ballast 12 inches deep and average width of 10 feet will be $1960 - 293 = 1667$ cubic yards per mile, which will cost from 60 cents to $1 per cubic yard. The above quantity may be taken as a minimum.

Burnt clay has been used for ballast, and it is claimed that it is both cheap and suitable.

Ballast is used only on the earthen foundation-bed, as a rule. It is, however, being introduced on bridges, where it possesses many advantages as far as safety and speed are concerned.

1025. Cross-ties are used for the rails to rest upon, and to which

they are spiked or bolted. These keep the parallelism of the rails, the alignment both horizontally and vertically, and distribute the heavy concentrated weights on the rails over a broad bearing surface. They are usually of some kind of timber, and commonly white, post, or chestnut oak, or pine. The durability of the tie depends largely upon character and quantity of ballast used. A tie, however, often cuts and rots out at the rail-bearing and spike-holes, while remaining comparatively sound and solid in other parts. Oak ties are liable to warp and split, but hold the spikes better. The number of ties per mile vary from 2112 when spaced 2½ feet centres to 2640 when spaced 2 feet centres. The dimensions of ties vary from 8 to 9 feet long, from 6 to 8 inches deep, and from 8 to 10 inches wide. On bridges and trestles they are commonly sawn; on other portions of the road hewn ties are used. The life of ties may be taken between extreme limits of 4 and 8 years, depending on material, ballast, and drainage.

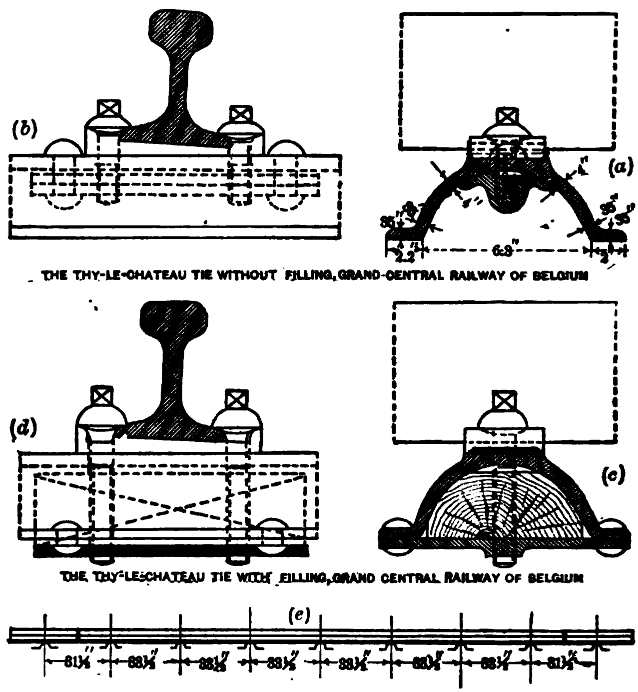
The life of ties can be greatly prolonged by the resort to some of the means of protecting timber from decay. But little attention has been given to this subject in this country, owing to the expense necessary and the abundance and cheapness of suitable timber, the price per tie varying from 20 to 40 cents, according to quality and abundance of material convenient to line of the road.

In European countries much attention has been given to this matter, with satisfactory results. But, notwithstanding this, much attention has also been given to the subject of the substitution of iron ties. Hundreds of patents have been issued, and some useful types invented. At present, in this country at least, they have not come into general use. Drawings and description of one type is given in Figs. 386 and paragraph 1027.

1026. Some of the general advantages are: (1) increased life as compared with timber ties; (2) the width of gauge is better maintained; (3) the cost of maintenance remains practically constant after one or two years' use, whereas this cost increases rapidly with age for timber ties; (4) the fastenings can be made safer and more permanent; (5) the old material has greater value; (6) it can be manufactured in that form giving the greatest strength with the least material. They can be made of varying thickness and have the slope (1 in 20) for the rail formed on them. The metal can be disposed where it will do the most good. The minimum cross-section is maintained for about two thirds of its length, and thickened under and on each side of the position of the rail. (7) It is

easily packed with any kind of ballast; (8) a broad bearing for the base of the rail can be secured.

Among the disadvantages accruing from the use of iron ties may be mentioned: (1) increase of first cost; (2) the holes for fastenings reduce the effective area of section; (3) the brittleness around the bolt-holes arising from punching; (3) the foot of the rail eats in time



FIGS. 886.

into the top of the tie; (4) the tie is more injuriously affected by impulse and shock at the rail bearing; (5) the earlier types were made too light, not over 55 to 70 pounds in weight, bending and shaking, attended with the working out of the ballast, and consequent expensive maintenance, cracking of the ties lengthwise and crosswise under the rail; (6) straining of the material at the rail-bearing during manufacture.

Increased weights from 100 to 160 pounds or more; the use of mild steel having a tensile strength of from 50,000 to 60,000 pounds per square inch; contraction of area from 30 to 40 per cent; care and

precautions in forming bolt-holes; closing the ends of the sleeper; etc.,—do away with most, if not all, of the above-mentioned disadvantages.

It is found better, while filling under the tie, that the ballast should not be too firmly compacted near the middle of the tie; but under and for a short distance on either side of the rail it should be well compacted.

Rusting and corrosion are not considered to be of any great injury to the metal tie.

1027. *Metallic Ties.*—It is not proposed to enter fully into the discussion of the actual value of metallic ties, or of their value as compared with timber ties. The following illustrations and remarks are taken from *The American Engineer and Railroad Journal*, December, 1893. Only one form of cross-section of tie is given; heavy forms of section have been designed, a great deal of experimenting has been done, and, as might be expected, the results have been both favorably and unfavorably interpreted. One form of metal tie under one set of conditions has given satisfaction, while another form used under the same or different conditions has not proved satisfactory.

They have been built up of channel-irons and brackets riveted to them, of rolled steel with riveted chairs, rolled iron riveted, hollow ties of the Post and Bract outlines, etc.

There are over 25,000 miles of track laid with metal ties, which, according to Mr. B. E. Fernow, U. S. Department of Agriculture, has proved satisfactory and successful. He says: "It is not a consideration of initial cost that makes the substitution of metal ties desirable or profitable. It is the superiority of track, permanence of roadbed, safety, greatly reduced cost of maintenance, and hence ultimate saving and economy, that recommend the metal tie."

The Grand Central Railway of Belgium have tested two types; namely, one without wooden filling-pieces, and one with such pieces. They laid a length of line 6.2 miles long in the years 1886–8. The ballast was composed of broken stones, cinders, and river gravel. The rails, of Bessemer steel, were 19 feet 8 inches long, weighing 83.8 pounds per yard. Under each length there were placed seven ties, spaced as shown in Fig. 386 (*e*). The ties are 8′ 6.3″ long; the first used weighed 131 lbs., but in later patterns 133.6 lbs. each. They are made of iron, as are also the rivets and clamp bolts; the tie-plates and bolts are of steel. The heaviest engines used weigh 52.3 tons, their rigid wheel-base is 14′ 1″. Maxi-

mum speed allowed, 37.3 miles per hour for passenger trains and 15.5 miles for freight trains.

This track has proved satisfactory, being more economical, more stable and easily maintained, as compared with those portions laid on wooden ties.

At a recent session of the Railway Congress, held at St. Petersburg, the following conclusion was reached:

"The use of metallic ties will permit of a saving of expense of maintenance when they fulfil the conditions for rational use; in other words, if their form has been so designed and their weight settled by paying careful attention to, first, the conditions of the traffic—that is to say, the speed and weight of the trains; second, to the conditions of the structure of the track and the nature of the subsoil; and, third, to the kind of ballast used;" etc. The engineers of the Belgian State Railway modified the above to the following: "By only using metallic ties on lines where the traffic was comparatively light and run over by trains at a low speed."

These conclusions were based, it seems, upon the unfavorable results of tests made in Belgium. The defects developed were to some extent traceable to defective manufacture, such as cracks resulting from punching instead of drilling the rectangular holes for the fastenings; and the steel used not being very mild.

Again: "In consequence of the alternating up-and-down movement of the hollow tie, it causes a hammering out of the ballast, which is transformed into a powder, and later into mud by the rain. Each time that the tie rises it sucks in this mud and the ballast loaded with it, forming a kind of macadam, which finally forms a hard and compact core, necessitating the removal of the ballast from beneath the support. This is the first period of maintenance, which is universally recognized as expensive.

"Under the action of the passage of trains the ties were subjected to a scraping which caused changes both in the longitudinal and transverse directions.

"These disturbances, which exist for all types of hollow ties, increase very rapidly and very notably with the speed of the trains. While for a maximum speed of from 47 to 50 miles per hour changes in level are small, and while these small movements exert hardly any influence upon the ordinary track, it has been shown by experiments that at speeds of about 56 miles per hour they become dangerous, and may even cause derailments, in consequence of the hammer-blows and depressing effect of the rolling-stock upon a

track which has been thus deformed; and it is therefore absolutely necessary, in order to bring the track up to good alignment, to demolish the hard cores of ballast which are formed inside the ties. Then the period of expensive maintenance begins. Finally, it does not appear to be desirable that the hollow form of tie should be used. In other words, it is well to avoid the use of such ties as permit of the formation of a compact core beneath them, unless they can be easily removed and given great lateral strength."

The following are the essential conditions to be fulfilled by the metallic tie for lines which are to be traversed by high-speed trains:

(1) They should not require any tamping into the hollow space; that is to say, they ought to be able to be moved longitudinally and transversely, just as a wooden tie can be which has a flat bottom. It is even objectionable to give it any curvature, bendings, or variations of section, or to provide it with bosses or have projecting rivet-heads. *Its shape should be perfectly prismatic.*

(2) It should weigh from 165 to 176 pounds, and be made of steel, in order to have a strength of from 54,000 to 58,000 pounds. Iron is not strong enough. The U-form, right side up or inverted, is not adapted to resist bending stresses, and ought to be excluded. The outline should be symmetrical above and below the neutral axis.

(3) The strength should be obtained with a full outline; that is to say, the shearing strength of the riveted joints should be excluded.

(4) The fastenings should not require rectangular-punched holes, but all holes should be drilled and sound.

Fig. 386 (*a*) shows cross-section of hollow tie and fastenings, (*b*) longitudinal sections of the same. Fig. 386 (*c*) shows cross-section of the tie and fastenings with the filling-pieces, (*d*) longitudinal section of the same. Fig. 386 (*e*) shows a longitudinal section of the rail, cross-section and spacing of the ties, and position of the suspended rail-joint.

TRACK.

1028. We have seen that the ballast is intended (1) to distribute the load over a large bearing-surface below, (2) to maintain the ties in position, (3) to thoroughly drain the road-bed, thereby protecting the formation from becoming soft and muddy in wet weather and from heaving in freezing weather, and (4) to give elasticity to the

road-bed. The cross-ties support the rail directly, maintain the gauge, and distribute the bearing over the ballast.

The rails, which were formerly made of iron, are now made to a great extent, or exclusively, of steel. Until somewhat recently the heaviest rails were only about 60 pounds per linear yard, or $94\frac{3}{10}$ tons of 2240 pounds for two rails per mile. The weight now often used is from 80 to 100 pounds per yard. The rail must have sufficient strength and stiffness to act as a beam between the ties, and as a lateral support or guide to the wheels of the engine and cars. Its top surface must have sufficient size and hardness to resist the action of the heavy rolling weights. The bottom surface must give a good broad bearing on the tie. The old 60-pound iron rail was $4\frac{1}{4}$ inches high, $2\frac{3}{16}$ inches width of head, 4 inches width of base; thickness of stem or web, $\frac{1}{2}$ inch.

According to the recent recommendation of a committee of the American Society of Engineers, the head should contain 42 per cent, the web 21 per cent, and the lower flange 37 per cent of the metal in the section. The important factors are: top radius, 12 inches; top corner radius of head, $\frac{5}{16}$ inch; lower corner radius, $\frac{1}{16}$ inch; corner of flange, $\frac{1}{16}$ inch; side radius of web, 12 inches; top and bottom radius of web, $\frac{1}{4}$ inch; angle between under side of head and top of flange, 13°; width of head of 80-pound rail, $2\frac{1}{2}$ inches. Wearing qualities of rail depend largely upon the closeness of grain of the steel in the heads. In order to have a smooth track it is necessary that the surface and line of rails shall be without lumps and kinks; hence the metal in head and flange of section should nearly balance, thereby permitting the hot metal in the just-rolled rail to cool with the least internal strain, and when cold to be as nearly as practicable straight in all directions; the importance of this increases with the quantity of metal in the section. The larger percentage of straight track governs questions connected with form and dimensions of rail sections. The length of a rail varies between 20 and 30 or more feet. The several lengths of rail are now almost exclusively connected by the angle, fish, or splice plates, and bolts through the plates and the web of the rails. The suspended joint, that is, when the joint between the rails is between two ties, is generally preferred. The joint ties are placed somewhat closer to each other than the intermediate ones.

Sufficient space should be left between the ends of adjacent rails to allow for the expansion due to the difference between the temperature at which the rail is laid and the temperature to which it

may be subjected. The expansion of iron is from 0.0000068 to 0.0000069 of its length for each degree Fahr., which for a 30-foot rail and variation of 100° in temperature is 0.021 foot or 0.25 inch. An extreme high temperature of 130° Fahr. may be taken. A full-spiked rail has two spikes, one on each side of rail, not exactly opposite, in each tie.

It is a bad practice to lay down only a few ties, to which the rail is spiked, and then run heavy loads over the rails; they will spring and bend. The full number of ties required should be in position; it is not absolutely necessary that the rails should be fully spiked.

Rails that are to be laid on curves should be bent to the proper curve before laying. This can be done by lifting and dropping the rail on some form of narrow support, or by supporting the rail near its ends and using blows from sledges; or special machines can be used. The proper curvature is determined from the length of the middle and quarter ordinates of its lengths—found in the usual tables for ordinates. They are often sprung into position by crow-bars after securing one end. This is not good practice on any but very easy curves. On straight portions of the track the top surfaces of rails should be in the same horizontal plane, except that near the ends of the curve the outer rail is gradually raised until at the points of curve it has the full elevation required for the rail on the curve.

1029. *Elevation of Outer Rail on Curves.*—It has always been a disputed point both as to the advantage to be gained by elevating the outer rail, and the amount of elevation that should be used. The avowed purpose is to balance the tendency of the wheels to bear against the outer rail when running around curves at high speeds. Theoretically the force tending to press the wheels against the rail acts horizontally, and varies directly as the square of the velocity and inversely as the radius of the curve, and is expressed by the equation

$$f = \frac{wv^2}{32.16R}. \qquad (634)$$

f = the so-called centrifugal force; w, the weight of the loaded car in pounds; v = velocity in feet per second, and R the radius of the curve in feet. Placing the loaded car on an inclined plane whose base is the gauge g of the track, and whose rise is the differ-

ence in level of the rails $= e$, it is easily proved that $f = \frac{we}{g}$, g being the distance between rail centres, not, as is usually taken, between inside faces of rails. Then

$$\frac{we}{g} = \frac{wv^2}{32.16R}. \quad \text{Hence elevation } e = \frac{gv^2}{32.16R} \quad . \quad (635)$$

on a 2° curve. $R = 2865$ feet; and if $v = 30$ miles per hour $=$ 44 feet per second; $g = 4.7$ feet. Then $e = \frac{44^2 \times 4.7}{32.16 \times 2865} = 0.099$ foot $= 1.19$ inches.

Mr. Rankine gives, on account of the parallelism of the axles of wheels, and the difference in length between outer and inner rails on a curve causing the outer wheel to slip over a distance equal to the difference in length of the rails, which would produce a tendency to press against the outer rail, an additional elevation in inches, $= 7200 \times R$, in feet.

1030. *Coning of Wheels.*—In order to obviate the latter elevation required, the tread of the wheels was made conical instead of cylindrical. By this means a larger diameter was brought over the outer rail, thereby rendering any slipping unnecessary. For very sharp curves the coning would be so decided as would produce unsteady motion on the straight portions of the line, and it is considered better that the flanges should press steadily against the outer rail, reducing this tendency to a certain extent by increased elevation of the outer rail. For these and other reason sthe practice now is to make the tread of the wheels cylindrical, or nearly so.

It was mentioned that the top surfaces of rails should be in the same horizontal plane on tangents. It should be noted that, with a well-defined coning on the wheels, the rails should have a cant or inward inclination towards each other. This has been taken at 1 in 20, as was mentioned in discussing the proper form and manufacture of iron or steel ties; otherwise the bearing instead of being full on the head of the rail will be concentrated on one edge.

1031. There has been much discussion in recent years in regard to the question of open joints between rails as compared with continuity over greater or less lengths of track in connection with expansion and contraction. Some experiments have been made with rigid joints, giving a continuous rail of two, three, or more miles.

It has been claimed that where made the results have been satisfactory. The joints are necessarily the weak points in a line of rails, increase greatly the cost of maintenance and repairs, and often are causes of serious accidents.

On curves having a shorter radius than 1000 feet, it has been customary to widen the gauge by about ½ inch; under 750 and over 500 feet, ¾ inch; under 500 feet, 1 inch.

The elevation of the outer rail above the inner rail is better obtained by depressing one rail one half the amount and elevating the outer rail one half, with respect to the mean or normal surface.

1032. *Transition Curves.*—The obvious objections of passing direct from a straight track to a curved one have suggested many more or less crude methods of making the passage more easy and gradual.

The curve of sines has been suggested as a substitute for circular curves; also, while maintaining the proper circular curve, to introduce near the ends short lengths of curve approximating to the elastic curve, thereby making the change of curvature by degrees.

In a work entitled "The Theory of Deflections," by Isaac W. Smith, a work in the form of an engineer's field-book, a full discussion of this subject, under the head of *Compound Arcs, with regularly Increasing Degrees of Curvature,* can be found, with methods of application and examples. This subject can also be found discussed in one or more works very recently published. These are called also "curves of adjustment."

The following examples for a street-car line will illustrate the principle. Width of street, 45 ft.; outer rail elevated, 1¼ inches.

1st curve:	radius 200	feet,	chord 5	feet,	Total angle, 8° 36′.
2d "	" 100	"	" 5	"	
3d "	" 66⅔	"	" 5	"	

Radius of centre curve on inner rail, 40 feet.
Length on chord, 15 feet.

On most railways, no matter how much care has been bestowed on the careful laying-out of curves, simple or compound, and in the elevation of the outer rail, the maintenance of these conditions is left entirely to the track-foreman, who does all the adjustments of line and levels by the eye alone, and it requires only a short period of use before all the nice adjustments are thrown out.

1033. *Turnouts.*—A turnout is a curved track by which a train

can pass from one track to another. The one is commonly called the main track, the other a side track. A railway yard has many side tracks, either connected directly with the main track or with each other by turnouts.

The point where the outer rail of the turnout crosses the rail of the main or other track is called the frog-point. For unimportant turnouts the rail of the turnout may be sufficiently raised to allow the flange of the wheel to pass over the rail of the main track, the turnout rail being cut at this point, leaving a sufficient gap for the wheels of cars on the main track to pass freely. This is an awkward and unsatisfactory arrangement. Therefore the intersecting rails of the two tracks end at a short distance on either side of the frog-point, and between these is placed a casting or an arrangement made of pieces of rail, by means of which a junction is made with rails of the tracks, and at the same time two intersecting channels or grooves are formed for the passage of the flanges of the wheels when on either track. It is a double wedge-shaped construction, as shown in Fig. 387, the whole being commonly called a frog. There are

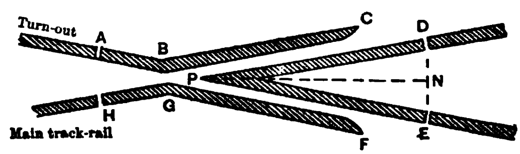


FIG. 387.

several special designs; the more common and simplest form is alone described and illustrated. *ABCDEFGHP* is the entire frog. The portion *DPE*, whether made of pieces of rail or of a solid casting, is called *the tongue* of the frog; the sharp end *P* is called *the point*, and the angle *DPE* is called the *frog angle.*

The sides of this angle, *PD* and *PE*, are in the prolongations of the inner edges of the respective main and turnout rails, or nearly so. The angle of frog may be any number of degrees within certain limits, and the frog may be designated by this angle, but more commonly by a number which is the ratio of the length of the tongue to its base, that is, the number of frog.

$$n = \frac{PN}{DE} = \frac{PN}{2ND} = \tfrac{1}{2}\cot \tfrac{1}{2}DPE. \quad . \quad . \quad . \quad (636)$$

Although under special conditions any number of frog may be used, for single turnouts the numbers used vary from No. 7 to No.

9, inclusive, for single turnouts. For these numbers the angles of frog are 8° 10′ 16″ and 6° 21′ 35″, respectively.

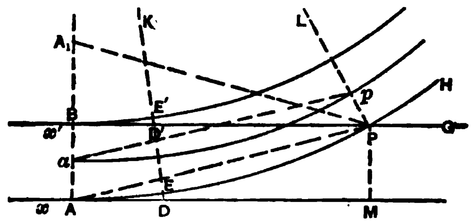


FIG. 888.

1034. In Fig. 388 the rail xD is called the *switch-rail.* It is used for both tracks. The end A is fixed; D is the free or movable end. When it is required to use the turnout the rails AD and BD' are thrown into the positions AE and BE' respectively. DE or $D'E'$ is called the *throw of the switch;* D is the *point of switch;* A, the tangent point of the turnout, is called the *heel of switch;* AD the length of switch. The portion of the switch-rail xA is spiked firmly; the portion AD will then spring into the proper curve if its length has been properly taken. The point of frog is at P. From A to P should be a simple curve. The throw DE varies from 4½ to 6 inches; it is usually made about 5 inches. The general problem is as follows:

1035. Given the frog-angle P, and a straight main track; required the length of switch AD, the radius R of the centre line ap of the turnout curve, the length of the chord ap, and the straight-line distance from the heel of switch A to point of frog $P = BP$. Let the lines AB, EE', and PL intersect at the centre C of the turnout curve, not given in drawing; P, the frog-angle, $= HPG$; g, the gauge of track AB; R, the radius, $= aC = pC$ (C is not shown in Fig. 388); DE, the throw of switch. Then the radius of the outer curve $AEP = R + \frac{1}{2}g$.

$$AB = R \text{ vers in centre angle} = g = (R + \tfrac{1}{2}g) \text{ vers } P. \quad (637)$$

Since $HPG = P =$ central angle C, then

$$\left.\begin{aligned} &R + \tfrac{1}{2}g = \frac{g}{\text{vers } P}; \quad \text{angle } APB = \tfrac{1}{2}P; \\ &BP = AB \cot \tfrac{1}{2}P; \quad BP = (R + \tfrac{1}{2}g) \sin P; \\ &\text{Chord } ap = 2R \sin \tfrac{1}{2}P; \quad \text{versin } ACD = \frac{DE}{R + \tfrac{1}{2}g}. \end{aligned}\right\} \quad (638)$$

Since the inside rail has the same throw, while its radius is $(R - \frac{1}{2}g)$, we may drop the $\frac{1}{2}g$. Then length of switch

$$AD = R \sin ACD. \quad . \quad . \quad . \quad . \quad . \quad . \quad (639)$$

All of these relations will be clear if AB, EE', and PL are prolonged to intersection at C.

By introducing the number of the frog n in the above equations they can be put in a more convenient form. From eq. (636) cot $\frac{1}{2}DPE =$ cot $\frac{1}{2}HPG = 2n =$ cot $\frac{1}{2}P$. This substituted in eq. (638) gives

$$BP = 2n \times AB = 2gn. \quad . \quad . \quad . \quad . \quad (640)$$

From Fig. 388,

$$AP = \sqrt{\overline{AB}^2 + \overline{BP}^2} = g\sqrt{1 + 4n^2}. \quad . \quad . \quad (641)$$

Now make $BA_1 = AB$ and draw A_1P. Then $APA_1 = HPG = C$ central angle; angle A common in the triangles APA_1 and APC. Hence $AA_1 : AP :: AP : PC$, as $PC = AC = R + \frac{1}{2}g$. Then $R + \frac{1}{2}g = \dfrac{AP^2}{AA_1} = \dfrac{g^2(1 + 4n^2)}{2g} = \frac{1}{2}g(1 + 4n^2)$. Hence

$$R = 2gn^2 = BP \times n. \quad . \quad . \quad . \quad . \quad . \quad (642)$$

Again: chord ap : chord $AP :: R : R + \frac{1}{2}g$. Hence

$$\text{chord } ap = \frac{AP \times R}{R + \frac{1}{2}g} = \frac{R \times g\sqrt{1 + 4n^2}}{\frac{1}{2}g(1 + 4n^2)} = \frac{2R}{\sqrt{1 + 4n^2}}. \quad (643)$$

For small angles the tangent offsets vary as the square of their distances from the tangent point, as was shown in paragraph 17, equation (9).

$DE = \dfrac{AD^2}{2R}$ and $PM = \dfrac{\overline{AM}^2}{2R}$; hence $AD^2 : \overline{AM}^2 :: DE : PM$, or

$AB = g; \quad \therefore AD = AM\sqrt{\dfrac{DE}{g}}.$

$$AM = BP = 2gn; \quad \therefore AD = \sqrt{4n^2g \times DE} = \sqrt{2R \times DE}. \quad (644)$$

All of the required quantities have been found. Knowing R, we find the degree of curvature; the angle to be turned is one half the frog-angle; the centre line of the turnout can be located by deflections from a, using short chords of not over 20 feet. But since such curves are usually very sharp, the distance for the given

deflection must be measured on the arc; that is, the actual chord for a given length of arc of 20 feet must be used. Or with a table of chords we can find the proper deflection for an assumed length of chord by finding the arc sin of the length of chord divided by twice the radius.

Length of chord $= 2 \sin \frac{1}{2}\alpha \times R$, $\sin \frac{1}{2}\alpha = \frac{\text{chord}}{2R}$, and $\frac{1}{2}\alpha$, or the deflection

$$= \text{arc} \sin \frac{\text{chord}}{2R} = \frac{20}{2R} = \frac{10}{R}, \quad . \quad . \quad . \quad . \quad (645)$$

for a 20-foot chord, whatever may be the degree of curvature.

Assuming a No. 8 frog, frog-angle will be $7° \ 09' \ 10''$. Substituting in the proper equations given above, we find $BP = 75.328$; chord $ap = 75.181$ feet; switch $AD = 22.418$; $R = 602.624$ feet; degree of curve, $9° \ 31' \ 07''$, when the gauge $= 4' \ 8\frac{1}{2}'' = 4.708$ feet and throw $= 5$ inches $= 0.417$ feet; and similarly for any other frog-angle or number of frog. Thus all of the elements for locating point of frog and turnout curve are known. There is no need of such great nicety of calculation and labor in laying out the turnout with transit deflections. The distance BP being measured on the main-track rail from the point B opposite the heel of switch A the point of frog is located. Then measuring a distance equal one half the gauge $= \frac{1}{2}g$ from the points B and P, we fix the points a and p on the centre line of the turnout. This locates and gives the length of the chord ap, or it can be calculated exactly from eq. (638). Whatever may be its length, the middle ordinate from it to the curve is always $\frac{1}{4}g$, and its quarter-point ordinates $= \frac{3}{4} \times \frac{1}{4}g = \frac{3}{16}g$, which enables us to locate three intermediate points on a curve whose chord will seldom be over 100 feet in length.

For turnouts from curved main tracks the principles involved are the same as for straight lines. Two cases arise: (1) when the turnout is on the inside of the curve, and (2) when it is towards the outside. For ordinary values of the radius of the main track curve, the lengths BP and ap are practically the same as for turnouts from straight lines. The degree of curve for the turnout is in case 1 about equal to the sum of the degree of curve for the main track and that for the turnout from a straight line for the same frog-angle, and in the case 2 the difference between the two degrees of curvature.

For a double turnout three frogs are required. The two where turnout rails cross the rails of the main track are placed opposite to each other, and have the same frog number. The middle one, where the rails of the turnout curve intersect, has a different frog-angle and number. If the two first frogs are n and n', and $n = n'$, then the middle-frog number will be approximately $n'' = 0.7071n$. With, then, two frogs whose equal number is $n = n' = 8.0$, then the centre frog number will be $n'' = 0.7071 \times 8 = 5.6568$, or No. $5\frac{2}{3}$, or the nearest standard number to it.

For a full discussion of the many problems connected with switches, frogs, and turnouts, see field-books for engineers, by the following named authors: Searles, Henck, Smith, and Shunk.

ART. LV.

TORSION OR TWISTING STRAINS.

1036. The bridge engineer has very little to do with torsional strains, as all such structures as a whole as well as in their parts are designed and connected to avoid the occurrence of such strains, and they are not generally considered in such structures; for this reason little discussion or space will be given to this subject. But as, in the experience of many engineers, screw-piles of iron and timber are frequently used, a short notice of this subject may not be inappropriate.

1037. If two equal and opposite couples be applied to a rectangular, circular, or other form of prism at two cross-sections, the planes of the couples being perpendicular to the axis of the prism, the tendency is to cause that portion of the prism between the planes of the couples to rotate or twist in opposite directions about the axis of the shaft or prism. One of these couples is usually that of some externally applied force; the other and opposite couple is that of some resisting force. The moment of these couples is called a twisting moment or moment of torsion. Mr. Burr states that the coefficients of elasticity for shearing and torsion are the same, that those two stresses are identical in character, and that the coefficient of elasticity for shearing lies between one third and one half of the coefficient of elasticity for tension. The value of E, the coefficient of elasticity for cast iron, may be taken as between 6,000,000 and 7,000,000 pounds per square inch; for wrought iron 8,000,000 to 9,000,000 pounds, and steel 8,000,000 to

14,000,000 pounds; for white pine 220,000, yellow pine 500,000 to 600,000, and for oak 570,000; locust, 1,225,000, pounds per square inch. These are for shearing or torsion, and are only average values. Let AB represent a prism with a circular, elliptical, square, or rectangular cross-section, and fixed in position at the bottom, the axis of the cylinder coinciding with the co-ordinate axis x. If now, a couple be applied at any cross-section,—the top section in the figure,—the plane of the couple being perpendicular to, or the axis of the couple coinciding with, the axis of the prism, the moment of this couple $= P \times ab$ is the twisting moment or moment of torsion, in which P is the externally applied force in pounds, and its lever-arm ab is in inches: so the moment of torsion M_t will be in inch-pounds. We then wish to find the condition of strain and stress at any cross-section of the prism, and also the angle of torsion, which is measured by the angle between two diameters, drawn in any two sections of the prism, originally parallel or in the same longitudinal plane. Assuming, first, a cylindrical prism: then, as the prism is of uniform section and the moment of torsion is also uniform, the condition of strain and stress in every cross-section is the same, as will be also that of every indefinitely small portion at the same distance from the axis. If, then, two cross-sections A_2 and A_1, distant from each other by dx, be taken, the entire surfaces or any particles in the same longitudinal plane containing the axis of the prism will move relatively to each other through an indefinitely small angle di, the one point having passed over this angle more than the other, without regard to the actual angular distance passed over by either. The original distance between the two points being dx, the distance apart after twisting will be vdx, in which v is the strain or distortion in a plane perpendicular to the radius r_1 passing through the point, and di being the arc or angle passed over by one point in excess of the other.

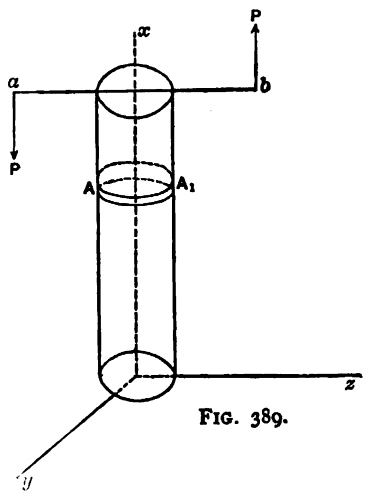


FIG. 389.

The distance passed through at a distance from the axis r will be rdi; hence

$$vdx = rdi; \quad \therefore \text{ the strain or distortion } v = r\frac{di}{dx}, \quad . \quad (646)$$

which varies as the distance of the point from the axis of the prism. This strain is similar to a shearing strain. There must be developed, then, an internal shearing resistance or stress at each point whose intensity is proportional to its distance from the axis. Hence

$$s = Ev = Er\frac{di}{dx}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (647)$$

E being a constant coefficient.

If, then, f represents the ultimate, proof, or working resistance to shearing and r_0 the greatest radius of the prism, then f is the coefficient or modulus of resistance at the outside fibres, and at any point r distance from the radius it will be

$$\frac{fr}{r_0} = s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (648)$$

If, then, any cross-section A be divided into a series of concentric rings dr in width, the area of one of these rings will be $2\pi rdr$; the total stress on this area is $2\pi rdrs$, and its distance from the axis being r, the moment of this pressure is $M_t = P \times ab = 2\pi r^2drs$. Substituting value of s from equation (648),

$$M_t = P \times ab = \frac{2\pi f}{r_0}\int r^3dr, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (649)$$

which, integrated between the limits of $r = 0$ and $r = r_0$, gives

$$M_t = P \times ab = \frac{2\pi f}{r_0}\int_0^{r_0} r^3dr = \frac{\pi f r_0^4}{2r_0} = \frac{\pi f r_0^3}{2}. \quad . \quad (650)$$

And for a hollow cylinder, integrating equation (649) between $r = r_1$ and $r = r_0$, then

$$M_t = P \times ab = \frac{2\pi f}{r_0}\int_{r_1}^{r_0} r^3dr.$$

r_1 = internal radius, $r_o - r_1$ = metal thickness. Then

$$M_t = P \times ab = \frac{\pi f(r_o^4 - r_1^4)}{2r_o}; \quad . \quad . \quad . \quad . \quad (651)$$

or, expressing equations (650) and (651) in terms of the diameter, for a solid axle or shaft,

$$M_t = P \times ab = \frac{\pi f d^3}{16} = \frac{fd^3}{5.1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (652)$$

for a hollow axle or shaft,

$$M_t = P \times ab = \frac{\pi f(d_o^4 - d_1^4)}{16 d_o} = \frac{f(d_o^4 - d_1^4)}{5.1 d_o}. \quad . \quad (653)$$

$r_o = \frac{1}{2}d_o$; $r_1 = \frac{1}{2}d_1$; $\pi = 3.1416$; $\frac{\pi}{16} = \frac{1}{5.109} = \frac{1}{5.1}$, nearly.

In equa. (137) the moment of resistance of solid and hollow cylindrical beams to cross-breaking was found to be, for solid beams,

$$\left.\begin{array}{l} M_o = \dfrac{fI}{r} = \dfrac{\pi f d^3}{32.} = \dfrac{1}{10.2} f d^3; \\ \text{to resist twisting,} \\ M_t = \dfrac{1}{5.1} f d^3; \end{array}\right\} \quad . \quad . \quad . \quad . \quad (654)$$

For hollow beams, resistance to cross-breaking,

$$\left.\begin{array}{l} M_o = \dfrac{1}{10.2} \dfrac{fd_o^4 - d_1^4)}{d_o}; \\ \text{for hollow beams, resistance to twisting,} \\ M_t = \dfrac{1}{5.1} \dfrac{f(d_o^4 - d_1^4)}{d_o}. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (655)$$

From these equations, for the same value of f, we see that the resistance to twisting, whether under working, proof, or ultimate strain, is about double that of the same beam or prism to cross-breaking.

1038. As the value of f is the same for shearing and torsion, we can take, as given in previous articles, for ultimate resistance of wrought iron $f = 0.8$ its tensile strength, or, say, from 40,000 to 48,000 pounds; and for working strength or resistance one fifth of 40,000 or 48,000 = from 8000 to 9600 pounds per square inch. For

cast iron the ultimate may be taken as equal to the tensile strength, from 16,000 to 20,000 pounds, and working resistance to torsion, 3400 to 4000 pounds, per square inch for f in the above equations. For steel the ultimate resistance to shearing is about equal to three quarters of its tensile strength $= \frac{3}{4} \times 72,000 = 54,000$ pounds per square inch, or for working stress $f = \frac{1}{4} \times 54,000 = 13,500$ pounds per square inch. Then, from eqs. (652) and (653), if P, ab, and f are given, we can find the diameter of the prism or shaft. For a solid shaft we have

$$d^3 = \frac{5.1 \times P \times ab}{f}; \quad \therefore d = \sqrt[3]{\frac{5.1M_t}{f}}; \quad . \quad . \quad (656)$$

or, with f, d, and $ab = y$,

$$P = \frac{M_t}{y} = \frac{fd^3}{5.1y}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (657)$$

From eq. (657) we see that the force required to produce rupture by twisting or torsion is independent of the length of the shaft. This is true for pure torsion, but it is almost always the case that there will exist some normal force which will result in causing a bending moment on the prism, which will increase with the length; and hence for long shafts or prisms the value of d in eq. (656) will be too small.

1039. *Examples.*—Required the diameter of a solid shaft to resist with safety a twisting force of 3000 pounds acting with a lever-arm of 18 inches, assuming a safe resistance (for wrought iron) of 9000 pounds $= f$, $M_t = P \times y = 3000 \times 18 = 54,000$ inch-pounds.

$$\therefore d = \sqrt[3]{\frac{5.1 \times 54000}{9000}} = 3.125 \text{ inches, nearly,} = 3\tfrac{1}{8} \text{ inches.}$$

For cast iron and steel substitute for f, 4000 and 13,500 pounds, respectively, instead of 9000 pounds. For a round-timber log or pile a safe value of f will be about 500 pounds. Substituting this in eq. (656),

$$d = \sqrt[3]{\frac{5.1M_t}{500}} = \sqrt{\frac{5.1 \times 192000}{500}} = 12.5 \text{ inches, nearly.}$$

$P = 2000$ pounds; $y = 8' = 96$ inches; $M_t = 96 \times 2000 = 192,000$ inch-pounds.

For hollow shafts or piles the material employed would usually be cast iron. To find the diameter, use eq. (653), $M_t = \dfrac{f(d_0^4 - d_1^4)}{5.1 d_0}$. Assuming the outer diameter $d_0 = 10$ inches, the inner $d_1 = 8$ inches, then $d_0^4 - d_1^4 = 5904$, $f = 4000$ pounds; $M_t = \dfrac{4000 \times 5904}{5.1 \times 10} =$ 463,840 inch-pounds $= P \times y$, if the lever arm y (or its equivalent gearing) is 10 feet long = 120 inches; then $P \times 120 =$ 463,840. $\therefore P = \dfrac{463840}{120} = 3865$ pounds (nearly) as the value of the safe externally applied force.

If the power is applied to a shaft so as to produce any number n of turns or revolutions per minute, and HP is the number of horse-power to be transmitted, r the radius of the shaft, then the work performed in foot-pounds will be $\dfrac{2\pi rn \times P}{12} = \dfrac{2\pi n \times Pr}{12}$; $Pr = M_t$; and $1HP = 33{,}000$ foot-pounds per minute. Hence

$$\frac{2\pi n \times M_t}{12 \times 33000} = HP, \quad . \quad . \quad . \quad . \quad . \quad . \quad (658)$$

the number of horse-powers; hence

$$M_t = \frac{12 \times 33000 \times HP}{2\pi n};$$

and for a solid shaft,

$$d = \sqrt[3]{\frac{5.1 \times 12 \times 33000 \times HP}{2\pi n f}} = 3.43\sqrt{\frac{HP}{n}}, \quad . \quad (659)$$

when f is taken at a safe working load of 8000 pounds per square inch. From this last value of d we see that as HP remains constant and n increases, the smaller will be the value of the diameter required.

For Square or Rectangular Shaft or Pile.—Mr. Trautwine says that the strength of a square shaft to resist torsion is one and one-fifth times that of a round shaft whose diameter is equal that of one of the sides of the square, or about one fifth less than that of a round one having the same area of cross-section. Mr. Rankine

states the value of the moment of torsion for a square shaft is $M_t = 0.281fh^3$, h being the side of the square, or

$$h = \sqrt[3]{\frac{M_t}{0.281f}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (660)$$

Substituting for M_t and f any of the preceding values, we find h, the side of the square.

1040. It is easy from eq. (647) to find the angle of torsion under any given safe moment of torsion, $\frac{di}{dx} = \frac{s}{Er}$; since $\frac{di}{dx}$ is constant, we have $\frac{di}{dx} = \frac{i}{x}$. $\therefore i = \frac{sx}{Er}$; but from eq. (648) $\frac{s}{r} = \frac{f}{r_0}$;

$$\therefore i = \frac{fx}{Er_0} = \frac{2fx}{Ed_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (661)$$

Taking for wrought iron $f = 9000$ and $E = 9{,}000{,}000$, $i = \frac{2x}{1000d_0} = \frac{x}{500d_0}$ for solid shafts, as $\frac{f}{E} = \frac{1}{1000}$.

The moment of resistance to cross-breaking being $M = \frac{fI}{r_0} = \frac{2fI}{d_0}$, and I for a solid cylinder being $\frac{\pi d_0^4}{64}$, $M = \frac{\pi f d_0^3}{32}$; but $M_t = 2M = \frac{\pi f d_0^3}{16}$. Hence

$$f = \frac{16M_t}{\pi d_0^3}, \text{ substituting in eq. (661)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (662)$$

$$i = \frac{32M_t x}{\pi E d_0^4} \text{ in terms of the moment, and coefficient of elasticity, length, and diameter.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (663)$$

Substituting proper values of $I = \frac{\pi(d_0^4 - d_1^4)}{64}$ in $M = \frac{fI}{r_0}$, and reducing,

$$i = \frac{32M_t x}{\pi E(d_0^4 - d_1^4)} \text{ for hollow shafts.} \quad . \quad . \quad . \quad (664)$$

From eqs. (663) and (664) we see that the angle of torsion varies as the moment of torsion and length of prism directly, and inversely as the coefficient of elasticity of torsion and the fourth

power of the diameter. For the same material, same moment of torsion, and same diameter, it varies with the length of the shaft or pile, and although the force required to produce rupture is independent of the length, it (the force) must pass over a greater angular distance in the case of a long prism to twist the shaft off, or strain it beyond a safe limit, than with a short one. Hence the moment of torsion must not be so great as to make the angle of torsion exceed a certain number of degrees of arc. This angle i should not exceed 1°, or at most 2° or 3°. As stated before, the principles established and formulæ deduced seldom come in the practice of bridge-building, except in putting down timber or iron screw-piles, and in these cases the above equations enable us to determine the safe twisting force or moment in case of piles of any given dimensions. The diameter is generally fixed so as to ultimately carry a certain working load.

1041. The resistance to screwing piles in sand or gravel is very great, and it is so easy to multiply the moment of the force applied by suitable gearing that there is always great danger of straining screw-piles beyond a safe limit in endeavoring to force the piles to great depths. In such cases the resistances should be reduced by the water-jet. (See work on Foundations, by the author of this volume.) The writer in putting in iron piers composed of eight screw-piles to each pier, in two rows of four piles each, for a bridge across the Mobile River, where it was intended to screw the piles into the bed of the river to a depth of from 30 to 40 feet, found it impracticable to screw them to a greater depth than 15 to 18 feet in a rather compact and fine sand without perceptibly and injuriously twisting the shafts, which were of wrought iron, solid cylinders of 6 inches diameter.

The power was applied to the crank handles of a worm-screw shaft by men, the number of men varying from eight to sixteen, half of these numbers working at each end of the worm-screw spindle, which geared into a large cog-wheel, in the centre of which the iron pile was so gripped that it turned with the cog-wheel. By these means the force applied by the men was multiplied many times.

In one or two instances, the resistance becoming very great at the above depths below the bed of the river, the length of the shaft from the screw-disk at the bottom being about 30 feet and unsupported length above the bed of the river about 15 feet, the piles were twisted through a considerable angle, and further screwing

was stopped to prevent serious and permanent injury to the pile, and the sinking of other piles regulated accordingly.

That the applied force was very great was evident by the fact that before perceptibly twisting, a number of blocks with steel teeth would shear off iron shavings from the surface of the shaft, and on two or more repetitions of this shearing the sinking was stopped. Referring to eqs. (661) and (663) for solid shafts, we have for a given value of stress f the angle of torsion

$$i = \frac{2fx}{Ed_0}; \quad \therefore f = \frac{iEd_0}{2x};$$

and for a given value of the twisting moment

$$i = \frac{32M_tx}{\pi Ed_0^4}; \quad \therefore M_t = \frac{i\pi Ed_0^4}{32x} = \frac{iEd_0^4}{10.2x};$$

Substituting for wrought iron solid shafts, $E = 9{,}000{,}000$; $d = 6$ inches; $x = 15' = 180$ inches. These formulæ become $f = \dfrac{9000000 \times 6xi}{2 \times 180} = 150{,}000i$. If i is one degree of arc, then arc $i = \dfrac{2\pi r}{360} = 0.0551$; $\therefore f = 150{,}000 \times 0.0551 = 8265$ pounds. With the same limiting value of i, we have from eq. (663)

$$M_t = \frac{0.055 \times 9000000 \times 1296}{10.2x} = 348{,}867 \text{ inch-pounds.}$$

Substituting these values in $d = \sqrt[3]{\dfrac{5.1M_t}{f}}$,

$$d = \sqrt[3]{\frac{5.1 \times 348867}{8265}} = 5.995 \text{ inches or 6 inches.}$$

The above shows the application of the various formulæ which, although far from being absolutely accurate, can be useful guides in giving safe working results, as a factor of safety varying from 4 to 6 is or should always be allowed.

1041*a*. *Resilience* or *Spring* is defined either as the quantity of mechanical work required to produce a given state or condition of strain in a body, or as the quantity required to produce the proof strain. In either case it is the product of the strain or alteration in form by the mean intensity of the stress acting in the same

direction and developed during the production of the strain, this mean stress being nearly or exactly equal to one half of the stress corresponding to the strain.

The work performed by any force against a resistance is the product of the resistance or equal force by the distance through which the resistance is overcome. Hence if P be the force and x the distance through which it acts, the work expended is $= Px$ foot or inch pounds, without reference to the time taken to perform the work. Power, however, is used to denote the rate of doing work, such as so many foot-pounds per minute, the usual term horse-power denoting 550 foot-pounds raised 1 foot per second, 33,000 pounds 1 foot per minute, or 1,980,000 pounds raised 1 foot per hour. The work performed by a force gradually increasing from 0 to P is evidently equal to that of a constant force of $\frac{P}{2}$ acting through the same space. Or if the same loads are first applied suddenly or all at once, and again applied gradually, increasing uniformly from 0 to the same amount, the strains caused and stresses developed will be twice as great in the former as in the latter case; and to carry the same ultimate loads the strength also will have to be doubled, and this is true whether the ultimate proof or working strain is one of compression, tension, torsion, or bending.

If, then, we take a bar under a tensile stress p whose original length is x, its length under a tensile strain will be $x + \alpha x$, α being its elongation or stretch per unit of length, or as commonly called its strain. Then E being the coefficient of elasticity of tension, we have $E = \frac{p}{\alpha}$ or $\alpha = \frac{p}{E}$; $\therefore \alpha x = \frac{px}{E}$ will be the entire elongation or stretch of the bar. Then if f be the proof or working value of p, and A the area of cross-section of the bar, then the force acting through the space or elongation $\frac{fx}{E}$ will increase from 0 to fA with a mean value of $\frac{fA}{2}$. Hence the resilience or work performed in stretching the bar to the strain f will be $R = \frac{fA}{2} \times \frac{fx}{E} = \frac{f^2}{E} \cdot \frac{Ax}{2}$. From this we see that the work expended is composed of two factors, viz., $\frac{Ax}{2}$ = one half the volume of the bar, and the factor $\frac{f^2}{E}$, called the Modulus of Resistance, in which f is the greatest

intensity of the stress allowed. This principle explains the necessity of making the factor of safety much larger for moving loads than for steady or dead loads. The actual conditions caused by a rapidly moving load are not exactly the same as a suddenly applied load, but approximately so.

The above principles are applied to beams under a transverse strain in the following manner: For a single load, applied at the end for a beam fixed at one end and loaded at the other, or at the centre in case of a beam supported at both ends.

As the strain in a beam is the deflection of the beam v_1, the resilience or spring of the beam is, for a load W, $R = \frac{Wv_1}{2}$. Then, for a beam fixed at one end, we have $W = \frac{fI}{rl} = \frac{nfbd^2}{l}$; and for the maximum deflection, $v_1 = \frac{n''fl^2}{Ey_0} = \frac{n''fl^2}{m'Ed}$. Hence the resilience $= \frac{Wv_1}{2} = \frac{nfbd^2}{2l} \times \frac{n''fl^2}{m'Ed} = \frac{nn''}{2m'} \cdot \frac{f^2}{E} \cdot lbd$. This is composed of three factors $\frac{nn''}{2m'}$, which depends on the form of the beam and manner of supporting it. For beams of uniform rectangular cross-section, as seen in Arts. XXXII and XXXV, Deflection of Beams, $n = \frac{1}{6}$, $m' = \frac{1}{2}$; for beams fixed at one end and loaded at the other, $n'' = \frac{1}{3}$ or $\frac{nn''}{2m'} = \frac{1}{18}$; and when uniformly loaded, which could hardly be possible for a suddenly applied load, $n'' = \frac{1}{4}$; hence $\frac{nn''}{2m'} = \frac{1}{24}$. The second factor is $\frac{f^2}{E}$, or modulus of resilience; and the third, lbd, is the volume of the beam.

For beams supported at both ends, and with the same length of span as before, and if strained to the same value of f, the load would have to be $= 2W_1$; hence $W_1v_1 = \frac{nn''}{m'} \cdot \frac{f^2}{E} lbd$.

Substituting for a single weight at the centre, $n = \frac{1}{6}$, $m' = \frac{1}{2}$, $n'' = \frac{1}{3}$; then $\frac{nn''}{m'} = \frac{1}{9}$. And for a uniformly distributed load, $n'' = \frac{5}{12}$, $\frac{nn''}{m'} = \frac{10}{72}$.

The suddenly applied load produces twice the stress and twice the strain as when the same load is applied by gradual and uniform increase from 0 to W.

1041*b*. In addition to the causes already given for decrease of strength in metal structures, or other deteriorations resulting from fatigue of metal, vibrations, shocks, alternation in kind and amount of strain from changes of temperature and the effects of exposure to ordinary atmospheric influences, moisture, and various gases,—metal structures, such as the roofs of stations, notably large union or central stations, tunnel-linings, when of metal, overhead highway or railway structures or bridges under which many trains pass daily, have always been assumed to be more or less injuriously affected by exposure to the gases or solid particles projected with considerable force from the smokestacks of locomotives; but apparently little serious attention has been given to this matter. The structures are painted from time to time with asphaltic, red oxide, or some of the many prepared paints, and no special examinations made to determine the extent of deterioration or the probable cause. That this is a matter of more than passing notice would seem to be indicated by an account found in the *Eng. News*, Jan. 19, 1893, of the actual extent of corrosion in certain members of iron bridges. The bridge was constructed in 1879, examined in 1892,—hence in use thirteen years. The floor of the bridge was close-timbered with oak stringers and plank. The corrosion seemed to be confined entirely to those members or portions of members below the flooring, and greatest at the connections between the truss members and lower chords, thereby indicating the direct cause of corrosion to be due to engines passing under the bridge, and not to ordinary atmospheric influences. The corrosion may be caused partly by chemical and partly by mechanical action, the gases discharged by the engine, heat, moisture, force of the blast, driving solid particles of grit, cinders, etc., against the members. It is well known that lace patterns are cut on glass by the use of the sand-blast without destruction to the lace patterns. It is known that sulphurous-acid vapors with atmospheric moisture, forming sulphuric acid, with its corroding effects, and also volatile compounds of iron and carbon monoxide, are formed at relatively low temperatures. All of these conditions and results may arise from the smokestack-discharges. Which one of these has the greatest effect is not known; probably all combine to cause a serious reduction in metal areas after the lapse of years, as the following examples show: In the exposed parts of the structure above mentioned many angle-irons were reduced in area of cross-section and weight, and consequently in strength, from 25.9

to 61.5 per cent, with an average of over 40 per cent reduction, which is about 3 per cent per annum, showing the danger of working structures on a too small factor of safety under such circumstances. Web-plates were reduced from a thickness of $\frac{3}{8}$ inch to $\frac{1}{4}$ and $\frac{3}{16}$ inch, or nearly or fully one half. The timber stringers, on the other hand, seemed to be preserved, as no material decay or deterioration after thirteen years' use was observed, except around bolts or spikes. If these facts are reliable—as doubtless they are—they show a reduction in the working strength of iron bridges under such conditions of from 40 to 50 per cent in the short period of thirteen years.

1042. The following extracts from a paper read at the New York meeting of the Society of Naval Architects and Marine Engineers by Mr. Russell W. Davenport, Second Vice-President Bethlehem Iron Co., and published in full in the *Engineering News*, November 23, 1893, are particularly interesting as showing the present development of the manufacture of heavy steel forgings in the United States, and as showing the special requirements of steel for the many purposes demanded at the present time, the defects likely to arise in the process of manufacture, the difficulties encountered, and the means of meeting and overcoming them.

It has only been in the last few years that suitable facilities for the production of the heavy steel forgings required either in the construction of modern marine engines and cannon of the larger calibres or for forged armor-plate have been provided in the United States. Heavy shafting, cranks, connecting-rods, etc., had been made of wrought iron. These have a low elastic limit, probably not averaging over 20,000 pounds per square inch, and though having a high degree of toughness and uniformity, the danger of imperfect welds and the occurrence of porous spots, especially in large forgings, inclosing slag and scale, are always a menace to the uniform strength so desirable in such pieces. So long as steel was made by the crucible process, and was melted in crucibles containing from 60 to 90 pounds apiece, the production of large ingots required for heavy forgings presented many and great difficulties, though forgings of moderate size have been made of steel produced by the Bessemer process. Fried. Krupp of Essen exhibited crucible-steel ingots weighing 5000 and 115,000 pounds.

The usual charge of the Bessemer converter does not exceed about 8 tons of steel, and it is difficult to combine several charges in one large mould.

By the open-hearth process a uniform product can be obtained, and the product of several furnaces can be conveniently united in one mould.

The history of the development and the perfecting of suitable and powerful machinery in England, France, and Germany, and the introduction of this machinery into this country by the Bethlehem Iron Co., almost regardless of the cost, is fully described in the paper referred to above—such as forging-hammers having a falling weight of 100 to 125 tons; hydraulic forging-presses, complete, with engine and pumps, having a capacity of from 1500 to 4500 tons; complete fluid-compression plant, including a press of 7000 tons capacity, and a 125-ton hydraulic travelling-crane for serving it (the upper and lower heads of this press weighing, respectively, about 135 and 120 tons); designs of open-hearth furnaces; large machine-tools, such as lathes, boring-mills, and everything essential to the greatest efficiency and range of work. Subsequently a double-cylinder forging-press of 14,000 tons capacity, with pumps, driven by 15,000-horse-power engines, was designed and built by Mr. John Fritz, the general superintendent, and Mr. E. D. Leavitt, consulting engineer for engines and pumps.

A short discussion of the conditions necessary and the most improved methods employed for producing uniformly reliable forgings of such quality as to offer maximum resistance to strains is given under the following heads: (1) The Casting of Ingots; (2) The Conditions of Shaping or Forging; (3) Treatment after Forging; (4) The Introduction of Unusual Ingredients into the Composition of Steel intended to give to it Desirable Qualities.

1043. *Casting of Ingots.*—Certain defects are likely to occur in all steel ingots, especially in large and heavy ones.

They are developed, for the most part, during the solidification and cooling of the fluid steel, and caused by the great change of temperature which then takes place.

They can be classified as follows: (1) Interior shrinkage or piping, caused by the outside of the ingot cooling more rapidly than the inside, and producing cavities and a porous condition of metal along the central axis or line of last cooling. These defects assume serious proportions, and concentrate within conical lines in about the upper third of the ingot. (2) Blow-holes or cavities, due to the evolution of gas during cooling and solidification, which under certain conditions of melting and composition occur

throughout the mass, but especially near the surface and toward the upper part of the ingot. (3) External or surface cracks, caused by rapid shrinkage of outside or skin of ingot, and at times due to hydrostatic pressure of the internal and fluid portion. (4) Internal cracks, due to internal strains set up by too rapid cooling, and occurring most frequently in ingots of hard steel. (5) Segregation, which is the name given to the change which takes place in the chemical composition of a mass of steel in cooling, and is due to the liquation or concentration of certain ingredients—principally carbon, phosphorus, sulphur, and, to a less degree, silicon and manganese—toward the central and upper portion of the ingot, where cooling and solidification of the metal last takes place. A number of expedients are resorted to in order to prevent the occurrence of these defects, with greater or less success. The best is, undoubtedly, subjecting the fluid steel, immediately after casting and during solidification, to a heavy hydraulic pressure, usually known as fluid compression, and the substitution of the hydraulic press for the hammer as a forging-machine, by which the forging hollow on a mandril of long lengths suitable for gun-tubes, jackets, and all descriptions of shafting, is rendered practicable, the forging in this case being practically free from all defects.

1044. *Conditions of Shaping and Forging.*—When a mass of steel is to be shaped or forged, the first requisite is careful and uniform heating. The next requisite, and one of prime importance, is the use of forging machinery of proper design and power. The presence of internal strains and defects can be traced to the shaping being done with hammers of insufficient power, and especially if the power be developed by a high velocity of impact rather than by weight of falling mass. The pressure applied in shaping a body of steel should be sufficient in amount and of such character as to penetrate to the centre and cause flowing throughout the mass. This flowing of the metal requires a certain amount of time, and therefore the requisite pressure should be maintained during a corresponding period. The quick blow of a hammer of insufficient power is absorbed near the surface of the mass struck, and there causes a local movement of extension. Repeated blows of this kind, stretching the exterior more rapidly than the interior, bring a tearing strain on the core or central portion, producing at times actual cavities, which are defects of the most dangerous kind. This action is shown by the ends of the forging showing a concave or cupped form. The conditions of forging attained by the use of

the hydraulic press are the reverse of those above mentioned. The pressure is definite and constant, and acts slowly but uniformly throughout the distance traversed by each stroke; consequently there is ample time for the pressure to distribute itself, and, if sufficient in amount, it causes flowing throughout entire section, the tendency being to squeeze out and extend the central portion even more rapidly than the exterior. A similar effect may be produced by a very heavy hammer falling by the force of gravity only, thereby reducing the velocity of impact to a minimum. This effect is shown by the ends of the forging assuming a convex shape instead of a concave shape.

With the hydraulic press long lengths can be forged hollow over a mandril; and this class of forgings is especially adaptable to marine shafting and the parts of built-up guns. A hole of suitable size is bored throughout the central axis of the unforged ingot, thereby removing portions rendered defective by piping and segregation, and disclosing any defects that may still exist; also, the bored ingot is more easily heated uniformly, and practically removes the danger of internal cracking during the heating process. The forging can be turned out at a low and uniform heat, thus fixing a uniformly fine or amorphous grain. A solid forging, on the other hand, of the same outside diameter would be much hotter toward the central axis than on the outside, and a gradual loss of this high internal heat will tend to coarsen the grain by crystallization and set up internal strains; and, moreover, internal defects are hidden.

1045. *Treatment after Forging.*—There are two processes employed in the treatment of steel forgings after they leave the press or hammer, viz., annealing and tempering or hardening, or a combination of the two.

The primary object of annealing is to relieve internal strains set up by forging, and by rapid and irregular cooling during and after forging. Annealing also alters the molecular conditions of the steel, and when properly applied has a tendency to break up crystallization and fix a finer or more nearly amorphous grain, whereby the toughness of the material is increased. The general effect of annealing is to lower the tensile strength and elastic limit, and increase the elongation and contraction of area.

Hardening or tempering of steel forgings consists in cooling them rapidly, usually by immersion in oil, from a red heat varying in degree according to conditions. The object of this treatment

is, first, to break up the irregular and more or less laminated and coarse crystalline structure produced by forging, and to fix a fine or amorphous condition of grain; and, second, to modify the physical properties of the metal with a view of obtaining the most desirable combination practicable.

The sudden cooling in oil or otherwise naturally produces strains which, unless properly guarded against and relieved, are hurtful, and under certain conditions may be so great as to cause actual rupture. To avoid this latter danger precautions both as to shape and composition are necessary, and when possible the removal of metal along axial lines should be done before tempering, hence forging hollow furnishes a product well adapted to tempering. Annealing is resorted to after tempering; first, to relieve strains, and, secondly, to soften the metal to a degree required to obtain the physical properties desired. The effect of this double treatment, tempering and annealing, is in general to increase the elastic limit relatively to the tensile strength, and when the hardness is "drawn" by annealing to increase materially the elongation and especially the contraction of area.

Treatment by tempering has been applied in the manufacture of steel gun-forgings and armor-plates for a number of years with highly satisfactory results. Comparatively little has been done toward the treatment by tempering of marine shafting and engine-forgings, notwithstanding the growing importance of decreasing and increasing the strength of such forgings.

1046. *Introduction of Unusual Ingredients.*—The introduction into steel of ingredients other than those usually present, for the purpose of imparting to the metal desirable physical qualities, is a subject upon which much study and experiment has been bestowed.

Results of practical value have been obtained with chromium, tungsten, manganese (in more than usual quantities), aluminum, and nickel, and some experiments have been made with copper.

Chromium and tungsten, in moderate amounts, are, for the most part, used in high-carbon crucible-steels, to which they impart special hardening properties found useful for cutting-tools. They are useful also in the manufacture of armor-piercing projectiles. Considerable amounts of chrome steel are now produced in open-hearth furnaces.

Manganese when introduced in considerable quantities, say 3 to 15 per cent, imparts to steel remarkable toughness, together with such great hardness as to render machining impracticable. Its useful application is therefore limited.

Aluminum in small quantities acts in a remarkable manner to increase the fluidity of steel when cast, and prevents the formation of blow-holes during solidification, and has consequently found wide use in the production of solid steel castings.

Nickel has been found to impart to the metal highly desirable qualities, and as this alloy, containing definite proportions of nickel, can be successfully produced and of uniform quality by the open-hearth process, it is specially applicable to the classes of large forgings now under consideration. It has a high degree of suitableness for armor-plates, and has been adopted for the armor of the navy.

Nickel-steel armor-plates have been regularly produced in large quantities. A steel containing about $3\frac{1}{4}$ per cent of nickel has been generally used for armor-plate. It increases the hardness, i.e., the tensile strength and elastic limit, without causing a corresponding reduction in elongation and contraction; and the elastic limit is also increased relatively to the tensile strength. These properties indicate toughness. The presence of this amount of nickel appears also to hinder crystallization after forging; favors a finely granular or amorphous condition; renders the material more susceptible to the effects of tempering, as shown by the ballistic tests of armor-plate; increases resistance to shock; and, in short, improves in all respects the physical qualities of mild steel. The use of nickel-steel should not be confined to armor-plate.

1047. *Gun-forgings.*—The high working strains to which modern heavy ordnance is subjected has called for the highest attainable qualities in the material of which the parts are made. Steel gun-forgings have, up to the present time, been in general made of simple steel. Chrome has been used to a limited extent for parts of small dimensions where great hardness and high elastic limit are desirable. But the use of nickel offers the best promise of improvement in the physical qualities of gun-forgings. From some experiments on nickel-steel forgings in transverse specimens $2\frac{1}{4}$ inches in diameter and 2 inches long, the following results were obtained:

	Tensile Strength, pounds per sq. in.	Elastic Limit, pounds per sq. in.	Elongation, per cent.	Contraction of Area, per cent.
Tube.... ...	93,200	58,300	21.2	42.0
Jacket.......	99,900	60,000	20.4	45.9
Hoops	109,100	68,200	20.5	46.9

These results show an increase of 10 per cent in tensile strength and 22 to 28 per cent in elastic limit over these for simple steel, while the elongation and contraction are but slightly reduced. Nickel-steel has been contracted for by the ordnance department of the army, which is to have a tensile strength of 85,000 pounds per square inch, an elastic limit of 53,000 pounds per square inch, an elongation of 18 per cent, and a contraction of area of 35 per cent.

1048. *Armor-plate.*—Simple steel plate offers a much greater resistance to penetration than wrought iron, or even than compound plates, when struck with steel armor-piercing projectiles; but the great trouble in all-steel plate lies in cracking. This can be corrected by the use of nickel, as described above.

The product obtained by surface or case hardening in the Harvey and other processes has checked experiments with nickel-steel for armor-plates.

The introduction of carbon by cementation into the face of the plate, with subsequent water-hardening, as proposed by Harvey, and known as the "Harvey Process," has up to the present time given the best results in this direction. A plate manufactured by this process resisted and broke up a shot which would have penetrated between 19 and 20 inches of wrought iron.

1049. *Marine Shafting and Engine-forgings.*—The present physical requirements for shafting, including cranks, are: Tensile strength not less than 58,000 pounds per square inch; an average elongation of not less than 28 per cent in longitudinal specimens ½ inch diameter and 2 inches long, cut from full-sized prolongation of forgings.

For connecting and piston rods a somewhat harder steel is required: tensile strength not less than 65,000 pounds, and an elongation of not less than 25 per cent. No treatment other than annealing is required, and no stress is laid on the elastic limit. In other words, a distinctly soft steel is used, in which the elastic limit will not exceed from 27,000 to 30,000 pounds per square inch. Soft steel presents some marked advantages as the standard material for shafts and engine-forgings. In the ingot form and during forging soft steel can bear with safety rougher treatment than harder steels; it is less sensitive to the hurtful effects of irregular and repeated heatings, and dangerous internal strains and defects are less apt to be developed thereby. Further, the cost of machining, and consequent cost of finished forgings, is reduced to a minimum. It would seem

that greater importance should be attached to the question of elastic limit, as experience indicates that many soft-steel forgings of excellent quality in other respects have failed on account of a low elastic limit.

The importance of reducing the weight of shafts is very great. This cannot be done in solid shafting without a diminution in the diameter, with a corresponding decrease in stiffness. It then can only be done by the use of hollow forgings. This practice has been followed in designing the shafting of nearly all of the ships of the new navy. There is, however, the danger of too great a reduction in sectional area, and to avoid this the diameters of the axial holes have not been made sufficiently great to allow of advantageous hollow forging on a mandril. Solid forging, with subsequent boring, has therefore been necessary, whereby a distinct loss in the quality of the metal has resulted.

It is, therefore, evident that to reduce the relative weights, as well as to increase the absolute strength, of the parts, the designer of marine engines needs a stronger material than that now employed; i.e., a material having a greater elastic limit, but at the same time possessing such a degree of toughness as to insure resistance to sudden strain or shock. Simple steel of the proper natural hardness, strengthened and toughened by tempering and annealing, will show, in specimens cut from the centre of sections, say, 3 to 6 inches thick, an elastic limit of about 45,000 pounds per square inch, an elongation of about 23 per cent in 2 inches, and a contraction of area of from 50 to 55 per cent.

For safe and effective tempering forgings must be made hollow wherever possible, and sharp re-entering angles and sudden changes from thin to thick sections avoided; and in order that the metal walls shall not be too thin, while providing bore-holes sufficiently large, both outside and inside diameters should be increased. A further and very pronounced improvement in strength and toughness can be obtained by the use of nickel steel, tempered and annealed. The use of nickel allows a reduction of carbon, makes the steel more sensitive to temper, and facilitates the tempering of irregular shapes.

Specimens from nickel-steel forgings, tempered and annealed, will show uniformly an elastic limit of from 50,000 to 55,000 pounds per square inch, an elongation of 23 per cent and above in specimens 2 inches long and ½ inch diameter, and a contraction of area of from 55 to 60 per cent.

In cases where, owing to thickness of section and irregular shape, tempering is not advisable, nickel-steel will still show a higher combination of elasticity and toughness than any other material known, under the same conditions.

The diameters of the "Brooklyn's" propeller-shafts will be 17 inches outside and 11 inches inside, giving walls 3 inches thick. The line shafts of the "Iowa" will be 15¾ inches outside and 9¾ inches inside diameter, with walls 3 inches thick. These shafts are to be oil-tempered; and to have a tensile strength of 85,000 pounds, an elastic limit not less than 50,000 pounds, an average elongation of 23 per cent in specimens, and no specimen to fall below 20 per cent. From calculations based upon the qualities determined by experiments on specimens, it is found that by the use of the hollow shafts made of the stronger steel there is a gain in strength of 3 to 1, and a reduction of weight of more than one half as compared with solid soft-steel shafts, when the diameters and weights are so proportioned that the strength of the two is the same while their weights per unit are different, or that while the weight per unit of length is the same their coefficient of strength is different. This gain of strength and reduction of weight is not obtained at a dangerous sacrifice of toughness, for the stronger-tempered steel is also extremely tough, as shown by cold bending and by the extension and contraction of area of tensile specimens.

ART. LVI.

RIVERS AND HARBORS.

1050. THE improvement of existing rivers and harbors is, and has been, one of the most important questions in engineering. The ablest engineering talent has been brought to bear upon the solution of the problems presented, and enormous sums of money have already been expended and will continue to be expended in surveys, experiments, and construction. While some good has been accomplished, yet it must be admitted that much of the labor, time, and expenditure of money have been ill applied and misdirected. This has been the result, no doubt, to a great extent, from unavoidable causes inherent in the nature and difficulties of the problem. Even if it could be said that the principles underlying this important branch of engineering were understood and formulated like

the simple principles of hydraulics as applied to the flow of water in pipes or open channels, each and every river presents so many different conditions, and these ever varying in each river, and even in each short section of the same river, that any attempt to apply the same laws and formulæ to any one river will usually result in disastrous failure. Each special case and condition has to be solved separately and independently. Much of the failure and of the actual waste of money are due to a failure to fully and thoroughly study out and understand the requirements and constructions proper to each case.

Every proposed improvement of a river or harbor, no matter how able, experienced, and learned an engineer is, or whatever may be his admitted reputation, who presents a plan for the same, is met with criticism and dire prediction of failure.

Every report on any river or harbor improvement will give ample evidence of the force of the above statements. The following is a fair sample of assertion and contradiction. A competent engineer reported: "All that can be said is that the attempt to maintain six feet is a very doubtful experiment. A study of the subject in the light of additional information shows the degree of improvement needed is impracticable, and that the attempt to make any improvement at all is a costly and doubtful experiment."

The paper states that this report was based upon data "very meagre and misleading." Among other things, that "the river is a non-sediment-bearing stream, because there is no delta at its mouth; and that at low river it is like a series of pools separated by sand-bars, which are often dry. *Nothing could be further than this from the real facts of the case.*"

Again: "The floods and the 'northers' completely sweep away that portion of the bar formed during low water in the river, and drive the material so swept off into the deep water of the Gulf." The reply is: "This is really the reverse of what does occur. The bar in its natural condition is formed by sediment deposited by the river during floods and by sand drift moving westward along the coast by the current and wave-action."

These are but samples of the statements and their contradiction found in almost all such papers.

They are not merely honest and reasonable differences of opinion as to methods of construction, materials used, length and direction of jetties, width between lines of jetties, or whether twin jetties or a single line is best. But they are differences as to the actual con-

ditions and facts existing in regard to the most important questions involved.

They are not introduced for purposes of criticism, but to impress the great importance of surveys, maps, observations, and experiments, in order that there may be no room for differences of opinion so far as the essential characteristics of a stream are concerned before either condemning as impracticable, or recommending as practicable and feasible, at a reasonable expenditure of time and money, any proposed improvement.

RIVERS.

1051. *Natural Features of Rivers.*—Rivers having their sources in the highlands and more or less gradually descending into some larger river, gulf, or sea, have their general direction of flow and beds modified in accordance with the character of the soil and lay of country through which they flow. But in general, near their sources, their beds are narrow, irregular, and rocky; the fall rapid, and current correspondingly swift; changes of direction may not be so frequent, but are often through a large angle, as the cause of such changes arise from permanent obstructions. Approaching their outlets the beds become wider and more regular, the declivity less, and the current more gentle and uniform. Their directions of flow became more tortuous, forming frequent bends called elbows: these may or may not be permanent in form or position; and here also are formed bars, which are composed of the materials arising from the wear of the banks by the current, and deposited in the bed when the current is no longer swift enough to hold them in suspension or roll them along the bottom.

The relations which are found to exist between the cross-section of a river, the volume of water passing, the longitudinal slope, and the nature of its bed constitute what is termed *the regimen of the river.*

1052. *Régime or Stability of a Water Channel.*—A water channel is said to be in a state of stability when the materials of its bed are able to resist the tendency of the current to sweep these forward. The following, according to Du-Buat, are the greatest velocities of the current close to the bed consistent with stability:

Soft clay	0.25	feet per second.
Fine sand	0.50	" " "
Coarse sand, and gravel as large as peas	0.70	" " "

Gravel as large as French beans	1.00	feet	per	second.
" 1 inch in diameter	2.25	"	"	"
Pebbles $1\frac{1}{2}$ inches in diameter	3.33	"	"	"
Heavy shingle	4.00	"	"	"
Soft rock, brick, earthenware	4.50	"	"	"
Rock, various kinds	6.00 and upwards.	"	"	"

The condition of the channels of streams having a rocky bed is one of stability. When the bed is strong or gravelly, the condition is one of stability in ordinary stages of the river, and of instability in floods.

When the bed is of ordinary earths the condition is one of being just stable, or being permanently unstable.

When the bed, that is, banks and bottom, are unstable, the river channel undergoes a continual alteration in form and position. If at first straight, the banks soon become curved; one becomes concave by the removal of the material, and the earthy matter suspended in the water is carried by the water to the opposite side and deposited, making this bank convex. A curved portion of the river tends to be more and more curved, the velocity of current being greater on the concave side than on the convex. This continues until some material is reached on bed and bank at the concave side that the current cannot wear and sweep away, or until the course of the stream is lengthened and the declivity so far reduced that the corresponding velocity can no longer erode the banks, and stability is established.

The formation of elbows occasions also variations in the depth and velocity of the flow. The greatest depth is found at the concave side, and the depth along the straight portion is found to decrease, while the velocity increases. The bottom of the bed is thus found to present a series of undulations, with shallows and deep pools, and with corresponding rapid and gentle currents. The bars are formed at those points and places where the velocity receives a sudden check, and the suspended material is thus deposited. This continues to accumulate until the velocity is so far increased that the suspended matter is carried over and beyond the bar.

The points at which bars are most likely to be formed are: (1) at the straight portions between two elbows; (2) at the junction of tributaries, and (3) at the mouth or outlet of the river to the sea. As a rule, the bars in the higher portions of the streams will be

composed, relatively speaking, of larger and heavier particles than found in the lower stretches and near the outlets. Rivers carrying large quantities of fine material or sediment deposit these at and near their outlets, forming in time extensive shallows which often so far obstruct the flow of the stream during floods that a portion of the water cuts for itself other channels through the more yielding material, forming thereby a series of outlets. Such a condition is commonly termed a Delta.

1053. *River Improvements.*—The two main purposes for which works of river improvement are undertaken are: (1) to make and maintain the river channel so that it may fully subserve the requirements of navigation and the demands of commerce; and (2) to protect the bordering land from gradually wasting away and from the destructive effects of inundations.

These two requirements may coexist over a part or over the whole course of a river, or one may be required at one portion of the river and the other at some distinct and separate portion. Where they coexist, especially, the works of improvement necessary to subserve one purpose may be injurious or disastrous as regards the other.

While many works of improvement may be beneficial in both directions, it will be simpler to treat the two general divisions of the subject separately.

1054. *River Channels.*—The defects in a river channel are, in the main, as follows: (1) The channel may be too shallow either in one or many places; (2) it may be too narrow either generally or in certain places; (3) it may be too wide in some places, the effect of which is to enfeeble the current, resulting in the formation of shoals or bars; (4) its declivity may be too flat, caused by the existence of obstructions, such as shoals, islands, weirs, piers of bridges, etc., which enfeeble the current above, and cause it to be too rapid under, around, and below; (5) it may contain one or many sharp turns or bends, which are injurious to the banks and bottom, and render the navigation both difficult and dangerous.

If these defects existed permanently in character and position; one great difficulty to improvement would be removed; but constant changes are being made. These may be gradual and extending through a period of many years, or they may be sudden and caused by a great flood in the river. It is a recognized fact that the cutting down of the forests has changed the condition of many of our rivers, from that having a good depth, comparatively a uniform

and easy current at all times, to that of rivers having periods of extremely low water, almost entirely preventing navigation, and periods of high and disastrous floods, with relatively only a short period of what is considered a condition favorable to navigation.

Again, the removal of a bar or shoal at one point results in the formation of another lower down; the protection of one bank which has been yielding to the influence of a current only results in its destructive action on the opposite bank.

One or a series of freshets may change entirely the character and dimensions of the bed, and give an entirely new direction to the channel.

1055. *Protection of Banks.*—Whatever may be the nature and extent of other improvements desired, the full protection of the banks from the wearing and erosive action of the current is of the first importance, if not essential, to the success of other improvements.

If such protection were required at only a few points, there would be but little difficulty and cost; but when the great distance along many of our rivers is considered, the work becomes one of gigantic cost and proportions, and of necessity becomes essentially a work only to be undertaken by governments, and even then it is necessarily limited and confined to certain localities.

Many means of protecting river-banks have been suggested and tried with more or less success and over greater or less lengths.

Water-plants.—Probably the most efficient and economical protection is afforded by the growth of certain water-plants on the banks. The common water-willow is excellent for this purpose. All that is necessary to start a willow plantation is to stick a number of short pieces of willow into the bank; these will grow and spread rapidly, especially above average water-level. Such a growth not only protects the bank by enfeebling the current, but results in a rapid deposition of sediment and the making of new ground. They are especially advantageous in preventing the destructive action of waves caused by winds or passing steamboats.

For some reason this simple and cheap protection is used to only a very limited extent, but has been effective where tried.

River-banks do not always yield from the simple wearing action of currents, but often to a great and injurious extent on account of the variation in the nature of the strata forming the banks of streams. It is not infrequent to find a firm and stable material underlaid with strata of gravel or sand. When the water rises

above these strata a greater or less quantity of water is carried into the layers of sand or gravel, which on the fall of the river pours out with considerable velocity, bringing with it considerable of the material near the slope of the bank, and resulting in the caving and sliding of large masses of the overlying material. Such strata, when above the lower line of willow growth, will be protected by it, provided a good thick growth can be obtained before the caving. This may require some temporary protection; but often these strata are found below the willow growth. In this case the willow plantation will not be of much use.

Timber Revetments.—Several methods of protecting banks with timber have been used, and are effective while they last, which will not be more than six to eight years.

Frequently a single or double row of piles, sheeted with plank along the outer row, and filled in with gravel, broken stone, or other material, when placed at the foot or toe of a bank will afford support to the bank above, and prevent sliding and caving, though offering no protection against ordinary wear above their tops. The front row is fully tied back by long timbers inserted well into the bank, or to the inner row of piles; both rows tied to the bank if necessary. Or a timber crib may be built and sunk along the foot of the slope, and filled in with some heavy material, and anchored back, which answers the same purpose, though not presenting the same degree of permanence and stability as the use of piles.

When necessary to prevent erosion above the foot-wall, it becomes necessary to cover the slope of the bank with logs or fascines anchored back into the bank; or, leaving out the foot-wall, logs or fascines can be laid with their lengths up and down the slope, their lower ends reaching well out into the water and weighted down, their upper ends anchored to the bank; above these layers of logs or fascines are placed horizontally and anchored; or, again, the surface of the bank may be covered with a network of interlaced willow branches, called a mattress, and anchored well to the bank. All of these expedients can be regarded only as of a temporary nature, and are used at the present time to but a very limited extent.

Wing-dams, or, as they are sometimes called, *Jetties* or *Groins*.—These consist of timber or stone walls projecting at right angles to the bank to be protected. Each groin protects a portion of the bank equal to five times its own length. While such works have proved effective in many instances, they may often prove injurious

to the bottom and banks of streams, due to scour, and cutting the banks at their ends. Works of this kind have in the main met with little favor, and they are but little used. Sometimes dikes or jetties are constructed nearly parallel to the current, and have produced satisfactory results in improving the channel and protecting the banks.

Riprap or Dry-stone Paving.—The most satisfactory and permanent protection of river-banks is secured by facing the bank with roughly squared or rubble-stone. The bank should be graded to a uniform and rather gentle slope, and at the foot of the slope a foot-wall of some kind should be constructed; or, as a substitute, a trench can be excavated and filled with broken stone, this trench extending below the scour-line. This affords a support to the paving on the slope. The paving-stone should be laid with care, firmly bedding the stone and bringing the surface to a uniform slope. This forms an effective and, if properly constructed, a permanent paving. It may or may not be expensive, according to the abundance and cheapness of stone suitable for the purpose. Its use is, however, in this country limited mainly to paving important landings or the river fronts of towns and cities.

1056. *Removal of Bars.*—Bars and shoals can be removed by excavation or dredging. The common method is by machine-dredging. The dredging-machine consists essentially of a large, strong barge, upon which is mounted the necessary machinery and gearing (which may be worked by steam or horse-power). This is connected with a bucket, attached to a long beam sliding between strong guides, by chains geared in such manner that the bucket can be lowered to the bed of the stream, forced forward into the material, and lifted when full; then the framework is turned around on a pivot, and by pulling a chain the hinged bottom of the bucket is unlatched, and its content emptied into a barge alongside, which when full is towed to some suitable place, commonly deep pools, and discharged.

This is a rapid method of removing material, and when sufficient quantities are to be removed to justify the purchase of the best dredging-machines, the cost per unit is small, being only a few cents per yard, contract prices being often as low as 8 or 10 cents per cubic yard. It may only be necessary to dredge a channel of sufficient width for the steamboats or vessels to pass through.

This process will, when the material of the bed is stable, pro-

duce a permanent channel; but often it will only be temporary, the same causes operating to refill it that formed the original bar or shoal, and periodical dredging will be required to keep the channel open. If the dredged material is not properly disposed of, it will aid in forming another bar at some other place.

When dredging is not deemed wise or economical, owing to the character of the material or other causes, a series of coffer-dams may be constructed, then pumped out, and the material excavated by blasting, if necessary, as in any kind of coffer-dam work. This method is especially applicable to shallows with rock beds, and has been used in some European rivers. In this country such works on a large scale are carried out by submarine drilling and blasting. The improved machine-drills worked from barges are capable of very rapid work. A single drill will average, even in hard rock, as much as 5 feet in an hour. Dynamite cartridges in water-proof cases are readily lowered into the holes and exploded. The material thus loosened will have to be removed, as a rule, by some expensive means. In certain cases the bed-rock to be removed is tunnelled at some depth below the bed, a network of galleries being thus formed; these are charged with dynamite cartridges in the proper number and positions, the explosion of which brings down the roof above.

All of these methods are expensive, but are effective in removing obstructions and securing channels of proper width and depth.

1057. *Regulating Dikes or Jetties.*—Any means of sufficiently increasing the velocity of the current over the bars or shoals will remove silt, sand, gravel, or other similar material, but will have little effect on stiff clay, and practically none on a bed of large bowlders or solid rock. The more common mode of increasing the velocity is by the construction of dikes or jetties. One line may prove efficient in many cases, but commonly two parallel or slightly converging lines are constructed. The direction of these lines should conform as nearly as practicable with that of the existing current.

These dikes may be constructed of broken stone, both of large and small dimensions. The facing stones should be large enough to resist displacement by the force of the strongest current to which they may be exposed; the slope of the face may be about 1 to 1, or the natural slope assumed by a dry-stone wall. The interstices will probably fill up with sediment. Or the dikes may be constructed by driving double rows of piles, the width between the

rows about one and a half times the depth of the water. Wale-pieces may be fastened to the piles in each row, and sheeting-plank driven against them and into the bed of the river; or the piles in each case may be connected by a network or wattling of small willow branches or twigs. The two rows in each dike should be tied together transversely, the space between them filled with broken stone or gravel. The tops of these jetties should not be much above average low-water surface, so that in floods the surplus water may flow freely over them. The purpose in view is to increase the velocity of the stream by forcing the entire ordinary flow of the stream through a contracted channel. Mr. Rankine states that this method of scouring out bars "should be adopted with great caution, and only where the excessive width of channel is an undoubted cause of shallowness." Broken-stone jetties have the advantage of simply sinking if any underscour occurs, and can be raised to their normal height by piling stone on top.

If the river separates into several channels at the bar or shoal, dams should be built across all of them except the main channel, thereby forcing the entire flow through this channel, and accomplishing the same result as the jetties.

In some cases attempts have been made to shorten and straighten the channel of a river by cutting across the convex spurs or tongue of land, thereby turning the water into a new channel. Care should be taken not to make the course too direct, as the current may be made so rapid that the bed will not be stable. The form of cross-section usually adopted is the trapezoidal, with uniform bottom and sloping sides. The probabilities are that such an artificial channel will fail to serve the purpose intended, and sooner or later the stream will form a new bed similar to its original one, unless the bottom and banks are well protected.

The construction of weirs or overflow dams may be resorted to either to furnish storage for water-supply or water-power purposes, and in neither of these cases can this subject be discussed under River Improvements. They may, however, be constructed purely to promote the navigation of streams which do not otherwise admit of it. In such cases locks are usually necessary. This view of the subject will be discussed under the head of Canals.

The foregoing discussion of river improvements has special reference to the upper portions of rivers, where the regular flow of the river and its effects upon its bed are the only questions considered.

The improvement of rivers near their mouths or outlets, especially where the conditions are modified by the existence of tides and storms, will now be briefly considered.

IMPROVEMENTS AT THE OUTLETS OF RIVERS.

1058. Any attempt to apply a hard-and-fast rule to the improvement of the mouths of rivers without a careful and thorough study of all the conditions existing will, unless by a lucky accident, invariably result in failure, and in the waste of immense sums of money, as the statistics of the cost of such works fully show.

The almost universally adopted method for improving the outlets of rivers into bays, gulfs, or seas is the construction of jetties of some kind, unless the obstruction is solid rock, in which case the channel must be blasted out as already indicated; or in the case of stiff compact silt and clay beds that will not scour, a channel must be dredged out and maintained by periodical dredging, or kept clear by the use of jetties. As has been stated, each case must stand by itself. If the water empties into the sea through several channels, all of the openings except the main one may be closed by weirs, which will force all or nearly all of the flow through one channel, and in the case of tidal streams making provision for the flow of the flood-tide through gates and sluices, which can be closed on the ebb-tide, thereby impounding the tidal water and forcing it to flow out of the main channel; or the top of the dam may be at a low enough level to allow the water at high tide to flow over, which impounds a portion of the tidal waters. Where the surrounding ground surface is low and flat, and the material composing it is of a silty and alluvial kind, it will generally be found necessary to combine with these means two lines of jetties, frequently of great length, and extending well into the estuary or gulf, thereby forming a well-defined and somewhat narrow channel for the flow of the water. A full discussion of the combined effect of the natural current of the stream, the currents due to the rise and fall of the tide, the construction of tidal weirs over adjacent channels, the construction of jetties and their effect on the travelling of the materials of the beach along the shore, under the influence of oblique waves and of the flowing tide, etc., would fill a volume of itself; and where conditions vary, requiring a different treatment, a brief general discussion of the subject would be of little value, and afford but little instruction. It therefore seems that a somewhat

extended description of the existing conditions and the construction of the works based upon these conditions, in one or two cases, will afford more valuable information, and at the same time direct the mind more clearly and fully to the kind of information required and the designs best suited to accomplish the purpose in view.

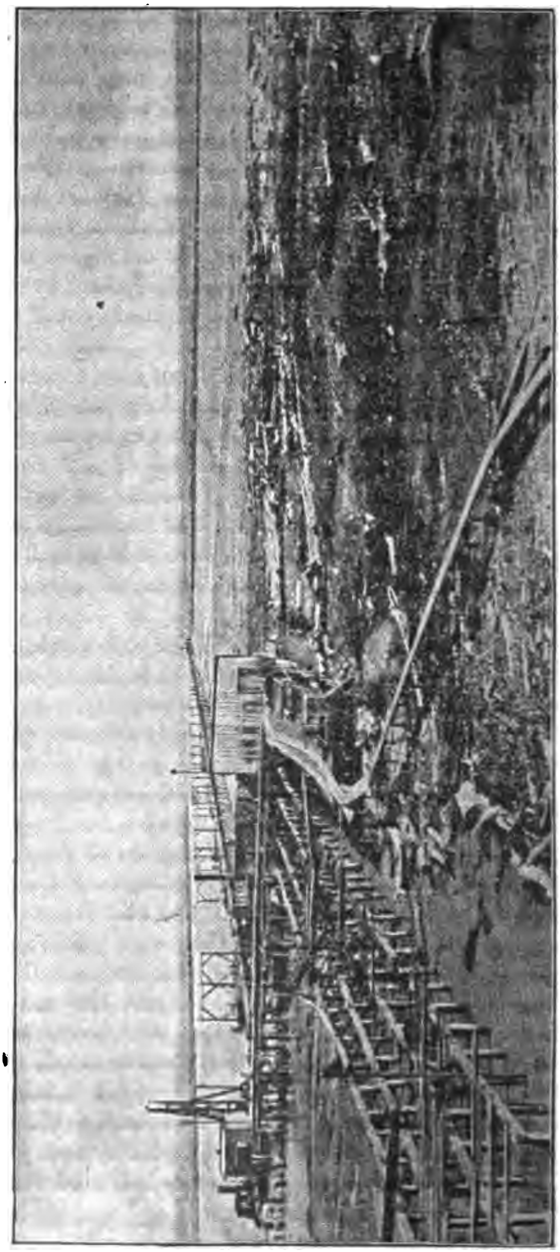
1059. *The Brazos River Harbor Improvement.*—The improvements at the mouth of Brazos River, in the State of Texas, have been selected as an illustration on account of the ultimate success of the undertaking, a full report of which is embodied in a paper read before the American Society of Civil Engineers by Mr. G. Y. Wisner, and especially on account of the full discussion of the paper by a number of eminent engineers.

About the first definite plan proposed was the construction of two jetties of closely driven piles, starting from the headlands at the mouth of the river, and converging so as to give an opening of 400 feet between the outer ends of the jetties on the crest of the bar, at an estimated cost of $286,484. In order to leave the entire river channel open to the free passage of vessels up and down, this plan also contemplated the construction of a system of docks, communicating with the river by suitable locks, in order to provide ample harbor room.

This plan was never carried out. The objections, however, raised to it were: (1) that the teredo would have destroyed the piles in one season; (2) that the volume of discharge is such that the velocity of the current through a channel only 400 feet wide would be over 7 feet per second at times of floods in the river; (3) that the action of the sea and river currents would have swept the structure from its unstable foundation in one night.

The next plan contemplated the construction of parallel jetties of brush and stone, and an increase of width between the jetties to 700 feet, at an estimated cost of $522,890.44. A contract was awarded on this plan, with the following unit prices: Brush in place for the jetties $6.75, stone ballast $8, and small concrete blocks $7, per cubic yard. Under this contract the east bank of the river, inside the shore-line, was revetted with brush mattresses and ballasted with sand, and the east jetty commenced. Under a new contract the east jetty was continued. Cost, brush-mattress work in place, $4.35, and gravel and concrete ballast $7, per cubic yard. Later the east jetty was extended and the west jetty commenced. This consisted of brush mattresses ballasted with plastic concrete of natural cement in bags.

FIG. 390.—BRAZOS RIVER HARBOR IMPROVEMENT. (WISNER.)



View of the Jetty Channel from the head of the West Jetty, showing the New Shore Line to the West of the Works, and in the Left Foreground the Mud Deposit produced by Wing-dams.

These works were destroyed in a short time, before producing any effect upon the bar.

It is stated that these works were constructed under a misapprehension as to the actual conditions existing, especially as to the quantity of sediment carried by the stream, there being no delta at its mouth, and that at low stages the bed of the river consisted of pools and intermediate sandbars.

The following facts are stated, as a result of a careful and thorough survey, as to the general characteristics of the river:

The lower 50 miles of the river is a tidal stream which, at low stages of the water, has a channel from 15 to 20 feet deep for a distance of over 30 miles from its mouth. The total length of the river is about 1000 miles, and with its tributaries the drainage area is 36,000 square miles. The annual rainfall along the coast is from 50 to 60 inches, in the interior from 30 to 40 inches, with a general average over the drainage area of about 42 inches per annum. Assuming that one seventh of the rainfall on the drainage area finds its way into the Gulf through the river, the annual discharge will be about 501,811,200,000 cubic feet, or an average of 15,900 cubic feet per second. If this were a constant flow, the problem of improvement would be a very simple one; but during the low-water flow—a period of from one to three months each year—the discharge is only from 1000 to 3000 cubic feet per second, while at the flood stage the discharge often exceeds 60,000 cubic feet per second. The amount of sediment carried in suspension by the river varies from nothing at low water to four ounces per cubic foot of discharge during freshets. The sediment is mostly alluvium, with a very small amount of sand.

The banks of the river are comparatively stable, except on the lower twenty-six miles of its course, and nearly all the sand carried to the Gulf by the freshets comes from caving of banks on this lower reach. These banks are composed of clay to a depth of 10 to 20 feet, resting on a strata of fine sand,

This sand is washed out when the river falls after a flood, and in the next rise is swept on towards the Gulf. This could be prevented by some of the bank protections already described, in which case the remaining fine sediment would be swept far out to sea. The fineness of the sediment and the strong littoral currents in the Gulf are the real reasons that there has never been a permanent advance of the bar seaward, nor any delta formation at the mouth of the river.

The next question is the cause of the formation of a bar at the mouth of the river, and the changes observed in its height and alignment; that is, whether the deposition of sediment is caused by the want of velocity in low stages, and the changes effected during floods and storms or "northers," or whether the bars are formed by sediment deposited by the river during floods and by sand drift moving westward along the coast by the current and wave action. The latter was determined by the engineer to be the real cause, for the following reasons: (1) The depths on the bar have varied but little, except in cases when violent storms have closed the main channel, leaving only depths of from 3 to 4 feet, and continuing in this condition for several weeks before the river could break through the barrier in some other place. (2) The extension of the bar seaward during freshets was not due to the pushing out of the old material on the bar, but simply to an accumulation of sediment carried by the stream and deposited on account of the slackened current. (3) The low-water season is that likewise of heavy easterly winds, during which the littoral currents have a velocity of from 1 to 3 miles per hour. The effect of these currents was to wear away the outer face of the bar, and deposit the material to the westward of the entrance to the harbor, as shown by the shoaling to the westward, the bending seaward of the 10 and 20 fathom curves at this place proving this. (4) The 18-foot contour in 1858, outside of the bar, was 1200 feet further seaward than in 1881, whereas after a big rise in 1889 the position of the bar was practically the same as in 1858. (5) The borings on the bar showed a heavy clay bed, overlaid in order with the following layers: a layer of sand 8 feet thick, a layer of clay 2 feet thick, a thick top layer of fine sand and shell, while beyond the bar is found a flat slope, 1 in 500, composed mainly of clay and soft mud deposited from the river. Over this area during storms this mud is disturbed, and imparts the same color to the water that the river has in high stages, and causes changes of depth from 1 to 2 feet. (6) The movement of the sand drift does not extend beyond the 18-foot contour. Prior to the construction of the jetties these sand drifts tended to narrow the channel by piling up the sand on the east or windward side of the entrance during low-water stages. This sand point was swept out at each rise of the river, causing the cross-section of the channel to be somewhat variable. (7) From 8,000,000 to 12,000,000 tons of sediment are annually deposited in the Gulf, and no permanent change in the deep-water contours

occurs in front of the harbor,—which could only be the case with strong littoral currents and winds from the eastward side, as winds from the south and west only cause very light surface currents to the eastward.

To determine the cross-section for the maximum discharge it was necessary to compare the sections and slopes at two or more points along the river. The banks of the river for 50 miles above the mouth have approximately the same slope as that of the river at flood stage, The width of the river at 25 miles above the Gulf is only about one half that at the mouth, while the depth is twice as great. The cross-section for the maximum discharge is approximately the same. This fact establishes the section of channel to be made and maintained between the jetties.

The slope of the river surface is not affected by the distance that the jetties are extended seaward. The only effect is to increase the rise in the river for any given stage by an amount equal to that due to the slope in the jetty channel.

At about one-half flood stage, the plane of high-water surface being well defined by marks on the timber along the banks, a careful determination of mean velocity, discharge, and slope was made, and by substitution in Kutter's formula the coefficient of roughness for the section of the river used was computed; from which, with the slope of high water, the maximum discharge was calculated. Subsequent observations have shown this determination to be in error less than one half of one per cent.

1060. *Brazos River Jetties.*—After a careful collection and collation of the above facts and data the following plan was recommended: two parallel lines of jetties were to be constructed, extending into the gulf to a depth of 18 feet outside the bar. The works were to be brush-mattress ballasted with stone. It was at first proposed to construct the jetties from a trestle to be built seaward as the work progressed, the mats to be built on tilting ways placed on a platform-car running on a double-track railway on two parallel pile bridges. It was found to be impracticable to carry on the pile-driving and the jetty construction at the same time without serious interference and difficulty. It was therefore determined to let the pile-driving proceed without interruption, and to suspend the mats during construction directly beneath the caps and stringers of the completed portion of the trestle, and when a mattress was completed it was allowed to slide down the piles to its proper position by loosening the suspending and supporting ropes.

In this mode of construction all materials were delivered direct where needed. The work could be carried on simultaneously over any desired length, and at as many points as desired.

The trestle was built with four rows of piles, in bents 16 feet apart, and spaced laterally to suit the width of the jetty; each bent had 8 × 10 caps driftbolted to the piles and 10 × 12 inch stringers.

The mats were constructed by suspending strips of 2½ × 6 inch timbers, made continuous by splicing, and supported by ropes, above the line of wave-action, fastened to the trestle timbers. Brush was then piled crosswise on these plank or binders to the thickness of one half of that of the completed mat. It was then loaded with sufficient clay or shell to sink the entire mat. The mat was completed by piling on another layer of brush to make the required total thickness. This upper layer was placed at right angles to the former. The mat was then compressed between the mattress strips with binders of iron wire or rods. The latter are preferred. The rods were half-inch iron. These extended upwards through holes in the lower and upper binding strips, their upper ends being caught by a shackle-bar. By lifting the rod a pressure of about one ton was developed, and the mat was held compressed by means of iron wedges driven into a washer resting on top of the mattress strip.

The ballast used on the mats constructed inside the bar, where not subjected to heavy wave-action, was either clay, shell, or gravel, placed directly in the mats before binding and compressing them. This was less expensive than the use of broken stone, and not only effected the same purpose, but made a more compact, impervious jetty, and rendered it safe from the attacks of the teredo. Outside of the bar, where the force of the waves was great enough to wash the clay and shell ballast from the mats, only stone was used for ballast. This was generally placed on top the completed mat, and not between the upper and lower layers of brush

Mattress-work made as above described is flexible, and when not subjected to strain from wave-action can be constructed continuous over any desired length and sunk as completed, thus forming an incline rising from its final position to the unfinished portion suspended from the trestle. Where liable to be disturbed during heavy storms, the mats were made in separate lengths of from 100 to 300 feet, and allowed to sink by loosening all suspending lines simultaneously.

No difficulty was experienced in making and sinking the mats for the one-half mile inside of the bar, but beyond mats ballasted

Fig. 391.—Brazos River Harbor Improvement (Wisner).

View of Pile-driver, Trestle and Suspended Mats near the outer end of the West Jetty.

with 600 pounds of stone to each cord of brush were readily torn loose by the waves and the material carried for miles along the shore. After completing the jetties for a distance of 3000 feet the work was temporarily abandoned. The pile trestle had, however, been completed for a length of from 1000 to 2000 feet beyond the mattress-work; this resulted in a great deal of scouring along the unprotected portion. This shows the importance of carrying on both trestle and mattress work simultaneously.

Even when the jetties were only about 800 feet long, a channel 20 feet deep was scoured between the jetties during a rise in the river; but the ends of the jetties being inside the bar, the only effect was to push the bar seaward several hundred feet.

Before recommencing the work the following year, the trestle beyond the mattress-work had been completely wrecked, and a trench from 16 to 18 feet deep had been scoured out its entire length, the normal depth being only 10 feet. The outer face of the bar had been cut away, bringing it approximately to its original position, with only a channel 8 feet deep over its crest.

On resumption of the work an earnest effort was made to complete the east jetty, in order to stop further scour, concentrate the river current along its line, and to act as a breakwater for the protection of the west jetty from easterly storms. Before the completion of this jetty one of the heaviest floods ever known occurred, the discharge of the river exceeding 60,000 cubic feet per second, and carrying in suspension from 400,000 to 600,000 tons of sediment per day. In addition, over a million cubic yards of clay and sand was scoured from the bed of the channel at the mouth of the river, and through the jetties this heavy material could be carried but a little way beyond the bar. The result of this rise was a channel from 25 to 30 feet deep from the inside of the harbor to the outer end of the mattress-work, 20 feet for 1500 feet beyond, and 13 feet across the bar to the Gulf. The face of the bar was built seaward 500 feet by the deposit of material on the line of the east jetty and 1300 feet on the line of the west jetty, the alignment of the bar making an angle of about 45° with the axis of the jetty channel. Beyond the bar shoaling to the extent of 1 or 2 feet extended seaward for a half-mile.

To the east of the jetties no shoaling whatever occurred, showing conclusively that the Gulf currents, during the entire time of the freshet, were to the westward. This is an important point for consideration everywhere along the Gulf coast.

The east jetty was then extended to a distance of 5000 feet from the shore on the outer edge of the bar in 14 feet of water, and 500 feet inside of the 18-foot contour, the point originally intended to be reached. The east jetty was then ballasted with rock weighing from 1000 to 3000 pounds.

The west jetty was then completed to a distance of 4000 feet from the shore, the bottom course of mats extending to 5400 feet, opposite the outer end of the east jetty. The work was again suspended, only sufficient work and attention being given to maintain the jetties above the plane of average tide.

Some damage was done by washing the stone from the crown of the east jetty.

No rise of any importance occurred during this period, and all changes of the bar were due entirely to low river currents and the littoral currents in the Gulf. Before the extension of the west jetty the action of southerly winds caused shoaling to 11 feet; but when the mattress work was completed the shoaling was stopped, and the channel, under the action of natural forces, was again deepened to 13 feet. With the commencement of easterly winds the erosion of the outer face of the bar commenced, and continued until the 20-foot contour was only 100 feet outside the jetty entrance; and the depths beyond were from 1 to 2 feet greater than at any time since the commencement of the work. (See Fig. 392.)

The original width of the river at its mouth was 700 feet and depth 14 feet, shoaling towards the Gulf to 6 feet of depth on the crest of the bar, while the depth of the river one-half mile above its mouth was about 45 feet.

The river channel above the jetties has been corrected by wing-dams or groins at right angles to the channel, leaving an average width of 475 feet. The new channel forms a tangent to the curve of the river above, thus avoiding any liability of a cross-over bar being formed. This straight reach, including the jetties, is 1½ miles long, and is such that the momentum of flood discharge of the river, after removal of the bar, will *probably* scour for at least 1500 feet beyond the end of the works.

To prevent undermining by the formation of a deep channel along the jetties, spur-dikes 40 feet long were built at intervals of about 400 feet along each side of the jetty channel. The deposits produced by these spurs have built solid walls of mud along each jetty, making them impervious, preventing undermining, and protecting them from destruction by the teredo.

FIG. 392.

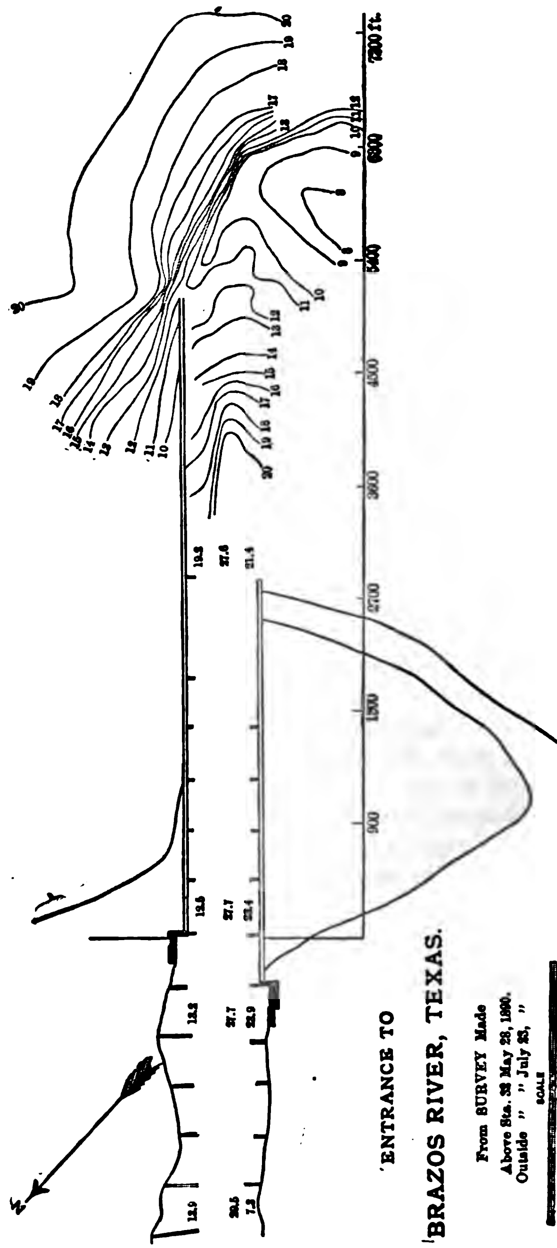


Chart showing the extension of the bar seaward caused by the freshet of May, 1890. Later the 20-foot curve was only 100 feet in front of jetty entrance.

The inner edge of the river-bar was originally about 3000 feet from the shore on the channel line, and uniting with the shore a half mile west of the entrance. All this portion to the westward of the west jetty has been filled up by deposits and silting, so as to be bare at low tide, thus making the inner 3000 feet of the west jetty practically a river-bank. No river deposit has accumulated to the eastward of the east jetty, but the movement of sand-drift along the shore has formed a high concave beach in the angle of the east jetty and extending 1200 feet seaward. Sand is banked against the outside of the jetty for a distance of 3000 feet from the shore, but slopes rapidly down to the original depth from the jetty.

When the next rise occurred in the river about 2000 feet of the east jetty had been denuded of the broken stone, as already stated, the top of the jetty being 2 feet or more below mean Gulf level.

A large amount of river discharge was thus allowed to escape laterally; but, notwithstanding, a channel 15 feet deep was scoured over the bar, and the inner face cut away so that the 18-foot contours on opposite sides were only 400 feet apart. This channel has been subsequently increased to a depth of 16 feet, under the action of currents, at a medium stage of water. The bar in front of the west jetty has been pushed seaward several hundred feet, but no change has occurred east of the axis of the jetty channel. A slight deposit of mud, 1 to 2 feet deep, has been made on the outer slope, which was being rapidly removed by the currents. Had the jetties been maintained above the water surface, confining the discharge, it is probable that a still better result would have been secured.

It was the original intention, so soon as the jetties had sufficiently settled, to build concrete coping-walls on those portions exposed to wave-action. The advantages to be derived from these coping-walls have been fully shown at the mouth of the Mississippi, South Pass Channel, and at the Sulina mouth of the Danube. In the former case a depth of only 24 feet was maintained between the jetties until the concrete coping was placed, when it increased to 30 feet; in the latter the increase was from 15 feet before to 22 feet after the placing of the coping. In addition, without this coping the jetties are in constant danger of being wrecked by storms.

With average weather, three bents of four piles each were driven and trestle completed per day. From an average for the season, it was found that the labor required in the construction of mats was one and one-third hours for each cord of brush used, and one hour for each ton of rock distributed in place in the jetties. The brush

and rock is delivered alongside in barges. Two cubic yards of brush, as measured on the barges, when compressed in the mats and consolidated with heavy stone ballast, made one cubic yard of completed jetty work. The following table gives the cost of one cubic yard of completed jetty on a distance of 2000 feet, exposed to heavy sea action:

Trestle-work	$0 20
Scaffold and mattress frame	0 05
Brush	0 70
Labor on mat	0 10
Rock ballast	1 40
Labor placing ballast	0 08
Engineering and superintendence	0 07
Cost of one cubic yard in place	$2 60

Brush cost $1.50 per cord delivered on board of barges, and the rock ballast $3.50 per ton on barges alongside of the works.

1061. The following principles are stated by Mr. Wisner as governing jetty and harbor construction:

(1) With tidal harbors the slope of the surface in the jetty channel is inversely as the length of the pass; and consequently, on the Gulf, where the tides are very small—about 1 foot—and the distance to deep water very great, the plan of improving harbor entrances by confining tidal currents between jetties is a somewhat doubtful experiment.

This view or principle is controverted by another eminent engineer, Mr. Ripley, who—while admitting that such a plan is an expensive and unscientific method of improving tidal harbors, inasmuch as the improvement which can be effected by such means, where possible, will be only a fraction of that attainable by other means, and where the form of the tidal curve is a sinusoid, like that on the Atlantic coast, the experiment is exceeding doubtful—believes that, where the form of the tidal curve is such as is found on the Texas coast during great declinational tides, it is possible, by confining tidal currents between jetties, to greatly increase the slope, and hence the scouring force due to tidal action, over its normal amount, especially during ebb currents; and in support of these views gives a large number of observations on the tidal phenomena of Galveston harbor, and concludes that the contraction of the entrance will increase the ebb slopes, and, in case of

extreme contraction, to the extent of 108 per cent, and that the duration would also be increased to the extent of 2 per cent. The average ebb slope exceeds the flood slope by 50 per cent, while the average duration of the flood slope exceeds the ebb by 50 per cent. He thinks the effect would be much greater if the contraction was one-half of that proposed for the improvement of Galveston harbor, namely, jetties 7000 feet apart. Mr. Lewis Haupt says that the attempt to obtain deep water by twin jetties 7000 feet apart must result in a "calamitous failure," as such jetties cannot coact to produce scour, and are too remote from each other to protect the channel.

(2) Jetties, to produce a maximum result at a minimum cost, must be completed beyond the bar in a single season. The bar then acts as a submerged weir with the strongest current on the outer crest, thus transporting all eroded material to a safe distance from the entrance.

(3) Delays cause great increase in the cost of construction from damage to the works by storms, and the much greater distance the jetties have to be built seaward.

(4) The success of jetty improvements depends largely on the existence of strong littoral currents in front of the harbor entrance; otherwise the advance of fore shore and bar would soon close any channel formed.

(5) In jetties at the mouth of rivers the strongest currents are at the outer end of the channel, and in no case is it necessary to build the works to a greater depth beyond the bar than that required in the channel.

(6) When scour takes place in the channel-bed its action is first noticeable at the lower end of the section, and gradually works up stream.

(7) In fresh-water streams flowing directly into the Gulf salt-water currents up-stream often exist, where the surface current is flowing seaward; and consequently surface velocities are no sure measure of the discharge or scouring action.

This statement is amply confirmed by direct experiments for determining the direction and velocity of currents at different depths in tidal streams.

(8) When the east jetty was completed 2000 feet in advance of the west jetty, the main current at ebb-tide flowed past the end of the unfinished jetty at nearly right angles to the jetty channel. The shoaling which then took place on the bar, and the subsequent

deepening when the jetty was extended, very plainly indicate that one jetty would not be very effective for channel-making at ports of this class.

As to the statement in the last paragraph, Mr. Haupt states that the Brazos River is the only one between Sabine and Corpus Christi that has no lagoon or estuary, and which debouches directly into tidal water, and therefore offers the most favorable conditions for improvement by parallel jetties, properly spaced and located, while the other rivers between the points mentioned are entirely dependent upon the tidal volume for their action; that the location and width between the jetties were the best for that river, and the direction was selected with reference to the prevailing seas, requirements of navigation, discharge of fresh water, and the littoral currents.

The case is peculiar in its physical conditions and its resemblance to the few successful instances of jetties in pairs at the mouths of rivers with weak tides; but where there is a large tidal movement, even though the range be small, and especially where there is a prevailing direction of littoral drift, he has advocated the construction of a single line of defensive works to protect the channel on the outer bar of inlets from travelling sands, as being a better expedient for tidal entrances than the usual pair of jetties intended to produce concentration of ebb currents.

An example of this kind is to be found at the mouth of the Columbia River, Oregon, where the single jetty which has been built on the south side of the entrance has intercepted the drift of sand and shingle, and thus improved the depth several feet, although the bar has been pushed seaward, and is still in an unstable condition, because of the improper location and form of this work. The construction of another jetty would still further increase the shoaling which has taken place, and reduce the tidal prism entering the estuary.

Upon this same point Mr. J. F. Le Baron said that a different plan might have been adopted, by which at least a portion of the west jetty might have been dispensed with, thereby saving a considerable portion of the cost. The well-known fact that the concave bends of rivers and training-walls always induce a scour, and are deeper than other reaches of the river, has been taken advantage of in planning jetties. One of the mouths of the Oder River, Prussia, on the Baltic Sea, is of about the same size as the Brazos, with a drainage area of 50,000 square miles, and empties into an almost tideless sea. The prevailing storms come from the northwest, and

there is at other times a somewhat strong littoral current to the eastward.

The east jetty projects about one quarter mile beyond the west jetty, the two being nearly parallel, and curved to westward. The littoral current strikes the concave side of the east jetty, which would be considered unfavorable; but the river current is strong enough to sweep away all its sediment.

With a similar construction at the Brazos the littoral current would strike the convex side of the east jetty; that is, when built on a curve concave to the west. The waves rolling in at the mouth will expend themselves on the concave jetty, thus insuring tranquil water in the largest part of the channel, instead of rolling up the whole length of the channel. The construction of the windward jetty on a curve entails no increased expense; if not effective alone, another jetty can be constructed parallel to it. In the case of the Oder, the radius of the curve of the east jetty is 5500 feet; this is too short, the result being that too much scour has been developed alongside this jetty.

1062. The question of the strength and stability of brush-and-stone jetties in very exposed situations is also raised. If there is a sufficient quantity of stone, and heavy enough to remain on the jetties, there would seem to be no reason for doubt on this point, and a preference is shown for log mats rather than brush mats by some engineers.

Log mattresses consist essentially of a series of logs not less than 9 inches diameter at the small end, formed into a raft, which are held together by binders of logs not less than 4 inches at the small end, and bolted to the main logs. Brush or poles and sawmill slabs in layers are placed on the log mats.

The thickness of these mats was from 16 to 22 inches; such mats are easily made. Seven men would make a mat 100 × 100 feet in three days, at a cost of about 45 cents per square yard. Stone cost \$1.50 to \$1.75 per cubic yard; lime rock cost \$2.45 per ton. These mats, when completed, were towed out to position, the stone barges being alongside the mat, and when in position held by heavy anchors. The mat is held up by bridle lines at the corners. It is carefully located by masts built in the mat being brought into line with ranges on shore, and position with respect to other mats already sunk are indicated by their masts. It is best to place adjacent mats overlapping a few feet; otherwise gaps will be left.

The stone is then thrown on the mat from the barges, and the

lines loosened so as to allow a gradual sinking to the bottom. The bottom layer was rapidly covered with sand to a considerable depth, protecting them from the teredo; the teredo ate into the exposed ends of the upper layers, but did not attack those portions covered with rock.

Several jetties have been constructed in this manner on the Atlantic coast, at Cumberland Sound, and the mouth of St. John's River. The rock was in a short time completely covered with oysters, barnacles, etc., which effectually protected the logs from the teredo and induced the deposit of silting sands. The width of such jetties varies from 50 to 120 feet, according to their position in shoal waters or in the breakers. The number of courses of mats varies with the depth of water. Greater solidity, strength, and simplicity, as compared with brush mattresses, is claimed.

1063. As against lines of jetties on curves, it is argued that the increased depth is always attained at the sacrifice of a wide channel, and that a curved narrow channel at the entrance to a harbor is both difficult and dangerous whenever the current in the river is strong. At South Pass the course near the outer end of the jetties has a tendency to make the channel too narrow, and requires constant dredging to remove the deposits from the convex side of the channel which are invariably formed at certain stages of the river.

1064. It is not only required that a certain depth of channel, but also a certain minimum width, shall be maintained—not less than 20 feet deep nor 100 feet in width.

A comparison of the Brazos River and its improvements with those of the Sulina mouth of the Danube, and South Pass, Mississippi River, shows the following points of resemblance: In each case there is a sediment-bearing stream of fresh water debouching into a sea of salt water, a littoral current flowing past the mouth in one direction, and a similar arrangement of jetties extending normally, or nearly so, to the direction of the littoral current. In each case there has resulted deep water close to the end of that jetty on the side from which the littoral current comes, and a new shore-line and an immense shoal beyond the end of that jetty on the opposite of the jetty channel from that which the littoral current flows, the new shore-line extending far out from the old one in the angle between this and the jetty.

1065. The ballast used in jetties is not only required to sink them, but must be of sufficient weight to hold the mattresses in

place; for this latter purpose about $1\frac{1}{4}$ yards of heavy limestone were used for every cord of brush.

1066. *The Mississippi River.*—The importance of the proper improvement of the Mississippi River throughout its great length as well as at its mouth has naturally led to an immense amount of labor, expenditure of money, and thought, not only in regard to improving the channel of the river in the interest of navigation, but for the protection of the adjoining country from inundations, with their consequent devastation and interference with and injury to the efficient cultivation of the lands affected by them.

Entirely opposite conclusions have been reached from the observations made, both in regard to the causes of the obstructions to navigation, the changeableness of the magnitudes and positions of these obstructions, as well as in regard to the proper remedies to be applied.

It is not the purpose in this place to discuss these different observations or the conclusions drawn from them, or to take sides in advocating the theories or views of any particular engineer. Some state that the sediment in suspension is greater during periods of low velocity, and others during periods of high velocity; that the bed is composed of stiff, compact clay, and others that it is composed of light, easily scoured material; some that the bed is for a depth of two or more feet composed of a moving mass of sand. For a full discussion of these and other important questions the reader is referred to the reports of Messrs. Abbott and Humphrey, U. S. Engineers; Capt. James B. Eads, Robert E. McMath, and others.

We will in this place simply describe in brief the construction of the works employed in the improvements of the mouth of the Mississippi River by Capt. Eads.

1067. While admitting the efficacy of jetties in producing a sufficient scouring action to cut a channel through the bar existing at or near the mouth of the river, over the crest of which a navigable channel at low tide of only 7 feet in depth existed, the opponents of Capt. Eads' plan, though not novel as far as simple jetty construction was concerned, doubted the bearing-power of the soft and silty soil upon which the jetties would have to rest, and argued that the only effect of the jetties would be to push the bar farther seaward, and require the continual lengthening of the jetties to secure any permanently beneficial results. In answer to this latter objection Capt. Eads claimed the existence of littoral currents on

the line of the ends of the proposed jetties, which would effectually sweep away all sediment passing out of the sea end of the jetties, and effectually prevent the formation of a new bar; and that properly constructed jetties extending to the proper distance into the Gulf would produce a permanent channel, the sediment being deposited to one side and behind one of the jetties, while the travelling sands of the beach would form to some extent behind the other jetty, thus adding to the stability and permanency of the jetties.

As to the bearing-power of the soft, marshy soil composing the delta, it seems somewhat strange that as late as the year 1875 any question should have been raised on this point, as at that time more than one railroad had been built across this same material and in full operation. A board of engineers, however, reported that, "after four months of actual operations, the work of pile-driving extending from the East land's end to 26 feet depth beyond the crest of the bar, along a line two and a quarter miles in length, covering nearly the whole length of the eastern jetty, and an examination of the texture of the bar and of the shoals on which the works are to rest, furnish the most satisfactory evidence of a bottom material not only adequate to bear all the necessary works, but even to suggest that but for motives of economy the jetties, as at the Sulina mouth of the Danube, might be made wholly of stone." The broad lateral shoals, almost bare at low water, afford an excellent protection for the works, until the outer edge of the bar is reached, leaving only the comparatively short portions of the jetties, outside the bar, exposed to the force of the waves.

1068. The South Pass is the middle one of three passes into which the river divides a few miles before it finally discharges its waters into the Gulf of Mexico. It is about 12 miles long, with no sharp bends, no shoals, forming a straight uniform channel averaging 700 feet in width, having an average depth of 30 feet or more.

At the head of the passes a shoal lies in advance of its entrance: this presented no serious difficulty in cutting a channel through it. About 10 miles below this, where the land margins of the delta end, the current diffuses itself, the depth diminishes over a distance of about 2 miles, and the bar is found, having over it only a channel 7 feet deep. Seaward of this the depth increases rapidly from 6 to 24 fathoms, and more.

The improvements consist in the construction of two parallel jetties over these 2 miles, from land's end to the outer slope of the

bar, thereby prolonging to the sea, beyond the bar, a channel having the same uniformity of section and depth as exists in the first 10 miles of the length of the pass. The success attending the construction of two parallel jetties at the Sulina mouth of the Danube, where a depth of 20½ feet had been secured, gave every assurance of equal, if not much better, results at the mouth of the Mississippi River, which, though similar in many respects to the physical characteristics of the delta and bars at the mouth of the Danube, presented some advantages (1) in the greater depth immediately beyond the crest of the bar; (2) the existence of tide-water; (3) the greater drifting and abrading force of the littoral currents; (4) the fineness of the sand composing the bar; (5) the volume of discharge being three times as great, notwithstanding the greater turbidity of its current.

1069. As late as 1873 a board of engineers reported as follows: "Upon a review of the practical difficulties which the adoption of the jetty system of improvement at the mouth of the Mississippi would entail, and a due consideration of the original cost of construction and of annual extension, entertaining doubts, moreover, of the successful issue of the attempt, the board do not consider it advisable to recommend it;"—recommending instead the construction of a ship-canal, with locks.

1070. The work of construction was commenced about June 14, 1875. In November of the same year it was reported that the lines of the jetties were distinctly marked out by the rows of piles extending seaward on the east side beyond the crest into 26 feet of water, and on the west side to about 20 feet depth. This could produce little scouring action; but with only about 4400 feet in length of the eastern jetty raised to the water surface, considerable scouring effect was noticeable, notwithstanding an opening of 600 feet at the head of the west jetty.

The crests of the parallel jetties are 1000 feet apart. The jetties proper consist of mats or mattresses of brush ballasted with stone, and sunk along the line of piles already mentioned.

In about one year after commencing work, although the work had not been completed on any part of the jetty channel, extending over a length of nearly 2½ miles, the result was very satisfactory. At a point 10,000 feet from the inner or land end of the jetty the depth in the channel had increased from 9.7 feet to 17.1 feet; and with the exception of a length of 75 feet, the depth was 25 feet, where there was less than 9 feet a year before this date. An

immense amount of material had been taken up and carried seaward by the confinement of the flow between the jetties, and there was a decided shoaling on the outer sides of both jetties, protecting them from lateral wave-action, and rendering them more secure. In addition considerable deepening in front of the jetties at distances between 2400 and 4000 feet was observed.

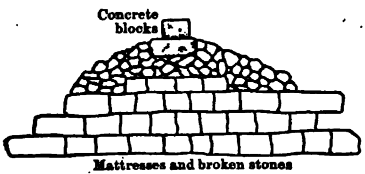
In less than two years' time a channel depth of 30 feet was reported through nearly the whole line of jetties; and just at the sea end of the jetties a channel depth over the bar of 21½ feet. "The jetties are protected from the sea by miles of sand-bars which have formed behind the jetties along the whole line, and the channel is now like a canal 1000 feet wide cut through sand-bars and walled."

Again, in 1879, the following description and report was made by a board of Government engineers: From land's end (East Point) of eastern shore to 35 feet depth in the Gulf, which was 300 feet beyond the crest of the bar, was a distance of 11,941 feet, which figures define the length of the east jetty as originally designed and marked by piles. The natural bank on the west side of the South Pass extended seaward 4000 feet farther than the natural eastern bank: the initial point of the west jetty was therefore taken about that distance below the origin of the eastern one; hence the required length of this jetty was 8000 feet, nearly. The average width between the banks of the pass itself is about 700 feet, and this width was fixed upon as the minimum between the jetties. The origin of the west jetty was established some 600 feet from the west bank, and this end is connected with the west bank by a dam built at right angles to the jetty; this is known as the Kipp dam. Fig. 393 shows general alignment of jetties, dams, wing-dams, and headworks, soundings, etc.

The original design, therefore, consisted of an east jetty 11,941 feet in length, or 2¼ miles; a west jetty, 8050 feet in length; and the Kipp dam. The construction of the jetties consists in a broad foundation layer of willows or other suitable brush, formed into mattresses, on top of which was built a superstructure of tapering section of alternating layers of mattresses and stone or gravel, with the exception of about 300 feet in length at the extreme ends. The jetties had been built up to the level of average high water, or somewhat above that level. Owing to settling and sinking, mattresses and stone were added from time to time, and at the time of the report about 1500 feet of the seaward ends were overflowed at high tide, and a portion of this distance at low tide.

The extreme end of the east jetty was about 11 feet below average flood-tide, this depth decreasing to 1 foot and less higher up-stream. These depths were partly due to settling, and partly to superficial destruction by storms. The actual maximum settlement and compression of willows was estimated at 3½ feet.

Up to the time of this report there had been used on the jetties and the dam 310,830 cubic yards of mattresses and willows and 54,565 cubic yards of stone—mostly small stone.



PART CROSS SECTION OF SEA END OF SOUTH PASS JETTIES
FIG. 393a.

To remove the bar at the head of the pass the channel east of the island was closed, and dikes, called T heads, were run out from the island and from the west bank in order to define the proper channel. These constitute the works at the head of the pass.

The other two passes were partially closed by mattress-sills, which simply consist for each pass of a single mattress 70 feet wide and about 30 inches thick, sunk to the bed and weighted with stone.

In these various head-works there was consumed 141,100 cubic yards of mattresses and willows and 10,755 cubic yards of stone.

A number of wing-dams were constructed, intended to accelerate the deepening of the channel, and regarded only as temporary works.

It may be stated at this point that it was apprehended that the construction of the jetties, dams, and head-works in South Pass, and the so-called mattress-sills placed across Southwest Pass and Pass à l'Outre, would so far obstruct the free discharge of the total volume of water as to materially increase the height of the flood-line in the river at New Orleans, about 100 miles above the head of the passes, and 115 miles above the mouth of the passes. And the dangerous high-water conditions at New Orleans in April, 1892, was attributed to the refluence of waters caused by the above works. Mr. E. L. Corthell, who is thoroughly familiar with the hydraulic problems involved, and who had immediate charge of the construction of the jetties and other improvements, states without hesitation that the condition at New Orleans was in nowise the result of the jetties or their auxiliary works.

The top of the maximum flood at New Orleans is about 17 feet above mean Gulf-level found at the mouth of the passes; this mean Gulf-level is scarcely 2 feet below the level of the narrow ridge of

FIG. 893.

Chart of a part of South Pass, Mississippi River, showing the location of Jetties.

land along the immediate banks of the river in its lower reaches, and is about the level of the mud flats just back of this narrow river ridge of deposits. There were two free outlets within a short distance above the head of the passes: one, "Cubits" crevasse, which at the time of building the jetties was one-half mile wide and 60 feet deep; the other was nearly as large. If the jetties and sill-mattresses at the three main outlets of the Mississippi had raised the water to any extent, it would have at once sought relief through these crevasses and kept them to their full size—which it did not; or it would have made other channels in the very soft and easily scoured materials forming the banks and bed of the river, as these materials are moved by any acceleration of the current.

1071. The almost immediate effect of the works at the head of South Pass was the deepening of the two large passes on each side, which carry to the sea 90 per cent of the volume of the main river.

The mattress-sills did no more than restore the normal depth in those side passes, as they were only about 30 inches thick, and the deepening of these channels was fully equal to this thickness. The only purpose of this sill-dam was to restore the proper and normal flow in the South Pass.

Returning to the report of the board, dated 1879, it proceeds:

(1) The top of the east jetty is to be raised about 1¼ feet above average flood-tide to a point 9200 feet from the land end, and finished by a rounded paving of riprap stone. The next 1000 feet in length to be capped by a low wall of rubble masonry. The remaining portion of this jetty—a distance of about 1550 feet, the extreme sea end of which is to be raised 7¾ feet above average flood-tide—is to be capped with large blocks of concrete built in place, on which ultimately a continuous parapet of concrete is to be built. The river and sea slopes of this jetty and its sea end are to be protected by mattresses strengthened at the sea ends and for some distance back by cribs of palmetto logs filled with stone. Similar additions are to be made along the west jetty at corresponding positions. The protection-works for the sea ends are not so extensive as those for the east jetty, which is more exposed.

(2) The training-walls or T heads are to be raised above the water surface.

(3) The dam or mattress-sill is to be raised by laying other mattresses superimposed, until the cross-section of this pass is made about 12,000 square feet less than it was after the original sill had been laid.

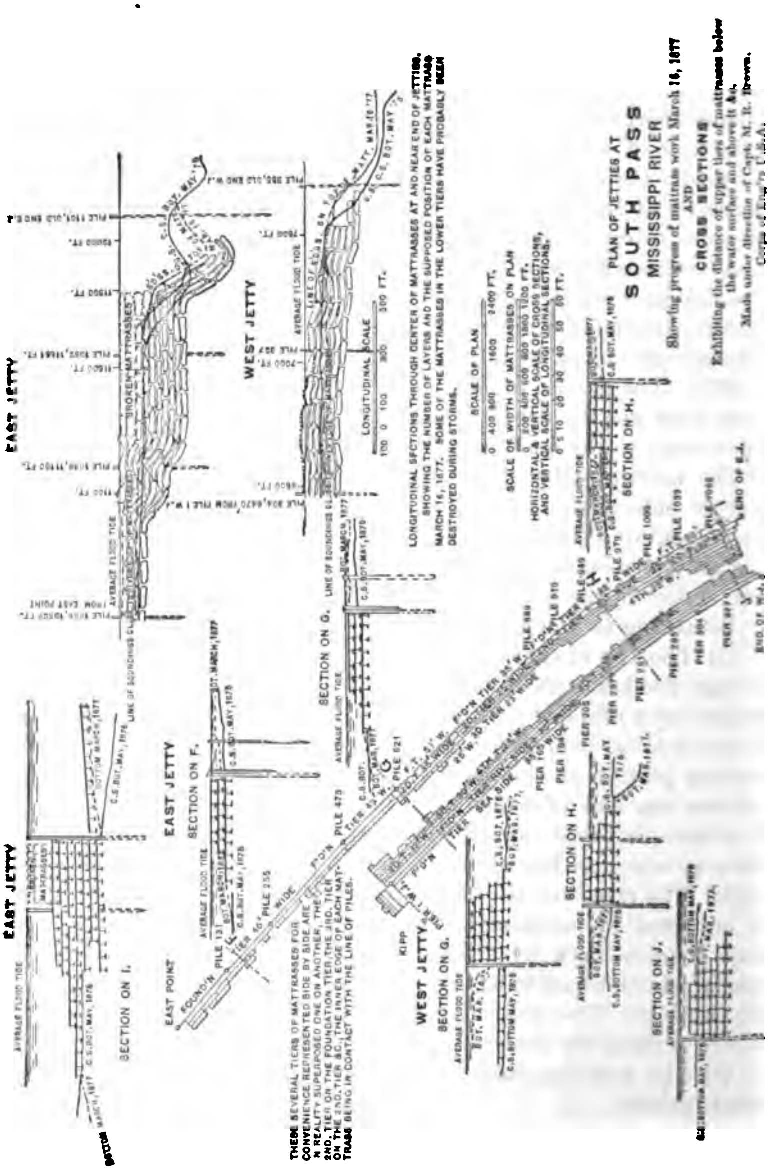

PLAN OF JETTIES AT
SOUTH PASS
MISSISSIPPI RIVER
Showing progress of mattrass work March 16, 1877
AND
CROSS SECTIONS
Exhibiting the distance of upper tiers of mattrasses below the water surface and above it &c.
Made under direction of Capt. M. R. Brown.
Corps of Eng'rs U.S.A.
EAST JETTY
WEST JETTY
SECTION ON I.
SECTION ON F.
SECTION ON G.
SECTION ON H.
SECTION ON J.
LONGITUDINAL SCALE
SCALE OF PLAN

FIG. 393½

The design and construction of these works were left entirely to Capt. Eads, with the sole limitation that the minimum width between the jetties "shall not be less than 700 feet. He was to be paid on the attainment of certain results," the test of permanent completion being the creation and maintenance for 20 years of a channel 30 feet deep and 350 feet wide.

At that date, at East Point, where the width was increased to 850 feet, to the crest of the bar 2¼ miles distant, the depth gradually diminished to about 9 feet (average flood-tide). The results produced by the works are given as follows: A depth of 24 feet, with a channel width of 300 feet, extended down to within 2000 feet of the jetty ends, and the same depth with a channel width of 200 feet almost to the very ends; thence to the same depth outside was a distance of but 60 feet, with a navigable channel of 23 feet intervening. The 25-foot channel is not materially different from the 24-foot one, the intervening navigable channel over the bar being greater—about 160 feet.

The 26-foot channel had a width of from 100 to 150 feet, and in the widest parts from 350 to 700 feet.

The depth of 27 feet is found at various points in the channel down to very near the jetty ends.

Marked and progressive improvement in depth and width was noticeable as compared with the condition found the preceding year, the depths having increased from 2 to 10 feet.

The works at the head of the passes had resulted in an increased depth of from 7 to 8 feet, changing the depths from 14 or 15 to 22 feet over the shoal.

These were accompanied by rectification and widening of areas of lesser navigable depths.

Between the training-walls a more equal diffusion of the current over the intervening space had occurred.

These results were obtained in less than four years from the commencement of the work, with jetties and dams not fully consolidated, permitting considerable lateral leakage, and owing to settling of the jetties their tops were under water over considerable portions of their length, allowing a great amount of overflow. This waste-water was estimated at 25 per cent of the entire volume.

"The actual results, therefore, so far as we know them, do not justify the predictions of accelerated bar advance; on the contrary, they show a disappearance of bar material from the point of the jetties."

1072. Wave or storm action of the sea, and destruction of the willow mattresses by the teredo, are the main considerations affecting the permanency of jetties.

Only about 1500 feet of the sea end of these jetties are liable to injury or destruction from winds and waves. Even on the sea end the effect of these forces had been mainly superficial, the upper courses of mattresses and stone, mostly small, having been repeatedly washed off and scattered, and requiring the placing of additional mattresses ballasted with stone. This will stop when the jetties are finally capped with concrete and the slopes protected by the cribs of palmetto logs, referred to already.

The teredo only eats the exposed ends of timber and brush, as they do not attack timber where the free access of sea-water is impeded. Those portions imbedded in sand or mud, or which are packed around with these materials, are free from their attack; and there is no evidence of the teredo penetrating into the interior of the mattresses.

1073. In 1889 Mr. Wisner published an article in the *Railroad Gazette,* from which the following facts and illustrations are taken:

As previously stated, the sediment discharged by the river was deposited behind and outside of the jetties; this effect has continued so far as the west jetty is concerned, and the shore-line has advanced to within less than 600 feet from the sea end of the jetty, this being on the opposite side of the jetty channel from the direction of approach of the littoral currents.

On the east of the east jetty the original shore-line was about 5400 feet from the end of the jetty as it now stands. As the work advanced this shore also advanced seaward over 2000 feet; but after the completion of the jetties beyond the bar the volume of the sediment was discharged into deep water of the Gulf, and was swept by the littoral currents to the westward, and deposition and shoaling east of the east jetty ceased. Erosion then commenced and is still going on, until the shore-line has receded to a line 7400 feet from the end of the east jetty, or 2000 feet back of the original shore-line. The depth of the water varies from 8 to 12 feet, and as nearly all storms come from the east, this portion of the structure is subjected to very heavy shock from wave-action. On the outer 5400 feet of this jetty a concrete sea-wall was built in 1879–80. The inner 4000 feet of this wall had a base 4 feet $4\frac{1}{2}$ inches thick and 3 feet $3\frac{1}{2}$ inches high, and carried a rubble parapet-wall 3 feet thick and $2\frac{1}{2}$ feet high. (See Fig. 394 (a).) The outer 1400 feet of

wall was from 6 to 12 feet wide or thick, and was surmounted by a parapet-wall 4 feet thick and 3¾ to 4 feet high. By a succession of storms between 1880 and 1889, the inner 4000 feet of this wall had been totally wrecked, and about 300 feet of the jetty swept away to the depth of 10 feet below water-level. The wider blocks of the outer portion of the wall had remained in place, but the larger portion of the parapet-wall had been broken up.

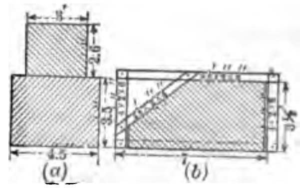


FIG. 394.

It was evident that a wall with a vertical face built on a pliable foundation was inadequate to withstand the shock of the waves. It was then determined to use concrete blocks presenting a sloping surface towards the exposed side, thereby presenting a minimum surface to the wave-action. (See Fig. 394 (*b*).)

The foundation for this wall was then built upon the old jetty, either by laying loose rock where the depth of water was not great, and where it was, layers of mattresses and stone. (See Fig. 395 (*b*).)

The lengths of the concrete blocks are from 50 to 100 feet, the breaks being made where foundations changed from stone to mattress-work. The only bottom used for the moulds was burlap cloth fastened loosely across the bottom; this, while preventing washing out the cement, allowed the concrete to conform to the inequalities of the base upon which it rests. Divisions between the blocks were made by timber partitions of inch plank in order to take up the expansion of the blocks from changes of temperature. The concrete is carried direct from the mixer alongside and deposited in the moulds. The old concrete blocks having slid downward, form a toe for the riprap of the new wall, and prevent undermining. (Figs. 395 and 395 (*b*).)

The original jetties were 1000 feet apart, but interior jetties were subsequently built 150 feet from the old ones, thus narrowing the channel to the minimum limit of 700 feet allowed by act of Congress.

These interior jetties were constructed by piling brush between rows of piles and paving with riprap.

It was these interior jetties that saved the channel when the outer jetty gave way. (See Fig. 395 (*a*).)

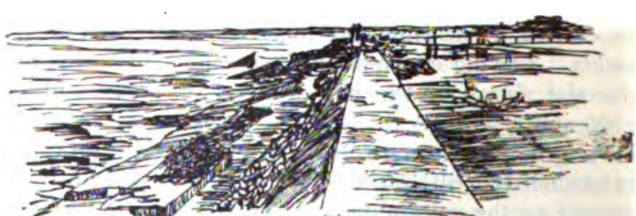
In Fig. 395 is shown a view of a portion of the wrecked jetty.

In 395 (*a*) is shown a portion of the original jetty, with the inner line of jetty under construction just emerging above water. This inner line is 150 feet inside from the original line.

Fig 395 (*b*) shows a portion of the completed jetty, with concrete coping, which was built on the wrecked portion of the old jetty.

Fig. 395.

(*a*)



(*b*)

From the report of the Chief of Engineers, U. S. Army, for the year 1892 the following notes are taken:

1074. *East Jetty.*—The concrete wall on this jetty remained in fair condition during the year, the average subsidence being $\frac{8}{100}$ of a foot; total since 1889, $\frac{28}{100}$ foot.

West Jetty.—This jetty is completely buried; the concrete wall is entirely out of sight, covered by sand and mud in which the marsh-grass is growing. This jetty is secure for all time, and will consequently require no more work upon it.

A few new wing-dams were constructed during the year in the usual manner. Two or three rows of piles are driven—generally two; the piles in each row are connected by 12 × 12 inch timber bolted to them about 7 feet above the water, and to these are bolted 12 × 12 inch transverse timbers to prevent the rows spreading when willows are forced between them. Between the rows of piles willows loaded with stone or earth are placed.

During this period the depth from the main river into South Pass has not been less than 29 feet, and at all times there has been a very wide 26-foot channel. The depth at the time of the report was 34.4 feet, the 30-foot channel 620 feet wide and the 26-foot channel 880 feet wide. This is at the head of the passes.

The required depth from the head to South Pass Lighthouse of 26 feet over a width of not less than 200 feet, was maintained during the year.

Channel through the Jetties.—The required depth of 26 feet and width not less than 200 feet, and having throughout its length a central depth of 30 feet without regard to width, was maintained by the aid of dredging, except for about thirty days.

The Channel beyond the Ends of the Jetties.—The 26-foot channel for a width varying from 110 to 630 feet has been maintained, with a central channel of 30 feet deep and from 40 to 100 feet wide.

The channel turning to the eastward beyond the end of the east jetty maintained the required width and depth during the year.

The shoal area east of the east jetty is gradually but slowly deepening, and the land formation east of this jetty is rapidly receding, averaging 250 feet during the preceding year.

This shows what has been accomplished in a comparatively short period by a system of judiciously located and constructed jetties.

In Fig. 393½ is shown several sections, longitudinal and transverse; also alignment and plan, with details.

1075. The following interesting information concerning the Mississippi River is given by Captain Eads:

(1) Quantity of water discharged by the river annually, 14,883,-360,636,880 cubic feet.

(2) Quantity of sediment discharged annually 28,188,083,892 cubic feet.

(3) Area of the delta of the river, 13,000 square miles.

(4) Depth of the delta of the river, 1056 feet.

(5) The delta therefore contains 400,378,429,440,000 cubic feet, or 2720 cubic miles.

(6) It would require for the formation of one cubic mile of delta five years and eighty-one days;

(7) For the formation of one square mile of the depth of 1056 feet, one year and sixteen and a half days;

(8) For the formation of the delta, 14,268¼ years.

(9) The valley of the Mississippi from Cape Girardeau to the delta is estimated to contain 16,000 square miles of 150 feet depth. It therefore contains 66,980,160,600,000 cubic feet, or 454½ cubic miles.

IMPROVEMENTS OF CHARLESTON HARBOR.

1076. In the improvements at the mouth of the Mississippi and Brazos rivers the solution of the problem consisted in the construction of high jetties and forcing the flood discharge of the river through a narrow channel confined by parallel jetties, the large quantities of sediment held in suspension by the water and removed from the channel between jetties by the scouring action of the current being carried by the current well beyond the ends of the jetties, and there swept from the immediate front of the jetty channel and deposited westward, resulting in a shoaling outside of the west jetty, strengthening this jetty, and ultimately forming of itself a natural and permanent jetty over and outside of the artificial one. In both of these cases the rivers debouch directly into the Gulf, having only a small rise of tide—about 14 inches, and no tidal basin which could be arranged to store and let loose a large tidal discharge at the proper time, so as to materially aid in, if not of itself, scouring a channel through the bar, and in addition strong littoral currents capable of preventing the formation of a new bar beyond the ends of the jetties.

The problems presented at the sea entrance of Charleston harbor were therefore different in many respects. Here we have a large tidal basin containing about fifteen square miles of surface, having a few short tributary streams. Hence the production and maintenance of a deep-water channel through the bar at the mouth of the harbor must be caused mainly by the scouring action of the ebb-

tide flow from the basin, aided by the water derived from land drainage during the ebb flow of the tide.

1077. The following description of the conditions existing and the methods and constructions adopted to make a channel to the sea is taken from the report of General Q. A. Gillmore, U. S. Army, and published in September number of *Van Nostrand's Magazine* in the year 1878. A general view of the harbor and alignment of the proposed jetties is seen in Fig. 396, to which reference is made in the following description and discussion:

The bar, which stretches bow-shaped across the entrance into Charleston harbor from Sullivan's Island on the north to Folly Island on the south side, has existed practically as found at the time of this report from the earliest records on the subject. Measured along its crest or line of least depths, the bar is 10 miles long. At its north end it is close up to the entrance or throat of the harbor, while its south end is 6 miles distant therefrom. Its average width between the 18-foot curves is about $1\frac{3}{4}$ miles. In some portions of its length the average depth of water along the crest is only 3 or 4 feet, but greater at other portions.

South of the main entrance its direction is parallel to the shore, and at a mean distance of two miles from it, and had never been traversed by practicable ship-channels. There never has been, however, less than four nor more than six channels across the bar at any one time. The greatest depth of water, which was rarely less than $11\frac{1}{2}$ or more than $13\frac{1}{2}$ feet at mean low tide, was found sometimes in one channel and sometimes in another.

The northern and southern extremities of the bar are formed by rather sharp curves, which connect the straight portion with the shore above and below the harbor.

These channels were always grouped two and two or three and three near the extremities of the bar in the curved portions, leaving a straight, deep, and broad anchorage abreast of Morris Island. This anchorage, called the main channel or outer harbor, had an average width from one third to two thirds of a mile between the 18-foot curves, and in maximum low-water depths from 20 to 45 feet. The direction of its central line is about north and south, and its length from the throat of the harbor between Morris and Sullivan's Islands to its southern terminus, where it spreads out in various channels and shoals in crossing the bar, is fully five miles. At the two extremities of this anchorage are found the two groups of

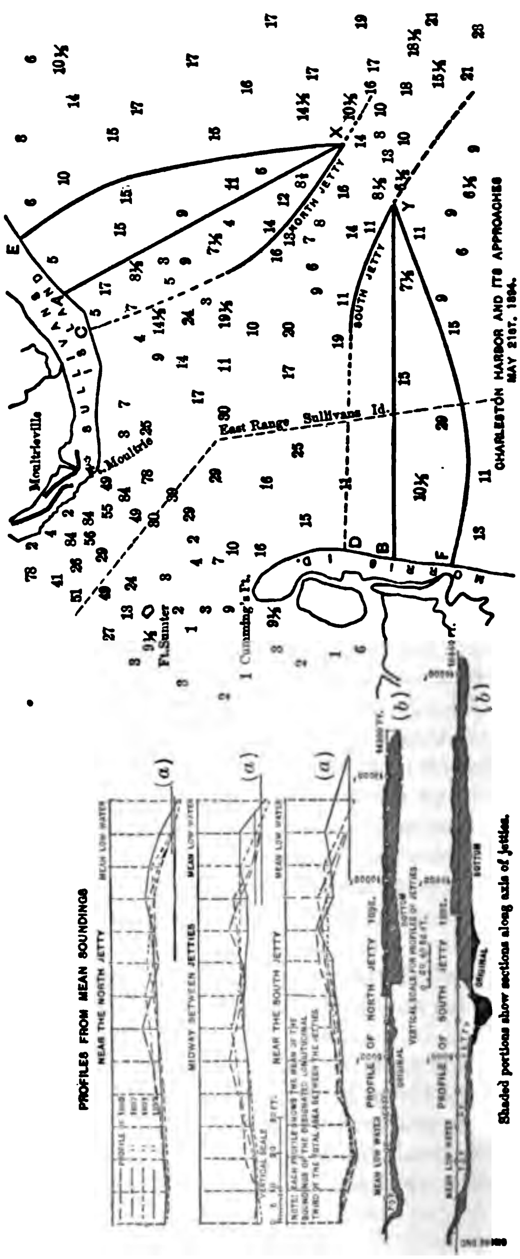

CHARLESTON HARBOR AND ITS APPROACHES
MAY 21ST. 1894.
Moultrieville
SULLIVANS ISLAND
Ft. Moultrie
Ft. Sumter
Cumming's Pt.
East Range Sullivans Id.
NORTH JETTY
SOUTH JETTY
PROFILES FROM MEAN SOUNDINGS
NEAR THE NORTH JETTY
MIDWAY BETWEEN JETTIES
NEAR THE SOUTH JETTY
PROFILE OF NORTH JETTY
PROFILE OF SOUTH JETTY
MEAN LOW WATER
BOTTOM
ORIGINAL
Shaded portions show sections along axis of jetties.

FIG. 396.

channels, already mentioned, across the bar, the northern group being directly in front of the gorge of the harbor.

The bar is essentially a drift-and-wave bar, produced in part by the upheaving action of the waves when they approach the shore, which are converted by breaking into waves of translation, and in part by drift material carried along the coast by surf currents, especially those produced by northeast storms. The materials composing the surface of the bar are shells or silicious sand, or a mixture of the two; it is easily thrown into suspension by waves, and is moved by a moderate current. Borings indicated some layers or lumps of mud, as well as mud mixed with sand or shells. All save one of the channels are ebb-tide channels, produced and maintained mainly by the scour of the ebb current. One of them, the most northerly of the channels, which lies close to Sullivan's Island, is evidently a flood-tide channel, the characteristics of which are: (1) that the least depths are always found near their inner ends, and therefore in comparatively quiet water; (2) from the cross-section of shoalest soundings inward toward the harbor the descent into deep water is sharp and sudden, while outward, toward the ocean, it is gradual and gentle; (3) the ebb flow through the same channel is but little over one half of the flood flow, the volume of flow being diverted in the tidal basin, and a large portion of it finding an outlet at some other point.

1078. *Capacity of the Tidal Basin.*—The area, 15 square miles, of this tidal basin is assumed to be filled, during each mean flood-tide, by a volume or layer of water 5.1 feet in height above the mean low-water level. In addition, the adjacent tributary reaches will be filled by a wedge-shaped mass resting on the sloping low-water line of the tributary, and extending up to a point where the influence of the tidal wave ceases to produce a rise and fall of the surface of the water.

The total volume of outflow during each ebb-tide will be measured by the volume contained in these tidal prisms, augmented by the volume derived from land drainage during the period of ebb flow. Upon data of this kind the computed discharge through the throat of the harbor on each ebb during the period of mean rise and fall of tides was 3,655,443,686 cubic feet; of this volume only about 76,571,000 cubic feet is supplied by the land drainage, on the assumption that one half the rainfall reaches the sea. For two or three days during the period of spring-tides the average ebb discharge will be about 4,228,846,000 cubic feet. The neap discharges

being in smaller volumes than those pertaining to mean tides, even if slight shoaling ensued during this period, the mean and spring tidal flow would restore the original depths of channel.

The mean duration of the ebb flow is taken at 6 hours, the longest flow being 6 hours 20 minutes, and the shortest 5 hours 25 minutes.

The average ebb discharge per second through the gorge of the harbor, during the period of mean rise and fall of the tides, is therefore $\frac{3,655,443,686}{21,600} = 169,233$ cubic feet, and during the spring-tides 195,780 cubic feet, the average rise and fall at ordinary spring-tides being 5.9 feet. During very high spring-tides the discharge will be much greater. With a rise of 10.3 feet the prism amounted to 341,780 cubic feet per second, or 7,382,562,000 cubic feet per tide; and in addition, owing to the flooding of the marshes over an area of 8 square miles, the actual capacity of the tidal basin is increased. Every 3 inches of rise over them will add 2590 cubic feet per second to the average discharge. The above amounts of discharge have been determined from the cubical capacity of the tidal basin and the rainfall upon the drainage area, and not by gauging the flow through the gorge of the harbor by means of current velocities. It is well known that surface velocities are no indication of those at depths below, and the difficulty in determining these, especially close to the bed, is very great. (In fresh-water rivers flowing directly into the Gulf, salt-water currents up-stream often exist where the surface current is flowing seaward.) In tidal harbors it has been observed that, while during flood-tides the surface velocity is 1.8 miles per hour, undercurrents at considerable depths were not less than 4 miles per hour, and similarly during ebb-tides.

1079. General Gillmore lays down three important conditions that must be fulfilled in order that the success of a work, where the facts are as above stated, may be assured:

(1) They (the works) should not impede the inflow to such degree as to prevent the tidal basin being filled, as now, at every influx of the tidal wave.

(2) They should control the outflow to such degree and in such manner that a channel of the required depth will be maintained through the bar.

(3) They should not to any considerable extent cause a movement seaward of the main body of the bar; that is, the general position

of the bar should be independent of the effects produced between and beyond the heads of the jetties.

The general plan of improvement recommended consists in constructing two lines of low jetties (see Fig. 396), the one springing from Morris Island and the other from Sullivan's Island, and converging towards each other in such manner that their outer ends on the crest of the bar shall be one half to five eighths of a mile apart, the opening between these ends approximately enclosing that one of the northern channels more nearly having its axis in the prolongation of the axis of deep-water flow through the gorge of the harbor. He argued that, assuming X, Y as the positions of the sea ends of the jetties, it was to a great extent immaterial whether the lines of the jetties were straight, presenting their concave sides to each other, or finally presenting their convex sides to each other, as indicated by AX, BY, EX, FY, or CX, DY respectively. He considered that those convex to each other, CX and DY, would better fulfil the conditions required in training-walls, and therefore on the whole to be preferred.

To meet the first requirement or condition, namely, allowing a free inflow of tide, the inner half of each jetty should be kept several feet below the water, and built on a curve having a radius of about $1\frac{1}{2}$ miles in length. The outer half, nearly parallel to the direction of flow, is straight, and built higher; the sea ends for a distance of several hundred feet may be carried up to high water, or above it.

To meet the second condition, namely, to direct and concentrate the ebb flow, although a portion of this will escape over the top of the lower jetties, the curved outline will deflect much of the ebb flow and carry it directly from the gorge of the harbor through the opening left at the ends of the jetties, and especially with the last half of ebb flow.

He also considered that the low jetties would have the effect of keeping the main body of the bar in its then position, though it might be raised higher north and south of the ends of the jetty; while the effect of high jetties would be to advance the gorge of the harbor to the sea ends of the jetties two and one half miles distant, and move the shore-line to the same point by the filling in behind and outside of the jetties, and a drift and wave bar would be formed to the seaward of the present bar, in front of the jetties, necessitating an extension of the jetties to keep the channel open. With the inner half of the jetties below water surface, a portion

of drift material might be carried over those portion of the jetties and deposited in deep water of the main channel: this would be taken up by the ebb flow and rolled out through the jetty channel, and there taken up by the littoral currents and carried southward with a velocity accelerated by the storm, and finally deposited where it could do no harm.

The length of the north jetty was to be 7450 feet long, inner half curved with a radius of 1½ miles, the outer half straight; The length of the south jetty 11,650 feet, the shore end curved with a radius of 3 miles for over one half the total length, the remaining portion nearly straight.

For fully one fourth of their lengths the sea ends will be built above the level of high water.

1080. *Probable Effect produced by the Jetties.*—The following computations are given only as an illustration of the use of formulæ and methods of calculation.

All of the formulæ for flow of water in open channels have the following general form:

$$V = C\sqrt{ri}.$$

V = velocity in feet per second; r = mean hydraulic radius; i = slope, or ratio of horizontal length to vertical descent; C = constant, which varies with the character of the bed of the channel and its condition as to roughness or smoothness.

In paragraph 1059 the method of determining C for a particular case was shown, and in table, page 112, are given various values as determined by experiment for use in Kutter's formula. As the application of the formula is now the principal object in view, without reference to the relative merits of the several formulæ, we will take $C = 100$. The formula is then known as that of D'Aubuisson-Downing, and $V = 100\sqrt{ri}$.

We now desire to find the slope i, assuming the following data. Referring to Fig. 396, the sectional areas of the gorge between Morris and Sullivan's Islands, as determined by ordinary methods are:

Area of low-water section,	159,550	square feet.
" high-water "	195,350	" "
Mean of mean ebb-tide section,	176,600	" "

The width of the surface at half-tide, corresponding to mean ebb-tide area, was 6825 feet, and the wetted perimeter 6927 feet.

$$\text{The hydraulic mean radius } r = \frac{176,600}{6,927} = 25.49 \text{ feet;}$$

$$\text{and at mean low water } r = \frac{159,550}{6,851} = 23.29 \text{ "}$$

The area enclosed between the gorge line, the jetty lines, and the gap or opening between the ends of the jetties is 2.16+ square miles. Assuming this area to be covered to a depth of 5.1 feet in flood-tide, the prism of water contained would be, at each tide, 307,108,434 cubic feet, and the discharge per second $\frac{307108.434}{21600} =$ 14,218 cubic feet; and the low-water discharge through the gorge has been found to be 169,233 cubic feet per second; giving a total discharge over the jetties and through the opening at the ends of 183,451 cubic feet per second.

The following were the existing sectional areas along the lines of the proposed jetties and across the gap or opening between their ends:

	Low-water, sq. ft.	Mean half-tide, sq. ft.
Along line of north jetty,	59,900	78,880
" " " south	171,720	201,365
" " " gap,	22,840	29,572
Total,	254,460	309,817

The formula $Q = VA$ gives the relation between the amount of discharge Q, the velocity V, and the sectional area A; hence

$$V = \frac{Q}{A}.$$

The mean velocity of flow through the gorge of the harbor is $V = \frac{169233}{176600} = 0.958$ feet per second.

The mean of the velocities of flow over the lines and across the ends or gap of the proposed jetties is

$$V = \frac{183451}{309871} = 0.59212 \text{ feet per second.}$$

The wetted perimeter along these respective lines are:

Along line of north jetty,	7,450 + 124.3 =	7,574.3 feet.
" " " south "	11,650 + 175 =	11,825 "
Along gaps at ends of jetty (½ mile wide)	2640 + 79.7 =	2,679.7 "
Total wetted perimeter	=	22,079 "

The hydraulic mean depth of the aggregate section is $r = \frac{309817}{22079} = 14.0322$ feet. Substituting values of V and r in $V = 100\sqrt{ri}$, $i = \frac{V^2}{10000r} = \frac{(0.59212)^2}{10000 \times 14.0322}$; $\therefore i = 0.000002498$, which gives the surface slope over the lines of jetties and gap, assuming it to be the same throughout the entire section.

The total discharge over these lines is 183,451 cubic feet. Having the areas of the several divisions and the wetted perimeters, we can find the respective hydraulic mean radii as follows:

$$\text{Over line of north jetty, } \frac{78800}{7574.3} = 10.41 \text{ feet} = r;$$
$$\text{" " " south " } \frac{201365}{11825} = 17.03 \text{ " } = r';$$
$$\text{" " " gap, } \frac{29572}{2679.7} = 11.03 \text{ " } = r''.$$

Then substituting the common value of the slope i and the values of r, r', and r'' in $V = 100\sqrt{ri}$, we find the corresponding mean velocities V over the several divisions, namely:

$$\text{Over line of north jetty, } V = 100\sqrt{10.41 \times 0.000002498} = 0.51 \text{ foot.}$$
$$\text{" " " south " } V = 100\sqrt{17.03 \times 0.000002498} = 0.653 \text{ "}$$
$$\text{" " " gap, } V = 100\sqrt{11.03 \times 0.000002498} = 0.525 \text{ "}$$

Having then the mean velocities of flow and the areas of the three divisions, we find the respective amounts of discharge through them by substituting in $Q = VA$.

Over line of north jetty,	$Q = 78,880 \times 0.51 =$	40,229	cubic feet.
" " " south "	$Q = 201,365 \times 0.653 =$	131,481	" "
" " " gap,	$Q = 29,572 \times 0.525 =$	15,525	" "
Total discharge,		187,235	" "

This, however, owing to slight inaccuracy in the value of V, exceeds the actual estimated discharge, 183,451 cubic feet. A slight reduction in the velocities will give, respectively, 39,418 + 128,806 + 15,227 = 183,451.

The several quantities will represent the then distribution of the outflow per second through the section selected for the sites of the jetties and the opening between them at the narrowest point.

The change of regimen which the jetties will tend to produce, and the area of the waterway which, once established, they would be expected to maintain between and beyond the sea ends of the jetties, will now be considered.

On the proposed plan only the tops of the inner half of each jetty being submerged, and the depth below the surface as fixed upon, the area of the half-tide waterway over north jetty is reduced from 78,880 to 41,593 square feet, and wetted perimeter to 5480 feet, and consequently the hydraulic mean radius becomes $\frac{41593}{5480} =$ 7.59 feet; and that of the south jetty from 201,365 square feet to 94,684 square feet, and wetted perimeter to 8791.4 feet, consequently new hydraulic radius to 10.77 feet.

It is assumed that the tops of the submerged portions of the jetties will not be disturbed by waves or current, hence the above hydraulic radii will be permanent. Erosion and scour will, however, take place in the jetty channel. The original sectional areas of waterway were: at half-tide, 29,572 square feet; and at mean low tide, 22,840 square feet. When the equilibrium of flow is restored by scour, aided by dredging where clay-beds are encountered, the original general average slope, $i = 0.000002498$, will be also restored; and the aggregate average discharge per second will be the same as before the construction of the jetties. Under these conditions the average discharge over the waterway of north jetty is found as before, and reduces to—

Over north jetty.................	18,113	cubic feet.
" south "	49,113	" "
Or total over jetties............	67,223	" "

The remaining amount of discharge, 183,451 − 67,223 = 116,228 cubic feet per second, will flow out through the jetty channel, where at first the sectional area is 29,572 square feet, and mean discharge 15,227 cubic feet per second; and substituting value of $i =$ 0.000002498 in $V = 100\sqrt{ri}$, it becomes $V = 0.15807 \times \sqrt{r}$.

As yet r is unknown. The width at jetty end is one-half mile = 2640 feet. The estimated wetted perimeter, by Gen. Abbot's rule, will be $2640 \times 1.015 = 2680$ feet. Hence $r = \frac{A}{2680}$, substituting $V' = 0.15807 \frac{\sqrt{A}}{\sqrt{2680}}$.

The calculated average discharge is 116,228 cubic feet; hence $116{,}228 = AV' = A \times \sqrt{A} \times \frac{0.15807}{\sqrt{2680}}$; reducing

$$A = \sqrt[3]{\frac{(116228)^2 \times 2680}{(0.15807)^2}} = 113{,}160 \text{ square feet.}$$

Hence mean hydraulic radius $r = \frac{113160}{2680} = 42.22$ feet at mean half-tide, or 39.71 at mean low water.

By comparison of other conditions of discharge through channels in the same harbor, with corresponding surface widths, sectional areas, discharges, and hydraulic radii, with the above determined discharge through jetty channel, area of waterway, width, and hydraulic radius, we may predict that there will exist, after completion of the jetties, a channel having 24 feet of depth over a large portion of the gap, with depths of 75 feet or more in mid-channel, and that these depths will be maintained.

The average velocity, from which the general average slope (i) is derived, is of course less than the velocity that will prevail in the deep-channel compartments of the profile, since with unaltered slope the velocities in different portions of the profile may be considered to vary as the square root of depths.

The grand average in a profile, with a mean hydraulic radius of 25,46 feet, is 0.958 feet per second; in the 50-feet compartments the average velocity would be 1.33 feet per second; while during the second and third quarters of ebb the velocities will vary between two and three feet per second. From many observations near Fort Sumter the bottom velocities will generally be but little less.

1081. The determination of the effects produced to the seaward of the jetties upon the outward slope of the bar, by so large a volume of outflow, is rendered difficult and uncertain on account of the exact form and degree of spread that will occur. For the purpose of following the line of investigation, assume that there will be a spread in a fan-shaped form through an angle of 60 degrees. The width of the profile, 1½ miles to seaward, through which the outflow from the jetties is supposed to pass, is 10,933 feet. Then, adding the fan-shaped water-prism to theoutflow at the ends of the jetties, we find the average volume passing through the outer profile to be 128,916 cubic feet per seeond. The half-tide sectional area found by the preceding method will be 172,312

square feet, its wetted perimeter 11,097 feet, its hydraulic radius at mean half tide = 15.52 feet, and at mean low water about 13 feet, which implies more than ample mid-channel depths through the outer slope of the bar for vessels of the deepest draught. These depths could be increased by a moderate extension of the jetties.

If the gap between the jetties is widened, the submerged portion of the jetties must be raised to a greater average height, thus diminishing the area of waterway over them, in order that a channel of the same mean depths in the seaward profile near the outer 18-foot curve may be maintained.

1082. A similar calculation of the hydraulic elements could be made with an opening of, say, $\frac{5}{8}$ mile in width at the jetty ends. The calculation shows that it would be necessary to raise submerged jetties 14 inches in order to maintain the same hydraulic mean depth at $1\frac{1}{2}$ miles beyond the jetty ends as for the half-mile width of gap. The hydraulic mean depth at the jetty ends would be 4.45 feet less than for the half-mile gap.

Under both suppositions the sea ends of the jetties rise to high-water level for a length of 1500 feet on the north jetty and 2000 feet on the south jetty.

Substituting $r = 39.71$ in $V = 0.15807 \sqrt{r}$, $V = 0.93$ foot per second, the mean average ebb velocity through the gap.

By raising the submerged portions of the jetties above the calculated heights, greater ebb flow and velocities would be established in the gap, with corresponding increase of power and outward reach, and consequently increased depths through the outer slope of the bar. But this would give no greater depths in the gap, assuming the scour to have denuded the bed of clay, by carrying away the overlying shell, sand, soft mud, and mixtures of these.

If the submerged portions of the jetties be raised, the overflow waterway is diminished. The volume of discharge remaining the same, there will result in the gap a banking up of the waters, and consequent increase of slope and of velocity. The calculations, on a certain total waterway area, show that the slope will be increased from $i = 0.000002498$, or about $\frac{5}{32}$ inch to the mile, to $i = 0.000004963$, or about $\frac{5}{16}$ inch per mile, and the mean average velocity from 0.93 foot to 1.09 feet per second.

The above computations have been made on the basis of outflow of 3,655,374,296 cubic feet, through the gorge of the harbor, on each ordinary ebb-tide.

By an exactly similar calculation on the basis of 4,834,000,000 cubic feet, it was found that the hydraulic radius in the gap between the jetties was the same in the two cases, which is due to having only the calculated slopes and mean velocities to deal with, and that these vary with the volume of flow through the same section. The actual slope and velocities may be assumed to lie between those found for the two cases, and therefore to correspond to the deduced hydraulic radius.

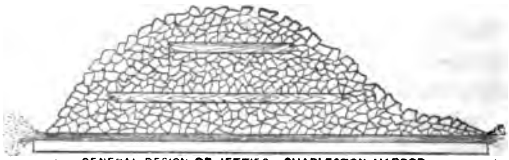
The above rather theoretical discussion is given to show clearly the lines of study required in connection with the problems to be solved, where it is desired to determine approximately the lengths, directions, and proper heights of different portions of jetties, and the proper width of opening at the jetty ends, mainly as a basis of estimates as to cost, and possible or probable results. The velocities should be determined by complete and accurate observations, and the calculations revised before their practical value can be depended upon.

Accurate and full borings are also essential to determine the character of the material desired to be removed before the effect of the current in scouring a channel can be determined, and also to estimate the probable amount of dredging necessary to establish the required depth and width of channel. This once established, the current will probably be capable of maintaining the channel.

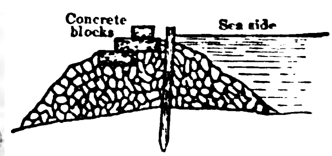
1083. *Design and Construction of the Jetties.*—The tops of the jetties over the inner half of their lengths are at varying depths below the water surface.

Their sea ends, for a length of 3000 feet on the north jetty and 3500 feet on the south jetty, have their crests at the level of half flood of spring-tides, or 3 feet above mean low water. The lengths probably required were, respectively, 8480 feet and 13,040.

The construction (see Fig. 397) consists essentially of a platform



GENERAL DESIGN OF JETTIES, CHARLESTON HARBOR.
FIG. 397.



General Section of Sulina Jetty.
FIG. 398.

of round logs from 11 to 12 inches in diameter, placed and held close together side by side, at right angles to the axis of the jetty.

This log platform was overlaid by a compact layer of stout brush to a depth of 12 to 14 inches, the ends projecting 4 feet beyond the ends of the logs. As much brush was used as the logs would bear up and float. Upon this wooden substructure was to be placed good sound stone of varying sizes, none less in weight than 30 pounds, to a depth of at least 2 feet. All of this stone was placed within the limits of the log base, none being placed on the projecting ends of the brush. The greatest bottom width of platform 92 feet, the least 33 feet, and varying between these limits according to the depth of the water and height of the jetty. On the interior faces of the jetties, except near the sea ends, the side slopes are 1 to 1½; on the exterior faces, and also on the interior faces for one-half mile from the sea ends, the slopes are 1 to 2.

For the north jetty the minimum width on top is 15 feet, and increases to 24 feet in width on the outer lengths which are above low water.

The south jetty had a minimum width on top of 12 feet, and increasing outwards to 24 feet for the higher portions of the jetties.

The greatest pressure per square foot of base did not exceed one ton, and generally less than one-half ton. No settlement was therefore apprehended.

After locating and sinking the bottom mattress, and loading it with stone, a second mattress was used, resting on the stone, and additional stone placed on top, the thickness of the whole being that required to bring the top of the jetty to the required height, the width of the second mattress is uniformly less by 32 feet than the bottom, in order that its ends might be entirely covered by the loose stone. The use of the additional mattresses in the hearting reduced the cost, without otherwise affecting the integrity of the work, this costing at the rate of $5 per linear foot, or $1.66⅔ per cubic yard; whereas the cost of the stone was from $3 to $4 per cubic yard, and for the necessary facing-stone from $5.50 to $6 per cubic yard. The total estimated cost was from $1,500,000 to $1,800,000.

The widths top and bottom of jetties were fixed so as to allow an increase of height if it should be found necessary in order to produce the desired depths.

So far as the destructive effects of the teredo are concerned, it is found that the bottom mattresses are soon covered with sand, and that those in the hearting and covered with stone are soon protected by the growth and cementing effect of barnacles and other

shellfish accretions, resulting in a dense and compact mass. Fig. 396 (*a*) gives profiles from mean soundings at different dates. Fig. 396 (*b*) shows the heights of the jetties with respect to mean low water in 1892.

VERTICAL CROSS-SECTION.

1084. The results thus far attained have not come up to the expectations of the engineers. This has been attributed to two main causes: (1) Submerged jetties failed to accomplish the results intended; and (2) the distance between the jetties being excessive, the scour expected and predicted never occurred. It has been necessary to maintain a channel by dredging. Consequently a change has been decided upon. It is claimed by eminent engineers that the theory upon which submerged jetties are based is fallacious, and their construction is practically abandoned by its advocates, after many costly experiments. It is but just to Gen. Gillmore (now deceased) to say, that while designing the Charleston jetties he admitted it to be an experiment, and laid his plans for raising the jetties if subsequent events should render it necessary.

The great width between the jetties was intended to facilitate the rapid filling of the tidal basin at each tide.

The amount of work any given stream is capable of performing is equal to the volume of flow into its fall in a unit of time; therefore to increase its slope is to increase the scouring effects. With a wide entrance the difference in level per tidal flow must always be very small. The width between the jetties should therefore be as little as a safe entrance will allow. Another error claimed is that the scouring effect or force of a stream increases with the square of the velocity, without regard to the depth. In a shallow, steep stream a 3-mile current may cut its way through the hardest materials—even some rocks; while in a deep river a current of the same velocity often fails to disturb the lightest silt.

The depths near shore ends of jetties have been increased from 1 to 2 feet, and beyond ends of jetties decreased from 5 to 7 feet. The outer face of the bar has been pushed seaward over a mile; a narrow dredged channel about 20 feet deep has been cut through the bar to deep water outside the bar.

The amount expended to date is (about)......	$2,427,000
The amount under contract................	1,953,000
Total allowed by Congress	$4,380,000

This estimate is based on building the jetties up to mean low water, and will be \$5,300,000 if brought throughout their lengths up to a level 3 feet above mean low water.

A hydraulic dredge has been used since 1891. The dredge is 122.5 feet long, 30 feet beam, and 12 feet depth of hold. The pumping machinery consists of a 230-H.P. compound engine, coupled direct by a shaft to a centrifugal pump, with a 14½-inch suction and a 15-inch discharge-pipe. The maximum bin capacity is about 270 cubic yards. The required crew is 16 men.

The contract prices for stone, which was a hard and heavy granite, varied from \$2.25 to \$3.20 per ton; for mattress-work, \$1.25 per square yard; and for dredging, 28 cents per cubic yard. The rock is taken out in blocks from 20 pounds to 7 tons weight. Total cost of jetties, \$70.46 per linear foot. The cost of dredging by Government plant was only 11½ cents per cubic yard.

The width of jetty foundation has varied from 40 to 206 feet, depending on the depth of the water. As many as three courses of mattress have been used. Up to June, 1892, 306,585 square yards of mattresses and 212,301 cubic yards of rock (at 2600 pounds per cubic yard) had been used in the south jetty. On the north jetty foundation course, from 40 to 118 feet in width, composed of one or two courses of mattresses, up to June 30th, 175,155 square yards of mattresses and 159,369 cubic yards of stone had been used. (See Figs. 396 (*b*) and (*a*).)

1085. *Improvements at the Mouth of the Danube.*—This improuement has been alluded to in several places, and it will only be necessary to add a few additional statements.

The Danube enters the Black Sea by three mouths. The Sulina mouth was selected as the best suited for improvement on account of the relatively small proportion of alluvial matter discharged through it, being only 70,000 tons out of a total of 900,000 tons every 24 hours.

The improvement consisted in two parallel piers or jetties of stone, having a length of 8000 feet, at a cost of about \$900,000. The result has been an increase of channel depth from 11 to 20 feet, and it has been maintained at the latter depth.

A general vertical cross-section of jetty at the Sulina mouth of the Danube is shown in Fig. 398.

1086. *Jetties, Yaquina Bay, Oregon.*—Jetties built of crib-work and filled with stone were first tried. The cribs were in sections 48 feet long and 25 feet wide, formed with timbers of Oregon

pine, 12 × 12 inches, built 23 courses high. These failed entirely. It was then determined to build pile trestles, extending out on the lines of the proposed jetties, from which were swung mattresses 60 feet wide and 2 feet thick. These were made of two layers of fascines, each 1 foot thick, crossing each other and bound together between two grillages of 6-inch fir poles; on this was placed a covering of two or more feet of coarse-grained blocks of sandstone, weighing 140 pounds per cubic foot. Upon this a second mattress, 30 feet wide, was placed, and the whole well riprapped with stone. These jetties were only built up to midtide.

In some portions of the work the jetties were built simply by dumping stone under and around the trestle. These stone jetties were built up to high tide in order to provide for settlement, and were to be faced and coped with large stone after full settlement had occurred.

Fir piles 30 to 50 feet long cost 6 to 8 cents per lineal foot; fir lumber, from $10 to $12 per 1000 feet B.M.; iron, 3 cents per pound; rock placed on jetties from 82 cents to $1.18 per ton of 2000 pounds; pile trestle, from $4.76 to $8.54 per linear foot, double track, and $2.52 for single track. Total cost per linear foot of jetties, $90.

IMPROVEMENT OF HARBORS.

1087. Harbors may be classed under two general heads:

(1) Harbors of refuge, which may be either natural or artificial. These serve mainly for the protection of ships during storms.

(2) Harbors for commercial purposes. Such harbors must necessarily provide safe and secure anchorage for ships, and must therefore have either natural or artificial protection from storms, and in addition must have such constructions, as quay walls or wharves, as will facilitate the loading and unloading of ships.

Natural harbors of refuge are those more or less extended areas with sufficient depth to float a number of ships during all stages of tide, and at the same time protected by natural obstructions, from dangerous action of winds and waves, while affording easy and safe access from the open sea in all states of the weather and tide.

Artificial harbors of refuge are those formed by the construction of works called breakwaters, which form a barrier, more or less perfect, to the progress of the waves and winds, and must serve the same purposes and fulfil the same conditions and requirements as for natural harbors of refuge.

1088. *Breakwaters.*—We will now briefly discuss the location, design, and construction of outer breakwaters erected in deep water, and which are constantly exposed to the action of winds and waves.

The location of a breakwater must be determined with reference to: (1) enclosing a sufficient area for the accommodation of the greatest probable or possible number of ships that may seek its protection at any one time; (2) the angle at which the heaviest waves impinge upon the coast-line; (3) the nature and character of the bed of the sea upon which the structure will rest, as well as the depth of the water along the proposed site of the structure; (4) the perfect ease and safety of entering into the protected area; (5) the cost of the structure; for although strength, permanency, efficiency, and suitableness for the purpose in view should be the controlling factors in the construction of such works, yet a judicious selection of the proper location may result in a large saving of expense, while fully satisfying all other conditions required.

The first condition, namely, enclosing a sufficient area, having a proper depth of water, ample protection during storms, and good holding ground, is purely a local question, and must be determined from reliable data obtained in each case.

As to condition (2). The amount of shelter which is produced by a breakwater must be measured by the length of the portion of the wave which is either destroyed or reflected by it. The amount of work done by it decreases from a maximum when the waves come upon it at a normal angle of incidence, to zero when the waves come upon it end on, in which case it ceases to be a breakwater, strictly speaking. If a breakwater be so situated that the waves strike obliquely its inner face, any increase in its length, although enlarging the area cut off from the sea, may during certain winds increase the sea within the harbor instead of reducing it. Hence when such lengthening is necessary to acquire increased area, either the direction of that portion of the breakwater must be changed, or a separate breakwater must be built out from the shore in order to shelter the inner side.

The nature and character of bed, and the depth of water, as mentioned in condition (3), are important as affecting both the permanency and cost of the work, as it would obviously be useless to build works on a shifting bottom, and the depth at which disturbance or scour will be likely to occur is of great importance. The surface disturbance of the sea caused by storms is known to

extend to great depths. Shingle is moved during heavy winter storms, as observed by divers, at depths of 8 fathoms; and it has been maintained that waves may excavate the bottom at depths of 100 or more fathoms in the ocean, and to the depth of 20 to 30 fathoms in seas and channels. The depth at which mud reposes is of great value in judging of the exposure of a coast; this depth has been found to vary from 12 to 90 fathoms.

The line of maximum exposure, that is, the line of greatest fetch or reach of open sea, which can be readily obtained from charts, is important in connection with the height of waves.

1089. *Heights of Waves.*—It has been found by experiments that the heights of waves are proportional to the square roots of their distances from the windward shore, and can be expressed by $h = a\sqrt{d}$, in which h = height of wave in feet, d = distance or length of fetch in miles, and a a coefficient varying with the strength of the wind. It has been found that $a = 1.5$ indicates very nearly the heights of waves, during heavy gales, in deep water. Where the water is not of sufficient depth to allow the waves to be fully formed, or where it becomes so shallow as to reduce their height after they are formed, the above formula is not applicable. Applying the above formula, we find, when $a = 1.5$ and d = length of fetch in miles = 9 miles (nautical), $h = 1.5\sqrt{9} = 4.5$ feet; the actual observed height on the Clyde = 5.0 feet. With $d = 45.5$ miles, $h = 1.5\sqrt{45.5} = 10.2$ feet; actual height = 10.0 feet; place of observation Macduff. With $d = 165.0$ miles, place of observation Sunderland, $h = 1.5\sqrt{165} = 19.3$ feet; actual observed height 15.0 feet.

In short fetches, as in lochs or narrow arms of the sea, waves are raised higher than indicated by the above formula. For short reaches and violent squalls the following formula may be used:

$$h = 1.5\sqrt{d} + (2.5 - \sqrt[4]{d}).$$

Giving the same values to d, we find $h = 5.25$, 10.0, and 18.4, respectively. For the longer fetch use the first formula, and for the shorter the second. Waves only 14 feet high have been observed with a fetch of 600 miles, and with a similar exposure at another place waves 40 feet high have been seen to strike the breakwater, and heights as great as from 60 to 108 feet above the hollow have been reported.

During storms thousands of tons of water are elevated and maintained above the sea-level, and a breakwater has to stop their onward progress within a given space, or else change the direction of their movement. The whole construction of breakwaters, therefore, turns on the theory of waves, as they are structures designed to break the force of waves. They may be temporary or permanent in character, and constructed of timber, stone, iron, or of other material.

1090. Solid breakwaters may be erected so as to oppose a direct resistance to the force of the waves : this can be effected either by means of a vertical, or nearly so, faced wall, which alters the direction of the moving water, causing it to ascend vertically and then allowing it to descend vertically, by which process the waves are reflected and sent back seawards; or the undulations may be arrested by a long sloping surface, which allows the mass of elevated water to fall down upon the slope, if however, not long enough to enable the waves to destroy themselves, they will, though reduced in height, pursue their original direction and pass over the top of the breakwater, in which case only imperfect shelter is obtained. (See Figs. 399, 400, 401, 402.)

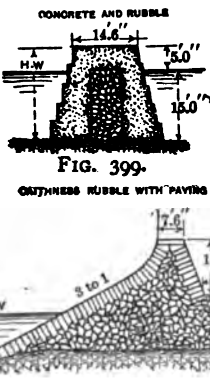


FIG. 399.

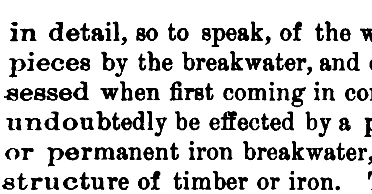
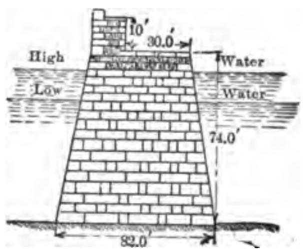


FIG 400.

Floating breakwaters may be placed which do not oppose a direct resistance to the force of the waves, and instead of defying and withstanding these violent forces, yield to them and allow them to be expended gradually by being divided and broken up, as it were. The important object to be attained is the gradual but complete disintegration in detail, so to speak, of the wave ; it should be absolutely cut to pieces by the breakwater, and entirely deprived of the power it possessed when first coming in contact with the structure. This may undoubtedly be effected by a properly braced and open-work fixed or permanent iron breakwater, but better by an open-work floating structure of timber or iron. The advantages and disadvantages of those types above mentioned will be briefly discussed. A few remarks on the force of waves will be first made.

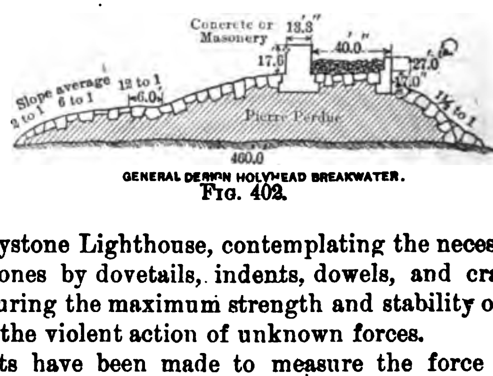
1091. *Force of Waves.*—Smeaton says: " When we have to do

with and to endeavor to control those powers of nature that are subject to no calculation, I trust it will be deemed prudent not to omit in such a case anything that can without difficulty be applied, and that would be likely to add to the security." This important principle was laid down in connection with the construction



DOVER BREAKWATER.

FIG. 401.



GENERAL DESIGN HOLYHEAD BREAKWATER.

FIG. 402.

of the Eddystone Lighthouse, contemplating the necessity of bonding the stones by dovetails, indents, dowels, and cramps, with a view of securing the maximum strength and stability of a structure exposed to the violent action of unknown forces.

Attempts have been made to measure the force of waves by means of marine dynamometers, which consist essentially of a flat disk on which the waves impinge directly. These disks are from 6 to 9 inches in diameter, and are connected by rods, with a strong spring enclosed in a cylinder so that the stretching or drawing out of the spring can be determined, from which the force of the wave can be measured or calculated. In this manner the force of the waves has been determined, according to locality, to be from $1\frac{1}{2}$ to $3\frac{1}{2}$ tons per square foot of exposed surface. It is more satisfactory, however, to arrive at the immense power of waves by observing the work done by them.

At Barra Head, one of the Hebrides, a block weighing 50 tons was moved by the sea. At Whalsey, in Shetland, at a level of 72

feet above the sea, a block weighing $5\frac{1}{2}$ tons was broken out of its bed in the mass of rock and moved from its place. "In 1872, at the harbor works of Wick, a huge monolithic block of concrete, weighing in all 1350 tons, was removed *en masse* out of its position and carried to leeward of the breakwater. The first portion of this wall was founded at 12 feet below low water, but later portions to 18 feet, and the rubble has been washed or scoured down to a depth of 15 feet below that level. The following year a mass weighing 2600 tons was moved in like manner. Again, in another case a mass of 300 cubic feet, weighing 28 tons, was carried a distance of 90 feet. Many similar instances might be given.

In order that the waves may be completely formed, there must be a sufficient area to admit of the wind acting fully on the water, the sea must be unobstructed by shoals, and there must be a sufficient depth of water. "If in front of harbors shoal water extends to a considerable distance, forming an extensive flat fore-shore, the depth of water above it becomes the true limit of the maximum wave, whatever may be the general depth of the sea outside."

1092. *Depth of Water in which Waves Break.*—It is generally stated that waves break when they come into water of a depth equal to their height. Observation shows that in some cases the height above mean level is as much as two thirds of their height, whereas in other cases waves from 5 to 6 feet broke in water from 10 to 14 feet in depth. In such cases $h = \frac{1}{2}d$ nearly, where d is the depth below mean level and h the height of wave from hollow to crest.

1093. We have now considered the conditions determining the height of waves and the magnitude of the forces developed under certain conditions. We are next to consider the effect of these on walls or breakwaters at different angles to the direction of the wave, and having different forms of cross-section and presenting either a complete or partial resistance to the force of the wave.

Deep-water Harbors.—In deep water it is claimed that oceanic waves are purely oscillatory, and exert no impact on vertical-faced piers or breakwaters, and that it is only necessary to consider the hydrostatic pressure due to the height of the impinging waves, which are reflected without breaking. There seems to be, however, ample evidence that waves break in deep water from various causes, and that consequently waves of translation are generated, subjecting any form of barrier to heavy impact. The theory of the purely oscillatory character of waves, and consequent hydrostatic pressure alone exerted, is the one upon which vertical-faced walls are advo-

cated. Even if this is true after the completion of the wall, it is evidently not tenable during the construction of such barriers.

Mr. Rankine says "that every wave is more or less a wave of translation, setting down each particle of water, or of matter suspended in the water, a little in advance of where it picked that that particle up, and thus by degrees producing that heaping up of water which gathers on a lee shore during a storm. This property of waves accounts for the facts, that although they tend to undermine and demolish steep cliffs, they heap up sand, gravel, shingle, or such materials as they are able to sweep along, upon every flat or sloping beach against which they directly roll; that they carry such materials into bays and estuaries; and that when they advance obliquely along the coast they make the materials of the beach travel in the same direction."

Vertical-faced Breakwaters.—Assuming, then, that a vertical-faced wall is thus acted upon, waves, when they roll straight against it, are reflected, and the particles of water for a certain distance in front of the wall have motions compounded of those due to the direct and to the reflected waves. Those particles adjacent to the wall move up and down through a vertical height, double the original heights of the waves. as also do those at half wave-length from the wall; at quarter of a wave-length the motion is backward and forward in horizontal directions, while the intermediate particles oscillate in lines having various angles of inclination.

Such reflection of waves will take place on surfaces sloping even to an angle of 45°. A sunken breakwater thus reflects the layers of water below its top, and causes the sea to break over it, thereby diminishing the energy of the advancing waves.

"The greatest wave-lengths in the ocean are estimated at about 560 feet, corresponding to a speed of about 53 feet per second and a period of 11 seconds, and the greatest height of waves at 43 feet.

"In smaller seas the waves are both lower and shorter and less swift; and waves in an expanse of shallow water of nearly uniform depth never exceed in height the undisturbed depth of water.

"But the concentration of energy upon small masses of water, which occurs on shelving coasts in the manner already stated, produces waves of heights greatly exceeding those which occur in water of uniform depth." Greatest height of breakers on the south-west coast of Ireland is given at 150 feet.

1094. Such walls (see Figs. 399–401) are designed, as in the case of reservoir-walls, to resist the greatest pressure to which they may

be subjected. The facing may be made of dressed masonry backed with coursed rubble or concrete, or the entire wall may be made of concrete in large blocks. The outer edges of the joints should be laid in the best cement mortar, while the body of the structure may be laid in ordinary or hydraulic mortar. The great danger arises from the jumping out of the stones or blocks, caused by the pressure and elastic reaction of air and water, which may penetrate into the joints under the blow of a wave; and, in addition, the undermining action of the waves at the foot of the wall is very great. This is best prevented by a paving of stones in front of the wall, but not bonded into it. The front of the wall may be built in steps, which break the vertical descent of the water and diminish the danger of undermining. The rear face is also vertical, or nearly so.

In Fig. 401 is shown a cross-section of a vertical-faced breakwater at Dover, England. This was constructed of solid masonry, having a uniform slope of $\frac{1}{4}$ to 1 on both outer and inner faces. The base of the wall is founded 45 feet below low water, and the top 23 feet above high water. The thickness or width at base is 82 feet; at high-water line, 57 feet; and at 74 feet above the bottom, 45 feet thick. Above this level it was surmounted with a parapet wall 10 feet high, giving a total height of 84 feet. Above high-water surface the work is faced and covered with stone and filled with concrete.

The parapet-wall is not an essential feature of breakwaters, the slopes being carried often to the extreme upper or top surface. The cost of the Dover Breakwater was about $5200 per lineal yard.

1095. *Breakwaters with Long, Sloping Faces.*—Breakwaters with long slopes have the advantage of causing the waves to break. "When a series of waves advance into water gradually becoming shallower their periods remain unchanged, but their speed, and consequently their length, diminishes, and their slopes become steeper. The front of each wave gradually becomes steeper than the back, the crest, as it were, advancing faster than the trough. At length the crest of the wave falls forward, and it breaks into surf on the beach."

"The energy of motion is successively communicated to smaller and smaller masses of water, tending to throw those masses into more and more violent agitation; this is usually counteracted by the loss of energy which occurs through the production of eddies and

surge at sudden changes of depth, and through friction on the bottom."

The inclinations given to the slopes vary from 1 to 1 to 3 to 1 below low water and from 4 to 1 to 7 to 1 above low water, as these are more violently acted upon by the waves. Waves partially break in passing over the line where the inclination changes. For this reason a series of level benches or berms, alternating with flat slopes of about the same length with the berms, are very effective in breaking the waves and exhausting their energy. (See Figs. 400, 402.)

Such breakwaters may be constructed of a hearting of earth and gravel or of loose stones, depending upon the situation, and faced or paved with large blocks, each having sufficient weight to withstand independently the lifting action of the waves. This paving may be curved upward at the toe of the slope, as described for weirs, so as to prevent the undermining action of the returning current or undertow from the breakers. The top of the slope is sometimes curved upwards and outwards, presenting a concave surface to the waves; it will be preferable, however, to finish the work with a level berm or top surface paved with large blocks, and upon this erect a strong parapet so placed that the slope will pass, if prolonged, above the top of the parapet-wall.

A common construction now is to use large and small stone, which are either thrown overboard from barges or carried out on staging made of timber or iron piles, and dumped over from these, the stones thus forming a base or substructure assuming its natural slope; or a flatter one, if so desired, may be formed. This construction is known as the *pierre perdue*. This kind of work is not usually expected to stand the action of waves, and must consequently be paved with large blocks of sufficient weight. This facing need not extend below that depth at which the *pierre perdues* are able to resist displacement. This depth will vary according to the height of the waves and the sizes of stone used. It may not, on an average, exceed from 12 to 15 feet below low water; but, as stated, at Wick harbor it extended to a much greater depth. In all such cases, as already mentioned, a parapet-wall should be built on top.

1096. *Combined or Composite Breakwaters.*—In the case just mentioned it may be well to form a *pierre perdue* substructure, which may be built as already described, or say up to or near low-water mark, and upon this, as on a beach in shallow water, build a steep wall well back from the edge of the slope.

In all such masonry walls it is desirable to use very large blocks of stone, or else take special precaution to tie these together by dowels or cramps, and to close the joints with cement mortar.

Concrete lends itself especially for such purposes, as immensely large blocks can be made in place, saving thereby the great cost in handling and transporting large blocks of natural stone.

Such blocks can be set on beds of concrete laid either on the natural bed or on top of the artificial substructure, these beds being arranged and levelled by divers, the blocks also being bedded and the joints filled with mortar by divers; or better still, when concrete blocks are used horizontal and vertical semi-cylindrical or rectangular grooves can be formed on the beds and sides a little back from the edges of the blocks. These are then set in juxtaposition, forming the proper vertical or sloping face.

The joints are then calked on the exposed face by divers, a liquid cement is poured into the vertical holes, through pipes extending above the surface of the water, or into the holes if the masonry reaches above the surface; the pressure-head thus obtained forces the grout into the joints, filling them and the grooves, which at once fills the joints and joggles the blocks together. If desired, pressure can be applied to the grout, thereby insuring the flow of the grout into very thin joints.

This same method can be applied to simple *pierre perdue* or gravel substructures by building a series of pipes into the mass, provided the slopes are paved and calked sufficiently to prevent the lateral escape of the soft mortar or grout. The quantity of cement required in such case would, however, be very great, and need not be resorted to below that depth at which the loose stones would remain undisturbed.

Breakwaters of this combined construction are shown in section in Figs. 400 and 402.

The parapet-wall as shown in Fig. 402 is often omitted, and the entire breakwater simply paved with large stones. In this case the waves can usually flow over the top of the breakwater, after expending a large portion of their energy.

In Fig. 400 the drawing shows a section of breakwater with a quay on the inner side. This construction is also a pier.

1097. The heights of breakwaters should be such that their tops may be above the crests of the highest waves, augmented as they are in height by the reflection; or else the coping-stones should be heavy enough to prevent lifting by the pressure due to the greatest

height of the waves above its beds, and dowelled to each other so as to act as a unit. The cope should in no case project over the face of the wall. This is for walls which are intended to stop the waves entirely. Where it is not necessary to entirely stop the waves, the height is regulated by the extent to which interference is necessary or desired. There will always be in such cases more or less commotion of the water surface behind the breakwater. Such conditions may secure safe anchorage, but not for commercial purposes, such as loading and unloading vessels. Special precautions are necessary to prevent the top paving from being washed off.

1098. *Maximum Effective Force of Waves.*—"It does not follow, however, that the line of the maximum exposure is in every case the line of maximum effective force of the waves, for this must depend not only on the length of fetch, but on the angle of incidence of the waves on the walls of the harbor. What may be termed the line of maximum effective exposure is that which, after being corrected for obliquity of impact, produces the maximum result, and this can only be ascertained from the chart by successive trials."

Let P be the greatest force that can assail the pier; h the height of waves which produce (after being corrected for obliquity) the maximum effect, and which are due to the line of maximum effective exposure; α the azimuthal angle formed between the direction of the pier and the line of exposure. Then, when the force is resolved normally to the line of pier, $P \propto h \sin^2 \alpha$.

And if the force be again resolved in the direction of the waves themselves, $P \propto h \sin^3 \alpha$.

The great difference in the effect of the waves impinging at right angles and those which have even a slight amount of obliquity was shown unmistakably at the Wick Breakwater, where all attempts to make the work stand when exactly at right angles to the waves were unsuccessful; whereas with a small angle, 9°, of obliquity the effect of the waves was greatly diminished.

"An important advantage of a sloping wall is the small resistance which it offers to the impinging wave; but it should also be borne in mind that the weight resting on the face-stones in a talus-wall is decreased in proportion to the sine of the angle of the slope. If we suppose the waves which assail a sloping wall to act in the horizontal plane, the component of their impulsive force at right angles to the surface of the talus will be proportional to the sine of the angle of inclination to the plane, while the effective force esti-

mated in the horizontal plane will be proportional to the square of the sine of the angle of inclination. But if we assume the motion of the impinging particles to be horizontal, the number of them which will be intercepted by the sloping surface will also be reduced in the ratio of the sine of the angle of inclination, or of the inclination of the wall to the vertical. Hence the tendency of the waves to produce horizontal displacement, on the assumption that the direction of the impinging particles is horizontal, will be proportional to the cube of the sine of the angle of elevation of the wall. If it further happens that, owing to the relative direction of the pier and of the waves, there is an oblique action in azimuth as well as in altitude, there will be another similar reduction in the ratio of the squares or cubes of the angle of incidence, according as the component of the force is reckoned at right angles to the surface of the pier or in the direction of the waves.

Let p = force of the wave on a unit of surface for normal incidence; p' = force on unit of surface at vertical incidence ϕ and azimuthal incidence ϕ'; then $p' \propto p\ (\sin\phi \sin\phi')^2$. See Encyclopædia Britannica.

1099. A breakwater may be isolated, and in the midst of the entrance of the bay, or it may run out from the shore into deep water. In the latter case the best position for the junction of a single breakwater with the land is in general at the up-stream corner of the entrance to the inlet or harbor, or that side where the currents are strongest as regards the flood current along the coast, for in that position it opposes the strongest flood current and does not interfere with the strongest ebb current.

This location presents a barrier to the waves of the prevailing storms, and especially to those which come along with the flood current.

1100. *Piers and Sea-walls.*—The distinction between piers and sea-walls on the one hand and breakwaters on the other is, (1) that breakwaters, properly speaking, are constructed in deep water, and have the sea on both sides, where, although constantly exposed to the waves, the exact character of the force or pressure to which they are subjected is not so well defined; and (2) that piers or sea-walls are placed within the range of the breaking surf, where they are only exposed to the force of the waves for a limited period, being sometimes left nearly or entirely dry by the receding tides, and, moreover, the waves exert admittedly a true percussive force.

In such cases there are obviously two methods of resisting this force: (1) a resistance dependent on mass and weight; and (2) a

comparatively light structure, the stability of which depends upon the strength of its members and their connections with each other and with the bottom. The general weight of authority and practice seems to be in favor of the first, although there are examples of the second kind of construction which show both strength and stability in a considerable sea.

The impact of the waves against the outside of a sea-wall or pier gives rise (1) to a direct horizontal force, which tends to loosen and drive in the blocks of stone in the structure; (2) a vertical force which tends to lift the blocks or portions of the wall by acting against projecting points or rough surfaces; (3) the downward force caused by the receding wave striking upon the toe of the talus or sloping wall, or passing over the top of the parapet and falling upon the pitching behind so as to scour it out; and (4) the back draught, which tends by reaction from the wall to remove the soft bottom, and thereby undermine the structure, or to suck loose stones out of the wall. Provision must be made to resist or counteract each one of these tendencies.

Sea-walls are frequently constructed expressly for the purpose of preventing encroachments upon the land, and as such will be considered in the next article, on the Reclamation of Land.

Piers are really a kind of breakwater combined with quays, which, while affording protection as a breakwater to vessels which lie under the lee of the wall, have a construction called a quay on the lee side, by which vessels are enabled to load and unload without their having to beach or take ground. (See Fig. 400.)

A quay or wharf alone is simply a structure either parallel or at right angles to the shore-line. In the former case no protection or shelter is afforded, whereas in the latter there is some shelter, provided the wind does not blow direct upon the shore.

By running a pier directly out from the shore for a certain distance, then changing its direction for a certain length; or by running two parallel piers for a certain distance, and then canting or inclining their directions toward each other; or, again, running one pier straight outwards, and parallel to this another pier for a certain or the same distance, then turning and extending it towards the free or sea end of the first pier; or in fact any combination of piers by which a large area is nearly closed, leaving only a narrow entrance, so placed that ships can enter or leave in all kinds of weather and stages of tide,—we have what are usually called Tidal Harbors for Commercial Purposes.

The location of the line of the pier depends on the nature and configuration of the shore and of the bottom. The proper location requires a careful survey in order to lay down a series of contours of the bottom; then with due regard to the direction of the motion of waves, and enclosing the proper area with sufficient depth of water, the pier or piers can be traced out on the shoal ground.

A single straight pier will answer in many cases when the waves strike upon it obliquely and glide along it landwards. But special precautions must be taken that a sea work nowhere presents to the sea a surface with a concave. horizontal outline, or what is worse, abrupt faces forming a re-entrant angle, for the waves will then act with great violence. The breaking of a free wave is a very different thing from the breaking of a wave confined by a barrier of masonry.

The entrance should be fixed seaward of every part of the works, and its direction, unless where the sea is very heavy and the enclosed area small, should be made to coincide with that of the heaviest waves, so that they may run along with and guide vessels into the harbor.

The outer pier should be extended sufficiently far seaward of the end of the inner pier to allow a ship to shape an easy course before taking the entrance to the harbor. The introduction of steamers and powerful tugs permits of entrance in other directions than above stated.

There should be a sufficient distance landward of a harbor mouth to allow a vessel having full weight on her to shorten sail.

1101. The tranquillity of close harbors with the same exposure depends on the relative widths of the entrance and the interior, the depth of water, and the form and direction of the entrance in relation to the line of maximum exposure.

It is essential when the exposure is great that there be either a considerable internal area, or a separate basin opposite the entrance to the inner basin, for the waves to destroy or spend themselves.

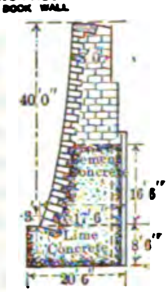
When there is not over a fetch of about five miles, which, by the formula $h = 1.5 \sqrt{d}$, gives a wave of about 3.8 feet high, it has been found that quays unprotected by covering piers will admit of carrying on traffic successfully.

The ordinary cross-sections of quay-walls, without protecting piers, are shown in Figs. 403, 404, 405, and with protecting piers in Figs. 400. These are supposed to be constructed of masonry or concrete, or a combination of these materials.

It is customary to protect vessels from rubbing against masonry quay-walls by using vertical timbers, called fenders, fastened to the masonry by bolts or straps.

As to the character of construction of piers combined with quay-

FIG. 404.



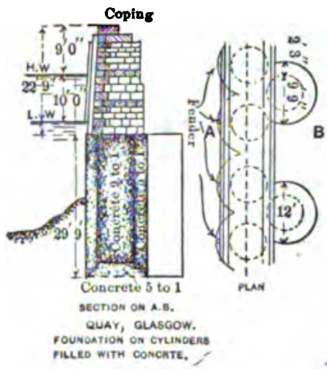


FIG. 403.

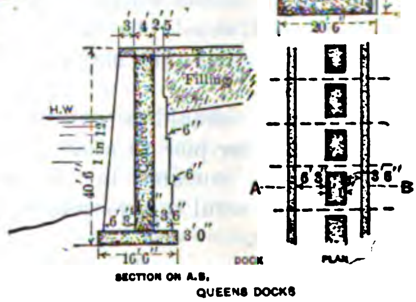


FIG. 405.

walls, there is no essential difference from that employed in the construction of vertical-faced breakwaters, or those with ordinary slopes.

1102. The internal effects of the force of waves on masonry which tend to the destruction of the works are various. The masonry is constantly subjected (1) to more or less vibrations produced by the shock of the waves, which may be transmitted through the body of the work to the inner or quay wall; (2) by the direct communication of the impulses through the films of water occupying the interstices in the masonry acting upon the inner quay; (3) by the sudden condensations and expansions of the air in the masonry loosening and blowing out the face stones; (4) by the hydrostatic pressure transmitted through the small prisms or films of water in the masonry exerting a considerable force at the back of the quay-wall.

The thickness of piers fully exposed to the action of ordinary ıves should not be less than from 35 to 45 feet at the high-water 'el, and greater thickness where the waves are very high.

1103. *Piers of Iron.*—Both cast and wrought iron have been ›d either in the form of screw-piles or of hollow cylinders, and ık by the ordinary methods appropriate to such structures. ese have been used for piers, quays, and for lighthouses.

It is known that such structures stand a considerable sea, and effective to a great extent in breaking somewhat in detail the :e of waves.

Although iron piers immersed in sea-water may last a long ;, they nevertheless corrode quite rapidly, reducing greatly the ıgth of the component parts,—as much as 50 per cent in fifty s in some observed castings.

luch constructions are doubtless best used as foundations or :ructures, surmounted with masonry or concrete walls, when are designed for ports or harbors having a large commercial e. Otherwise the entire structure may consist of piles over h a flooring or platform is placed. Fig. 403 shows a masonry wall on cylinders filled with concrete.

:rew-piles can, with proper precautions, be forced to a suffi- depth in the softer varieties of rock so as to give ample ity.

ıe exact extent to which a screw-pile pier, composed of a er of rows of piles, will be effective in breaking the force of ıves is not definitely known. It is a form of structure capa- exhibiting a high degree of strength, especially when first ınd new. It is difficult, however, to so connect the parts ɔntinual vibrations will not tear apart or loosen the connec- ɔ such a degree as to materially diminish its powers of resist- eing neither rigid nor pliable enough to withstand the shocks ɔh it will be exposed without experiencing a gradual and ıous depreciation in stability and strength, and ultimately ıg in its total destruction, to say nothing of deterioration ›rrosion. Such structures can therefore be regarded mainly , strong structures of a temporary nature.

rapidity with which such piers can be constructed and the ıtive cheapness in cost are their principal recommendation iewed as permanent structures for important purposes. ıin, it sometimes happens that it is practically impossible

to build and maintain any other kind of structure without enormous cost and consumption of time. Sometimes timber can be advantageously substituted for iron piles.

1104. *Sea-piers of Timber.*—In sheltered bays or river fronts of cities, where a deep-water landing is required, timber piers or quays can be used to great advantage, and even in exposed situations they can be used for tidal harbors.

But when exposed alone to sea-water all kinds of timber are liable to be destroyed very rapidly by the *Teredo navalis* or *Limnoria terebrans.* Some kinds of wood, such as memel, greenheart, African oak, and teak, are to a certain extent free from their attacks.

Some of the many methods of protecting timber from the attacks of these worms and preventing decay in exposed portions can be effectively employed.

The methods of constructing such piers, quays, or wharves vary in different localities, mainly depending upon the character of the bottom. Where rock or very firm earth is found along the shore or site of the piers, simple timber cribs of proper height and width can be constructed and sunk in position with gravel or broken stone, and properly paved or covered over to answer the purposes intended.

Where the bottom is soft or liable to be scoured, it becomes necessary to remove, by dredging or otherwise, the soft and loose material before founding the cribs; or in some cases, where the depth of water is sufficient, a pierre-perdue foundation may be placed, on which the superstructure may be erected. Or rows of piles can be driven, the outside sheeted with timbers, and the enclosed space filled with earth, shells, gravel, or broken stone. Where structures of this kind are wide and stable enough to resist an excess of pressure caused by an earthen embankment, or where, as in piers proper, they are subjected to an equal pressure from both sides, no special precautions are necessary to insure stability of position.

In quays proper, which may consist of two or more rows of piles driven along or close to the shore-line, whether surmounted by cribs or not, the excess pressure from the filling behind may be considerable, especially at low tide. In such cases stability can only be secured by some form of land ties.

These may consist of additional rows of piles driven parallel to

and in rear of the quay, well up on shore, which are tied to the quay by a system of transverse and longitudinal timbers; or in some cases long iron rods are extended well back to landwards, and held by timber walls or iron plates well imbedded in the firm earth. These matters have been fully discussed in other parts of this volume. Fig. 403 shows masonry quay on foundations of cylinders; Fig. 404 shows masonry quay on concrete foundations; Fig. 405, masonry quay with concrete hearting; and Fig. 406 shows timber-pile quay with land ties.

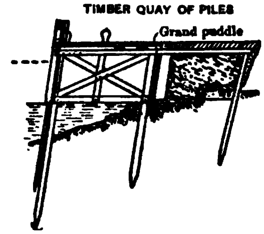


FIG. 406.

1105. *Floating or Yielding Piers or Breakwaters.*—It is not the purpose to discuss piers or quays at which vessels can load and unload, but those floating or yielding structures more properly termed breakwaters, the purpose of which is to cause comparatively tranquil water over a given area, and whose effect is not entirely to break or destroy the force of the waves, but only to such an extent as to make it reasonably safe for the anchorage of vessels, and admit of carrying on the construction of many important public works, such as piers or lighthouses, which could not be constructed unless at least partially protected from the violence of the waves.

It would be useless to multiply examples of the wonderful effect of a pliable or yielding body in destroying the momentum of moving bodies impinging on it. The motion of water moving with the velocity of torrents is checked or entirely stopped by a matting of brush, and resulting in the deposition of large quantities of earthy matter, sand, and gravel.

It is well known that trailing hawsers attached to floating bodies are effective in breaking the violence and heights of waves when cutting through their upper surfaces. From these and many other considerations it is obvious that if a barrier is formed of several rows of timber or water-tight iron columns, properly proportioned so as to float at given depths, with their upper portions projecting well above the surface of the sea, strongly connected, but not by rigid braces or connections, and at the same time firmly held by heavy anchors,—the effect of such a strong structure, yielding gradually, while developing an increased resistance, will be to greatly diminish the energy of the waves, dividing, splitting, and

partially or entirely dissipating it. There seems to be no doubt of this result being produced, and it is admitted to be the case provided the entire structure is not bodily torn from its moorings. The apprehension of failure from this cause does not seem to be well founded. The writer is clearly of the opinion that no great difficulty will be experienced in holding the component parts of such a structure, and consequently the structure itself, in place. The great difficulty will be found in a proper system of bracing and connections, by which a uniform but rather rapid increase of resistance is developed by bringing each row of floating columns into action, supporting and relieving those in front, the columns in each row being opposite, the space (of a few inches, or feet), between the columns in rows in its front and rear. For light seas from 3 to 4 such rows will doubtless be sufficient and efficient; for heavier seas, from 4 to 8 or 10 rows. The columns used should have a diamond-shaped section, and set with their sharp edges to seaward.

While such structures may be regarded as only serving a temporary purpose, there is no reason for not so constructing them as to be permanent if desired, and to afford almost as safe accommodations for shipping as the more massive and heavy, and vastly more costly, breakwaters.

They are cheaply and rapidly constructed in place, and if only employed for a temporary purpose, can be readily removed and used at other places for the same or for similar purposes.

In view of the great difficulty in constructing massive breakwaters, piers, and lighthouses on exposed coasts, and the enormous waste of time and money thus involved, and in the light of the recent failure to construct a lighthouse on Diamond Shoals, off the exposed coast of North Carolina, the writer has designed, a structure of the kind above described, which he believes will at a small outlay enable this important and necessary lighthouse to be constructed without serious difficulty.

In addition, there are good reasons for believing that a considerable degree of silting up will occur under and around both to the windward and leeward of such an open-work barrier, the lighter matters, such as mud, silt, and sand, being deposited to the leeward; the heavier matters, as coarse sand and gravel, under and on the windward side.

The following are examples of the more modern breakwaters

and piers. They have been specially selected as showing the more recent constructions and designs, and particularly in regard to the use of concrete in large blocks.

1106. *Bilbao Breakwaters, Spain.*—One of these breakwaters is 4760 and the other is 3516 feet long. They reach out from opposite shores near the head of the bay, nearly perpendicularly to their respective shore-lines, and are so constructed with respect to each other that while the space or entrance between them is 2100 feet long, the entrance is perpendicular to the direction of the prevalent winds, the angle formed by the directions of the breakwater being such as would bring about this condition. The enclosed space of harbor thus formed covers an area of 700 acres. Over a very large portion of this the depth of the water at spring-tides will vary from 16 to 49 feet. Quays are to be constructed on enclosed shore-lines, provided with railway tracks, at which large ships will be able to load and unload at all stages of the tide and in all kinds of weather. The general construction of the breakwater is shown in Fig. 407. The foundation is formed of a

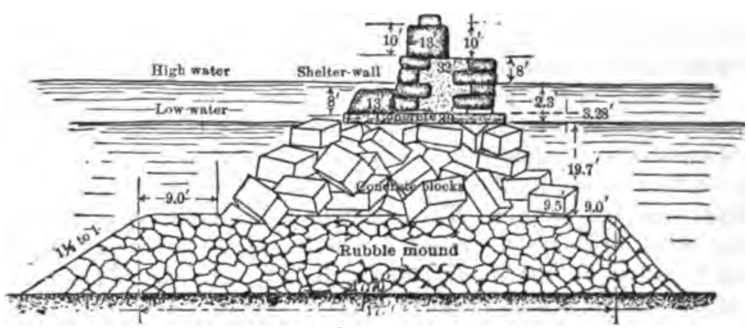


Fig. 407.

rubble mound up to a level of 19.7 feet below low water. The width of this mound at the crest is 177 feet, and side slopes are 1½ to 1. The outer slopes are formed of blocks weighing not less than 1 ton, within these slope-walls, and adjacent to them a wall made of stones weighing not less than 880 pounds, and the interior space or core is formed of smaller stones having a minimum weight of 44 pounds. The top of the mound is protected by concrete blocks measuring from 39 to 65 cubic yards, and weighing from 60 to 100 tons, the larger blocks being placed seaward.

The voids between these blocks are filled with rubble. The top surface is then levelled with a layer of concrete 3.28 feet thick. On this foundation the superstructure is erected. The top of the concrete is 3.28, above low water. The superstructure is built up from this level to a level 23 above low water, with concrete blocks weighing 10 tons each, laid so as to break joints; the interior is filled with quick-setting cement. The dimensions of this portion of the structure are 39 feet thick at the base and 32 feet at the crest. Above this is a shelter wall 10 feet high and 13 feet thick, crowned by a parapet nearly 5 feet thick and 3.28 feet high. Both the shelter wall and parapet are made of concrete moulded in place. The base of the superstructure is protected by a concrete toe 13 feet broad and 8 feet thick. The proportions used for the 60 and 100 ton blocks of concrete were 1 Portland cement, 3 shore sand, and 6 broken stone; for the 10-ton facing-blocks 1 Portland cement, 3 sand, and 5 broken stone; for the levelling course of concrete, 3 parts quick-setting Zumaya cement, 4 sand, and 9 broken stone.

The breakwaters will terminate in a circular pier-head, 65.6 feet in diameter at high-water level, on which will be placed light-houses. The cost of this work will be $4,100,000.

The large blocks of concrete, 39 to 65 cubic yards each, were formed by using the Carey concrete-mixer. The broken stone, sand, and cement were placed in the mixer and mixed dry, by means of an endless screw, in the upper part of the drum, the water being added near the bottom. The concrete when mixed was emptied into small wagons and carried to the moulds. It was then lifted and deposited in the moulds and well rammed. Each block was completed in one day, in order to avoid seams or planes of weakness. The moulds were removed at the end of four or five days, but the blocks were allowed to harden for three months before further handling. They were then lifted by means of machinery worked by electricity. Steel eye-bolts were imbedded in the concrete to enable them to be more readily handled.

An interesting and somewhat novel construction of a breakwater and connecting quays in the harbor of La Guaira has recently been completed.

The works consist of a breakwater 2050 feet long, its inner side formed into a quay 1608 feet in length, and quays running perpendicular to these and then in a parallel direction, having an aggregate length of 3150 feet. The first work consisted of a wall 443 feet

long to the end of the breakwater proper; this portion was to have been built of blocks of concrete, weighing 12 tons, in jute sacks, forming a retaining-wall, subsequently to be backed up with a loose filling to a height of 6 feet 6 inches above mean low tide. When within 8 to 10 feet of the surface these blocks were frequently swept bodily into deep water, ripping the sacks and scattering the concrete. It was found necessary to deposit broken stone to form a slope or toe on the sea front. The original contract called for a top width of 26 feet of concrete.

A subsequent storm destroyed a good portion of the work completed at that time. It was then determined to increase the top width and to carry the structure to a height of 12 feet above water-level. According to final plans, the area of water directly sheltered was 60 acres; the breakwater to be run into a depth of 46 feet of water, and having three jetties, respectively 180, 220, and 680 feet in length; and some other modifications in length and positions of quays were made.

The construction of the breakwater was to be made of large blocks of concrete in sacks, and capped with concrete in mass. These blocks weighed as much as 160 tons. The lengths of the lower tiers of sacks were 48 feet; these when deposited stretched to 54 feet. The next series of blocks, which brought the structure to within 8 or 10 feet of water-level, were 40 feet in length, stretching to 46 feet. These were deposited from hopper-barges. Blocks of 70 tons in weight were used to complete work to water surface; and were deposited from large tippers running on six lines of rails.

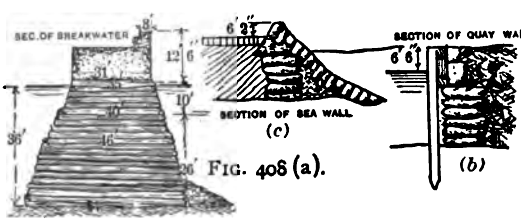


FIG. 408 (a).

These blocks were 32 feet in length, stretching to 35 feet, having a base of about 35 square feet, and extended to about 3 feet above water-level. Upon the top of this concrete in mass was used to a height of 12 feet above water-level, finishing 31 feet in width. This mass concrete was bonded into that below in a shaft or trench cut into it. The concrete cap was built in sections of 40 feet in length, each requiring from 1¾ to 2 days to complete. Along the

quays and berths for vessels timber fenders were placed at intervals not exceeding 8 feet.

English Portland cement was used, the requirements being a tensile strength of 350 lbs. per square inch, and a fineness not exceeding 10 per cent residue on a No. 50 sieve (2500 meshes per square inch). The concrete was 1 cement, 8 ballast; the average was 1 to 6, based on total quantity of cement used, namely, 150,000 barrels. The concrete was mixed on shore and delivered in wagons, which were run by a locomotive to the end of the breakwater. The sack blocks lay with great accuracy, and the large ones did not roll into deep water to any great extent. Fig. 408 (*a*) shows clearly the construction of the breakwater; (*b*) a section of one of the quays; and (*c*) a section of the sea-wall at the shore end of the breakwater.

Breakwater at Copenhagen, Denmark.—A novel construction for a breakwater is shown in Figs. 409 (*a*) and (*b*). The details of construction, form of concrete blocks, etc., are shown in figures (*c*), (*d*), (*e*), and (*f*). There are no tides at Copenhagen, and the variation in waterl-evel above and below the mean, under the strongest winds, is only about 35 inches.

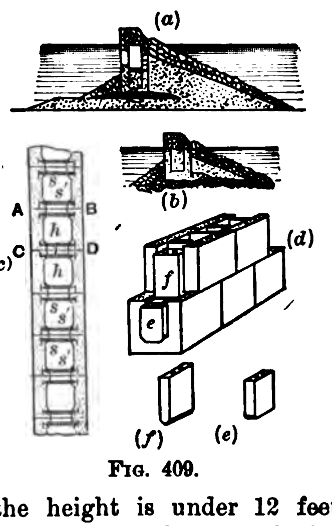


FIG. 409.

The entrances to the two basins are, respectively, 134.5 and 361 feet, and these are protected by breakwaters. A 300-foot channel is maintained by dredging. The quay-walls in the basins are partly of wood; some of granite, or concrete with granite facing, on pile foundations where necessary.

The breakwater is now under construction; the submerged portions are built of concrete blocks. Where the height is under 12 feet up to mean water-level only one course of blocks is used, above this up to 25 feet in height two courses are used, as indicated in Figs. 409 (*b*) and (*a*), respectively. The concrete blocks rest on a bed of broken stone of 2 feet in thickness, the upper surface being levelled off by divers to receive the blocks. In figure (*a*) the broken stone is placed on a bed of gravel, filled in with the dredged material, about 4½ feet thick.

Above the concrete blocks a superstructure wall is composed of quarry-faced granite blocks. It is 6.16 feet thick at top; the crest line is 6.16 feet above mean water-level. The gravel slope in front of the wall is paved with rough granite stones as indicated, placed on a layer of broken stones from 1 to 2 feet in thickness. The slope of the paving is 2 to 1. On the inside at the foot of the wall a footing of riprap is placed. The general construction is clearly shown in Figs. 409 (*a*) and (*b*).

The blocks of concrete for the lower courses are 10¼ feet high, 10¼ feet thick, and 8¼ feet measured along the wall. They are built hollow, as shown in figure (*c*). Each block is of the outside dimensions as given above. The hollow space is 5.65 wide between the side walls, which are respectively 31 inches and 24 to 28 inches. The bottom, of concrete, is 18 inches in thickness, joining the side walls. These blocks are shown in the bottom course figure (*d*). Each block weighs 39.13 tons.

The side walls were tied together by two partitions to each block. These partitions are 2⅜ inches thick (see *s* and *s'*, figure (*c*)), and are composed of a network of iron rods or wires, the vertical ones 0.22 inch in diameter, and the horizontal ones in pairs 0.28 and 0.41 inch in diameter alternately. The rods are spaced 2¾ inches centre to centre, and tied with wire at their intersections. The wires are built into the side walls, as shown at *s* and *s'*. Thus when the blocks are placed end to end, instead of a long hollow trough, there is formed by the monier partitions alternately large and small pockets, as shown at *h* and *s'*. The wire partition is filled and covered with cement mortar, 1 cement, 3 sand, and allowed to set for a week or more before the partitions are put in place. The blocks of concrete are moulded between frames or moulds, the interior frames having no bottom so that the sides and bottoms can be moulded together, the inner frames being supported on sticks or rods, which extending through the concrete mass forming the bottom, will leave holes when removed, into which 2-inch steel bolts are inserted by which to lift the blocks.

These bolts bear against wooden strips moulded into the under surface of the blocks. The bottom of the blocks and the front wall up to 3 feet below mean water-level are of concrete, 1 cement, 4 sand, and 7 broken stone. The remainder of the concrete is 1.3 and 6 respectively. The concrete blocks are faced near their tops with granite set in cement mortar, 1 cement to 2 sand, that is, where

only one course of blocks is used, or in the upper course where two are required.

The large hollow spaces *h* are filled with sand, which increases the weight of each block to 53.8 tons. The joints between the concrete blocks in each course are between adjoining partitions of adjacent blocks. A bond is made at the joints by inserting the key-blocks (*e*) of concrete, as indicated in figure (*d*). This key-block weighs 4 tons, and is about 2 feet thick, and moulded to fit between the wire and mortar partitions.

The blocks of the upper course are similar to those of the lower course. There is no bottom between the partitions and ends of the block, they therefore weigh a little less, the weight of each being 34.2 tons. These blocks have the same length and width as for the lower course, but are only 9.83 feet high. They are set so as to break joints with those of the lower course. They are keyed together and bondnd to the lower course by concrete blocks (*f*), which extend through the upper course and to about 18 inches into the lower course, as shown at (*f*) in figure (*d*). Those keys weigh about 7.3 tons.

The drawings clearly indicate the shape of blocks, keys, etc., and method of construction.

The New Dock-wall, Oakland, California.—In Fig. 410 is shown a simple construction for a dock-wall, which consists essentially of three rows of piles, anchored back by horizontal timbers to a

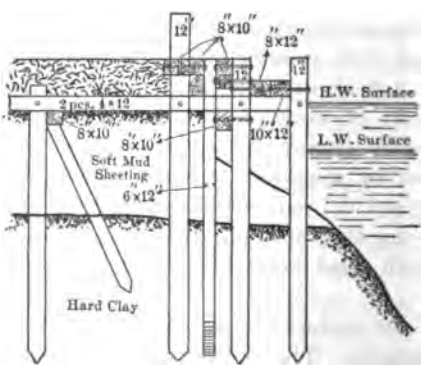


FIG. 410.

row of land-tie piles, supporting a string-piece or wale near the top, and by a strut brace-pile. The main piles are of the usual dimensions. All wale-pieces are 8 × 10 or 10 × 12 inches. The course

of sheet-pile is made of 6 × 12 inch timbers. The novel features are: (1) filling in behind the pile bulkhead with the soft mud dredged from the front to a depth of 23 feet below tide, and (2) the means of protecting the piles from the attacks of the teredo. Creosoting has been found to be unsatisfactory. In this work paraffine has been adopted; every stick is coated with paraffine on the parts of its surface between mud and low-water lines. These portions of the piles are then covered by strips of burlap soaked in paraffine, and held in place with galvanized nails; finally another coat of paraffine is applied, and strips of wood, soaked also in paraffine, are nailed over the burlap to protect it from abrasion. This work is now under construction.

The foregoing sections of breakwaters, piers, and quays have been selected as typical structures, fully illustrating the many designs that have been used, and suggesting such modifications as may be required by local conditions and special purposes of construction.

ART. LVII.

SEA-COAST DEFENCES—RECLAMATION OF LAND.

1107. While structures known as sea-coast defences may not have exactly the same object in view as those for the reclamation of land, yet they are similar, and the general character of the works are the same for both. They will therefore be considered in the same connection.

Sea-coast defences have for their main objects the protection of the coast from the undermining action of the sea, and preventing the continuous changes in the configuration of the coast-line, as well as enabling the utilization of certain portions of the land rendered useless by the incursion of the sea in high tides and storms.

It may be stated generally, that all coast-lines are in a continual state of change from the action of the sea. The general effect is to wear down promontories and to fill up bays and estuaries. The opposite effect is sometimes observed under the action of special currents, combined with small wave-power.

These materials after being ground down into small fragments and sand are drifted and transported by the sea currents and deposited in various localities, where the currents are checked either by natural or artificial obstructions, or by meeting with other oceanic or river currents. At such places these suspended materials are

deposited in the order of their sizes and specific gravity, resulting in the formation of sand banks, bars, etc. The result is the retreat of the land in some places and advance seawards in others.

1108. Coast currents are caused by differences in temperature, which are greatly lessened near the land. The main cause of strong coast currants, such as move sand and gravel and shingle, is the wind. Such currents therefore change with the direction of the winds, but are stronger in the direction of the prevailing winds, so called. Coast currents thus formed often have a velocity of as much as 6 feet per second, and extend to a depth of 30 feet.

When the wind blows nearly perpendicularly on a sandy coast, it drives the sand onward, which, when stopped from any cause, forms into sand-hills or regular chains called *dunes*. If an oblique wind blows on such dunes, it erodes the seaward face, carrying a portion of the sand inland and another portion along the face; if there are openings in the front dunes the sand rushes through them, resulting in the formation of other dunes inland. In this manner are formed sandy wastes of great extent. This effect can best be prevented by planting the dunes with vegetation, or by careful protection of them with mattresses or other means already described for preventing erosion, scour, or caving, or by the construction of works such as groins, which are run out at right angles to the coast; these not only interrupt the travelling of the materials of the beach along the shore caused by oblique waves, but also cause a permanent deposit of such materials, and if gradually extended seaward in shallow water result in a gain of ground from the sea.

After the spaces between the groins are filled the travelling of sand and shingles goes on past their ends as before. They can only be of temporary advantage, especially when not distributed over a long line of the coast.

Sea-walls either with long slopes or with vertical or slightly sloping faces have been described and discussed in the preceding article.

A sea-wall is sometimes made of earth, and is given a long flat slope seaward, this slope varying from 3 to 1 to 12 to 1. The top is usually flat and wide enough for a carriageway and foot-walks. Its top should not be less than about 6 feet above high-water surface of spring-tides, and generally should be above the reach of the waves. The inner slope is steeper than the outer, and varies from

$1\frac{1}{2}$ to 1 to 3 to 1. The seaward slope should be paved with stone, fascines, or mattress-work. The inner slope should also be paved with stone if the waves break over the top of the dike, otherwise it may be sodded. It is common to build a hearting formed of layers of fascines, forming a rectangular wall, over and atound which the earth is placed and packed.

In constructing a sea-wall along a sinuous coast the wall may follow the coast-line along or near the high-water line. It will, however, be often found advantageous to carry the wall straight across, for although greater depths of water will be found and consequent higher and more stable walls required to withstand the heavier surf, there will be a saving in the length, and portions of the wall presenting concave surfaces in horizontal directions will be avoided, also preventing thereby the concentration and destructive action of wave-force.

1109. *Reclamation of Land.*—The reclamation of dry, arid, and waste lands by means of irrigation has been fully discussed under the head of Irrigation. The present reference is made to the reclamation of lands where the main object is to keep away the excess of water caused by overflow from rivers and seas, and the removal of water from rainfall on such protected areas and drainage from surrounding highlands.

1110. Large areas of land exist having no present worth, which can be rendered valuable and productive by simple drainage. Often a simple system of underground tile-drains, properly arranged with respect to main and lateral drains, will answer every purpose; and extensive areas are thus treated in many parts of the country.

An intersecting network of small open drains or ditches is often resorted to with good results. Large tracts of what is often called marshy lands can be reclaimed by one or two main canals, into which lead smaller canals or ditches, and into these latter either small open drains or underground tile-drains, each proportioned in number, dimensions, and capacity to the areas served by them. All cases present little if any difficulties, and only require the exercise of good judgment and the expenditure of more or less money.

The reclamation, or rather protection, of land along rivers which annually or oftener overflow their banks and flood the adjacent lands on either side presents no special difficulties when situated above tide-level, and only involves the necessary outlay in the construction and maintenance of embankments or levees along

the banks of the stream of sufficient height and stability to prevent the water flowing over their tops or breaking a passageway or crevasse through them. And the failure thus to protect the lands along many of our rivers, notably the Mississippi (apart from any consideration of the effect of levees in maintaining a navigable channel in the river, and from any considerations as to whether levees raise the bed of the streams and the level of high-water surface), is not due to the system adopted, but mainly arises from the ignorance, and doubtless often from the dishonesty, of those entrusted with their construction, in using either inferior materials or in the improper design of the works as to heights, thickness, and inclination of the slopes of the levees, or from an attempt to make a limited sum of money cover too much ground. That such lands can be protected by properly designed and constructed works is not denied. Along such rivers it is commonly found that the land is higher and more valuable along the river than at distances from it. This sloping of the ground from the rivers towards the interior is favorable for the drainage of the surface water, which is thus carried into low and less valuable land, where it can find some outlet, or where it remains without any special damage until the flood in the river subsides, and is then carried off through the natural channels.

Where such is not the case, it is a matter of no very great expense to cut canals or storage-basins of sufficient capacity to hold the usual quantity of local rainfall during the period of high water, and discharge the same through conduits and sluices at a lower stage of the water in the river.

Such levees usually follow close to the banks of the stream, for the reason that being on the highest ground their construction is less costly, and the further reason that the most valuable land is near to the river. Lands whose surface is below the tide-level, in addition to the usual submergence resulting from floods, are daily overflowed by the rising tide. The method of reclamation is still by means of levees; andunless the lay of the ground is such that the surface water can be stored and freely discharged at low tide, it becomes necessary to remove this water by pumping, which entails an additional and permanent annual expense. For this reason it is recommended, and often resorted to, that the land be raised as much as possible by *warping*, or deposition of sediment from the tidal water, which is accomplished by covering the land to be reclaimed by a system of wattled longitudinal dikes and

transverse groins. When thus raised, or, as is sometimes the case, raised by filling in with material obtained by dredging from the adjacent stream or elsewhere, the land is enclosed with levees or sea-dykes.

Where tributary streams flow through the land to the main stream, it will be necessary to build levees along both of its banks also.

1111. The most difficult conditions for land reclamation exist where the land is well below the high-water sea-level—notably in Holland. In such cases high and expensive sea-walls have to be constructed and maintained, and in addition all drainage-water has to be pumped out. This calls for an extensive system of drainage-canals and storage-basins, an expensive pumping-plant, and an annual expense.

The following gives an idea of the lay of the lands in relation to each other and to the sea-level, with the conditions and systems of drainage adopted in Holland.

The total area which drains into the general bosom of Rhineland is 302,600 acres. Of this only 26,740 acres ($\frac{1}{11}$ of the whole) consists of highland, and this lies within the district of the sand-hills along the coast. Also, 35,689 acres are bosom-lands—called highlands in Holland, although their average level is ten inches below the mean level of the sea. Then there are 231,170 acres called polders, or lowlands, which lie from $3\frac{1}{2}$ to 5 feet below mean sea-level; and, lastly, 9000 acres which constitute the area of the bosom canals and lakes.

The bosom-lands contain the canals and lakes. The water is pumped from the polders or lowlands into the bosom-canals, and is carried by these to the rivers or sea, passing through sluices at the sea ends. The bosom-lands are sometimes flooded, thereby securing additional reservoir space, supplementing that afforded by the bosom canals and lakes. These together form the general bosom of a district. Pumping from the polders or lowlands never ceases, unless the water rises to such a height on the bosom-lands that there is danger of breaking through the banks or levees which separate them from the lowlands. Should these banks give way, all the water stored over the bosom-lands would at once flow over and inundate the polders.

Rather than run this risk the pumps are stopped and the rainfall is allowed to accumulate on the lowlands for a time; consequently the flooding of the polders is of the rarest occurrence. No diminu-

tion in the area of the bosom-lands would be allowed in Holland, as by reducing the storage capacity the whole of the lowlands would be liable to be inundated; whereas now, by the simple expedient of stopping the pumps, this can be prevented, except on rare and unusual occasions. All questions of stopping the pumps and other matters connected with the drainage and protection of the country from overflow is strictly and rigidly regulated by law, and proper boards and officers are appointed to see that the law is strictly enforced and complied with.

1112. *The Reclamation of Haarlem Lake.*—Haarlem Lake, a commune of the Province of North Holland, has an area of 46,000 acres; the portion originally covered by water, forming a lake, was 42,000 acres. The average depth of the lake was about 13.1 feet. As there was no natural outfall by which the drainage of the lake could be effected, it became necessary to resort to pumping to accomplish the object sought.

The first step was to dig a canal around the lake for the reception of the water and the accommodation of the large amount of traffic which had been previously carried on. This canal is 38 miles long, with a depth of 9 feet, with widths varying from 115 to 130 feet, and enclosing an area of 70 square miles. It was estimated that 1,000,000,000 tons of water would have to be raised by mechanical means. An engine having a capacity of 112 tons at each stroke and discharge of 1,000,000 tons in 25½ hours was erected, the cost of machinery, pumps, etc., was about $180,000. Two other engines of equal size and power were subsequently constructed. Work was commenced in the year 1840 A.D.

The pumping commenced in 1848, and the lake was dry by the middle of the year 1852.

The total area recovered from the water, 42,000 acres, was at a cost of about $5,400,000, and was sold for about $3,900,000, leaving an actual expenditure on the part of the government of $1,500,000 nearly—a trifling expenditure compared with the valuable results attained.

1113. The size of Holland has been and is in a state of perpetual increase and diminution. From 1833 to 1877 there were more than 1,000,000 acres reclaimed by impoldering and drainage from inundations which have taken place in some portions of the country only at long intervals varying from 40 to 150 years; but, taking the country in general at periods of 11 years' interval; there has been lost nearly 1,500,000 acres. Between 1850 and 1864 some

680,000 acres have been reclaimed by endiking. There is now a system of dikes or ramparts by means of which this large area has been reclaimed, and behind which the country lies in comparative security. The present coast defences consist: (1) of sand banks or dunes indicating the direction of the ancient lines of dunes; (2) of dikes along the low coasts of sea clay, enclosing also land reclaimed from the sea; and (3) of some highlands which rise high enough above the sea-level to render the employment of dikes unnecessary.

1114. The breadth of the line of dunes along the west and northwest coast is from 600 to 7000 feet; the average height of the dunes is from 50 to 60 feet, though in some cases as much as 195 feet.

As already mentioned, the tendency is to wear away the seaward slope of the dunes, the material being carried or drifted on the landward side, except where this has been prevented by planting vegetation or sand oats. These dunes are practically continuous; in some places they have been replaced by dikes.

The sea-dikes are found along the northern coasts, the coasts of the provinces which border on the Zuyder Zee, and the coast of the islands of Zealand and South Holland, where not protected by dunes. Practically the entire country is protected by dikes or dunes, there being only a few places that are high enough to need no protection.

The earthen dikes are protected from the action of winds and waves by paving the slopes with stones and by piles; and also at the more exposed and dangerous portions by "*zinkstukken*," which are structures made of bulrushes, reeds, and branches, sunk and weighted with stone—something similar to the crib or mattress construction used in jetties. These are sunk in circuits measuring some 400 yards, by means of which the current is to some extent turned aside.

The West Kappel dike is 12,468 feet, or 2.4 miles, long and 23 feet high, with a seaward slope of 300 feet, which is protected by rows of piles and basalt blocks. Its top width is 39 feet, on which is a roadway and also a service railway. There are some 1550 or more miles of sea dikes, requiring an enormous expense for maintenance.

RECLAMATION OF THE POTOMAC FLATS.

1115. The Potomac flats, near Washington, D. C., have increased in area to a considerable extent, and have been rendered a great nuisance by the discharge of a large amount of sewage upon them.

The question was: (1) Should the flats be filled up above the level of overflow, (2) dug out so as to form a large area of deep water, or (3) should they be diked in and pumped out?

It was finally decided to fill the flats to a level about 6 feet above low tide, and protect them from overflow by an embankment, and to construct over a portion of the area to be reclaimed a high grade filling and sluicing ponds, between Long Bridge and Easby's Point. The low grade-filling below Long Bridge was modified subsequently to a fill of 3 feet above the freshet line, along the axis of this area, sloping off to about 6 feet at low tide. For the flushing ponds a single lake or reservoir was substituted.

The total area reclaimed was 625 acres, calling for 12,000,000 cubic yards of material, the greater part of which was to be taken from the bed of the river.

The dredging was done at first in the ordinary way, the material being emptied from the dredge-buckets into bottom-dumping scows, then carried to a receiving-basin, again dredged up and deposited into cars holding 10 cubic yards each, and hauled in train-loads of 10 cars by a locomotive running on trestlework built for the purpose to the final place of deposit. The trestle costs at the rate of 2 cents per cubic yard of material hauled over it, when deposited to a depth of 6 feet. It was attempted to spread the material over the flats after being dumped from the cars by means of a Worthington force-pump, the water being forced through a 4-inch wrought-iron pipe with hose and nozzle attached. This did not prove satisfactory. The first lift of the material was from a depth of 18 feet below tide to 8 feet above. After depositing into the receiving-basin, a lift of 28 feet was required to place the material in the cars, which, adding the grade on the trestle, required an aggregate lift of 60 feet, whereas the required effective lift only averaged about 21 feet. The price, including profits, was 21.2 cents per cubic yard, scow measurement. The protection embankments were built at the same time. They were econstructed in water from 2 to 4 feet deep at low tide. A light-draught clam-shell dredge was first used to cut a trench along the line of the bank or levee, depositing the material on the proper site. Stone was then thrown into the trench, forming a ridge 9 feet high, with slopes of 1 to 1. The trench was then widened and deepened, the material being carried in chutes resting on a scow, or a dredge with a long boom was used. With the chutes the material, softened by handling and aided with a jet of water, would run 200 to 300 feet beyond its end from an embankment with a flat slope.

After the first cut was made with the ordinary dredge an endless chain dredge was used, by which the material was lifted to a height of about 12 feet and dropped into a hopper, from which it was fed to a conveyor and carried to the place of deposit. The conveyor was a line of little cars forming the links of an endless chain, supported on a frame built on scows. By this method the embankments were raised 12 to 15 feet above low tide.

This work extended over a period of seven or eight years, about $8\frac{1}{2}$ miles being built. The material was mainly soft alluvium with some sand. Where the material was very soft, time had to be given for it to harden, causing the progress to be slow.

The stone in the trench formed a footing for the embankment, and a foundation for a dry wall constructed above. This wall was built 6 feet above low water, with the back vertical; it was 5 feet thick at base and 3 feet at top. The cost was $1.70 for the stone and $1.45 for the labor per cubic yard, or a total of $2.80 per linear foot of wall.

The hydraulic dredges used are described below. These materially reduced the cost of lifting and spreading the material.

1116. *Hydraulic Dredging.*—In this method of dredging the material is excavated from the bed of a river or other body of water and deposited on shore with one operation.

Mounted on a scow is a large rotary pump, from which is led an iron suction pipe with flexible joint to the bottom of the stream. That part leading under the water is capable of being moved about a centre, the end describing an arc of a circle. The discharge pipe is made in sections connected with flexible joints and carried to the shore on pontoon supports. Rubber joints are expensive, but are much more durable than canvas or other cheap material. Under favorable circumstances 30 to 40 per cent of the volume pumped would be solid matter, but such large quantities did not give good results. With engines making 125 revolutions per minute, the material was more easily handled when containing only 10 to 20 per cent. See also supplement.

Working in soft materials, one dredge averaged 350 cubic yards per hour for days, the length of the discharge-pipe being 3000 feet, and height above water-level 6 to 12 feet. On one occasion 800 cubic yards per hour were deposited at a height of 10 feet above mean low tide and at a distance of 1200 feet. Clay in passing through the pipes would be formed into balls about 5 inches in diameter, and bowlders as large as a man's head have been forced through. A certain amount of sand and gravel could be handled

when mixed with mud or clay; but pure clay could not be handled with advantage, as it cut away the shell of the pumps very rapidly. In sand the discharge-pipes were also apt to fill up so as to largely reduce their capacity, sometimes entirely choking up the pipes. Under ordinary circumstances the velocity of discharge of water was 10 to 15 feet per second.

By this method 5,000,000 cubic yards of material were dredged and deposited on the flats at a cost of 12.37 to 15.45 cents per cubic yard. With the Riker pump the cost was from 13 to 14.5 cents per cubic yard.

Converting place measurement into scow measurement on the basis of 20 per cent excess, the average price for the whole work, 9,437,523 cubic yards, would be 11½ cents, including contractors' profit, or a little more than one half the cost of the same work done by means of scows and railroads on trestles. The filling of marshes in proximity to cities will often pay the cost from the enhanced value of the land created, in addition to the benefits derived from improved sanitary conditions.

With a moderate quantity of sand the settlement of the material after being deposited was small; but with pure mud the settlement was from 2 to 3 feet in three years, and 1½ feet additional in seven years.

See an article in *Engineering Record*, Dec. 16, 1893.

ART. LVIII.

CANALS.

1117. CANALS may be classed under the following heads: (1) Navigable Canals, (2) Irrigation Canals, and (3) Drainage Canals.

Navigable canals may be divided into two classes: (1) Ordinary or Boat Canals, and (2) Ship Canals.

1118. *Ordinary or Boat Canals.*—Small canals of great length, and calling forth, perhaps, the highest order of engineering ability, skill, and energy, have been constructed in many countries, and enormous sums of money spent in their construction.

They served a valuable purpose in their time, and in many cases are still doing good and remunerative service. Many have been entirely abandoned, some have been enlarged, or the enlargement is contemplated. The Legislature of New York has recently appropriated $9,000,000 for the improvement and enlargement of the Erie Canal. But it may be said with safety that the construction of new canals of this kind has almost entirely ceased,

and is not likely to be taken up again in the near future, if ever. Such means of carrying on traffic having been almost entirely superseded by railways. It is not, therefore, deemed necessary to discuss the question of small navigable canals, particularly as the general principles involved, the character of construction, and the difficulties encountered and to be overcome are similar in every respect in the case of large ship-canals, to which subject more than the usual discussion will be given, as this is a living and important question and one likely to be developed to a very great extent. And in the main, except in the magnitude of the works required, what will be said will be equally applicable to small canals.

1119. *Ship-canals.*—The construction of ship-canals, though confined to a comparatively recent period, is by no means a new question, as more than one ship-canal was contemplated, surveyed, and even partly constructed, thousands of years ago. The more important works of the kind have, however, been commenced and completed in the past century, mainly in the last thirty or forty years.

While ship-canals are generally considered as large canals cutting across some narrow neck of land in order to afford a short passage from one sea to another, or between two portions of the same sea, thereby shortening by many thousands of miles the distance by water between important commercial centres, and in avoiding long and dangerous trips around continents, as the name implies, they are equally applicable to inland navigation, to and from the sea, recent examples of which will be given.

For convenience ship-canals will be considered under three classes:

(1) Canals which, in passing from sea to sea, or from a sea to some inland point of importance, have to traverse high districts requiring the construction and use of locks to overcome differences of elevation;

(2) Canals, commonly called sea-level canals, which only have to traverse low-lying districts having a uniform water-level from end to end—having storm or tidal locks only at the ends, which defend the canal from waves and storms, and retain the water in the canal at low-tide level;

(3) Canals, without locks even at the ends, which communicate freely with the sea, from which its water-supply is derived.

Examples of the first class are found in the Languedoc Canal in France; the Caledonian Canal in Scotland; the proposed and partly constructed Panama Canal, Central America, under the last plan of construction; and the Nicaragua Canal, Central America, which last

canal has been carefully surveyed, and a considerable sum of money, \$6,000,000 to \$8,000,000, has been spent upon its construction, and is now the one canal for the early construction of which earnest efforts are being made to raise the requisite funds; and the recently completed Manchester Canal, connecting the city of Manchester with the Atlantic Ocean.

Of the second class: the canals of Holland; the proposed Panama Canal under one plan of construction, and several other proposed routes connecting the Caribbean Sea and the Pacific Ocean across the narrow neck of land connecting Central and South America; and the North Sea and Baltic Sea Canal, Germany, recently completed.

Of the third class, the more notable examples are the already constructed Suez Canal, connecting the Mediterranean Sea and the Red Sea across the Isthmus of Suez, and the Corinth Canal, Greece.

1120. *The Caledonian Canal.*—The location of this canal was selected on account of a chain of fresh-water lakes which offered great facilities for inland navigation. The total distance across is about 60 miles, of which the lakes constitute about 37 miles, leaving 23 miles as the length of canal required. The canal was to be 120 feet wide at top-water surface, 50 feet at bottom, and 20 feet deep. The locks are 170 feet long and 40 feet wide. The general lift at each lock is about 8 feet. There are about 25 locks used in the ascent and descent together. Vessels 160 feet long, 38 feet beam, and 17 feet draught can be accommodated. The work was completed in 1823, and at a cost of about \$5,000,000.

The Manchester Ship-canal, England.—Table of distances and water-levels:

Section.	Distances from Eastham Locks, miles.	Length of Sections, miles.	Rise in Locks.	
			Lift at each Lock.	Above Mean Tide in Mersey Estuary, feet.
Eastham............	21	21		9.5
Latchford...........	$28\frac{1}{2}$	$7\frac{1}{2}$	16.5	26.0
Irlam...............	$30\frac{1}{2}$	2	16.0	42.0
Barton....	$33\frac{3}{4}$	$3\frac{1}{4}$	15.0	57.0
Mode Wheel........	$35\frac{1}{2}$	$1\frac{3}{4}$	13.0	70.0

Minimum depth of canal = 26 feet.

From Manchester to Barton, bottom width 170 feet; from Barton to Eastham, bottom width 120 feet, top width 172 feet.

Dimensions of three tidal locks, placed side by side at the Eastham entrance in the Mersey River: One 600 feet long by 80 feet wide; one 350 feet long by 50 feet wide; one 150 feet long by 30 feet wide.

Similar locks are placed at other points, except that they are without storm gates.

This canal is crossed by a number of swing-bridges. These have generally a long and short arm, 148 and 111 feet respectively. The swing-bridge carrying the Bridgewater Canal over the ship-canal is the largest and most novel in construction. This is a trough built of $\frac{3}{8}$-inch plates and angles; extreme length of main girders, $234\frac{1}{2}$ feet; width, centre to centre of girders, $22\frac{1}{4}$ feet; the depth at the middle is 33 feet, tapering to $28\frac{3}{4}$ feet at the ends. The tank is 19 feet wide and 7 feet deep to the timber fender rails.

This aqueduct is always swung full. This is accomplished by iron gates on the bridge and at the shore ends of the canal. The water-tight joint on the bridge is made by a steel V piece closing on a flat steel bar. To make a joint between the bridge and the canal a U-shaped wedge, 2 feet wide and corresponding to the section of the canal, is used. To open the swing-bridge the gates on the canal and bridge are closed. These leave a short section between the gates on the bridge and canal. From these sections the water is allowed to escape. The U-shaped wedge, which weighs 12 tons, is then lifted clear from the tapering seat upon which it rests by means of four hydraulic jacks; the clearance thus afforded allows the bridge to swing.

The total amount of excavation required was 53,500,000 cubic yards. Of this 12,000,000 was rock of various degrees of hardness, mainly removed by regular excavation, but partly by dredging. The remaining softer material was partly excavated and partly dredged. The rate of excavation varied from 750,000 to 1,250,000 cubic yards per month.

All quays and walls were built of brick or concrete. The total brickwork contains 175,000 cubic yards, and concrete 1,250,000 cubic yards; in addition 220,000 cubic yards of masonry in piers and abutments.

Six years were consumed in the construction. The total cost has been given at from $65,000,000 to $80,000,000.

The two just described are good examples of canals of the first class.

Considering the want of information and experience derived from studying the construction of similar works, the difficulties of rendering available the Highland lakes, and the works surmounting the summit level, the Caledonian Canal is a monument to the skill and originality of Telford, under whose supervision the canal was designed and constructed.

The main purpose in constructing this canal was the saving of 400 miles along the coast and avoiding the stormy passage through the Pentland Frith.

The purpose in constructing the Manchester Ship Canal was avowedly in the interest of the commerce of the city of Manchester, and was constructed not to save distance or provide means of transportation, which already existed by rail, but to give this city the same facilities, as near as practicable, of a seaport, and on account of the high rates charged by the railways, which discriminated in favor of the rival and more favorably situated city of Liverpool.

The design and general construction of locks and canal excavation and embankments are the same for all canals, and will be described and illustrated in another paragraph.

Canals of the second class, as those traversing low-lying districts, or combined with open excavation or tunnels through high dividing ridges, will be illustrated by the following examples.

1121. Some of the best examples of canals of this class are found in Holland, of which the North Holland Canal is a good example. In this case the difficulties of constructing the canal are not due, as in the case of the Panama or Caledonian Canal, to protecting the works from assaults of mountain and river torrents, but rather from those of waves, as the level of the canals is below the sea, and vessels have to be locked down from the sea into the canal, or up from the canal to the sea-level. The length of this canal is 50 miles. The cross-section is a simple trapezoidal form, with a width at water surface of $123\frac{1}{2}$ feet, at bottom of 31 feet, and having a depth of $18\frac{1}{2}$ feet, with uniform side slopes of 2.5 to 1.

A more recently constructed canal in Holland is that which connects the city of Amsterdam, on Lake Y, with the North Sea, rendered necessary by the largely increased traffic requiring better communication with the North Sea than was afforded by the North Holland Canal. The length of the canal is only $16\frac{1}{2}$ miles. Fig. 411 shows the location and profile along the waterway, also the

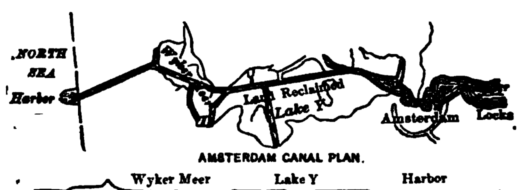


FIG. 411.

piers forming the harbor at the North Sea end, and the dam across Lake Y at the Amsterdam end.

The piers for the harbor are built of blocks of concrete founded on a bed of basalt. These piers are 5069 feet in length each, and enclose an area of 260 acres. Over about 140 acres of this a depth of 26½ feet will be obtained by dredging, leaving the remaining portion of its natural depth. From the harbor a deep excavation was made through the sand-hills which protect this portion of the coast. The cross-section of this portion of the canal is as shown in Fig. 412. This excavation is about 3 miles in length; the maximum depth from the surface to the bed of the canal is 78 feet, and

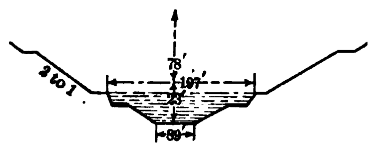


FIG. 412.

the amount of material excavated 6,213,000 cubic yards. It then passes into the Wyker Meer, a large tract of tide-covered land. Then follows another excavation of 327,000 cubic yards through the dividing ridge between Wyker Meer and Lake Y, another tide-covered area; thence through Lake Y as far as Amsterdam. The excavated material was used to form two embankments parallel to each other and 443 feet apart across the tide-covered areas and for the banks of branch canals, as indicated in Fig. 413. The total length of these banks is 38½ miles. These banks are formed mainly of sand, and faced with clay. During the construction the banks were protected with fascines, and when advanced the proper depth and width of channel is obtained by dredging.

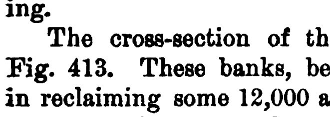


FIG. 413.

The cross-section of these portions of the canal is shown in Fig. 413. These banks, besides forming simple canal-banks, aid in reclaiming some 12,000 acres from the lakes. The water is removed and kept from these reclaimed areas by pumping. In addition pumping is required, when necessary, in order to maintain the water-level in the canal at about 1 foot 7 inches below average high-water level—a requirement imposed by law. The ordinary lift in pumping is only 3½ feet, whereas the maximum is 9¾ feet.

There are at each end a set of locks. At the North Sea end the locks are placed at a distance of three quarters of a mile from the harbor. This set of locks is shown in Fig. 414, and consists of three locks. A similar system was described in case of the Manchester Canal. The central or main lock is 390 feet long and 60

feet wide. This is provided with two pairs of gates at each end, pointing in opposite directions, and one pair in the centre. The locks on each side are smaller. On one side, for the passage of barges, is a lock 30 feet long and 34 feet wide, with three pairs of gates, and on the other is a lock 227 feet long and 40 feet wide, with five pairs of gates.

The arrangement and details are clearly shown in the drawing.

At about two miles eastward of Amsterdam it was necessary to

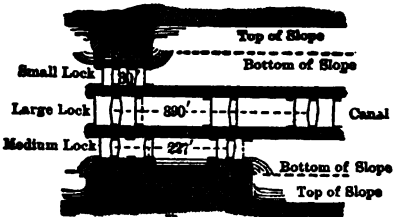
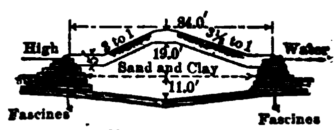


FIG. 414.
PLAN OF LOCKS IN SETS OF THREE
AMSTERDAM CANAL



SECTION OF DAM
ACROSS LAKE Y
FIG. 415.

build a dam across Lake Y, at a narrow of 4265 feet in width, with a set of locks. It was necessary to build the locks before completing the dam. For this purpose a large circular coffer-dam 590 feet in diameter was constructed with a double row of piles, within which the locks were built. These locks have three large or main passages, each with five pairs of gates, and one smaller passage with three pairs of gates, arranged in a similar manner to those at the North Sea end.

The entire masonry-work was founded on piles. The dam across Lake Y consists of clay and sand, placed on and protected at the sides by large masses of wickerwork or mattresses, loaded and covered by basalt. A cross-section of the dam is shown in Fig. 415.

All of the lock-gates at both ends of the canal pointing seawards are of malleable iron; those pointing inward toward the canal are of wood.

These locks were required for the purpose of effecting drainage of the land, and of maintaining the water-level in the canal below the sea-level, the sea-level being at high water several feet above the level of the canal. The contract price for constructing this canal was $11,250,000.

Of this class of canals are several of the many schemes for constructing a canal across the Isthmus of Darien—not because the entire country between the two oceans was low-lying ground, as there is the high range of the Cordilleras Mountains between the two, through which it would be necessary to cut a tunnel having sufficient clearance in height and width to pass freely the largest vessels with their high masts, but to avoid the use of locks, which are a great hindrance to navigation, accompanied as they are with great loss of time.

It may safely be said, however, that a sea-level canal with only storm locks and gates at its ends have been shown to be impracticable across this isthmus.

The Corinth Canal.—This canal, very recently completed, has a length of 21,441 feet, top-water surface width of 70.8 feet, bottom width 68.9 feet, and depth of 26½ feet. It saves a distance of 185 nautical miles for vessels from the Adriatic Sea and bound for Constantinople, and 95 miles for those from Mediterranean ports. It obviates the necessity of making the long and dangerous passage around Cape Matapan, and is expected to facilitate greatly the communication between Europe and the East.

It is notable as the canal commenced by Nero, abandoned during eighteen centuries, and finally completed in the last year. This canal has locks only at its ends. It has one rock cut 1.2 miles in length, with a maximum depth of 250 feet. The amount of traffic is estimated at 4,500,000 tons annually, for which one franc is charged from the Adriatic and one-half franc from elsewhere, and one franc for each passenger.

It was constructed by a private company, but becomes the property of the Greek Government on paying 5,000,000 francs at the expiration of a lease for 99 years.

The Nord-Ostsee ship-canal is also of this class. This canal is being constructed between the Baltic and the North Seas, through the Schleswig-Holstein territory. It is 61 miles long; bottom width, 72 feet; width at top-water surface, 213 feet; mean depth, 30 feet. It was commenced in 1886, and is now completed.

Its object is to avoid the long and tedious as well as perilous

voyage around the Danish peninsula and through the Skagerrack. It will afford a passage through German territory to the North Sea, and will be consequently of great importance and advantage to the German Government from a military point of view. There will be double tidal locks at each end, but otherwise the canal will be free from locks. There will be six passing-places at intervals of 7½ miles, each 1476 feet in length and 197 feet in width. The cost was estimated at $39,000,000; actual cost, $37,440,000. The allowed speed is 5.3 miles per hour. The toll will be 18 cents per net registered ton, loading capacity. Ships bound to Bremen will save 322 miles, and those to Hamburg 424 miles.

1122. An example of ship-canal of the third class is the Suez Canal. In Fig. 416 is shown the location, alignment, and profile

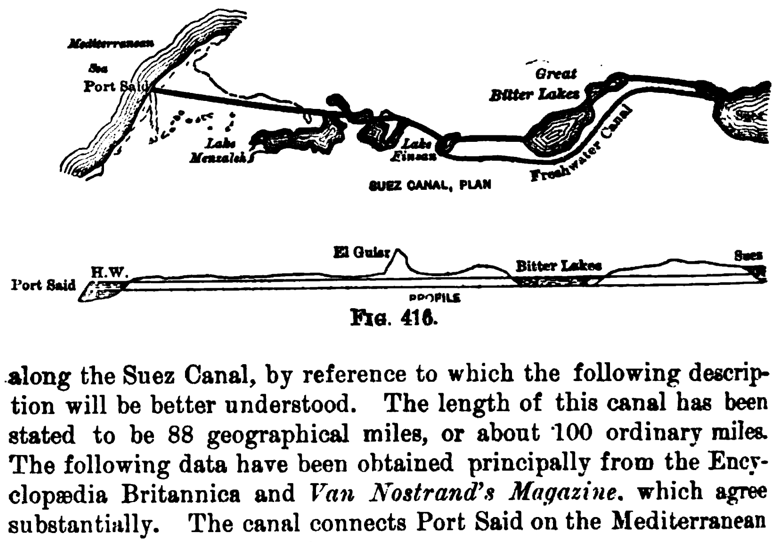


FIG. 416.

along the Suez Canal, by reference to which the following description will be better understood. The length of this canal has been stated to be 88 geographical miles, or about 100 ordinary miles. The following data have been obtained principally from the Encyclopædia Britannica and *Van Nostrand's Magazine*, which agree substantially. The canal connects Port Said on the Mediterranean Sea and Suez on the Red Sea.

This canal belongs to the third class; it has neither locks, gates, nor reservoirs, and no pumping of any kind is required; strictly speaking it is simply an artificial strait, affording a short passage for vessels between two seas, which are practically on the same level, both furnishing the water for the canal, and the rise of tide is small. It reduces the distance, by water transportation, between Europe and India from 11,379 miles to 7628 miles, and causing a saving of 36 days on the voyage.

In the general lay of the country between Port Said and Suez, no mountains or even high hills intervened. The route selected took advantage of certain valleys or depressions, called lakes, but really low-lying tracts of country, at some places below the level of the seas; these were covered with deep deposits of salt. Work was commenced in 1860, and the canal was opened to traffic in 1869.

The ground being low, some portions required no excavation or works of any kind; the natural depth was equal to that required for the canal, the main excavation being only where the sandy dunes attain an elevation of 50 or 60 feet, near El Guisr.

The material excavated was mainly alluvial and easily removed; the only rock was near El Guisr, which was a soft gypsum, removable to a considerable extent by dredging. The excavation through the low-level portions, lakes, and sand-hills was done principally by dredging, though a considerable amount of work was done by hand-labor, under a forced-labor system, the number of men employed at times being from 25,000 to 30,000. As many as 40 dredges, each costing some $200,000, have been worked; these delivered the sand to barges to be carried out to sea and discharged, or piled it up on the banks often to a height of 50 feet. The hand labor was performed by men carrying on their heads baskets filled with sand, and also 1000 or more donkeys at one place walking along in long lines, carrying on their sides mat-baskets filled with sand, these being emptied in banks along the sides of the excavation.

The cross-sections of the canal are similar in every respect to those shown in Fig. 412, for those portions excavated through the sand, and as in Fig. 413, for those across the lakes. The dimensions for the Suez Canal were varied as follows:

(1) Width at water surface, 196 feet; at bottom, 72 feet; depth, 26 feet. The slopes are 2 to 1, with one or more horizontal benches or berms 10 feet wide, according to the depth of the excavation.

(2) Width at water surface, 327 feet; at bottom, 72 feet; depth, 26 feet. The lower part of the excavation had slopes 2 to 1, but the slopes above and below the water are 5 to 1, with a horizontal bench between the steep and flat slopes of 58 feet in width. This arrangement of section is such that the slope both above and below the water surface can be faced with stone and made permanent and secure against wash from waves, while at the same time by diminishing the width of the berm the navigable portion of the canal can be increased without disturbing the flat-paved slopes beyond the berm.

In those portions of the canal having only uniform side slopes

of 2 to 1 from bottom to top, which was adopted to save excavation, no paving can be used until the greatest required width is ascertained, and the widening executed, as all the paved portion would have to be cut away.

There were many predictions to the effect that the canal would be a failure from silting up, and filling up with sand drift from the surrounding sand desert; that the Bitter Lakes would be filled up with salt; that the canal would become a stagnant ditch; that shipping would not risk the dangers of the Port Said approach, nor the dangerous and difficult navigation of the Red Sea; that the Mediterranean entrance could not be kept open, etc. It will be sufficient to say that each and every one of these dire forebodings have failed to be realized.

There is ample current caused by the tides, and the evaporation from the surface causing a tendency to flow from one or both seas; the salt in the lakes has been gradually dissolved; shipping has increased greatly in the number of vessels and their tonnage.

The largest daily evaporation is estimated at 250,000,000 cubic feet of water. This can be readily supplied from either sea, but particularly from the Red Sea (which is nearer the Bitter Lakes, covering 100,000 acres), which has a tidal range of 6 feet in spring-tides, and 2 feet at neap tides, while the tidal range is much less in the Mediterranean Sea. But it has been found that from May to October the winds cause a rise of level at Port Said and a fall at Suez, the difference of level being as much as 15.5 inches, which leads to a current from the Mediterranean to the Red Sea; although interrupted by the tides, a large volume of water flows from north to south. In winter the winds cause a higher level of 12 inches in the Red Sea than in the Mediterranean, which results in a flow from south to north. About 400,000,000 tons of water thus pass yearly from one sea to the other, which, coupled with the tides, tend to supply the loss from evaporation from the surface of the lakes, and dissolve the basis of salt (about 32.8 feet in depth), which is gradually taking place. These local currents vary from 0.5 foot to 1.3 feet per second between Port Said and Lake Timsah, and in the broader portion between Suez and the Bitter Lakes it is from 2 feet to 3.6 feet per second.

In the year 1876 only 52,700 cubic metres of material was removed from the canal, and it was navigated with facility by steamers 400 feet long and drawing as much as 27 feet of water, and with the increase of vegetation along the banks of the canal the quantity of drifting sand collecting in the canal will be greatly diminished.

With respect to the gross tonnage: In 1870 486 vessels passed through the canal; gross tonnage 654,915, receipts \$1,031,865. In 1874 1,264 vessels 2,423,672 tonnage, \$4,971,875 receipts; that is, the tonnage was quadrupled in five years. In 1880 the gross tonnage was 3,446,431 tons, showing a regular increase up to that date. In 1885 M. de Lesseps' report shows a traffic of 3624 vessels of 6,335,753 tons, exceeding that of the preceding year by 340 ships and 464,253 tons. The passengers numbered 205,951 for 1885, against 151,916 in 1884. The average time of transit was 43 hours. Owing to a temporary obstruction caused by a dredger being run down, 123 ships assembled, all of which were passed through in three days.

The estimated total excavation was 78,000,000 cubic metres. In addition to the necessary excavation for the ship-canal, and accessory canals for fresh-water supply, etc., terminal harbors had to be constructed. At Port Said a harbor has been constructed by running out into the sea two piers or breakwaters, built of concrete blocks; the western one extends from the shore 2400 yards in a straight line towards the north, then with a slight angle to the east 330 yards farther. The eastern one extends 2070 yards nearly north; at this point it is 760 yards from the western pier, which gives the width of the entrance to the harbor. It contains about 500 acres of enclosed area. The average depth of water over this area is only about 13 to 14 feet, but the depth of channelway leading into the canal is from 25 to 28 feet. The piers are 1530 yards apart at their shore ends, and converge to 760 yards, as above stated.

Owing to the greatly increased traffic up to the year 1885, a commission was appointed to report upon the best method of enlarging or otherwise improving the capacity of the canal, so as to meet fully the exigencies of a traffic exceeding 10,000,000 tons per annum. The commission considered three methods of increasing the carrying capacity of the canal, namely: (1) Widening the existing canal; (2) Construction of a second canal; (3) Doubling the capacity of the canal by a combination of the first two methods.

With the large vessels of 50 feet or more in width, which propel themselves through the canal, a bottom width of 230 feet has been proposed for the 81 miles from Port Said to the southern end of the Bitter Lakes, where the tidal currents do not exceed 1 knot an hour, and 262 feet for the rest of the distance to Suez, where the currents often exceed 2 knots an hour, in order that the vessels may pass each other freely. The cost of this widening was estimated at

$41,200,000, assuming the depth of the canal to remain at 26¼ feet below low-water of ordinary spring-tides, but an additional cost of $4,876,000 if the depth was increased to 29½ feet, unless a reduction in width of 18 feet was made, or a total of $46,076,000.

The construction of a second canal, similar to the old canal, 72 feet wide at the bottom, widened to 131 feet through the small Bitter Lakes, would cost from $44,500,000 to $48,500,000.

Taking into consideration the greater danger of collisions where the velocity was greater, a third plan was suggested enlarging the canal in the northern portion, and the construction of a second canal from the Bitter Lakes to Suez.

The commission decided upon an enlargement of the canal from sea to sea, as it would enable the speed of passing vessels to be increased from 5½ to 8 knots an hour, which could never be accomplished with two separate canals: and further, there would be only two banks to maintain instead of four. This increase of speed would greatly facilitate the steering, and there would be less danger of vessels stranding on the banks in a wide canal. The danger of collisions could be reduced to minimum by reducing the speed while passing. The widening would ease the curves existing in the old canal, which is an important consideration with the long ships of the present day. And finally, during the construction of widening, each additional enlargement could be at once utilized as passing-places, which would immediately increase the carrying capacity. The completion of such extensive excavations as would be required takes a considerable period of time.

It was also decided to increase the depth to 28 feet, and decrease the bottom widths to 213 and 246 feet, measured at a depth of 26¼ feet below water surface. The widths were to be increased on curves as follows: In the northern portion the width in curves exceeding 8200 feet in radius was to be 246 feet, and 262 feet on sharper curves; while in the southern portion the widths at the curves were to be 262 feet, all curves exceeding 8200 feet in this portion of the canal. All slopes were to have a berm about 6½ feet below water surface, and the slopes from that depth up to a line 3¼ feet above the water surface was to be paved or pitched, in order to protect the slopes from the wash of waves caused by passing steamers.

With the increase of speed admissible in the enlarged canal the normal period of transit, including stoppages, should not exceed 12 hours, as against about 40 in the single-width canal.

The total cost of all the works above described, and in addition a breakwater at Suez and two large basins and a dry-dock at the same place, is stated to have been $100,000,000.

This interesting and in many respects difficult work, constructed in the face of many discouragements and predictions of ultimate failure and financial disaster, is worthy of a careful study, not only as concerns the construction of this work itself, but for the important bearing upon other similar works of the many questions settled. More space cannot be given in this volume to this subject.

INTEROCEANIC CANALS BETWEEN THE ATLANTIC AND PACIFIC OCEANS.

1123. Ever since Nuñez of Balboa, in 1514, crossed the Isthmus of Panama, and thereby discovered that the Atlantic and Pacific Oceans were only separated by a narrow neck of land, determined efforts have been made either to find a natural strait connecting the two oceans, or some short line along which a canal could be constructed at a reasonable cost, or, finally, almost regardless of the cost to construct a waterway between the two oceans; and in contemplation of this latter purpose numerous examinations have been made to determine the best and most practicable route along the narrow strip of land known as Central America connecting North America with South America.

So long as the commerce of the world was carried on in sailing-vessels, propelled by wind-power alone, the first and most important questions, in connection with the selection of the site of the canal, were the character and direction of the prevailing winds, or the extent of calms as to time or duration. So much so, that that eminent officer Capt. M. F. Maury said in a letter written to Capt. Pim, and dated July, 1866, "If Nature by one of her convulsions should rend the continent of America in twain, and make a channel across the Isthmus of Panama or Darien as deep and as wide and as free as the Straits of Dover, it would never become a commercial thoroughfare for sailing-vessels, saving the outward-bound and those that could reach it with leading winds. Steamers would, and coasters might, use it; but homeward-bound vessels in the China, India, or Australian trade, rarely."

These considerations, now that steamers are so much employed in the carrying-trade, and sailing-vessels can enter and leave any

port by the aid of powerful steam-tugs, are no longer worthy of more than incidental discussion, and the question of practicable construction turns almost entirely upon the cost, and the probable revenue to be derived.

Of the many routes examined with the view of constructing a canal or railway, only a few of those which have been viewed in the more favorable light will be mentioned.

(1) The Tehuantepec route has its Pacific terminus in the gulf of that name, and its Atlantic terminus at the mouth of the Coatzocoalcos River. A grant with munificent franchises was obtained from Mexico. This route failed to meet with great favor on account of the difficulties and expenditure required to build a safe harbor for its terminus on the Pacific, and deepening the water on the bar at the mouth of the Coatzacoalcos. Climate, interior resources, and the distance were all in its favor, but it lacked harbors.

(2) The several Nicaraguan routes, though having a longer distance from ocean to ocean, have always been looked upon with more or less favor, and have been no mean competitors of the Panama route, with now vastly more chances in favor of its being constructed than the latter.

In opposition to the Panama Railway, a land route was established across Nicaragua; passengers were carried partly on mules, stage-coaches, river and lake steamboats; and even with these disadvantages, discomforts, and necessary delays, the passenger traffic was divided with Panama. This company was finally bought off by the Panama Company.

Next a railway across this route was projected. Nothing of importance came of this.

Again the construction of a ship-canal has come prominently to the front. Careful surveys and examinations have been made, and in addition a considerable sum of money has been spent in construction; this money was raised by private subscription. The completion of the work will necessarily depend upon the favorable action of the United States Government.

1124. The advantages claimed for the Nicaraguan route are:

(1) Sailing-vessels can better make the ports on the Atlantic and Pacific.

(2) The rainy season is not so long.

(3) The route is more exempt from dampness and disease.

(4) Its climate and soil are well adapted to the cultivation of coffee, sugar, rice, tobacco, indigo, and the like; while in the forests

can be obtained ornamental and dye woods of rare beauty and excellence, and drugs and spices can be gathered.

(5) The country will be rapidly and easily developed, and there will be opened up a large local interior trade.

(6) The harbor accommodations will be superior, and more safe.

"To conclude, you see the sum of all these disadvantages (contrasted with the advantages above mentioned) of the Panama route expressed by the road itself. It has been opened about 12 years (1854–1866), but sailing-vessels go and come by the old routes. Few are the cargoes of merchandise to or from the East that have found their way across that road, and though its earnings are enormous, it has as a commercial highway disappointed the world. It has not altered a single old route of commerce, but it makes enormous dividends, for all that." (See Capt. Maury's letter to Capt. Pim.)

1125. *The Panama Route.*—The demands for some means of rapid and ample transportation between the two oceans led to the construction of a railway across the isthmus, and for economic reasons the line across the State of Panama from Aspinwall or Colon on the Atlantic to Panama on the Pacific was selected. The road was opened in 1854 and has been in full operation ever since. It has been exceedingly remunerative to its owners. Its inefficiency or inadequacy to meet the demands of commerce has been amply demonstrated by the recent attempt to construct a canal close to and approximately parallel to the railway. This route was selected in the face of the many and well-known disadvantages of the route, looked at from almost every point of view.

The disastrous failure to complete the canal is replete with useful lessons to engineers. The enterprise has been abandoned, and but little evidence remains of the work done and the enormous sums of money spent, except the wrecked reputation of many men, some of whom at least merited a better fate.

1126. *The Darien Routes.*—Of these routes the only one that will be mentioned here is what is known as the San Blas route, for which the following advantages are claimed: (1) Except near the mountains, the entire line is nearly level. (2) The distance calling for the actual construction of an artificial channel or canal is only 30 miles.

The following data are given: The tides in the Pacific rise from 12.65 to 22 feet; on the Atlantic the tides are insignificant—only from 1 to 1½ feet. Summit of the Cordilleras is 1500 feet above the

level of the Pacific Ocean. The saving in distance from New York to various ports over the Cape Horn route will be from 8000 to 14,000 miles.

The plan proposed was a canal maintained at the level of high tide in the Pacific. The canal is to be fed entirely from the Pacific. A canal of the usual width and section, with a depth of 25 feet for the greater portion of the distance, and a tunnel of 7 miles in length, 100 feet wide, and 115 feet high through the Cordilleras. The excavation will be mainly through rock, covered a few feet in depth with loam. A tidal lock having a wall elevation of 45¼ feet on the Pacific, a lift lock being required on the Atlantic side to raise vessels to the level of the water in the canal.

1127. The only two routes, however, that have at all met with popular favor are the Panama and Nicaragua. The Panama route has been the favorite with the French and the Nicaraguan one with the Americans. It is not the desire or purpose of the writer to disparage the one or advocate the other. But he will try to give the facts as they have been presented and occurred. The unfortunate condition of the Panama route is such that at present at least there is but little encouragement for even its most ardent advocates. And the results obtained correspond so fully with the predictions made, that a common impression prevails that this route is impracticable. The facts are as follows:

1128. *The Panama Canal.*—M. de Lesseps, the builder of the Suez Canal, by the ability, energy, and skill displayed in the construction of the Suez Canal as a purely sea-level canal, in the face of the most violent opposition, discouragements, and dire predictions of failure, immortalized his name, and so far as he himself is concerned the breath of scandal and slander has been repressed in connection with the facts developed.

He, no doubt, overestimating his own ability, underestimating the difficulties, confiding in friends who ultimately betrayed their trust, and who doubtless misrepresented the facts and conditions, conceived the idea of building a sea-level canal similar in every respect to the Suez Canal, and convinced himself, as he said, "that unless the Atlantic and Pacific can be united by simply piercing the Isthmus from sea to sea without locks, as at the Suez Canal, the proposed scheme cannot possibly succeed as a commercial enterprise, because of the inadequacy of a canal with locks to pass the traffic that will frequent it, and also of the uncertainty of sufficient water to supply the lockage and the evaporation." He easily con-

vinced the French people, whose confidence he possessed to a degree vouchsafed to few, before or since.

It has been openly proclaimed that this gigantic work was undertaken without sufficient and proper surveys and examinations, in ignorance of the conditions existing and of the difficulties to be encountered, and under the belief that a sea-level canal was entirely practicable, and that the cost would be reasonable. The facts, however, were soon discovered to be so far different from those stated, that the original plan had to be radically changed. Instead of a flat sandy country, as between the Mediterranean and Red Seas, in which the rise of tides were insignificant and almost equal, with no torrential streams which would deluge the country and destroy in one night, works costing millions, it was realized that there was a rugged, mountainous country to build over; mountain streams, especially the Chagres River, whose sudden and high floods rendered the construction of the canal impracticable until this stream could be absolutely controlled during periods of floods—a work of gigantic proportions in itself; and that the difference in rise of tide in the two oceans varied from 12 to 20 feet or more, necessitating beyond question both tidal and lift locks; and further, the objection raised to the employment of locks, namely, want of water-supply at the summit, was shown to be of no value, as ample water could be stored to supply all the demands of the canal. An immense mountain barrier had to be pierced with a tunnel, having great width and clear height. The projectors were either ignorant of these things or ignored them. The following is a partial record of what followed.

The route selected was the narrowest width between the two oceans—a distance of about 46 to 50 miles. A cross-section of the canal similar to those already given, having a bottom width of 72 feet, top-water surface of 131 feet, with two berms 6 5 feet each a little below the water surface, and the usual depth of water from 26 to 30 feet. These small dimensions were taken in order to save excavation.

For the purposes of traffic these dimensions were evidently too small. At the water surface 197 was the least that should have been considered, and even with this width, as at Suez, the canal would have to be enlarged at no distant day.

In addition, the character of the material to be excavated, when not rock, was of the most treacherous and uncertain kind. The trenches were liable to be filled up by the soft, flowing material as

fast as they could be excavated. It was impracticable to form any estimate of the amount of the material that would have to be handled, or to maintain any definite widths and depths.

The climate is considered one of the most oppressive and unhealthy in the world.

Enormous sums of money, however, were expended in purchasing the necessary plant, numbers of large and expensive dredges were purchased. Actual work commenced in 1883, and according to the estimates the work was to have been completed in five years, namely, in the year 1888.

In the year 1884 reports showed $104,000,000 cash expended, liabilities $153,000,000. May 1st, 1885, less than one tenth of the excavation was done,—12,376,500 cubic yards out of a total now estimated at from 125,000,000 to 150,000,000; original estimate from 80,000,000 to 100,000,000 cubic yards. No suitable foundation for the Chagres River dam had been found at a depth of 60 feet below the surface. The estimated cost of the completed canal was at this time $600,000,000, original estimate about $120,000,000.

In the year 1887 less than one quarter of the excavation done; cost to date $180,000,000. The work done was mainly on the less expensive portions of the work, and the great Culebra cut hardly commenced. Supposed final cost $800,000,000. The damming or diversion of the Chagres River was not commenced, nor even a plan decided upon.

In the year 1888 the determination was reached to abandon the construction of a sea-level canal, and to pass through the section of greatest excavation by a temporary high-level canal, with a series of huge iron locks at either end. The level of the canal was from 125 to 150 feet above the sea-level. This was recommended as only a temporary expedient to reduce the immediate cost of construction and the time required for completion. Ultimately this high-level canal was to have been lowered, so that the canal would in time become a sea-level canal, as originally intended. There is some doubt and conflict of testimony as to the extent of the work executed on this plan, which is, however, a matter of but little moment to any but those who had subscribed to the stock of the company, as the entire enterprise shortly collapsed. The works have been abandoned, and recent reports state that there is but little evidence of the nature and character of the work done, except a few large dredges and other machinery partly submerged in the old excavations.

1129. In 1889 a pamphlet was published by a French engineer, G. Sautereau, a colaborer with M. de Lesseps in the construction of the Suez Canal, and freely consulted with by him in regard to the Panama Canal. M. Sautereau seems, however, to have favored the Nicaraguan route, and to have condemned in unmeasured terms the proposed construction of a sea-level canal at Panama. What has been said above is more than confirmed in this pamphlet, and in fact no severer criticism has been made of the Panama route by any American than is to be found in the pages of this pamphlet, written by a French engineer.

He, however, advocated for both the construction of the Panama or Nicaraguan route the formation of a great interior lake (see Fig. 416½), forming a long level reach entered from either ocean by

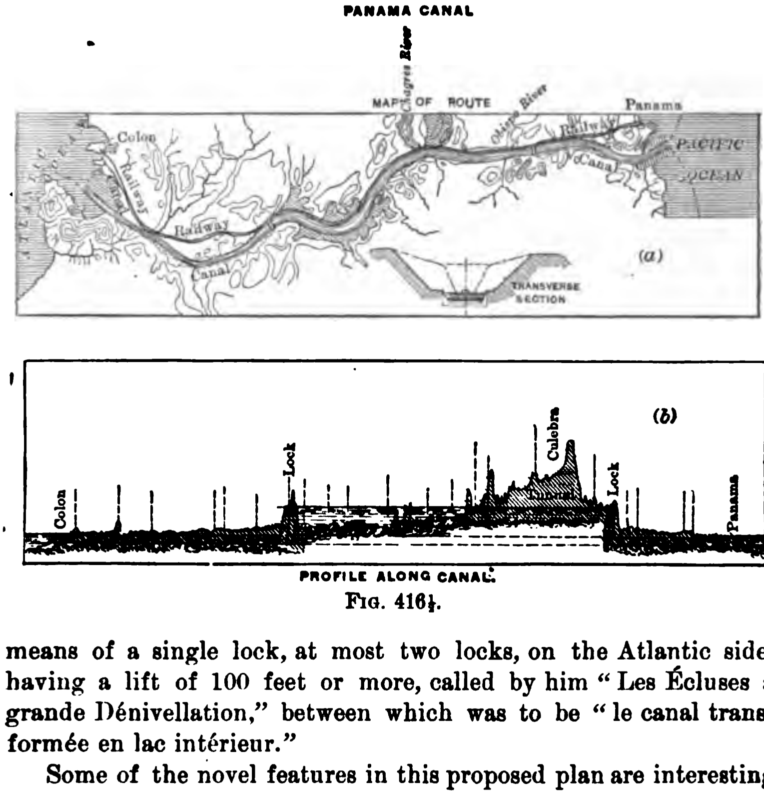


FIG. 416½.

means of a single lock, at most two locks, on the Atlantic side, having a lift of 100 feet or more, called by him "Les Écluses a grande Dénivellation," between which was to be "le canal transformée en lac intérieur."

Some of the novel features in this proposed plan are interesting and instructive, especially as only at a very recent date M. Bartissol

has proposed to revive the construction of the Panama Canal by adopting at least a part of M. Sautereau's idea. What follows is taken from M. Sautereau's pamphlet, entitled "Le Canal de Panama transformé en lac intérieur."

The modified plan of M. de Lesseps consisted in the construction of five locks on each side of the mountain, with level reaches between each two locks. According to the report mentioned, the expenditure had amounted in 1889 to $266,000,000, and to complete the canal as proposed $114,000,000 more was estimated as required, and to carry out the ultimate purpose of lowering the level of the canal to that of the sea $190,000,000 more, and in the end they would find themselves in this position: "with a mass of water retained by a dam without stability [*consistance*], the dam of the Chagres at Gamboa, and which, suspended above a narrow canal in which would be passing or assembled [*engagés*] a number of vessels, would become a perpetual danger to navigation" (translated from the French by the writer).

"It is certain that the proposed project does not constitute an acceptable solution, and that its execution will never respond to the immutable principles laid down by M. Ferdinand de Lesseps. It is not doubtful that, in order to triumph at Panama, as at Suez, over all obstacles, it is necessary to abandon the traditions of the school and the antiquated projects of the éléves of the school of Talabot and d'Enfantin, and proceed to take examples from nature.

"The Strait of Gibraltar, that is to say, a direct opening from sea to sea, has served as an example for the execution of the Suez Canal. The communication established by nature at the other extremity of the Mediterranean, with an intermediary basin, the Sea of Marmora, which unites the Ægean with the Black Sea, ought to be the type in order to establish the interoceanic canal (at Panama or Nicaragua).

"To dam the Chagres at Buhio-Soldado with a single lock having a high lift, which will inundate the superior valley, and raise the water-level sufficiently in order to admit the flood-water of the Chagres in the canal; to dam the Rio Grande on the side of Panama by a second lock of the same type, and cut the culebra with a trench 197 feet wide at water surface in the interior lake—such is the solution that alone can give to the interoceanic canal conditions of navigability comparable to those that are given by the Suez Canal. Each lock to have two chambers, one for ascending and the other for descending vessels. The masonry to be in a full compact mass,

containing 450,000 cubic metres, that would not be disturbed by the trembling of the earth, so frequent in that country. Such an arrangement would assure the passage of 40 ships a day, whereas the proposed plan would only admit of the passage of 10 ships in the same time.

"These works, one on each side of the mountain, can be built in one or at most two years, at a cost, for both together, of not over $32,000,000."

This construction then provides a sea-level canal from each ocean to the locks, in which vessels easily reach these locks; they are then lifted by one lift to the level of the water in the interior lake. The estimated total cost on this basis was about $80,000,000.

"The Culebra attacked by the Chagres by means of galleries tunnelled through it at the level of the bottom of the canal, through which the water of the lake or basin on the Atlantic slope will be forced to flow towards the Pacific Ocean, carrying with it all material easily excavated, as well as those constantly thrown into it by blasting, and disposing these materials on the low plane bordering the ocean—by this means the Culebra would be levelled in a year

"The high locks and dams are placed where the limestone and hard sandstone first crop out, the valley of the Chagres being very narrow at these places; and the effect will be, with locks having a rise of about 100 feet, and with a diversion dam to raise the level of the Chagres to this same level, that the valley will be inundated up to the approaches of the Culebra, and there will be created a lake which will be extended from 9½ to 12½ miles in the valley of the high Chagres, forming a liquid surface of several miriamètres square, covering virgin land and without inhabitants. A similar effect will be produced on the Pacific slope. The distance through the solid Culebra from water to water will be reduced to about 5 miles.

"Thus ample water-supply will be secured to supply the locks and replace loss by evaporation, which would be difficult, if not impracticable, by the other plan without the use of steam-pumps.

"The excavated material can readily be removed by transports. The Chagres River, the most formidable obstacle to the execution of the works, forever subdued and enslaved, will become the principal assistant (*adjuvant*) of the canal, instead of a constant menace to the construction and maintenance of it. The excess of water in time of floods will be spread over the surface of the lake, and allowed to escape by diversion channels as may be deemed necessary.

"In the locks, each chamber will be about 650 feet in length between the gates, in width 66 feet, and in depth about 138 feet."

The details of construction of the locks, gates, etc., are set forth fully in the pamphlet. The above is translated by the writer from the French.

Such is the latest scheme for the construction of the Panama Canal. Whether work will ever be resumed on this route is at present a matter of doubt.

The following is a brief account of the suggestions and plans of M. Bartissol:

1130. M. Bartissol's method for completing the Panama Canal is similar in many respects to that of M. Sautereau, described in paragraph 1129. He, however, proposes the construction of two locks on each side of the Culebra cut, with a total lift of 75 feet, instead of the one lock with a lift of 135 feet. He therefore forms an interior lake of much lower surface level. He also proposes to use the waters of the Chagres river to remove the material to be excavated in the Culebra, but in a somewhat different manner. He proposes to control the waters of the upper Chagres by dams and reservoirs, and lead the water in an open channel to the line of the canal. He then proposes to build a conduit 13 feet in diameter, starting just inside of the side slope of the completed canal, towards the Pacific. The length of this conduit to be about 6¼ miles, lined throughout with masonry or metal, and having a uniform slope of 1 in 1000. At intervals along the canal shafts are to be excavated, opening into the conduit, for the purposes of throwing the excavated material into the conduit; there it is to be taken by the water flowing through the conduit at a velocity of 10 feet per second, and deposited where desired.

The conduit is to be filled to the height of 10 feet with the water. The duty of the conduit is estimated at 30 cubic metres per second (1 cubic metre = 1.31 cu. yds.), or about 1,000,000 cubic metres in a day of 10 hours., with 100 shafts and 400 cubic metres of débris thrown in each. The day's work would represent a cube of 40,000 cubic metres of débris, or 4 per cent of the total flow. According to the experiments of Guillemain and Durand-Claye:

A velocity of	0.5	feet	per second	will	carry the	heavier clay;
" "	0.65	"	"	"	"	fine sand;
" "	2.28	"	"	"	"	small gravel;
" "	5.86	"	"	"	"	flat stones;
" "	5.31	"	"	"	stones	4 inches in diameter;
" "	7.5	"	"	"	"	8 " " "

M. Duponchel says a stream of water with a velocity of from 6.5 to 10 feet per second carried 23 per cent of its own volume in rock material, and that in a conduit of circular cross-section the material would not only be carried forward by rolling, but it would be in a state of complete suspension, and there would be but little friction on the sides of the canal. Similar results were obtained in a 20-inch pipe at the Culebra. M. Bertissol claims that by this method the canal can be completed in four years, at an expense of $100,000,000.

This proposition was declined, as being uncertain in its result, and not sustained by any precedent.

In Fig. 416½ (*a*) is shown a map of the railway and proposed Panama Canal connecting the Atlantic and Pacific oceans at Colon and Panama respectively; and in Fig. 416½ (*b*) is a profile along the axis of the canal, showing the two portions of sea-level canal at the ends, and the interior lake formed by single locks, one on each side of the culebra, having a lift of great height, as described above. The drawings are readily understood from the foregoing discussion and description. Instead of the single locks as shown, the last plan of M. de Lesseps contemplated several locks on each side of the culebra.

1131. *The Nicaraguan Canal.*—The year 1888 was no less noted for the closing period of work on the Panama Canal than it was for the commencement of actual work on the Nicaragua Canal. Careful preparation was made for this work by careful surveys and examinations, and so far as practicable by plans and designs of the work to be done and of the manner of doing the work.

The route of the canal is from San Juan del Norte, or Greytown, on the Caribbean Sea, to Brito, on the Pacific Ocean, a distance of 169.8 miles; but of this distance there is only 29 miles of actual canal excavation. The remaining distance is by free navigation of the San Juan River, Lake Nicaragua, and the basins of the rivers Descado, San Francisco, and Tola. (See Fig. 417 for plan and profile.)

The lake is a body of water 2600 square miles in area, with a drainage area of 8000 square miles, and a mean daily flow through its outlet, the San Juan River, of 1,272,153,600 cubic feet—considerably more than is necessary for the works it is to feed. The San Juan River will be navigated for 64.5 miles, and the lake for 56.5 miles. The summit level, 110 feet above the sea, will be maintained for a distance of 150 miles, beginning at the eastern divide, some 16 miles west of Greytown, and continuing to the west side of the

Tola basin, less than four miles from the Pacific Ocean. The ascent from the level of the sea on the east side to the summit level will be effected by three single locks, and on the west side by one double lock and one single lock. Those portions of the line in river, lake, and basin will have width and depth sufficient to render navigation

(a)

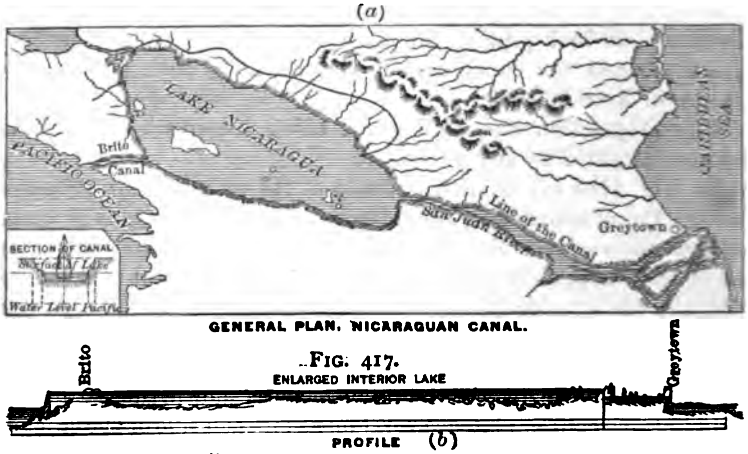


FIG. 417.

(b)

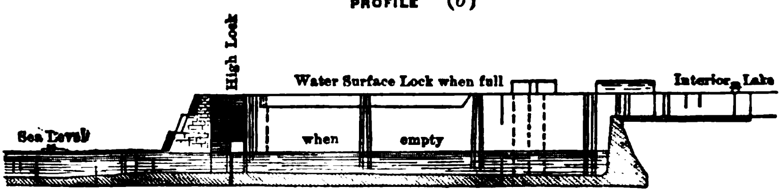


FIG. 417½ (a).

almost as convenient as on the high seas. The cross-sections of the excavated canal will have a depth of 30 feet, width at water surface of from 174 to 288 feet, and bottom width from 80 to 120 feet.

The harbors are to be improved by means of breakwaters and piers, and the eastern and western extremities of the canal are to be widened and deepened to afford convenient access, safe anchorage, and dockage. The rock cut to be made is 3 miles long, with an average depth through that distance of 149 feet. The cost of this excavation is estimated at $12,000,000. A dam at Ochoa, 1255 feet in length on its crest and 52 feet high, will convert the upper 64.5 miles of the San Juan River into an extension of the lake. The

dam at the extremity of the Tola basin, near the Pacific, will be 2100 feet long on the crest and 71 feet high.

For impounding the waters to form the San Francisco basin five dams will be required, varying in length from 1200 to 1700 feet, but of no great heights, and some secondary embankment along the crest of the impounding ridge from 5 to 30 feet in height.

This arrangement is a substitute for a single dam of 6000 feet in length, intercepting the Rio San Francisco near its junction with the San Juan.

There will also be a dam and a waste-weir at the east end of the Descado basin. It was believed that the work could be completed in six years, at a cost of $50,000,000. Latest estimate, $100,000,000.

The climate is not at all unhealthy. The necessary labor can be drawn from the neighboring States, which will also supply all the fresh provisions needed. There is an abundance of excellent timber, stone, clay, sand, and limestone found along the line.

The locks are to be 650 feet long between the gates, 70 feet clear width between side walls. The locks will be built of concrete and masonry.

The estimated traffic is 6,000,000 tons. With a toll-rate of $2.50 per ton, the income would be sufficient to pay the interest on a capital of $200,000,000. The saving of distance from principal points on the Atlantic and Pacific will be very considerable. The following table gives the distances by several routes between some points of importance:

TABLE LXXIXa.

DISTANCES IN MARINE MILES OF 1852 METRES (6076.5 FEET).

	By Panama.	By Cape Horn.	By Cape Bonne Esperance.	By Suez.
From Havre to—				
San Francisco	8,000	15,000		
Callao (Lima)	7,000	11,500		
Valparaiso	8,000	10,000		
Yokohama	11,400		17,000	18,000
Shanghai	13,000		16,000	12,000
Hong Kong	13,600		15,500	11,000
From New York to—				
San Francisco	6,000	14,900		
Callao	4,000	11,000		
Valparaiso	5,000	9,800		
Shanghai	11,000		15,000	14,000

M. Sautereau also suggests the employment of locks with high lifts for the Nicaragua Canal. He recommends on the Pacific slope a single lock with a fall of about 109 feet at Flor, and on the Atlantic slope two locks and dams (*deux écluses barrages*), the one of 42.6 feet fall and the other of 65.6 feet, which will be sufficient to overcome the difference of level between the oceans and the lakes. The adoption of these locks reduces the works to their simplest expression, and admits of making all the necessary excavations by means of hydraulic forces arising from the excess of the water in the lake. It admits of the utilization of the old bed of the San Juan River without the resort to manual labor or mechanical appliances.

The question as to the number of locks is not only one of cost, but also of time and expense of passing through them.

The following analysis of cost of locks is given for what it is worth, as indicating the lines upon which comparisons can be made. This was made in connection with the Nicaragua Canal.

Assuming locks 600 feet long between gates, 70 feet clear width, and 25 feet draught: in locks of different sizes, and well proportioned in capacity for doing the necessary work, approximately proportional to the product of areas and draught, the following data and dimensions are usually given:

Total length of side walls .... 700 feet.
" " " lock floor .... 700 "
" height of side walls = lift + draught + 4 feet.
Length of thin portion of wall .... 552 "
Width or thickness on top of thin portion of wall .... 10 "
Width or thickness at bottom of thin portion of wall = 10 feet + 22% of height.
Length of thick portion of wall .... 148 "
Thickness at top of thick portion of wall .... 15 "
" " bottom of thick portion of wall, 15 feet + 32% of height.
Length of mitre wall on curve .... 75 "
Height of mitre wall on curve, lift + 12 feet.
Thickness of mitre wall, two-thirds lift.

On this data the following estimate of cost is made on the basis of cut-stone masonry, cut stone with rough backing, and with concrete; only details for the least expensive and most expensive ar-

rangements to overcome the total elevation of 110 feet, with the same number of locks on each side of the lake.

With four locks, two on each side, lift 55 feet each:

	Cut stone.	Cut stone and backing.	Concrete.
Cost of floors	$692,000	$692,000	$692,000
" " masonry	5,808,000	3,706,000	2,968,000
" " gates	1,306,000	1,306,000	1,306,000
" " metal	491,000	491,000	491,000
Totals	$8,297,000	$6,195,000	$5,457,000

With twelve locks, six on each side, lift 18.3 feet each:

	Cut stone.	Cut stone and backing.	Concrete.
Cost of floors	$1,449,000	$1,449,000	$1,449,000
" " masonry	7,502,000	4,562,000	3,656,000
" " gates	586,000	586,000	586,000
" " metal	1,186,000	1,186,000	1,186,000
Totals	$10,723,000	$7,783,000	$6,877,000

For an intermediate number of locks the lifts and quantities vary somewhat, in the proportion of the number of locks. The following are the totals:

		COST.		
No. of Locks.	Lift.	Cut stone.	Cut stone and backing.	Concrete.
2	110 feet.	$10,967,000	$8,501,000	$7,587,000
4	55 "	8,961,000	6,691,000	5,894,000
6	36.7 "	9,211,000	6,747,000	5,924,000
8	27.5 "	9,776,000	7,124,000	6,264,000
10	22 "	10,611,000	7,719,000	6,809,000
12	18.3 "	11,581,000	8,406,000	7,437,000

The above includes four culverts and other adjuncts.

The cost of operating and maintenance, when capitalized, was found, on the St. Mary's Canal locks, to be $562,000 for each lock, and, as this will be practically proportional to the number of locks, the equivalent total cost would be found by adding to the above sums $1,124,000 for two locks, $2,248,000 for four locks, and so on. On the basis of concrete locks, the total equivalent cost will be:

For 2 locks,	$8,711,000	For 8 locks,	$10,760,000
" 4 "	8,142,000	" 10 "	12,429,000
" 6 "	9,296,000	" 12 "	14,171,000

There is no advantage to be gained by using locks having different lifts in the same set.

The cost of locks alone, with a total rise of 110 feet, is least when the lift is 55 feet. The cost of three locks in a set, when built of concrete, six in all, is only $30,000 more than two locks in a set, or four in all; whereas single locks, two locks in all, according to the above estimate, cost $1,693,000 more as compared with four locks. This does not correspond exactly with M. Sautereau's statement, though he recommends two locks on one side and only one on the other. But it appears from the two estimates that the number of locks should be kept as low as practicable, as the cost of operating and maintaining increases with the number.

Duplicate locks are necessary in order to accommodate a large gathering of ships at any one time, and also to admit of repairs without impeding navigation.

A final conclusion was drawn, that "the usefulness and value of the canal to commerce is practically the same with or without locks."

This proposition can hardly be maintained, except on the ground that such a shortening of the distance will command the traffic regardless of the time and expense of passing through the canal. It is argued, on the contrary, that the success of the enterprise depends upon the construction of a canal without locks. But this is certainly impracticable at Nicaragua, and seemingly so at Panama.

In Fig. 417 (*b*) is given the profile along the Nicaragua Canal, and in Fig. 417½ (*a*) the high locks and interior lake formed as proposed by M. Sautereau.

1132. *St. Mary's Falls Canal.*—Length of the locks of this canal is 515 feet between gates; width, 80 feet, reduced to 60 feet at gates; mean draught, 14.6 feet; mean lift, 18.6 feet. Lock-walls near gates are thicker than the central portion in order to give weight to resist thrusts of gates, and length of thick wall is 4 feet more than width of lock. Part of wall between mitre-walls is 10 feet thick on top. Batter on back, 1 to 4.9 feet in old lock, 1 to 4.4 feet in new lock; width of wide wall at bottom 39 per cent greater than thin portion of wall. In new lock width at gates 100 feet, and wide wall 80 per cent greater than narrow wall at bottom.

Facing masonry 4 feet deep, coping 2 feet. Entire mitre-wall and lining of gear-passages are of cut stone, remainder backing. Thickness of mitre-wall 12 feet. Gates of oak, with ties and straps of iron. Conduits for water, 8 feet square, passing through walls

and under the floor of the locks. Water admitted into lock through 29 small apertures. Floors and conduits of timber and concrete bolted to rock. Cost per square foot covered by conduits and floor, $2.38. For new lock, $2.00; cost of bolts alone, 45 cents per square foot; cut stone, $27.84 per cubic yard; backing, $8.10; concrete, $6.52 per cubic yard. Time filling lock 12 minutes, emptying 8 minutes.

Metal-work of lock cost $83,800; valves, $2000; gate anchorage, $700.

Cost of engineering and superintendence, 8 per cent of total cost.

1133. It will be noticed that ship-canals are constructed mainly with the view of shortening the distance between important navigable bodies of water and to avoid difficult and perilous routes, and are encouraged and supported to a great extent by all governments as constructions of common and public interest. The Manchester Canal was constructed to transform the city into a seaport, and was rendered almost necessary, it is stated, on account of the excessive charges and tariffs imposed by another and rival city and intervening railway corporations, both of whom opposed and obstructed in every manner the construction of this canal.

The construction of drainage-canals will now be briefly considered.

1134. *Drainage-canals.*—The main considerations in drainage-canals are to have sufficient volume of water to dilute sewage and thereby avoid offensive odors, and a sufficient velocity, to prevent deposition of solid matter contained in sewage, which should not be less than 2 feet per cent, but may be considerably greater. If the excavations and embankments are of earth, the velocity should not be so great as to scour the bed and slopes, otherwise they should be protected in some manner. Large heavy masses of organic refuse should be destroyed by cremation, and not allowed to settle in the canal, where practicable to avoid it. Drainage-canals are sometimes used, or constructed to be used, as navigable canals; this at once brings in the conflict of velocities, as for drainage a high velocity is desirable, while for navigation as low a velocity as practicable is desirable.

There is now being constructed in this country a canal which primarily has a different object and purpose from either of those already described, but which in all probability will form a link in the often-discussed scheme for connecting the waters of the Lakes

and the Gulf of Mexico with a navigable canal connecting the Lakes with the Mississippi River, thence by this river to the Gulf.

The far-reaching effect upon the commerce of Chicago and the Lakes will be immense, giving uninterrupted navigation between Chicago and New Orleans, relieving the former from the tolls that her commerce has to pay to the Canadians, freeing her from the delays and inadequate facilities of the New York and Erie Canal, and affording a less rate for tonnage than she has to pay for railroad transportation. It will make New Orleans the entrepot of her foreign trade, and the port of distribution for her cereal and provision products. In this respect the underlying object is the same as that which led to the construction of the Manchester Canal. Though this may be the ultimate result, the immediate object of this canal is for drainage purposes.

1135. *Drainage-channel of Chicago.*—A work of the magnitude and importance of this drainage-channel calls for more than a passing notice in a work on engineering. This canal has assumed a national importance, in that it is designed and is to be constructed to form an important portion of a great water-channel connecting the Great Lakes of North America with the Mississippi River, and by it with the Gulf of Mexico. (See map at the end of this volume.)

It is an accepted fact that railroads cannot compete with water-carriage in hulls of 1000 tons or over. In few instances have railways won against deep interior waterways. The day of small trunk canals is passed, that of deep wide channels is recognized.

The economical advantages to Chicago and the people along the sixteen hundred miles to the Gulf from the construction of this canal will be incalculable.

General Poe, in charge of the 20-foot waterway improvement of the Lakes, is authority for the statement that "the saving to the public in 1890 alone was $135,000,000, two thirds of all the expenditures of the Government upon rivers and harbors up to date. Water carriage adds but 6½ per cent to the average value of goods, while railway freightage adds 45 per cent. Who can doubt that this explains the enormous relative increase in wealth and population of deep-water ports over those of St. Louis, Cincinnati, Louisville, and New Orleans. A 14-foot waterway to the Gulf, if it follows the precedent of Lake development, will be lined with manufacturing towns supplying the wants of near consumers. The South especially will receive a new impetus; the increased demands

upon her forests alone will bring hundreds of cargoes of lumber and ties to Northern markets annually. There will also result a great impulse to ship-building; with the increase of national security, the new route by way of the Gulf will become a strong competitor in the hands of Lake ship-owners for much of the grain and provisions now transported by way of the St. Lawrence and the Erie Canal."

With the foregoing facts in view it was a wise forethought on the part of the projectors of the Drainage Canal to make proper provision for its employment as a navigable ship-canal throughout its length.

The Sanitary District of Chicago.—This district was created by an act of the Legislature of the State of Illinois. This act described the capacity of the channel to be built, the functions of the trustees, and the possible financial resources of the district.

Under this act the following-named gentlemen were elected to the Board of Trustees: John J. Altpeter, William Boldenweck, Lyman E. Cooley, Bernard A. Eckhart, Arnold P. Gillmore, Thomas Kelly, Richard Prendergast, William H. Russell, Frank Wenter.

Officers—Frank Wenter, President; Thomas F. Judge, Clerk; Melville E. Stone, Treasurer; Isham Randolph, Chief Engineer; George E. Dawson, Attorney; U. W. Weston, Superintendent of Construction; T. T. Johnson, Hydraulic and Assistant Chief Engineer,—together with a full corps of division and assistant engineers.

As soon as practicable after the organization of the Board of Trustees the necessary surveys were made and important data for the rapid prosecution of the work were secured. Actual work of construction was commenced in September, 1892, and has been prosecuted with unparalleled vigor up to the present time.

"While the channel is primarily built to answer the sanitary requirements of the present and the future, yet nothing has been left undone to make it possible that the same can be used for navigation as an outlet to the South, on the assumption that the United States Government will ultimately construct the necessary link from Lockport to the city of La Salle, a distance of 65 miles, and from La Salle to the mouth of the Illinois River, a distance of 220 miles." (See President's Report, December 4, 1894.)

Location of the Canal.—The channel leaves the West Fork of the South Branch of the Chicago River, near Robey Street, and

extends in a southwesterly direction to Lockport, a distance of 28 miles. The alignment is practically straight, there being only one or two curves with radii of great lengths. It follows the general direction of the Desplaines River, and occupies for a considerable distance the old bed of this river. This, of course, necessitated the excavation of a new channel for the river over a distance of 21 miles, called the Diversion Channel.

Construction of the Canal.—We will first consider the construction of the canal proper. Owing to the enormous amount of material to be excavated and handled, the distance of 28 miles was divided into about one-mile sections, each section being let to contract separately. The entire distance is now under contract. According to the character of the material to be excavated, the mile sections are known as rock or earth sections.

For the earth sections a trapezoidal cross-section is adopted, having a bottom width of 202 feet, and side slopes both above and below water surface of 2 to 1, with a low-water depth of 22 feet, fall of 1 in 40,000, and corresponding velocity of flow of nearly 2 feet per second.

For the rock sections practically a rectangular section is adopted. The sides are vertical, but owing to the manner in which the excavation is made, there are two 6-inch offsets on each side. The bottom width is 160 feet, top width 162 feet, with a low-water depth of 22 feet, and corresponding velocity of flow of nearly 3 feet per second, the fall being 1 in 20,000.

In addition to the materials of earth and rock, a conglomerate known as glacial drift is found in large quantities. This material consists of a compact and tough cemented gravel, sand, and clay, containing in large quantities bowlders of all diameters from two to three inches to as many feet. There are fourteen sections of earth and glacial drift, six sections of earth, glacial drift, and underlying solid rock, and nine sections practically of solid rock. The average depth excavated is about 35 feet.

The average cross-section of this canal is greater than that of the Suez, Manchester, or North Sea and Baltic canals.

The canal is entirely in excavation. The material is excavated from the proposed channel and deposited in spoil-banks alongside.

Excavating and Handling Machinery.—The interesting and instructive features of this work consist in the great variety, novelty in design, and mammoth dimensions of the machines and appliances employed in excavating and handling the material.

The strong competition to secure the work necessarily resulted in securing bids by the trustees at bottom prices, and this incited contractors and others to the invention and use of many novel appliances in order to reduce the cost of construction to a minimum and to enable them to complete their contracts within the specified time. It is not too much to say that there exists on this work, concentrated in a distance of 28 miles, a greater variety of machines for excavating and handling the material, a larger collection of these machines and a greater expenditure in procuring them—estimated at $3,000,000—than was ever before employed on one piece of work.

A detailed description of these machines and their records of work in cubic yards would require a volume to contain it.

General Description of Machines.—On the earth or glacial-drift sections can be seen all kinds of ordinary excavators, such as drag and wheel scrapers, carts, wagons, a variety of cars on tramways hauled by horse or steam power, steam-worked hoisting-drums, steam-shovel excavators of every known type, and a great number of inclines, built of timber or iron, of various designs. On some portions of the work covered with water were found large quantities of silt or muck, which was handled mainly by hydraulic dredges, but in part by the ordinary bucket-dredge.

On the rock sections are employed mainly large derricks, cable-hoists, cantilevers, channellers, steam-drills, etc., each type of machine being used on that portion of the work where experience indicated it was best adapted and could be most economically operated. Only a brief description of some of the more novel and expensive machines will be given in this volume.

Hydraulic Dredgers.—These consist essentially of a large centrifugal pump connected to and operated by engines, and the suction and discharge pipes leading to and from the pump, respectively, together with suitable means of cutting up the material and feeding the same to the end of the suction-pipe. The suction-pipe can be from 9 to 18 inches in diameter and from 10 to 20 feet in length. It is supported in a frame which can be lowered or raised and turned horizontally through any desired arc. Commonly, however, this horizontal motion is imparted by swinging the suction end of the barge, upon which the entire machinery is placed, through the desired angle around a spud at the other end as a pivot. The discharge-pipe can be of sizes corresponding to the suction-pipes, and can be of any desired lengths, reaching from the

pump to some suitable place of deposit, selected, or prepared by levees, so that silt and water discharged cannot return to the channel. Where the discharge-pipes extend over water to silt basins, they have to be supported on pontoons or some similar apparatus. The bucket-dredger consists simply of a large bucket attached to the end of a beam sliding between strong, stiff guides. By means of suitable machinery and gearing the bucket is lowered to the bed of the stream, pushed forward into the material, and when full it is raised and swung over a barge alongside, into which its content is dumped. The barge, when full, is taken to the proper place and discharged.

The more novel and expensive machines are employed in excavating and handling the material in the rock sections. These sections are entirely in solid rock, to the full depth of about 36 feet, except a few inches or feet of soil at the surface. This is taken out in three "lifts" of 12 feet each. The general operation is as follows: After removing the surface soil and exposing the rock, a series of steam-power drills or channellers running on tracks built for this purpose are set to work along the outer side lines of the canal. These cut a vertical channel about 2 to 3 inches wide, 12 feet deep, and of any desired length, thus separating the core, which is the full width of the canal, from the rock outside its limits. This work of the channeller is continuous along the line of the canal. Upon the core thus separated is placed a number of ordinary steam-drills, so called. These are now operated by compressed air almost exclusively. Plants for supplying compressed air are placed at convenient points near the excavation. The air is carried along the canal in large pipes, from which small pipes lead to the drills. After drilling a number of holes, these are charged with dynamite, the explosion of which brings down large masses of rock—often in large blocks, which have to be broken by small blasts into smaller pieces, of sizes convenient to handle.

In this manner any desired portion of the first "lift" is excavated. The channellers and drills are then set to work on the second "lift," and finally on the third or bottom one. In this manner this canal is being excavated for miles of its length. As the material is thus thrown down by the explosions of dynamite it is removed from the canal excavation and deposited on one or both sides. For this purpose some mammoth iron derricks are employed. A car, capable of running on a track, carries a turntable similar to that of a swing-bridge, upon which is carried the neces-

sary machinery, and from which projects a stiff iron boom 160 feet in length. The general operation of this machine is similar to that of any ordinary derrick. The material, however, is mainly handled by means of cable-hoists and cantilevers.

Cable-hoists.—The cable-hoist consists essentially of two strong platforms or cars capable of moving on tracks, one on each side of the canal, upon which rests all the necessary machinery, and upon each of which is erected a strong timber tower about 60 feet in height. A strong wire rope, called the carrier-cable, $2\frac{1}{2}$ inches in diameter, passes over saddles on the tops of the towers, its ends being firmly fastened to the platforms. Upon this is placed an iron frame with grooved wheels. To each end of this frame is attached one end of a smaller cable, the middle portion of which is wrapped around a hoisting-drum. This cable passes over sheaves placed in the top of the towers, and with the frame constitutes an endless cable, and is used for hauling the frame along the carrier-cable. As one portion of the hauling-cable winds around the drum another unwinds, and in this manner the frame can be moved and stopped at will. The hoisting-cable is attached at one end to a hoisting-drum and at the other to the movable frame, after passing over sheaves in the top of the towers and in the movable frame, a suspended or movable sheave is raised or lowered by means of the hoisting-cable, and carries chains and hooks for taking hold of the skips or large iron buckets into which the broken rock is placed. There is also a dumping-cable attached at one end to the hoisting-drum and at the other to the rear end of the skip. This cable is so connected that while the frame is being run along the cable and the skip is being raised or lowered its speed is that of the hoisting-cable, and the skip holds its load; but by a very simple device it can, when desired, be made to run a little faster, the effect of which is to raise the rear end of the skip, and in this manner discharge its load. This device is the invention of Mr. Charles Locker, one of the contractors on the canal work, and by it the cable-hoist has been advanced as a strong competitor of the cantilever for this kind of work.

Cantilevers.—The cantilever consists essentially of a trussed bridge built on a slope and resting on top of an iron or steel tower, the whole resting on a car or platform, movable on a track built for the purpose. The necessary machinery also rests upon the same car. On one side of the tower the cantilever-arm slopes downward and over the material to be excavated, and on the other side, up-

ward and over the place for depositing the material. Carried by this truss or cantilever is a system of cables and pulleys, by means of which an iron frame can be carried from one end to the other of the truss, as well as raised or lowered at will. The cables are operated by the machinery on the car.

The operation is as follows: The frame is run along the cantilever until a point is reached vertically over a loaded skip or bucket. It is then lowered to the bottom of the excavation, and connected to the skip by means of hooks; the motion is then reversed, the skip is raised nearly to the floor of the truss, and thence along it to or near the other end, where arrangements are made to tilt and dump the skip automatically. The empty skip is then run back to some suitable point and lowered to the bottom of the excavation, where it is reloaded, connection made with another skip, and the same operation repeated. The cantilever as a means of handling excavated material has been used for the first time on this work, and has given entire satisfaction. It may be said that without the invention and use of this apparatus the execution and completion of this work could not have been accomplished in so short a time. This is said without any disparagement of the cable-hoist as now at work. It did not at first adapt itself to this class of work, and without the invention of the automatic dumping arrangements it could not have been advantageously used, either considered from an economic or speed standpoint, while well adapted to many other important purposes.

There has been considerable competition between the manufacturers of the two machines. It is seemingly admitted that the cantilever is capable of handling a greater quantity of material in a day, but, owing to its greater cost, it is claimed that the cable-hoist handles its material more economically, can handle larger blocks, and reduces the danger to the workmen to a minimum.

The total estimated quantities of excavation are:

Solid rock.............	12,000,000	cubic yards.
Glacial drift...........	28,000,000	" "
Aggregate...........	40,000,000	" "

Of this amount 44.43 per cent has been removed at this date (Jan. 1, 1895). The Chief Engineer believes that the entire work will be completed by the fall of 1896.

COLLATERAL WORKS.

It might be expected in the execution of a work of this nature and magnitude that there would be a number of works necessary to be constructed not included in the canal proper. Only the more important of these will be described.

River Diversion.—As has been stated, the canal occupies the old bed of the Desplaines River for a distance of twenty-one miles. This necessitated the diversion of the river into a new channel. The work required consisted in excavating a new channel for the river, and the construction of levees to prevent the flood-waters of the river from flowing into the canal. The levees are constructed of the material taken from the new river channel and the canal. These and the banks of the river will be protected by riprap or masonry where necessary. A considerable portion of the levees had to be constructed by dumping earth from cars run out on a pile-trestle built for the purpose.

"The work of the past year includes flood-measurements of the Desplaines, which gave results corroborative of the curves previously established, and demonstrated the correctness of the calculations on which the grades and sections of the diversion channel had been computed to insure sufficient capacity for the service it must perform. The total cost of the river diversion, including the spillway and levees, is $1,079,397.40."

Spillway.—The spillway is a concrete dam 397 feet long, 5 feet wide on top, with a front face batter $\frac{1}{2}$ to 1, rear face vertical; total height 10 feet. This is connected at its ends with massive concrete abutments. The coping of both dam and abutments is of large limestone blocks, obtained from the quarries of the Western Stone Company located along the line of the canal.

The necessity of this dam or weir will be clearly appreciated from the following: During the spring floods the current in the Chicago River, whose waters are loaded with and poisoned by a large quantity of sewage, sets strongly towards the lake. This effect has heretofore been intensified by a large inflow, during periods of floods, through the Ogden pass from the Desplaines River. As a consequence the water of Lake Michigan, as far out—4 miles—as the intake for the water-supply of the people of the city of Chicago, has been almost annually rendered unfit for drinking and cooking purposes by its admixture and impregnation

with the sewage of the city. It was therefore deemed advisable to check this flow from the Desplaines into the Chicago River by constructing a dam across the pass mentioned, and thereby force the waters of the Desplaines to follow its own channel. Owing, however, to the inadequate dimensions of the channel through the city of Joliet, it was not considered safe or wise to divert at the present time the entire flood discharge of the Desplaines. The height of the spillway was therefore adjusted to the carrying capacity of the present channel through Joliet. When the contemplated improvements are made in this portion of the channel the weir will be raised so as to compel the entire flood discharge of the river to flow in its own proper channel. The water flowing into the canal from the Chicago River will carry the sewage along the canal, and not, as now, into the lake. The result will be that not only will the source of water-supply in the lake be kept pure, but the Chicago River itself will be relieved of its load of sewage and changed from its present foul condition and appearance into a clear-pure stream.

The river diversion and the spillway are the two more important hydraulic problems which have thus far been satisfactorily solved and consummated. In this connection the Chief Engineer says: "The spillway was completed during the early fall, at a total cost of $20,518.40. The flood of March last demonstrated the great value and efficiency of this work, and Chicago then reaped the first substantial benefit from the great scheme, which she has entrusted to your wisdom and loyalty to her best interests. This flood, but for the intervention of the spillway, would have swept pollution and disease into the fountain of her drinking-water, as has been done year by year in the past; but its turbid waters were turned aside, save for a limited flow of about sixty hours, which was not sufficient to cause any damage." As stated, this dam will be raised to a sufficient height to divert the entire maximum flow as soon as practicable.

Controlling Works.—"These works, of such vital importance to the success of the great enterprise you have in hand, have been carefully considered during the past year, and alternate plans have been gotten up and the merits of each carefully weighed and contrasted. The plan which combines the greatest advantages with the least cost is a combination of lifting-gates and a section of bear-trap dam. There will be eight of these lifting-gates, working between solid masonry piers; they will be lifted vertically by mechan-

ism, operated from bridges spanning the spaces between the piers, each space or opening will be 30 feet wide. These gates are a modification of what is known as the Stoney Gate, with special features designed for this work. The bear-trap dam will be 160 feet between bulkheads. This dam will be unlike either the Lang, the Parker, or the common bear-trap, as conditions obtaining on this work admit of a departure from those examples in several particulars. The designs for this work have been prepared under the immediate charge and direction of Mr. T. T. Johnson, First Assistant Chief Engineer. The estimated cost of this work is $160,000."

Tail-race.—The maps and cross-sections of this work have been completed from careful and accurate surveys, and its route finally determined upon. The estimated cost is $425,260.

Work between the Waste-weir of the Tail-race and the Upper Basin at Joliet.—This work consists of removing obstructions from the channel of the Desplaines, deepening same at different points and building such levees as are required. Estimated cost of this improvement, $150,000.

Work through Joliet.—This work consists of deepening and enlarging the channel of the Desplaines, making proper provisions for maintaining the navigation of the Illinois and Michigan Canal, and affording necessary protection to the city of Joliet against flood damages. Estimated cost, $1,760,000.

These works and improvements involve many difficult and intricate problems in hydraulics. The full details of the necessary constructions have not as yet been made public.

"The construction of the drainage-canal, unlike the other great canals of the world, involves the continual flow through it of a large volume of water. The design of the work, therefore, has involved the consideration of an inclined grade, together with the influence of varying flow, varying slope, and varying elevation of Lake Michigan. The volume of flow contemplated is such as to truly class the canal as a great river, the hydraulic ramifications of which involve the permanent level of the Great Lakes, the depth and overflow of the valley of the Illinois River, and in fact the navigable low-water depth of the Mississippi River."

Bridges.—A large number of swing-bridges for carrying both highways and railways over the canal will be ultimately required. The question at present involved is whether it is not better to build temporary fixed bridges of some cheap design, and when the canal is open to navigation remove these and build swing-bridges, rather

EXHIBIT 3—CHIEF ENGINEER'S REPORT, 1894.

Statement of Amount of Work Done to and Including December 31st, 1893, and During 1894; also Total to Date as per Vouchers, Regular Contracts.

Section.	Main Channel, Glacial Drift.			River Diversion, Glacial Drift.			Main Channel, Solid Rock.			River Diversion, Solid Rock.			Rubble Masonry.
	Jan. 1, 1895.	Jan. 1, 1894.	Difference.	Jan. 1, 1895.	Jan. 1, 1894.	Difference.	Jan. 1, 1895.	Jan. 1, 1894.	Difference.	Jan. 1, 1895.	Jan. 1, 1894.	Difference.	Jan. 1, '95 (built in 1894.)
O	518,821	00	518,821	7,600	00	7,600							
N	71,300	00	71,300										
M	343,800	00	343,800										
L	458,100	00	458,100										
K	414,600	00	414,600										
I	680,400	00	680,400										
H	141,444	00	141,444										
G	527,906	00	527,906										
F	504,293	244,712	259,581	158,284	134,386	23,898							
E	518,382	462,075	256,307	95,718	83,426	12,292							
D	1,117,872	227,317	890,555										
C	792,987	135,828	657,159	162,537	162,537	00							
B	600,477	000	600,477	208,626	204,626	4,000							
A	859,322	90,859	768,463	120,786	108,228	12,558							
1	423,166	99,332	323,834	5,876	5,876	00							
2	576,527	91,000	485,527	29,516	29,500	16	32,242	00	32,242				
3	351,288	94,510	256,778				200,852	00	200,853				
4	672,879	131,400	541,479				6,491	00	6,491				
5	567,000	218,700	348,300				15,100	00	15,100				1,800
6	540,900	136,100	404,800	112,700	111,900	800	43,100	00	43,100				1,800
7	169,600	67,800	101,800	97,000	96,700	300	363,400	67,200	296,200	41,800	41,000	800	
8	41,600	7,500	34,100	56,600	52,900	3,700	622,000	76,600	545,400	96,900	95,500	1,400	
9	71,700	50,700	21,000	37,700	37,700	00	568,400	80,600	487,800	16,000	16,000	00	
10	29,200	25,900	3,300	27,400	27,400	00	848,200	217,400	630,800	56,500	56,500	00	
11	44,032	40,300	3,732	5,756	5,756	00	635,300	267,100	368,200	11,483	11,483	00	
12	27,400	27,400	00	11,739	00	11,739	622,500	295,800	326,700				
13	32,822	32,822	00				873,700	376,800	496,900				
14	343,700	43,066	300,634				295,800	119,305	176,495				
15	29,500	00	29,500				9,700	00	9,700				
Totals	11,471,018	2,227,321	9,243,697	1,137,788	1,060,885	76,903	5,136,285	1,500,805	3,635,480	222,683	220,483	2,200	3,600

than incur at present the far greater expense necessary to construct swing-bridges of a permanent nature for which there will be no use for several years to come. This is purely an economic question as to which there is room for a difference of opinion.

The foregoing description has been abstracted from the reports of the president and chief engineer for 1894, supplemented from personal examinations of the work now under construction.

EXHIBIT 4—CHIEF ENGINEER'S REPORT, 1894.

Estimated Cost of the Main Channel and Auxiliary Works of the Sanitary District of Chicago.

Designation of Work.		Designation of Work.	
Section.	Cost.	Section.	Cost.
O	$344,783.93	2	$922,256.36
N	256,183.89	3	836,769.37
M	156,858.45	4	1,022,199.72
L	217,070.56	5	765,746.66
K	288,989.25	6	755,580.17
I	284,962.25	7	827,485.88
H	812,339.28	8	1,028,130.72
G	881,941.00	9	820,185.69
F	848,749.51	10	1,022,850.47
E	606,618.14	11	840,519.32
D	686,997.02	12	857,007.05
C	486,819.17	13	866,740.75
B	485,120.38	14	928,593.20
A	913,789.02	15	472,158.00
1	1,286,190.98		
Total of Main Channel under contract			$18,973,025.64
Levee, Lock 4, to Dam No. 1			18,052.85
Total Main Channel and Auxiliary work now under contract.			$18,991,078.49

Further Work Contemplated but not Contracted for.

Regulating Works	$160,000.00
Tail-race	425,260.00
Tail-race to Upper Basin	150,000.00
Between Section 15 and Upper Basin	$735,260.00
Head Upper Basin through Joliet	1,760,000.00
	$2,495,260.00
Fixed bridges	1,388,689.32

Total work yet to be contracted for on basis of fixed bridges..........................	$3,883,899.32	
Total covered by existing contracts............	18,973,025.64	
Total estimated cost of construction on basis of fixed bridges		$22,856,924.96
Total expenditures for Engineering and Superintendence up to December 31, 1894.......	$596,575.89	
Estimated future cost of Engineering and Superintendence........................	405,066.12	
Total cost of Engineering and Superintendence.		1,001,642.01
Total cost of Engineering, Superintendence, and Construction..........................		$23,858,566.97
Expended to Dec. 31, 1894, for Construction...	$7,027,294.78	
Expended to Dec. 31, 1894, for Engineering and Superintendence..........................	596,575.89	
Total for Construction, Engineering, and Superintendence..............................		7,623,870.67
Balance needed on basis of fixed bridges........		$16,202,644.85

The foregoing estimates will, I believe, be borne out by the actual cost of the completed work of the Sanitary District.

(Signed) ISHAM RANDOLPH,
Chief Engineer.

CHICAGO, January 16, 1895.

The preceding tables show the amount of work done to and including December 31st, 1893, during 1894, also total to date as per vouchers, regular contracts, and estimated cost of the main channel and auxiliary works of the Sanitary District of Chicago. The writer desires to express his thanks for the uniform kindness and courtesy of the officers of the Sanitary District, who have afforded him every opportunity of examining their works and records.

1136. *Irrigation Canals.*—Having fully discussed dams for storing water for irrigation purposes, waste-weirs, the general construction of regulating works at the entrance of the canal into the reservoir, which are intended to prevent the flood-waters from entering the canal and at the same time regulate the admission of the requisite quantities of water into it to furnish the proper irrigation supply, it would be beyond our scope to enter into too great detail of the construction of regulating-gates, which may be of timber or iron, and opened by sliding in grooves or turning on axles. It is generally advisable to provide for the proper discharge by using a number of small gates rather than one or two

large ones. The supply can be better regulated, and there is less danger of accidents, causing disastrous results; hand-power can be used to open and shut the gates; renewals and repairs can be better executed; provision can be made to meet a varying or increased demand more conveniently and satisfactorily. What follows will be confined to a few general directions and remarks on the actual construction of the canal in earth and flumes.

If the material through which the excavations and with which the embankments are to be made is too light and porous, the bed and slopes must be made water-tight by means of clay puddle. This may be effected by a layer of puddle forming a continuous sheet in the slopes and under the bottom. This may be entirely imbedded and covered over with other material to the depth of one or more feet. This covering-layer may be porous, but should have stability against scour from the current. If such material is not available, it will then be better to make the main body of the banks of any heavy material convenient, and place the layer of puddle on the outside of the slopes and over the bottom bed. It is a question whether this latter plan is not better both for canal-banks and reservoir-banks. It is more liable to be disturbed or disintegrated by the action of frost, unless always covered by water. It is not uncommon to rely entirely upon a puddle or concrete core in the centre of bank, supported by a sufficiently heavy mass on the outer face to resist the pressure from the water and saturated material on the inner face. It has been found practically impossible in some cases, especially in India, where such canals have very high embankments, to give stability to the banks as ordinarily constructed. Water will find its way into the interior of the embankment, resulting in softening, or, little by little, scouring a channel through it, and ultimately in entire failure of the work.

1137. *The Soonkêsala Canal.*—This long canal, intended both for irrigation and navigation, constructed in India under great difficulties, requiring unusually high cuttings and embankments, has been selected as illustrating what are and should be the character of construction for canal-banks. The following abstracts are taken from a paper by John Herbert Latham, M.A., C.E., and discussed by other members of the Institution of Civil Engineers. See *Van Nostrand's Magazine*, November and December, 1876.

The canal is about 190 miles long. The first 75 miles is intended to carry 400,000 cubic yards of water per hour, and after gradually parting with 100,000 cubic yards the remaining 300,000

is carried on, parting with and acquiring water, and finally carrying at its lowest extremity about 100,000 cubic yards.

The diversion weirs are either of solid rubble masonry, of gravel concrete faced front and rear with rubble, or, as in later types of weirs, of rubble masonry faced with limestone ashlar on the lower face, averaging 3 feet in thickness where exposed to concussion. The thickness at top of these weirs is 8 feet under coping. The coping is limestone 1 foot in thickness, and every other stone is a through-stone, weighing about $1\frac{1}{2}$ tons. These stones are joggled in some cases.

The heights of these weirs vary from 7 to 26 feet. In some cases there is a batter of 1 in 8 on each face, in others 1 in 4 on the rear or up-stream face, and vertical on the front. The greatest height of water over the crest is $7\frac{1}{2}$ feet.

These weirs are founded on a rocky river-bed. Portland cement mortar is only used in the joints of the coping. The headworks weir is a solid limestone rubble-masonry wall, 6 feet thick under coping and a batter of 1 in 4 on up-stream face, vertical on exposed face. As these weirs are built on a soft shale, the overflow water would undermine the weir; therefore an excavation is made in the shale and lined with masonry, thus forming a water-cushion to break the shock of falling water.

The bottom width of the canal is from 45 to 90 feet. The fall per mile is adapted to deliver the requisite quantity of water along the several reaches, without increasing the depth beyond 8 or 9 feet. The proper locks are placed where required; these are 120 feet long, and have a depth of 5.5 feet.

The grade or fall of the canal where it is taken down the irrigating channel varies between 18 inches a mile in one or two deep cuttings and a level in the tanks formed where the canal is carried over an open valley by damming it across. In other places it is usually from 4 to 6 inches a mile. The last seems to be the most suitable fall where the banks and cuttings are of cotton soil unprotected by revetments.

The more rapid the fall the less silt deposited, but the greater the current against ascending boats.

The chief interest and novelty in the construction of the canal is the mode of forming the banks. These range up to a height of 50 feet above the ground along their crest-line, but being constructed usually on sloping ground, the height above the toe of the lower slope is much greater. A number of different forms of cross-

section and character of construction have been used—simple filling, masonry wall, masonry face-wall, masonry revetment-wall, puddle face-bank, and puddle core-bank. Some of these have failed or given much trouble, and required watching and costly repairs.

The strongest and best sections are shown in Figs. 418, 419, the choice between the two being only one of cost. Then follows a lengthy discussion of the theoretically perfect bank. The following are the essential points:

Failure will take place either from the soil taking a flatter slope, when acted upon by the water, than that originally intended, or from leaks forming holes through the bank and ultimately destroying it.

It is claimed that soil acted upon by water does not become viscous, i.e., chemically semi-fluid, and that for all practical purposes the action is mechanical.

The flattest slope taken by dry material is that of ground quicklime; as it runs dry from the grindstones it assumes a nearly flat slope; and the natural slope which ordinary dry material deposited at hap-hazard will stand is steeper in proportion as the particles are angular and rough, and flatter in proportion as they approximate to polished spheres. The effect of a little water or moisture is virtually to increase the roughness of the particles, supposing no chemical action; the cohesion of small surfaces in contact being increased so much that if the particles are small the material may stand vertically when damp, or at an overhanging slope. Excess of water may have a very different effect, which will depend upon the nature of the material. Thus a mass of rubble or gravel consists of materials of such size as to be unaffected by the presence of water, as far as regards the relative positions the pieces are disposed to take, except when the flow of water is so violent as to displace them by mere force of impact. In clean sand, composed of a collection of insoluble particles or grains so small that though after being thoroughly wet much of the water if allowed will drain off, yet enough will remain to increase cohesion, and so much as to equal or exceed the force of gravity on any particle. Clayey earth has its particles so fine as to retain by cohesion a sufficient quantity of water to separate or nearly separate them, and to render the mass viscid; and so it will remain until the water is squeezed out by pressure or dried up by heat; it may require an intense pressure or heat to render such a material really dry, but a moderate press-

ure or heat may render it very hard. When really dry the material will have become stone, or so like stone that the particles will not separate if again exposed to water. But if only moderately dry, as it is found in banks, it is possible that when water is admitted into the crevices or pores of the material it may so loosen the particles as to make them easy to separate, and incapable of resisting the slightest action of running water.

In case of water flowing through a crack or burrow, no danger will result unless the material be tough clayey soil; a sandy soil soon fills in. In a tough, loamy clay a crack is much more dangerous, for the soil is nearly impervious, and the effect is simply to cut galleries or caverns often 5 or 6 feet in diameter.

The slipping of the banks arises from the water being forced into the interstices or fissures by pressure faster than it can drain or dry out—as where the outer slope is made of sand, or where a bank is made entirely of a moist clay, not rammed or puddled; and the greater weight on the centre of the mass squeezes out water and forces it into the slopes. In both cases slips of the slope are likely to occur, then flattening, and only stopping when a much flatter slope than the original is attained.

A given quality and stability may be secured with less risk of defective workmanship in earthwork with at least as much certainty as in masonry.

A good bank should satisfy the following conditions:

(1) The greatest possible leakage must not carry any material out of the bank.

(2) It must be impossible for any burrow or creak to be made right through the bank.

(3) The impervious part of the bank must be secured in its place so that neither settlement can breach it nor leaks through cracks which will occur in it carry the material out of place.

(4) The impervious part must be encouraged to settle or shrink, so as to close any cracks which may occur in it.

(5) It must be easy to detect and overhaul any bad puddle.

It will be found, in practice, that when a bank fulfils these conditions the entrance of water into any part of the bank is more difficult than its exit, and that in consequence the head of water upon the bank tends to press upon and tighten up every layer of material in the bank,—a result that tends to increase the stability of every part.

1138. A theoretically perfect filter is a trough of triangular

cross-section with sides containing an angle of 60°. By its being perfect is meant that whatever depth of water is placed upon it the water alone moves as it passes through the filter; and that the material is arranged in horizontal layers, beginning with fine sand at the top, and then becoming coarser and coarser towards the narrow bottom, the lower layer containing only large rubble blocks in such accurate proportions that the water flows through freely enough not to resist that above it, but not so freely as to leave any apertures.

If the upper layer of sand be as deep as the water placed upon

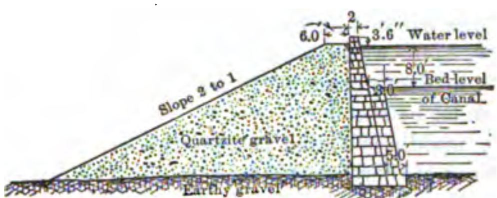


FIG. 418.

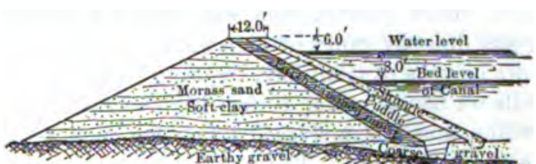
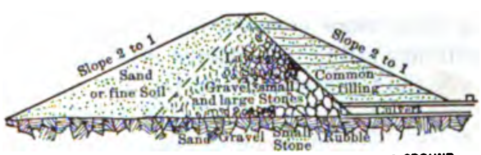


FIG. 419.



THEORETICALLY PERFECT BANK ON STRONG IMPERVIOUS GROUND

FIG. 420.

it, and is clean, the quantity of water flowing through the filter, or, in other words, the leakage, would be usually calculated at 700 gallons per day for each square yard of surface, which for a filter 200 feet high would give 13½ cubic yards of leakage an hour per lineal yard of filter; an amount so small that it would flow away in a stream an inch deep with a velocity of little over one quarter of a mile per hour. And this leakage would represent the work done by 200 feet head of water in forcing a passage for such a

stream through the obstruction presented by the filter, and imparting to it the velocity with which it issues from the lower edge.

If, now, this prismoidal filter be laid on its side upon an impervious soil, and the casing be removed and replaced by slopes of fine sand in front and of ordinary filling in rear, care being taken to keep open a drain of dry stone or of masonry arches from the outer rubble toe of the filter or core, the result is a bank (see Fig. 420) which fulfils the five conditions required for a safe bank, but without any puddle. No crack or burrow can be made in the material of this bank either by sun or vermin, and it only requires its surface protected from the action of rains and waves to make a durable bank. If the bank is higher and the layer of sand thinner on the front slope, more water would flow through the bank. Still there could be no displacement of the outer toe of rubble blocks, and the bank must be stable.

In practice puddle is added to stop the leakage, and the durability of the bank is thereby also increased. The best position for the puddle is on the front slope, and should be protected with rubble.

The puddle under construction was always kept flooded, and the tamping was done by gangs of men walking backwards and forwards on short lengths of it all day, treading each layer stiff.

The puddle on banks under 40 feet high was made 3 feet thick at the water-line, and was increased 1 foot for each 8 feet vertical depth below the water-line. The puddle must be carried down into a trench, and sheet-piled, if necessary, to prevent too great leakage under the toe of the bank. It should be laid on a layer of rammed filling a little more compressible than the puddle itself, and not on an ordinary filling, the change in consistency being too great. In Fig. 420 the inner slope is on the left; in Fig. 419 it is on the right.

The form of construction recommended is shown in Fig. 420, but faced on the inner slope with puddle as shown in Fig. 419.

1139. In the discussion of this paper other eminent engineers contended that banks constructed as described were not only contrary to good precedent and good practice, but absolutely dangerous, and should not be taken as good examples.

The first objection raised was in regard to placing such banks on sandy or clayey foundation-beds without any puddle trench carried below the surface in order to prevent undermining and destruction of the dam.

The next objection raised was placing the puddle on the slope where it would be liable to be fissured by exposure to the sun and

burrowed by vermin, with the additional liability of sliding away, and maintaining that the puddle wall would be more effective and less expensive placed as a central core; that a canal or reservoir embankment should be constructed to prevent leaks, and not to encourage them; and notwithstanding the fact that such banks had been constructed in this manner and had to the time of the discussion been efficient and safe, there seemed to be a rather general disapproval or even condemnation of the plans proposed.

Without expressing an opinion as to the comparative merit of placing the puddle core on the face or in the body of the wall, upon which point the most eminent engineers differ both in opinion and practice, the question after all resolves itself into what is a sufficient thickness of less impervious material that should be placed between the puddle layer or core and the water. Regarding the puddle as the sole reliance to prevent leakage through the body of the embankment as well as under it, it would seem that from reading of the failure of reservoir embankments outside and beyond the water-tight layer, stability and security against disastrous failures similar to those that have occurred can only be secured by some such arrangement of fine and coarse material supported at and near the outer toe by a heavy mass of open-work rubble of large stone. As has been said, water abhors angles; and if the flow of water can be impeded by being split up into a number of small streams and these made to change their direction a number of times, and not held in large volumes under heavy pressure, there can be but little doubt that even large initial volumes under considerable pressure will flow away harmlessly. And the question at issue seems to be, which is the better—to permit the embankments to be swept away bodily, with the fearful loss of life and property which almost universally results, or to run the risk of having a leaky dam which will at least not give way suddenly and with violence. It is often possible to stop such leaks when first developed if it could only be known beforehand that they existed; but the effects of the leaks when held and confined within a mass of earth are to gradually but surely, without their existence being known, gather and accumulate pressure sufficient to entirely destroy the embankment without a moment's notice.

1140. On ordinary irrigation-canals with low embankments, either a bank made entirely of puddle, with a puddle core, or a layer of puddle on the water slope, will serve every purpose; and even if a break occurs, no very disastrous results can follow, as only

a limited amount of water would be discharged, and this spreading out over an ever-widening surface would do little harm, the main damage results from the failure of the canal to do the work expected and required of it, together with the cost of repairs. But with high embankments, especially for storage-reservoirs, the importance of determining in what manner and with what materials the banks should be constructed cannot be overrated; and for this reason considerable space has been given to the discussion of this subject.

Flumes.—In carrying canals over deep valleys or rivers, the water must be confined in some kind of channel, made of iron, timber, concrete, or masonry; this construction is known as an aqueduct or flume, the latter term, however, being usually applied to irrigation canals. Such flumes are usually made of timber or iron, supported on a series of piers or trestle-bents.

The following drawings show the general construction of the diversion-weirs, head-works, canal along a hillside, and a timber flume, as employed on the Pecos Valley Irrigation-canal. They

Fig. 421.—Pecos Valley Dam and Head-works.

Fig. 422.—Timber Flume, Pecos Valley Irrigation-canal.

are merely intended to give a general view, as the principles of construction have been sufficiently well described in connection with other similar works.

Fig. 421. Shows a dam for storage-reservoir, and Fig. 422, a timber flume for carrying a canal over streams, ravines, etc.

ART. LIX.

HYDRAULICS AS APPLIED TO BUILDING CONSTRUCTION.

1141. It is not intended in this volume to discuss fully or even to any great extent the subject of hydraulics, upon which volumes have been written. The discussion will be limited to a few general principles and their application to the flow of water in closed pipes and open channels; by which some of the more common and simpler problems can be understood.

FLOW OF WATER IN PIPES.

The principles and equations and their applications are taken almost exclusively from Notes on Building Construction, published by J. B. Lippincott Co. The several volumes of these notes are full of the most valuable information. The author's name is not given. They purport "to explain not only the calculations that may called for in the Honours Examination at South Kensington, England, but also all that can be required in connection with ordinary buildings."

Hydraulics is a subject which is related to building construction only to a limited extent, and in fact only two subdivisions of the subject need be considered, namely: (1) The motion of liquids through pipes in connection with water-supply and disposal of sewage; (2) The delivery of water from a jet in connection with standpipes. First, let us define some of the terms in common use.

The hydrostatic pressure at any point in a liquid mass has been shown to be of equal intensity in all directions around that point, and equal to the weight of a column of water whose base is unity and whose height is the depth of the point below the water surface. The pressure of water at a depth of 20 feet below the surface is $20 \times \frac{1}{144} \times 62.4 = 8.67$ pounds per square inch.

Head of Pressure.—The depth of the point below the surface is also called the *head of pressure* at the point, or simply the *head*, and is generally expressed in feet.

Head of Elevation.—The height of the point above some datum level is called the *head of elevation* of the point.

Loss of Head.—When a liquid is in motion each molecule or particle is constantly moving from a place of greater head to a place of lesser head, and the difference between the two heads is called the *loss of head.* This loss of head may be entirely a loss of head of pressure, or entirely a loss of head of elevation, or partly a loss of head of pressure and the remainder a loss of head of elevation. This is illustrated in the following figures. In Fig. 423 a

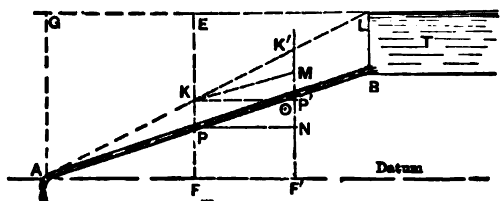


FIG. 423.

pipe *AB* opens into a tank *T*; at *A* there is a spigot; when this is closed we have the conditions due to hydrostatic pressure. The head of pressure at any point *P* in the pipe is *EP*, and if a pipe opened into the pipe *AB* and extended vertically to or above *E*, the water would rise to this elevation, which is the same as that of the surface in the tank. The head of elevation is *PF*, the height of the point above the datum *AF* passing through the lower end *A* of the pipe *AB*. At the point *A* the head of pressure is *GA*, and the head of elevation with respect to *AF* is zero, and with a vertical pipe at *A* the water would rise to the height of the surface *GEL*.

If now the spigot be opened so that the water flows freely through it, the water would not rise in the pipe *GA* at all. In other words, the pressure is zero at that point *A*; and if the resistance to the flow of the water in the pipe *AB* is uniform, since *L* and *A* are points of no pressure, and the head of pressure is entirely consumed in overcoming a uniform resistance, it follows that the straight line *AKL* is the line of no pressure, and the ordinates at any points between the pipe *AB* and the straight line *AKL* would represent the pressures at those points; at *P* it is *PK*. Or, in other words, if an open pipe *PE* existed, the water now would only rise to the point *K* and not to *E*, as was the case when the spigot at *A* was closed.

The head of pressure is *KP* and the head of elevation is *FP*, by which it is seen that the head of pressure is reduced by the height *KE*.

Taking any other point P', the head of pressure is $K'P'$, and the head of elevation is $F'P'$. The loss of head between the two points is $K'O$. Drawing KM parallel to PP', it will be observed that the total loss of head is made up of two parts, viz.: $K'O = K'M + MO = K'M + P'N$. $K'M$ is the loss of the head of pressure, and $MO = P'N$ is the loss of head of elevation.

It is evident that if the pipe AB extended horizontally along the line AF', and communicating with the tank at or vertically below B, the points P and P' coinciding with F and F', the line of no pressure would still be AL, and the loss of head between the two points F and F' would be entirely a loss of head of pressure, there being no loss of head of elevation.

1142. *Flow of Water in Open Channel.*—If the water is flowing in an open channel, the loss of head is entirely a loss of head of elevation. The water is not flowing under pressure at all. (See Fig. 424.)

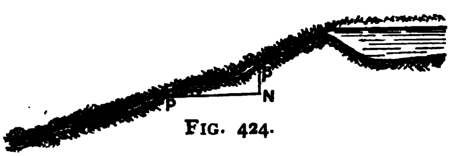


FIG. 424.

Wetted Perimeter.—In an open channel, shown in cross-section in Fig. 424 (*b*), or in a pipe when not flowing full, as shown in Fig.

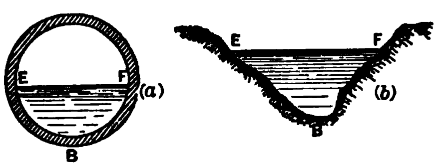


FIG. 424.

424*a*, that portion of the cross-section of the channel, or pipe, wetted by the liquid EBF is called the *wetted perimeter*.

Hydraulic Mean Depth.—The quotient arising from dividing the area of the cross-section of the liquid by the wetted perimeter is called the hydraulic mean depth or hydraulic mean radius

$$= R = \frac{\text{area } EBF}{\text{length } EBF} \quad . \quad . \quad . \quad . \quad . \quad . \quad (665)$$

1143. *Total Head and Effective Head.*—The total head of pressure is equal to the effective head increased by what is commonly called the velocity-head.

The effective head is equal to the loss of head between A and B, Fig. 423, due to the resistance of the straight pipe, while the velocity-head is required to overcome such resistances as are opposed to the entry of the water into the discharge-pipe, and in causing the water in the reservoir or tank which is at rest, to acquire the velocity of flow through the pipe; and when the pipe is not straight, special resistances are developed at the bends, which must be overcome, requiring an increase of head.

We have then the total head = to the head required to overcome the resistances along a straight pipe + the head to impart velocity of flow + head to overcome resistance of entry into orifice + head to overcome resistances at bends. As the total head is thus consumed or lost, it is called loss of head due to resistances in a straight pipe and at bends or elbows; loss of head due to orifice of entry; and loss of head due to velocity. These several losses of head will be discussed separately.

It will be noted that the line LA, Figs. 423 and 425, called the line of no pressure, is drawn on the supposition that GA is the effective head. If we have decided upon a definite velocity of flow, it is easy to determine the head due to this velocity from the well-known formula

$$v = \sqrt{2gh} \quad \text{or} \quad h = \frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \qquad (666)$$

Assuming a velocity of 6 feet per second or 4.1 miles per hour, which is unusually high, $h = \frac{36}{64.4} = 0.56$ feet, or 6.72 inches: and not considering for the present the resistance to entrance into the pipe or from elbows or bends, we should raise the surface of the water 6.72 inches above L in Fig. 425 in order to provide the velocity-head. The line AL is called the hydraulic mean gradient or hydraulic grade-line. Or, what is the same thing, with a fixed water surface at L, in order to maintain the required velocity it would be necessary to lower the line AL to AL', 6.72 inches between L and L'. AL' is then the hydraulic grade-line. It is of the greatest importance to understand the relations between the line or lines of pipes and this hydraulic grade-line in regard to their relative positions throughout their entire lengths.

In Fig. 425 is shown a section of a reservoir which is assumed to be maintained full to the surface SL by a sufficient supply from some source, and it is desired to discharge, at some distant point

A, through some one of the several pipes *OA*, O_1A, O_2KA. The total head of pressure is the vertical distance *BA* between the surface of the water in the reservoir and the point *A* of discharge into the open air. The total head *BA* is composed of the effective head *AC* consumed in overcoming the several resistances to flow through the pipe, and the portion *BC* which imparts the velocity of flow.

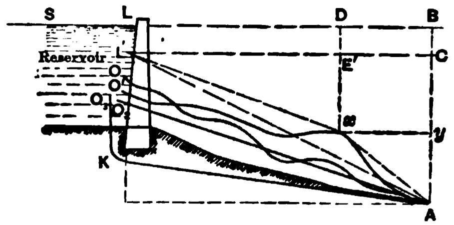


FIG. 425.

The resistance now considered arises from friction, and in pipes of the same diameter running full varies in the same proportion as their lengths. The line *L'A* will be the hydraulic grade-line. And not taking into consideration the small difference of length and the slight bends, any one of the pipes whose upper or entry end is below the point *L'*, regardless of its distance below *L'*, and whose entire length is below *L'A*, the hydraulic grade-line, the quantity of discharge and velocity of flow will be the same from any one of the pipes.

When the water first enters the pipe its velocity will be considerable, as there is but little resistance. The resistance increases and the velocity diminishes as the far end of the pipe *A* is approached. After passing out from this end the resistance and effective head balance, and the remaining portion *BC* of the total head imparts a uniform velocity of discharge.

If pipes opened into any of these pipes and reached above the line *L'A*, it would be found that the water would rise in them to the line *L'A*, and no higher; and the pressure due to these heights will be the bursting pressures at these points of the closed pipes when running full. If while running full the pipes be partly closed by means of valves or by accumulation of sediment or other obstruction, the velocity of flow would be reduced, and a portion of the velocity-head would become pressure-head and increase the bursting pressure, as could be shown by a vertical pipe placed at any point along the main pipe. The obstruction in this case would be at some point between this vertical pipe and the discharge end.

If, on the contrary, the water falls below the grade-line, then the obstruction is between the vertical pipe and the reservoir. If a pipe thus running full be suddenly closed, thereby stopping the flow, the pressure will be sufficiently great to burst any ordinary pipe.

All valves should therefore be closed gradually, and when finally and fully closed the maximum bursting pressure will simply be the hydrostatic pressure due to the depth of the point below the surface of the water in the reservoir.

It is seen that so far as velocity of discharge is concerned for any given pipe, it is only necessary that the upper extremity of the pipe shall be below the point L', provided the water surface SL remains unchanged. The effect of a lowering of this surface is simply to reduce velocity of discharge.

If a pipe is laid exactly coinciding with the grade-line $L'A$, the velocity of discharge would still be the same, and the bursting pressure would be zero at all points of the pipe. Such a condition can rarely exist. On the contrary, it will often be necessary for a line of pipe following the natural lay of the ground to pass above the hydraulic grade-line: this, while not desirable, is not seriously objectionable within a certain limit. This condition is shown in Fig. 425, where $L'A$ is the hydraulic grade-line, and OxA is a pipe passing above $L'A$ at and near x. In this case, while $L'A$ is still called the hydraulic grade-line, it is evidently necessary to consider separately the two portions of the pipe from O to x and from x to A, each having its own hydraulic grade-line, $L'x$ and xA respectively.

The result of this condition is that the effective head for the portion of the pipe Ox is $E''x$, and for the portion xA is yA. The velocity of flow in the portion Ox would be reduced, while that in xA would be increased; and if the pipe is of the same diameter throughout, the total head would be Dx and not BA, and the discharge at A would be due to the head Dx. In other words, the pipe would flow full to x, and only partly filled from x to A, the water running as in an open channel or gutter. Or we could increase the diameter from O to x or diminish it from x to A, the areas of the sections of the pipes being in the inverse ratio of their respective velocities of flow. While no greater quantity would be discharged at A, the pipe would run full throughout its entire length.

If the pipe has the same diameter throughout, it may still run full, as a tendency to vacuum can be maintained at the highest

point x, provided this point in the pipe is not more than 30 feet vertically above the hydraulic grade-line $L'A$. If this vacuum cannot be maintained, the total effective head will only be $E'x$, and the portion xA will only run partly filled.

MOTION OF LIQUIDS IN PIPES.

1144. Thus far we have not considered the causes or the character of resistances to the motion of water in pipes. It has been seen that this motion is entirely due to gravity; if there were no resistances the velocity would increase indefinitely. These resistances arise from the roughness of the inner surface of the pipe, and it is found to be dependent upon the diameter of the pipe. This resistance is not considered in many of the formulæ in general use. The velocity of the liquid is not uniform throughout the cross-section, being much greater in the centre than on the sides. Close to the sides the liquid does not flow freely, but is disturbed by numerous small eddies. The sides become quickly coated with sediment, which removes the roughness of the inner surface. All formulæ for the flow of liquids in pipes are empirical, and few if any of them can be considered as accurate from a scientific point of view, although many of them are accurate enough for all practical purposes.

The two cases to be considered are:

(1) When the pipe is flowing full and the liquid is therefore impelled by the pressure of the head of liquid; (2) when the pipe is flowing partially full. In this case the liquid is not under pressure, and simply flows down, owing to the slope of the pipe, as it would in an open channel.

1145. *Discharge from a Pipe flowing full under Pressure.*—The following are some of the more common formulæ:

Eytelwein's formula is

$$V = 108\sqrt{\frac{D}{4} \times \frac{H}{L}} - 0.13, \quad . \quad . \quad . \quad . \quad (667)$$

in which V is the mean velocity of the water in feet per second; H is the effective head of water in feet, that is, the total head reduced by the various losses of head due to bends, etc.; L is the total length of the pipe in feet; and D is the diameter of the pipe in feet.

The following formula, known as Neville's, is said to be more accurate than Eytelwein's:

$$V = 140\sqrt{\frac{D}{4} \times \frac{H}{L}} - 11\sqrt[3]{\frac{D}{4} \times \frac{H}{L}}. \quad . \quad . \quad . \qquad (668)$$

Thrupp's formula is

$$V = \frac{R^{x+y}\sqrt{\frac{z-R}{R}}}{c} \times S^{\frac{1}{n}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (669)$$

in which R is the hydraulic mean depth (which is equal to $\frac{D}{4}$ for pipes flowing full), S is the slope of the pipe $\left(\text{i.e., } \frac{H}{L}\right)$, and x, y, z, and c are constants depending on the material of which the pipe is made and on the surface of the pipe. For wrought-iron pipes over 3½ inches diameter

$$V = \frac{R^{0.65}}{0.004787} \times S^{\frac{1}{1.80}}.$$

For wrought-iron pipes 1 inch diameter

$$V = \frac{R^{0.657}}{0.004787} \times S^{\frac{1}{1.80}}.$$

For new cast-iron pipes

$$V = \frac{R^{0.63}}{0.006752} \times S^{\frac{1}{2}}.$$

Darcy's formula is

$$V = C\sqrt{\frac{D}{4} \times \frac{H}{L}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (670)$$

The coefficient C has the following values:

TABLE LXXX.

Diameter in inches = $12 \times D$	½	1	2	3	4	5	6	7	8
Value of C.	65	80	93	99	102	103	105	106	107

The maximum value of C for very large pipes is 113.3.

The following example shows the results by the above formulæ:

Find the velocity in a pipe 1 inch diameter, 100 feet long, the effective head being 10 feet.

By equation (667), Eytelwein's formula,

$$V = 108\sqrt{\tfrac{1}{4} \times \tfrac{1}{12} \times \tfrac{10}{100}} - 0.13 = 4.8 \text{ feet per second.}$$

By equation (668), Neville's formula,

$$V = 140\sqrt{\tfrac{1}{4} \times \tfrac{1}{12} \times \tfrac{10}{100}} - 11\sqrt[3]{\tfrac{1}{4} \times \tfrac{1}{12} \times \tfrac{10}{100}} = 4.99 \text{ ft. per second.}$$

By equation (669), Thrupp's formula,

$$V = \frac{(\frac{1}{4} \times \frac{1}{12})^{0.657}}{0.004787} \times \left(\frac{10}{100}\right)^{1.80} = \frac{0.0786}{0.004787 \times 3.594}$$
$$= 4.59 \text{ feet per second.}$$

By equation (670), Darcy's formula,

$$V = 80\sqrt{\tfrac{1}{4} \times \tfrac{1}{12} \times \tfrac{10}{100}} = 3.65 \text{ feet per second.}$$

And similarly for any other heads, diameters, and lengths. In all cases Darcy's formula gives the lowest velocity. It is simple in application, and it is better to underestimate than to overestimate the velocity.

1146. Having found by either of the formulæ the mean velocity of flow, the quantity of discharge is simply the product of this velocity by the area of cross-section of the pipe.

$$\left.\begin{array}{ll} \text{Discharge in cubic feet per second} = V \times \dfrac{\pi D^2}{4}. & (a) \\ \text{Discharge in gallons per minute} \quad = V \times \dfrac{\pi D^2}{4} & \\ \qquad \times\ 6.25 \text{ (or 7.5)} \times 60. & (b) \end{array}\right\} \quad (671)$$

When quantity in gallons is required we use the factor 6.25 for English gallons and 7.5 for American gallons.

The quantity of discharge from a 1-inch pipe with a velocity of 3.65 feet per second, as found by Darcy's formula, gives cubic feet per second $= 3.65 \times \dfrac{3.1416 \times 1}{144 \times 4} = 0.0199$, English gallons per minute $= 0.0199 \times 6.25 \times 60 = 7.45$, United States gallons per minute $= 0.0199 \times 7.5 \times 60 = 8.95$. The U. S. gallon contains 231 cubic inches, the English gallon 277.274 cubic inches.

The more common problem is to find what diameter of pipe is required for a given discharge, the effective head and length being

given. Combining equations (670) and (671(*b*)), calling Q the discharge in U. S. gallons per minute,

$$Q = V\frac{\pi D^2}{4} \times (6.25) \text{ (or } 7.5) \times 60,$$

and from equation (670)

$$V = C\sqrt{\frac{D}{4} \times \frac{H}{L}}.$$

Substituting value of V in first equation, squaring and reducing, we find

$$D^5 = \frac{Q^2 L}{31201 C^2 H}; \quad \therefore D = \left(\frac{1}{7.37} \text{ or}\right) \frac{1}{7.93}\left(\frac{L}{H} \cdot \frac{Q^2}{C^2}\right)^{\frac{1}{5}}. \quad . \quad (672)$$

D being expressed in feet, with D inches, $d = 12 \times D$.

$$d = (1.63 \text{ or}) \ 1.513\left(\frac{L}{H} \cdot \frac{Q^2}{C^2}\right)^{\frac{1}{5}}. \quad . \quad . \quad . \quad . \quad (673)$$

Combining equations (670) and (671*a*), we would get a similar result, which can be put in the form, calling Q' the quantity of discharge in cubic feet per second,

$$\frac{L}{H}Q'^2 = C^2\frac{\pi^2}{64}\left(\frac{d}{12}\right)^5. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (674)$$

Similarly, equation (673) can be put in the form

$$\frac{L}{H}Q^2 = C^2\left(\frac{d}{1.513}\right)^5. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (675)$$

In these last two equations the value of the second member depends alone on d; in other words, for every value of d there is a corresponding value for the first members, namely, $\frac{L}{H}Q'^2$ and $\frac{L}{H}Q^2$. Equations (674) and (675) are used for pipes varying from 4 to 30 inches in diameter. The second can be used for all sizes up to 10 inches. This limitation is only to avoid the use of very large numbers.

Tables can be computed which give the diameters corresponding to a range of values of $\frac{L}{H}Q'^2$ and $\frac{L}{H}Q^2$.

In order to facilitate the calculations for pipe diameters by Darcy's formula, the following tables are copied from Notes on Building Construction, Part 4:

TABLE LXXXI.

FLOW OF WATER IN PIPES RUNNING FULL, IN GALLONS PER MINUTE. CALCULATED FROM DARCY'S FORMULA.

Value of the expression $\frac{L}{H}Q^2$.	Corresponding diam. of pipe, in inches.	Internal diam. of pipe, allowing for incrustation, in inches.	Value of the expression $\frac{L}{H}Q^2$.	Corresponding diam. of pipe, in inches.	Internal diam. of pipe, allowing for incrustation, in inches.
1	0.32	3/8	34,600	2.14	2 1/2
5	0.43	1/2	60,000	2.36	2 3/4
18	0.54	5/8	92,000	2.57	3
48	0.64	3/4	140,000	2.79	3 1/4
117	0.75	7/8	208,000	3.00	3 1/2
250	0.86	1	296,000	3.21	3 3/4
370	0.96	1 1/8	420,000	3 43	4
810	1.07	1 1/4	760,000	3.86	4 1/2
1,410	1.18	1 3/8	1,340.000	4.29	5
2,260	1.29	1 1/2	2,150,000	4.71	5 1/2
3,400	1.39	1 5/8	3,400,000	5.14	6
5,020	1.50	1 3/4	7,500,000	6.00	7
7,400	1.61	1 7/8	16,500,000	7.00	8
10,200	1.71	2	32,800,000	8.00	9
20,800	1.93	2 1/4	60,200,000	9.00	10

L = length of pipe; H = available head; Q = discharge in gallons per minute.

The allowance for incrustation is:

1/8 of diameter for pipes under 6 inches in diameter.
1 inch " " over 6 " " "

TABLE LXXXII.

FLOW OF WATER IN PIPES RUNNING FULL, IN CUBIC FEET PER SECOND. CALCULATED FROM DARCY'S FORMULA.

Value of the expression $\frac{L}{H}Q'^2$.	Corresponding diam. of pipe, in inches.	Value of the expression $\frac{L}{H}Q'^2$.	Corresponding diam. of pipe, in inches.
6.6	4	14,380	18
17.6	5	18,800	19
53	6	24,900	20
115	7	31,500	21
227	8	39,600	22
427	9	49,400	23
730	10	61,500	24
1,190	11	75,700	25
1,850	12	91,700	26
2,740	13	110,600	27
4,020	14	133,400	28
5,740	15	158,300	29
7,880	16	188,000	30
10,650	17		

L = length of pipe; H = available head; Q' = discharge in cubic feet per second. No allowance is made for incrustation.

LOSS OF HEAD.

1147. We have seen that the effective head is the total head diminished by certain minor losses, which will now be determined.

Loss of Head due to Entrance into Orifice.—The orifice of entry obstructs to a certain extent the flow of water into the pipe, thereby causing a loss of head. This loss depends on the form of

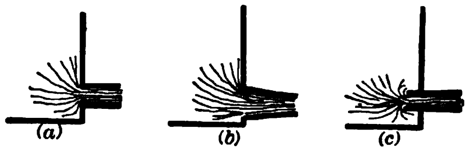


FIG. 426.

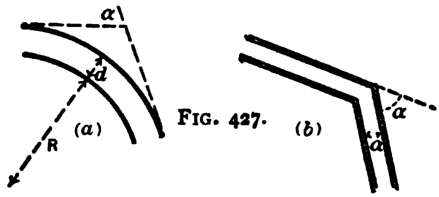


FIG. 427.

the orifice, and is indicated in Fig. 426 (*a*), (*b*), and (*c*). Calling h this loss of head, then $h = V^2 \times C$.

The following are values of C:

For round orifices, such as the end of a pipe, Fig. 426 (*a*),

$$C = 0.007849.$$

For bell-mouthed orifices, Fig. 426 (*b*),

$$C = 0.000444.$$

For pipes of uniform diameter projecting into cistern or reservoir, Fig. 426 (*c*),

$$C = 0.014846.$$

Taking the more unfavorable case, Fig. 426 (*c*), and assuming a velocity of 3 feet per second in the pipe, find the loss of head.

$$h = 3^2 \times 0.014846 = 0.134 \text{ feet.}$$

Loss of Head due to Velocity.—The water in the cistern or reservoir being at rest, a certain amount of head is lost in causing the water to take up the velocity in the pipe. In other words, a

certain amount of energy of position (or potential, which is measured by the head) has to be converted into energy of motion.

This energy of motion is expressed by $\frac{V^2}{2g}$; and if h' is the loss of head due to velocity, $h' = \frac{V^2}{2g} = \frac{V^2}{64.4}$.

Find the loss of head due to a velocity of 3 feet per second.

$$h' = \frac{V^2}{64.4} = \frac{9}{64.4} = 0.14 \text{ foot or } 1.68 \text{ inches.}$$

Loss of Head due to Bends and Elbows.—Calling loss of head due to bends h'', then

$$h'' = b\alpha V^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (676)$$

b is a constant depending on the ratio of the radius of the bend to the internal diameter of the pipe, or $\frac{R}{d}$; α is the change of direction at the bend measured in degrees. (See Figs. 427 (*a*) and (*b*).)

Find the loss of head in a pipe 1 inch diameter; $\alpha = 30°$; $R = 3\frac{1}{2}$ inches; $d = 1$ inch; velocity 3 feet per second.

The value of b is found from the following table:

TABLE LXXXIII.

$\frac{R}{d}$	Loss of head for each degree in change of direction, with mean velocity 1 foot per second.
1	0.000025
1.25	0.000018
1.5	0.000015
2.0	0.000013
3	0.000011
4	0.000011
5	0.000011

Above the ratio $\frac{R}{d} = 5$ the loss of head due to bends may be neglected.

Then $\frac{R}{d} = 3\frac{1}{2}$ and $b = 0.00011$ from above table, and $h'' = b\alpha V^2 = 0.000011 \times 30 \times 9 = 0.003$ foot.

For elbows or abrupt changes in direction, the formula for loss of head,

$$h'' = eV^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (677)$$

e is found from the following table for several values of angle α. (See Fig. 427 (b).)

TABLE LXXXIV.

$\alpha =$	10°	20°	30°	40°	50°	60°	70°	80°	90°
$e =$	0.0001	0.0005	0 0011	0.0022	0.0036	0.0056	0.0083	0.0115	0.0152

For an angle of $\alpha = 30°$, $e = 0.0011$; and for a velocity of 3 feet per second, loss of head $h'' = eV^2 = 0.0011 \times 9 = 0.0099$ foot.

1148. The foregoing losses of head constitute all of the minor losses for which provision must be made.

The effective head, or the loss of head due to the resistance of the pipe in any given length, assuming the water flowing with a velocity of 3 feet per second in a pipe 1 inch diameter and 100 feet long, is, by Darcy's formula, eq. (670), $V = C\sqrt{\frac{D}{4} \cdot \frac{H}{L}}$, with C from table for 1-inch pipe = 80, and $V = 3$;

$$3 = 80\sqrt{\frac{1}{4} \times \frac{1}{12} \times \frac{H}{100}}; \quad \therefore H = \frac{9 \times 4 \times 12 \times 100}{(80)^2} = 6.75 \text{ feet.}$$

Then in the case taken above the total head will be

$$H_0 = H + h + h' + h'' = 6.75 + 0.134 + 0.14 + 0.003 = 7.027 \text{ feet,}$$

whereas the sum of the minor losses is only 0.277 foot, which is less than 4 per cent of the total. Therefore the loss due to bends, entry of orifice, velocity, etc., unless the bends are numerous and sharp and the velocity considerable, are insignificant as compared with the loss due to the resistance of the pipe.

PRACTICAL EXAMPLES.

1149. Case 1. Find the diameter of the service-pipe required to meet the following conditions:

The head in the water-main at the junction with the service-pipe is 50 feet. Assume the condition of intermittent service, on account of which 500 gallons is to be delivered per diem, and

only flowing during a period of four hours. The cock in the service-reservoir or cistern is 40 feet above the main. The length of service-pipe required is 130 feet.

To fill the cistern in 4 hours, or 240 minutes, requires a discharge into the cistern of $\frac{500}{240} = 2.08$ gallons per minute. The available head at the cock is the head of pressure less the head of elevation, $H = 50 - 40 = 10$ feet. As the pipe will be small, we will take the value of C in Darcy's eq. (670), $\left(V = C\sqrt{\frac{D}{4} \cdot \frac{H}{L}}\right)$, to be 65 (see Table LXXX); and from eq. (672), $D = \frac{1}{7.93}\left(\frac{L}{H} \cdot \frac{Q^2}{C^2}\right)^{\frac{1}{5}}$. Substituting values,

$$D = \frac{1}{7.93}\left[\frac{130}{10} \cdot \left(\frac{2.08}{65}\right)^2\right]^{\frac{1}{5}} = \frac{1}{7.93}(0.017981)^{\frac{1}{5}} = \frac{1}{7.93} \times 0.42 = 0.0531 \text{ ft.,}$$
$$\text{or } 0.64 \text{ inches} = d.$$

The value assumed for $C = 65$ was for a ½-inch pipe, whereas the pipe is 0.64 or ⅝ inch, nearly. $C = 70$, a proportionate value between 65 and 80, the respective values corresponding to ½ and 1 inch pipes. Substituting this new value of $C = 70$ and reducing, we find $D = \frac{1}{7.93} \times 0.41 = 0.53$ foot, or 0.63 inch $= d$.

The two values are practically the same. Adding one sixth for incrustation, we have $d = 0.74$ inch, or say a ¾-inch pipe.

Applying equation (675) in connection with Table LXXXI, $\frac{L}{H}Q^2 = \frac{130}{10} \times (2.08)^2 = 56.24$. The nearest value in the table to this is 48, which corresponds with a ¾-inch pipe, as above found. By the use of such a table the labor of calculation is greatly reduced.

The effect of imperfect valves or cocks is to reduce the effective area of the service-pipe, if the area is reduced the velocity at the cock must be increased.

The number of cubic feet of flow is $V\frac{D^2\pi}{4}$ per second, or $V\frac{D^2\pi}{4} \times 7.50 \times 60$ gallons per minute; that is,

$$Q = V\frac{D^2\pi}{4} \times 7.50 \times 60 = 353.43\,VD^2;$$

$$V = \frac{1}{353.43}\frac{Q}{D^2} = 0.00283\frac{Q}{D^2},$$

where D is in feet. Or

$$V = 0.41\frac{Q}{d^2},$$

where d is in inches.

Where the full bore, $\frac{3}{4}$ inch in diameter at the cock, is maintained, the velocity of flow required to deliver 2.08 gallons per minute is

$$V = 0.41 \times \frac{2.08}{(0.75)^2} = 1.52 \text{ feet per second.}$$

If d is reduced to $\frac{1}{2}$ inch, $V = 3.41$ feet per second.

If d is reduced to $\frac{1}{4}$ inch, $V = 13.65$ feet per second.

While small reductions are immaterial, a considerable change is a matter of importance.

In the last case the velocity required is 13.65 feet to deliver the required flow. The loss of head to produce this velocity is $h' = \frac{(13.65)^2}{64.4} = 2.9$ feet.

To find the effect of this loss of head. The available head, instead of being 10 feet, is now $10 - 2.9 = 7.1$ feet; then $\frac{L}{H}Q^2 = \frac{130}{7.1} \times (2.08)^2 = 79$, which would require a $\frac{11}{16}$-inch pipe, or a market size of 1 inch diameter. The effect of any other reduction could be found in the same manner.

If we assume that in the length of pipe, 130 feet, there are 10 bends of 90° each and radius of 3 inches, to be on the safe side the diameter of the pipe should be the actual diameter required, and not that increased to allow for incrustation. In this case the least diameter is 0.63 inch, and not 0.75 inch. Then from eq. (676), of loss of head from bends, $h'' = b\alpha V^2$; the ratio $\frac{R}{d} = \frac{3}{0.63} = 4.92$, the value of b corresponding in table $= 0.000011$.

The required velocity for discharge of 2.08 gallons per minute, through a pipe 0.63 inch diameter, is $V = 0.41\frac{Q}{d^2} = 0.41\frac{2.08}{(0.63)^2} = 2.15$ feet per second; then $h'' = 0.000011 \times 90 \times (2.15)^2 = 0.0045$ foot, loss of head at each bend, and for 10 bends 0.045 foot, which

is so small that it need not be considered, as has already been intimated would be the case.

1150. *Water-supply to a House with Constant Service.—Case* 2. Diagram Fig. 428 shows a main and service-pipe leading to the house. Assuming that the supply is required to be 4 gallons per

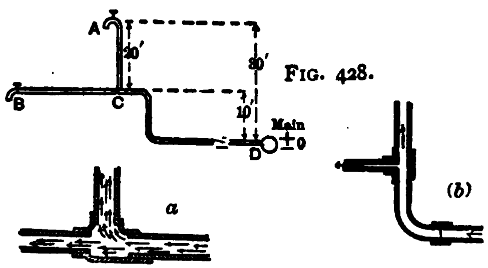


FIG. 428.

minute at the spigot *A* and 3 gallons at *B*, when both spigots are open. Required to find the diameter of the pipes, and also the discharge from each spigot when the other is closed. The elevation of the spigot *A* above the main is 30 feet and of *B* 10 feet. As the pipes will not be required of a great diameter, assume the pipe *BC* to be ½ inch diameter and 30 feet in length; then from Table LXXXI $\frac{L}{H}Q^2 = 5$, $L = 30$ feet, $Q = 3$ gallons; hence $H = \frac{30 \times 3^2}{5} = 54$ feet will be the loss of head from *B* to *C*. *C* and *B* being at the same level, the available head at *C* must be 54 feet, and since the spigot *A* is 20 feet above *C*, the available head is $54 - 20 = 34$ feet. Then with length of pipe $AC = 20$ feet $= L$, $Q = 4$ gallons, and $H = 34$ feet. $\frac{L}{H}Q^2 = \frac{20}{34} \times 16 = 9.4$. The diameter corresponding in Table LXXXI is between ½ and ⅝ inch. Without considering the minor losses of head, as was seen, the loss due to bends proper in a line of pipe are so small that they may be neglected here. But if the junction of pipes at *C* is as shown in Fig. 428 (*a*), there will be a loss of head required to impart the velocity in the pipe *CA*, and a further loss of head caused by eddies, as indicated by the arrow-heads; this is taken at three times that due to the velocity. As 4 gallons per minute is to be discharged at *A*, the velocity required, assuming that the larger-sized pipe, ⅝ inch,

is used, is from equation $V = 0.41 \frac{Q}{d^2} = 0.41 \times \frac{4}{(0.54)^2} = 5.63$ feet per second, not allowing for incrustation. (See table.)

Loss of head due to this velocity $h'' = \frac{(5.63)^2}{64.4} = 0.492$ foot. The total loss on account of this connection will then be $0.492 \times 3 = 1.476$ feet.

If the bore of the spigot is only 0.5 inch, the velocity of discharge must be $V = 0.41 \times \frac{4}{(0.5)^2} = 6.56$ feet, and a further loss of head $= \frac{(6.56)^2}{64.4} = 0.67$ foot; but as the head required to produce the flow in the pipe is 0.492 foot, the loss due to the contraction at the spigot is $0.67 - 0.492 = 0.178$ foot. The total minor loss of head is then $1.476 + 0.178 = 1.654$ feet. The available head is therefore $34 - 1.654 = 32.346$ feet, or say 32 feet; then $\frac{L}{H} Q^2 = \frac{20}{32} \times 16 = 10$. This from Table LXXXI calls for $\frac{9}{16}$-inch pipe, or say $\frac{5}{8}$ inch.

It only remains to determine the size of the pipe leading from the main at D to the point C (see Fig. 428), CD being taken 100 feet long. We found above that the available head at C should be 34 feet, and as there is a rise of 10 feet between DC, and the available head at the main D being 90 feet; there remains to overcome the resistance in the pipe CD, an available head of $90 - 44 = 46$ feet.

$$\frac{L}{H} Q^2 = \frac{100}{46} \times (3 + 4)^2 = 106.5,$$

which in table corresponds with a $\frac{7}{8}$-inch pipe, or the nearest market or standard size 1 inch diameter.

If the connection at C is made as shown in Fig. 428 (b), then the loss of head due to eddies, etc., 1.476 feet, would not have occurred, and the loss would occur only in branch BC.

To find the discharge from one spigot when the other is closed, Assuming A open, B closed, the available head is evidently $90 - 30 = 60$; or allowing 2 feet for all minor losses, $H = 58$ available head. L is now equal to $100 + 20$; then $\frac{L}{H} Q^2 \; \frac{100 + 20}{58} Q^2 = 117$ from Table LXXXI, corresponding with a $\frac{3}{8}$-inch pipe; then $Q^2 = \frac{117 \times 58}{120} = 56.55$ and $Q = 7.5$ gallons per minute.

When spigot A is closed and B open, we have from C to D a $\frac{3}{8}$-inch pipe and from C to B a $\frac{1}{2}$-inch pipe. Whatever may be the required head at C, call it h_x, then the available head in the portion $CD = 90 - 10 - h_x$.

Then in the pipe CB, $L = 30$ feet, head $= h_x$; and since the pipe is $\frac{1}{2}$ inch diameter, $\frac{L}{H}Q^2 = 5 = \frac{30}{h_x}Q^2$, and for the pipe CD, $L = 100$ feet, $H = 80 - h_x$; then for a $\frac{3}{8}$-inch pipe from Table LXXXI

$$\frac{L}{H}Q^2 = 117 = \frac{100}{80 - h_x}Q^2.$$

The discharge Q must be the same through both pipes. Dividing the last two equations into each other,

$$\frac{30}{100} = \frac{5h_x}{117(80 - h_x)}; \quad \therefore h_x = 70.02 \text{ feet.}$$

Substituting this value in $5 = \frac{30}{h_x}Q^2 = \frac{30}{70.02}Q^2$, we find $Q^2 = 11.67$ and $Q = 3.41$ gallons per minute. The same result is obtained by substituting in $117 = \frac{100}{80 - h_x}Q^2$. $Q = 3.41$ gallons per minute.

1151. *Water-supply to Eight Houses in a Street.—Case* 3. The following example is given substantially as worked out in Notes on Building Construction. The exact solution, involving the use of several quadratic equations, would be long and tedious.

Figs. 429 (a) and (b) show the main, the branch main, and position of the eight houses. The levels of the branch main and spigots are shown in Fig. 429 (b). The connections in each house are shown in Fig. 428.

Find the diameter of the branch main capable of supplying each house with 5 gallons of water per minute.

The highest spigot in Fig. 429 (b) is 29 feet above the main, and is connected with the main by 120 feet of $\frac{3}{8}$-inch pipe. The discharge from it is 4.9 gallons per minute when the head in the main is 90 feet. This example is worked out in every respect similar to the one already worked out in Case 2, the diameters of the pipes being somewhat different, and the English gallon used instead of the U. S. gallon.

It will be seen that the discharge is almost the same when both spigots are running. The lower spigots are therefore not considered in this example.

Size of the Main. Half of the Spigots Opened Simultaneously.—It is very unlikely that all the houses will require water at the same time, and a very safe assumption to make is that half the number of houses require the full amount of water at the same time. Supposing, therefore, that the upper spigots in houses 1, 2, 1', and 2' are opened simultaneously, and that an average of 4.9 gallons is running out of each, then the head at A_2 is $90 - 29 = 61$ feet more than the height of the spigot, that is, $H_2 = 61 + 31 = 92$ feet. That is, the available head required to produce a discharge of 4.9 gallons from the spigot 29 feet above the main is 61 feet, and for a spigot 31 feet above the main the available head must still be 61, while the total must be 92 feet.

As the pressure in the main at A is due to a head of 92.5, as shown in Fig. 429 (a), the available head in the portion of the branch main AA_2 is $92.5 - 92 = 0.5$ foot. The discharge at A_2 must be $4.9 \times 4 = 19.6$ gallons per minute for the four houses. Then

$$\frac{L}{H}Q^2 = \frac{600}{0.5} \times (19.6)^2 = 460{,}000, \text{ nearly.}$$

Referring to Table LXXXI, the nearest value of $\frac{L}{H}Q^2$ corresponds with a pipe 4 inches in diameter.

This main branch might be reduced in size after the junction with each house, but for a series of not more than eight houses no such reduction is made.

All Upper Spigots Opened Simultaneously.—The discharge from each spigot will be reduced very little. Then for a first approximation assume that the average discharge per spigot is 4 gallons per minute when all taps are running. Then the discharge at A_4 must be $8 \times 4 = 32$ gallons per minute.

From Table LXXXI, for a 4-inch pipe, $\frac{L}{H}Q^2 = 420{,}000, L = 120,$ $Q^2 = (32)^2$; $\therefore H = \frac{120 \times (32)^2}{420{,}000} = 0.29$ foot, which is the loss of head between A and A_4. The available head at A_4 is $H_4 = 92.5 - 0.29 = 92.21$ feet, as A and A_4 are at the same level.

The discharge from the spigot A in house No. 4 with $\frac{3}{4}$-inch pipe, from Table LXXXI,

$$\frac{L}{H}Q_4^2 = 48 = \frac{120}{(92.21 - 29)}Q_4^2; \quad Q_4^2 = \frac{48(92.21 - 29)}{120},$$

or $Q_4 = 5$ gallons per minute, and for houses Nos. 4 and $4' = 5 \times 2$ $= 10$ gallons. The discharge at A_3 must then be $32 - 10 = 22$ gallons.

$$\frac{L}{H} Q_3^2 = 420{,}000 = \frac{240}{H} \times (22)^2; \therefore H = \frac{240 \times (22)^2}{420{,}000} = 0.28 \text{ foot},$$

the loss of head between A_3 and A_4, a distance of 240 feet.

In figure (*b*) the branch is 5 feet below the main; hence the

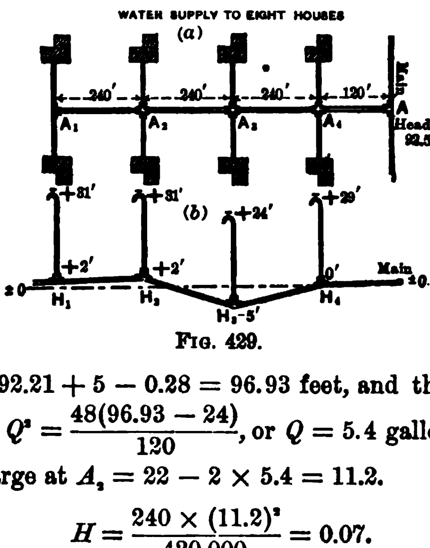


FIG. 429.

head at A_3 is $92.21 + 5 - 0.28 = 96.93$ feet, and the discharge in house No. 3 is $Q^2 = \dfrac{48(96.93 - 24)}{120}$, or $Q = 5.4$ gallons per minute.

The discharge at $A_2 = 22 - 2 \times 5.4 = 11.2$.

$$H = \frac{240 \times (11.2)^2}{420{,}000} = 0.07.$$

And since A_3 is $5 + 2 = 7$ feet above A_2,

$$H_2 = 96.93 - 7 - 0.07 = 89.86,$$

and discharge in No. 2 is

$$Q_2^2 = \frac{48(89.86 - 31)}{120}; \quad \therefore \; Q_2 = 4.8 \text{ gallons nearly.}$$

The discharge at A_1 is $11.2 - 9.6 = 1.6$.

$H = \dfrac{240 \times (1.6)^2}{420{,}000} = 0.014$, the loss of head from A_2 to A_1 and the head at $A_1 = 89.86 - 0.01 = 89.85$.

Discharge in house No. $1 = Q_1^2 = \dfrac{48(89.85 - 31)}{120}$, and $Q = 4.8$ gallons per minute.

The actual quantity of discharge in the eight houses required by the available head and size of branch main is $(5 + 5.4 + 4.8 + 4.8)$ $2 = 40$ gallons, while the estimated average discharge is only 32 gallons. If, then, for a second approximation we take the average at 4.9 gallons for each spigot per minute, the discharge at A_4 must be $4.9 \times 8 = 39.2$ gallons. Then, following the same steps as before, $\frac{L}{H}Q^2 = \frac{120}{H} \times (39.2)^2 = 420{,}000$; $H = \frac{120 \times (39.2)^2}{420{,}000} = 0.44$, loss of head between A and A_4. Hence head at $A_4 = H_4 = 92.5 - 0.44 = 92.06$. Then $Q_4^2 = \frac{48(92.06 - 29)}{120}$, and $Q_4 = 5$ gallons. Discharge at $A_3 = 39.2 - 10 = 29.2$. Loss of head between A_4 and A_3,

$$H = \frac{240 \times (29.2)^2}{420{,}000} = 0.48 \text{ foot.}$$

Head at $A_3 = 92.06 + 5 - 0.48 = 96.58 = H_3$.

$$Q_3^2 = \frac{48(96.58 - 24)}{120}, \quad Q_3 = 5.4 \text{ gallons.}$$

Similarly, $H_2 = 96.58 - 7 - 0.19 = 89.39$ and $Q_2 = 4.8$ gallons. $H_1 = 89.39 - 0.04 = 89.35$; $Q_1 = 4.8$ gallons.

From which we see that the assumed discharge agrees practically with the computed discharge. The calculated discharge is, however, practically the same in the two cases. This is readily understood when we consider that a small alteration in the head produces a considerable alteration in the discharge of a large 4-inch pipe, but causes no practical difference in the discharge of a small $\frac{3}{4}$-inch pipe.

Case 4. Required to find the discharge in the pipe connecting house No. 3 when only this spigot is opened. As yet we do not know the head at $A_3 = H_3$; but whatever it is, we find by referring to the preceding values of Q_3^2 the discharge through the $\frac{3}{4}$-inch service-pipe is

$$Q_3^2 = \frac{48(H_3 - 24)}{120};$$

and for the 4-inch pipe,

$$Q_3^2 = \frac{420000(92.5 + 5 - H_3)}{240 + 120}.$$

Q_1^2 is the same in both equations, hence

$$48(H_1 - 24) = \frac{420000(97.5 - H_1)}{3}.$$

Hence $H_1 = 97.5$ feet, nearly. That is to say, that the flow of water from A to A_1 is so little that there is no appreciable loss of head. Therefore

$$Q_1^2 = \frac{48(97.5 - 24)}{120} \quad \text{and} \quad Q_1 = 5.42 \text{ gallons per minute;}$$

and the discharge is practically as found before. In the above example though the service-pipe is ¾ inch diameter, the quantity 48 corresponds to an effective diameter after incrustation of only 0.64 inch. When the pipes are new, however, the effective diameter is 0.75 inch, corresponding to the value of $\frac{L}{H}Q^2 = 117$. Substituting this in the last equation, (48), we find

$$Q_1^2 = \frac{117(97.5 - 24)}{120} \quad \text{and} \quad Q_1 = 8.4 \text{ gallons per minute.}$$

In other words, the discharge will be considerably greater when the pipes are new.

FLOW OF WATER IN OPEN CHANNELS.

1152. *Discharge from Open Channels and from Pipes flowing partially Full.*—As has been already stated, water flowing in an open channel is not flowing under pressure, and the loss of head is entirely a loss of *head of elevation.*

Gravity is the cause of the flow of water, whether in closed pipes or in open channels. We have seen that in closed pipes,

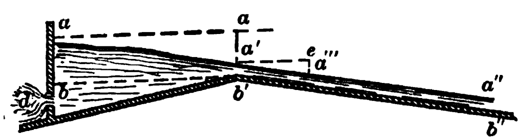


FIG. 430.

when running full, the water flows under pressure, and so far as discharge is concerned the flow may be the same whether the pipe is horizontal, inclined downwards, or inclined upwards, pro-

vided the head of pressure is the same. It is also essential that the entry end of the pipe shall be immersed below the surface of the water in the cistern or reservoir, at a depth below the surface so that the head of pressure may be sufficient to overcome the resistance and impart the required velocity in the pipe, this depth being measured to the centre of gravity of the section of the pipe at its end.

If, however, as shown in Fig. 430, water issues from an orifice at d into an open channel $db'b''$, it will simply rise until attaining the level bb', and if no more water issues from the orifice it would remain at rest. If, however, this condition of equilibrium is destroyed by allowing a constant volume of water to issue, the water surface will rise above bb', and the fluid particles, under the action of the force of gravity, will flow towards b'. When the motion of the water is fully established and the flow past the point b' has become uniform, the surface of the water will slope downwards towards a''.

This inclination is at once the cause and effect of the flow, and, being a resultant of a constant force, gravity, may be used as a measure of the portion of that force which is consumed in maintaining the velocity of flow.

Let the channel be extended with a uniform inclination from b' to any desired length, as from b' to b''.

1153. *Resistances to Flow.*—Fig. 430 represents a vertical longitudinal section along the channel, which may have bottom and sides of any material—of timber, stone, iron, or of earth as in an open ditch or canal. But for the resistances developed to the flow, the velocity would increase indefinitely under the accelerating force of gravity. The resistances are due to friction and adhesion of the particles to the bottom and sides, and to the obstructions caused by projections, and to the air resistance on the surface.

If, then, S is the area per unit of length of the wetted surface or perimeter of the channel, S_1 the area of the water surface per unit of length, A the area of the cross-section of the water column, v the mean velocity of flow of the stream in feet per second, all of the above quantities in feet or square feet; and, finally, $L =$ length of the channel in feet, measured from a', where the flow is uniform, —the total resistance is found to vary directly as S, directly as a certain part S_1 taken at $\frac{1}{10}S_1$, directly as the length, directly as the square of the velocity multiplied by some coefficient m, and inversely as A, the area of the stream.

If R is the sum of all of these resistances in foot-pounds per second, then

$$R = \frac{S + 0.1S_1}{A} \times L \times mV^2. \quad . \quad . \quad . \quad . \quad (678)$$

When the surface of the water is level the entire force of gravity acts through it as pressure, but when the surface is inclined a portion of this pressure is converted into motion.

If in Fig. 430 $a'a'''$ is a length unity, the vertical and horizontal components of this inclination of the surface per unit of length are $a'''e = h''$ and $a'e = l$, respectively.

The effective action of gravity g, in maintaining motion or velocity of flow, varies with the slope, and the slope i is measured by the sine of the inclination, that is, taking the angle for the sine, $i = \frac{h''}{l}$. If there were no resistance to flow, the velocity V would be accelerated in the distance l by an amount $= \sqrt{2gh''}$; but since the flow is uniform, the sum of the resistances in the length l prevent any acceleration, and the velocity continues at the rate established at a', which is due to some height,

$$aa' = h = \frac{v^2}{2g}, \text{ or } v = \sqrt{2gh}.$$

The following considerations and principles will aid in a clearer understanding of this problem.

The velocity referred to in all formulæ is what is known as the mean velocity at any section.

1154. *Greatest and Least Velocities.*—The greatest velocity of the same cross-section of a stream is found at some central point, and the least close to the sides and bottom of the channel. In open channels, like those of rivers, the ratio of the mean velocity to the greatest or central velocity is given approximately, by Prony's formula, as follows:

$$\frac{\text{Mean velocity}}{\text{Greatest velocity}} = \frac{\text{greatest velocity} + 7.71 \text{ feet per second}}{\text{greatest velocity} + 10.28 \text{ feet per second}}.$$

The least velocity is about as much less than the mean velocity as the greatest velocity is greater than the mean. In ordinary cases, the least, the mean, and the greatest velocities may be taken as bearing to each other nearly the proportions of 3, 4, and 5. In very slow currents they are nearly as 2, 3, and 4.

1155. *Steady Flow.—General Principles.*—Steady motion of a mass of fluid means that kind of motion, as distinguished from unsteady motion, in which the velocity and direction of motion of a particle depend on its *position* alone, and not jointly on position and time, so that each particle of the series of particles which successively come to a given point assumes a certain velocity and direction of motion proper to that point. It is, in short, the motion of a permanent current, as distinguished from that of a varying current, or that of a wave.

In order to acquire velocity from a state of rest, or an increase of velocity, a fluid particle must pass from a place of greater total head to a place of less total head.

This it may do either by actual descent from a higher to a lower level, or by passing from a place of more intense pressure to a place of less intense pressure, or by both those changes combined. The loss of head thus incurred is connected with the velocity produced by the following laws:

(1) In a liquid without friction the loss of head in producing a given increase of velocity is equal to the height of vertical fall which would produce the same increase of velocity in a body falling freely; in other words, the loss of head is equal to the height due to the acceleration; and if the particle starts from a state of rest, that height is called the height due to the velocity, or height in feet $= h = \dfrac{v^2}{64.4}$.

(2) If the motion of the liquid is impeded by friction, there is an additional loss of head, bearing to the height due to the velocity of flow a certain proportion, depending on the figure and dimensions of the channel and openings traversed by the stream, including resistances at surface, sides, beds, eddies, roughness of surface, and other circumstances. This loss is expressed by

$$Fh = F\frac{v^2}{64.4} = h''. \quad . \quad . \quad . \quad . \quad . \quad . \quad (679)$$

The combination of these two losses is expressed by

$$H = \frac{v^2}{64.4} + F\frac{v^2}{64.4} = h + h'' = (1 + F)\frac{v^2}{64.4}, \quad . \quad (680)$$

in which F is a factor, determined by experiment, and expressing the proportion which the loss of head by friction, etc., bears to the height due to the velocity.

In an open channel the loss of head H consists wholly in diminution of the *head of elevation*, and is the actual fall of the upper surface of the stream.

In a close pipe it may consist wholly or partly of a diminution of *the head of pressure*, and is then called *virtual fall*, as has been already explained and illustrated.

If the water, instead of starting from a state of rest, has a sensible velocity of flow at the starting-point, the loss of head required is diminished to the extent of the height due to the *velocity of approach*, as it is called. If v_0 is the velocity of approach, the loss of head due to this velocity $= \frac{v_0^2}{64.4}$, and H becomes

$$H' = (1 + F)\frac{v^2}{64.4} - \frac{v_0^2}{64.4}. \quad . \quad . \quad . \quad . \quad (680a)$$

When a stream flows with a *uniform speed* down a *uniform channel*, and two cross-sections of that channel are compared together, the velocities v_0 and v are equal. In this case the whole loss of head between the two cross-sections is expended in overcoming friction and other resistances; then eq. (680a) becomes $H' = F\frac{v^2}{64.4}$.

F in eq. (679) is the sum of the quantities already mentioned, that is, $F = \frac{S + 0.1S_1}{A} \times lm$.

If the sum of the resistances in the length $a'a''' = l$ balance the accelerating force due to the head $a'''e = h''$, then

$$h'' = \frac{v^2}{2g}\frac{S + 0.1S_1}{A} \times lm;$$

$$v^2 = 2g \times \frac{A}{S + 0.1S_1} \times \frac{h''}{l} \times \frac{1}{m}. \quad . \quad . \quad (681)$$

$\frac{A}{S + 0.1S_1} = r$, the hydraulic mean depth, and $\frac{h''}{e} = i$; then

$$v = \sqrt{\frac{2gri}{m}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (682)$$

$$h'' = \frac{lmv^2}{2gr}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (683)$$

The total head $H = h + h'' = \frac{v^2}{2g} + \frac{mvl^2}{2gr}$, and

$$v = \sqrt{\frac{2gH}{1 + m\frac{l}{r}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (684)$$

In long canals and rivers with slopes not exceeding 3 feet per mile the velocity-head h is insignificant as compared with the friction-head h'', and may be neglected in the equation.

When the rate of flow is uniform, h is constant and independent of the length. The friction-head h'' increases with the length.

Taking, then, the value h'' (and neglecting the value of $h = \frac{v^2}{2g}$, which has given the stream its resultant motion), the head-balancing the resistance to flow,

$$h'' = \frac{lmv^2}{2gr}; \quad v^2 = \frac{2gr}{m}\frac{h''}{l} = \frac{2gri}{m}; \quad \text{and} \quad v = \sqrt{\frac{2g}{m}} \times \sqrt{ri}. \quad (685)$$

Eq. (685) gives the mean velocity of flow in feet per second.

If the flow is to be at some predetermined rate, it is necessary to find the inclination or slope i; then from eq. (685)

$$i = \frac{mv^2}{2gr}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (686)$$

The coefficient m is of very uncertain value, and depends on a variety of conditions and circumstances.

It seems impracticable to find any fixed relation between the slope and cross-section of a running stream and the resulting mean velocity, though for a considerable distance above and below the section there may be a relation between the bed of a stream, the surface slope, and the resulting velocity at the section, as applied to any one stream, and any one portion of that stream. No two streams have similar beds, nor the same stream in different portions of length, and these are constantly changing. Again, the slope is seldom uniform over any considerable distance, and it is seldom that the velocity at any section corresponds to the slope across that section.

The value of formulæ for determining the mean velocity is therefore not much to be relied upon, unless in case of uniform bed and slope.

Experience shows that the coefficient m is less for large or deep streams, for high velocities, and for smooth channels, than for small or shallow streams, for low velocities, and for rough channels.

Kutter's formula is

$$v = c\sqrt{ri}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (687)$$

A fuller discussion of this formula, with tables giving the value of the coefficient c, was given in Art. IX. As will be noticed, c corresponds with $\sqrt{\frac{2g}{m}}$ in eq. (685).

Kutter's formula seems to have been accepted as the most reliable and valuable for general use, when tables for the value of c are available. For some practical examples see Supplement.

DISCHARGE FROM PIPES FLOWING PARTIALLY FULL.

1156. The formula already deduced, eq. (687),

$V = C\sqrt{ri}$;
C, a coefficient to be determined by experiment;
$r =$ the hydraulic mean radius or depth;
$i =$ the slope of the pipe;
$V =$ the mean velocity in feet per second.

For pipes running full,

$$r = \frac{\pi D^2}{4} \times \frac{1}{\pi D} = \frac{D}{4}, \text{ and } i = \frac{H}{L}; \quad \text{hence } ri = \frac{D}{4} \cdot \frac{H}{L}.$$

This shows that the formulæ for pipes running full are similar to those for pipes running only partially full, the former being a special case of the latter. This distinction must be drawn, namely, that $\frac{H}{L}$ is simply the head of water divided by the length of pipe, whether the pipe is straight or not, when running full, while the slope i may be different at different points when the pipes are running only partly full.

In Fig. 423, $\frac{H}{L}$ of the equations for pipes running full is $\frac{H_1}{l+l'}$

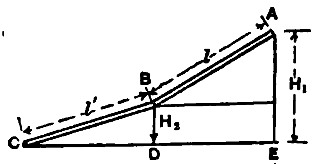


FIG. 431.

in Fig. 431, and the velocity of flow would be the same throughout the length of the pipe.

But when the pipe is only partially full, each portion AB and BC must be considered separately. For the portion AB,

$$i = \frac{H_1 - H_2}{l}, \text{ and for } BC, \; i = \frac{H_2}{l'}, \quad . \quad . \quad . \quad (688)$$

and the velocity would be greater in AB than in AC.

Let Fig. 432 be the section of a pipe running partially full. The discharge

$$Q = AC\sqrt{ri}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (689)$$

A being the area of discharge, that is, a segment of a circle. This area is area of the sector $KNOMK$ minus the area of the triangle KNM, or

$$A = \frac{\phi}{360} \times \pi\frac{D^2}{4} - \frac{D^2}{8}\sin\phi. \quad . \quad . \quad . \quad . \quad (690)$$

The wetted perimeter $NOM = S = \frac{\pi\phi}{360}D$; hence the hydraulic mean depth

$$r = \frac{A}{S} = \frac{\frac{1}{4}D^2\left(\frac{\pi\phi}{360} - \frac{\sin\phi}{2}\right)}{\frac{\pi\phi}{360}\cdot D}, \quad . \quad . \quad . \quad . \quad (691)$$

which substituted in eq. (689), we find the discharge Q.

The more important problem is to find, especially in sewer or drain pipes, whether, when conveying a certain quantity of sewage, the velocity is sufficient to keep the pipe clean without flushing; or, knowing the diameter of the pipe and the discharge, to find the velocity of flow.

From eq. (687), $v^2 = C^2ri$; $r = \frac{A}{S}$; $A = \frac{Q}{v}$; $V^3 = C^2\frac{Q}{Sv}i$;
hence

$$v^3 = \frac{C^2Qi}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (692)$$

In this equation (692) the value of S, the wetted perimeter, is as yet unknown. Assuming a trial value, S', we obtain a corresponding value for $v = v_1$, and from the following table a value A_1 corresponding to S'. The discharge

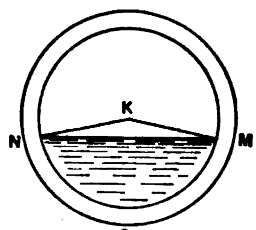


FIG. 432.

$$Q_1 = A_1V_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (693)$$

If Q_1 is greater than the actual discharge, Q, given, then v_1 is too great, or S' is too great. If, on the contrary, Q_1 is less than Q, the value of S' or v' is too small. The proper value of v ought to be found after two or three trials with a sufficient degree of approximation for all practical purposes.

The following table shows the relation between the wetted perimeter $S = \frac{\pi\phi}{360}D$, and the cross-section of flow.

$A = \frac{1}{4}\left(\frac{\pi\phi}{360} - \frac{\sin\phi}{2}\right)D^2$, where ϕ is the angle subtended at the centre by the wetted perimeter and D is the diameter of the pipe.

The table is readily calculated, can be readily extended, or any intermediate values calculated.

$$\frac{S}{D} = \frac{\pi\phi}{360}; \quad \frac{A}{D^2} = \frac{1}{4}\left(\frac{\pi\phi}{360} - \frac{\sin\phi}{2}\right).$$

Assuming $\phi = 57° 18'$, $\sin \phi = 0.84$,

$$\frac{S}{D} = 3.1416 \times \frac{57° 18'}{360} = 3.1416 \times 0.1592 = 0.5001 \text{ or } 0.5;$$

$$\frac{A}{D^2} = \tfrac{1}{4}(0.5 - 0.42) = \tfrac{1}{4} \times 0.08 = 0.020.$$

These results are recorded opposite each other in the above table. Similarly for any other value of ϕ.

TABLE LXXXV.

$\frac{S}{D}$	$\frac{A}{D^2}$	$\frac{S}{D}$	$\frac{A}{D^2}$	$\frac{S}{D}$	$\frac{A}{D^2}$
0.1	0.00025	1.1	0.174	2.1	0.633
0.2	0.00137	1.2	0.216	2.2	0.669
0.3	0.0064	1.3	0.260	2.3	0.699
0.4	0.0105	1.4	0.307	2.4	0.725
0.5	0.020	1.5	0.357	2.5	0 745
0.6	0.034	1.6	0.406	2.6	0.760
0.7	0.052	1.7	0.456	2.7	0.771
0.8	0.075	1.8	0.505	2.8	0.779
0.9	0.103	1.9	0.551	2 9	0.782
1.0	0.136	2.0	0.594	3.0	0.785

PRACTICAL EXAMPLES.

1157. *Case* 1. Having an 18-inch sewer or drain pipe, laid at a slope of 1 in 200, and discharging 0.3 cubic foot per second, required the velocity of flow.

Assume that $\frac{S_1}{D} = 0.6$ foot, $S_1 = 0.6 \times D = 0.6 \times$ (18 inches) or 1.5 feet $= 0.9$ foot. Then from eq. (692), $V^2 = \frac{C^2Qi}{S}$. Interpolating the constant C from Darcy's Table LXXX, par. 1145, for an 18-inch pipe $C =$ about 111; $Q = 0.3$ cubic feet; $i = \frac{1}{200}$; $S = S_1 = 0.9$. Then the corresponding velocity $V_1^2 = \frac{\overline{111}^2 \times 0.3}{0.9 \times 200}$ $= 20.54$, and $V_1 = 2.75$ feet per second.

In the above table LXXXV, corresponding to $\frac{S_1}{D} = 0.6$, we find $\frac{A_1}{D^2} = 0.034$, $A_1 = 0.034 \times (1.5)^2 = 0.0765$ square feet; then $Q_1 = 0.0765 \times 2.75 = 0.21$ cubic foot per second.

This quantity is less than $Q = 0.3$ cubic foot, the actual discharge. The first assumption, that $\frac{S_1}{D} = 0.6$, is too low. Try for a second approximation $\frac{S_2}{D} = 0.7$; $S_2 = 0.7D = 1.05$ feet. Then

$$V_2^2 = \frac{\overline{111}^2 \times 0.3}{1.05 \times 200} = 17.6, \quad \text{and} \quad V_2 = 2.6.$$

Corresponding to $S_2 = 0.7$ in table, $\frac{A}{D^2} = 0.052$, $A = 0.052 \times (1.5)^2 = 0.117$ square foot, and $Q_2 = 0.117 \times 2.6 = 0.304$ cubic foot, practically the same as Q; and therefore the proper velocity of flow $= V = V_2 = 2.6$ feet per second.

If this had proved much too large the proper value of $\frac{S}{D}$ would have been between 0.6 and 0.7, and a recalculation made in a similar manner.

In the case of small drains from 6 to 9 inches in diameter, the velocity should not be less than 3 feet per second; and for larger pipes, where the sewage is well diluted with water, the velocity ought not to be less than 2 feet per second, unless periodical flushing is resorted to.

1158. *Case* 2. The following examples are taken substantially as found in "Notes on Building Construction:"

SIMPLE SYSTEMS OF DRAINS.

Fig. 433 shows a main drain AB, 1283 feet long, and falling on a uniform slope 1.6 feet from B to A. Two branch drains lead into the main, the one, BC, falling on a uniform slope 3.1 feet in a length of 320 feet, and having a maximum discharge of 4 cubic feet and an average discharge of 0.2 cubic foot per second; the other, BD, falling on a uniform slope 12.2 feet in a length of 416 feet, having maximum discharge of 3 cubic feet, and an average of 0.03 cubic foot per second.

Required what size of pipes should be used, and whether any arrangements for periodical flushing should be provided.

For pipe BC. Maximum discharge $Q = 4$ cubic feet per second. $H = 4.7 - 1.6 = 3.1$ feet; $L = 320$ feet. Then $\frac{L}{H}Q^2 = \frac{320}{3.1} \times (4)^2 = 1650$. This number in Table LXXXII corresponds nearest to

1850, which calls for a 12-inch pipe. From which it is seen that this pipe will run very nearly full when having the maximum discharge. Assume it to be running full: then $\frac{L}{H}Q^2 = 1850$, $Q^2 = \frac{1850 \times 3.1}{320}$, and $Q = 4.23$ cubic feet per second.

$Q = AV = V\frac{\pi D^2}{4}$; hence $V = \frac{4 \times Q}{\pi D^2} = \frac{4 \times 4.23}{\pi \times 1^2} = 5.4$ feet per second. This velocity is well above that for flushing.

Find the velocity when the average quantity of sewage is flowing, namely, $Q = 0.2$ cubic foot per second. Proceeding as in Case 1 preceding, assume

$S_1 = 0.8D = 0.8$ foot, since $D = 12'' = 1'$;

$C = 109.5$ for a 12-inch pipe, by interpolation in Table LXXX.

Then, substituting in equation (692),

$$V_1^2 = \frac{\overline{109.5}^2 \times 0.2 \times 3.1}{0.8 \times 320}. \quad \therefore \; V_1 = 3.07 \text{ feet per second.}$$

Note that instead of a slope of $\frac{1}{300}$, as in Case 1, the slope is now $\frac{3.1}{326}$.

Corresponding to $\frac{S_1}{D} = 0.8$ in Table LXXXV, $A_1 = 0.075D^2$; hence $A_1 = 0.075 \times 1 = 0.075$ square foot. Then $Q_1 = 0.075 \times 3.07 = 0.23$ cubic foot per second. This is a little in excess of the actual 0.2 cubic foot, and being greater, for a second approximation $S_2 = 0.75D$. A somewhat smaller value might be assumed and a recalculation made if great accuracy is required. Recalculated on this basis $V_2 = 3.09$ feet, $A_2 = 0.64$ square foot, and $Q_2 = 0.198$ cubic foot, within 0.002 of the proper quantity. In either case, the velocity being over 3 feet per second, no flushing arrangements are required.

For the pipe BD, $Q = 3$ cubic feet; $H = 13.8 - 1.6 = 12.2$ feet: $L = 416$ feet; $\frac{L}{H}Q^2 = \frac{416}{12.2} \times \overline{3}^2 = 307$.

This calls for (see Table LXXXII) a pipe between 8 and 9 inches. Assume a pipe 9 inches = 0.75 foot in diameter.

Find how full this pipe will run when discharging maximum quantity. Again follow Case I. Assume $S_1 = 2.2D = 2.2 \times 0.75 = 1.65$ feet.

The slope i is 12.2 in 416 feet. Hence equation (692) becomes

$V_1^2 = \frac{\overline{108}^2 \times 3.00 \times 12.2}{1.65 \times 416}$; hence $V_1 = 8.5$ feet per second. $C = 108$ for 9-inch pipe. By interpolation in Table LXXX, $Q = 3$ cubic feet. $\frac{S_1}{D} = 2.2$, corresponding value of $\frac{A_1}{D^2} = 0.669$, Table LXXXV; hence $A_1 = 0.669 \times (0.75)^2 = 0.376$ square foot, $Q_1 = 0.376 \times 8.5 = 3.2$ cubic feet per second. For $\frac{S_2}{D} = 2.0$, $V_2 = 8.8$ feet per second, and $Q_2 = 2.95$ cubic feet per second.

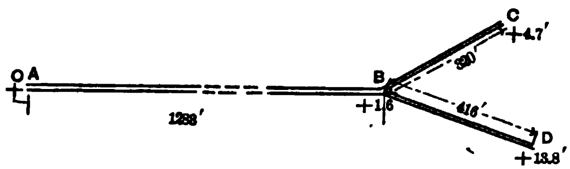
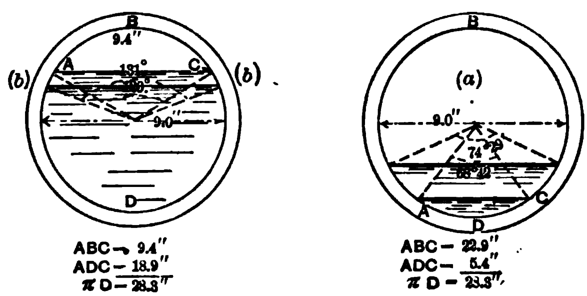


FIG. 433.

As one of these values is a little too large and the other too small, we may take the average, and make the wetted perimeter $S = \frac{S_1 + S_2}{2} D = 2.1D$; and as D is 9 inches, $S = 9 \times 2.1 = 18.9$ inches; In Fig. 434 (*b*), the entire circumference is $\pi D = 3.1416 \times 9 =$



FIGS. 434.

28.2744. The upper or unwetted perimeter will be $28.3 - 18.9 = 9.4$ inches, which corresponds to a central angle $= \frac{9.4}{28.3} \times 360° = 120°$, nearly. The pipe will be filled to the depth shown in Fig. 434 (*b*).

To find the velocity when the average discharge is flowing: Assume $S_1 = 0.6D = 0.6 \times 0.75 = 0.450$;

$$V_1^2 = \frac{\overline{108}^2 \times 0.03 \times 12.2}{0.450 \times 416} = 2.286.$$

Hence $V_1 = 1.32$ feet per second. From Table LXXXV for $\frac{S_1}{D} = 0.6$, $\frac{A_1}{D^2} = 0.034$, $A_1 = 0.0191$ square foot, and $Q_1 = 0.0191 \times 1.32 = 0.025$ cubic foot. This is somewhat less than Q, but near enough.

As the velocity is less than 2 feet per second, periodical flushing would be necessary during dry weather. The wetted perimeter, $S = 0.6 \times 9 = 5.4$ inches; the unwetted, $28.3 - 5.4 = 229$ inches. The central angle subtended by the wetted perimeter $= \frac{5.4}{28.3} \times 360° = 68° 42'$, nearly. (See Fig. 434 (a).)

For the pipe AB, as this is the main drain, it will have to carry the discharge from both BC and BD. Then $Q = 4 + 3 = 7$ cubic feet per second, $H = 1.6$ feet, and $L = 1283$ feet.

$$\frac{L}{H}Q^2 = \frac{1283}{1.6} \times \overline{7}^2 = 39{,}300, \text{ nearly,}$$

corresponding in Table LXXXII with a pipe 22 inches in diameter when running full. But not to run any risk of its being overcharged, and as a 22-inch pipe is rather an unusual size, we will use the nearest standard size, namely, 24 inches.

Assume $S_1 = 2.0D = 4$ feet; C_1 (see Table LXXX) for a 24-inch pipe $= 111.5$; $Q = 7$; $i = \frac{1.6}{1283}$. Then eq. (692) becomes $V_1^2 = \frac{\overline{111.5}^2 \times 7 \times 1.6}{4 \times 1283}$. Hence $V_1 = 3.00$ feet, nearly; $A_1 = 0.594 \times D^2$ (see Table LXXXV for $\frac{S_1}{D} = 2.0$); $A_1 = 2.376$; $Q = 2.376 \times 3 = 7.128$ cubic feet per second $= Q$, nearly.

For average discharge $= 0.2 + 0.03 = 0.23 = Q$; $S_1 = 0.6D = 1.2$ feet; $V_1^2 = \frac{\overline{111.5}^2 \times 0.23 \times 1.6}{1.2 \times 1283}$; $\therefore V_1 = 1.44$ feet per second; $Q = 0.034 \times 1.44 \times 4 = 0.196$ cubic foot.

Making a second trial, $S_2 = 0.65D$, we will find $V_2 = 1.40$ feet per second; $Q_2 = 1.40 \times 4 \times 0.042 = 0.235 = Q$, near enough.

Under maximum discharge the wetted perimeter is $S = 2 \times D = 48$ inches; the total circumference is $3.1416 \times 24 = 75.4$ inches: the unwetted perimeter $= 75.4 - 48 = 27.4$ inches; the central angle is $\frac{27.4}{75.4} \times 360° = 131°$, nearly; and the flow will be as seen in Fig. 434 (b), changing diameter to 24 inches.

Under average discharge, $S = 0.65D = 15.6$ inches, wetted perimeter; unwetted $= 75.4 - 15.6 = 59.8$ inches; central angle $= \frac{15.6}{75.4} \times 360 = 74° 29'$, as seen in Fig. 434 (*a*), changing diameter to 24 inches.

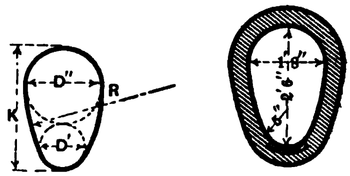
Owing to the low velocity with average discharge, which is well under 2 feet per second, and especially because the velocity is rather low when discharging the maximum quantity of sewage, periodical flushing will be necessary.

It will be also noted that even with the average discharge the wetted perimeter is large in comparison with the amount of liquid. This is one cause of the low velocity; and it is to avoid this condition existing when the pipes have to be of large size to carry maximum flow, and consequently necessitate a low velocity with average flow, that the egg-shaped sections are adopted for sewers under certain circumstances.

EGG-SHAPED SEWERS.

1159. The wetted perimeter can be reduced, the velocity of discharge increased, and at the same time the depth of the flowing liquid increased by using the oval or egg shaped sewer, while quantity of discharge remains unaltered. Sewers of this section are, then, suitable when the slope is small, average discharge small, and maximum discharge large.

Egg-shaped sewers are made of several forms of section. Experience shows that the curves of the top, bottom, and sides and total depth should bear some more or less well-defined relation to each other.



FIGS. 435.

Let D' be the diameter of the lower portion of the section; D'', that of the top of the section; R, the radius of the sides; and K, the depth. Then $D' = \frac{K}{3}$; $D'' = \frac{2K}{3}$; $R = K$, as indicated in Fig. 435.

Case 3. Assuming the same data as for the main pipe AB, Case 2, diameter of pipe 24 inches = 2 feet, slope 1.6 feet in a length of 1283 feet. The area of cross-section of a 24-inch pipe is $\frac{\pi D^2}{4} = \frac{3.1416 \times 4}{4} = 3.1416$ square feet.

The area of the oval is approximately that of an ellipse whose semi-axes are $\frac{1}{2}K$ and $\frac{1}{2}D'' = \frac{1}{2} \cdot \frac{2}{3}K = \frac{1}{3}K$, or area of oval $= \pi \times \frac{1}{2}K \times \frac{1}{3}K = \frac{3.1416}{6}K^2 = 0.5236\,K^2$. Placing this equal to the area of the 24-inch pipe $0.5236K^2 = 3.1416$, $K = 2.45$ feet, nearly. Or taking the depth $K = 2.5$ feet, $D' = \frac{2.5}{3} = 0.833$ feet or 10 inches, and $D'' = 1.666$ feet or 20 inches. From these dimensions Fig. 435 is drawn.

Taking the average discharge $= 0.23$ cubic foot per second, and as this amount will probably not rise above that line separating the lower circle or invert from the sides of the oval, we may consider this discharge as flowing in a 10-inch pipe.

Assuming, then, $S_1 = 1.2D' = 1.2 \times .833 = 1.0$ foot: For a 10-inch pipe $C = 109$, $i = \frac{1.6}{1283}$, $Q = 0.23$. Eq. (692), $V^3 = \frac{C^2Qi}{S}$, becomes

$$V_1^3 = \frac{\overline{910}^2 \times 0.23 \times 1.6}{1.0 \times 1283} = 3.403; \quad \therefore\ V_1 = 1.5 \text{ feet per second.}$$

From Table LXXXV, corresponding to $\frac{S}{D} = 1.2$, $A_1 = 0.216D^2 = 0.216 \times (0.833)^2 = 0.15$ square feet. Then $Q_1 = 0.15 \times 1.5 = 0.225$ cubic feet per second, which makes $Q_1 = Q = 0.23$, practically. The velocity $V_1 = V = 1.5$ feet per second, whereas in Case 2 for the 24-inch pipe $V_1 = 1.4$, showing a small increase in velocity.

The wetted perimeter being $S = 1.2D = 12$ inches, diameter 10 inches, entire circumference $= 31.4$ inches, then $\frac{12}{31.4} \times 360° = 137°\ 36'$ for central angle subtended by wetted perimeter. The versin of depth of the water area $= \frac{1}{2}D' - \frac{1}{2}D' \cos \frac{1}{2}\,(137°\ 36') = 3.19$ inches. Similarly for the 24-inch pipe the depth of flowing liquid $= \frac{1}{2}D - \frac{1}{2}D \cos \frac{1}{2}\,(74°\ 29') = 12 - 9.55 = 2.45$ inches. We therefore have under the same conditions of flow a slight increase of velocity and depth of stream, both of which are advantageous.

Neville's formula for velocity of flow is

$$V = 140\sqrt{ri} - 11\sqrt[3]{ri}.$$

$r =$ the hydraulic mean depth $= \frac{A_1}{S_1} = \frac{0.216 \times (0.833)^2}{1.2 \times 0.833} = 0.15$;

$i = \frac{1.6}{1283}$, as before.

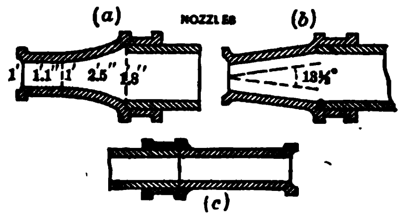
Substituting and reducing, we find $V = 1.92 - 0.63 = 1.29$ feet per second, which is somewhat lower than found above, but agreeing fairly well.

JETS OF WATER.

1160. It is often desirable to know the issuing velocity of a stream of water flowing from a nozzle attached to the end of the pipe or hose. Knowing this velocity we can find approximately the height to which a jet will rise when the nozzle is directed vertically upward, or the maximum height and horizontal distance reached when inclined at any given angle to the horizontal.

Issuing Velocity of Jet.—If the nozzle offered no obstruction to the issuing stream of water, the velocity would be that due to the head at the nozzle, but there is a loss of head due to the resistance in the nozzle, which depends upon the size and form of the nozzle. It has been found by experiment that for the following forms of nozzle the velocity is reduced as follows:

For the form of nozzle shown in Fig. 436 (*a*) the issuing veloc-



FIGS. 436.

ity is $V = 0.97 \times$ velocity due to the head.

For the form shown in Fig. 436 (*b*), $V = 0.94 \times$ velocity due to the head.

For the cylindrical nozzle shown in Fig. 436 (*c*):

When the diameter is $\frac{1}{2}$ to $\frac{1}{3}$ its length, $V = 0.81$;
" " " " $\frac{1}{4}$ " $\frac{1}{12}$ " " $V = 0.77$;
" " " " $\frac{1}{12}$ " $\frac{1}{24}$ " " $V = 0.73$;
" " " " $\frac{1}{25}$ " $\frac{1}{36}$ " " $V = 0.68$,

—multiplied by velocity due to the head.

This reduced velocity is that which must be used in finding the height of rise or range.

Neglecting the resistance of the air, a body falling from a state of rest through a distance h will have acquired a velocity V, such that $h = \frac{V^2}{2g} = \frac{V^2}{64.4}$, and if a body be thrown vertically upward with

an initial velocity V it will rise to the same height as that from which it fell in the first. In other words, the height to which it would rise *in vacuo* is $h = \frac{V^2}{64.4}$, V being the issuing velocity

1161. If the body instead of being projected vertically upward is thrown an angle less than 90° with the horizon, the body will follow a curved path rising to a certain height and then fall to the same level from which it started. This condition is indicated in Fig. 437, in which Ox and Oy are the co-ordinate axes to which the curved path is referred; OA is the direction in which the body is projected; θ is the angle between OA and Ox, the horizontal through the starting-point; OBC is the curved path described by the projectile; and the distance OC on the horizontal line is called the range. Then it is readily shown that

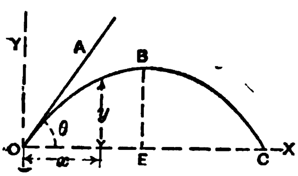


FIG. 437.

$$y = x \tan \theta - \frac{g}{2V^2 \cos^2 \theta} x^2, \quad . \quad . \quad . \quad . \quad (694)$$

y being the vertical and x the horizontal co-ordinates of any point on the curve OBC, V the velocity of projection, $g = 32.2$, and θ any angle.

Eq. (694) shows that the path of the projectile OBC is a parabola with vertical axis, touching OA in O.

To find the range or horizontal distance OC, make $y = 0$ in eq. (694), and x will equal $OC = R$. Solving with respect to x, we find

$$R = x = \frac{V^2}{16.1} \sin \theta \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad (695)$$

To find the maximum rise above OA, $= BE$, place $x = \frac{R}{2} = \frac{V^2}{32.2} \sin \theta \cos \theta$, and solving with respect to y, we find

$$y = BE = h' = \frac{V^2 \sin^2 \theta}{64.4}. \quad . \quad . \quad . \quad . \quad . \quad (696)$$

The resistance of the air prevents any actual projectile near the earth's surface moving exactly as an unresisted projectile. It will more nearly conform to it the slower the motion and the heavier the body, because the resistance of the air increases with the

velocity, and because its proportion to the body's weight is dependent upon that of the body's surface to its weight.

The foregoing relations would be applicable to a jet of water projected *in vacuo*, as for solid bodies, but for the same and even stronger reasons the formulæ are not applicable to a jet of water without the introduction of some factor, determined by experiment, by which the calculation is made as if the initial velocity is less than its actual value. In Table LXXXVI are found these reducing factors for several ratios $\frac{H}{d}$, H being the head at nozzle, and d the diameter of the nozzle.

TABLE LXXXVI.

$\frac{H}{d} =$	200	600	1000	1500	1800	2800	3500	4500
$J =$	0.98	0.95	0.92	0.89	0.84	0.77	0.71	0.50

From which it is seen that the reducing factor J diminishes as the ratio of the head to diameter increases.

PRACTICAL EXAMPLES.

1162. *Case* 4. A jet of water is issuing from a $\frac{1}{2}$-inch nozzle with a velocity of 90 feet per second. Required the height to which it will rise when directed vertically upward.

A velocity of 90 feet has a head $h = \frac{\overline{90}^2}{64.4} = 125.8$ feet; $\frac{H}{d} = \frac{125.8 \text{ feet}}{\frac{1}{2} \text{ inch}} = \frac{125. \times 12}{\frac{1}{2}}$ inches $= 3019.2$. This number corresponds to a value of J between 0.77 and 0.71, or say $J = 0.75$. By interterpolation in Table LXXXVI, $V = 0.75 \times 90 = 67.5$; consequently the true height $h = \frac{(0.75 \times 90)^2}{64.4} = 71.2$ feet.

If a jet is projected vertically upwards, when no wind is blowing, the water falls back on itself, still further reducing the velocity.

1163. *Case* 5. If projected at an angle of $30° = \theta$, then, from eq. (695), the range or horizontal distance $R = \frac{V^2}{16.1} \sin\theta \cos\theta =$ 122.8 feet $= OC$ in Fig. 437, and for the maximum rise $= BE$, $h = \frac{V^2}{64.4} \sin^2\theta = 17.7$ feet.

And similarly for other velocities and angles of elevation.

1164. *Case* 6. Assuming a hose 3 inches in diameter attached to a fire-plug, the following data given: Head at fire-plug = 100 feet, length of hose 300 feet, nozzle end of hose 10 feet higher than plug. The nozzle itself of the form shown in Fig. 436 (*a*). Diameter of nozzle 1 inch. Required the height to which a vertical or nearly vertical jet will rise above the nozzle end, and also the quantity of discharge in gallons per minute.

Let V = mean velocity of issuing jet, and d = 1 inch be diameter of nozzle; then (see page 1446) $V = 0.41\frac{Q}{d^2}$, $Q = \frac{Vd^2}{0.41} = 2.44\,V$.

The velocity due to the head h at the nozzle is $V_1 = \sqrt{64.4h}$. The actual velocity $V = 0.97\,V_1 = 0.97\sqrt{64.4h}$; hence $V^2 = 60.6h$. From $Q = 2.44\,V$, $V^2 = \left(\frac{Q}{2.44}\right)^2$. $\therefore \left(\frac{Q}{2.44}\right)^2 = 60.6h$. $Q^2 = 360.8h$, nearly. From Table LXXXI, for a 3-inch pipe, we have $\frac{L}{H}Q^2 =$ 92,000, after allowing one sixth for incrustation. As yet we do not know, therefore $H = 100 - 10 - h$ and $L = 300$. Hence $Q^2 = 92{,}000\frac{100 - 10 - h}{300}$. Placing the two values of Q^2 equal to each other,

$$360.8h = 92{,}000\frac{100 - 10 - h}{300}. \quad \therefore h = 41.3 \text{ feet.}$$

The ratio of h to the diameter of the nozzle is

$$\frac{h}{d} = \frac{41.3 \times 12 \text{ inches}}{1 \text{ inch}} = 495.6.$$

From Table LXXXVI, the factor J corresponding is about 0.96; and since $V^2 = 60.6h = 60.6 \times 41.3$, we have

$$h = (0.96)^2\frac{V^2}{64.4} = \frac{(0.96)^2 \times 60.5 \times 41.3}{64.4} = 39.0 \text{ feet, nearly,}$$

as we must make the reduced velocity $0.96\,V$, which is the height to which the jet will rise.

The area of the nozzle being $\frac{\pi \times d^2}{4} = 0.7854$ square inch, 0.00545 square foot, and $V^2 = 60.6 \times 41.3 = 2{,}503$, $\therefore V = 50$ feet. Then $Q = A \times V = 0.00545 \times 50 = 0.2725$ cubic foot per

second, and in gallons per minute $Q = 0.2725 \times 7.5 \times 60 = 122.7$ gallons. Or, by using the expression above, $Q^2 = 360.8h = 360.8 \times 41.3 = 122.7$ gallons per minute, as before found.

1165. *Case* 7. As an interesting and useful example, we will take the following: Assuming that it is desired to furnish a farm-house with a supply from a distant point. The supply comes from a small reservoir fed from a spring or running stream. Unless the spring or stream has sufficient flow to give the full supply, the dimensions of the reservoir must be such as to hold enough water to supply the deficiency during dry seasons. When the reservoir is thus supplying the deficiency, the level of its surface will be continuously lowered. The water surface then taken must be near the bottom of the reservoir and not at the high-water level. The capacity of reservoir required can be readily determined by ascertaining (1) the minimum daily supply from the spring or stream, (2) the maximum daily demand for all purposes at the house, recollecting that this demand will be greater during dry seasons, when the supply is the lowest. The difference between these two, daily supply and daily demand, will give the quantity in gallons or cubic feet to be supplied from the reservoir per diem; to this must be added loss by evaporation and seepage, if any. This sum, then, multiplied by 60 or 90 days, or whatever time may be considered necessary, will give the number of gallons to be stored.

Mr. Fanning, in his Treatise on Water-supply Engineering, gives the following rates of water consumption, in American cities, per day:

For ordinary domestic purposes............	20	gallons	per	capita.
" private stables......................	3	"	"	"
" commercial and manufacturing purposes........................	5 to 15	"	"	"
" fountains, drinking and ornamental,	3 to 10	"	"	"
" fire purposes.......................	1/10	"	"	"
" waste to prevent freezing during four months...........................	10	"	"	"
" waste by leakage, flushing, etc.........	5	"	"	"

For evaporation from reservoir surface from 20 to 30 inches per annum. The evaporation will not be considered further in the example now under consideration.

While the above quantities may be a proper basis of estimate for cities, or even for a single house, a much more liberal supply

would be advisable and desirable in the case of a supply for a single establishment.

We will then take the following quantities, distances, etc.:

Distance from reservoir to stable along pipe lines (see Fig. 438).

Main pipe RA = 5280 feet; branch pipe....	AB =	100 feet.
From stable to houses, along main...............	AC =	500 "
Along house branch..............................	CD =	50 "
From house to fountain along main...............	CF =	100 "
Elevation of point of delivery, at stable, above main	AK =	10 "
" " " " " " house, " "	CM =	25 "
" " nozzle of fountain above main.......	FN =	10 "
Rise of jet above nozzle........................	NT =	20 "

Discharge at stable 5 gallons per minute.
" " house 5 " " "

Required the head at each point, and the height to which the jet will rise when no water is being supplied to house and stable.

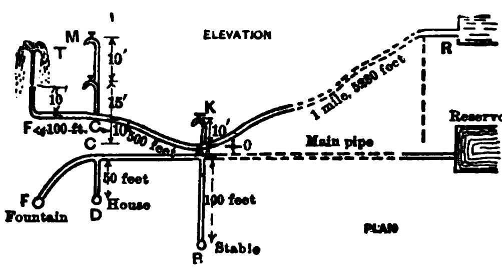


FIG. 488.

For a trial, assume the velocity-head required to give a jet 20 feet high, with a $\frac{3}{8}$ nozzle. Then ratio of head to depth (see Table LXXXVI)

$$= \frac{h'}{d} = \frac{30 \times 12 \times 8}{3} = 960.$$

The reducing factor J in table corresponding to 960 is about 0.93. Substitute $V = 0.93v$ in $h = \frac{V^2}{64.4} = \frac{(0.93\,V)^2}{64.4} = 20$, the rise of the jet, then $V = \sqrt{64.4 \times 20} \div 0.93 = 38.6$.

Using the form of nozzle shown in Fig. 436 (*b*), the issuing velocity is equal to 0.94 × the velocity due to the head; this velocity is then $= \frac{V}{0.94} = \frac{38.6}{0.94} = 41.1$ feet per second.

The actual head due to this velocity is $h' = \frac{(41.1)^2}{64.4} = 26.3$ feet, ich is less than the assumed head of 30 feet. For a second trial ume $h' = 26$, $\frac{h'}{d} = 832$, $J = 0.935$. Substituting and reducing above, we find $V = 38.4$, and $\frac{V}{0.94} = 40.9$; hence $h' = 25.98$ ›t, which gives a practical agreement with the assumed value $= 26$ feet.

To find the discharge $Q = \frac{Vd^2}{0.41}$ (see page 1446):

$$= \frac{38.4 \times 3^2}{0.41 \times 8^2} = 13.2 \text{ gallons per minute, or from}$$

$$= AV = \frac{\pi d^2}{4 \times 144} \times 38.4 \times 7.5 \times 60 = 13.2 \text{ gallons per minute,}$$

›ing a $\frac{3}{8}$-inch nozzle, $= d$.

If now we assume an elevation at the reservoir of 120 feet above ›e lowest point A of the main, we can proceed to determine the iameter of the pipes. The approximate or trial method will be dopted. Assuming then the following diameters (see Fig. 438):

From R to A a $3\frac{1}{2}$	inch	pipe;	length	= 5280 feet;
" A to C a 2	"	"	"	= 500 "
" C to F a $1\frac{1}{4}$	"	"	"	= 100 "

The following quantities of water are required at the ends of ›hese sections:

At A, gallons per minute	$= 13.2 + 5 + 5$	$= 23.2$
" C, " " "	$= 13.2 + 5$	$= 18.2$
" F, " " "	$= 13.2$	

For loss of head between R and A,

$$\frac{L}{H} Q^2 = \frac{5280 \times (23.2)^2}{H} = 208{,}000,$$

number in Table LXXXI, corresponding to $3\frac{1}{2}$-inch pipe. Hence $H = 13.7'$.

From A to C, number in table corresponding to a 2-inch pipe $= 10{,}200$; hence $\frac{L}{H} Q^2 = \frac{500 \times (18.2)^2}{H} = 10{,}200$; $\therefore H' = 16.2$ ft., loss of head in pipe AC.

Similarly in CF

$$\frac{L}{H}Q^2 = \frac{100 \times (13.2)^2}{H} = 810; \quad \therefore \quad H' = 21.5 \text{ feet, loss of head.}$$

Then recollecting that the nozzle of the fountain is 10 feet above F, the head at the nozzle will be

$$120 - 13.7 - 16.2 - 21.5 - 10 - 10 = 48.6 \text{ feet} = h.$$

The corresponding velocity $V = \sqrt{48.6 \times 64.4} = 55.9$ feet per second. The issuing velocity $V' = 0.94 \times 55.9 = 52.5$.

The reducing factor J has already been found $= 0.935$. Hence height of jet $= h' = \dfrac{(0.935 \times 52.5)^2}{64.4} = 37.1$ feet, whereas only a jet of 20 feet rise is called for. No account has been taken of the minor losses of head, which may or may not be of moment.

A recalculation could be made on somewhat smaller pipes, consuming more head in friction, etc., thereby saving something in the cost; or the reservoir might be located at a somewhat lower level, resulting in a shorter main pipe AC.

But let us see, as matters stand, what sizes of pipe are required from the main to the house and stable. The head at the point F is $48.6 + 10 = 58.6$, the loss of head between F and C is 21.5 feet. Therefore the head at $C = 58.6 + 21.5 = 80.1$ feet, the point of delivery for the house pipe is 25.0 feet above the main; hence the head is $80.1 - 25.0 = 55.1$ feet $= H$ and $Q = 5$. Then $\dfrac{L}{H}Q^2 = \dfrac{75 \times 25}{55.1} = 34.0$, which corresponds to ⅝ or ¾ inch pipe, say standard size ¾ inch (see Table LXXXI).

If we change sizes of pipes or height of reservoir to bring the rise of the jet to 20 feet, the head at F becomes $26 + 10 = 36.0$ ft., $36 + 21.5 = 57.5$ feet at C, $57.5 - 25.0 = 32.5$,

$$\frac{L}{H}Q^2 = \frac{75 \times 25}{32.5} = 57.0,$$

corresponding to a ⅞-inch or nearest standard 1-inch pipe.

The head at A is $80.1 + 10 + 16.2 = 106.3$ feet, or subtracting 10, the rise to the point of delivery K, 96.3 feet; then

$$\frac{L}{H}Q^2 = \frac{110 \times 25}{96.3} = 27.5,$$

corresponding to a ¾-inch pipe (see Table LXXXI).

When no water is being supplied to the house or stable the loss ıead will be somewhat less, and the jet will be still higher. The ply of water will now be only that required at the fountain. To ›erfectly accurate, as the issuing velocity will be increased, the harge will be somewhat greater than 13.2, but we will make the ulation on the basis of 13.2 gallons.

Then loss of head from R to $A = \dfrac{5280 \times (13.2)^2}{208,000} = 4.4$ feet;

" " " " " A to $C = 10 + \dfrac{500 \times (13.2)^2}{10,200} = 18.5$ ft.,

from C to N the same as before, $21.5 + 10 = 31.5$ feet, and 1 at nozzle $120 - 54.4 = 65.6$ feet; then

$$\frac{h'}{d} = \frac{65.6 \times 12 \times 8}{3} = 2099,$$

esponding in Table LXXXVI to $J = 0.83$. Then

$$20 = \frac{(0.83 \times V)^2}{64.4}; \quad \therefore \quad V = \sqrt{20 \times 64.4} \div 0.83 = 43.3,$$

ing velocity $= 43.3 \div 0.94 = 40.7$.

$$\text{Rise of jet} = \frac{(46.1)^2}{64.4} = 33.0 \text{ feet.}$$

$$\text{Discharge} = \frac{40.7 \times 3^2}{0.41 \times 8^2} = 14.0 \text{ gallons per minute.}$$

Such problems can be varied to any extent.

With purely empirical formulæ and uncertain values of conıts, it would seem useless to resort to the long and tedious ulations required by many if not all of the formulæ. The trial approximate methods and results are probably as accurate as ld be obtained by the more complicated formulæ. These should checked at some stage of the calculation, using what are purted to be the most accurate methods and formula.

SHIP-RAILWAYS.

1166. A ship-railway, as its name implies, is a railway concted in such manner and with such terminal machinery and liances that a loaded vessel can be lifted bodily out of the er at one terminus, comfortably and safely supported on cars

or cradles, transported on tracks to the other terminus, and lowered into the second body of water.

Space forbids more than a brief allusion to this proposed substitute for ship-canals.

As in many other grand conceptions, projects, and actual constructions, Captain James B. Eads stands either as their originator or, at least, the first to carry them to a point of assured success or to final completion. While his great ship-railway scheme has never been carried out, there is but little doubt that had he lived, it would have been constructed, if not on the exact line proposed, at some other location. As an evidence of this fact there is now under construction the Chignecto Ship-railway, between the Gulf of St. Lawrence and the Bay of Fundy. This work was commenced in 1888, and is now about three-fourths completed. The basin is 500 × 300 feet, with gate 60 feet wide and 30 feet high. The lifting-dock is 230 × 60 feet, of first-class masonry, containing twenty hydraulic presses, with 40 feet lift over a gridiron or cradle. The vessel and cradle rest on wheels, by which the load is distributed over the rails. The weight to be lifted is 3500 tons, including that of the cradle and loaded vessels of 2000 tons displacement.

The railway is double-tracked, seventeen miles long, level and straight; the heaviest gradient is 1 in 500; rails of steel, 110 pounds per linear yard. The vessel is carried over the track on the same cradle used in lifting it out of the water. Two locomotives are to be used, and the speed intended is ten miles per hour. Time required in lifting, transporting, and lowering the vessel is two hours. The traffic is estimated at 12,000,000 tons annually.

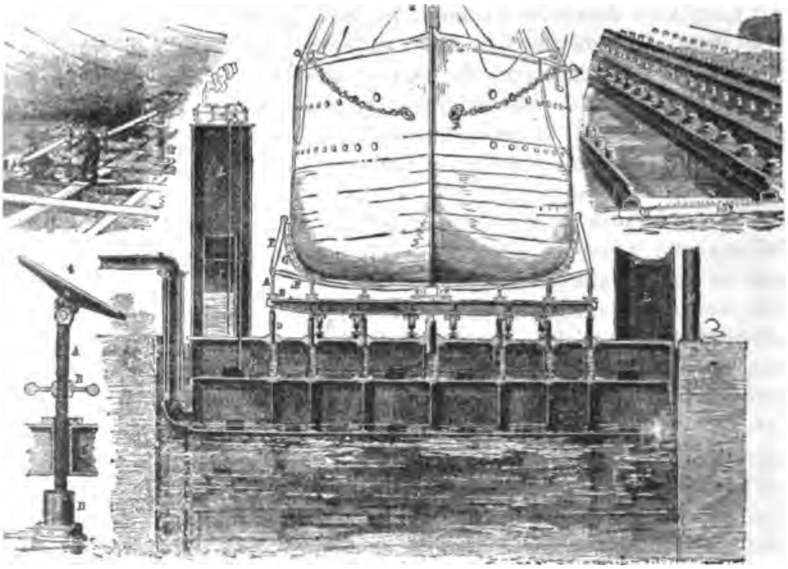
The following description of the proposed Tehuantepec Ship-railway is taken from the *Engineering News*, February 14, 1885:

THE TEHUANTEPEC SHIP-RAILWAY.

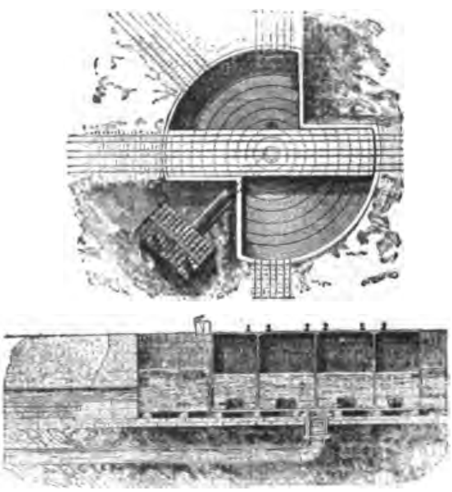
1167. For the illustrations accompanying the following sketch of this proposed ship-railway from the Atlantic to the Pacific we are indebted to the officers of the company; they were engraved by and first issued in the *Scientific American* of Dec. 27, 1884. The description is condensed from data supplied by the company and from London *Engineering*.

The Isthmus of Tehuantepec lies immediately southwest of the promontory of Yucatan, and is the narrowest part of the Isthmus of Mexico. A line drawn between the two termini of the proposed

THE INTEROCEANIC RAILWAY.—SECTIONAL ELEVATION OF PONTOON AND RAILWAY CRADLE.



FIGS. 439, 440, 441, AND 442.



FIGS. 443 AND 444.—ILLUSTRATIONS OF THE TURNTABLES.

railway is almost due north and south. The ground has often been surveyed with the idea of cutting a canal, and notably in 1774, by Don Augustin Cramer; in 1824, by Don Tadeo Ortiz and Don Juan de Orbegoso; in 1842–43, by Señor Moro; and in 1852, by Mr. J. J. Williams, on the part of the Tehuantepec Railway of New Orleans. This last engineer is associated with the present project, and several of his colleagues have been engaged in previous undertakings in the same neighborhood. The concession granted to Mr. Eads gives a right of way across the country about one quarter of a mile in width, upon which he may construct a ship-railway and a line of telegraph. He also enjoys a free right of way on public lands; exemption from all duties on ships, passengers, and merchandise in transit; free importation of all materials required for the railway; exemption from all taxes and contributions on capital stock and property; a grant of 1,000,000 acres of public lands; and a guarantee of protection by the military and naval forces of the country without expense. The terminus of the railway on the north is on the banks of the Coatzacoalcos River, at the town of Minatitlan, about twenty-five miles from the mouth of the river. This is a broad, deep stream, and requires improvement by artificial means only at one point. At the mouth of the river is a bar formed by deposit, and this it is designed to deepen by jetties similar to those at the mouth of the Mississippi. There is now about 15 feet of water over the bar. The line ascends by easy gradients of 42 feet to the mile over the Atlantic plains for about 35 miles. It then enters a gently undulating table-land, from which it passes by a series of broad valleys to the summit-level, the Tarifa plains, 725 feet above the sea. The descent from this point to the Pacific plains has a uniform grade of 1 in 100, and requires three deflecting turntables, since curves of less than 20 miles radius are inadmissible from the form of the carriage, as will be seen later on. From the base of the mountains to the Pacific terminus the line extends over a nearly level country, the station being either at Salina Cruz or on one of the lagoons, as may be found convenient. The climate of the country is quite salubrious, and the surveying parties have been at work there for seventeen months, part of them in the rainy season, without any sickness.

The entire line has been carefully and conscientiously surveyed under the direction of E. L. Corthell, C.E., formerly in charge of the construction of the South Pass jetties, and, later, Chief Engineer of the West Shore Railroad, and Martin Van Brocklin, Resi-

dent Engineer on the Isthmus. As a consequence, sufficient data is at hand to enable a reliable estimate of cost to be made, and the means of overcoming possible obstacles to be well studied, and the difficulties of the way provided for. To further illustrate and practically demonstrate the novel problem, an elegant working model, too large to be regarded as a toy, was made for Mr. Eads by the Messrs. Holtzapffel & Co., of London. In this model the steamer itself is 7½ feet long, and the car, dock, and basin in pro-

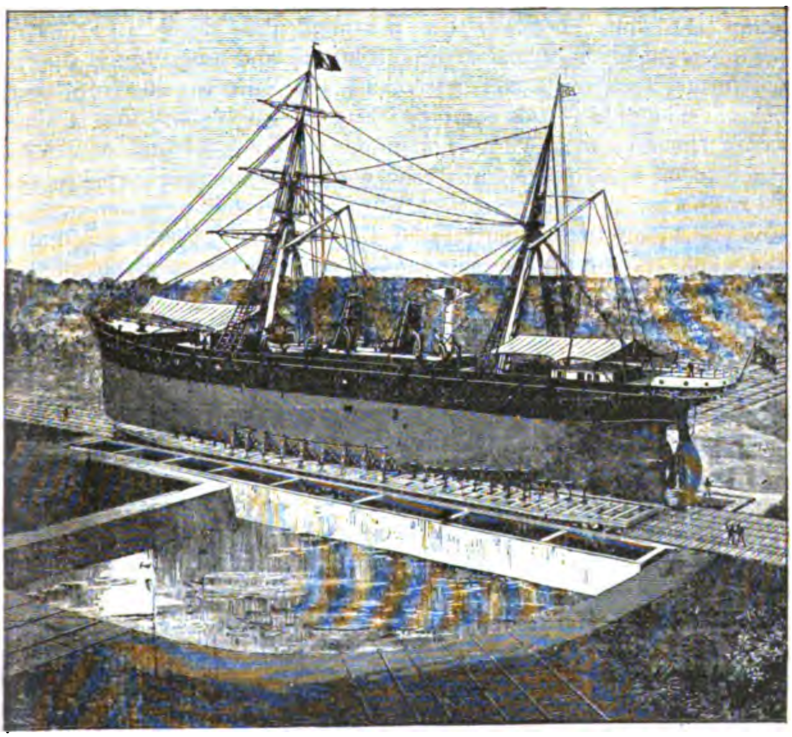


FIG. 445.—THE INTEROCEANIC SHIP RAILWAY—THE FLOATING TURNTABLE.

portion. From this model, while on exhibition in New York, the present engravings were made.

We have already described the details of the working of this ship-railway,* but we reproduce this description from the text accompanying the original engraving.

* In a preceding number of *Eng. News*.—AUTHOR.

The pontoon or floating-dock (see Figs. 439 to 442) is of the same general construction as those in use all over the world, save in some important modifications, rendered necessary to fit it for its special work. For it is not enough that the vessel should be docked and lifted out of the water, but that it shall be caused to rest upon a cradle in such a manner that its weight shall be equalized fore and aft, and thus enable the carriage with its load to move easily and safely. This is effected by means of a system of hydraulic rams arranged along an intermediate deck, about 6 feet below the upper deck of the pontoon. (See Fig. 440.) The arrangement of the rams is in both lateral and longitudinal lines, the former standing a little less than 7 feet apart, the one from the other. The area of the combined rams in each lateral line is the same; the area of the one ram under the keel forward or aft is equal to the area of the five or seven rams amidships. They may be connected and made to work in unison, so that the same pressure per square inch of surface of the rams will exist throughout the whole system, or they may be disconnected by valves, so that a greater pressure may be brought upon the rams in a certain section or on a certain line.

It is no part of the duty of these rams to lift the vessel. They are designed only to resist its weight as it gradually emerges from the basin. They get their power from a powerful hydraulic pump placed on a tower affixed to the side of the pontoon, and rising and sinking with it, but of such a height that, even when the pontoon rests upon the bottom of the dock, it is not entirely submerged. The pontoon itself is directed by powerful guides, which cause it to descend and emerge from the water always in the same position.

A ship having entered the mouth of the Coatzacoalcos River, on the Atlantic side, and come up to the basin, the carriage with its cradle is run on to the floating dock, when water is let into the compartments of the pontoon, and dock and cradle gradually sink to the bottom. Then the ship is brought in from the exterior basin, and so adjusted as to position that her keel will be immediately over the continuous keel-block of the cradle, and her centre of gravity over the centre of the carriage. The water is then pumped out of the submerged pontoon in the manner employed in floating-dock systems, and it rises gradually, bringing the cradle up under the ship's hull (see Fig. 440). As soon as the keel-block of the cradle is close to the ship's keel, the hydraulic pump is called into action, and pushes up the pendent rods and posts of the sup-

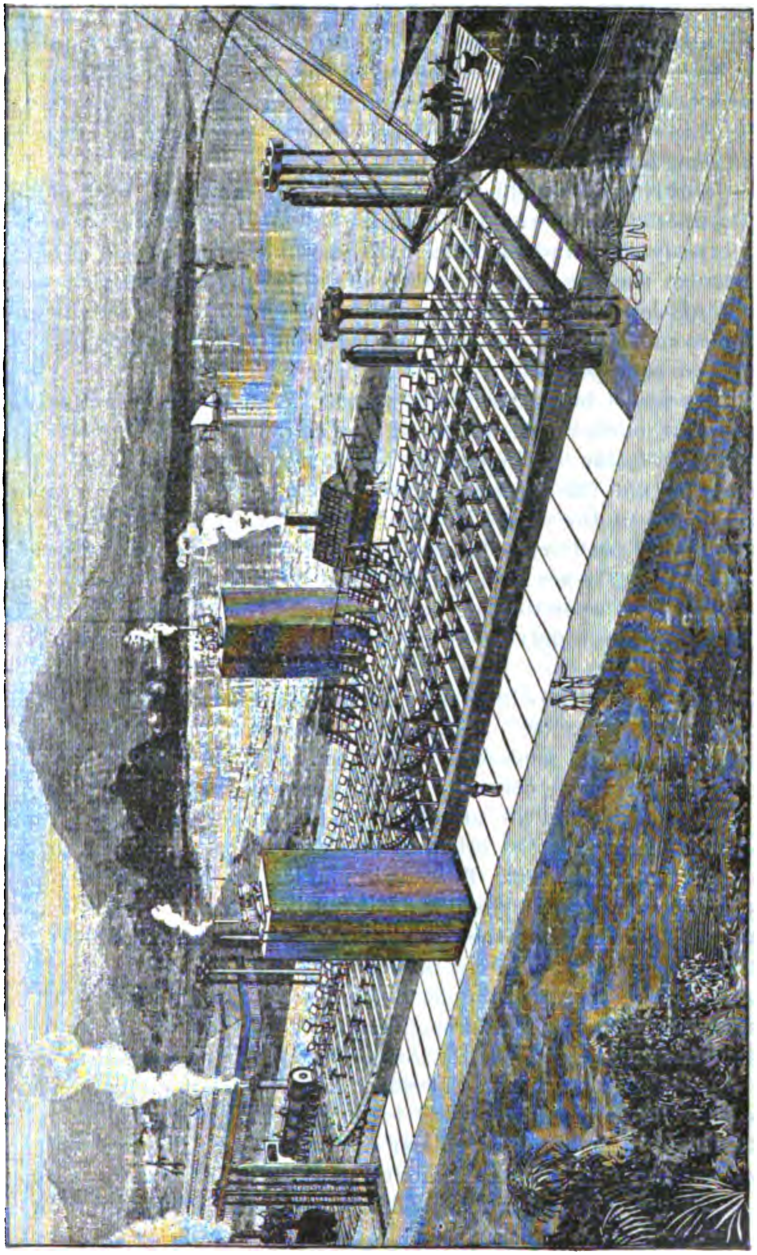


Fig 446.—The Interoceanic Ship Railway—The Lifting Pontoon and Railway Cradle.

ports gently against the vessel, closely following the lines of her hull and the run of the bilge. The pressure upon the rams increases as the vessel emerges from the water, but the water-pressure under them being prevented from escaping by the closing of the valves, the ship's weight, when she stands clear of the water, is borne by the rams by means of the supports.

In the case of a ship weighing five thousand tons, each of the fifty lines of rams would, of course, be called to sustain a burden of exactly one hundred tons; and these lines being placed at equal distances the one from the other, it will readily be seen that each unit of the ship's weight is equally distributed. The weight and displacement of the vessel is learned from the pressure-gauge on the hydraulic pump.

The vessel being clear of the water, hand-wheels or adjusting-nuts that move in threads cut in the columns of the supports are run down to the bearings in the girder plates, whereupon the valve is opened and the rams withdrawn, leaving the girders to support the weight of the ship. Now each girder has the same number of wheels, and, as described above, bears its just proportion of weight and no more; hence each of the multitude of wheels under the carriage is called upon to bear the same weight. This weight has been calculated to be only from eight to nine tons, though tested to twenty.

One of the many ingenious contrivances in the scheme is the "hydraulic governor," so called, and by which the unevenness of the plane of the pontoon, when it comes to the surface with its load, can be readily corrected. This apparatus is thus described:

"Two cylinders are attached to each corner of the dock, one being upright and the other inverted. Plungers attached to the pontoons move in them. These two cylinders are connected by pipes, and all spaces in the cylinders and pipes are filled solid with water. As the pontoon rises, the water, forced out of one cylinder by the ascending plunger, is forced into the inverted cylinder on the diagonal corner where the plunger is being withdrawn. Now, if there is, say, one hundred tons preponderance on one end of the pontoon, one half this weight, or fifty tons pressure, will be exerted by each plunger on that end upon the water in its cylinder. This pressure is instantaneously transmitted through the pipes to the water in the top of the upright cylinder in the opposite diagonal corner, which acts with the same amount of pressure as a water-plunger upon the metal-plunger to hold it down; thus an equilibrium is

aintained, and the pontoon compelled to rise and fall perfectly vel. It is possible, by aid of a pressure-gauge attached to the pes, to ascertain the exact amount of the excess of weight, so that, iould this gauge show too great a preponderance, the pontoon ust be lowered and the ship placed in a new position."

The pontoon cannot elevate the rails on its deck above what ould be a prolongation of the rails ashore, because of the heads of ie anchor-bolts or guiding-rods, and these will also prevent any pping of the pontoons when the ship-burdened cradle is moving f. The carriage, with its cradle, which comes up upon the sub-erged dock, is calculated to hold a ship even more firmly than ie launching cradle used at the ship-yards, with its shores and ays. This carriage moves upon six rails, three standard-gauge acks each of 4 feet 8½ inches. Ships themselves are girders, and ust of a necessity be so, from stem to stern, because in the tem-stuous seas in which they are designed to roam, the one part is nstantly being called upon to support the other; now her bow ojects over a great billow with nothing under to support it, and ;ain she is poised upon a huge wave, leaving the midship section support in great measure both the bow and the stern, and were e not constructed as a girder fore and aft, her back would be oken in the first big sea she encountered. Comprehending this, e designers of the ship-carriage make its strength reach its maxi-um in the cross-girders, which are spaced like the lateral lines of e rams already described; that is to say, 7 feet apart, and having fficient depth and material in their plates to insure an equal posit of weight upon all the wheels. These latter are double-nged and are placed close together, each being hung indepen-ntly on its own journals, and having its own axle. Under an dinary railway car the four- or six-wheel trucks move together out a central pin. But in the ship-carriage, which is not designed move off from an almost straight line, this is not required, and eater strength is obtained by adhering to the rigid principle; isticity being had by placing a powerful spring over each wheel. iese springs will, as said before, bear a weight of twenty tons and ve a vertical movement of about 6 inches, while the maximum ight they will be called upon to bear will not depress them more an three inches, and allow for crossing irregularities without inging an undue weight upon the wheels.

There is also a system of supports for the vessel, each having justable surfaces hinged to the top of the supports by a toggle-

joint in such a way that they may be made to closely follow every depression and yield easily to every protuberance or bulging. They pierce the girders of the carriage, and are exactly pendent over the hydraulic rams when the carriage is on the pontoon and rests in its proper position. Thus, as will be seen, the ship when crossing the Isthmus rests upon what might be called a cushion, and indeed she will have experienced far rougher treatment, both in the Atlantic and Pacific under only ordinary conditions of weather, than that had while *in transitu* by rail across the Isthmus.

As said before, the road is designed to be almost exactly straight, since there will be no curves having a radius of less than twenty miles, for the carriage is four hundred feet long, and rests upon wheels which, as already explained, are not set on trucks swinging to a common centre. There are only five places in the whole line where it is necessary to deviate from a straight line, and at each of these places a floating turntable (see Figs. 443, 444 and 445) will be built. These turntables in design resemble pontoons, for they rest upon water, and will be strong enough to receive the carriage and its burden. The turntable pontoon will be firmly grounded, when the carriage is run upon it, by the weight of water upon the circular bearers of the basin.

The water is pumped out by a powerful centrifugal pump, the water being emitted through an opening in the cylindrical pivot of the pontoon and discharged into the basin. Now, the pontoon has been made sufficiently buoyant to be turned easily upon its pivot by steam-power, and the ship-carriage is quickly pointed in its new direction. The valves then permit the water to enter once more, and the pontoon turntable again rests on its bearings. These turntables may be made to serve another purpose. By their means a ship can be run off on a siding, so to speak, where she can be scraped, painted, coppered, calked, or otherwise repaired without removal from her cradle, and thus be saved the heavy expense of going on a dry-dock.

The locomotives for hauling the ship-carriage over the Isthmian Railway will not differ from those in ordinary use. The big freight-engines of the day have no difficulty, as we know, in drawing freight trains of a total of two thousand tons; and as the ship-carriage moves along three tracks it would be easy, if such a course were necessary, to place three locomotives in front of it and three behind. The time estimated for crossing from ocean to ocean is only sixteen hours.

Accepting the engineering features of the scheme as feasible, and they have been approved by some of the best engineering and naval experts of England and America, the next point to be considered is the cost of construction and the future success of the project from a commercial or dividend-paying standpoint.

As we have said before, there is reliable data upon which to figure the cost of the Eads railway, and experts estimate the expenditure required at $60,000,000, or at $75,000,000 at the utmost. In this point of ultimate cost the plan of Mr. Eads differs widely from the schemes of some of his competitors, notably the Nicaragua-canal project, where the estimates range from $40,000,000 to $140,000,000, and none of these are based upon reliable engineering information.

In comparing the ship-railway with a canal with locks, or even at sea-level, the advantage lies with the former. Speed in transit is the measure of the commercial success of any scheme for passing from one ocean to the other. The plan by which the greatest number of vessels can be safely transported in a given time and with the least tonnage-tax, due cost of construction, will be the one selected by the owners of vessels. The speed of ships through the Panama Canal is put down at 2½ miles per hour, but the railway can carry them over 10 miles in the same time, once the vessel is on its cradle. The roadway, necessarily well built in the beginning, will require less expenditure comparatively than any canal, with the liability to the latter of damage by flood or the filling of the channel from the side-washing in the rainy season. As the ratio of paying cargo to dead load would be so much in excess of that on an ordinary railroad, the operating expenses should be correspondingly decreased; and conservative estimates put the current estimates and cost of maintenance at 50 per cent of the gross receipts, while others figure it as low as 40 per cent.

As to probable business across the Isthmus, the Paris Interoceanic Canal Congress of 1879 estimated the gross tonnage at 7,250,000 tons in 1889. Assuming 6,000,000 tons as a safe basis of estimate, and a tonnage-tax of only $2.00, and deducting 50 per cent for operating expenses, there remains a net profit of $6,000,000 or 8 per cent on $75,000,000; at $3 per ton, the net profit as above would be 12 per cent. While all estimates on the financial future of any project involving so large an outlay must be more or less unreliable, there certainly seems a sound basis for this one when we consider the great success achieved at Suez, where the dividend

in 1882 was 17 per cent and the owners look with confidence for 30 per cent in 1890.

U. S. Engineer Improvement of the Great Kanawha River, West Virginia.—Description with Detailed Drawings of Lock and Dam No. 2.

1168. The completed improvement of the Great Kanawha River, as proposed, will consist of ten locks and dams, carrying the slack water from the mouth of the river to the foot of Loup Creek Shoal, a distance of 90½ miles. Eight of these dams, or all but the two upper ones, are "movable," being kept up in low and down in medium and high stages of the river. The two upper (numbered Lock No. 2 and Lock No. 3 on the profile) are fixed or stationary dams.

A profile of the whole river, with tables, etc., showing the relative locations of all of the locks and dams, with general references and description of each, is shown on the drawings.

LOCK AND DAM NO. 2.

No. 2, as shown by the profile and explained above, is the uppermost lock and dam on the river. It is 85 miles from the mouth and 27 miles above Charleston. The building of the lock, or of the coffer-dam to inclose it, was begun in July, 1883. The lock and dam were completed and put in operation in December, 1887.

Foundations.—The works all rest on solid rock—a hard, medium-grained sandstone, found from 12½ to 14½ feet below extreme low-water mark. The general character and depth of the river bed and bank material is shown by the original cross-section on Fig. 449.

THE LOCK.

General Description.—The lock is 308 feet long between quoins (377 feet from out to out of masonry), and is 50 feet wide in chamber at level of mitre-sills. The top of coping is uniformly 31 feet above lower mitre-sill, and from 37 to 39 feet above foundations. The upper mitre-sill rests on head-bay walls, 11 feet above the lower sill.

The maximum lift of the lock is 12 feet.

Filling and Discharge Valves.—The lock is filled by eight wrought-iron valves, each 5 feet by 2½ feet area, set horizontally in

ought frames in the platform of head-bay. They are manœuvred pairs with endless chains and capstans.

The discharge-valves are in the lower gates, five valves in each .f. A culvert was built through the river wall at the lower te recess, for use with a "Stoney" valve or like device, case additional discharge-way should ever be needed. This lvert has never been used, but is stopped, as shown, by a ınk bulkhead.

The lock has been in operation now over four years, and the rking of the valves has been very satisfactory. The filling-valves easily and rapidly manœuvred; one man by a turn and a half of capstan easily opens two connecting valves in 7 seconds. They ve never been out of order, and there is no noticeable leakage out the valves or any part of the head-bay.

The lock is filled or emptied at maximum lift in 4 minutes. eamboats without tows are locked either way in from 6½ to 8 nutes.

Head-bay Trestles.—Six heavy movable iron trestles are placed the head-bay, anchored to the upper cross-wall, and connected, en standing, by I beams. In case of an accident to the gates, or enever needed for repairs, the trestles will form the support of a n, made either of scantlings (Poirée needles) resting against the per cross-sill and connecting beams, or of plank placed horizonly against the trestles.

Lock-gates.—The gates are of white oak, built without heel or tre posts, the main beams running through and the ends and itres made solid by filling-blocks, assembled with bolts and keys shown. They are suspended at heel on steel gudgeons, and at by fastenings and anchorage all below level of coping. The charge-valves in the lower gates are manœuvred by racks and nions, and the gates themselves by spars and capstans, as shown. e material for the gates was procured by contract, Ainslie, Cochı & Co., of Louisville, Ky., furnishing the ironwork, and W. D. wis, of Malden, W. Va., the timber. They were built and hung hired labor. The cost of the gates complete was $7965.

Quantities and Prices.—The lock, with the exception of the tes and ironwork and the three upper guide cribs, was built der two contracts, there not being funds available at first to conct for all of the masonry. The first contract, made in June, 83, with Mr. Frank Hefright, of Huntingdon, Pa., embraced the sonry, including excavation, coffer-dams, etc., up to the level of

the upper mitre-sill. The second, dated February 27, 1885, with Chas. H. Strong & Son, of Cleveland, Ohio, covered the completion of the lock, except the gates. Both contracts included the furnishing of everything, except the irons built into or attached to the masonry; these were supplied by the United States and put in place by the contractor.

The quantities and prices in the two contracts are given in the table on the opposite page.

THE DAM.

The dam, shore abutment, etc., are shown in Figs. 447 and 454. The dam is a squared timber—white oak, crib filled with stone. It is 524 feet long; average height from bed-rock to crest about 26 feet. The front, from an elevation of 2.75 feet above minimum surface of lower pool, is built in steps. The back is vertical to within 3 feet of top and then sloped, 3 feet to 11, to crest. The width on the bottom and up to first step is 38 feet. The dam is sheathed at the back 15 feet down from the top and banked with heavy dredged material up to within about 3 feet of bottom of slope. The dam and abutment rest on solid rock about 14½ feet below low-water mark.

The site for the dam was dredged out to bed-rock and the bottom cribs, up to above lower pool level, sunk in 24-foot sections. The arrangement of the longitudinal centre sticks, to assist in sinking and keeping the sections in line, is shown in plan, Fig. 454. The sheathing of the steps, in addition to the drift-bolts, is secured at down-stream ends by iron straps and screw-bolts as shown.

The Abutment, etc.—The abutment is about 35 feet high from bed rock; face 48 feet long, with right-angle wings 40 feet. It is low with reference to the dam, the coping being but 8.25 feet above the crest, and was placed well back into the bank (see cross-section, Fig. 447) to lengthen the dam as much as possible and reduce the disturbance of the water and erosion of banks on that side in high stages. Owing to the low abutment the paving on the retaining crib and slope below was made strong enough to stand considerable overfall when the water is above the coping. The bank back of the abutment, as shown by the cross-section, was raised to keep the water from running over it in floods, the embankment extending back to that of the Chesapeake and Ohio Railroad. The material for the fill was taken from the excavation for the abutment.

Kind of Work and Material.	First Contract.			Second Contract,			Totals.	
	Quantities.	Prices.	Amount.	Quantities.	Prices.	Amount.	Quantities.	Amount.
Grubbing and clearing	....	$100.00	$100.00	....	....	....	....	$100.00
Lineal feet of crib logs in coffer-dam	59,740	.25	14,935.00	....	....	....	59,740	14,935.00
Feet B M. sheathing	34,800	20.00	696.00	....	....	....	34,800	696.00
Cubic yards, excavation	30,210	.55	16.615.50	1,562	$0.75	$1,171.50	31,772	17,787.00
Cubic yards, excavation rock	1,326	2.00	2,652.00	....	....	....	1,326	2,652.00
Cubic yards, embankment	5.763	.25	1,440.75	11,416	.28	3,196.48	17,179	4,637.28
Cubic yards, puddling	237	1.00	237.00	77	.80	61.60	314	298.60
Cubic yards, concrete	187	6.50	1,215.50	....	....	....	187	1,215.50
Cubic yards, backing masonry	7,182½	6.50	46,686.25	2,823	6.00	16.938.00	10,005½	63.624.25
Cubic yards, rock-face masonry	2,057½	10.00	20 575.00	1,011½	9.50	9,609.25	3.069	30,184.25
Cubic yards, pointed masonry	861½	11.50	9.907.25	1,037	11.00	11,407.00	1,898½	21,314.25
Cubic yards, cut stone	381	15.00	5,715.00	477½	16.50	7,878.75	858½	13,593.75
Cubic yards, sills	105¾	15.00	1,586.25	....	....	....	105¾	1,586.25
Cubic yards, quoins	28¾	20.00	575.00	65½	22.00	1,441.00	94¼	2,016.00
Cubic yards, coping	....	....	....	393½	28.00	11,018.00	393½	11,018.00
Cubic yards, stone filling	2,095	1.50	3,142.50	1,601	1.40	2,241.40	3.696	5,883.90
Cubic yards, riprap, hand-placed	....	....	....	631	1.40	883.40	631	883.40
Cubic yards, paving	....	....	....	1,189	4.50	5,350.50	1,189	5,350.50
Feet B. M. Timber in permanent construction	110,606	35.00	3,871.28	26,533	30.00	795.99	137,141	4,667.27
Pounds of anchor-bolts and attachments	5,182	.05	259.10	....	....	....	5,182	259.10
Lineal feet, bolt-holes in masonry	....	....	....	1,046	.20	209.20	1,046	209.20
Totals	....	....	$130,209.38	....	....	$72,202.07	....	$202,411.45

Contract for Dam—Quantities and Prices.—The dam, including the abutment and abutment crib and part of the shore protection on that side, was built under a contract with L. M. Petitdidier. The work was begun in March, 1887, and finished the following December. The quantities and payments on this contract were as follows:

	Prices.	Amounts.
Grubbing and clearing site	....	$1,500.00
Excavation, common, 38,445 cubic yards	$0.75	28,833.75
Excavation, rock, 199 cubic yards	1.50	298.50
Embankment, 7872 cubic yards	.35	2,755.20
Puddling, 418 cubic yards	1.00	418.00
Rock-face masonry, 1160 cubic yards	9.50	11,020.00
Coping, 31 cubic yards	16.00	496.00
Stone filling, 15,333 cubic yards	1.55	23,766.15
Hand-placed riprap, 112 cubic yards	2.50	280.00
Paving, 455 cubic yards	4.00	1,820.00
Timber in permanent construction, B. M., 991,773 feet	30.00	29,753.19
Iron in place, 67,755 pounds	.05	3,387.75
Total	....	$104,328.54

MISCELLANEOUS.

Guard-cribs.—Additional guard-cribs were found advisable at the head of the lock, to enlarge the harbor and overcome the danger of boats being drawn over the dam in high water, and the three upper ones were built by contract with Mr. Layton Williams in 1888. In building them 70,837 feet B. M. of square white oak, 1358 cubic yards of stone, and 3230 pounds of drift-bolts were used.

Ironwork.—The principal irons for the lock were procured in two contracts: The first with the Snead & Co. Iron Works, of Louisville, Ky., embracing the filling-valves and frames, gate anchorage, etc., and the second with Ainslie, Cochran & Co., of Louisville, for irons for the lock-gates, and for the trestle-dam at the head of the lock. The two contracts embraced 58,408 pounds of wrought iron, 29,905 pounds of cast iron, and 2122 pounds of steel.

Buildings.—A double lock-house, including also an office and storeroom, was built by contracts with David Eagan at a cost of $4947.82. Two more single lock-houses are yet to be built, and the estimate for them, $900 each, is included in the following summary of cost. The shops and storehouse are cheap buildings used by the

contractors, that were purchased of them after completion of the work and fitted up by the lock hands.

Bank Protection.—Owing to the want of funds in the winter of 1887–88, when the contract for the dam was finished and the lock put in operation, the shore protection was much curtailed. The top-soil of the banks, nearly all of it in fact above the gravel at this site, contains a good deal of sand, and the waves and "back-lash" made by the dam in high stages have washed the banks considerably and made it necessary to strengthen and extend the riprapping from time to time. This has been done by hired labor, assisted by the regular lock hands. About 7200 cubic yards of stone have been used for this since the lock has been in operation. Below the work the riprap now extends down-stream on the abutment side 1210 feet, and on the lock side 1370 feet from the line of the dam. The river was originally somewhat narrowed below the work by a projection of the left bank. This was purposely left unprotected until the scour had straightened the bank and increased the waterway. The high bar below the dam on that side has also been much reduced by scour, assisted by the removal of the large surface stone each season in low water. This increase in the waterway has materially modified the action of the waves and current below the dam in medium and high stages.

Total Cost of Lock and Dam.—The cost of the work complete is as follows:

The lock, as per details of two contracts given above	$202,411.45	
Irons in lock exclusive of gates	1,844.35	
Lock gates complete	7,965.00	
		$212,220.80
Dam as per details of contract given above		104,328.54
Riprapping in addition to lock and dam contracts		7,450.00
Guard-cribs above head of lock		4,000.54
Land at site	$4,013.87	
Buildings and fences	7,285.00	
Inspection, engineering, and incidentals	14,310.00	
		25,608.87
Total		$353,608.75

S. ENGINEER OFFICE,
CHARLESTON-KANAWHA, W. VA., *March*, 1893.

DESCRIPTION, WITH DETAILED DRAWINGS, OF LOCK AND DAM No. 7.

BRIEF DESCRIPTION OF THE RIVER.

1169. The Great Kanawha River empties into the Ohio 262 miles below Pittsburg and 205 miles above Cincinnati. It is a continuation of New River, which rises at the base of Grandfather Mountain between the Blue Ridge and Smoky ranges in Watauga County, North Carolina. The length of the New and Great Kanawha together is about 425 miles.

The Great Kanawha is generally spoken of as being formed by the New and Gauley, the latter joining the main stream two miles above Kanawha Falls, but the Kanawha is commonly regarded as beginning at the Falls. The distance from the foot of the Falls to the mouth of the river, measuring the surveyed line along the shore, is 95.25 miles.

Slope.—The low-water fall of the river is shown in detail by the profile (not given); the total descent from the foot of Kanawha Falls to the mouth of the river (95¼ miles) is 107 feet. It will be noticed that over 46 feet of this occurs in the first 15 miles. The fall may be divided, using round numbers, as follows, viz.: From the foot of the falls to the foot of Loup Creek Shoal, distance 4⅝ miles, fall 22 feet. From the foot of Loup to the foot of Paint Creek Shoal (Lock No. 3), distance 10½ miles, fall 24 feet. From the foot of Paint Creek Shoal to Charleston, 21½ miles, 16 feet. From Charleston to the mouth of the river, 58½ miles, fall 45 feet. The most of the fall in low stages, as shown by the profile, occurs at the shoals and ripples, the natural pools between them having but little descent. As the river rises the slope becomes of course more uniform, and the effect of the shoals and ripples is reduced and finally obliterated.

Bed, Banks, Floods, etc.—The bed of the river is composed of bowlders and gravel, with some sand and mud, getting naturally finer towards the mouth. It is underlaid with rock, found at distances varying from about 7 to 18 feet below low-water mark. The depths to rock as far as ascertained are shown on the profile. The

banks are from 35 to 50 feet high, composed mainly of heavy clay, but with frequent mixtures and strata of sand.

Ordinary Kanawha floods rise about 30 feet in the upper part of the river, and 40 feet near the mouth, above low-water mark. The highest recorded rise at Charleston (September, 1861) was 46.87 feet above low-water. The extreme high-water line in the lower part of the valley, from about Charleston down, is effected by back-water from the Ohio. The highest water at the mouth and for about 25 miles above, as appears by the profile, was caused by the extraordinary flood in the Ohio of February, 1884. This rose at the mouth of the Great Kanawha and for several miles above, as shown, to about 61 feet above low water.

The average width of the river at low water is about 600 feet.

The "hard-pan" overlying the rock at Lock and Dam No. 7, has been found nowhere else in any quantity, and may be considered as peculiar to that site; except as to this, cross-section *AB*, Fig. 449, is a fairly characteristic one of the river, and of the bed and banks, in the pools, or between the shoals. The natural low-water depth between the shoals is generally from about 3 to 7 or 8 feet; in many places, as shown by the profile, it is much deeper. On the shoals there was originally but a few inches of water in low stages, scarcely enough on many to float a loaded canoe or skiff.

(For fuller description of the natural features of the river and valley, reference may be made to the following annual reports of the Chief of Engineers: Report for 1871, page 625; for 1873, page 836; for 1875, part 2, pages 91 and 95; for 1876, part 2, page 163; for 1877, pages 307, 320, and 744.)

Discharge—Charleston Gauge.—The discharge of the river for different stages with reference to the Charleston gauge is given below. This gauge was established in June, 1873, at the beginning of work on the river by the United States. It is set to show the available water for open navigation in the Great Kanawha below Charleston. The zero of the gauge is about one foot below ordinary low water. The extreme low water of 1881 fell one tenth of a foot below the zero.

The discharge of the river in cubic feet per second for the different gauge-readings is as follows. All of these measurements were made at or near Charleston. All but the first and last were

made above the mouth of Elk River; the first was made just below the mouth of Elk, and the last includes the flood discharge of Elk.

Gauge one tenth of a foot below zero (extreme low water), discharge 1130 cubic feet per second; gauge-reading 3.00 feet, discharge 2942 cubic feet per second; gauge 3.90, discharge 4925; gauge 5.28, discharge 8613; gauge 6.80, discharge 12,733; gauge 8.10, discharge 18,562; gauge 9.20, discharge 28,798; gauge 14.40, discharge 47,120; gauge 19.60, discharge 76,851; gauge 32.60, discharge 118,291; gauge 34.60, discharge 155,388; gauge 34.60, discharge, including Elk River, 188.347 cubic feet per second. (See Report of Chief Engineers, 1876, part 2, page 160; for 1878, page 473, and for 1883, page 714.)

Ice.—The Great Kanawha, as a rule, is but little obstructed by ice. In the last 19 years navigation has been suspended by it, wholly or in part, an aggregate of 146 days, an average of less than 8 days per year. The longest suspension was in the winter of 1876–77, when navigation was stopped above Charleston by ice-gorges 39 days. Aside from an occasional exceptionally cold winter, accompanied with low stages of water, the river seldom freezes over, and the moving ice is rarely more than a couple of inches thick. In the winter of 1878–79 it was covered nearly everywhere, except on the shoals, with stout ice from 10 to 12 inches thick. This ice was carried out by a rise from the head-waters of New River; it made remarkable gorges in the Kanawha, and did a good deal of damage to steamboats, barges, coal-tipples, etc.

The Fixed Dams, Lifts, and Number of.—The first project, as stated above, contemplated three fixed dams of 15 feet lift each, above the foot of Paint-Creek Shoal, carrying the improvement to the foot of Kanawha Falls. It was afterwards deemed advisable to change the plan by reducing the lifts of the fixed dams to 12 feet, and locks and dams Nos. 2 and 3 have been so built, the No. 2 pool reaching to the foot of Loup Creek Shoal, as shown on the profile. This is nearly or quite to the upper line of the best coal-deposit on the Great Kanawha (being about where the Lower Coal Measures run out and the thick top sandstone of the Conglomerate series appears), and it is proposed not to continue the improvement further up-stream until the locks and dams are all completed below. The reduction in the lifts will make two more fixed dams necessary

if the slack water is carried to the foot of the falls, making four in all instead of three as first proposed. It may be added, that the experience at Nos. 2 and 3, particularly in regard to the scour of the banks below the works, has fully justified the change of plan, and shown that the height adopted (for maximum 12-foot lifts) is as great as either of these dams should have been built.

Detailed drawings of Lock and Dam No. 2, uniform with those of No. 7 herewith, are now under way for publication by the Department.

The Size of the Locks.—The first project and estimate was for locks with "clear interior dimensions of about 48 to 50 feet in width and from 285 to 300 feet in length." The locks above Charleston are 50 feet wide in the clear and from 300 to 311 feet long between quoins. Before Lock No. 6 was built, the first below Charleston, it was determined, in order to better accommodate the coal trade, particularly large-sized towboats in the lower river, to build all of the locks below Charleston 55 feet wide in the clear and 342 feet long between quoins. The coal-barges are from 24 to 26 feet wide and about 130 feet long. The locks are designed to pass four barges at once, or three barges and a towboat.

The building of the locks and dams was begun, as before stated, in 1875. Progress made to date, relative locations, lifts, etc., of each lock and dam are shown on the profile. This, with some other important features and dimensions of each work, is also given in the following table:

No of Lock and Dam.	Style of Dam.	Max. Lift in feet.	Length of Dam—feet.			Lock—Dimensions—feet.		Location—miles from Mouth.	
			Navigation Pass.	Weir.	Total.	Clear Width.	Length between Quoins.		
No. 2	Fixed	12	....	....	524	50	308	85	Finished in 1887.
No. 3	Fixed	12	....	....	564	50	311	80	Finished in 1882.
No. 4	Movable	7	248	210	458	50	300	78¼	Finished in 1880.
No. 5	Movable	7	250	265	505	50	300	67¾	Finished in 1880.
No. 6	Movable	8½	248	310	558	55	342	54½	Finished in 1886.
No. 7	Movable	8	248	316	564	55	342	44½	To be finished in 1892.
No. 8	Movable	8½	248	292	540	55	342	36	To be finished in 1892.
No. 9	Movable	6¼	248	300†	548†	55	342	25¼	Not begun yet.
No. 10	Movable	7½	248	290†	538†	55	342	18¾	Not begun yet.
No. 11	Movable	10	248	420†	668†	55	342	1¾	Not begun yet.

† Approximate.

THE MOVABLE DAMS.

The movable dams are of the Chanoine wicket type, operated from trestle service-bridges. In general features they are all like Dam No. 7, illustrated and described herein. Dams Nos. 4 and 5 were completed and put in operation in 1880, and were the first movable dams in connection with slack-water improvement built in America. Dam 6 was completed in 1886. Nos. 7 and 8 are now building; both are well along, and will probably be completed during the present season, 1892.

The operation of these dams on the Great Kanawha, the number of and time taken in the manœuvres, difficulties met with, number of days the dams are kept up, the cost of operating and maintaining, etc., each year, are fully described in the annual reports of the Chief of Engineers.

The experience with movable dams on this river has, on the whole, been very satisfactory. They are easily and rapidly manœuvred (in these respects Dam No. 6 and those now under construction have considerable advantage over those first built), the expense of operation and maintenance is but little if any more than with fixed dams, and they prove highly satisfactory to the river interests.

Advantages over Fixed Dams.—The movable dams are kept up whenever there is not water enough in the river for coal-boat navigation, and down at other times. Their advantages over the ordinary fixed dams for a commerce and river like the Great Kanawha are decided, furnishing the benefits of the usual slack water without its most serious drawbacks. With fixed dams everything must pass through the locks; with them navigation is entirely suspended, too, when the river is near or above the top of the lock-walls. With movable dams the locks are only used when the discharge of the river is so small as to make them necessary. At all other times they are down, practically on the river-bottom, out of the way, affording unobstructed open navigation. This is of great advantage to all classes of commerce, and is particularly so with coal, transported as it is, and empty barges returned in "fleets" of large barges. More barges can, of course, be taken by a towboat, and much better time made by all kinds of

craft in "open river," when there is water enough for such navigation, than when the stage or discharge compels the use of the locks.

The movable dams being down in high water, there is comparatively little difficulty in protecting the banks about the works from scour. In this respect they have considerable advantage, too, over the fixed dams.

Modifications, Cost of Operating, etc.—Experience with the dams has naturally suggested improvements, and No. 6, the last one completed, has considerable advantages over those first built in strength and durability of construction, facilities for rapid manœuvring, and cost of operation and maintenance. Dams 7 and 8 have been still further improved in some of their details.

No. 6 has been in operation over five years. The average cost of operating and maintaining the lock and dam has been $2515 per year. This covers wages, supplies, repairs, including considerable addition to the riprapping, and all expenses connected with the work. The entire cost during the five years of repairs on the dam proper and on all of its apparatus, including paints, one of the principal items, has been something less than $250, or an average of $50 per year.

This dam is put up by 4 or 5 men in from 7 to 12 hours; the usual time is about 8 hours. It is lowered with the same force in about 2 hours. No material difficulty has ever been met with in any of the manœuvres at No. 6.

Four men are employed regularly at each work, the same as at the fixed dams. In raising and lowering the dams one or two extra men are often hired.

Manœuvring the Dams.—The operation of raising and lowering the dams is generally understood, or will be inferred from the drawings, but may be briefly described as follows: In raising the pass the bridge is first put up, trestle by trestle (they are connected by chains as shown), beginning at the lock. As the trestles come up, and with them the aprons that make the walk, the rails forming the connections and winch-track are placed. In raising the trestles, the winch is used by means of the small top-crane and sheave. After the bridge is up, the wickets are pulled up one by one with the winch and wicket chains until the props drop into the hurter-seats. The wickets are not erected or

"righted" as fast as pulled up, but left "on the swing" (*en bascule*), that is, with the horse erect, the end of the prop in the hurter-seat, and the wicket in a horizontal position at the top of the horse. In this position the water passes freely under the wicket. If righted as fast as pulled up, the head of water becomes so great that the last wickets cannot be safely handled with the winch. After being put on the swing clear across, they are all rapidly righted; this is done with the drum and brake on the winch and wicket chain, the butt of the wicket being held against the pressure of the water and let against the sill without shock. In lowering the pass, the wickets are pulled up-stream a few inches with the winch by a simple line and grab connection at the top of the wicket. This carries the foot of the prop out of the seat into the descending channel of the hurter, when the grab is disengaged and the wicket falls. After the wickets are lowered the bridge is put down. The manœuvres briefly described above refer particularly to the navigation pass. The weir is manœuvred on the same general plan, but the weir-wickets, being smaller than those of the pass, they can be raised or lowered, put on the swing, or righted with full head whenever desired. The manœuvre of the weir when the dam is up is governed by the stage or discharge of the river, it being kept wholly or partly raised, as required, to regulate the surface of the pool. A pass-wicket, for reasons given above, is never lowered or swung unless the whole dam is to go down.

Telephone Line, Equipments, etc.—Concert of action is necessary in manœuvring the dams and regulating the pools; and the different works are connected with each other and with the central office at Charleston by telephone. The line is also extended to Kanawha Falls, to give notice of floods; and daily communication by mail, and by telegraph when necessary, is had with Hinton, at the mouth of the Greenbrier, 60 miles above the Falls.

A light-service boat, furnished with a derrick, capstan, and and cabin, is required at each movable dam to assist in the manœuvres, transport bridge-rails, tools, etc. A complete diving outfit is also necessary at each.

On the bank, in addition to the houses for the men, a drum-house and tramway, to handle apparatus and tools, a carpenter-shop, blacksmith-shop, and a storehouse are required. Such

buildings, except the drum-house, are in use at the fixed dams as well. All of the ordinary repairs are made by the regular lock hands.

LOCK AND DAM NO. 7.

Lock and Dam No. 7 is located 14 miles below Charleston and 44½ miles from the mouth of the river. The building of the lock, or of the cofferdam to enclose it, was begun in April, 1889. The lock was completed, except the gates, in February, 1892. The foundations and masonry for the dam were begun in the spring of 1891, and it is expected the dam will be completed during the present season, 1892.

Foundations.—The foundations are fully shown by the drawings. The bed-rock at this site is from 11 feet to 15¾ feet below low-water mark. It is overlaid with "hardpan," a tough, indurated clay, varying in depth from 3½ to 8½ feet. On the hardpan is the river-bed of bowlders and coarse gravel, mixed with some sand and mud.

The foundations of the lock, except the upper cross-sill, all extend to solid rock; those of the dam resting partly on the rock and partly on the hardpan, as shown.

THE LOCK.

The lock is shown on Figs. 448 to 455. It is 342 feet long between quoins, with a clear width in chamber of 55 feet. The total length, not including guard-cribs, is 411 feet. The walls, including concrete foundations, are from 27 feet to 31.75 feet high; they are uniformly 20 feet above top of mitre-sills. The maximum lift, when Dam No. 8 is up and the pools full, will be about 8 feet; with No. 8 down, the lift in low water would be about 10 feet.

The Coffer-dam.—The coffer-dam for the lock and guard-cribs was 536 feet long up and down stream, with shore ends 152 and and 134 feet long. It was built of round-timber cribs, sunk in sections to the hardpan, the top bowlders and gravel being dredged off, filled with heavy dredged material, sheathed on the outside, and banked with clay and gravel. The cribs forming the sections were 15 feet wide, 21 feet long, and about 19 feet high, the top of the coffer being about 11 feet above low-water mark.

Foundations.—The hardpan was excavated to solid rock, and in main part replaced by concrete up to within 5.50 feet of top of mitre-sills, at which level the masonry of the walls begins. For the purpose of anchorage, part of the masonry under the mitre-sills extends to the rock as shown (Figs. 451 and 452).

Masonry.—The stone used at No. 7 is yellowish and bluish gray, medium and fine-grained sandstone (probably the "Morgantown" and "Mahoning") from three quarries along the river, from one to seven miles above the site. It weighs about 150 pounds per cubic foot, and the crushing load of two-inch cubes varies from 25,000 to 46,000 pounds.

The chamber-faces of the walls are of pointed-face ashlar, and the other faces generally, except the back of the land-wall, of rock-faced ashlar. The chamber-corners, quoins, sills, and coping are dimension-stone, bush-hammered. The interior of the walls and the back of the land-wall and wings were classified as "backing," described in the specifications as follows: "The backing of all the walls shall be of sound, good-sized, vertical-sided stones. It shall generally be shaped up and bedded top and bottom, and made to correspond with height of front stone before being brought on the wall. The beds of backing not to exceed one inch in thickness. It shall be laid in full beds of mortar, so as to thoroughly bond and break joints. The spaces between backing-stones, due to irregularities of form not to exceed 8 inches at the widest point, and are to be filled solid with selected hammer-shaped stones and spalls, carefully laid and settled in mortar." The use of grout was prohibited.

Concrete.—The concrete was mixed in batches of broken stone, 33 cubic feet; sand, 15 cubic feet; cement (Rosendale), 2 barrels. This made 36 cubic feet of concrete rammed in place.

Back of Walls, etc.—The back of the land-wall was hammer-dressed and laid in offsets. For drainage behind this wall, loose stone were placed between the back and embankment, leading to a culvert in the lower wing. Communication between the pools was guarded against by puddle about the upper wing.

Irons in Masonry, Anchorage, etc.—The mitre-sills are anchored by $1\frac{1}{4}$-inch wedge-bolts that reach into the bed-rock.

Lock-gates.—The gates and gate irons are shown on Fig. 450 The gates are of white oak, built without heel or mitre "posts,"

the main beams running through and the ends and centre made solid by filling-blocks, assembled with horizontal and vertical bolts and keys, and spaces planked as shown. They are suspended at the heel on steel gudgeons, and by top fastenings and anchorage, all below the level of the coping. Each leaf weighs complete about 37½ tons.

The lock is filled and emptied by valves in the gates, each leaf having five cast-iron valves hung horizontally in a wrought frame. The net filling and emptying areas are each close to 68 square feet. The valves are manœuvred by racks and pinions and the gates by spars and capstans.

The valve areas are the same as at Lock No. 6, and the chamber contents, owing to the faces of the walls being vertical instead of battered, somewhat less than Lock 6. At No. 6 the lock is filled and emptied at maximum lift in about 4 minutes. Steamboats without tows are locked either way in from six and one half to eight minutes.

Quantities and Prices.—The lock was built under a contract that covered the lock complete except the gates. It included coffer-damming, pumping, and bailing, and the furnishing of all work and of all materials, except the irons built in the masonry; these irons being supplied by the United States, and placed by the contractor. The quantities in the contract and the prices per unit for the work in place were as follows:

Grubbing and clearing complete, $2000; crib-logs in coffer-dam, 53,903 lineal feet at 28 cents; sheathing for coffer-dam, 34,870 feet B. M. at $30.00; coffer-dam filling, 7361 cubic yards at 60 cents; excavation, common, including dredging, 22,387 cubic yards at 55 cents; hardpan excavation, 3064 cubic yards at $1.25; rock excavation, 11 cubic yards at $4.80; embankment, 11,031 cubic yards at 50 cents; puddling, 233 cubic yards at $1.50; concrete, 600 cubic yards at $5.00 and 2713 yards at $6.25; backing masonry 5224 cubic yards at $4.40; rock-face masonry, 2017 cubic yards at $8.60; pointed-face masonry, 1836 cubic yards at $9.00; "cut stone" (corners, etc., bush-hammered), 351 cubic yards at $12.00; sills, 219 cubic yards at $18.00; quoins, 76 cubic yards at $25.00; coping, 377 cubic yards at $25.00; riprap, hand-placed, 1987 cubic yards at $3.00; paving, 1171 cubic yards at $5.00; stone filling, 3165 cubic yards at $1.50; bolt-holes drilled in masonry, 1493 lineal feet at

50 cents; timber, white oak in guard-cribs, etc, 60,150 feet B. M. at $40.00.

The aggregate of the contract was $160,630.24.

THE DAM.

The dam is of the Chanoine wicket type, operated from a trestle service-bridge. It is divided in two main parts, the Navigation Pass and the Weir, or in four parts, beginning at the lock, viz.: The Navigation Pass, Centre Pier, Weir, and Abutment.

The foundations all rest on concrete, the latter extending to bed-rock under the upper and lower or exterior walls, and at wicket and trestle anchorage, and to hardpan elsewhere, as shown. The foundations are 50 feet long up and down stream, between neat lines of walls. The surface or apron of the pass is entirely of masonry except the wicket sill and the timbers for the horse and trestle boxes.

The pass is 248 feet wide. It is closed by 62 wickets spaced 4 feet between centres; the wickets are of oak with pine panels, framed, ironed, and hung, as shown on the drawings. They are 3 feet 8 inches wide, the space between them being 4 inches, and 14 feet $\frac{1}{8}$ inch long. The axis of rotation is 6 feet 10 inches from the butt of the wicket and 5 feet 11 inches vertically above the top of the sill. The top of the wickets stands 13 feet vertically above the sill. The inclination with the vertical is 20° and the lap on the sill 5 inches. These wickets are a few inches longer than any before built on the river. The details of the wicket and wicket-irons and of the horse, prop, and hurter are shown on Fig. 455.

The service-bridge of the pass is made by 30 wrought-iron trestles, with attached aprons for walk and connecting rails. The trestles are 8 feet apart between centres. The floor of the bridge is 16 feet $9\frac{1}{4}$ inches above the centre of bottom axis of trestles and 2 feet 6 inches above top of wickets or normal pool level. The trestles are connected by chains for use in raising, the aprons forming part of this connection, and have forged stops to fasten the wicket chains in.

The wickets and bridge are anchored by $1\frac{3}{4}$-inch rods and cast disks, built in the foundations, spaced 4 feet apart for both wickets and bridge.

The masonry of the down-stream wall of the weir extends to

bed-rock, the remainder of the foundations resting on the hardpan, as shown generally by cross-section on Figs. 451 and 452. The space between the upper and lower walls is filled partly with concrete and partly with clay and gravel, the concrete being used about the anchorage and immediately under the surface masonry, as shown.

The weir is 316 feet wide, closed by 79 wickets set 4 feet between centres. The wickets are 3 feet 9 inches wide (the space being 3 inches) and 9 feet 2⅛ inches long. The axis of rotation measured on the wicket is 4 feet from the butt and vertically 3 feet 4⅝ inches from top of sill. The top of the wicket is 8½ feet vertically above the sill. The inclination with the vertical is 20° and lap on sill a fraction less than 4 inches. These weir-wickets (those for the weir of Dam 8 now building are the same size) are from 1½ to 3½ feet longer than at the other dams on the river.

The weir-service bridge is made by 39 trestles-spaced, except at ends, 8 feet between centres. In general form of construction it is like the pass-bridge, the trestles having attached iron aprons, connecting-chains, and stops for wicket-chains, etc.

The upper surfaces of the weir-foundations of the Great Kanawha dams are all a little above natural low-water mark. On account of this, in recent construction, beginning with Dam 6, the surface is made entirely of masonry (except the upper guard-stick and wicket-cushions, both easily renewed), and the trestle-boxes, wicket-sill, and hurters are fastened directly to the coping by wedge-bolts.

The wicket sill is of cast iron, made in sections, with the horse-boxes attached. The sill is anchored by rods and disks.

Centre Pier, Abutment, etc.—The foundations of the pier all rest on bed-rock, and are mainly of concrete; about the down-stream end the masonry proper extends to rock. The concrete foundations of the abutment extend to rock throughout. The masonry of the pier and abutment is principally rock-faced ashlar with dimension, bush-hammered corners and coping. The circular recesses in both, for the adjoining trestles of the bridge to fall in, are shown. For drainage back of the abutment, loose stone are placed between the wall and embankment leading to an opening in the lower wing. Puddle is used back from and about the upper wing to prevent communication between the pools.

Quantities and Prices.—The foundations and masonry of the dam are being built by a contract that covers the work complete, ready for the wickets and trestles. It embraces coffer-damming, pumping, and bailing, and the furnishing of all work and of all materials, except the iron built into or attached to the work; these irons being furnished by the United States and put in place by the contractor. The approximate quantities and contract prices are as follows:

Grubbing and clearing site complete, $500.00; coffer-dam logs, 80,000 lineal feet at 20 cents; sheathing for coffer-dam, 53,000 feet, B. M., at $30.00; coffer-dam filling, 13,000 cubic yards at 60 cents; excavation, common, including dredging, 16,000 cubic yards at 90 cents; hard-pan excavation, 1960 cubic yards at $1.80; rock excavation, 100 cubic yards at $2.50; embankment, 2000 cubic yards at 60 cents; puddling. 875 cubic yards at $1.50; concrete, 2830 cubic yards at $7.25; rock-face masonry, 2270 cubic yards at $10.00; pointed-face masonry, 450 cubic yards at $13.00; cut stone (bush-hammered corners, etc.), masonry, 100 cubic yards at $15.00; sills, 310 cubic yards at $16.00; coping, 430 cubic yards at $15.00; stone filling, 938 cubic yards at $1.50; riprap, hand-placed, 700 cubic yards at $2.50; drilling bolt-holes in masonry, 5000 lineal feet at 30 cents; timber, white-oak, in permanent construction, 110,000 feet, B. M., at $50.00.

The aggregate of the contract, as above, is $118,215.00.

Total Cost of Lock and Dam.—The estimate for the lock and dam complete is as follows:

The Lock, as per details of contract given above	$160,630	
Irons built in masonry of lock	1,093	
Lock gates complete	7,800	
		$169,523
Foundations and masonry of Dam, as per details of contract given above	$118,215	
Ironwork in anchorage and fixed parts of dam	5,500	
Ironwork in movable parts of dam	12,700	
Woodwork of wickets	2,350	
Diving apparatus and service-boats	1,300	
		140,065
Land at site, buildings, engineering, superintendence, and incidentals		34,012
Total		$343,600

In this estimate the cost of unfinished details and work not contracted for is based mainly on the actual cost of same at Lock and Dam No. 6.

ADDISON M. SCOTT, *Resident Engineer.*

U. S. ENGINEER OFFICE,
CHARLESTON—KANAWHA, W. VA.,
March, 1892.

Quantities and Prices.—
dam are being built by a con
ready for the wickets and
pumping, and bailing, and
materials, except the iron
irons being furnished by th
contractor. The approxim
follows:

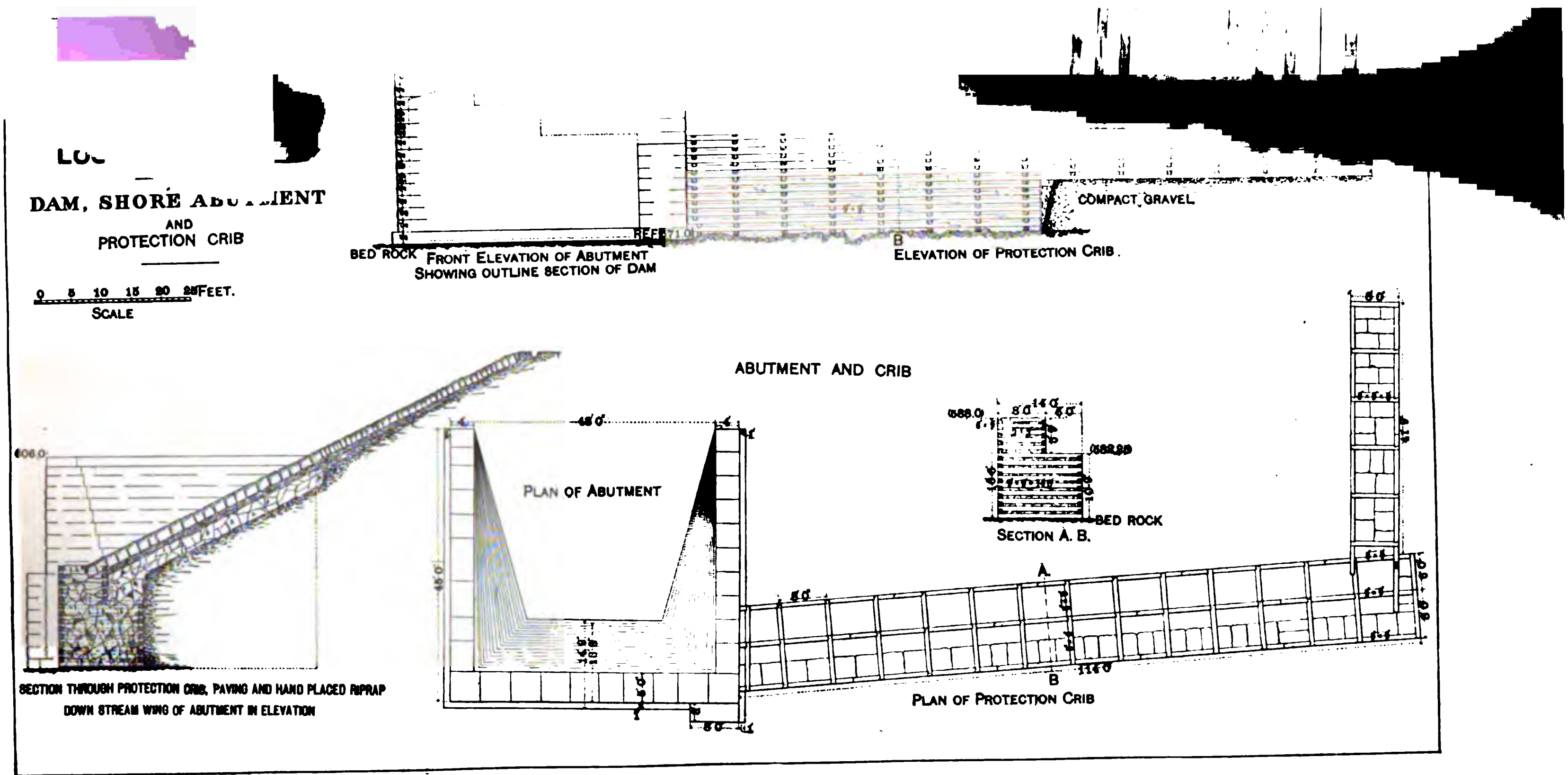
Grubbing and clearin
80,000 lineal feet at 20 c
B. M., at $30.00; coffer-
excavation, common, in
cents; hard-pan excava
tion, 100 cubic yards
60 cents; puddling. 8
yards at $7.25; rock
pointed-face masonry
hammered corners,
310 cubic yards at $
filling, 938 cubic y
yards at $2.50; dri
30 cents; timber,
feet, B. M., at $50

The aggregate
Total Cost of
dam complete is

The Lock, as per
Irons built in mas
Lock gates compl

Foundations an
tract given
Ironwork in a
Ironwork in
Woodwork
Diving

Lan



DAM, SHORE ABU ENT
AND
PROTECTION CRIB
0 5 10 15 20 25 FEET.
SCALE
BED ROCK
FRONT ELEVATION OF ABUTMENT
SHOWING OUTLINE SECTION OF DAM
COMPACT GRAVEL
B
ELEVATION OF PROTECTION CRIB.
ABUTMENT AND CRIB
PLAN OF ABUTMENT
SECTION A. B.
BED ROCK
PLAN OF PROTECTION CRIB
SECTION THROUGH PROTECTION CRIB, PAVING AND HAND PLACED RIPRAP
DOWN STREAM WING OF ABUTMENT IN ELEVATION

FIG. 447.

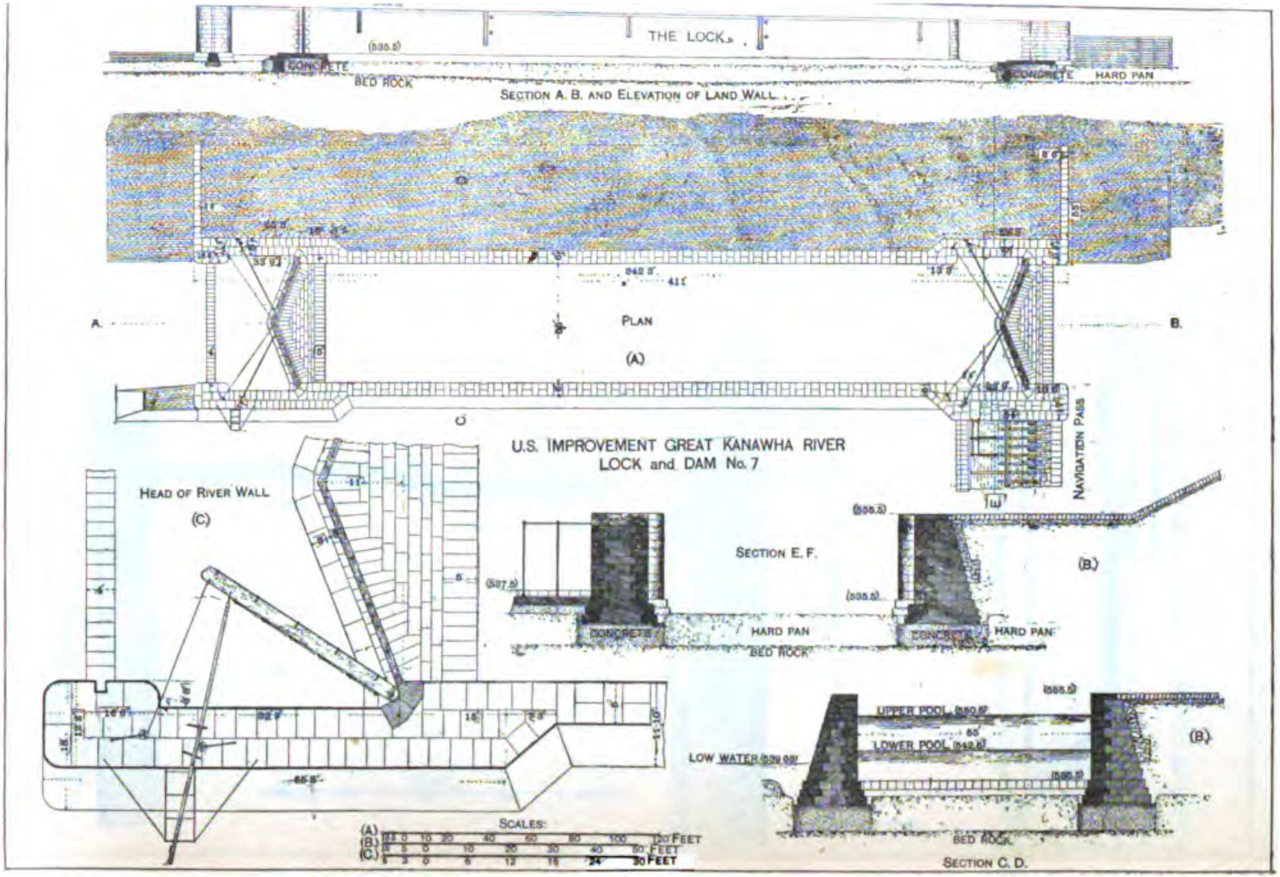

THE LOCK.
CONCRETE
BED ROCK
CONCRETE
HARD PAN
SECTION A. B. AND ELEVATION OF LAND WALL.
A.
B.
PLAN
(A)
U.S. IMPROVEMENT GREAT KANAWHA RIVER
LOCK and DAM No. 7
NAVIGATION PASS
HEAD OF RIVER WALL
(C)
SECTION E. F.
(B)
CONCRETE
HARD PAN
BED ROCK
CONCRETE
HARD PAN
UPPER POOL
LOWER POOL
LOW WATER
(B)
BED ROCK
SECTION C. D.
SCALES:
FEET
FEET
FEET

FIG. 448.

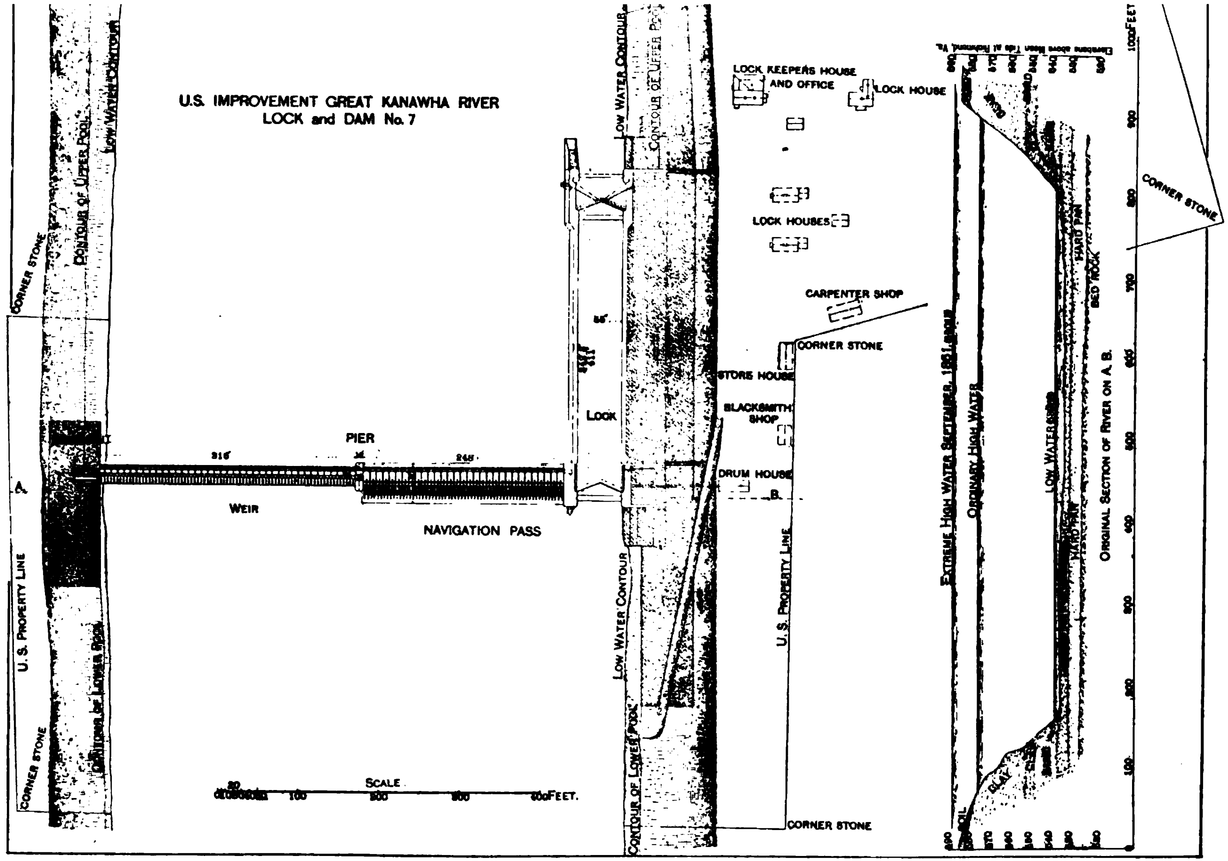

U.S. IMPROVEMENT GREAT KANAWHA RIVER
LOCK and DAM No. 7
LOCK KEEPERS HOUSE AND OFFICE
LOCK HOUSE
LOCK HOUSES
CARPENTER SHOP
CORNER STONE
STORE HOUSE
BLACKSMITH SHOP
DRUM HOUSE
U. S. PROPERTY LINE
LOW WATER CONTOUR
CONTOUR OF UPPER POOL
CONTOUR OF LOWER POOL
LOCK
PIER
WEIR
NAVIGATION PASS
SCALE
FEET.
EXTREME HIGH WATER SEPTEMBER, 1861
ORDINARY HIGH WATER
LOW WATER
HARD PAN
BED ROCK
ORIGINAL SECTION OF RIVER ON A. B.
SOIL
CLAY
SAND
1000 FEET

FIG. 449.

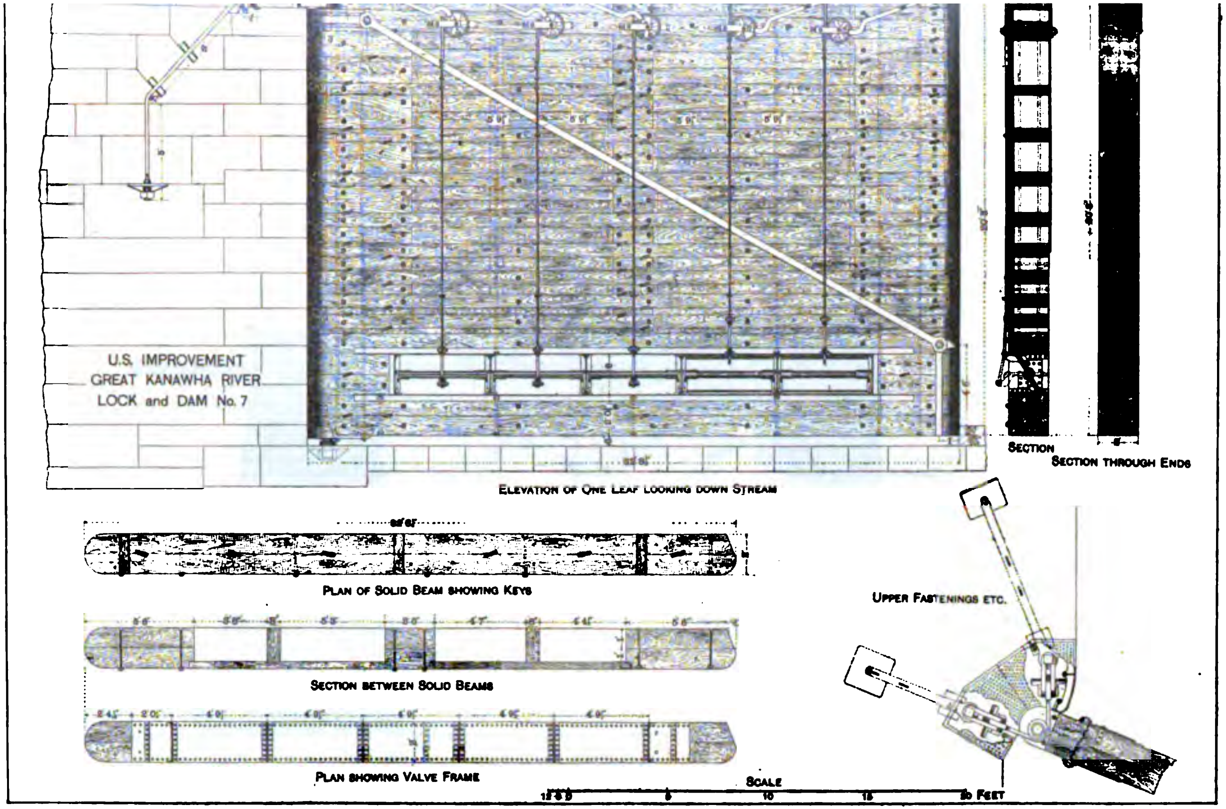
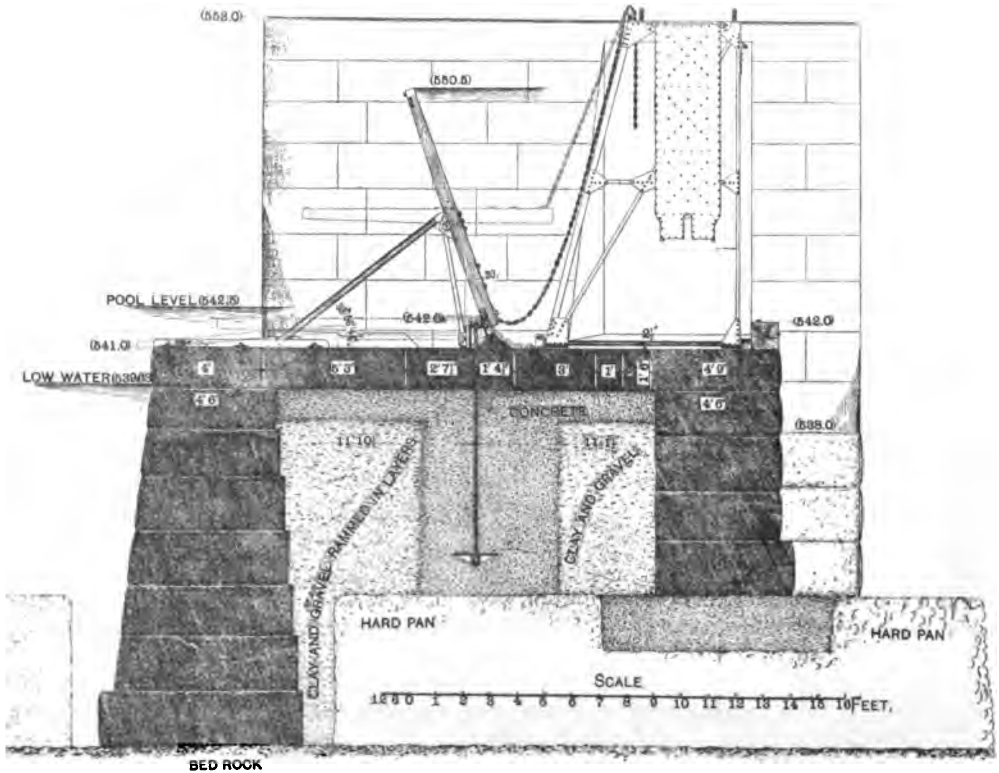

U.S. IMPROVEMENT
GREAT KANAWHA RIVER
LOCK and DAM No. 7
ELEVATION OF ONE LEAF LOOKING DOWN STREAM
SECTION
SECTION THROUGH ENDS
PLAN OF SOLID BEAM SHOWING KEYS
SECTION BETWEEN SOLID BEAMS
PLAN SHOWING VALVE FRAME
UPPER FASTENINGS ETC.
SCALE
5
10
15
20 FEET

FIG. 450.

LOCK and DAM No. 7



GENERAL SECTION OF WEIR ABUTMENT IN ELEVATION.

FIG. 451.

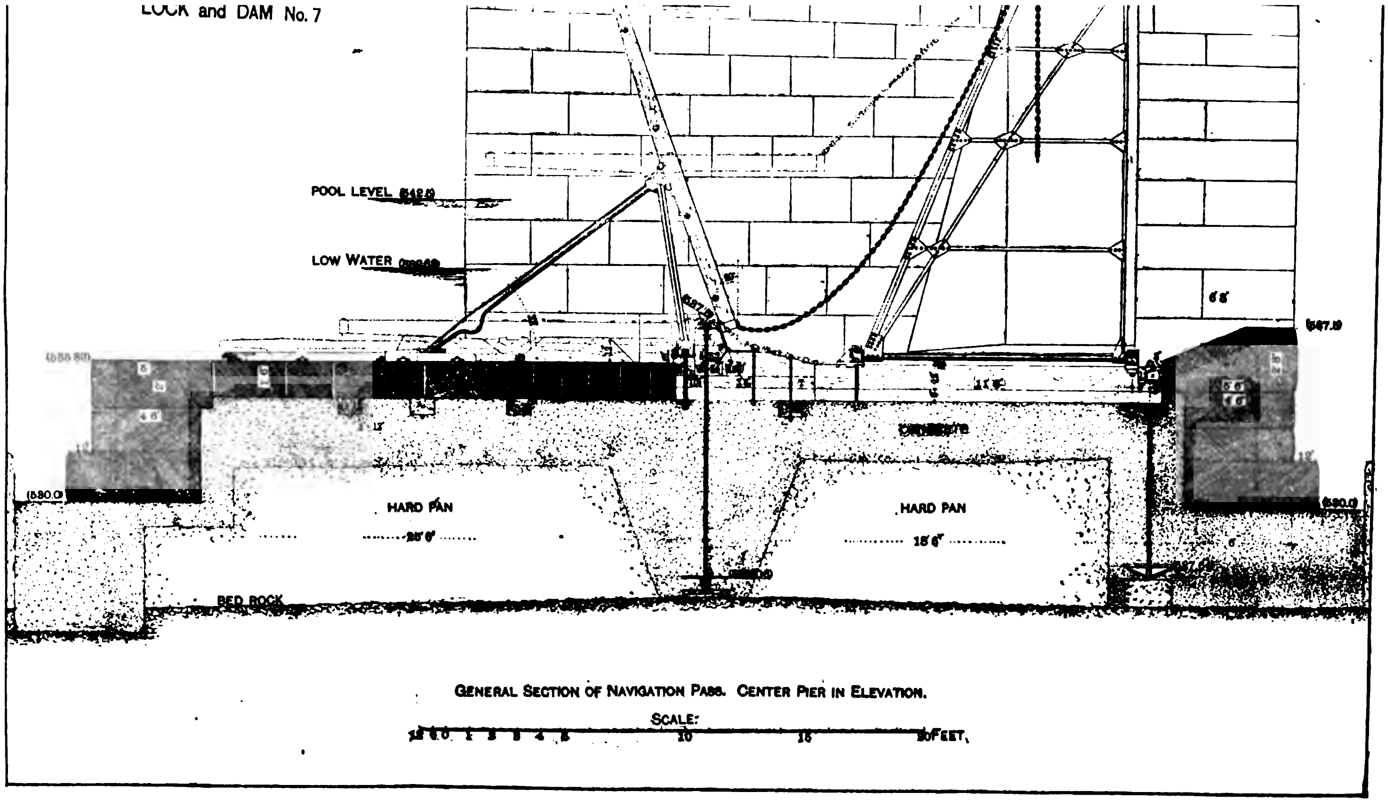

LOCK and DAM No. 7
POOL LEVEL
LOW WATER
HARD PAN
HARD PAN
BED ROCK
GENERAL SECTION OF NAVIGATION PASS. CENTER PIER IN ELEVATION.
SCALE:
FEET.

FIG. 452.

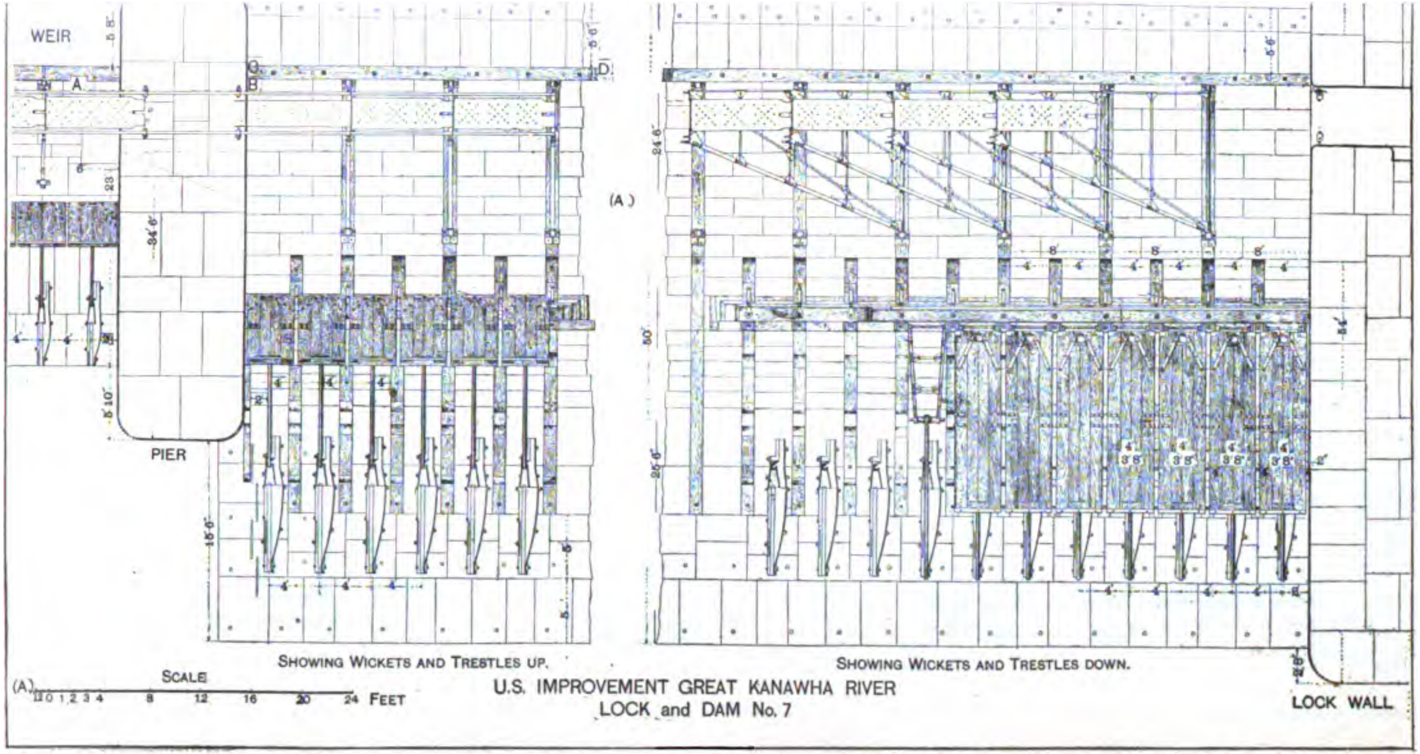

WEIR
PIER
LOCK WALL
(A)
SHOWING WICKETS AND TRESTLES UP.
SHOWING WICKETS AND TRESTLES DOWN.
SCALE
FEET
U.S. IMPROVEMENT GREAT KANAWHA RIVER
LOCK and DAM No. 7

Fig. 453.

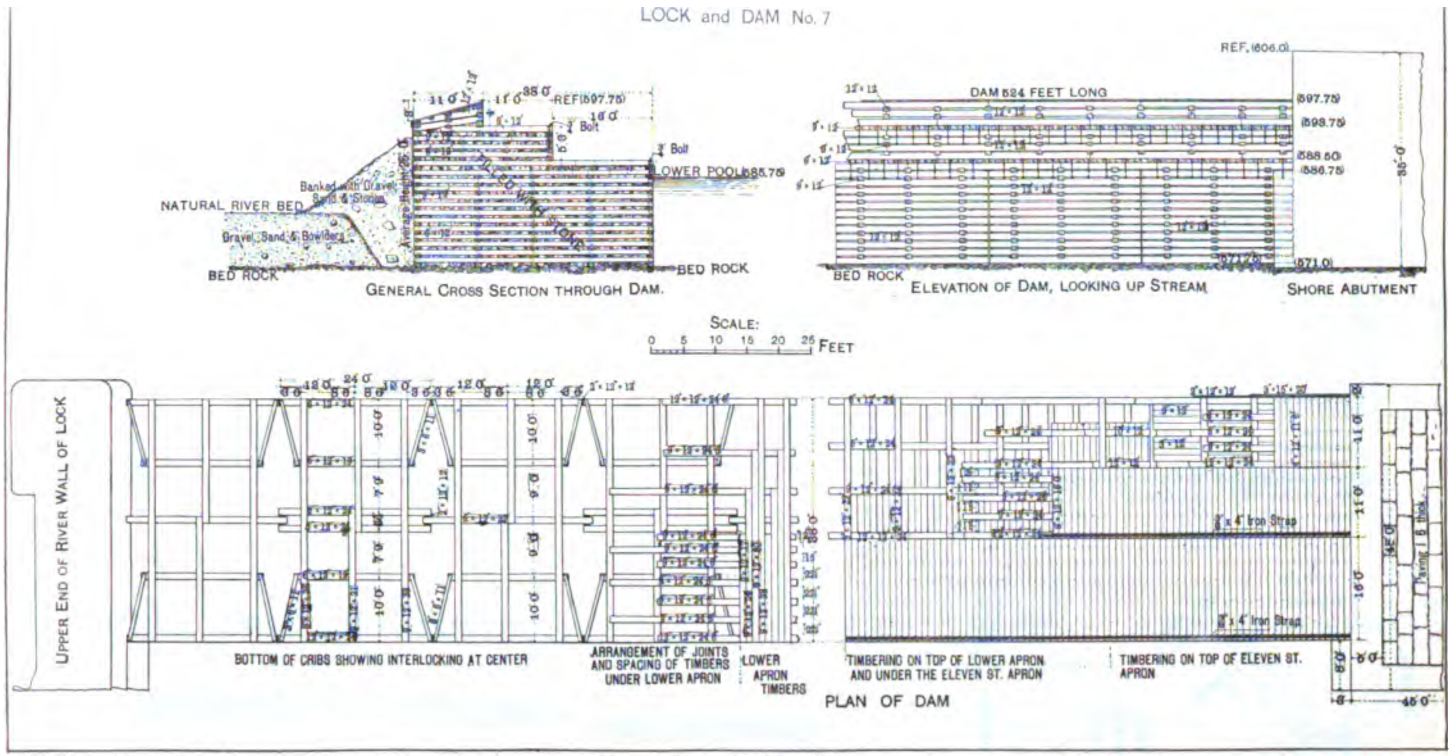

LOCK and DAM No. 7
REF. (606.0)
REF (597.75)
LOWER POOL (585.75)
NATURAL RIVER BED
Banked with Gravel Sand & Stone
Gravel, Sand & Bowlders
FILLED WITH STONE
BED ROCK
BED ROCK
GENERAL CROSS SECTION THROUGH DAM.
DAM 524 FEET LONG
(597.75)
(593.75)
(588.50)
(586.75)
(571.0)
BED ROCK
ELEVATION OF DAM, LOOKING UP STREAM
SHORE ABUTMENT
SCALE:
0 5 10 15 20 25 FEET
UPPER END OF RIVER WALL OF LOCK
BOTTOM OF CRIBS SHOWING INTERLOCKING AT CENTER
ARRANGEMENT OF JOINTS AND SPACING OF TIMBERS UNDER LOWER APRON
LOWER APRON TIMBERS
TIMBERING ON TOP OF LOWER APRON AND UNDER THE ELEVEN ST. APRON
TIMBERING ON TOP OF ELEVEN ST. APRON
PLAN OF DAM

Fig. 454.

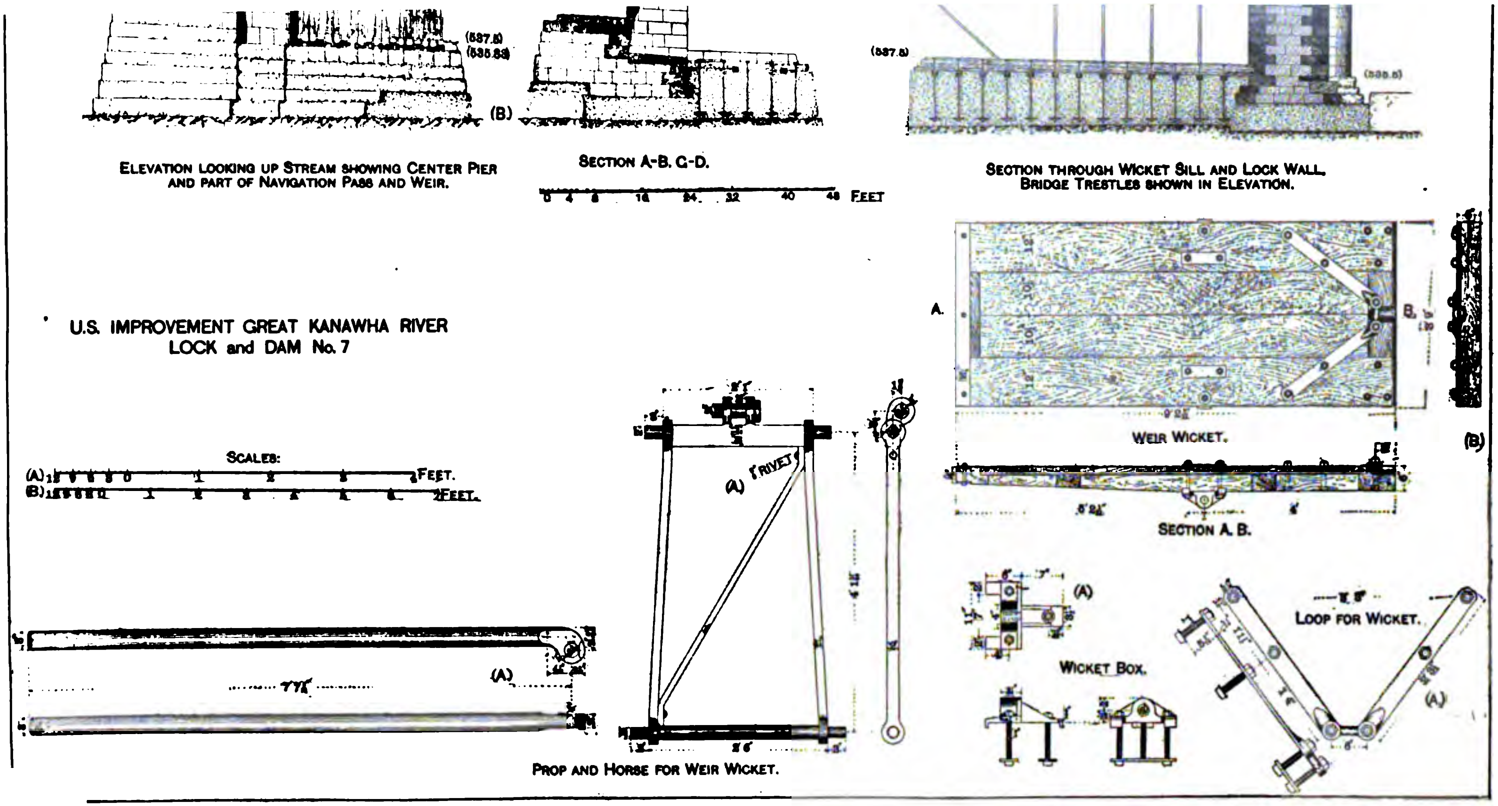

(537.5)
(535.83)
(B)
ELEVATION LOOKING UP STREAM SHOWING CENTER PIER AND PART OF NAVIGATION PASS AND WEIR.
SECTION A-B. C-D.
0 4 8 16 24 32 40 48 FEET
(537.5)
SECTION THROUGH WICKET SILL AND LOCK WALL. BRIDGE TRESTLES SHOWN IN ELEVATION.
U.S. IMPROVEMENT GREAT KANAWHA RIVER
LOCK and DAM No. 7
A.
B.
(B)
WEIR WICKET.
SECTION A. B.
SCALES:
(A)
(B)
FEET.
FEET.
(A)
RIVET
(A)
WICKET BOX.
LOOP FOR WICKET.
(A)
PROP AND HORSE FOR WEIR WICKET.

Fig. 455.

SUPPLEMENT.

RIVER AND HARBOR SURVEYS.

THE author's attention was kindly called by W. M. Black, Captain of Engineers, U. S. A., to the omission, from the earlier portions of this volume, of any description of River and Harbor Surveys (though considerable space had been given to River and Harbor Improvements). At the author's request Capt. Black furnished him with a number of references in the Reports of the Chief Engineer, U. S. A., from which the writer has selected the following examples as fairly typical surveys of their class. These surveys were carefully made in accordance with the best methods known, and the reports of them are very complete. Want of space will prevent their introduction in full. What is said, however, is substantially found in the reports and in letters received from Capt. Black. River surveys vary so in their object that it is difficult to find a typical case. They may be summed up as follows:

(*a*) "Where a thorough survey and maps of the entire river is required. For three methods of such a work see the Reports of the Surveys of the Mississippi and Missouri Rivers; Reports of the Chief Engineer for extensive work; for less elaborate work, see Reports, 1888, Part 2, pages 1095 and 1109.

(*b*) "Where a meander of the river and a survey of shoals and obstructions is desired, see Survey of Upper Savannah River, Report 1890, Part 2, page 1332.

(*c*) "Where a still less elaborate map is required and where bank work is impracticable, see Survey of Upper Manatee River, Reports, 1888, Part 2, page 1110, and 1891, Part 2, page 1621."

The general purpose of river and harbor surveys may be briefly stated as follows: (1) To deepen, to widen, to narrow rivers, or to canalize them; (2) to determine and locate the navigable channel, if one exists, or to straighten or rectify it, or to determine the cost of making such a channel and the best means of doing so; (3) or the purpose may be to divert some portions of the river into entirely new channels, and to determine the best means of maintaining the old or new channels; (4) to determine the extent and position of shoals, bars, rocks, or other obstructions to the use of rivers and harbors for purposes of navigation; (5) to determine upon the character of artificial works, such as breakwaters, jetties, dock-walls, etc., which will be necessary to effect the objects in view, and also their location and cost.

More or less elaborate surveys are required. The banks of rivers and coast-lines of harbors and their outlets have to be carefully surveyed or meandered. This work can be done by running meander-lines as close as practicable to the shore-lines, and offsets taken from them to upper crest-line of banks and to the water-line. These lines may be run on one or both sides; in the case of one side being used, points on the other or opposite side are to be located by triangulation or stadia-measurements. Rock-bound coasts or rivers with steep and precipitous

banks, or those bordered with dense jungle, marshy lands, or other similar obstructions, are best surveyed from boats. Four boats can be used to advantage in such surveys. Two points, one near each shore-line at the mouth of the river, are located, and the direction and distance between them accurately determined : two of the boats are anchored at these stations ; the other two move up-stream to convenient points on the opposite side of the stream. These points would be selected at the first bend, if not too great a distance from the first two points. At these positions the boats are anchored, and angles are measured from each of the boats to the other three by the use of a sextant. The first two boats then move up-stream, beyond the second two, to convenient points, and angles are again measured, thus triangulating thei way up-stream from point to point.

All such surveys should be connected with some base-line, either one already established by the U. S. Coast Survey, or one having its ends determined by astronomical observations.

The positions of rocks, shoals, bars, and other obstructions can be located by triangulation from the ends of subsidiary base-lines whose positions relatively to other lines of the survey are known, or by stadia-measurement from predetermined points ; or, finally, from the positions of the obstructions, or from boats anchored over them, by means of the sextant, measuring the angles to any three predetermined points on shore.

The positions of soundings can be located in the same manner, or these can be determined by taking time soundings on a line whose total distance has been determined, and a uniform rate of rowing maintained. For a complete hydrographic survey, tidal and current observations have to be made ; also, borings to determine the character of the underlying material.

Though possibly somewhat out of place, a few remarks will be made on mapping, which, together with the foregoing general remarks on surveying and locating points, is substantially as found in Text-book of Science, by T. G. Gribble.

The object of mapping is to produce a correct graphic representation of the field-work upon paper. The only absolutely true map is a terrestrial globe, but (as quaintly remarked) as we cannot carry globes about us we have recourse to the principles of projection, which are artificial representations of a spherical, or more properly spheroidal, surface upon a plane. If the survey extends over a large area, it becomes necessary to adopt some method of projection by which, in the first place, the distances are reduced to the sea-level, and in the second place the meridians are converged or distorted so as to allow for curvature. When the survey is a continuous traverse of a railway route, or some similar kind of work, this is not necessary. It is not the object of the railway engineer to know the sea-level dimensions ; he needs the actual length of the road, wherever it may be. The difference in length between a degree of latitude at the sea-level and at 528 feet ($\frac{1}{10}$ mile) elevation is only about 9 feet, at an elevation of 5280 feet (one mile) it would be about 92 feet, and the distance measurement practicable does not come nearer than that. Nor does the engineer want a distorted map, but one to which he can apply a scale throughout ; he therefore does not need to take account of the earth's curvature, but plots his traverse on a horizontal plane. When the area over which a triangulation extends is not large, the engineer is still able to adopt one of two methods of plane construction. In the first the meridians and parallels of latitude are parallel straight lines at right angles to one another. In the second, the parallels of latitude are straight lines parallel to each other, but the meridians are converging straight lines, or, if great accuracy is needed, curved lines—for the first a limit of, say, 1000 square miles, and for the second 100,000 square miles. When the survey is in high

latitudes the spheroidal form of the earth much more affects the map than near the equator, in which region a belt could be projected all round the globe by the first method without sensible error.

Assuming a mean latitude of 32°, the length of a degree of latitude at 32° is 68.90 statute miles, and the length of a degree of longitude is 58.70 miles at the same latitude. If then we draw two lines at right angles to each other and from the intersection of these lines lay off on the up and down line above and below 68.9 miles to any scale, and through the points thus located draw two horizontal lines, these will correspond to parallels 31° and 33° of latitude. Then from the intersection of the first lines drawn we lay off to the right and left distances of 58.7 miles, and through the points thus located we draw two vertical or meridian lines. There is thus formed a large rectangle whose sides are 2° of arc, and embracing an area of about 16,000 square miles, as a square of about ½° on a side will contain 1000 square miles. The correction for 1° longitude for each change of one degree of latitude will be 0.61 north and 0.62 south, that is, for latitude 33° and 31° respectively; therefore, by the second method, using converging meridians, the length of the north parallel 33° for 2° arc will be less than in the first by 1.22 miles, and in parallel 31° greater by practically the same amount. The distances on lines running north and south will remain the same, and for the ½° square covering an area of 1000 square miles the expansion on the top of the sheet (with parallel meridians) will be 50 (nearly) yards greater than with converging meridians, and a similar error of contraction at the bottom, in practically 29.8 miles. The method of straight converging meridians may be used with sufficient accuracy up to a latitude of 65° for stretches of 100,000 square miles. It may be said, therefore, that the method of plane construction meets all the ordinary requirements of the engineer.

The principles of reducing extensive surveys to atlas scale will be briefly explained.

Conical Projection.—A globe may be conceived to be wholly contained inside a cylinder, or partly contained inside a hollow cone. The cylinder must have a diameter equal to that of the globe. The cone must be of such dimensions that its sides will be tangential to the radius of the sphere at the point of contact. A belt of a few degrees on either side of the equator might be conceived to be unwrapped or developed on the cylinder without sensible error; this much would be plane parallel construction. Similarly a belt of the cone may be developed with converging meridians and with curved parallels. Maps of continents are drawn in the atlas upon this principle, and being of large extent the apex of the cone is determined and the radial parallels of latitude drawn direct upon it with trammels.

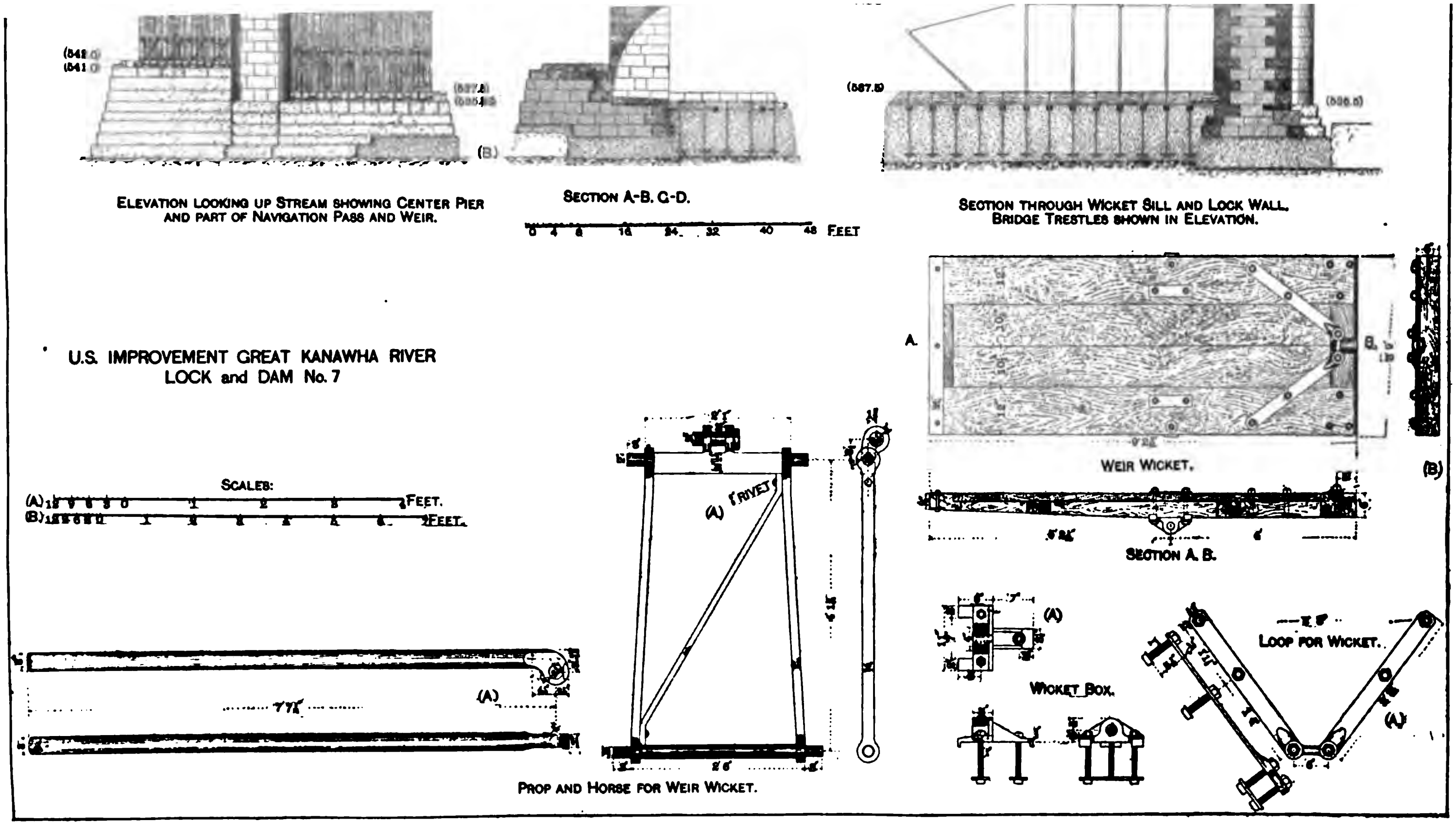
EXAMPLE.—It is required to project, by conical projection, a belt of 10° longitude, say from 20° to 30° east, whose middle parallel of latitude is 50°; the width to be 10°, i.e., from 45° to 55°; the scale 100 miles to the inch. Draw the scale of miles at the foot of the paper. Draw a horizontal base-line in the middle of the paper, fix its centre, and draw a perpendicular through from top to bottom. The base will represent the chord of the middle parallel and the perpendicular the central meridian 25° longitude E. Calculate the length of the chord by the following formulæ:

Radius of cone = radius of earth × cotan lat. $= 3950 \times \cot 50° = 3{,}314R$;

Central angle = total longitude × sin latitude $= 10° \times \sin 50° = 7°\ 66' = A$;

$$\frac{\text{Chord}}{2} = R \sin \frac{A}{2} = 3{,}314 \times \sin 3°\ 63' = 221 \text{ miles};$$

$$\text{Versed sine} = R - R \cos \frac{A}{2} = 7.4 \text{ miles}.$$



(542.0)
(541.0)
(587.8)
(B)
Elevation looking up Stream showing Center Pier and part of Navigation Pass and Weir.
Section A-B. C-D.
0 4 8 16 24 32 40 48 Feet
Section through Wicket Sill and Lock Wall. Bridge Trestles shown in Elevation.
U.S. IMPROVEMENT GREAT KANAWHA RIVER
LOCK and DAM No. 7
Scales:
Feet.
Feet.
A.
B.
Weir Wicket.
(B)
Section A. B.
(A)
Rivet
(A)
Prop and Horse for Weir Wicket.
(A)
Wicket Box.
Loop for Wicket.
(A)

Fig. 455.

meter). The place selected for this had an inappreciable current, a uniform depth of 7 feet, and convenient for obtaining a base for a run of 1000 feet. The party consisted of two assistants, two recorders, an engineer, and a helmsman. Having selected the line for the run, a base was measured on the shore approximately parallel to it; the stakes marking the run were then readjusted so that they would be equidistant from the base-line, by turning off equal angles from each end of the base, intersecting lines at 90° from the other. Stakes were set 15 feet inside of these, with which two others, beyond each end of the run, were ranged. The second set of stakes was then removed, leaving the other two so as to enable the observers on the launch to know about when they were approaching either end of the run. The meter was suspended from the bow of the launch on a frame of gas-pipe; this was arranged so that it was quite rigid, and the entire apparatus could be turned back over the bow of the launch or lowered into the water at any time with ease; when in use the meter was 6 feet ahead of the bow and about 3 feet below the surface. An observer with a transit was stationed at each end of the base-line, which was 1000 feet long, and one at the register, and another with a time-piece on the launch. The launch was started, the helmsman getting in range with the stakes at the other end and preparing to run with a uniform speed; when within 50 feet of the stake at the near end of the run the proper signal was given; at the instant the pipe on which the meter was suspended passed the cross-wires of the telescope the transitman called "time." The time and reading of the register were noted. The same was done at the other end. The velocity of the launch varied from 1.2 to 8.3 feet per second. The day for rating was calm and the water smooth; fifteen round trips were made, going down and back, so as to eliminate the effect of current, if any existed. Taking the mean of each trip down combined with the next trip back, and the mean of each trip down combined with the previous trip back, twenty-nine results were obtained. These were plotted with reference to a system of rectangular co-ordinates, the velocity (v') in feet per second being the abscissa, and the revolutions (n) per second the ordinate. The line showing the relation between them is practically a right line, whose equation is $v' = an + b$, in which a is the tangent of the angle which the line makes with the axis of abscissas and b the distance from the origin to the point of intersection with the axis of ordinates. Substituting the observed values of v' and n in the above equation, forming twenty-nine equations, and each one being compared with all the others, 435 values of a and b were obtained. The method of least squares was applied to these, and 3.772 was obtained for the value of a, with a probable error of 0.00099, and 0.091 for the value of b, with a probable error of 0.00402.

The party for the survey consisted of two assistants, two recorders, one leadsman, two tidal observers, and five boatmen.

Tide-gauges.—A box tide-gauge was established at the Northwest Light, which was in successful operation during the entire survey. A gauge was also established at Fort Taylor, the object being to obtain the difference in times of high and low water between the two places. No tidal constant is given for the Northwest Bar in the Coast Survey tide-tables.

Base-line.—A base-line was measured by driving stakes, 3 x 4 inches, in the sand, cutting them off to the same elevation; they were placed 100 feet apart, and measurements made with steel tape, the exact points indicated by knife-marks on copper tacks. The length of the base was 6999.3 feet.

Signals.—Platforms were erected for the transits at East Base and West Base; elsewhere tripods were built of 4 x 4 inch scantling, 25 feet long, nailing the edge of one on to the side of the other. The schooner was anchored fore-and-aft and the signal built on the windward side; it was then lifted with the proper tackle, the lower ends spread out; it was then lowered to the bottom, their

sharpened points driven into the bottom of the stream with mauls, and the whole lashed and braced.

Triangulation.—Stations East Base, West Base, and Northwest Light were occupied by transits. To the principal triangulation-points the angles were repeated at least five times and both verniers read. The remaining stations were occupied with sextants, each station being occupied by two observers separately. From every station angles were observed to the two sides of all the keys visible, as well as to the lighthouses and the buoys.

Soundings.—The soundings were all taken from the schooner while sailing. Whenever possible, natural objects were used for ranges. The principal lines radiated from the lighthouse. The use of a steamer would have enabled directions to be maintained more satisfactorily. The soundings were located with a transit at the lighthouse and a sextant on board the schooner, the position of the sextant with respect to the leadsman carefully noted, and topographical points on the coral reef were located by triangulation.

Current Observations.—In using the current-meter the yawl boat was anchored by three 100-pound anchors. The bow of the boat was towards the waves. The meter was clamped on to a piece of ½-inch gas-pipe, 3 feet in length, the lower end of which was loaded with 10 pounds of lead. To the upper end of the pipe a graduated line was securely fastened in such a manner as to be easily detached when required. The meter cable, a ⅜-inch tarred manilla cable, which was fastened to a 100 pound mushroom-anchor on the bottom, was rove through the gas-pipe and led up to the boat, together with the graduated line, by which the meter was suspended at the required depth. An apparatus had to be designed to counteract the rise and fall of the boat in the seaway. It consisted of a frame with two arms, 10 feet long, working outside the boat. The arms were about 7 feet apart at the forward end and 2 feet at the after end, with a crosspiece at the after end and one about 4 feet from the forward end. The second crosspiece rested in chocks on the gunwale of the boat, and the other swung clear of the stern. Counterpoises weighing about 50 pounds each were placed on the forward end of the arms. The meter cable, etc., was fastened to the crosspiece at the after end. The arms were halved into and rested on the longer crosspiece, and the shorter crosspiece was halved into and rested on the arms. It was fastened together by pins, which could be easily removed, and the frame taken apart and stowed in the boat. The counterpoises were fitted over the arms and fastened with a pin, for which holes were provided at intervals along the arms—the weight being slipped forward or back, thus adjusting it to give the proper strain on the meter cable. With this arrangement practice demonstrated that observations could be conducted without hindrance when the boat rose and fell as much as 6 feet. At each station occupied, observations being begun at slack-water, if possible, readings of the register were noted every five minutes, and the meter was kept at mid-depth until it was estimated that the maximum velocity of that tide was reached, when observations were begun at different points in the vertical plane. The meter was retained at each depth for two minutes, and the reading of the register noted each minute. The results of these observations were plotted on cross-section paper, the mean of the two results at each depth being used, and a curve constructed that would coincide with these points the nearest. This curve was adopted as the vertical velocity curve of that station. The ratio of the mean velocity to the mid-depth velocity as determined from the curve, was used as the coefficient for the reduction of all mid-depth velocities at that station. From each curve results were taken for each 0.2 foot in depth, and the mean velocity for the stations determined from these results. The velocities at different depths are taken off graphically as required. A comparison was made between the mean velocity (V_m) and the mid-depth velocity ($V \frac{1}{2} D$); between the depth

of mean velocity fillet (M) and the whole depth (D); and between the depth of maximum velocity fillet (d) and the whole depth. The mean results of these comparison for all stations are as follows:

$$\frac{V_m}{V_{\frac{1}{2}D}} = 0.925, \quad \frac{M}{D} = 0.597, \quad \frac{d}{D} = 0.193.$$

The first current observations were taken with a view of ascertaining the volume of discharge during the ebb and flood tides in the main channel. In computing the discharge, each tide was divided into ten stages of equal duration, and the cross-section into seven divisions of such size as the shape of the bottom required. The coefficient used in reducing the velocity at the meter station to that at each division is the ratio of the velocity on the velocity curve at the same proportionate distance from the axis of the curve as the middle point of the division was from the meter station to the velocity of the axis. The mean velocity, the discharge for each division for each stage of the tide, and the total discharge were obtained in this way.

The following notes are taken from Report of Chief of Engineers, 1889:

The current apparatus consisted of a mariner's wooden log for surface velocities, float rod for mean, and double float for bottom velocities. The floats were of yellow pine 1¼ inches square, with weight placed in metal tube also 1¼ inches square suspended from bottom of float, space between the two being left for the addition of shot to properly adjust the weight. These floats were made in sections of 2 and 15 feet, securely connected by ferrules of yellow metal 5 to 10 inches long. The double floats consisted of hollow yellow metal surface float, an oblate spheroid in form, with axes of 2½ and 8 inches, and a hollow cylindrical subsurface float 8 inches in diameter and 8 inches high, also of yellow metal, suspended by means of cross-wires midway between ends. Spar buoys were moored at stations; a float line 80 feet long was graduated into 20-foot distances from ends and then into 25-foot distances. This was fastened to the log, which was set with its face normal to the current, and when the 20-foot mark on the line was reached "time" was called, and when the whole line was run out time was called again; by a sudden jerk, the bottom of the log was released from its lines, and floating flat on the water it was drawn back to the boat. The magnetic bearing of the direction taken by the floats or log was observed. A similar method was adopted for the single and double floats. The lower of the double floats could be immersed to any depth; the single float reached nearly to the bottom. In this manner the surface velocities were obtained, also those at any depth from surface to bottom, and also the mean velocities.

Soundings.—The soundings were taken on radial lines centring at the lighthouse; the angle between adjacent lines was 3°. In order to avoid confusion by lines too close together near the lighthouse, the launch was directed on one of the outside lines radiating from the lighthouse; it then returned on the adjoining line, but some distance from the lighthouse it turned off the second line on to the third, and went forward on this line as far as desired, then returned on the fourth line close up to the lighthouse, and for the next set of lines the same operation was repeated. The launch was kept on the lines by means of signals from the lighthouse. Soundings were taken at intervals of 2 minutes, and their positions located by transit angles from the lighthouse and sextant angles from the launch taken simultaneously. The progress was from 3 to 4 miles per day. Some cross-lines were run and soundings taken.

From report for 1889, Part II, page 1307:

Survey of St. John's River.—In observing velocities with the current meter, the instrument was placed 1 foot from the bottom, and register read; at the

end of 3 minutes, register read again, instrument then raised 3 feet, and the operation repeated; and so on. In heavy seas weighted poles and floats attached to graduated lines were used, the former for mean velocities and the latter for surface velocities.

Direction of Current.—This was obtained at bottom, mid-depth, and surface by means of surface and subsurface floats connected by fine cords. The floats were dropped from a boat and allowed to run for a distance of 150 feet, and the direction noted.

Borings were made with small pipe inserted in a larger one and sunk by water-jet; the material passing up between the large and small pipe, 1½ and 1 inch respectively, furnished samples of the underlying strata.

Water Samples.—Water samples were taken by an apparatus invented by Lt. D. D. Gaillard. Two iron plates 1 foot square and ¼ thick were held by four iron pins, leaving a 1-inch space between them. A pipe 1 foot long, for anchor-line, was fastened through the centre of the plates. The pump was connected with a hose coupled to a nipple passing through the upper plate. By this arrangement only the water found between the plates at the time and depth could be pumped up. The anchor-line passed through the short pipe and was fastened at the bottom; the apparatus could be raised or lowered along the anchor line.

RIVER SURVEYS.

The following general description of the stadia method of making river surveys is abstracted from the *Engineering Record*, February 9, 1895, containing an article written by J. L. Van Ornum: "For a complete survey of a river, where latitudes and longitudes as well as the most exact location of all the salient features of the river and its surrounding territory are desired, no method can supplant a complete triangulation system. But for any ordinary river survey the stadia method furnishes the best system, both because of its economy and efficiency; not so accurate, of course, as triangulation, yet its accuracy is amply sufficient for survey, for navigation, or for improvements, or for any purpose up to the most rigorous, if it is in the scope of the method. This scope limits it to rivers not much more than one half mile in width, because the location of the soundings as well as the meanderings of the river banks is made by the stadia, and the sights must be kept within practicable limits of length. While in the stadia surveys one party or more may be employed, the use of one party is only applicable to the smaller navigable rivers, and for ordinary purposes on them is reliable. The writer, however, recommends for complete surveys five parties, distributed as follows: two transit parties, two level parties, and one hydrographic party. Each transit party consists of a transitman, two stadiamen, and a boatman with a boat. All measurements are taken with stadia and directions by successive azimuths. Reciprocal sights for azimuths and distance should be exchanged between transitmen every 2 or 3 miles. Such sights furnish data from which the latitudes and departures of these circuits are computed, thus checking the work in the field. If an error of magnitude in either azimuth or in latitudes and departures is discovered, the field-work in the circuit is retraced and the error found and corrected. The average error of closure should not exceed $\frac{1}{100}$, and of azimuth not more than 3 feet per mile. The extreme limits allowed should not exceed double these amounts. The topographical points and points located for the hydrographic and level parties will be made by secondary (side) sights from the occupied stations of the closed lines. If practicable one transit party works on each bank of the river, meandering its own bank, and taking the topography of the adjacent country back to the highest

flood-line, where practicable. The transitman has the general supervision of the survey, while the sounding-boat is under the immediate charge of the recorder.

A level party works on each bank, carrying forward its level line. Each party has a levelman, one rodman, and one boatman with a boat. Each party should, on its own bank, keep as nearly as possible abreast of the hydrographic party, and take the elevation of water surfaces (located by the transit parties) at the time and place of sounding as frequently as is necessary—i.e., very frequently at bars or shoals, and less so at pools.

The hydrographic party is charged primarily with making the soundings, which are located by angle and stadia. It consists of a transitman (with his own boatman and boat) and a large sounding skiff carrying a recorder, steersman, leadsman, stadiaman, and two or three oarsmen. The leadsman is in the bow of the boat, and the stadiaman close to him, with his stadia-rod always vertical and facing the transitman. The steersman keeps the boat on the desired line, and the leadsman takes the sounding (with lead-line or sounding-pole) as frequently as desired, calling to the recorder the depths and character of the river-bed.

At the same time the transitman is taking his observations of stadia reading and angle as frequently as possible, each location being timed to coincide with the vertical lead-line, and the vertical hair being directed to the lead-line for the angular reading; the stadia-rod being so close to the lead-line makes it practicable to read the stadia interval, although it is not in the centre of the field. The transitman records his observed stadia interval and then reads and records the corresponding angle, thus fixing the position of the sounding by polar co-ordinates. The accuracy of the readings on the stadia-rod held in the boat depends mainly on care on the part of the transitman; the stadia-rod should be well braced at the bottom, and the stadiaman should have a high and firm seat to enable him to hold his rod steadily in a vertical position. Experience and practice will enable all parties to perform their approximate duties effectively.

The positions of the transit are arbitrarily chosen to the best advantage, as determined by the meanderings of the river, and are from one-fourth mile to one mile apart. Whenever the sounding-lines have been extended a sufficient distance beyond the transit station, it is marked by a flag, and the transitman with his transit is carried in his own boat to the next advantageous location for his station, while the sounding-boat remains in position, and observations are made from the new station, as already described. Preparatory to measuring the location angles of soundings, the zero of the horizontal limb of the transit is always set either on one of the stations previously occupied by the transit or on a station established for the purpose and marked by a flag.

The rate of progress for the complete survey varies from a mile to a mile and one half per day. The cost is from $60 to $70 per mile.

"The advantages of the stadia system are, then, economy, elasticity of adaptation to circumstances, more numerous and accurate location of soundings than usual, and an adequate and complete instrumental oversight of the work at the time."

The following detailed description of the surveys of the Savannah River will be interesting and instructive in this connection:

SURVEY OF THE SAVANNAH RIVER.

The following description of the survey of the Savannah River is taken from the report of the Chief Engineer for the year 1890:

For the first five miles the party consisted of the assistant-engineer, rodman, two chainmen, and two boatmen or laborers. In general the transit-line was run

parallel to the river, beginning at the City Bridge in Augusta, and ending below Blue-House Bar. In running the line both bearings and intersecting angles were taken, and the bearings were corrected to agree with the angles. Distances along the line were measured with the steel tape, and plugs with tacks in them set at every 100 feet or oftener, and also at angle-points. Where the line crossed the river the distance across was determined by triangulation. The shore-line on one side of the river was determined by direct measurements from the transit-line; the opposite shore and bank lines were run in by angles and stadia distances. Wharves, bridges, and jetties were also accurately located with respect to the transit-line.

The soundings were taken upon ranges, the lines running about at right angles to the river, at from 50 to 600 feet apart. These ranges were laid off at definite angles from the meander-line. Soundings at the less important places were spaced by time, checked by proportional distances to each bank.

At Blue-House Bar, where a number of soundings were necessary in order to show the exact nature of the bottom, the soundings were accurately located by intersections in the following manner: Two flags were established on the sand-bar, higher up the river, in such positions that from all points upon the area to be sounded one of them was projected against the railway bridge above. With each sounding on the cross ranges was also recorded the point with reference to the piers and panel points of the various spans against which one of the flags was projected.

From the old bench-mark and the river-gauge at Augusta a line of check-levels was run with a "wye-level" to a point below Blue-House Bar. The slope of the river at various stages of the water was carefully determined, and one or more permanent bench-marks were established at each bar.

Below Blue-House Bar the party consisted of the assistant or transitman, rodman, pilot, four boatmen, and a cook, who lived on a barge or floating house constructed for the purpose. When detailed surveys were made the entire crew was employed. The general survey was conducted as follows: The transit was set up near the edge of the river at a convenient point with reference to clear and unobstructed back and fore sights of convenient lengths. A sight was taken on a rod upon a stake at the point of beginning, and the compass bearing, stadia distance, and level reading carefully noted. With the transit remaining fixed, the rodman went ahead, taking a turning-point on a stake or fixed point near the edge of the water, in such a position that it could be readily observed from both that and the subsequent position of the transit. The transitman would then, as before, note bearing, distance, and level reading on the rod in this position, after which the transit would be moved to a position below the rodman, and take the back bearing, distance, and level reading. The rod and instrument were moved from position to position precisely as in levelling. At each position of the transit its elevation above the water-level adjacent was determined by means of the sounding-pole used as a level-rod, and the time noted. The elevation of each turning-point above the water-level was determined by direct measurements, and also noted. Turning-points were always taken upon stakes or fixed points. In cases where it was not possible to take a sight from or to the bank, stakes about 12 feet long were driven in deep water upon which to hold the rod. Besides the location of the turning-points, there was always noted from each position of the transit the bearing of the shore both above and below for estimated distances, the direction of points and of lines passing tangent to points or rounds with estimated distances, and the directions and distances to islands or other objects, which would aid in procuring an accurate map.

In going from one position to the next the rodman would pass the transit near the opposite shore with rod held vertically in the bateau, and the width of

the river would be obtained directly by stadia, with no delay of the work. From data thus obtained sketches were made in the field, and the positions of the channel, and of snags, overhanging trees, and other obstructions, were sketched.

In passing from one position to the next below, the transitman's boat was placed at once in the channel and kept there under the direction of the pilot, who took soundings every 50 to 500 feet, as was deemed necessary. The soundings were taken with an iron-shod pole 16 feet long, graduated to feet and tenths. In passing down the river the transitman would note the height and character of the banks, the size and kinds of trees and brush, the size and depths of streams, the positions of landings, bluffs, shoals, rocks, etc. The instrument used was a Brandis transit (No. 584), with level bulb under the telescope and a $4\frac{1}{2}$-in. needle. The stadia-rod used was 16 feet long and 4 inches wide; upon the white background of the rod every alternate tenth was black, and one tenth of a foot square. Modifications of the black squares, which would quickly catch the eye, were made to show the even feet and half feet. The tenths of the even-numbered feet were on the left side of the face of the rod, and those of the odd-numbered feet on the right side.

The figures on the rod were large enough to be easily read at a distance of 2000 feet. For the purpose of distinguishing the feet more clearly on the long sights, the edges of the rod were painted alternately black and white in feet, the edge adjacent to the black tenths on the face being black. With the corners of the rod presented to the instrument the foot graduations would then become very prominent.

The stadia-wires in the transit had not been adjusted, and a slight correction was found necessary. From a series of observations made at the beginning of the work, the length of the sight in feet was found to be $102x + 1$, in which x was the stadia interval in feet. Sights from 1600 to 3200 feet were measured by doubling the half interval, it having been found from the observations mentioned above that the intervals on either side of the middle wire were equal. With the telescope used the rod could be read to the nearest tenth with certainty at a half mile or more. Sights up to 3200 feet were taken, when from the nature of the surroundings positions could not be taken nearer without a great sacrifice of time. The ordinary sight was about 1000 to 1200 feet. At this distance the rod could be easily read to $\frac{1}{100}$ of a foot in elevation. No attempt was made to keep back and fore sights of equal length. All sights were corrected for curvature and refraction, using the formula

$$\text{Correction} = -\left[\left(\frac{x}{5280}\right)^2 \times 0.5714\right] \text{ feet,}$$

in which x = length of sight in feet. This is based upon a correction for curvature of $-$ 0.667 foot per mile, and a mean correction for refraction of $+\frac{0.667}{7}$ foot per mile. From the formula a table was constructed, giving the mean correction to hundredths of a foot for all distances up to 3500 feet. The instrument was kept in perfect adjustment during the entire survey.

LEVELS.

It was found that the elevation determined by this survey is 3.16 feet lower than that given by a line of levels running from Augusta to Milledgeville along the Georgia Railroad, a distance of about 90 miles, and down the Oconee and Altamaha rivers to the sea, a distance of about 281 miles. How closely the levels were run on the Georgia Railroad is not known, but if the probable error of railroad levels can be measured by any considerable portion of a variation of 40

feet in 140, among five roads, we can place little confidence in their results. A discrepancy of only 3.16 feet in a circuit of about 90 miles of railroad, nearly 500 miles of river, and about 100 miles of ocean shows what may be called excellent work. Considering the purpose for which the levels were taken, an accumulative error of 5 to 10 feet on the Savannah River would by no means vitiate the results.

The line of transit-levels was checked on the upper five miles and on the lower 14 miles with a wye-level. In the 5-mile section a difference in the two lines of less than 0.1 foot was found, and on the 14-mile section the levels checked within 0.34 foot. The wye-level determinations were in each case taken as the correct ones.

DETAILED SURVEYS.

Below Blue-House Bar twelve detailed surveys were made. The usual method employed in making these surveys was as follows: The transit was set up at some point commanding the whole bar. A sight on the stadia-rod at the last turning-point located the instrument. The sounding-crew consisted of the rodman, as recorder, in charge of the boat; the pilot, with a sounding-pole; a boatman to hold up the stadia-rod; and two oarsmen. The rodman was instructed where to sound, and occasionally for his guidance ranges or flags were established on shore. The work would then proceed, the position of second or third sounding being located by transit angle and stadia distance. The transitman would give a signal with each position located; the signal being seen by the man holding the stadia-rod would be entered in the sounding-book opposite the depth at the position. Instead of identifying the signals by means of time, as is usual when two transits are used, different-colored flags were employed for that purpose. A number of modifications of this system were adopted as the case required. In a few places, where the river was narrow, a number of flags were located by compass and stadia along either shore, and soundings taken from flag to flag. In all cases where time soundings were taken they were checked by proportional distances from either shore.

In most cases where detailed surveys were made bench-marks were established near the bars. These were also established along the river at convenient intervals. They were cut as usual upon the roots or trunks of trees; the "sap" in all cases was cut away to prevent "growing over." The bench was cut so as to leave a definite projecting point, into which four nails were usually driven. The cuts were so made that no water, to cause decay, would stand in the notches.

GENERAL MAPS.

Except as to levels taken, the general maps exhibit about all the information collected on the survey. A dotted line shows the position of the best steamboat channel. The soundings given are not reduced to low water, but at the beginning and ending of each day's work are notes giving the stage of the river existing at the time. In places where the bottom could not be reached by a pole 16 feet long the sounding was entered as "N. B." Positions of trees or logs in the channel are shown by "t" or "l." Marginal notes indicate particularly bad obstructions, whether logs, snags, or overhanging trees. The traverse-line of the survey is given in a light, full line; transit points are indicated thus, △, and rod points ⊙. The former are usually accompanied by numbers and the latter by letters. The numbers begin anew each day, or as often as necessary to avoid higher numbers than 10 or 12. Bench marks are shown thus ◎ with their descriptions given in marginal notes. Their elevations are given above mean low

water at Fort Pulaski, as determined by the United States Coast and Geodetic Survey in 1874. All the more important landings on the river are located and named. The larger creeks, branches, and "breakovers" are shown on the maps. with estimated low-water dimensions. Systems of hachure-lines are shown along the river-banks; where they are above 20 feet in height, each system represents a difference in level of about 20 feet. The bank heights given are estimated above the existing stage of water. Those parts of the river or of its branches not in sight from the main channel are sketched in from the best information available, and are not presumed to be correct, except in the lower 13 miles of the river, where the parts remote from the channel were taken from the U. S. Coast Survey Chart of 1857.

DETAIL MAPS.

On the detail sheets the top of the immediate banks and low-water lines are shown. The soundings on these sheets are reduced to low water. Bottom contours of 2, 4, 6, 8 feet, and often of 5 feet, are shown.

Lines at the centre and sides of the best channel, 80 feet wide, are also shown. Accompanying the plat in each case is a longitudinal section of the channel at the shoal. Upon these sections are shown the surface of the water at the time the soundings were taken; the assumed low-water line, a line representing the bottom of the river in the centre of the channel, and another line showing the least depth in the channel 80 feet in width.

In addition the 2, 4, 5, 6, and 8 foot contours are projected on the section. Positions every 100 feet along the channel-line are marked with letters, agreeing with the same positions shown on the section. The height of local low water above the sea and the description and elevation of bench-marks are also given. The detail maps between Augusta and Blue-House Bar show the exact positions of all jetties constructed. Where the bank and low-water lines of 1886 and 1887 differ from those of this survey they are shown in colored lines. The cutting or filling of the banks is thus shown at a glance.

RIVER-GAUGES.

Several gauges were established along the river and gauge-readers employed. The zero was determined with reference some pre-established datum. Also, while the survey was in progress a river-gauge was kept at the house boat. As soon as the boat stopped, for whatever length of time, a gauge was put out and read at such intervals as would give the changes in the stage of water. The amount of rise or fall in the river during the night was determined instrumentally each morning on continuing the survey.

GAUGINGS.

The methods employed in gauging the river were, in general, as follows:

A suitable place having been chosen, a base-line 200 feet in length and as nearly parallel as possible to the main current was laid off along the river-bank. From either end of the base-line sections at right angles to it were marked out. and soundings on these lines carefully taken. The position of every second or third sounding was located from shore by means of transit and stadia. The velocities of the stream were determined by means of loaded rod-floats. The floats were dropped from a boat a short distance above the upper section ranges; the position of crossing the upper range was determined by a transit on the range at the upper end of the base-line with a stadia-rod in the boat closely following the path of the float; the position of crossing the lower range, 200 feet below, was determined by an angle, taken with the transit, between the base-line and float. The time of crossing each range was taken to the nearest second.

Floats were run across these sections every 10 to 25 feet, depending upon the section of the river bottom. The height of the surface of the stream during the sounding of the section and the gauging were taken from a temporary gauge, referred in each case to one or two permanent bench-marks. A plat is made to proper scale, showing the ranges and so much of the river as is necessary. The path of each float is shown upon the plat. Cross-sections of the river-bed are shown at both upper and lower ranges. A third and mean cross-section is constructed on a line midway between the ranges, with depths of water, where each float crosses this intermediate line, equal to a mean between the depths of the water under the points where the float intersects each of the ranges. In general, the longest floats which would not drag the bottom were used. The mean velocity for any vertical is determined from the rod velocity by the following formula:

$$V^m = V_r[1 - 0.116(\sqrt{D} - x)];$$

in which

V^m = mean velocity for the vertical;

V_r = velocity of the rod;

$$D = \frac{\text{distance between the bottom of the rod and the river-bottom}}{\text{depth of river}};$$

and x is a function of $\frac{l}{d}$, l being the length of the rod and d the depth of the water. For the various values of $\frac{l}{d}$ the following values of x are used:

When	$\frac{l}{d} = 0.9$	$x = +0.10$
	0.8	+ 0.05
	0.7	0.00
	0.6	− 0.05
	0.5	− 0.10

When $\frac{l}{d} = +0.9$ or more, the formula is that given by Mr. James B. Francis in the "Lowell Hydraulic Experiments."

From the mean of the velocities and mean section the discharge is determined. When the river-bottom was not reasonably regular or when greater accuracy was desired, a middle section was also sounded out and a mean section determined in the manner mentioned above, double value being given to the middle section.

Hydrographical Survey of the Lower Savannah River.

TIDAL OBSERVATIONS.

Tide-gauges were established at fourteen places, and upon these continuous readings every five minutes were made from sunrise to sunset during the first three weeks of the survey, and for some time subsequently, at high and low water. As many as 250 high and an equal number of low water observations of time and height were obtained on each of the standard gauges. These were considered as sufficient for forming a satisfactory basis of the tidal part of the survey.

From these were deduced the corrected establishment; mean lunitidal interval of low water; mean duration of rise and fall; height above mean low water at Fort Pulaski of both high and low water; mean rise and fall. The highest high water and lowest low water observed at the standard gauges were recorded.

From the distances between the tide-gauges and the times of high and low water the duration and rate of propagation were found.

CURRENT OBSERVATIONS.

"Turning now to the discussion of the most important part of the survey, viz., the current observations, it seems pertinent first of all to mention the general conditions under which they were made and the method adopted to reduce their results to uniform mean conditions. The two factors determining the flow are the volume of the tidal prism and the amount of fresh water flow, both of which are variable and will, therefore, in their combination produce an almost endless variety of results. Any measurement of ebb outflow or of flood inflow can, therefore, be of value only if the conditions are stated under which they were executed; and, in order to utilize a number of such results obtained at various times, they should be reduced to the uniform basis of mean volume of tidal prism and mean fresh-water flow. The first correction has been made by assuming the rise and fall of tide to be directly proportional to the volume of the tidal prism, and consequently to the entire outflow or inflow through a given cross-section. The second correction has been made by reference to the observations at the Augusta gauge, a reading of 10 feet on the same having been assumed as representing a mean state of the upper river, and consequently of the fresh-water flow.

"It is generally accepted that the conditions existing in the river at Augusta make themselves felt at Savannah eight days later, and with this assumption the figures given in a table of gauge-readings and corresponding discharge in cubic feet per second at Augusta were applied to the results of current observations; in this way any excess of fresh-water flow above the mean was subtracted from the ebb outflow and added to the flood inflow, while any deficiency was added to the former and subtracted from the latter. The weak point in the method consists in the difficulty of determining accurately the time when a given condition of the river at Augusta makes itself felt at Savannah, and furthermore in the fact that only the variations of fresh-water flow in the upper river were considered, while those below Augusta were ignored."

The first deficiency will depend upon whether the changes in the river during the time are slight and gradual, or great and sudden. In the survey under consideration changes were slight, ranging from 8 to 10 feet by gauge-readings; and the corrections for fresh-water flow were small. The second deficiency depends upon the relative dimensions of the river-basin above and below Augusta (2 to 1), and the character of the basin. In this case the country below is flat, and the conditions are such that the evaporation and absorption are large. and the direct flow from the drainage area into the river correspondingly small.

The instruments used for making the current observations were

(1) Ellis meter "A";
(2) " " "B";
(3) Stackpole propeller-meter, No. 1;
(4) " " " No. 3;

—all provided with electrical recording attachment.

The rating of the meters "A," "B," and 3 was done by moving them through still water at various rates of speed. The results obtained were expressed by the equation

$$y = ax + b,$$

in which y = velocity of current in feet, x = number of revolutions per second, and a and b constants found from the process of rating by plotting the results obtained, making x the abscissas and y the ordinates, measured from rectangular

co-ordinate axes. The equation is that of a straight line cutting the axis of y at b distance from the origin, and making an angle with the axis of x whose tangent is a.

The record of the wheel was given by an ordinary electric sounder or by a bell register inserted in the electric conduit. The general arrangement of the apparatus in the boat and the method and appliances for lowering and raising the meter are similar to those described in the Report of the Chief Engineer U. S. Army for 1875, pages 308 and 309. The meter was always suspended from the bow of the boat, which was invariably headed against the current. As soon after turn of tide as the current was sufficiently strong to swing the boat, the bow and stern lines, by which it was anchored, were shifted, the former to the stern and the latter to the bow, in order to head the boat into the current again. The plan followed during the work was to locate, by means of instrumental observations from the shore, points from 120 to 200 feet apart on each line to be gauged, and to observe at each of such points one practically complete ebb and flood.

The observations were commenced at the surface, then the instrument was lowered to 3 feet below, then to 6 feet, then to 9 feet, and so on until the bottom was reached; the duration of each observation generally being about one minute. After bottom had been reached the meter was hauled up, examined, and then a new series of observations was commenced. Frequent examinations are necessary to keep the instrument clear of weeds, etc., which wrapping themselves around it interfere materially with its action. By commencing early in the morning and continuing till dark, it was generally possible to complete ebb and flood observations on one vertical in one day. But there were many delays due to examining the instrument, and to the weather, which prevented any such rapid prosecution of the work.

On seven cross-sections 50 verticals were located and about 15,700 single observations made.

The notes during observations were made according to the following schedule:

Time, P. M.	Depth below Surface, feet.	Duration of Observations. M. S.	Observed Number of Revolutions.	Direction.	Computed Velocity in ft. per sec.	Wind.
3.48	1.0	0.40	59	Ebb	2.056	S. E.
3.50	3.0	0.40	65	"	2.258	Mod-
3.51	6.0	0.40	55	"	1.922	erate
etc.	etc.	etc.	etc.	etc.	etc.	

The reductions and application of the corrections necessary to determine the flow, from mean volumes of tidal prism and mean fresh water, are long and somewhat complicated. The conditions affecting any particular case would be different from any other. Different elements would enter into the computation, and these would have different weight depending on local relations and surroundings. A typical example, worked out in full, can be found in the Report of the Chief Engineer U. S. Army for 1890, pages 1263 and 1333, to which the reader is referred. The following system of notation, adopted for the precise designation of the quantities to be dealt with, will direct attention to the elements entering into the computation, and also to their great number and variety.

NOTATION ADOPTED.

a = area of mean low-water cross-section;
a_f = average area of mean flood inflow;
a_e = average area of ebb outflow;
p = length of wetted perimeter;
$r = \frac{a}{p}$ = mean hydraulic radius;

$Q = \begin{cases} \text{mean ebb discharge in cu. ft. per sec} \\ \text{mean flood inflow in cu. ft. per sec;} \end{cases}$

$V = \frac{Q}{\alpha}$ = mean velocity;

D = depth of river at any given point;

Δ = maximum of mid-channel depths.

All velocities designated by V are mean velocities of the entire duration of either ebb or flood flow, the depths at which they exist being indicated by a right-hand and lower index as follows:

V_0 = mean surface velocity at a given point;

$V_{\frac{D}{2}}$ = mean mid-depth velocity at a given point;

V_D = mean bottom velocity at a given point;

V_m = mean of mean velocities of a given vertical.

The upper index m indicates the mean of all the mean velocities across the width of the river as follows:

V_0^m = mean of mean surface velocities of a given cross-section;

$V_{\frac{D}{2}}^m$ = mean of mid-depth velocities of a given cross-section;

V_D^m = mean of mean bottom velocities of a given cross-section;

V_m^m = mean of all V_m's found in a given cross-section;

D_m = depth in fraction of total depth, at which V_m is found.

All velocities designated by C are velocities at the time of either ebb or flood maximum current, the further designation by indices remaining the same as before.

C_0 = surface velocity at a given point at maximum current;

$C_{\frac{D}{2}}$ = mid-depth velocity at a given point at maximum current;

C_D = bottom velocity at a given point at maximum current;

C_m = mean velocity at a given point at maximum current;

The addition of the upper index m has the same meaning as before. All velocities designated by U are the greatest velocities existing in a cross-section at the time of either ebb or flood maximum current.

U_0 = greatest surface velocity of cross-section at the time of maximum current;

U_D = greatest bottom velocity, etc., as just given for U_0.

A NEW CURRENT-METER AND A NEW METHOD OF RATING CURRENT-METERS.

By W. G. Price, U. S. Engineer.—(*Eng. News*, Jan. 10th, 1895.)

This meter was constructed for use in measuring velocities of water in shallow rivers and canals, where it can be supported at the proper depth by being attached to the end of a metal rod or pipe, which is held in the hand of the observer. It was designed to meet the following conditions, viz.: The wheel must be strong enough to withstand quite hard knocks, which it is liable to receive while in use, without being injured. The bearings of the wheel must be so constructed that the friction will be very slight and a constant quantity. Means must be provided for counting the revolutions of the wheel, and the mechanism used must not appreciably add to the friction of the bearings.

The meter as constructed has a strong wheel, composed of six conical-shaped cups, bound together by a solid frame, which revolves in a horizontal plane, and is carried on two bearings, which are at the top of deep, inverted cups, which hold air and oil, which at all times entirely excludes the water and the grit or other matter which the water may contain. Just above the upper bearing there is a small air-chamber, and the shaft of the wheel extends up into it.

The water cannot rise up to the air-chamber, as it cannot compress the air sufficiently to do so. In the air-chamber there is a small worm-gear on the shaft, which turns a small wheel which has 20 teeth. The wheel carries a pin, which for every 20 revolutions of the shaft depresses and releases a pin-head spring-hammer which strikes a small diaphragm that forms the top of the air-chamber. The rod which is used to support the meter is hollow, and the diaphragm is at the bottom of the rod. The sound produced by the striking hammer is transmitted through the rod and through a connecting tube to the ear of the observer. The ear-piece is held in place by a band of elastic ribbon. The rod is in lengths of 2 feet, and is graduated to feet and tenths. The wheel is 5 inches in diameter and the meter, without the rod, weighs 17 ounces. A meter constructed in this way will not change its rate so long as it is well cared for and is not bent out of its original shape. A few drops of thin oil, such as will not become solid in cold water, must be placed in both bearings once every day it is used. Even though the water to be measured carries in suspension a large quantity of silt and other matter, it cannot injure the meter or change its rate.

The method of rating this meter, which is also applicable to the electric meter, is as follows: The rating should be made in a pond of still water, and a test with rod-floats should be made to determine whether the water is absolutely still or not. A small, deep pond is best. In many of the large crescent-shaped lakes, along the Mississippi and Missouri rivers, which have no inlet or outlet, there is a small oscillation of the water, probably caused by the wind, which will render a rating made in them quite inaccurate. The meter should be attached to the bow of the skiff, so that the supporting-rod will be vertical, and the wheel will be about 2½ feet below the surface of the water.

Take two metallic tapes, each 50 feet long, and lengthen one of them with 100 feet of steel wire—about No. 24. Take the tapes out of their boxes and wind them on two wooden reels which are about 6 inches in diameter and are placed side by side on a single shaft, on which they are free to turn independently.

The reels must be provided with pawls with sharp points, which can be pushed into the wood to stop them from turning. The shaft and pawls can be supported by a wooden frame, and this should be screwed or nailed to the forward seat in the skiff. The oarsman sits in the after seat. The observer sits just forward of the reels, and the rubber tube leads from the meter-rod to his ear. The tapes are unreeled far enough to pass the ends over the stern of the skiff and to a stake which is driven firmly at the water's edge, and which has a nail driven half its length into the top. Rings of about No. 20 cotton thread are tied in the ends of the tapes, and are passed over the nails in the stake. The oarsman then rows the skiff straight away from the stake, and the tapes are unreeled as the skiff moves along. A little friction from the hand of the observer on the reels keeps the tapes taut. When the observer hears the first click made by the hammer in the meter he instantly starts his stop-watch and presses a pawl into the wood of the reel which carries the 50-foot tape. This stops the motion of the reel and breaks the thread which connects the tape to the nail in the stake. When the meter-wheel has made 40 revolutions the observer will hear the third click, and at that instant he stops his watch and presses the second pawl into the second reel, which stops the long tape from paying out and breaks the thread which attaches it to the nail in the stake. The difference in the reading of the tapes at the reel, plus 100 feet, will be the distance the meter travelled while the wheel was making 40 revolutions, and the reading of the watch is the required time. The reading of the tapes must be corrected for any error there may be in their lengths.

Data should be obtained for very slow, medium, and high velocities; and to obtain the high velocities it will be necessary to pull the skiff with a ½-inch cotton rope, which can extend from the bow of the skiff to the opposite bank of the pond; but if this is done the oarsman should be retained in the skiff to stop it from running into the bank and to return it to the starting position. If the pond is very wide, a long stake may be driven out in the pond, and it can be made rigid at its top with small guy-ropes, which can lead diagonally down to the bottom of other stakes, which are driven around it, the ropes being tied to the stakes before they are driven, then all attached to the centre stake. The skiff can then start from this stake, and the pulling-rope can lead to the shore. If the skiff is propelled at all velocities with a rope in this way the rating will be more accurate, as the motion of the oarsman in rowing rocks the skiff and thus causes the meter to travel through a curved path which is longer than the distance measured with the tapes. Just before each trip is made over the base the meter-wheel should be turned till it is about midway between two clicks, so that the first click will be heard by the observer before all of the short tape is paid out. The wheel will make 40 revolutions in about 103 feet. A longer wire can be used to lengthen the tape, so that a measurement for 60 or more revolutions may be made.

The amount of error in such a rating, supposing the skiff to move at the same velocity at the instant the first and last clicks are made, depends entirely on the time intervals used by the observer in starting and stopping his watch, and in stopping the reels after he hears the click of the meter. In order to eliminate all error, the time consumed by him to do this must be the same at the beginning as at the end of the trip.

When the data are reduced to revolutions of the meter-wheel per second and velocity in feet per second, and then plotted on cross-section paper, taking the revolutions per second as abscissas, and the velocities in feet per second as ordinates, the plotted points will lie in a curve. A line can best be drawn through these points by bending a true steel straight-edge between three heavy paper-weights till one side of it coincides with the points. It has been the usual practice to assume that this line should be straight, and expressed by the equation

$$y = ax + b,$$

from which it is easy to compute a reduction-table; but when more accurate ratings are made, so that all the plotted points are in their true positions, it is found that the elastic curve formed by the bent straight-edge passes precisely through them all. This curve departs so much from a straight line, it is seen, that to compute the reduction-table upon the assumption that the line should be straight is a very inaccurate method. By an inspection of the curve, it will be seen that for successive higher velocities the revolutions per second increase faster than the velocities in feet per second. This indicates that the slip of the wheel or the screw, as in a steamship propeller, becomes less as the velocity increases; and we can think of a velocity so great that the water would become in effect so near a solid there would be no slip, and beyond that point the rating would be a straight line.

The friction of the bearings of a meter should be so slight that the wheel will begin to revolve when a very low velocity is attained. For the first pneumatic meter constructed this velocity was 0.042 foot per second. In order to make a reduction-table from a meter rating, form a three-column table and write in the first column the times in seconds of 100 revolutions of the meter-wheel, from 20 seconds to 400 seconds, or more if very slow velocities are to be measured. Then compute the revolutions per second corresponding to those times of 100 revolutions, and write them in the third column. Then from the plotted curve scale

off the velocity in feet per second corresponding to those revolutions per second, and write them in the second column. In using the meter, measure the time of 100 revolutions, and look in the table for the corresponding velocity. Such tables are usually furnished by the manufacturers.

In *Engineering News*, March 2, 1893, a current-meter with an electric registering attachment, was described invented by the author of the above article. The pneumatic meter above described is a much simpler and cheaper device, and is recommended as preferable for use in shallow rivers and canals.

The price of this meter is only about $50.

THE RITCHIE-HASKELL DIRECTION CURRENT-METER.

In conducting the hydrographic surveys for harbor and river improvements it is often desirable to know the direction as well as the velocity of the prevailing currents in order to calculate their value for good or harm. In tidal streams, for example, for some time before the turning of the tide, an "under run" of the flood-tide is going on while the surface is still ebbing. These subcurrents of tidal waterways are of extreme importance in their effects upon the sea bottom, and need to be determined with considerable accuracy both in direction and velocity. For determining the velocity of currents one of the common methods, viz., by floats, gauge-tubes, or current-meters, may be used, but the last two methods give nothing concerning the direction of the current. The Ritchie-Haskell meter is designed to record simultaneously both the direction and velocity of any character of subsurface currents.

The principal differences of appearance between this and the ordinary form of current-meter are the propeller-wheel and the fish-shaped body, inside of which is placed the magnetic needle and other mechanism for actuating the recording apparatus in the hands of the operator.

The velocity-wheel is of the screw or propeller type, and is made conical in form to prevent the catching of weeds, grass, and other floating débris.

This wheel may have any desired pitch to suit any kind of work, but thus far only two styles of wheels have been made, the first covering a range of from 0.2 foot to 6 feet per second in velocity, and the second a range of from 0.6 foot to 12 feet per second in velocity. Of course a meter can have both of these wheels furnished with it, so that either one can be attached as desired. The electric mechanism for transmitting the number of revolutions of the wheel to the velocity register is, as stated above, placed inside the fish-like body of the meter. In the velocity register starting the watch closes the circuit, and stopping the watch breaks the circuit, thus giving an absolute record of the time and the corresponding number of revolutions made in that time.

The apparatus for determining the direction of the current and actuating the direction register is also placed in the fish-like body of the instrument. This apparatus consists primarily of a compass, whose needle is free to assume the magnetic meridian at all times, immersed in an oil-chamber to prevent rust. An expansion-bag compensates for changes in temperature, and establishes equality of pressure between the inside and outside of the chamber when submerged. By the use of an electric current the angle, to the nearest degree, between the direction of the current and the compass needle is shown on a dial. This dial is graduated in azimuth from south to west, and also has the points of the compass given. The total length of the meter, which has a low-pitch wheel $7\frac{1}{2}$ inches in diameter, is 36 inches, and its weight is 30 pounds, exclusive, of course, of the lead weight, which can be adjusted to weigh 20 pounds, 35 pounds, or 50 pounds, as desired. The cable for suspending the meter is of galvanized Bessemer steel, $\frac{7}{16}$ inch in diameter, with a core formed by three insulated wires—three circuits

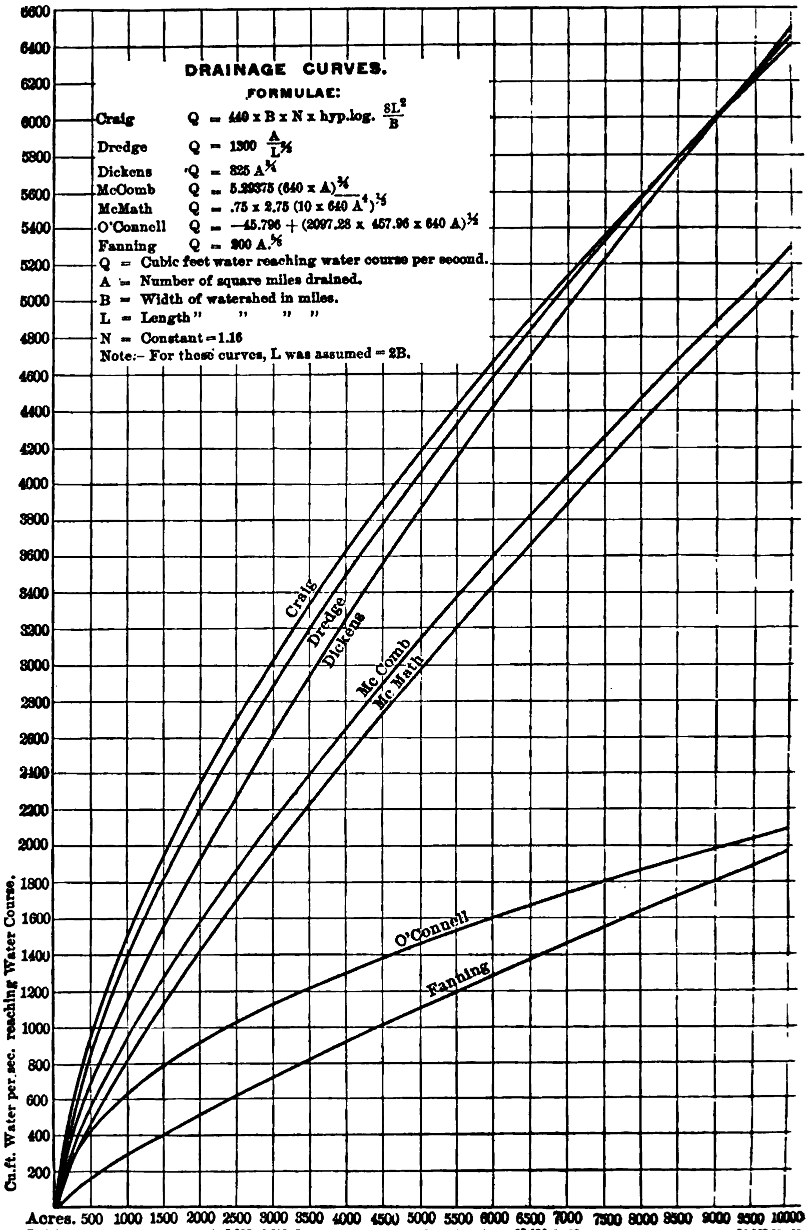

DRAINAGE CURVES.
FORMULAE:
Craig $Q = 440 \times B \times N \times \text{hyp.log.} \frac{8L^2}{B}$
Dredge $Q = 1300 \frac{A}{L^{2/3}}$
Dickens $Q = 825 A^{3/4}$
McComb $Q = 5.29375 (640 \times A)^{3/4}$
McMath $Q = .75 \times 2.75 (10 \times 640 A^4)^{1/5}$
O'Connell $Q = -45.796 + (2097.28 \times 457.96 \times 640 A)^{1/2}$
Fanning $Q = 200 A^{5/6}$
Q = Cubic feet water reaching water course per second.
A = Number of square miles drained.
B = Width of watershed in miles.
L = Length " " " "
N = Constant = 1.16
Note:- For these curves, L was assumed = 2B.
Craig
Dredge
Dickens
Mc Comb
Mc Math
O'Connell
Fanning
Cu.ft. Water per sec. reaching Water Course.
6600 6400 6200 6000 5800 5600 5400 5200 5000 4800 4600 4400 4200 4000 3800 3600 3400 3200 3000 2800 2600 2400 2200 2000 1800 1600 1400 1200 1000 800 600 400 200
Acres. 500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000 6500 7000 7500 8000 8500 9000 9500 10000
Sq.Miles. 0.781 1.563 2.344 3.125 3.906 4.688 5.469 6.250 7.031 7.813 8.594 9.375 10.156 10.937 11.718 12.500 14.843

FIG. 456.

being required for both recording registers—from the meter to the recording apparatus. The total cost of the apparatus, including both velocity and direction registers, is $280. Extra wheels cost $25 each. The meter, without the direction-recording apparatus, can also be secured in various sizes and styles.

In an instrument especially designed for gauging brooks, creeks, and irrigation ditches the suspending cable is replaced by a graduated hand rod, and a foot-plate is provided to prevent the wheel from sinking into the mud of the bottom. This instrument cost $130, is 14 inches long, with a 4-inch-diameter wheel, and weighs 7 pounds. It will register velocities varying from 0.4 foot to 5 feet, and from 1 foot to 10 feet per second, with the low and high pitch wheels respectively.

These meters are now used by many of the United States engineers in their hydrographic work, and have also been adopted by a number of large irrigation companies.

In the Government work Mr. Haskell has used the direction current-meter in measuring the currents in New York Harbor, Long Island Sound, the Gulf Stream, and the Gulf of Mexico since 1887, and states that he believes it to be reliable under all conditions.

CURVES SHOWING AMOUNT OF WATER REACHING STREAMS AND SEWERS.

The amount of water reaching streams and sewers will, for simplicity, be called the "Run-off."

The author is indebted to Col. W. E. Cutshaw, City Engineer of Richmond, Va., for the following remarks on the run-off from the ground surface, and also for the accompanying diagrams.

In Fig. 456 will be found the Drainage Curves, embracing a wide range of drainage surfaces in acres and square miles, together with the more common formulæ in use. This diagram shows the cubic feet of water per second reaching the watercourses.

Diagrams Figs. 457, 458, 459, 460 are intended to represent curves showing the number of cubic feet of water per second per acre reaching sewers from rainfalls of 1, 2, 3, and 4 inches per hour, together with the formulæ in more common use. These various diagrams fully explain themselves, and the formulæ will be readily understood.

Col. Cutshaw writes that these diagrams show what may be done about drainage and sewerage in the way of finding what allowances should be made for storm discharges.

The practice now is to determine by one formula the amount of storm-water to provide for, and then to calculate the sewer or channel to discharge it by another formula. Though most cities use this method, some have used a single combined empirical formula for determining the diameter or sizes of sewers.

The run-off depends on such conditions that the variations of formulæ as well as the difficulties of applying them to small city areas and to large country areas alike make the applications unsatisfactory. The same imperviousness of surface and the same degree of saturation will not obtain for like areas; and the surface slope varies to such an extent with the rolling features of the country that the constants introduced into all these formulæ are difficult to settle upon, even after comparisons with extended observations of rainfalls and channel discharges.

Formulæ now applying approximately well for city areas do not apply to country areas, where the storm discharges are carried off by creeks and rivers. The curves on the diagrams should be particularly noticed in this respect.

The best formulæ now used seem to be based on variable areas, variable slopes, and variable rainfalls, the powers, roots, and constants used in each giving it its special merit.

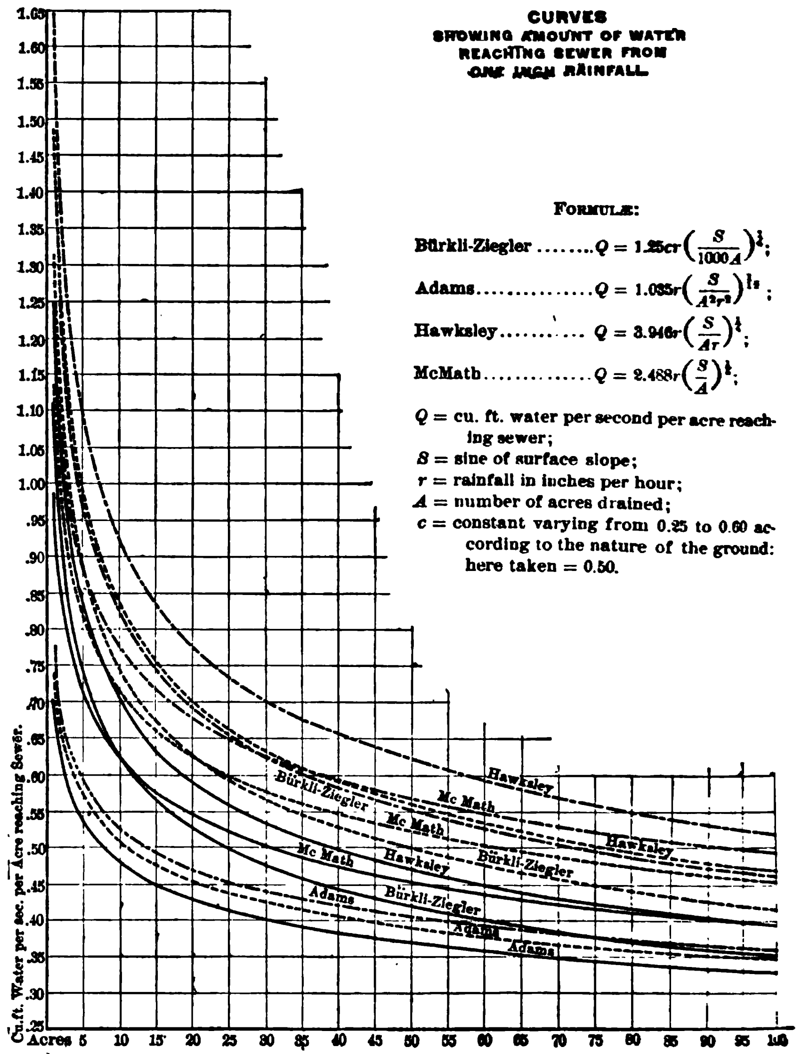


FIG. 457.

	Shown thus
Curves for 1/100 surface slope	————————
" " 2/100 " "	....
" " 3/100 " "	— — — — —

Automatic rainfall records for short periods of heavy rainfalls and the sewer gaugings of run-off corresponding are now being observed more closely in the cities where combined sewers are used, and these observations are beginning to

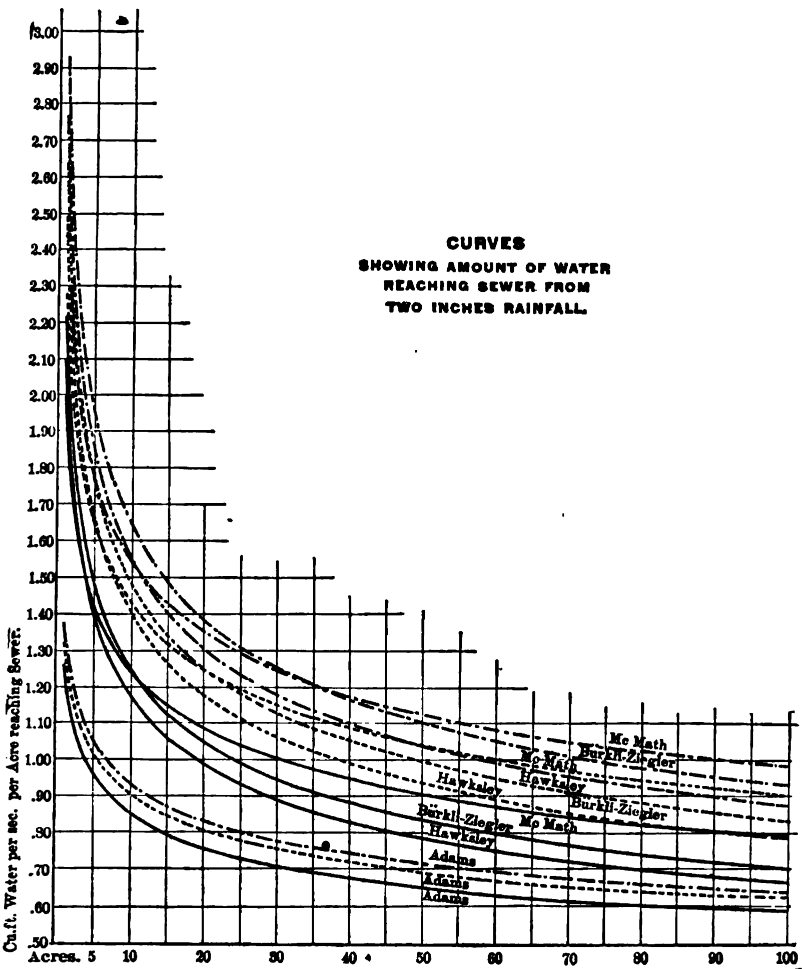


FIG. 458.

ettle which formula best applies to each city. Col. Cutshaw's purpose in preparng these diagrams was, by comparisons of curves of run-off, to determine which s best to use in Richmond. Even with the four best formulæ for run-off, three f them,—Hawksley's, McMath's, and Bürkli-Ziegler's,—it will be noticed, give urves for small areas (under five acres) showing more run-off than rainfall; and

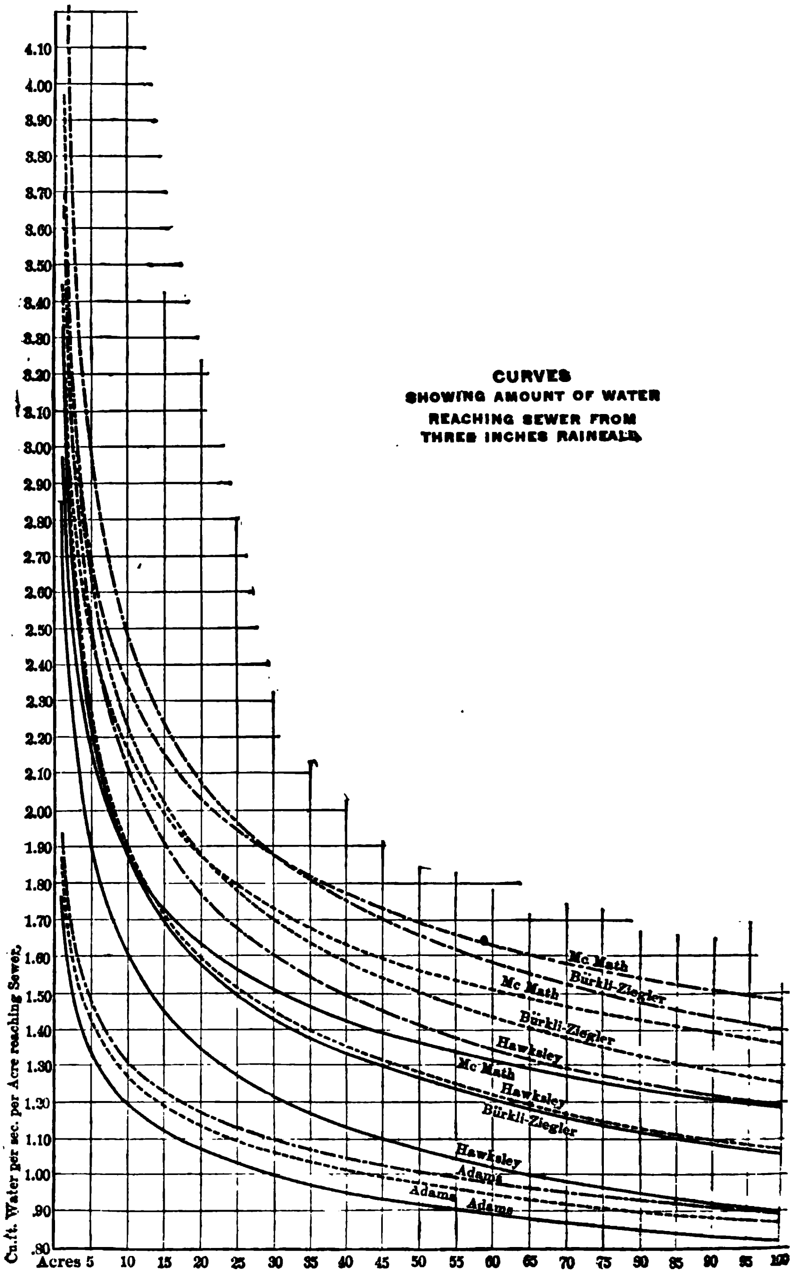

CURVES
SHOWING AMOUNT OF WATER
REACHING SEWER FROM
THREE INCHES RAINFALL
Cu.ft. Water per sec. per Acre reaching Sewer.
4.10
4.00
3.90
3.80
3.70
3.60
3.50
3.40
3.30
3.20
3.10
3.00
2.90
2.80
2.70
2.60
2.50
2.40
2.30
2.20
2.10
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
.90
.80
Acres 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Mc Math
Bürkli-Ziegler
Mc Math
Bürkli-Ziegler
Hawksley
Mc Math
Hawksley
Bürkli-Ziegler
Hawksley
Adams
Adams
Adams

FIG. 459.

yet Bürkli-Ziegler's and McMath's are more generally used, because of a better agreement with observed run-off from areas, say, above 50 or 60 acres. None of

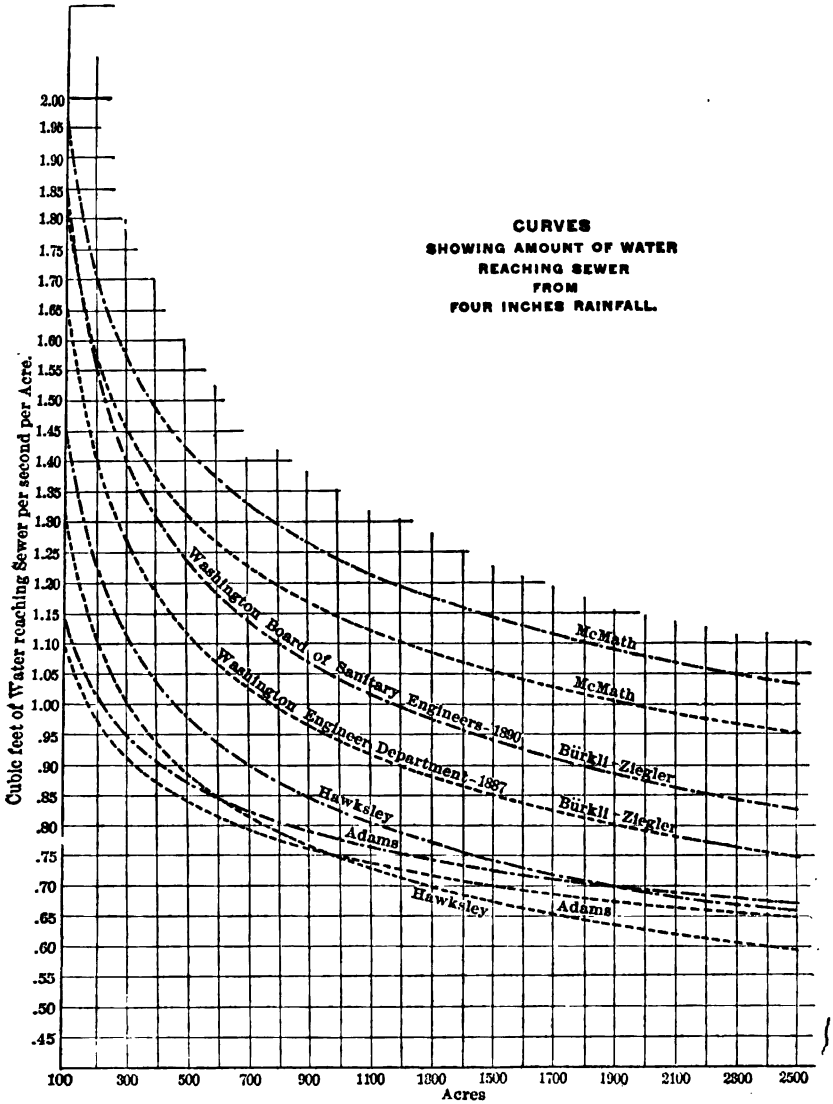


Fig. 460.

iese formulæ, and still worse, none of the various flood-discharge formulæ, are ıtisfactory in very large country areas, as will be seen from the attempt to ap-

ply them to Stony Brook, near Boston: there they were found to give from about 540 cubic feet per second in the lowest to 5500 cubic feet per second in the highest, and this discrepancy on an area of about 8000 acres. The Board of Engineers, investigating the drainage of the stream, discarded all the formulæ, and assumed, from a very unusual set of observations extended over a very large area, that 12 inches in 24 hours, or ½ inch per hour, was the rainfall over this area, and that three fourths of it ran off; in other words, that 3000 cubic feet per second should be provided for. A similar result was worked out for Rock Creek at Washington City, where some 49,000 to 50,000 acres were involved; and using some five or six flood-discharge formulæ, the cubic feet per second, as calculated, varied from 4707 to 25,640, and was finally taken at between 18,000 and 25,000 cubic feet per second, practically assuming ½-inch rainfall per hour, on the whole area of 49,363 acres. It is to be noticed that McMath has his formulæ modified so as to apply to city areas for sewers and to country areas for streams.

Kutter's formula for mean velocity from which to determine the sizes of sewers or channels seems to be about the best, and is almost exclusively used.

An interminable amount of detail and discussion is involved in this subject, and it is difficult if not impossible to put it in a concise and perfectly digested form for a general work on Engineering, and the reader must be referred to such works as Fanning's Water-supply Engineering, Latham's Sanitary Engineering, Staley and Pierson's Separate Sewers, Flynn's Irrigation Canals, etc., Stephenson's Canals and Rivers, etc., for full discussions under their appropriate titles.

APPLICATION OF KUTTER'S FORMULA TO THE FLOW OF WATER IN OPEN CHANNELS.

The author has not considered it necessary to introduce in this volume extended tables of the coefficient of discharge c, or of the square roots of the hydraulic radius r, or of the square roots of the slope i, to be used in the solution of problems of flow by Kutter's formula. Such tables are of great value to an engineer when working a number of problems, and can be found in such works as Flynn on Irrigation Canals and Works, also Wilson on the same, and to a limited extent in Merriman's Hydraulics and Trautwine's Engineer's Pocket-Book.

This article will be limited to the working out a few examples in full. From a clear understanding of these the reader will have little trouble in working out similar problems under other conditions as to roughness of bed and side surfaces, slope of bed, and hydraulic radius. In order to obtain very accurate results, all terms should be carried out to five or six places of decimals; for less important problems two places of decimals are sufficient. In Article IX is found a general discussion of the flow of water in open channels, and the formulæ of D'Arcy and Kutter are given, with the values of the coefficient of roughness n and a table containing a few of the coefficients of discharge. As will be noticed, D'Arcy's formula, eq. (40), depends upon and varies with the hydraulic radius r and slope i, while Kutter's depends upon and varies not only with r and i, but also with the condition of surface of the channel as represented in the coefficient n. There are a large number of formulæ in use, all of which have constant coefficients, except the four following, Kutter, Bazin, Molesworth, and Gauchler. Only the first two will be considered in this place. D'Arcy's formula will be referred to under the head of the flow of water in pipes.

BAZIN'S FORMULÆ.

For very even surfaces, such as planed planks, smoothly plastered sides and beds, and other surfaces in the same or similar condition,

$$v = \sqrt{1 + 0.0000045\left(10.16 + \frac{1}{r}\right)} \times \sqrt{ri}.$$

For surfaces of cut stone, brickwork, ordinary mortar, sawn plank, etc.,

$$v = \sqrt{1 + 0.000013\left(4.354 + \frac{1}{r}\right)} \times \sqrt{ri}.$$

For rubble masonry and similar surfaces,

$$v = \sqrt{1 + 0.00006\left(1.219 + \frac{1}{r}\right)} \times \sqrt{ri}.$$

For uneven surfaces, such as earth,

$$v = \sqrt{1 + 0.00035\left(0.2438 + \frac{1}{r}\right)} \times \sqrt{ri}.$$

In these formulæ it is to be noted that the coefficients of $\sqrt{ri}$ depend upon and vary with the character of the surface over which the water flows and also upon the hydraulic radius r, but are independent of the inclination or slope of the bed of the channel.

Mr. Flynn says that Bazin's formula for given channels agrees very nearly with Kutter, $n = 0.0275$ up to 3 feet in depth, and with Kutter, $n = 0.025$ from 3 to 5 feet in depth. It is also shown that Bazin's formula is almost a mean between Kutter with $n = 0.025$ and $n = 0.0275$; that is, that it almost suits *canals and rivers in earth of tolerably uniform cross-section, slope and direction, in moderately good average order and regimen and free from stones and weeds*, and also *canals and rivers in earth below the average in order and regimen.* The results again show that it gives too low a velocity for *canals in earth above the average in order and regimen* with $n = 0.0225$ or a smaller value of n, while it gives a too high velocity for *canals and rivers in earth, in rather bad order and regimen, having stones and weeds occasionally*, and obstructed by *détritus*, for which $n = 0.03$. With these remarks no further notice will be taken of Bazin's formulæ, as the practical application of them can be readily made after understanding the examples worked out with Kutter's formula:

The following is Kutter's formula :

$$v = \left\{ \frac{\frac{1.811}{n} + 41.6 + \frac{0.00281}{i}}{1 + \left(41.6 + \frac{0.00281}{i}\right) \times \frac{n}{\sqrt{r}}} \right\} \times \sqrt{ri} = c\sqrt{ri};$$

in which, as in other formulæ, n is the coefficient of roughness of the surface of the channel, and, as given in Art. IX, varies from 0.009 to 0.05, the smaller values from 0.009 to 0.02 applying to the smoother surfaces and from 0.0225 to 0.05 applying to the rougher surfaces, as more specifically described in Art. IX. The coefficient c, seemingly complex, depends upon the roughness of the surface as

represented by n, the hydraulic radius r, and the slope of the bed i. With the smoothest pipe it would be hardly advisable to use a value of n less than 0.013 in order to provide for increase of resistance from use; and it will rarely be justifiable to use a value greater than $n = 0.03$. It will serve the present purposes to determine the values of c for values of n between the limits of 0.017 and 0.025. The slopes will be taken at 1 in 1000, 1 in 10,000, and 1 in 20,000. The hydraulic radius will depend upon the area of the cross-section and the surface wetted by the water, or wetted perimeter, as it is called.

EXAMPLE 1.—Given the bed width, depth, and grade of a channel, to find the velocity and discharge. We will assume that the nature of the surface falls under the one or the other of the following conditions: *Rubble in cement; coarse rubble of all kinds; also coarse gravel carefully laid and rammed*, or *rough rubble where the interstices have become filled with silt.* (See Art. IX, par. 75, for which $n = 0.02$.) Since $r = \frac{\text{area}}{\text{wetted perimeter}}$, we will assume the width of the channel = 125 feet, and its depth 12.5 feet, with side slopes 1 to 1. (Here it may be noted that when the bed width is over from 60 to 70 feet the side slopes have very little effect on the velocity.) The area will be $(125 + 12.5) \times 12.5 = 1718.75$ square feet; the wetted perimeter will be $125 + 2 \times 12.5 \times \sec 45°$, or $125 + 2\sqrt{12.5^2 + 12.5^2} = 125 + 35.4 = 160.4$; then the hydraulic radius $= r = \frac{1718.75}{160.4} = 10.71$ and the $\sqrt{r} = 3.27$. The slope i will be taken 1 in 10,000 = 0.0001, $\sqrt{i} = \frac{1}{100} = 0.01$. We have now the values of $n = 0.02$; $\sqrt{r} = 3.27$; $\sqrt{i} = 0.01$. Substituting these values in the coefficient in Kutter's formula, there results

$$v = c\sqrt{ri} = \left\{ \frac{\frac{1.811}{0.02} + 41.6 + \frac{0.00281}{0.0001}}{1 + \left(41.6 + \frac{0.00281}{0.0001}\right) \times \frac{0.02}{3.27}} \right\} \times \sqrt{ri}.$$

Reducing,

$$c = \frac{\frac{1.811}{0.02} + 41.6 + \frac{0.00281}{0.0001}}{1 + \left(41.6 + \frac{0.00281}{0.0001}\right) \times \frac{0.02}{3.27}} = \frac{160.25}{1.4253} = 112.43.$$

Then

$$v = 112.43 \times \sqrt{ri}.$$

Substituting values of the square roots of r and i, respectively 3.27 and 0.01, $v = 111.43 \times 3.27 \times 0.01 = 3.67$ feet per second, which is the mean velocity. Then the discharge is $Q = \text{area} \times \text{velocity} = av = 1718.75 \times 3.67 = 6307.8$ cubic feet per second.

In the following examples so much detail will not be given. The processes will be indicated and only results written.

The slope will be taken at 1 in 20,000 = 0.00005, the roughness coefficient will be $n = 0.017$, which applies to the one or the other of the following conditions, ashlar masonry and brickwork. Assuming a width of channel = 160

feet, and depth of flow = 22 feet; then area = $a = 160 \times 22 = 3520$ sq. ft.; wetted perimeter = 160 + 44 = 204 ft.

$$\text{Hydraulic radius} = r = \frac{3520}{204} = 17.27.$$

Square root hydraulic radius,

$\sqrt{r} = 4.154$; Slope of bed = $i = 0.00005$;

$\sqrt{i} = 0.00707$;

$$c = \frac{\frac{1.811}{n} + 41.6 + \frac{0.00281}{i}}{1 + \left(41.6 + \frac{0.00281}{i}\right)\frac{n}{\sqrt{r}}} = \frac{\frac{1.811}{0.017} + 41.6 + \frac{0.00281}{0.00005}}{1 + \left(41.6 + \frac{0.00281}{0.00005}\right)\frac{0.017}{4.154}} = 146.0;$$

$v = 146.0 \times \sqrt{ri} = 146.0 \times 4.154 \times 0.00707 = 4.289$ feet per second;

$Q = av = 3520 \times 42.89 = 15{,}107$ cubic feet per second.

The last example to be given will apply to *earthen canals having tolerably uniform cross-section and slopes and those in rather bad condition having stones and weeds obstructing the channels*, for which $n = 0.025$ to 0.03. Taking $n = 0.025$, slope $i = 1$ in $1000 = 0.001$, width of channel = 70 feet, depth = 4.0 feet, side slopes 2 to 1, then the area of cross-section = $(70 + 8) \times 4 = 312$ square feet; wetted perimeter = $70 + 16.2 = 86.2$ feet.

$$r = \frac{312}{86.2} = 3.63; \quad \sqrt{r} = 1.9;$$

$$\sqrt{i} = 0.0316;$$

$$c = \frac{116.91}{1.59} = 73.5;$$

$$v = c\sqrt{ri} = 73.5 \times 1.9 \times 0.0316 = 1.753 \text{ feet per second};$$

$$Q = av = 312 \times 1.753 = 546.9 \text{ cubic feet per second.}$$

It is evident that from the formula $v = c\sqrt{ri}$, either one of the quantities v, c, r, or i can be determined when the other three are known, and the problems arising are: (1) to determine v when c, r, and i are known; (2) to determine c when v, r, and i are known; (3) to determine r when v, c, and i are known; (4) to determine i when v, c, and r are known.

Weisbach's formula is as follows:

$$v = (0.00024 + 8675ri)^{\frac{1}{2}} - 0.0154.$$

It will be observed that this formula has a constant coefficient, and therefore can only apply to one set of conditions. After understanding the preceding examples there will be no difficulty in applying it either to determine v, r, or s, any two of which are assumed or determined.

A complete list of all the formulæ in use will be found in Flynn's "Flow of Water in Irrigation Canals."

FORMULÆ FOR MEAN VELOCITY IN PIPES, SEWERS, CONDUITS, ETC.

A full list of formulæ for mean velocity in closed channels will be found in Flynn's "Work on Irrigation," together with a list of those used for both open

and closed channels. Only a few of these formulæ will be given, followed by a few practical examples.

D'Arcy's formula for iron pipes under pressure is

$$v = \left\{ \frac{ri}{0.00007726 + \dfrac{0.00000162}{r}} \right\}^{\frac{1}{2}}.$$

Flynn's modification of D'Arcy's formula is

$$v = \left(\frac{155256}{12d + 1}\right)^{\frac{1}{2}} \times \sqrt{ri},$$

in which d is the diameter of the pipe in feet.

D'Arcy's formula, as given by J. B. Francis, C.E., for old cast-iron pipe, lined with deposit, and under pressure, is

$$v = \left(\frac{144d^2i}{0.0082d + 1}\right)^{\frac{1}{2}}.$$

Flynn's modification of D'Arcy's formula for old cast-iron pipe is

$$v = \left(\frac{70243.9}{12d + 1}\right) \times \sqrt{ri}.$$

Weisbach's formula is

$$v = \left(\frac{2gh}{1.505 + c \times \dfrac{l}{d}}\right)^{\frac{1}{2}},$$

where $c = 0.01439 + \dfrac{0.016921}{v^{\frac{1}{2}}}$.

Eytelwein's formula is

$$v = 108\sqrt{rs} - 0.13 \text{ nearly,} \quad \text{or} \quad v = 50\left(\frac{dh}{l + 50d}\right)^{\frac{1}{2}}.$$

Kutter's formula is the same as already given.

In the foregoing formulæ

v = mean velocity in feet per second;
r = hydraulic mean depth in feet;
h = fall of water surface in any distance l;
l = length of water surface for any fall h;
$i = \dfrac{h}{l}$ = sine of slope;
c = coefficient of mean velocity;
the coefficient depending on the nature of the bed; that is, the lining or surface of the channel;
g = acceleration of gravity = 32.16.

Almost all the old formulæ have constant coefficients. These coefficients give too high a velocity for small channels and too low a velocity for large

channels. The modern and more accurate formulæ have varying coefficients, whose value increases with that of the hydraulic mean depth. The value of the coefficient in D'Arcy's formula varies with r, but is not affected by the slope s, while in Kutter's formula the coefficient varies with both r and s.

For diameters larger than 6 feet D'Arcy's coefficient changes very little, and for very large pipes does not exceed 113.8, being only a little greater for pipes 16 feet or more in diameter. Kutter's coefficient continues to increase up to the greatest diameter likely to be required in practice.

The great advantage in Kutter's formula is that it adapts itself to a change in the condition of the surface of the pipe exposed to the flow of water by changing the coefficient of roughness n, which may vary from $n = 0.011$ to 0.013, and more commonly the latter, to allow for the increasing roughness of surface.

Reducing all the foregoing formulæ to the form $v = c\sqrt{ri}$, the following table gives the value of c for the several formulæ named:

TABLE 87, GIVING THE VALUE OF c IN THE FORMULÆ NAMED. (FLYNN.)

Diameter. ft.	in.	D'Arcy, for clean cast-iron pipes.	Lampe. $i = 0.001$	Kutter. $n = 0.011$ $i = 0.001$	Kutter. $n = 0.012$ $i = 0.001$	Kutter. $n = 0.013$ $i = 0.001$	Blackwells.	Prony.	Downing.	D'Arcy.	Kirkwood.
	1	80.3	65.1	47.1	.....	.....	95.8	97	100	54.1	80
	2	92.9	74.8	61.5	.....	.....	95.8	97	100	62.5	80
	4	101.7	85.4	77.4	.....	.....	95.8	97	100	68.4	80
	6	105.3	92.8	87.4	77.5	69.5	95.8	97	100	70.8	80
1		109.3	106.2	105.6	94.6	85.3	95.8	97	100	73.5	80
1	6	110.7	115.0	116.1	104.8	94.4	95.8	97	100	74.5	80
2		111.5	128.5	123.6	111.3	101.1	95.8	97	100	74.9	80
3		112.2	133.2	133.6	120.8	110.1	95.8	97	100	75.5	80
4		112.6	139.0	140.4	127.4	116.5	95.8	97	100	75.7	80
5		112.8	145.2	145.4	132.3	121.1	95.8	97	100	75.9	80
6		113.	150.4	149.4	136.1	124 8	95.8	97	100	76.	80
7		113.1	155.0	152.7	139.2	127.9	95.8	97	100	76.1	80
8		113.2	159.1	155.4	141.9	130.4	95.8	97	100	76.1	80
9		113.2	162.7	157.7	144.1	132.7	95.8	97	100	76.2	80
10		113.3	166.1	159.7	146.0	134.5	95.8	97	100	76.2	80
11		113.3	169.2	161.5	147.8	136.2	95.8	97	100	76.2	80
12		113.3	172.1	163.0	149.3	137.7	95.8	97	100	76.2	80
14		113.4	177.3	165.8	152.0	140.4	95.8	97	100	76.3	80
16		113.4	182.9	168.0	154.2	142.1	95.8	97	100	76.3	80
18		113.5	186.1	169.9	156.1	144.4	95.8	97	100	76.3	80
20		113.5	190.0	171.6	157.7	146.0	95.8	97	100	76.4	80

In Kutter's formula the value of c depends upon those of r, n, and i, so that any change in the value of the slope s causes a corresponding change in c; but this effect is small, as the coefficient of discharge c is the same for all slopes greater than 1 in 1000, and in fact there is only a slight difference in the value of c up to a slope of 1 in 5000. And as the change is not very marked in such slopes as are commonly adopted for pipes, sewers, and conduits, it will usually be sufficiently accurate to compute c for only one slope, say 1 in 1000. For flatter slopes even to 1 in 2640, or 2 feet per mile, the coefficient based on a slope of 1 in 1000 will give an error of less than 2 per cent.

The following table shows the change in the coefficient c for use in the

formula of Kutter with $n = 0.013$ and with slopes from 1 in 1000 to 1 in 5000 $v = c\sqrt{ri}$:

TABLE 88. (FLYNN.)

$\sqrt{r}$	Slopes, s 1 in 1000 c	1 in 2500 c	1 in 3333 c	1 in 5000 c
0.6	93.6	91.5	90.4	88.4
1	116.5	115.2	113.2	113.2
2	142.6	142.8	141.1	141.2

Mr. Flynn gives in his work a very complete set of tables to facilitate the solution of all problems relating to pipes, sewers, and conduits by the formulæ of Kutter and D'Arcy. He first computed the value of c for different sizes of of channels and different values of n from his modified form of Kutter's formulæ,

$$v = \left\{ \frac{k}{1 + 44.41 \times \dfrac{n}{\sqrt{r}}} \right\} \times \sqrt{ri},$$

the quantity in brackets corresponding to c. By this means the complicated form of Kutter's formula is reduced to the Chezy form,

$$v = c\sqrt{r} \times \sqrt{i}.$$

The following table gives the value of k for several values of n:

TABLE 89.

n	k	n	k
0.009	245.63	0.017	150.94
0.010	225.51	0.018	145.03
0.011	209.05	0.019	139.73
0.012	195.33	0.020	134.96
0.013	183.72	0.021	130.65
0.014	174.77	0.022	126.73
0.015	165.14	0.0225	124.9
0.016	157.6		

And computing the value of the $\sqrt{r}$ for a large range of diameters, we have the necessary elements to determine the value of the term $\dfrac{k}{1 + 44.41 \times \dfrac{n}{\sqrt{r}}} = c$ in the last formula.

Let the value of n be 0.013, then from the above take $k = 183.72$, and if the pipe is 2 feet in diameter the area will be 3.1416 square feet, the circumference or wetted perimeter will be 6.2832 feet, the hydraulic mean depth

$$r = \frac{3.1416}{6.2832} = 0.5 \text{ and } \sqrt{r} = 0.707. \text{ Substituting, } c = \frac{183.72}{1 + 44.41 \times \dfrac{0.013}{0.707}} = 101.1.$$

In this manner values of c for any values of n, r, and k can be computed.

The value of $n = 0.013$ is the coefficient of roughness for ashlar and well-laid brickwork, ordinary metal, earthenware and stoneware pipe in good condition but not new, cement and terra-cotta pipe not well jointed nor in perfect order, plaster and planed wood in imperfect or inferior condition, and also surfaces of other material equally rough.

Let it, then, be required to find the mean velocity of flow in a pipe coming under one of the above conditions, 2 feet in diameter, and laid to a slope $i = 0.001$. Substituting in $v = c\sqrt{ri}$, $v = 101.1 \times 0.707 \times 0.031623 = 2.26$ feet per second. Since the area of cross-section is 3.1416 square feet, the discharge will be $Q = 3.1416 \times 2.26 = 7.1$ cubic feet per second.

In a similar manner the value of the constant c in D'Arcy's formulæ can be computed. This substituted, together with the values of r and i, similar numerical results can be obtained as in the above example. Flynn's modification of Kutter's and D'Arcy's formulæ are only intended to simplify the computations necessary to find the value of the constant c.

Mr. Flynn says: (1) By Kutter's formula the value of c, or the velocity, changes with every change in the value of r, i, and n, and with the same slope and the same value of n the value of c increases with r, that is, with the increase in diameter (the hydraulic mean depth r is one fourth the diameter d). It is on this variability of its coefficient to suit the different changes of slope, diameter, and lining of channel that the accuracy of Kutter's formula depends.

(2) According to Kutter's formula the value of c increases with the increase of slope for all diameters whose hydraulic mean depth is less than 3.281 feet—one metre, and with a hydraulic mean depth greater than 3.281 feet an increase of slope gives a diminution in the value of c. For example, with $n = 0.013$, a circular pipe 12 feet in diameter gives, for a slope of 1 in 1000, $c = 137.7$, and for 1 in 40, $c = 137.9$, whereas a circular pipe 20 feet in diameter, for 1 in 1000, $c = 146.0$, and for 1 in 40, $c = 145.7$. It will thus be seen that by Kutter's formula when $r = 3$ feet, that is, less than 3.281 feet, an increase in the slope from 1 in 1000 to 1 in 40 causes a slight increase in the coefficient, but where $r = 5$ feet, that is, more than 3,281 feet, the same increase in the slope causes a slight diminution in the value of c.

(3) By Kutter's formula, when the hydraulic mean depth (r) is equal to 3.281 feet, the value of c is constant for all slopes, and is $= \dfrac{\sqrt{3.281}}{n} = \dfrac{1.811}{0.013} = 139.31 = c$. This is the only instance, I believe, where Kutter's formula gives a constant coefficient with a change of slope.

EXAMPLE.—Given the discharge and dimensions of the cross-section of a rectangular (masonry) inverted siphon, to find its grade or fall from surface of water at inlet to its outlet, capable of discharging 100 cubic feet per second.

Let the area of cross-section $= 3 \times 4$ feet $= 12$ square feet; then $v = \frac{100}{12} = 8\frac{1}{3}$ feet per second; $r = \frac{12}{14} = 0.86$, nearly; $\sqrt{r} = \sqrt{0.86} = 0.93$, nearly. Assuming a rough lining corresponding to $n = 0.020$, and having v and r, it is required to find i. This operation by Kutter's formula will be long and laborious. So, using Flynn's modification, we find k corresponding to $n = 0.02$, in Table 89, equal to 134.96. Then

$$c = \frac{k}{1 + \left(44.41 \times \dfrac{n}{\sqrt{r}}\right)} = \frac{134.96}{1 + \left(44.41 \times \dfrac{0.02}{0.93}\right)} = 63.92;$$

and from $v = c\sqrt{ri}$ we have

$$i = \frac{v^2}{c^2 r} = \frac{(8\frac{1}{3})^2}{(63.92)^2 \times 0.86} = 0.0197 \text{ for the slope.}$$

Then the fall will be $\dfrac{h}{l} = 0.0197$. $\therefore h = 0.0197l$; and if the length $l = 1000$

feet, then the fall or difference in level between the inlet and outlet ends will be $h = 19.7$ feet.

This head of 19.7 feet can be applied in one of three ways:

(1) The culvert may have a level floor, and a head of pressure of 19.7 feet be maintained at the inlet end.

(2) A fall of 19.7 feet may be given between the two ends.

(3) A less fall than 19.7 feet may be given, and a sufficient head above the inlet end be given to maintain the required velocity. For the same area of channel a circle has the greatest hydraulic mean depth, and therefore requires the least head to give the same velocity. The above rectangular channel contains 12 square feet area. A circular pipe with the same area has a diameter of 3.92 feet, its hydraulic radius $r = 0.98$, and $c\sqrt{r} = 70.18 \times 0.99 = 69.48$.

$$\therefore \sqrt{i} = \frac{8\frac{1}{3}}{69.48} = 0.11965, \quad \text{and} \quad s = 0.01449 \text{ for the slope.}$$

The fall $h = 0.01449 \times 1000 = 14.49$ feet, as compared with 19.7 feet for the rectangular conduit.

EXAMPLE.—Given the grade, mean velocity, and value n, of a circular sewer to find its diameter.

Let the grade be 1 in 500, the mean velocity 3 feet per second, and $n = 0.015$. Required the diameter.

For a slope of 1 in 500, the $\sqrt{i} = 0.044721$, and $v = 3$; then $c\sqrt{r} = \frac{v}{\sqrt{i}}$ $= \frac{3}{0.044721} = 67.0$; c found by Flynn's modification of Kutter's formula will be about 88.0, and then the square root of r, i.e., $\sqrt{r} = 0.76$, and $r = 0.58$, and diameter $d = 4r = 2.32$ feet.

It is evident from the above that any assumed value of c will give a corresponding value of $\sqrt{r}$, but with a given value of n the relation between c and $\sqrt{r}$ is fixed. In such cases it is therefore necessary to tabulate a number of values of c based on values of n, r, and i, and from this table find that value of $c\sqrt{r}$ which corresponds with the number required—67.0 in the above case. For instance, Mr. Flynn has calculated a number of tables based upon the several values of n and d from $n = 0.009$ to 0.02 inclusive, and diameters of pipe d from 5 inches to 20 feet; the corresponding values of r are one fourth the diameters for circular pipes, sewers, and conduits, etc., flowing full. These tables are true for any value of i greater than 1 in 1000, and practically so for values up to 1 in 2640, which will cover the range of slope in practice. The value of $n = 0.015$ applies to second-class or rough-faced brickwork, well-dressed stonework, foul and slightly tuberculated iron, cement and terra-cotta pipes with imperfect joints and in bad order, canvas lining on wooden frames, and also surfaces of other channels equally rough. With this value of n, $i = 0.001$, and with assumed values of r from 5 inches to 20 feet substituted in

$$\frac{k}{\left(1 + 44.41\frac{n}{\sqrt{r}}\right)}\sqrt{r} = c\sqrt{r},$$

the table is formed from which the writer abstracts the following, reproducing only a few of the results for diameters corresponding:

TABLE 90.

$n = 0.015.$

d = diameter in ft. in.	a = area in sq. ft.	For velocity $c\sqrt{r}$	For discharge $ac\sqrt{r}$
6	0.196	20.21	3.9604
1	0.785	35.40	27.803
1 6	1.767	48.38	85.496
2	3.142	60.08	188.77
2 4	4.276	67.32	287.87
3	7.068	80.77	570.90
4	12.566	99.10	1245.30
5	19.635	115.7	2272.7
6	28.274	131.0	3702.3
7	38.485	145.3	5591.6
8	50.266	158.7	7978.3
10	78.540	183.7	14426.0
12	113.10	206.5	23352.0
14	153.94	227.8	35073.0
16	201.06	247.8	49823.0
18	254.47	266.6	67839.0
20	314.16	284.6	89423.0

As in the last example we found $c\sqrt{r} = 67.0$, we look in the table above for the nearest value to this, which is found opposite the diameter 2 feet 4 inches, agreeing practically with the value of $d = 2.32$ feet $= 2$ feet 3.8 inches.

The advantage derived from such tables needs no comment. Having the diameter, the area is found at once, and is $= (2.32)^2 \times \frac{\pi}{4} = 4.227$ square feet; the nearest in the table is 4.276. Since the velocity is 3 ft. per second, the discharge $Q = 4.227 \times 3 = 12.681$ cubic feet per second, and from the table $ac\sqrt{r} = 287.87$; consequently the discharge

$$Q = ac\sqrt{r} \times \sqrt{i} = 287.87 \times \sqrt{1/500} = 287.87 \times 0.04472 = 12.874 \text{ cu. ft. per sec.}$$

Similarly other tables can be formed.

EXAMPLE.—Given the diameter, discharge, and value of n of a circular conduit flowing full, to find the slope or grade. To enable us to use the foregoing table 90, let $n = 0.015$, diameter $d = 5$ feet, discharge $Q = 100$ cubic feet per second. Opposite 5 feet in table we find

$$ac\sqrt{r} = 2272.7 \quad Q = ac\sqrt{r} \times \sqrt{i};$$

$$\therefore \sqrt{i} = \frac{Q}{ac\sqrt{r}} = \frac{100}{2272.7} = 0.044, \quad s = 0.001936,$$

corresponding to a slope of 1 in 515.

The following examples are given to illustrate further the subject under consideration.

To find the diameter in three sections of an intercepting sewer, with increasing discharge, the grade or inclination being the same throughout, and the value of n also given.

A circular sewer with that condition of inner surface corresponding to $n = 0.015$ has to discharge, for 1000 feet of its length, 10 cubic feet per second; flowing full, for 500 feet of its length, 15 cubic feet; and for 800 feet, 20 cubic feet. The total fall available is 10 feet. Required the diameter and fall for each

section. The total length is 2300 feet, the total fall 10 feet; the rate of fall therefore is 1 in 230. This corresponds to a slope $i = 0.004348$, and $\sqrt{i} = 0.065938$.

From $Q = ac \times \sqrt{r} \times \sqrt{i}$; $ac\sqrt{r} = \dfrac{Q}{\sqrt{i}}$.

For first section, $ac\sqrt{r} = \dfrac{10}{0.065938} = 151.66$;

" second " " $= \dfrac{15}{0.065938} = 227.49$;

" third " " $= \dfrac{20}{0.065938} = 303.32$.

By interpolation we find from Table 90 that 151.66 corresponds to a diameter of 1 foot 10 inches, 227.49 to 2 feet 2½ inches, and 303.32 to 2 feet 4 inches, nearly. The table is not computed sufficiently close for accurate interpolation.

The slope i being 0.004348,

The fall of the first section = 0.004348 × 1000
" " " " second " = " × 500
" " " " third " = " × 800

Then

1st section, diameter	1′ 10″,	fall	4.348 ft.
2d " "	2′ 2½″,	"	2.174 ft.
3d " "	2′ 4″,	"	3.478 ft.
Total fall,			10.000 ft.

Mr. Flynn has also calculated the constants for use in egg-shaped sewers, for different values of n. The writer introduces here an abbreviated table for $n = 0.011$. This value of n applies to plaster in cement with one third sand in good condition; also for iron, cement, and terra-cotta pipes, well jointed and in best order, and also the surfaces of other materials equally rough. The egg-shaped sewer referred to has a vertical diameter equal to 1.5 times the greatest transverse diameter D, that is, the diameter at the top of the sewer. The first table is for a sewer flowing full depth.

Area of egg-shaped sewer $= D^2 \times 1.148525$
Perimeter $= D \times 3.9649$
Hydraulic mean depth $= D \times 0.2897$
$n = 0.011$

TABLE 91.

Size of Sewer. ft. ft. in.	Area in sq. ft.	For Velocity $c\sqrt{r}$.	For Discharge $ac\sqrt{r}$.
1 × 1 6	1.1485	58.8	67.5
2 × 3	4.5941	96.8	444.7
3 × 4 6	10.337	127.7	1325.1
4 × 6	18.376	154.7	2843.9
5 × 7 6	28.713	179.0	5140.6
6 × 9	41.347	201.0	8312.7
7 × 10 6	56.278	221.6	12473.0
8 × 12	73.506	240.8	17704.0
10 × 15	114.853	276.5	31754.0
12 × 18	165.388	308.7	51051.0

Egg-shaped sewers flowing two-thirds full depth. Vertical diameter 1.5 times the greatest transverse diameter.

$$\begin{aligned} \text{Area of water section} &= D^2 \times 0.755825 \\ \text{Wetted perimeter} &= D \times 2.3941 \\ \text{Hydraulic mean depth} &= D \times 0.3157 \\ n &= 0.011 \end{aligned}$$

TABLE 92.

Size of Sewer. ft.		ft.	in.	Area in sq. ft.	For Velocity $c\sqrt{r}$.	For Discharge $ac\sqrt{r}$.
1	×	1	6	0.7558	62.71	47.40
2	×	3		3.0232	102.9	311.2
3	×	4	6	6.8022	135.3	920.5
4	×	6		12.093	163.7	1979.6
5	×	7	6	18.895	189.0	3571.8
6	×	9		27.210	212.3	5776.3
7	×	10	6	37.035	234.0	8670.0
8	×	12		48.372	254.1	12293.0
10	×	15		75.582	291.3	22018.0
12	×	18		108.838	325.1	35387.0

Egg-shaped sewers flowing one-third full depth. Vertical diameter 1.5 times the greatest transverse diameter.

$$\begin{aligned} \text{Area of water section} &= D^2 \times 0.284 \\ \text{Wetted perimeter} &= D \times 1.3747 \\ \text{Hydraulic mean depth} &= D \times 0.2066 \\ n &= 0.011. \end{aligned}$$

TABLE 93.

Size of Sewer. ft.		ft.	in.	Area in sq. ft.	For Velocity $c\sqrt{r}$.	For Discharge $ac\sqrt{r}$.
1	×	1	6	0.2840	45.72	12.98
2	×	3		1.1360	76.26	86.63
3	×	4	6	2.5560	101.6	259.8
4	×	6		4.5440	123.6	561.5
5	×	7	6	7.1000	143.3	1017.8
6	×	9		10.224	161.6	1652.4
7	×	10	6	13.916	178.9	2489.4
8	×	12		18.176	194.8	3541.7
10	×	15		28.400	224.2	2366.4
12	×	18		40.892	251.3	10277.0

The foregoing tables are based on Kutter's formula with $n = 0.011$.

EXAMPLE.—To find the dimensions of an egg-shaped sewer to replace a circular sewer.

A circular sewer 5 feet in diameter and 5280 feet in length has a fall of 12 feet. It is desired to replace it with an egg-shaped sewer with a fall of 6 feet; that the discharge flowing full shall be the same, $n = 0.011$.

A slope of 12 in $5280 = 1$ in 440. For 1 in 440, $i = 0.0022727$, and $\sqrt{i} = 0.047673$. From a table computed with $n = 0.011$, we will find $ac\sqrt{r} = 3191.8$ opposite diameter of 5 feet. This table would be similar to Table 90.

Then $Q = 3191.8 \times 0.047673 = 152.16$ cubic feet per second for the discharge of the circular sewer.

The egg-shaped sewer is to have a grade of 6 in 5280 = 1 in 880; then $s = 0.00113635$, and $\sqrt{i} = 0.033710$. Then for the egg-shaped sewer

$$ac\sqrt{r} = \frac{Q}{\sqrt{i}} = \frac{152.16}{0.03371} = 4513.8.$$

The nearest value to this in Table 91 is 5140.6 opposite a sewer 5′ × 7′.6″. By interpolation the proper size is 4′ 10″ × 7′ 3″.

EXAMPLE.—To find the dimensions and grade of an egg-shaped sewer flowing full, the mean velocity and discharge being given.

Assume a velocity of 4 feet per second, and the discharge 100 cubic feet per second. $n = 0.011$. Required its dimension and grade.

Area $a = \frac{Q}{v} = \frac{100}{4} = 25$. The nearest area in Table 91 is 28.713, opposite 5′ × 7′ 6″; but by interpolation or from a more complete table it would be found to be 4′ 8″ × 7′ 0″. In the same line we find $c\sqrt{r} = 179.0$, and $ac\sqrt{r} = 5140.6$, or more accurately 171.0 and 4279.1, respectively. Then to find the slope,

$$\sqrt{i} = \frac{Q}{ac\sqrt{r}} = \frac{100}{4279.1} = 0.2337;$$

and $i = 0.0005464$, corresponding to a slope of 1 in 1830.

If the same discharge and velocity be assumed, with a sewer flowing two-thirds full, the process is the same, only using Table 92.

Area $a = 25$ as before; the nearest number in Table 92 is 27.210, corresponding to dimensions 6′ × 9′, or more accurately 5′ 10″ × 8′ 9″; $c\sqrt{r} = 208.6$; $ac\sqrt{r} = 5364.3$ by interpolation. Then $\sqrt{i} = \frac{Q}{ac\sqrt{r}} = \frac{100}{5364.3} = 0.01862$, and $s = 0.0003467$, or a slope of 1 in 2885.

Examples could be multiplied to any extent. The foregoing serves to show the method of making the computations when no tables are available, and the manner of using such tables when accessible.

WATER-SUPPLY OF SOME LARGE CITIES.

(Abstracted from an article in the Journal of the Association of Engineering Societies by Mr. Allen Hazen.)

FILTERED RIVER-WATER SUPPLIES.

Cities.	Population.	Million gallons daily.	Gallons daily per head.
London	5,030,000	190	38
Berlin	1,606,000	26	16
St. Petersburg	960,000	39	40
Hamburg	583,000	32	53
Warsaw	500,000	6	12
Breslau	335,000	7	22
Rotterdam	240,000	13	54
Magdeburg	200,000	5	24
FILTERED WATER FROM SOURCES OTHER THAN RIVERS.			
Amsterdam	515,000	10	
Liverpool	815,000	22	
Bradford	364,000	12	
Dublin	327,000	18	

The water-supply for the first is from dunes and carried in canals. For the last three from storage-reservoirs.

GROUND-WATER SUPPLIES.

City.	Population.	Million Gallons Daily.	Gallons Daily per head.	Sources.
Paris	2,500,000	53	21	From springs, wells, and filter-galleries
Vienna	1,000,000	23	23	
Budapest	500,000	22	44	
London, Kent Co	460,000	16	35	
Leipzig	360,000	5.5	15	
Munich	300,000	11.5	38	
Dresden	280,000	6.	21	
Cologne	255,000	11.5	45	
Frankfort-on-Main	186,000	6.7	36	

Paris uses 80,000,000 gallons of river water daily for public and manufacturing purposes.

UNFILTERED SURFACE-WATER SUPPLIES.

Manchester	968,000	24	24
Sheffield	324,000	..	..
Glasgow	794,000	50	64

The first two are from storage-reservoirs, the last from Loch Katrine.

WATER-SUPPLIES FOR SIX OF THE LARGEST AMERICAN CITIES.

Cities.	Population.	Million Gallons Daily.	Gallons Daily per head.	Sources.
Chicago	1,099,850	152	140	Lake Michigan
Philadelphia	1,046,964	138	132	Rivers
New York	1,515,291	121	79	Storage reservoirs
Brooklyn	776,838	55	72	Wells and rivers
Buffalo	255,664	47	186	Rivers
Boston	527,606	42	80	Storage reservoirs

Continental cities are almost invariably supplied with either ground-water or filtered river-water; impounding reservoirs are practically unknown. England and America have great dams and reservoirs. In England water from reservoirs is filtered as a rule, and is required by law to be so in Germany.

Amsterdam has 6200 acres of dunes, which are drained by 15 miles of open canals located below ground water-line. The yield of the land is 14 inches; total rainfall 28 inches. The summer precipitation is practically lost by evaporation and absorption; that of winter only used for water-supply.

Paris is supplied by aqueducts from springs in the valley of Avre. There are

12 miles, diameter $5\frac{1}{4}$ feet, fall 1 in 2500;
46 " " 6 " " 1 in 3300,

5 miles of siphon where two lines of 40-inch pipes are used; fall 1 in 800. Total length of aqueduct is 63 miles; total fall, 132 feet.

ABSTRACTS FROM AN ARTICLE ON WIND PRESSURES IN ENGINEERING CONSTRUCTION, BY W. H. BIXBY, CAPTAIN OF ENGINEERS, U. S. A.*

Bridge Spans.—The exposed area of spans is figured: First, of the unloaded bridge, the wind pressure being treated as a dead load; second, of the loaded bridge, the wind pressure on the train being considered as a live load; and then each portion of the wind bracing is dimensioned to stand the maximum strain that may come upon it under either condition of load. The exposed area of a girder is measured by taking the front surface of each part (such as the ordinary upper chords and posts) which stands by itself, 1.5 times the front surfaces of those parts (as ordinary ties) which are in pairs, and 2 times the front surface of those parts (as ordinary lower chords) which are composed of several bars, one behind the other.

The wind pressure on the unloaded bridge is calculated at 30 to 50 pounds per square foot on the exposed area of the floor system, and of either two open girders, or one closed or plate girder. The train is treated as a continuous surface of 10 feet height, with its bottom 2.5 feet above the rails. The exposed area of the loaded bridge is measured by adding together the exposed surfaces of all the windward girder of the floor system, of the train, and of all the leeward girder (or at least so much of it as is not completely and closely sheltered by the train). In calculating the stresses on each wind truss the loads are assumed as exerted on the windward side. The chords and end posts of the main trusses are not usually stiffened or increased in size on account of the assumed wind stresses, except, first, when such stress, alone or in combination with a temperature strain, may change the strain from tension to compression in a chord built to stand only tension; and second, when the wind stress on any member exceeds 25 per cent of the sum of the maximum stress due to the dead load added to that due to the live load.

Members subject to alternate tensile and compressive stresses are built to resist each, and proportioned so as to stand each stress alone, increased by an amount equal to 80 per cent of the lesser.

The sway-bracing which is placed between the vertical posts of the main trusses at each panel point is introduced in order to prevent independent lateral vibration and swaying of the vertical trusses; also, to stiffen the long vertical posts, as well as to assist in carrying some of the wind stresses to the leeward girder. In double-track railway bridges the sway-bracing also helps to prevent lateral distortion of the cross-sections of the bridge under a load on a single track; but the extra stress due to such work is usually comparatively slight. The main stress on the sway-bracing comes usually when the wind is blowing on the unloaded bridge, in which case the web members of the lateral system of the loaded chords are only about one-third loaded, while the web members of the other lateral system are fully loaded, giving rise naturally to unequal lateral deflection, and the transfer of some stress from the weaker system through the sway-bracing to the stiffer system. The parts of the sway-bracing are therefore usually dimensioned so as to be able to carry 50 per cent of the wind load due to each panel of the bridge.

On ordinary heavy railway bridges the exposed areas per linear foot may be roughly estimated at 10 sq. ft. for the train, 1 sq. ft. for ends of the ties and sides of the guard-rails, 4 sq. ft. for the longitudinal floor-girders (or stringers), and 5.0 sq. ft. per linear foot for each truss (or girder), or a total of 10 to 14 sq. ft. for the dead wind load on the two trusses and floor, or 2.5 to 3.5 sq. ft. for each chord. In ordinary double-track railroad bridges, with vertical sway-bracing, the weight of this bracing for both trusses may be roughly estimated at

* A very full and valuable discussion on Wind Pressures and Wind Bracing is found in *Engineering News*, March 14, 1895.

$\left(\frac{6Nl}{170} + \frac{1186}{p}\right)\frac{b}{15}$ in pounds per linear foot of track, in which l is length in feet, N is the number of panels, p is the panel length in feet, and b is the width of bridge in feet.

When bridges are built on a curve they must also be stiffened laterally against the centrifugal force of the moving train. This extra stress is estimated at $0.0000117\,VDP$, in which V is the velocity in miles per hour, P is the weight of the train in tons, D is the degree of the curve or the angle subtended at the centre by a chord of 100 feet of track. For a speed of 30 miles per hour this amounts approximately to 1 per cent of the weight of the train for each degree of curvature, and each additional 10 miles is assumed to add as much more. This centrifugal force is assumed to act at a level of 5 feet above the rails. This stress is carried by the floor-beams to the posts of the outer trusses, and thence through the lateral trusses to the rest of the bridge. As one of the lateral systems usually lies close to the flooring, most of the stress comes upon such system, and its amount is usually added to that of the wind stresses, and treated as if it were an extra and live wind load on one side of the bridge.

Initial Tension.—All of the rods of the wind trusses must be further strengthened to cover the initial tension (ordinarily given them by turnbuckles in bringing all pieces to their proper bearings); and this initial tension is usually estimated at 1.0 ton for 1.0 inch diameter round rods, plus 0.25 ton for each additional 0.125 inch of diameter, or 1.125 tons for 1.0 inch square rods, plus 0.281 ton for each additional 0.125 inch of side, or equal allowances for equal areas of other shapes.

Piers and Towers.—In the calculation of wind stresses on piers or towers the same rules apply, with slight modifications as follows: The wind pressure on the whole pier is calculated at that of the front surfaces of the train and of all floor systems of the pier, added to twice that of the trussing of the front face of the tower, assuming the wind pressure on the pier and train together at 30 pounds per square foot, and on the unloaded pier at 50 pounds per square foot, and providing against the most unfavorable of the two cases. In figuring the weight of the loaded pier the train weight is to be taken at that of the lightest train that would not be blown over by a 30-lb. wind pressure. The pier must then be given base enough to prevent its overturning, and also enough to prevent any tensile stresses in its posts.

On the open-work piers of the average railroad bridge the exposed area of each side truss of the pier is roughly estimated at 4.5 square feet per lineal foot of height. The inclined and horizontal bracing of piers and trestles is to be made strong enough to resist the stresses of all wind and centrifugal forces. and of the resistances to sliding horizontally on the foundations; while the columns are to be made strong enough to carry the vertical components of the stresses due to wind and centrifugal forces, as well as the loads of the train, track, girders, and pier itself.

Longitudinal Bracing.—Ordinarily no allowances are made for any longitudinal wind stresses on the bridge. The piers, however, need and receive longitudinal bracing to resist stresses resulting from a longitudinal force of about 20 per cent the dead load of the train (or 800 lbs. per lin. ft.), due to a possible sudden application of the train brakes, and this allowance is also expected to cover any longitudinal wind stresses; but usually the columns of the towers are not given any extra cross-section or strength to carry these infrequent stresses, since such stresses will probably not occur during high wind or during other maximum loads.

Where the end of the bridge slides on a bed-plate during extension of the bridge under temperature strains the stress of sliding friction is usually taken as

25 per cent of the dead load on the bed-plate, and is added in with the longitudinal strain due to the application of the train-brakes; but special arrangements may sometimes have to be made to take special care of such stresses.

Dimensioning.—In ordinary wrought-iron railway bridges (in which the wrought iron has an ultimate strength of 50,000 pounds per square inch, with a stretch of 12.5 to 18.0 per cent, and an elastic limit of 26,000 pounds) the stresses allowed per unit of structure are from 5000 to 6000 pounds per square inch of net area on tension floor-beam hangers and other members liable to irregular and sudden strains; 8000 pounds on floor-beams, stringers, and plate-girders; 15,000 pounds on tension pieces of wind and other lateral bracing; 9000 to 10,000 pounds (reduced for length) on compression post of lateral and other wind bracing; 7500 to 8000 pounds for other live loads, and 15,000 to 16,000 pounds for dead loads (reduced for length) on compression chords and posts of the main trusses; 4000 pounds for shearing of web plates; and 7500 pounds for shearing, 12,000 pounds for crushing, and 15,000 pounds for bending of rivets and pins. The reductions for length to be deducted from the above limiting stresses in compression pieces vary from $30l/r$ to $40l/r$ for ordinary live loads, from $50l/r$ to $60l/r$ for wind strains, and from $60l/r$ to $80l/r$ for ordinary dead loads, in which l is the length of the compressed piece in inches and r is the least radius of gyration of its section in inches. No compression member is allowed to have a greater length than forty-five times its least diameter or width. Girders are given depths of from $\frac{1}{10}$ to $\frac{1}{15}$ their spans.

Soft steel may ordinarily be considered as from 10 to 15 per cent stronger than wrought iron, and medium steel as from 20 to 22 per cent stronger than wrought iron.

New Formulæ.—Although the above represents fairly the average practice of to-day in the dimensioning of bracing, there is a rapidly increasing tendency amongst bridge engineers towards determining the dimensions of tension and compression members of all trusses, including those of wind bracing, by the use of new formulæ, which take into consideration for each piece its ultimate breaking strength (b) under a single applied tensile stress, its limit of elasticity (e) beyond which a single tensile stress may produce a permanent set, its safe limit for simple repeated strains (p) of either tension or compression alone, its safe limit for repeated reversal of strains (v) from tension to compression or *vice versa*, and its maximum (m) and minimum (n) strains of simple repetition of compression alone or tension alone, or its maximum (m) of the kind of strain which is the greater and the minimum (n) of the other kind which is the lesser in case of repeated reversal of strains. The repetition limit has been found by experiment to be a little less than the limit of elasticity, and at the same time to vary between one half and two thirds of the breaking strength of the metal; and the reversal limit (called vibration resistance by Woehler) has been found in the same way to vary between one half and two thirds of the repetition limit. In the case of an ordinary wrought iron whose ultimate strength or ordinary breaking limit is 50,000 pounds per square inch, the elastic limit may be about 60 per cent of this or 30,000 pounds, the repetition limit about 52 per cent or 26,000 pounds, and the reversal limit about 32 per cent or 16,000 pounds.

Launhardt and Weyrauch Formulæ.—From the results of the experiments of Woehler and others, Prof. Launhardt has deduced formulæ of the form of $S = \frac{1}{c}p\left(1 + \frac{b-p}{p} \cdot \frac{n}{m}\right)$ for repeated stresses of tension or compression alone, and Prof. Weyrauch has deduced formulæ of the form $S = \frac{1}{c}p\left(1 - \frac{p-v}{p} \cdot \frac{n}{m}\right)$ for repeated stresses of alternate tension and compression, in which S is the safe load,

c is a constant of safety taken usually at about 3.5, and the other quantities are as above described, all values being in the same unit—usually pounds per square inch.

If $p = \frac{b}{2}$ and $v = \frac{p}{2}$, as in grades of ordinary wrought iron, then these formulæ reduce to $S = \frac{1}{c} \cdot \frac{b}{2}\left(1 + \frac{n}{m}\right)$ for repeated stresses of either tension or compression alone, $S = \frac{1}{c} \cdot \frac{b}{2}\left(1 - \frac{1}{2} \cdot \frac{n}{m}\right)$ for repeated stresses of alternate tension and compression, and $S = \frac{1}{c} \,.\, b$ for unvarying dead-load stresses; $S = \frac{1}{c} \cdot \frac{1}{2}b$ for repeated stresses of equal intensity, if unreversed, $S = \frac{1}{c} \cdot \frac{b}{4}$ for repeated stresses of equal intensity. If $p = \frac{2}{3}b$ and $v = \frac{2}{3}p$, as in some special grades of wrought iron, then these formulæ reduce to $S = \frac{1}{c} \cdot \frac{2}{3} \cdot b\left(1 + \frac{1}{2}\frac{n}{m}\right)$ for simple repetitions, $S = \frac{1}{c} \cdot \frac{2}{3} \cdot b\left(1 - \frac{1}{3}\frac{n}{m}\right)$ for repeated reversals, and $S = \frac{1}{c} \,.\, b$ for unvarying dead-load stresses as before; $S = \frac{1}{c} \cdot \frac{2}{3} \,.\, b$ for repeated stresses of equal intensity, if unreversed, $S = \frac{1}{c} \cdot \frac{4}{9} \,.\, b$ for repeated reversal stresses of equal intensity. In the case of long struts the safe load must, of course, be further reduced by the usual formulæ applicable to such cases—either Rankine's, Gordon's, or those given above.

The use of these new formulæ is therefore equivalent to that of the old formulæ modified by a variable coefficient of safety, such as, in the most disadvantageous combinations of live loads, to allow on ordinary metal only one fourth and on the best metal only four ninths of the stress that would be allowed upon it if the stress were unvariable. Such formulæ seem to be far better than those of the older method for the safe and advantageous dimensioning of the different parts of trusses in ordinary work. The Pennsylvania Railroad formulæ for bridge trusses may be deduced from the above by making $p = \frac{b}{2}$, $v = \frac{p}{2}$, and $c = 3.5$, which allows no metal, even under quiet dead loads, to receive over two sevenths of its ultimate strength, and which allows metal under the worst combination of live loads (equal alternating tension and compression) to receive a stress of only $\frac{1}{14}$ of its ultimate strength. These formulæ presuppose that metal by repeated strain within the elastic limit is fatigued in the same general way as when strained beyond such limit—a supposition that still awaits decided proof.

Dynamic Formulæ.—Fidler, however, suggests that the results of the Woehler experiments may be thoroughly accounted for by the well-known laws of dynamic action in accordance with which a load instantaneously applied will produce stresses twice as great as will the same load if quiet and constant, and any instantaneous increment of load will cause an increment of stress equal to twice that due to an equal increment of quiet and constant load. In accordance with this point of view (the justness of which is very fully proved by him) Fidler has deduced formulæ which he calls *dynamic* formulæ and which (using the same nomenclature as in the preceding formulæ) give m = the stress produced by m acting quietly and constantly, $2m$ = the stress produced by m acting instantaneously, $2(m - n)$ = stress produced by $m - n$ acting instantaneously, $n + 2$

$(m - n) = m + (m - n) =$ the maximum stress resulting from instantaneously increasing the stress from n up to m, and $n + (1 + n)(m - n) = m + a(m - n)$ = the general expression for maximum stress in all cases, a being a factor introduced to allow for dynamic action of the force $m - n$ equal to 1 when the application is instantaneous, to some intermediate value, between 1 and 0 in case of gradual application of the force, and to 0 only when the force is applied so gradually as to be considered quiet and constant. This dynamic formula, arranged for comparison with the Launhardt and Weyrauch formulæ, therefore becomes $S = \frac{1}{c} \cdot b \cdot \frac{m}{m + a(m \pm n)}$ for all cases of varying stresses, the minus sign of n being used for unreversed and the plus sign for reversed stresses, thus giving $S = \frac{1}{c} \cdot b$ for unvarying dead-load stresses, $S = \frac{1}{c} \cdot b \cdot \frac{1}{2}$ for repeated stresses of equal intensity if unreversed, and $S = \frac{1}{c} \cdot b \cdot \frac{1}{3}$ for repeated reversed stresses of equal intensity, all of which are in perfect harmony with the results of the Woehler experiments.

For ordinary use in bridge construction the new *dynamic* formula takes the form of $A = \frac{m + a(m \pm n)}{\frac{1}{c} \cdot b}$, in which A is the desired area for safe loading of the member under consideration, and the other quantities as before given; the minus sign of n being used as before for unreversed and its plus sign for reversed stresses, and a being taken at 1.0 for floor-beams (both stringers and cross-beams), floor-beam hangers, web bracing, and all other pieces subject to very irregularly and suddenly applied strains, being taken at intermediate values of from 1 to 0.5 for main girders of spans of from 20 to 100 feet length, at 0.5 for main girders of ordinary spans of more than 100 feet length. This new dynamic formulæ recommends itself by its thorough rationality, its comparative simplicity, its easy adaptability to the varying conditions of actual practice, and its division of the old, single coefficient of safety into two distinct parts, of which one, c, depends merely on conditions of manufacture, inspection, and accident, and the other, a, depends on the more easily foreseen conditions of application of its definite rolling or live loads. It now seems quite probable that this new dynamic formula will, before many years, be used in preference to all the others.

Whenever either the new Launhardt and Weyrauch formulæ or the new Fidler *dynamic* formulæ are carefully applied, it would seem safe to diminish the coefficient of safety c from 3.5 to nearly 2.0, so as to utilize the strength of the metal up to nearly its elastic limit in the case of unchangeable loads, taking care, of course, that the maximum allowed load b/c never exceeds the minimum elastic limit, e, of such metal as might be expected to actually get into the finished structure, due allowance being made for the ordinary unevenness of manufactured products, the ordinary differences between the results of tests upon small pieces and those upon full-sized members, and the ordinary failures of attempted careful inspection.

Wind trusses are, of course, subject to the same variations of temperature as the rest of the bridge, and must be arranged with due regard to such changes. The usual allowance for elongation of iron is 1.0 inch per 100 feet under 150° of temperature change. In analyzing the wind stresses in the laterals it is well to consider the various wind loads as being transmitted through intermediate parts to the points of final support by such routes as require the least internal work, that is, routes such that the internal work done in producing strain will be

a minimum. In some cases the weakness or accidental loosening of some pieces of the main or lateral girders may cause the stresses to be transmitted by unexpected routes, and to throw unusual and excessive strains on other parts; and such opportunities must be considered and provided against, if possible.

Wind Bracing of Bridges.—From the above rules it is fairly easy to deduce the approximate wind pressures upon the different parts of a bridge or other engineering structure by taking up each part in detail. In this application, however, it is well to bear in mind the modifications due to diagonal directions of the wind, that is, to those not exactly horizontal and not exactly normal to the axis of the bridge. It is hardly necessary in modern well-stiffened ordinary and long bridges to consider any uplifting wind pressures on the bridge, for general observation testifies to these being of small amount, except in peculiar gorge-like localities or during cyclones or tornadoes. Even in such cases, all that is apparently necessary to the safety of the bridge is to so arrange the flooring that it may be torn up from the floor girders by the wind before the upward-lifting wind strain becomes great enough to throw serious strain upon the main bridge girders or trusses.

Downward-pressing winds are of still less frequent occurrence and still less force, and therefore need not be considered at all. The greatest wind pressures upon the ordinary bridge will therefore come from horizontal winds, and the greatest strains upon the bridge will be naturally from such as are either normal to the axis of the bridge or nearly so. However, as the wind may be inclined 20° either way from the normal before the normal pressure on the surface diminishes perceptibly, 45° before it diminishes as much as 6 per cent, and 60° before 20 per cent, a diagonal wind, by blowing in behind the front surfaces of vertical and diagonal bracing of all vertical trusses, and by thus reaching all their rear and otherwise protected parts, may throw on the bridge a much greater strain than that due to a normal wind. Therefore, if the strain on a bridge be computed from that of a normal wind, the effective area of all vertical trusses must include at least all surfaces that may be reached by diagonal winds of about 45° angle. This of itself will therefore usually prevent the necessity of considering the question of any possible shelter afforded to each other by paired tie-rods or other verticals or diagonals in the vertical trusses.

Gusts.—As to the inequalities of the winds, that is to say, gusts, and the sudden pressures thereby brought on the bridge, it seems reasonable to assume that such gusts will not extend at one time over more than 600 to 1000 feet length of any bridge, and such pressure should be treated as any other live load. However, it should be remembered that the maximum pressure of gusts during storms of maximum intensity will occur so rarely, and the strain of the metal will therefore be so slight, that in such cases it is reasonable to allow the stress to reach very nearly the elastic limit of such metal. Moreover, it is necessary to bear in mind the fact that before any gust can act upon the wind bracing of a bridge it must take the time and use up the energy necessary to overcome the great inertia of the bridge, this inertia increasing very rapidly with the length of the bridge span.

Wind Pressures.—Consequently in long bridges, where periodic oscillation of the bridge has been duly provided against, it would seem that wind pressures would be amply provided for by allowing for, first, a dead-load wind pressure equal to the average steady pressure of high winds over the entire effective area of the bridge, increased by a live-load wind pressure equal to the added effect of 30 per cent added wind velocity (or 70 per cent added wind pressure) over from 600 to 1000 feet length of the bridge; and that the value of the wind velocities may be taken at 70 to 90 per cent (according as the anemometer constant is 3.0 or 2.3) of the ordinary cup rotary anemometer records

of the neighborhood, converted into wind pressure by the use of the formula $p = \frac{V^2}{233} = 0.0043\,V^2$; or the value of the wind pressures of gusts on the large bridge surfaces may be taken at 60 per cent and the average steady wind on the bridge at 36 per cent of the maximum of the small plate-pressure anemometer records of the neighborhood. · Acting upon this basis and the records of 1868 to 1888 of the Bidston Observatory, long bridges near that observatory should be prepared to resist the theoretical pressure due to steady winds (computed at 0.9 of the velocity anemometer records) of as much as 83 miles per hour once in 16 years, of 83 to 64 miles perhaps once per year, of 64 to 54 miles perhaps twice in one year, and of less than 54 miles at shorter intervals; or to resist steady pressures (computed at 0.36 of the small-plate pressure anemometer records) of from 33 to 29 lbs. per square foot twice in 16 years, of from 29 to 18 lbs. perhaps once per year, or from 18 to 15 lbs. perhaps twice in one year, and of less than 15 lbs. at shorter intervals; or to resist steady pressures $\left(\text{as deduced from velocity anemometer records and the formula } p = \frac{V^2}{233}\right)$ of as much as 30 lbs. once in 16 years, of 30 to 18 lbs. perhaps once per year, of from 18 to 13 lbs. perhaps twice in one year, and of less than 13 lbs. at shorter intervals; so as to make necessary an allowance of at least 30 lbs. per sq. ft. steady-wind pressure over the whole bridge front, and 50 lbs. per sq. ft. gusty-wind pressure over from 600 to 1000 feet length. These same limits appear large enough for all ordinary localities in the United States except in those regions where occasional tornadoes are to be naturally expected and specially provided for.

Example.—The computation of wind stresses and bracing upon a large single-track bridge, adapted to the heaviest trains, and having panels of 25 feet length and 28 feet depth, and spans of 150 feet length, would therefore proceed about as follows:

Wind Area.—The effective wind area of the road-bed (including guard-rails, ends of the cross-ties, and track girders) may be obtained by treating the fronts of tie ends as forming a continuous grating with a fixed 75 per cent coefficient, the front guard-rails as short solids with a 90 per cent coefficient, the front track girders as H beams or short solids with a 90 per cent coefficient, considering the length of the ties as of neutral effect, considering the rear guard-rails as unsheltered, and considering the rear track girders as adding about 10 per cent to the area of the front girder; so that the total effective resistance of the road-bed per foot of length of bridge may as a rule be taken at about 2.0 sq. ft. for the two guard-rails and the ties, and about 5.0 sq. ft. for the two track girders, or about 7.0 sq. ft. in all.

The effective wind area of the main girders or trusses of the bridge may be obtained by treating each of its round vertical or diagonal tie-rods as a cylinder with a 60 per cent coefficient (none of these considered as receiving or giving any shelter to the others); each of its vertical or diagonal tie-bars as a flat plate with a 100 per cent coefficient (none of these considered as receiving or giving any shelter to the other); each horizontal bar or plate (at eight or more breadths distance in front of or behind another) and each group of horizontal tie-bars or plates (one behind the other and with two breadths distance of the broadest plate) as a single plate of the breadth of the broadest with a 100 per cent coefficient; each group of horizontal tie-bars or tie-plates (one behind the other and within about four breadths distance of the broadest plate) as a single plate of the breadth of the broadest and with a 150 per cent coefficient; square solid-sided hollow beams as short solids with a 90 per cent coefficient; I-beams and

square lattice-sided hollow beams when horizontal as a single flat plate with a 100 per cent coefficient; the same I and lattice beams when vertical or diagonal and with solid sides and open front as a single flat plate with a 100 per cent coefficient; the same I and lattice beams when vertical or diagonal and with solid front and open sides as two flat plates of sizes equal to their front and rear, each with a 75 per cent coefficient; other open-built beams and octagonal beams as flat plates of the size of the greatest cross-section parallel to the track with an 80 per cent coefficient; rough-surfaced cylinders as octagonal beams, smooth-surfaced cylinders and wire ropes and wire-wrapped cables as cylinders with a 60 per cent coefficient; horizontal cylinders as completely sheltering other similar cylinders touching them in their rear and as giving 50 per cent shelter to those at 2 diameters distance and nothing at 4 diameters distance; vertical and diagonal cylinders and octagonal posts as giving no shelter to others in their rear, and the rear main girders or trusses (if lattice-work or low solid work) as receiving no shelter from front girders or trusses, but both front and rear trusses being considered as sheltered by so much of the track girders (or stringers) as may be directly in front or rear of their actual surfaces: so that the total effective area of resistance of the main girders may be assumed per foot of length of bridge at about 0.8 square foot for each tension chord, about 1.4 square feet for each compression chord, about 1.0 square foot for both diagonals (after reduction of $\frac{1}{4}$ for shelter by track girders), about 1.5 square feet for each post (after $\frac{1}{4}$ reduction for shelter by track girders), or about 4.5 square feet for each girder, or about 9 square feet for both girders.

The effective wind area of the two trusses of the horizontal bracing of heavy bridges (so far as not already provided for above) may be taken at that of the diagonals of a single truss, one truss being usually entirely sheltered by the compression chords of the main girders, the other truss, however, being where its posts are sheltered, but its diagonals unsheltered; and these diagonals may be treated as short solids with a 90 per cent coefficient, so that the total effective area of the horizontal bracing per foot of length of bridge may be assumed at about 0 2 square foot.

The effective wind area (due to diagonal wind pressure) of the vertical sway-bracing may be taken at that of the diagonals alone, the other pieces either being sheltered or already provided for above; and these diagonals may be treated as cylinders with a 70 per cent coefficient, and as not sheltering each other, so that the total effective area of the sway-bracing per foot length of bridge may be assumed at not over 0.05 square foot.

For a heavy single-track railway bridge the total effective area of wind resistance may therefore amount to

$$7.0 + 9.0 + 0.2 + 0.05 = 16.25 \text{ square feet per foot length of bridge.}$$

The effective area of a double-track structure may be taken approximately as 40 per cent greater than that of a single-track bridge, or as about 25 square feet per foot length.

Piers or Towers.—The effective wind area of piers or towers may be determined in the same general way as that of the spans, being equal to that of its front truss, its rear truss, the front surfaces of its various platforms, and the front surface of the diagonals of its sway-bracing, so as to amount per foot height to fully 2.5 square feet per corner, or 5 square feet per front or rear side, or 10 square feet per foot-height for the entire pier of a single-track bridge, or 15 square feet per foot height for the entire pier of a double-track bridge. In calculating the stability of a pier against wind, it is to be remembered that the piers are held down upon their foundations by not only their own weight, but also by that of their share of the bridge and its load, and that they are pressed laterally

by not only their own wind load, but also by that of their share of the bridge and its load.

Trains.—The effective wind area of trains may be determined by regarding the train as 10 feet high, with its bottom 2.5 feet above the rails, and treating it as a short solid with a 90 per cent coefficient, and considering it as sheltering its own height of the verticals and diagonals of the two trusses, and all trains in its rear; so that the added wind area due to the train will be only about 7 to 8 feet per foot length of bridge occupied by trains. This load, however, must be regarded as a moving load, travelling gradually across the bridge. As some car of the train would probably be blown over by an 80-mile wind, and as a train, therefore, would not enter the bridge at such time, it is not necessary to consider the wind pressure on a train at times of maximum wind velocities, although such train area should be considered under the gusty effect of moderate winds.

Suspension Bridges.—In suspension bridges the effective area of resistance of the main cables and suspenders may be obtained by treating each main cable as a cylinder with a 60 per cent coefficient, and considering it as completely sheltering other cables directly in rear, if touching one to the other; or giving 50 per cent shelter at 2 diameters distance, centre to centre, but no shelter at 4 diameters distance to either other cables or other parts of the bridge, and by treating each suspender as a cylinder with a 60 per cent coefficient, and considering it as giving no shelter to any other suspender or other parts of the bridge; so that in bridges of 4000 feet in length of span the effective wind area of the main cables and suspenders alone may amount to 8 square feet per foot length of single-track bridge, or 12 square feet per foot length of double-track bridge and 4 square feet per foot length for each additional track.

Total Wind Pressure.—Having found the effective wind area of the span and train and piers, the total pressure to be provided against may be next figured at 30 pounds per square foot over the entire bridge with a train upon it, or 50 pounds per square foot over from 600 to 1000 feet of length of the unloaded bridge (including piers), increased by 30 pounds per square foot over the rest of the unloaded bridge, and then using in subsequent computations whichever of these totals produces the greatest stress upon the particular members under consideration.

Arrangement of Wind Bracing.—The wind bracing for ordinary bridges may then be arranged in the form of two horizontal trusses, one above and the other below the roadway, as in the usual arrangement of modern bridges stiffened by vertical sway-bracing. In case of very long bridges it may be desirable to supplement these by horizontal cables passing under the bridge floor and around the piers and out to the shore anchorage, the versine of the cable being from two thirds to three fourths the breadth of the piers. In suspension bridges these two methods are advisably further supplemented by the swinging in or "cradling" of the main cables to such an extent that this cradling may be not only enough to support all the wind pressure on the cables and suspenders, but also considerable of that on the stiffening truss and roadway, besides otherwise stiffening the bridge against lateral movement.

Dimensioning of Wind Bracing.—The dimensioning of the various parts of the wind bracing may be computed by the usual methods, the use of the new formulæ of Launhardt and Weyrauch being recommended as preferable to the older methods, and that of the new dynamic formula of Fidler as preferable to either. In these computations, especially under either of the new formulæ, it seems hardly necessary because of the infrequency of the maximum stresses to use a coefficient of safety much greater than that actually necessary to make certain that the maximum stress is not allowed at any time to go beyond the minimum elastic limit of such metal as may be naturally expected to actually

get into the finished structure, due allowance being made for the ordinary unevenness of manufactured products, and the ordinary failures of attempted careful inspection, this minimum elastic limit being deducible from the tests on full-sized members, as well as from those of a large number of smaller pieces of such metal as it is proposed to use. With ordinary grades of wrought iron and ordinary circumstances of manufacture and inspection the coefficient of safety may best be taken at 3.5, and the repetition limit at one half and the reversal limit at one quarter the ultimate strength of the metal, thus allowing dead-load stresses of 29 per cent and maximum reversal stresses (those under the most unfavorable conditions) of 7 per cent of the ultimate strength of the metal; but in exceptional cases of the best steel (bar or wire) and the most careful manufacture and inspection (admitted, however, only after careful tests) it may be allowable to reduce the coefficient of safety to 2.0 and to increase the reversal limit to one third of the ultimate strength, thus allowing dead load stresses of 50 per cent and maximum reversal stresses of 17 per cent of the ultimate strength of the metal.

In dimensioning of piers and towers the breadth of the pier at each of its horizontal sections must be sufficient to prevent overturning of the pier about either edge of such section, a coefficient of safety of 2.0 being used in the computation of such stability, and the breadth must also be such that no tensional strains shall ever occur in any of the vertical or main posts of the pier.

WIND PRESSURE.

It must be admitted that the exact amount of pressure exerted by the wind in any given locality upon a bridge truss of open construction is unknown, and the same may be said in regard to the precise effect of an assumed wind pressure per unit of area upon those members of trusses specially designed to resist it. The best that can be done is to assume conditions of pressure which, in the light of experience and observation, will subject the structure to greater pressures than will in all probability exist, and then proportion and connect the resisting members with the truss members so that they may most effectually subserve the purpose for which they are introduced.

As the result of this condition of facts many suppositions have been made in respect to actual total pressure on bridge trusses and also to the proper distribution of this assumed pressure over the members of the trusses, and much ingenuity and variety in design has been developed to meet the requirements of proper bridge construction. The conditions are further complicated by the effects of high-speed trains passing over the bridge, and the effect of the wind pressure on the train surface exposed to its action.

The following examples illustrate the variety of assumptions made. They are taken from "Stresses in Bridge and Roof Trusses," by Wm. H. Burr.

The Erie specifications are as follows:

	Per Lineal Foot.
Fixed load, roadway chord	150 lbs.
" " other chord	150 "
Moving load, roadway chord	300 "
Iron in tension at 15,000 pounds.	
" " compression, factor 4.	

The P., C. and St. L. Railway requires 300 pounds per foot for the train and 30 pounds per square foot on one truss only.

For the bridge over the Missouri, at Glasgow, 50 pounds per square foot on one truss and 300 pounds per lineal foot of train were used.

For the Eads Bridge, at St. Louis, 50 pounds per square foot on the structure alone was the specified pressure.

For the Kentucky River Bridge the wind pressure was assumed at 31¼ pounds per square foot on spans, train, and piers, and factor 4 was used in proportioning the bracing.

The Portage Bridge, New York, was built to resist 30 pounds per square foot on structure and train and 50 pounds per square foot on the structure alone.

The 520-feet span over the Ohio, at Cincinnati, was designed to withstand 50 pounds per square foot on structure alone, or 30 pounds per square foot on train and structure combined.

A fully loaded passenger train and the heaviest possible freight train will leave the track at the respective pressures of 31¼ and 56¼ pounds per square foot.

Engineers frequently specify 30 pounds per square foot of trusses and train combined, or 50 pounds per square foot of trusses alone. 300 pounds per linear foot of single track is also frequently used for moving wind pressure on train.

The following refers to the single-track bridge at Plattsmouth, Neb. The structure is also designed to resist a lateral wind pressure of 500 pounds per lineal foot on the floor and 200 pounds per lineal foot on the top chord of the through spans and the bottom chord of the deck spans. These quantities are about equivalent to 30 pounds on the bridge with train and 50 pounds on the empty bridge.

LATERAL BRACING.

With an assumed condition of wind pressure in amount and distribution the computation of the stresses in portal and intermediate braces involves no special difficulty. As in case of the wind pressure, special assumptions have to be made in regard to the distribution of the resistances at the feet of the end posts.

We have now to deal with horizontal forces in the planes of the horizontal trusses constituting the lateral systems, instead of vertical forces or loads in the planes of vertical trusses. The determination of the stresses with given pressures is not therefore different, so far as intermediate lateral braces are concerned, from that in vertical trusses. The two upper chords and the lateral bracing between constitute a horizontal truss, acted upon by a series of horizontal pressures, and supported at its ends at the upper extremities of projecting posts (the end struts of the main trusses), whose lower extremities are assumed to be incapable of sliding along the top of the piers, either by being fastened to the pier or by the development of frictional resistances due to the weight and load carried to them by the main trusses. The two end posts, braced near their upper extremities, constitute a projecting beam acted upon by a force equal to one half the total wind pressure on the two upper chords. This wind reaction therefore causes a bending action upon these end posts (whether inclined or vertical) similar to that caused by a weight suspended from the projecting end of a simple beam, except that the point of maximum is not at the fixed end, but at some point determined by the character of the portal bracing.

The lateral trusses may be of any of the usual type of bridge trusses—Pratt, Warren, or Howe trusses. In the example selected as indicated in Figs. 461, 462, the Pratt type is employed. Fig. 461 is the lateral-truss system between the lower chords, and Fig. 462 is that between the upper chords. According to this system of bracing the upper-chord lateral truss is $b_0'c_0$. c_0c_0', . . . $k_0'k_0$, k_0l_0', and the lower-chord lateral truss is $a'b$, bb', $b'c$ . . . $k'k$, kl', $l'l$, lm', assuming the wind to blow from the direction indicated by the arrows. The compression chords are $a'm'$ and $b_0'l_0'$. The tension chords are bl and c_0k_0, respectively. With the wind blowing in the opposite direction the compressed chords would be am and b_0l_0, and extended chords $b'l'$ and $c_0'k_0'$. The lateral compression braces are those normal to the chords, and are the same with the wind blowing from either direction; the diagonals are tension braces. Those indicated by full lines

are the ones acting with the wind blowing as indicated by the arrows. The dotted diagonals act when the wind blows in the opposite direction. The maximum stresses in the two sets are equal in the same panel. In the diagrams compression members are indicated by double lines, tension members by single lines, with the wind blowing as indicated by the arrows, and *vice versa*. The computation of stresses with the wind blowing in one direction is sufficient, as corresponding members in the same panels will be equally stressed with the wind in the opposite direction. All members must be proportioned and connected to sustain stresses developed by wind blowing from both directions. We will now proceed to find the stresses in the upper-chord lateral truss indicated by the letters $b_0'c_0k_0l_0'$. The following data are given:

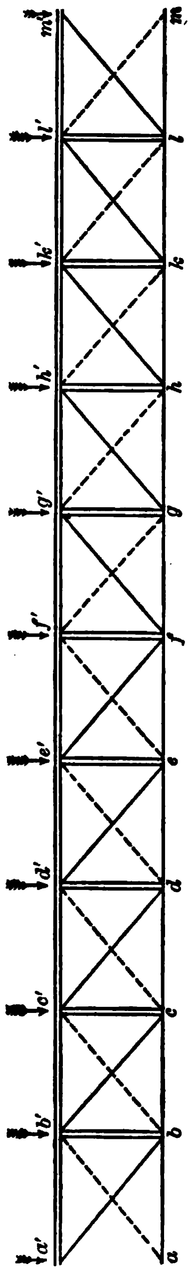
Upper-chord wind pressure, 150 pounds per lineal foot, considered as a fixed load and distributed as follows:

On windward chord, 75 lbs. per lin. ft.; on leeward chord, 75 lbs. per lin. ft.; load at each panel point of both trusses, $75 \times 20 = 1500$ lbs.; total length of span 200 feet, divided into 10 panels each = 20 feet; clear width between chords = length of transverse struts $c_0'c_0$, $d_0'd_0$, etc. = 14 feet; length of diagonals $d_0'e_0$, $c_0'd_0$, etc. =

$$\sqrt{20^2 + 14^2} = 24.41 \text{ feet;}$$

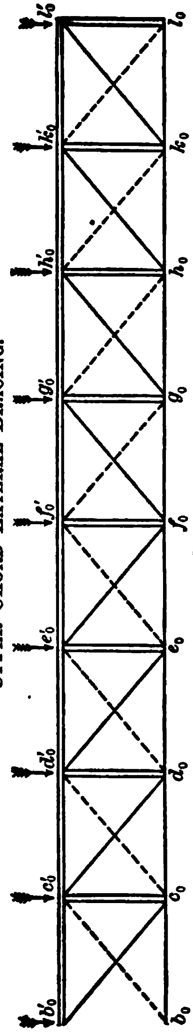
length of upper sway truss $b_0'l_0' = 160$ feet. (See Fig. 462.)

There is 1500 lbs. compression in $f_0'f_0$, the centre



LOWER CHORD LATERAL BRACING

FIG. 461.



UPPER CHORD LATERAL BRACING.

FIG. 462.

strut; at f_0 there is 1500 lbs. on the leeward truss—a total of 3000 lbs. concentrated at f_0. One half of this passes to the left and one half to the right; hence tension in $e_0'f_0$ =

$$1500 \times \frac{e_0'f_0}{f_0f_0'} = 1500 \times \frac{24.41}{14} = 1500 \times 1.741.$$

This transmits to e_0' 1500 lbs., and with 1500 acting directly at that point gives 3000 lbs. compression on $e_0'e_0$. At e_0 there is 4500 lbs., the whole of which passes to the left through e_0d_0', causing in it tension = 4500 × 1.741, and so on. Then write the following:

Compression in	$f_0'f_0$		= 1,500
"	" $e_0'e_0$		= 3,000
"	" $d_0'd_0$		= 6,000
"	" $c_0'c_0$		= 9,000
Reaction at	b_0'	= 9000 + 3000	= 12,000

Check on this 8 full panel loads, including panel load at b_0', 8 × 1500 = 12,000; the shear in the strut $f_0'f_0$ = 1500 lbs., and for each of the panels to the left 1500 lbs. more than the compression in the strut to the right. Hence

Tension in $e_0'f_0$	= 1500	× 1.741 =	2,612	pounds.
" " $d_0'e_0$	= (3000 + 1500)	× 1.741 =	6,836	"
" " $c_0'd_0$	= (6000 + 1500)	× 1.741 =	13,060	"
" " $b_0'c_0$	= (9000 + 1500)	× 1.741 =	18,284	"

Check the shear in the panel $b_0'c_0$ = 7 × 1500 = 10,500 lbs.; tension in $b_0'c_0$ = 10,500 × 1.741 = 18,284. What becomes of the reaction of 12,000 pounds at b_0' will be considered further on.

The chord stresses are now easily written, as in any panel it is the shear in that panel multiplied by the length of the panel and divided by the width between the chords, which will be taken as equal to 14 feet. It might more properly be taken as the distance between chord centres, about 16. The former gives a larger result, and will be used.

Compression in $e_0'f_0'$ or $f_0'g_0'$	$= 1,500 \times \frac{20}{14} =$	2,143	pounds.
" " $d_0'e_0'$ = tension in e_0f_0	$= 4,500 \times \frac{20}{14} =$	6,429	"
" " $c_0'd_0'$ = " " d_0e_0	$= 7,500 \times \frac{20}{14} =$	10,715	"
" " $b_0'c_0'$ = " " c_0d_0	$= 10,500 \times \frac{20}{14} =$	15,001	"

LOWER-CHORD LATERAL TRUSS.

The stresses in lower sway-truss, Fig. 461, are found similarly to the above, except that the fixed load is greater, and there is a moving-train wind pressure.

The panel loads are as follows: 1500 lbs. at each of the chord panel points, and a fixed load of 80 lbs. per lineal foot on the floor system, equivalent to 1600 lbs. at the leeward panel points or 1500 lbs. at each windward panel point and 1500 + 1600 = 3100 lbs. at each leeward panel. The stresses due to this wind pressure, considered as a fixed load, are computed precisely as for the upper-chord sway-truss. In addition the stresses due to the moving-train wind

load of 300 lbs. per lineal foot and acting on the leeward truss must be determined and combined with the fixed stresses.

FIXED-LOAD STRESSES IN LOWER LATERAL TRUSS. (See Fig. 461.)

$$\begin{array}{lll}
\text{Compression in } f'f = & & = 1{,}500 \text{ pounds.} \\
\text{“ \quad “ } e'e = & \frac{3000}{2} + 1500 + \frac{3100}{2} & = 4{,}550 \text{ “} \\
\text{“ \quad “ } d'd = & 4{,}550 + 1500 + 3100 & = 9{,}150 \text{ “} \\
\text{“ \quad “ } c'c = & 9{,}150 + 1500 + 3100 & = 13{,}750 \text{ “} \\
\text{“ \quad “ } b'b = & 13{,}750 + 1500 + 3100 & = 18{,}350 \text{ “} \\
\text{Reaction at } a' = & 18{,}350 + \frac{1500}{2} + 3100 & = 22{,}200 \text{ “}
\end{array}$$

$$\begin{array}{lll}
\text{Check, } 9\tfrac{1}{2} \text{ panel loads @ } 1500 = & 14{,}250 & \text{pounds.} \\
4\tfrac{1}{2} \text{ “ \quad “ @ } 1600 = & 7{,}200 & \text{“} \\
\text{at } a' \quad \tfrac{1}{2} \text{ “ \quad “ @ } 1500 = & 750 & \text{“} \\
\hline
 & 22{,}200 & \text{“}
\end{array}$$

$$\begin{array}{lll}
\text{Tension in } fe' = 1500 + \frac{1600}{2} = & 2{,}300 \times 1.741 = & 4{,}004 \text{ pounds.} \\
\text{“ \quad “ } ed' = & 6{,}900 \times 1.741 = & 12{,}013 \text{ “} \\
\text{“ \quad “ } dc' = & 11{,}500 \times 1.741 = & 20{,}022 \text{ “} \\
\text{“ \quad “ } cb' = & 16{,}100 \times 1.741 = & 28{,}030 \text{ “} \\
\text{“ \quad “ } a'b = & 20{,}700 \times 1.741 = & 36{,}038 \text{ “} \\
\text{Check.} \quad (9 \times 1500 + 4\tfrac{1}{2} \times 1600) \times & 1.741 = & 36{,}038 \text{ “}
\end{array}$$

STRESSES IN WEB MEMBERS DUE TO MOVING LOAD.

The load is supposed to come in from the right and to move along the leeward chord, *bl*, of the lateral truss. To determine whether counter stresses will exist, let the head of the load be at g; the total moving load will then be $4 \times 20 \times 300 = 24{,}000$ pounds, and reaction at a' and shear in panel $fg = 24{,}000 \times \frac{2\frac{1}{2}}{10} = 6000$ lbs.; the dead-load shear in this panel, as in fe, $= 2300$. The counter stress on the diagonal $fg' = 3700 \times 1.741 = 6442$ lbs., and this member should be so proportioned and connected as to resist this compressive force, if the other diagonal, $f'g$, in the same panel should be omitted. Both sets are used.

Maximum web stresses from moving load only occur when it has reached and passed the centre, f, of the chord. The following then will be the shears. A panel load = 6000 lbs.

$$\begin{array}{lll}
\text{Head of load at } f\text{, shear} & = 5 \times 6000 \times \frac{3}{10} = & 9{,}000 \text{ pounds.} \\
\text{“ \quad “ \quad “ \quad “ } e \text{ “} & = 6 \times 6000 \times \frac{3\frac{1}{2}}{10} = & 12{,}600 \text{ “} \\
\text{“ \quad “ \quad “ \quad “ } d \text{ “} & = 7 \times 6000 \times \frac{4}{10} = & 16{,}800 \text{ “} \\
\text{“ \quad “ \quad “ \quad “ } c \text{ “} & = 8 \times 6000 \times \frac{4\frac{1}{2}}{10} = & 21{,}600 \text{ “} \\
\text{“ \quad “ \quad “ \quad “ } b \text{ “} & = 9 \times 6000 \times \frac{5}{10} = & 27{,}000 \text{ “}
\end{array}$$

Compression ff' is zero, as the load acts at f.

Compression	$e'e$	= 9,000	pounds.
"	$d'd$	= 12,600	"
"	$c'c$	= 16,800	"
"	$b'b$	= 21,600	"

DEAD AND LIVE LOAD COMBINED.

Total compression in	ff'			= 1,500	pounds.
" " "	$e'e$	= 4,550 +	9,000	= 13,550	"
" " "	$d'd$	= 9,150 +	12,600	= 21,750	"
" " "	$c'c$	= 13,750 +	16,800	= 30,550	"
" " "	$b'b$	= 18,350 +	21,600	= 39,950	"

STRESSES IN DIAGONALS DUE TO MOVING LOAD.

Tension in	fe'	= 9,000 × 1.741	= 15,669	pounds.
" "	ed'	= 12,600 × 1.741	= 22,937	"
" "	dc'	= 16,800 × 1.741	= 29,249	"
" "	cb'	= 21,600 × 1.741	= 37,606	"
" "	ba'	= 27,000 × 1.741	= 47,007	"

DEAD AND LIVE LOAD COMBINED.

Total tension in	fe'	= 4,004 +	15,669	= 19,673	pounds.
" " "	ed'	= 12,018 +	22,937	= 34,950	"
" " "	dc'	= 20,022 +	29,249	= 49,271	"
" " "	cb'	= 28,030 +	37,606	= 65,636	"
" " "	ba'	= 36,038 +	47,007	= 83,045	"

Maximum chord stresses will exist in all panels when the moving load extends over the whole truss, i.e., from l to b.

The reaction at a' under this condition is 27,000 pounds and the shear in successive panels towards the centre will be less by 6000 pounds than in the one preceding. Then follows:

Comp. in $a'b'$ = tension in bc = $27{,}000 \times \frac{20}{14} = 38{,}576$ pounds.

" " $b'c'$ = " " cd = $38{,}576 + 21{,}000 \times \frac{20}{14} = 68{,}576$ "

" " $c'd'$ = " " de = $68{,}576 + 15{,}000 \times \frac{20}{14} = 90{,}006$ "

" " $d'e'$ = " " ef = $90{,}006 + 9{,}000 \times \frac{20}{14} = 102{,}866$ "

" " $e'f'$ = comp. " $f'g'$ = $102{,}866 + 3{,}000 \times \frac{20}{14} = 107{,}152$ "

To these must be added the chord stresses due to the fixed loads. The shears producing these stresses are the same as those used in producing tension in the diagonals.

CHORD STRESSES IN LOWER LATERAL TRUSS DUE TO FIXED LOADS.

Comp. in $a'b'$ = tension in bc = $20{,}700 \times \frac{20}{14} = 29{,}600$ pounds.

Comp. in $b'c'$ = tension in cd $= 29,600 + 16,100 \times \frac{20}{14} = 52,600$ "

" " $c'd'$ = " " de $= 52,600 + 11,500 \times \frac{20}{14} = 69,030$ "

" " $d'e'$ = " " ef $= 69,030 + 6,900 \times \frac{20}{14} = 78,890$ "

" " $e'f'$ = comp. " $f'g'$ $= 78,890 + 2,300 \times \frac{20}{14} = 82,176$ "

Total compression in $a'b' = 38,576 + 29,600 = 68,176$ pounds.
" " " $b'c' = 68,576 + 52,600 = 121,176$ "
" " " $c'd' = 90,006 + 69,030 = 159,036$ "
" " " $d'e' = 102,866 + 78,890 = 181,756$ "
" " " $e'f' = 107,152 + 82,176 = 189,828$ "

Total tension in $bc = 68,176$ pounds.
" " " $cd = 121,176$ "
" " " $de = 159,036$ "
" " " $ef = 181,756$ "

The same stresses found above apply to corresponding members in the other half of the truss, and, as stated for the wind blowing in the opposite direction, corresponding truss stresses will exist in opposite chords and diagonals.

As a check on the above total compression in $e'f'$ the aggregate panel wind loads is $6000 + 3000 + 1600 = 10,600$ lbs.; each total reaction at a' and m' is $10,600 \times 4\frac{1}{2} = 47,700$ lbs. Taking moments with respect to the centre f, $47,700 \times 100 - 10,600 \times 4 \times 50 = 2,650,000$ ft. lbs., and compression in $e'f' = \frac{2,650,000}{14} = 189,300$ lbs.

It will be observed that in the windward upper chord there is compression due to both the vertical loading and the wind pressure. The maximum compression is the sum of the two.

In the leeward upper chord there is compression due to the vertical loading, and tension due to the wind load. These need only be combined in order to determine whether or not there is a reversal of the stresses, which will not usually occur for upper chords.

In the windward lower chord there is also compression due to the wind pressure, while it is under tension due to the vertical loading. For instance, in the panel $a'b'$ at the end there is a compressive stress of 68.176 lbs., and in $b'c'$ of 121,176 lbs. These members are under tension due to the vertical loading. If in either of these panels the compression is greater than the tension, the bottom chord bars must be braced so as to be capable of resisting a compression equal to the difference between total compression and tension. This reversion will rarely take place in any panels except those near to the ends of the bottom chord. The leeward chord receives tension from both loads, the maximum being the sum of the two.

In the preceding discussion the moving wind load and the fixed leeward panel load have been assumed to act directly at the panel points. It is clear, however, that these pressures must be transmitted through the lower lateral struts or the floor-beams. The moving load acts at the track rails, and it is usual for the purpose now under consideration to assume it to be concentrated at the windward rail. The lower lateral struts bb', cc', etc., then have, in addition to the compressive stresses found from the lateral truss, first the total panel floor system wind pressure = 1600, and second a portion of the panel moving load.

It would seem that it would be as well to take it as equal to a panel load;

but Mr. Burr says that it is approximately equal to a panel load multiplied by the number of the strut from farther end of the truss numbered zero, and divided by the number of panels in the entire truss; or it is a simple application of the principle of the lever to the reaction or shear due to a single panel load at the head of the train extending over the longer segment. For example, assume the head of the load at d; then dd', cc', and bb' will have, in addition to the sway-truss compression found in it, $6000 \times \frac{6}{10} = 3600$ lbs.; dd' being at panel point 6 from m', zero being the number at m'. This completes the discussion of wind stresses in the lateral trusses, considered as horizontal and supported at a' and m' for lower chord, and at b_0' and l_0' for upper chord. We will now consider the stresses developed in the end posts and the portal bracing between them in the plane of the end posts.

In Fig. 463, is shown the type of lateral bracing employed with short spans and low main trusses. It consists of lateral struts and diagonal tension members in horizontal planes, connecting the panel points of the main chords. The portal bracing consists simply of two inclined members connecting the end

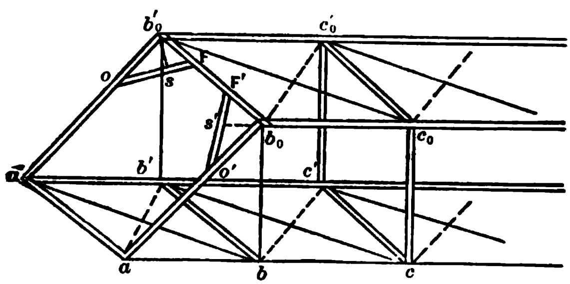


FIG. 463.—OBLIQUE VIEW OF LATERAL BRACING.

lateral struts and end posts of the vertical trusses. In this system the wind pressure is transmitted by the upper lateral truss to the ends, at the tops of end posts, and by them transferred to their lower extremities in the manner hereafter to be explained. The lower lateral truss transmits its wind load independently and directly to the lower extremities of the end posts.

Where the trusses, as in long spans, have considerable height, the type of lateral bracing used is shown in Fig. 465. Each pair of directly opposite posts in the two trusses are connected in the plane of the posts by two lateral struts and two tension diagonals, and similarly for the portal bracing in the plane of the end posts. The assumed effect of this system of bracing is to transfer, at each panel point, the wind pressure to the bottom chord. The entire wind pressure is thus transferred to the bottom chord and transmitted by it to the lower extremities of the end posts. (See Mr. Bixby's remarks on sway-bracing.) The depth of these vertical lateral frames or sway-trusses will depend upon the relative clearance required for the passage of trains and the height of the trusses. A clearance or head-room of from 20 to 22 feet must be provided.

We will now determine the stresses in the portal bracing of the first type. The dimensions and style of truss used is shown in Fig. 462 and 461. We will assume that the stresses developed in the upper chord sway-truss have been determined in precisely the same manner as already fully explained for the 200-foot span. All that we now need is the reaction of the total wind pressure on both upper chords, at the point b_0', the upper extremity of the windward post, causing this reaction we have one half of 14 panel loads, as an entire panel load is assumed

to act at each end of both chords; then $7 \times 18 \times 75 = 9450$ pounds $= H$, and acting as indicated by the arrow at b_0' (see Fig. 464). Assuming the stability of the structure, this force will be resisted by an equal force of friction developed at the lower extremities of the end posts, as indicated by the arrows H' and H_1 acting as indicated at the points a and a' respectively; a vertical upward force v at a, and an equal downward one at b_0', a portion of the weight of the windward truss. With this condition and relation of these forces we can write the equations of equilibrium as follows:

$$V + V' = 0;$$
$$H + H' + H_1 = 0;$$
$$(H' + H_1) \times 22 + V \times 16 = 0.$$

The last expression is the moment of the couple whose equal forces are $H' + H_1$ and H, whose lever-arm is the depth of the truss $= 22$, and the moment of the couple whose forces are V and V' and lever-arm the width centre to centre of chords $= 16$ feet, and expresses the condition that the structure as a whole shall not be overturned.

In the three equations there are four unknown quantities, namely, H', H_1, V, and V'; the magnitude of the forces H' and H_1 are therefore indeterminate.

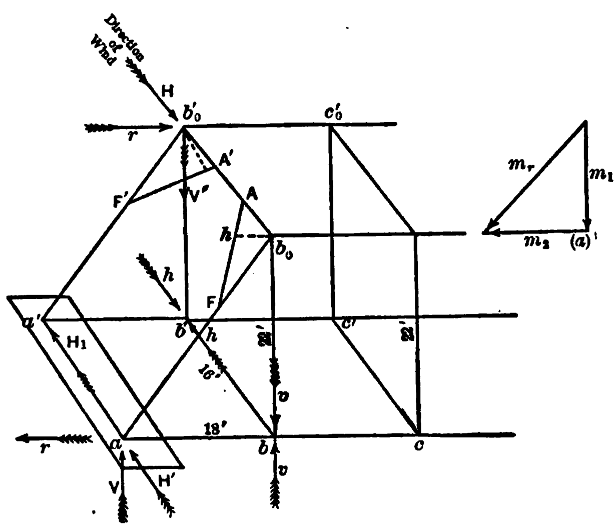


FIG. 464.—PRATT TRUSS.

Length of span = 144 ft.
Divided into 8 panels each = 18 "
Showing the positions and directions of action of the actual and assumed forces and couples.

We are therefore compelled to make one of three assumptions: (1) $H' = 0$; (2) $H_1 = 0$; (3) $H' = nH_1$, in which n is some fraction or part of unity, and since $H' + H_1 = H$, H' and H_1 can be determined.

In the following discussion we will make the first, namely, $H' = 0$.

If we now conceived two equal and directly opposed forces, $h = H = H_1$, to be applied at the point b_0' (see Fig. 464), and acting in a direction parallel to H and H_1, and also two vertical forces v applied at b, equal and opposite

to each other, and equal and parallel to V and V', the existing conditions of equilibrium are not disturbed. Instead then of single forces, we have a series of couples:

$$M = H(h) \times 22; \quad M_1 = H_1(h) \times 18;$$
$$M_2 = V(v) \times 18; \quad M_3 = V'(v) \times 16.$$

$m = m_3$, as seen in equations of equilibrium, and balances independently; and there remain m_1 and m_2, whose axes are perpendicular to each other—one vertical, the other horizontal. The resultant will be

$$M_r = \sqrt{M_1^2 + M_2^2} = 18\sqrt{H^2 + V^2};$$
$$M = M_3 = H \times 22 = V \times 16.$$
$$\therefore V = 9450 \times \frac{22}{16} = 13,000 \text{ pounds.}$$
$$\therefore M_r = 18\sqrt{9450^2 + 13,000^2} = \text{[illegible]}9,296 \text{ pounds.}$$

The plane in which this couple M_r acts contains the chords $b_0'c_0'$, etc., and a, b, c, etc.; and when viewed from the front will be right-handed, since m_1 is right-handed viewed from above, and m_2 is right-handed when viewed from the rear, as indicated in the moment diagram, Fig. 464(a). The forces of M_r are then r, r and act, as indicated by the arrows, causing compression in the chord $b_0'c_0'$, etc., and tension in the chord a, b, c, etc. The lever-arm of this couple is the diagonal

$$bb_0' = \sqrt{22^2 + 16^2} = 27.2.$$

Then

$$\text{Compression in } b_0'c_0' = r = \frac{M_r}{27.2} = 10,643 \text{ pounds.}$$
$$\text{Tension} \quad \text{"} \quad ab = r \qquad = 10,643 \quad \text{"}$$

As a check on this result

$$V \times \frac{18}{22} = 13,000 \times \frac{18}{22} = 10,640 \text{ pounds.}$$

With the following data given we can now find the stresses in the members of the portal:

Depth of trusses	= 22	feet.
Clear width	= 14	"
Length of $b_0F = b_0'F'$	= 8	"
" " $b_0A = b_0'A'$	= 5	"
" " normal b_0h	= 4	"
" " $AF = A'F'$	= 9.44	"
" " end post $ab_0 = a'b_0'$	= 28.42	"

Taking moment about an axis at b_0', after conceiving an ideal section cutting $b_0'A'$ and $A'F'$ and parallel to the windward truss, the only acting forces are $H_1 = H$ and the stress in $A'F'$. The lever-arm of the first is the length of the end post = 28.42, and of the second the normal from b_0' on $A'F' = 4$.

Stress in $A'F' = H \times \frac{28.42}{4} = 9450 \times \frac{28.42}{4} = 67,143$ pounds. Taking $F'b_0'A'$ as the triangle of forces, then stress in $b_0'A' = 67,143 \times \frac{b_0'A'}{A'F'} = 134,285 \times \frac{5}{9.44} =$

35,563 pounds, and in $F'b_0' = 67{,}143 \times \frac{F'b_0'}{A'F'} = 67{,}143 \times \frac{8}{9.44} = 56{,}900$ pounds. The kind of stress is found by considering the direction of the moments. That of H_1 is right-handed, consequently that of the stress in $A'F_1$ must be left-handed. The stress in $A'F'$ must act outward to the right, causing tension. Then following

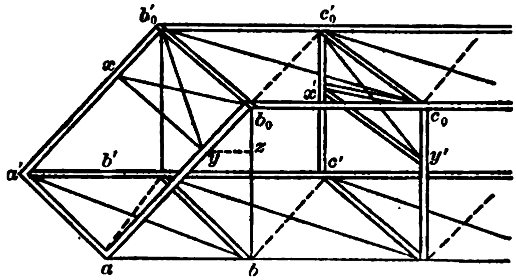


FIG. 465.—OBLIQUE VIEW OF LATERAL AND SWAY BRACING.

continuously around the triangle $A'b_0'F'$, the stress on $A'b_0'$ acts inward on b_0', indicating compression. The force in $b_0'F'$ is downward, causing compression. The end post is in the condition of a projecting beam supported at F' and acted upon by the force H' at its other extremity. The greatest bending moment is therefore $= H' \times a'F' = 9450 \times (28.42 - 8) = 192{,}969$ foot-pounds.

Length of span = 200 ft.
Divided into 10 panels each = 20 "
Showing the positions and direction of forces, when all the wind pressure is carried to the bottom chord.

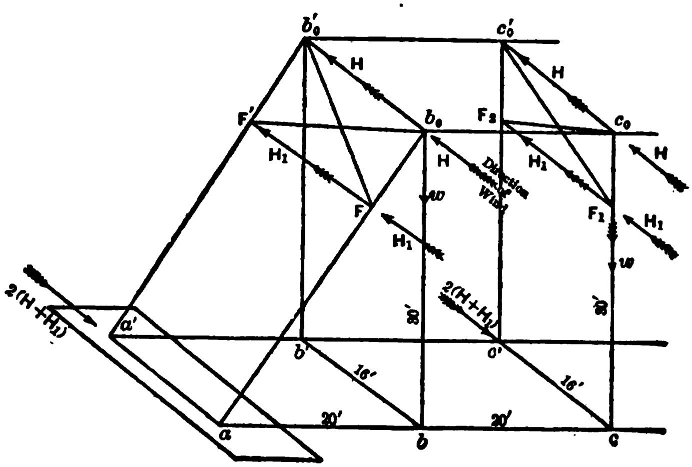


FIG. 466.

If the wind blows in the opposite direction to that assumed, corresponding stresses will be developed in am and b_0l_0 as were found in the chords $a'm'$ and $b_0'l_0'$ respectively. The stresses in lateral struts will not be changed. The dotted

diagonals will have a tension equal to that found for the full-line diagonals in the same panel.

So far as equilibrium is concerned in the foregoing discussion, we might have assumed $H_1 = 0$, and $H' = -H$. In this case the bending moment in ab_0 at F would be equal to that found in $a'b_0'$ at F', but in the opposite direction. The bending moment in b_0b_0' at A would be equal to that at A', but in the opposite direction. And the stresses in FA, Ab_0, and b_0F would be equal in amount but opposite in kind to those found in the members $F'A'$, $A'b_0'$, and $A'F'$ respectively. In other words, all the members of the portal should be proportioned and connected to resist both tension and compression of the amounts determined above. The end posts ab_0, $a'b_0'$, and b_0b_0' are subjected to both direct and bending stresses, and should be proportioned, and dimensioned accordingly.

If the portals are in a vertical plane the bending moment M_r and the resultant stresses r and r in the chords $b_0'l_0'$ and am would be zero. The lengths of the upper and lower lateral trusses would be equal.

If the bridge is of the deck design, the ends of the chords should be secured directly to the piers or abutments, as there will then be no bending in the end posts. Other stresses in and arising from the lateral trusses will be computed as for through-bridges. It must be noted, however, that the wind pressure on train and floor system will be found in the upper chord.

SWAY-BRACING.

Sway-bracing is not intended to nor does it do away with lateral bracing or trusses. It is sometimes stated in discussing this subject that its purpose is to transfer the entire wind pressure to the lower chord lateral truss, which would imply that the top lateral truss is omitted entirely, and this is the view taken in the following article. As Mr. Bixby says in a preceding article: "The sway-bracing, which is placed between the vertical posts of the main trusses at each panel point, is introduced to prevent independent lateral vibration and swaying of the vertical trusses; also to stiffen the long vertical posts, as well as to assist in carrying some of the wind stresses to the leeward girder. . . . The parts of the sway-bracing are therefore usually dimensioned so as to be able to carry 50 per cent of the wind load due to each panel of the bridge." But as for the present purpose it is immaterial in regard to the exact wind load employed, the author will follow the method of distribution of pressure as given by Mr. Burr. Fig. 465 is designed to show both the lateral and sway trusses. $aa'b'b$ and $b_0b_0'c_0'c_0$ are portions of lower and upper chord lateral trusses, respectively, xyb_0b_0' is the portal sway-truss, and $x'y'c_0c_0'$ is one of the intermediate sway-trusses. Fig. 466 shows only the sway-trusses, and the distribution of the pressures and resistances. In this discussion there is assumed to be no upper lateral truss, and consequently no transmission of intermediate panel wind pressures to the upper extremities of the end posts. Each panel wind load is transmitted at its own panel point to the lower chord. The end posts, therefore, except that they may be inclined as shown in Fig. 466, and are therefore longer, are in the same condition as any other pair of posts loaded with only one panel wind pressure. The members of the sway-truss FF', F_1F_2, b_0b_0', and c_0c_0' are in compression, and the inclined members between them in the same truss are in tension. The direction of the wind indicated in the sketch will relieve the truss ab_0c_0cba of a part of its weight carried by it, and increase that carried by the truss $a'b_0'c_0'c'b'a$ by an equal amount, and *vice versa* if the wind direction were reversed. Let w represent this relief (or increase) of truss load; it will act as though hung from b_0, c_0, etc. Let $F_1C_0 = F_2c_0' = d$, $cF_1 = c_1F_2 = a$, and $c_0c_0' = F_1F_2 = b$ applied to the sway-truss $c_0c_1{}^0F_1F_2$ (see Fig. 466). Also, let $H =$ total wind pressure per panel (for one truss) on $\frac{1}{2}(c_0F_1 + F_1c)$, and $H_1 =$ total wind pressure per panel (for one truss) on $\frac{1}{2}(c_0F_1)$. The total horizontal reaction $= 2(H + H_1)$ will

be taken as concentrated at a' and acting towards a. Taking moments about an axis at a', the acting forces are $2H$, $2H_1$, and w. Their respective lever-arms are $a + d$, a, and b, respectively. Hence for equilibrium

$$2H(a + d) + 2H_1a - wb = 0 \quad \text{and} \quad w = \frac{2[H(a + d) + H_1a]}{b},$$

which is the relief in the windward truss.

Compression in $c_0c_0' = H$.
Tension " $c_0'F_1 = w \sec c_0F_1c_0'$.
Compression " $F_1F_2 = H_1 + w \tan c_0F_1c_0'$.
" " $c'c_0' = w$.

The horizontal resistance $2(H + H_1)$ acting at c' produces a bending in $c'c_0'$, which has its maximum moment at F_2 or $M = 2(H + H_1)a$.

The above equations enable us to find all the stresses in the sway-bracing by a simple substitution of the proper values for H, H_1, w, and the angle $c_0F_1c_0'$. If the wind blows from the opposite direction the diagonal c_0F_2 will be stressed, and not $c_0'F_1$; but the amount in either case will be the same, and of the same kind. For the sway-truss in the plane of the portal it will only be necessary to make $b_0F = d$ and $aF = a$. No other changes will be necessary. Whatever may be the amount of bending moment at the points F, F', F_1, or F_2, arising from the lateral truss or sway-truss, the fibre stress must be determined at these points of maximum bending, which is done by solving the general equation $M = \frac{fI}{y}$, and there results $f = \frac{My}{I}$, in which M is the bending moment at these points of the posts, y is the greatest distance of any compressed fibre from the neutral axis of the cross-section, and I the moment of inertia of the section about an axis passing through its centre of gravity and lying in the plane of the truss. To f must be added the amount of increased weight w per unit of area of cross-section $= w/A$; hence total addition to be made to the regular truss stresses from vertical loading is $f + \frac{w}{A}$.

In deck bridges the sway-truss has the full depth of the main trusses, the transverse struts being in the planes of the bridge chords. The relief or increase of weight or direct stress on the posts is w. H_1 would be applied in the plane of the lower chord, and is the compression on the lateral strut, which at its leeward end combines with an equal pressure producing $2H_1$ which is carried by the sway diagonal to the windward upper chord. The tension in the diagonal is $= \sqrt{(2H_1)^2 + w^2}$. By these means all wind pressure is carried to the top chord. No parts or members are subjected to bending stresses from wind pressure.

STRESSES IN BRACED PIERS.

The following discussion in braced piers is taken substantially from Burr's treatise on "Bridge and Roof Trusses." It will be understood by reference to the accompanying drawings, Figs. 467 and 468. In Fig. 467 is shown an elevation of one bent of a high iron trestle or viaduct whose plane is normal to the centre line of track. This bent may be considered one of a pair in parallel planes, which, being braced together by longitudinal braces normal to the plane of the bent, constitute a complete braced pier. Commonly there is one span composed of girders on trusses, resting directly on the braced pier, and having a length equal to the distance between the bents forming it, and another span equal to the clear distances between the nearest bents of two adjacent piers or towers, these spans alternating in this manner over the entire length of the viaduct. This arrangement of braced towers and spans is adopted whether the

spans are equal or unequal in length, and whether the spans are long or short—say 30 or 60 feet. Where the spans are unequal, the longer is between the towers.

In Fig. 467 the inclined members AK and BL are columns or compression members, as also are the horizontal braces CD, FE, MG, and KL. The diagonals CF, DE, EM, FG, MK, and GL are tension members.

The bridge spans may be of the through or deck type. In the diagram Fig. 467 it is represented in section by the rectangle $ABO'O$, and is a deck or suspended truss. The rectangle $RQTS$ represents a skeleton section of the train.

The direction of the wind is indicated by the arrows, and acts on train, truss, and pier, or for the unloaded structure, on the truss and pier. The wind pressure on the truss or truss and train will be carried to the top of the pier in the same manner as has been fully discussed for wind pressure on any truss and train.

The train is supposed to cover the whole of the two spans adjacent to the bent. Let H represent half the total pressure against the trusses, and P half that on the train covering the two spans, h equal the height of the line of action of P above the top of the pier AB, and b equal the width of the pier AB. The effect of the wind pressure P on the train is to decrease the train reaction at A, and to increase that at B by an equal amount.

$$V_1 = \frac{Ph}{b}.$$

Let h_1 represent the vertical distance of the centre of action of H below AB. The wind pressure on the suspended truss $ABO'O$ will cause an increase of truss reaction at A, and equal decrease of that at B.

$$V = \frac{Hh_1}{b}.$$

Consequently the resultant reaction will be the half weight of truss and train, that is, $\pm (V_1 - V)$. If

$$t_1 = V_1 - V = \frac{Ph}{b} - \frac{Hh_1}{b},$$

the total horizontal force acting at A will be $(H + P)$ increased by the horizontal components of t_1 acting at A and B. Each of these horizontal components of t_1 is equal to t_1 tang a, a being the angle between either of the inclined posts AK or BL and a vertical line through A or B; and as one is decrement acting upward and the other an increment acting downward, their horizontal components act in the same direction, and the two together $= 2t_1$ tang a. Hence the total horizontal force acting at $A = (H + P - 2t_1 \text{ tang } a)$. To this, of course, must be added the wind pressure on the pier itself acting directly at A, the wind pressure transmitted to the bent at its top AB, and the wind pressures on the bent itself concentrated at the points A, C, E, G, K, B, D, F, M, and L, and noting that the wind panel pressures at A, B, K, and L are only one half those at the other points we are enabled to determine by the graphical method the stresses produced in the different members of the bent. Diagram Fig. 468 is the graphical construction for determining these stresses. Lay off the horizontal line cd to represent the total horizontal pressure at A, and since one of the components t_1 tang a is a pull, the compression in the horizontal member AB is $cd + t_1$ tang a.

The wind acting in the direction assumed, the diagonals sloping similarly to AD are not stressed. We have then acting at B the compression in AB + half panel wind pressure, the compression in BD, and the tension in CB. Taking dc' to represent the first force or stress, and drawing the lines $(BD)-$ and $(CB)+$ from its extremities parallel to the members BD and CB respectively,

Fig. 468, we have $c'od$ for the stress polygon, giving the stresses as indicated. Lay off to the right of d' the panel wind pressure acting directly at C, Fig. 467, and drawing the lines $d'o'$ and oo' parallel to CE and CD. $dd'o'o$ is the stress polygon for the panel point C, giving the stresses as indicated. Lay off $c'c''$ the panel wind pressure at D, and drawing $o'o_1$ parallel to DE, and $c''o_1$ parallel to DF. The stress polygon for joint or panel point D is $d'o'o_1c''d'$.

Continuing this method all the stresses in the members of the bent are readily determined. The members and the stresses are clearly indicated in the diagram, Fig. 468.

These are the stresses in magnitude and kind caused by the wind pressure on the train, truss, and pier. We are now to find the stresses in the several mem-

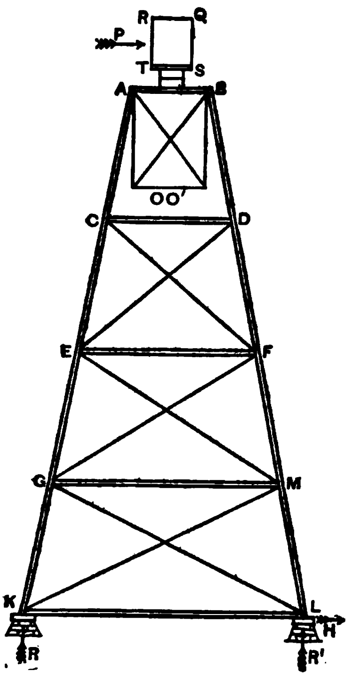


FIG. 467.—END ELEVATION OF HIGH IRON TRESTLE.

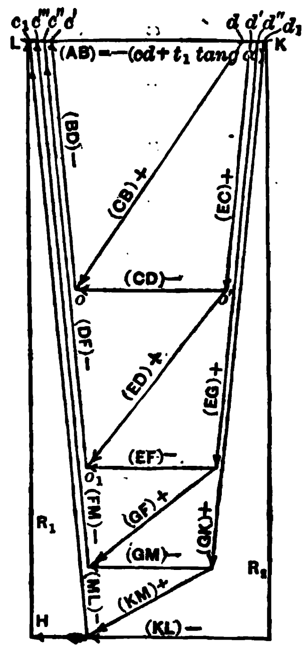


FIG. 468.—GRAPHICAL REPRESENTATION OF STRESSES IN HIGH IRON TRESTLES.

bers due to the weights of the train, truss, and pier itself, which must be combined for the same members in order to obtain the final maximum stresses.

Let W represent the total weight of adjacent trusses and moving load on the pier, W_1 the panel weight of the pier itself resting at the points C, E, G, D, F, and M, and $\frac{1}{2}W_1$ at the points A, B, K, and L; then the resultant reactions, with the wind blowing, will be at $A = \frac{W}{2} - t_1$ and at $B \frac{W}{2} + t_1$, assuming that V_1 is larger than V, which means that t_1 acts upward at A and downward at B.

The stresses produced by the vertical loads at A and B and the other panel points will be as follows:

$$\text{Compression in } AC = \left(\frac{W}{2} - t_1 + \frac{W_1}{2}\right) \sec a.$$

$$\text{" " } CE = \left(\frac{W}{2} - t_1 + W_1 + \frac{W_1}{2}\right) \sec a$$

$$\text{or } \left(\frac{W}{2} - t_1 + \frac{3W_1}{2}\right) \sec a$$

$$\text{" " } EG = \left(\frac{W}{2} - t_1 + \frac{5W_1}{2}\right) \sec a$$

$$\text{" " } GK = \frac{W}{2} - t_1 + \frac{7W_1}{2} \sec a$$

$$\text{" " } BD = \left(\frac{W}{2} + t_1 + \frac{W_1}{2}\right) \sec a$$

$$\text{" " } DF = \left(\frac{W}{2} + t_1 + \frac{3W_1}{2}\right) \sec a$$

$$\text{" " } FM = \left(\frac{W}{2} + t_1 + \frac{5W_1}{2}\right) \sec a$$

$$\text{" " } ML = \frac{W}{2} + t_1 + \frac{7W_1}{2} \sec a$$

In addition to the wind stress developed in the several sections of the inclined struts AK and BL, the wind has also the effect of relieving these columns on the windward side of a portion of the load of train and truss, and of increasing that on the leeward columns by an equal amount; consequently it was necessary to introduce $-t_1$ and $+t_1$ in determining the stresses due to weights of truss and train.

But having found the maximum effect of wind pressures on the transverse struts, the compression in these members due to the weights of truss and train are determined without regard to wind pressures, with which they must, however, be combined to determine ultimate maximum. The strut AB at the top of the pier receives therefore the horizontal component of the half weight of train and truss, plus the panel load of pier supposed to act directly at A, while each succeeding strut CD, EF, and GM receives only the horizontal component of the weight of the pier itself concentrated at its extremities, and the bottom strut KL receives the horizontal component of the total stress in GK. We can therefore write

$$\text{Compression in } AB = \left(\frac{W}{2} + \frac{W_1}{2}\right) \tan a.$$

$$\text{" " } CD = W_1 \tan a.$$

$$\text{" " } EF = W_1 \tan a$$

$$\text{" " } GM = W_1 \tan a$$

$$\text{Tension in } KL = \left(\frac{W}{2} - t_1 + \frac{7W_1}{2}\right) \tan a.$$

The horizontal component of the stress in $ML = \left(\frac{W}{2} + t_1 + \frac{7W_1}{2}\right) \tan a.$

The difference of the horizontal components in KL and the horizontal component of ML is therefore $2t \tan a$ and acts towards the right, and is an unbalanced force which must be resisted by friction or some special device.

The stresses due to wind pressures as determined by scale from diagram, Fig. 468, must now be combined with those due to the weights of train, truss,

and pier as determined from the foregoing equations. For simplicity let compression be indicated by the — sign and tension by the + sign; then write the following:

From Equations. Stresses due to Loads. From Fig. 468. Stresses due to Wind.

$$\overline{AC} = -\left(\frac{W}{2} - t_1 + \frac{W_1}{2}\right) \sec a$$

$$\overline{CE} = -\left(\frac{W}{2} - t_1 + \frac{3W_1}{2}\right) \sec a + (CE)$$

$$\overline{EG} = -\left(\frac{W}{2} - t_1 + \frac{5W_1}{2}\right) \sec a + (EG)$$

$$\overline{GK} = -\left(\frac{W}{2} - t_1 + \frac{7W_1}{2}\right) \sec a + (GK)$$

$$\overline{BD} = -\left(\frac{W}{2} + t_1 + \frac{W_1}{2}\right) \sec a - (BD)$$

$$\overline{DF} = -\left(\frac{W}{2} + t_1 + \frac{3W_1}{2}\right) \sec a - (DF)$$

$$\overline{FM} = -\left(\frac{W}{2} + t_1 + \frac{5W_1}{2}\right) \sec a - (FM)$$

$$ML = -\left(\frac{W}{2} + t_1 + \frac{7W_1}{2}\right) \sec a - (ML)$$

$$\overline{AB} = -\left(\frac{W}{2} + \frac{W_1}{2}\right) \tan a \qquad -(AB)$$

$$\overline{CD} = -W_1 \tan a \qquad -(CD)$$

$$\overline{EF} = -W_1 \tan a \qquad -(EF)$$

$$\overline{GM} = -W_1 \tan a \qquad -(GM)$$

$$KL = +\left(\frac{W}{2} - t_1 + \frac{7W_1}{2}\right) \tan a \quad -(KL).$$

The second terms in the above series of resultant stresses are to be scaled from Fig. 468, and as the diagonal tension members CB, ED, GF, and KM are stressed only by wind pressure, their maximum stresses are obtained by scaling directly from the stress diagram. With the wind coming from the opposite direction, these diagonals would not be stressed, but AD and those sloping in the same direction would then be under tensile stress of the same amounts as CB, ED, GF, and KM in the same panels.

The horizontal struts would have the same stress as already determined. The condition of the inclined struts would interchange, AK being stressed as already determined for BL, and BL as found for AK.

All the stresses may be checked by the method of moments, and such checks should always be applied.

The total reactions R and R' in Fig. 467 are:

$$R = \left(\frac{w}{2} - t_1 + 4w_1\right) + R_2;$$

$$R' = \left(\frac{w}{2} + t_1 + 4w_1\right) + R_1.$$

The terms in the parentheses are the total weights of pier, truss, and train on the windward and leeward columns of the bent respectively, and R_2 and R_1 and the respective vertical components shown in diagram Fig. 468. R_1 is taken as positive and equal to $-R_2$.

The lateral force H is the total wind pressure against the train, truss, and pier, and is to be resisted at the foot of the bent by friction or some special device. If f is the coefficient of friction at K and L, the lateral frictional resistance at K is fR, and that at L is fR'.

It is assumed that both the reactions R and R' are upward, for otherwise one of the inclined columns would be lifted from its bearing.

The expression $\overline{KL}$ has been written that all frictional resistance is developed at L. More properly the stress in KL should be $(\overline{KL})_1 = (\overline{KL}) - fR$, always assuming, numerically, $(\overline{KL}) > fR$.

If no train is on the structure, then W applies only to the weight of the trusses in the preceding relations. If, instead of a deck-bridge, suspended by its top chords from the top of the pier, a through-bridge is used, resting on top of the pier along the plane of its bottom chord, V_1 and V would have the same sign and $t_1 = V_1 + V$, noting that h is the distance from the centre of action of P to the top of the pier, and that h_1, now the distance of the centre of action H_1, the wind pressure on the truss above the same plane, i.e., the top of the pier.

Mr. Burr states that the web members of a braced pier, carrying a double-track railway, will receive their greatest stresses with the windward track only loaded. This fact he attributes to the investigations made by Mr. J. A. Powers, C.E. The discussion of this condition is so similar to the preceding, that the author will not introduce it here, but refers the reader to Mr. Burr's "Treatise on Stresses in Bridges and Roofs."

Hudson-river Bridge Specifications.

The following is taken from the *Engineering Record*, March 16, 1895. The specifications offer a concise outline of the bridge and the leading essentials of design and proportionment, and we reprint them in full as affording a complete *résumé* of the fundamental considerations for an enormous construction on advanced technical lines. . . .

The great importance and cost of this proposed bridge and the difficulty of foretelling the future demands on its traffic capacity require that it should be capable of carrying any traffic brought to it by the connecting lines without any restrictions as to speeds, loads, frequency of trains, or other limitations differing from those in force upon well-regulated railroads.

The specifications, therefore, define the capacity, strength, and legal restrictions under which the bridge is to be built. But upon all points where the bidder is at liberty to seek the best and most economical design the specifications are intentionally made very general. Before the acceptance of any proposal, detail specifications defining the methods proposed by the bidder, and which must, at least, equal the best practice of the day, will be prepared to supplement these general specifications.

Location.—The bridge will be located at such points between Fifty-ninth and Sixty-ninth Streets, New York City, as may be approved by the Secretary of War.

Span and Elevation.—The span will have a clear length of 3100 feet, the piers being located inside of the pier-head lines. The elevation shall be such that the bridge shall afford, under any conditions of load or temperature, a clear headway above high water of spring tides of not less than 150 feet at the centre of the span.

Capacity.—The bridge will have six standard railroad tracks upon one level.

General Description.—The general type of the proposed bridge will be a steel-wire suspension bridge, stiffened for moving load by longitudinal girders

extending from tower to tower. The main span only, or that portion between the towers, will be carried by the cables. The side spans, or that portion between the towers and anchorages, will be carried upon viaducts, independent of the cables. The towers will be steel skeleton structures, commencing at an elevation about 50 feet above high water, where the masonry piers end. All the connections must be riveted and all the bracing must be rigid. The stiffening trusses will be riveted lattice girders, with multiple systems of diagonal web bracing. They may be made continuous from tower to tower, or they may be made with a central hinge—at the option of the bidder. All the lateral and sway-bracing of the main span and the towers will be formed of rigid members, capable of resisting either tension or compression. The transverse and longitudinal floor-beams will be riveted plate girders. There must be two longitudinal girders under each track. The anchorage bars will be eye-bars of medium steel, and the anchorage pedestals will be built from forms of medium steel, and will rest on dressed granite blocks carefully bonded into the adjoining masonry.

Each bidder can select such depth of versine, number, and arrangement of cables, depth of stiffening trusses, and general details as he may deem best, within the requirements of these specifications. The main piers and anchorages will be founded upon the rock. The viaduct piers may be founded upon rock, or upon piles, or upon such other form of foundation as may be approved by the chief engineer. Bidders must submit with their proposals plans for the anchorages and piers, showing the proposed method of founding their general construction, and the character of the masonry.

The exposed faces of all masonry will be of granite or other approved stone.

LOADS.

The structure shall be proportioned to carry the following loads, the first two of which, taken together, shall constitute the "dead" or "permanent" load, while items 3, 4, and 5 will be known as the "live" loads.

First. The weight of rails, ties, guards, footwalks, etc., above the longitudinal track girders shall be taken as 400 pounds per lineal foot of each track.

Second. The weight of metal in the structure.

Third. Trains weighing 3000 pounds per lineal foot of track, covering all the tracks from tower to tower, at rest or moving slowly.

Fourth. Trains 1000 feet in length, and weighing 3000 pounds per lineal foot upon each of the six tracks; these trains being supposed to be moving at high speeds, either all in one direction, or the three on the north tracks in one direction and the three on the south tracks in the opposite direction.

Fifth. For the floor girders and other members, which will get their greatest stress from engine loads, a uniform load of 3000 pounds increased by load of 50,000 pounds upon each track. Items 3, 4, and 5 will be known as the "live" loads.

Sixth. *Wind.*—The structure covered with trains of cars will be assumed to be acted upon by a wind force of 25 pounds per square foot of surface of the bridge, or a wind force of 100 pounds per square foot of surface of the bridge only for a length of 300 feet. The wind will be supposed to act either horizontally, or at an angle of 30° above or below the horizontal; but the horizontal component only need be considered.

Seventh. *Temperatures.*—A range of temperature of 75° Fahr., above and below the mean, shall be considered in proportioning all parts of the structure. The modulus of elasticity of the cables will be assumed as 30,000,000 pounds, and for the trusses and other parts made from structural steel as 28,000,000 pounds. The coefficient of expansion for steel will be assumed as 0.0005 for 75° Fahr. All parts of the structure shall be proportioned to meet the maximum conditions produced by combination of the "dead" load with either of the "live" loads 3, 4, and 5, the wind, and temperature.

ALLOWED UNIT STRAINS.—MEMBERS SUBJECT TO TENSION.

Cables.—Under the above specified loads the allowed tension upon the wires in the cables shall not exceed a maximum of 54,000 pounds per square inch.

Suspenders.—The allowed tension in the suspenders shall not exceed 30,000 pounds per square inch of the section of the wires.

Cross-stays.—The wire ropes used for staying the cables together may be strained to 60,000 pounds per square inch of the wires.

Anchor-bars.—The anchor-bars may be strained to 20,000 pounds per square inch.

Floor Hangers, etc.—Forged-steel hangers and similar members shall not be strained more than 7500 pounds for "live" loads and 15,000 pounds for "dead" loads.

Floor Girders.—These girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing or web strains are resisted entirely by the web plates. No part of the web plates shall be estimated as flange area. Under these conditions the lower flanges of the longitudinal floor girders shall not be strained above 10,000 pounds per square inch of net section. The lower flanges of the transverse floor girders, when subject to tension only, shall not be strained above 15,000 pounds per square inch of net section. The top flanges of such girders will be made of the same gross section as the lower flanges, but the flanges must be stayed against transverse crippling at distances not exceeding 20 times their width.

Stiffening Trusses.—The chords of the stiffening trusses, when subject to tension only, shall not be strained above 18,000 pounds per inch of net section by the live loads, nor more than 22,500 pounds by the combined action of live loads, temperature, and wind. Tension members of the lateral and sway systems shall not be strained in tension above 18,000 pounds per square inch of net section.

Members Subject to Compression.—The columns of the towers shall be so proportioned that the maximum compression per square inch will never exceed that given by the following formula:

$$P = 20,000 - 70l/r.$$

For the compression struts in the lateral or sway system of bracing the unit strain must not exceed

$$P = 17,500 - 75l/r,$$

where P = allowed unit strains, l = length in inches, r = least radius of gyration in inches.

The chords of the stiffening truss, when subject to compression only, shall not be strained by live loads above

$$P = 17,500 - 60l/r,$$

nor by combined action of live loads, temperature, and wind above

$$P = 22,500 - 80l/r.$$

MEMBERS SUBJECT TO REVERSAL OF STRAINS BY LIVE LOADS.

Under the reversal of strains by live loads the chords of the stiffening trusses shall not be strained above the unit strains given by the following formulas:

$$t = \frac{T}{T + aC} . 20,000;$$

$$c = \frac{aC}{T + aC}(20,000 - 70l/r);$$

where t = allowed tension per square inch, c = allowed compression per square inch, T = total tension on the member, C = total compression on the member, a = value of the net section in terms of the gross section, l and r as before.

Should the transverse floor-beams be subject to reversal of strains by the live loads, the allowed strains will be deduced in a similar manner.

SHEARING STRAINS AND BEARING PRESSURES.

The shearing strains and bearing pressures upon the rivets or pins connecting any members shall never exceed 75 per cent and 150 per cent respectively of the maximum allowed tension or compression on these members for any particular loading. Pins shall not be subject to a greater bending strain than 18,000 pounds per square inch under the assumption that the middle halves of the bearing surfaces of the several members are considered as uniformly loaded with the applied forces. The webs of plate girders must be stiffened at intervals not exceeding the depth of the girders, whenever the shearing strain is greater than that given by the following formula

$$s = \frac{12,000}{1 + \frac{H^2}{3,000}},$$

where H equals the ratio of depth of web to its thickness.

Masonry.—The pressure transmitted to the masonry by the towers or anchor pedestals shall not exceed 40,000 pounds per square foot of surface under the maximum possible loading. The pressure within the masonry or upon the rock foundation shall not exceed 20,000 pounds per square foot, after allowing for displacement of the water, silt, and sand; the weight of water being taken at 63 pounds, silt at 100 pounds, and sand at 120 pounds per cubic foot. The pull of the cables must be resisted by that part of the anchorage masonry which is above mean high water, and its value for this purpose will be determined by the following factors: Weight per cubic foot of masonry to be taken at 150 pounds; coefficient of friction as 0.6; factor of stability as 2.

DETAILS OF CONSTRUCTION.

The arrangement of the anchorages must be such that all parts of the cables, anchorage links, and pedestals, other than the pedestal bearings, shall be readily accessible for inspection, cleaning, and preservation. The towers and cables must be treated on the supposition that the saddles are or may become immovable. The effects of the wind upon the cables may be provided for by cradling the cables and staying them together. The effects of the wind upon the floor and trusses must be provided for by a suitable system of lateral and sway bracing between the chords of the stiffening trusses. The suspenders may be made of wire ropes, or cables of straight wires, suitably wrapped. The cables must be so arranged that those on each side will be equally strained under any loading. The stiffening trusses shall be given a camber of 5 feet when at their lowest position from load and temperature. No compression member with a length greater than 125 times its least radius of gyration will be allowed. No

plate or shaped forms of steel of a less thickness than five-eighths of an inch shall be used. All details and connections shall be of such proportions that upon testing to rupture they may be of equal strength to the members they connect. All parts must be accessible to cleaning, painting, and inspection.

No closed forms will be allowed. Wherever it is possible, all rivets must be driven by power riveters. All bearing surfaces and abutting joints must be accurately tooled; all details of the structural work and all methods of manufacture must be equal in character to the best practice in American bridge work.

All the wires in the cables must be carefully and accurately adjusted for the positions they will finally occupy in the finished cables. The wires in each strand must be continuously spliced. The length of the wires shall be so arranged that the splices will be uniformly distributed throughout the strands and cables. The splices must be satisfactory to the engineer, but a splice which will give 75 per cent of the strength of the wire without any enlargement of the splice will be considered preferable to one which creates an enlargement with a greater percentage of strength.

The wires of each cable must be thoroughly compacted into a cylindrical form, saturated with a selected preserving preparation, and then closely and firmly bound together by a wire wrapping put on in the most approved manner. Parts inaccessible to the wrapping process must be protected to the satisfaction of the engineer. The unprecedentedly large diameter of the cable for a bridge of the size of this structure will demand that the strands be squeezed together and wrapped in two or more processes, the inner strands being treated as a separate cable, around which the additional strands will be placed, squeezed, and wrapped. The number of these processes shall be determined by the engineer after the size and manner of making the cable have been selected.

The pull of the suspenders in the direction of the cables must be taken up otherwise than by the friction of the cable bands produced by bolts through their flanges. Only 20 per cent of the normal pressure upon the cables, produced by the load on the suspenders, shall be considered as resisting this pull. The remainder must be taken up by supplementary cables or ropes, running from the saddles to the suspender connections, or some equally acceptable device.

MATERIALS OF CONSTRUCTION.

Masonry.—The kind and quality of all stone, cement, sand, and other materials used in the foundations or the masonry must be to the satisfaction and approval of the engineer. The masonry of each part of the work shall be thoroughly compacted and cemented into one solid mass free from voids, and each class of masonry shall be in all particulars fully equal to the best accepted practice of the day for its class. For the main piers, where great strength is desired, the selected kind of masonry must be made of such materials and be so bonded that the required strength will be obtained. For the anchorages, where weight of masonry is the more important element, a cheap class of masonry may be used for the interiors; but externally this masonry must be of granite or other selected stone, of such forms, dimensions, and projections as will give a suitable appearance for a work of this magnitude at this location. Each bidder must clearly specify the kind and quality of masonry proposed for each part of the work.

Built Structural Work.—The stiffening trusses, floor, bracing, towers, anchor-bars, pins, hangers, and pedestals will be of "medium steel," which shall have an ultimate strength of 60,000 to 68,000 pounds per square inch, an elastic limit of not less than 33,000 pounds, and a minimum elongation of 20 per cent in 8 inches, when tested by samples cut from the finished bars having a length of 12 inches and a uniform sectional area of one-half square inch.

Before or after heating to a low cherry-red and cooling in water at 82° Fahr.,

this steel must stand bending to a curve whose inner radius is 1½ times the thickness of the sample without cracking.

The finished bars, plates, and shapes must be free from cracks on the faces and corners, and have a clean, smooth finish.

All sheared edges of plates and other forms must be planed off to a depth of one-fourth inch. All rivet-holes must be drilled after the several pieces forming one member are assembled together, or they may be punched, and, after assembling, reamed to a diameter one-eighth inch larger.

All forged work must be finally annealed. All rivets shall be of soft steel. The saddles may be steel castings.

Wire.—The wire for the cables shall not be less in diameter than No. 8 Birmingham gauge, or 0.259 inch. It shall be bright, "straight" wire, free from any tendency to coil when unrolled. Machine-straightened wire will not be accepted. It must be perfectly free from nipper marks. It must have an ultimate strength per square inch of 180,000 pounds, an elastic limit of 90,000 pounds per square inch, and an elongation of 4 per cent in a length of 1 foot; it must stand wrapping around a rod one-half inch in diameter at least four turns without evidence of fracture. Each wire must have a minimum length of 1800 feet without weld, joint, or splice. A variation in diameter not exceeding 0.006 inch will be permitted in the length of one wire, and a variation of the minimum size of the various wires not exceeding 0.002 inch. All wire as soon as made must be passed through an approved preparation of linseed-oil to protect it from oxidation. After testing and acceptance of the coils they will be jointed and coiled upon suitable drums for transportation, in lengths as great as can be practically handled or transported. The drums must be of such a diameter as not to injure the straightness of the wire.

All the wire shall be subject to the following tests:

1. Each ring may be tested for tensile strength, elastic limit, elongation, and bending, by samples cut from either end of the wire; or, 2, upon satisfactory evidence that the stock and method of manufacture are uniform one wire from each 40 may be selected as the basis of acceptance or rejection of the whole 40. In this case a length of wire of 40 feet may be cut from any part and subjected to test. Such piece must show an ultimate strength of 180,000 pounds per square inch, an elongation over its whole length of 2 per cent, and an elastic limit not less than 90,000 pounds per square inch. It must also answer the requirements of the wrapping test.

3. More frequent tests than the above may be made by direction of the engineer, but for such additional tests the loss of the material and expense of testing shall fall upon the bridge company in case the wire complies with the specifications, otherwise they shall be at the contractor's cost.

The wrapping wire will be No. 8 B.W.G. for the external wrapping and No. 10 B.W.G. for the internal wrappings. It must be annealed steel wire and have an ultimate strength of not less than 80,000 pounds per square inch. If the suspenders are made from straight wires wrapped, the wire must not be less in size than No. 6 B.W.G. and be equal in quality to the wires of the cables. If the suspenders are made from wire ropes, the ropes must be steel wire with a wire core. The ropes must have an ultimate strength of 180,000 pounds per square inch of the cross-section of the wires and an elastic limit of at least 90,000 pounds. The wires must not be less than No. 10 B.W.G., and must stand wrapping around a ¼-inch rod and being unwrapped without fracture.

Some ordinary clauses relative to inspection, testing, painting, proposals, etc., are omitted. The specifications were drawn by Mr. Theodore Cooper, the consulting engineer for the bridge company.

HUDSON RIVER BRIDGE, N. Y.

Extracts from the report of the Commissioners appointed by the President,

namely, George S. Morison, W. H. Burr, G. Bouscaren, Theodore Cooper, C. W. Raymond. For a full discussion see *Eng. Record*, Sept. 8, 1894.

Cantilever spans of from 2000 to 3100 feet are considered entirely practicable and safe. Comparisons of cost are made as follows: 1st, 2000-foot span.—The piers to consist of four cylinders, 200 feet centres in each direction. Weight on each cylinder, including the effects of wind pressure, 25,000 tons. At the east pier rock is found at a depth of 125 feet, at the west pier rock is found at a depth of 260 feet, below mean high water.

The allowed pressure between the metallic bed-plates and the top of the masonry is 20 tons to the square foot, and the pressure within the masonry 10 tons to the square foot.

The estimated cost of these piers, including excavation and sinking, is $1 per cubic foot above a plane 125 feet below water, and an increase of 8 mills to this for each additional foot of depth.

Each cylinder of the east pier contains 866,000 cu. ft., costing $866,000, or for the four $3,464,000.

Each cylinder of the west pier contains 1,880,000 cu. ft., making cost of each cylinder $2,427,500, and of the four $9,710,000.

The anchorage piers, finishing 150 feet above high water and 20 feet thick × 100 feet long, cost above water 75 cents and below $1.00 per cubic foot; cost of east pier $431,000, and west pier $1,038,000; or a total for substructure of $14,481,000. Weight of superstructure 230,000,000 pounds, having a total length of 4120 feet, and on another location 240,000,000 pounds and total length of 4320 feet—the load being 8000 pounds per lineal foot, and maximum stress 20,000 to 22,500 pounds per square inch, or one-third ultimate strength. 240,000,000 pounds at 4½ cents per pound = $10,800,000; giving total cost of 2000-ft. span $25,443,000.

2d. For the 3100-ft. span both main piers would be founded at a depth of 125 feet. The weight of the trusses of this span would be about three times that of the 2000-ft. span, and the weight of the floor and moving load 1½ times. The total reaction on the piers would be at least 2½ times. Hence each of the cylinders would have to carry 62,500 tons. The piers are so large that their volumes can be proportioned to the loads or weights carried, or the volume of each pier 2½ times those of the 2000-ft. span. So each of the two piers will cost $8,660,000; the cost of the two anchorage piers, estimated, $958,000. Total cost of substructure, $18,278,000. Weight of superstructure 730,000,000 pounds at 4½ cts., $32,850,000, or a total cost of $51,128,000. Hence considered impracticable on account of cost.

Then a 6-track suspension bridge is discussed, and pronounced a safe structure.

The essential differences between a cantilever and a suspension bridge are: First, that in the place of compression chords of the cantilever we have land anchorages built of eye-bars and masonry; second, in place of the tension chords of the cantilever we have cables built of wire of a superior grade of metal; third, in place of the web bracing of the cantilever we have a composite system of suspenders and stiffeners.

No question can be raised as to the safe and permanent character of the anchorages, if built with a sufficient factor of resistance and proper provisions for thorough protection of the anchor chains against rusting. They have the advantage over the compression chords of the cantilever that their weight is supported directly on the ground instead of forming a part of the dead load to be carried. Wire at least three times as strong as eye-bar steel is a merchantable article, and cables have less weight to be carried by the superstructure.

A cantilever bridge is a rigid structure, subject to those changes of shape only which are due to strains; it is well adapted to railroad uses, while it is claimed that suspension bridges cannot have a sufficient degree of rigidity for railroad purposes, and that the stiffening members cannot be properly propor

tioned, owing to the uncertainty which exists in the intensity of stresses due to changes of temperature and elastic deformation in the composite system.

Three principal methods have been employed to secure greater rigidity in suspension bridges:

First, by inclined stays extending from the top of the towers to the platform; second, by trussing the cables either with straight chords or by a system of braces between two cables; third, by a stiffening girder fastened to the platform and extending from one tower to the other.

The third method, that of the stiffening truss, is recommended.

The substructure consists of masonry piers, finishing at fifty feet above high water; the towers, of steel, 570 feet high from top of masonry to saddles, or 620 feet from surface of water. For towers of this height there is no question of the economy and expediency of using metallic construction. The anchorages would be of masonry, located about 1000 feet back of the towers; the clear span 3100, between saddles 3200, feet. Both towers and anchorages would have to be founded on rock; the cables are to be of wire, and the plans have been based on cable containing about 6000 No. 8 wires (0.259 in. in diameter), guaranteed strength 180,000 pounds per square inch, at moderate prices, and a much stronger wire at a higher price; the unit stress on cables of straight wire, 60,000 pounds per square inch, or one-third the breaking strength; a versed sine of 400 feet, or one-eighth of the span. In the East River bridge the versed sine is less than one-twelfth of the span, and about the same as in the other long-span suspension bridges. In the East River bridge the cables are of steel wire and the towers of masonry. With the introduction of steel towers the economical proportions were changed, and it became practicable to adopt a greater versed sine than has hitherto been considered wise.

A lengthy discussion of the duties and mode of action of stiffening trusses is given.

A stiffening truss is a girder supported by the cable and extending from one tower to the other. It is fastened to the platform, at the several points of suspension to the cables, and it may be fastened to the towers in two ways—it may be held in the vertical direction, anchored down as well as supported, and acting as a girder resting on two supports; or it may be fastened also in the horizontal direction, acting as a girder fixed at the ends. The first is recommended, as having greater simplicity in the computation of stresses without material sacrifice in economy. They also recommend as practicable and more economical the truss to be hinged at the centre.

Six cables are to be used on each side, the cables being twenty feet apart on the top of the towers, the two cables next to the centre on each side being in vertical planes, and the other cables cradled into planes which intersect in the lines of the pins which sustain the floor-beams. Cradling is not deemed necessary. A separate suspender reaches from each pin to every cable, the suspenders being in the same planes as the cables.

Weights and Cost.—

Structural Stress.	Pounds.	
Suspended structure	100,921,600	
Towers	76,047,000	
Chains	27,300,000	
Anchor-plates	2,400,000	
		206,668,600
Cost at 4½ cents	$9,300,087	
Wirework.	**Pounds.**	
Cables	79,647,800	
Suspenders	4,560,000	
		84,207,800
Cost at 8 cents	$6,736,624	
Cost of substructure	$17,480,960	

The design of the Union Bridge Company for this bridge, based on Mr. Cooper's specifications, has been accepted by the Secretary of War. Guaranteed cost not to exceed $25,000,000.

EXTRACTS FROM THE REPORT OF THE ENGINEERS ON THE PROPOSED HUDSON RIVER BRIDGE. PRESSURES ON MASONRY AND FOUNDATIONS.

Weight of granite masonry per cubic foot = 153 pounds; of concrete, 120.

Pressure on central shaft of East River bridge: 26 tons per square foot.

Cresy: 5¼ tons on 9 sq. in.; 1370 pounds per sq. in.

Two columns in church, Toussant d'Augers, 12 in. diameter, 25 feet high. carrying pointed arches: load on each, 25 tons, or 400 lbs. per sq. in.

Piers of dome, St. Peter's: 1022½ lbs. on 9 sq. in., or 113 lbs. per sq. in.

" Invalides:" 922 lbs. on 9 sq. in., or 102 lbs. per sq. in.

St. Genevieve: 1840 lbs. on 9 sq. in., or 204 lbs. per sq. in.

St. Paul's: 1190 lbs. on 9 sq. in., or 132 lbs. per sq. in.

Columns, St. Paul's, without walls: 1235 lbs. on 9 sq. in., or 137 lbs. per sq. in.

Church, Toussant d'Angers: 2767 lbs. on 9 sq. in., or 307 lbs. per sq. in.

Cylinders for bridge piers resting on firm sand in estuaries and bays: 5 to 5.6 tons per sq. ft.

Dutch engineers consider from 6 to 6.16 tons as safe loads on firm, clean sand: on very firm, compact sand foundations, at considerable depths, not less than 20 feet, and on sandy gravel, 6.7 to 7.84 tons; firm shale and clean gravel, 6.7 to 8.96 tons; on compact gravel, 7.84 to 10.08 tons per sq. ft.

Clean sand and homogeneous Thames gravel has been weighted with 280 cwt. per square foot at depths of 3 to 5 feet below the surface, with no signs of failure. Stiff clay, marl, sand, or gravel have been loaded with from 55 to 110 cwt., or 3.08 to 6.16 tons.

Gorai Bridge: close sand, Lock Kew gravel, Bordeaux gravel, loaded with 165 to 183 cwt.; Nantes sand, with 152 cwt., showing some settlement; Szegedin clay and fine sand, 133 cwt., or 7.4 tons, reinforced by driving piles in interior of cylinders and sheathing outside.

Charing Cross: 159 cwt.; including adhesion, 8.9 tons.

Cannon Street: 117 cwt.; including adhesion, 6.5 tons.

Roque Favour Aqueduct: 258 cwt., rocky ground; 14.4 tons per sq. ft.

Bunker Hill Monument: On hard sand and gravel, 5½ tons; no settlement.

Hardpan, under tower of brick church: 7 tons per sq. ft.; some settlement.

SPECIFICATIONS FOR ELECTRIC-RAILWAY CONSTRUCTION.

The road is to consist of 5½ miles of single track, standard gauge, with three cross-overs and double track sufficient for convenient switching. The overhead line is to be supported on the side-pole system, with curves, switches, etc. The feeder system is to maintain a maximum drop not to exceed 10 per cent under full load. Cars, motors, and other apparatus for the operation of the road, exclusive of the power-house equipment, are to be provided.

Ballasting and Levelling.—The road must be well ballasted with coarse gravel level with the top of ties, 6 inches deep underneath the ties, and brought to a grade, according to grade stakes furnished by the supervising engineer of the company. The track must be brought to alignment by tamping well under each and every tie, giving a solid foundation before any filling is done. At cross-roads, or where village grades are established, they must be strictly adhered to. At all curves the outer rail must be raised as specified in curve plan, so that the necessary high speed may be attainable with safety, and special care must be taken to have curves well tamped while ballasting or filling between the ties. Wherever marsh or soft ground is encountered a substructure of broken stone or other suitable material must be built, so that a safe and reliable foundation is

furnished before the ties are tamped or any levelling is attempted. Special care must be exercised by the contractor in ballasting and leveling that a perfect alignment is secured.

Ties.—The ties, except those for switches and curves, must be of sound, white, clear cedar, with two well-levelled surfaces, not less than 7 feet long, 7 inches face, and 6 inches deep. They must be spaced 2 feet centres, except at joints, where they must not be more than 6 inches apart. At curves or cross-overs the ties must be of white or burr oak, 8 feet long, 6 inches deep, and 8 inches face. These must be spaced 24 inches centres from the point of switch to a distance of four feet from outer end of frog. Ties must be free from rot, worm-holes, splits, or other imperfections affecting the strength or durability of the timber, must be subject to the inspection and approval of the supervising engineer of the company, and must be well tamped and firmly bedded with gravel, in order that each tie shall have a solid, unyielding foundation under it.

Rails.—The rails will be the Illinois Steel Company's 56-lb. standard T-rail, made of steel, with ends sawed square and smooth, and having a length of 30 feet per rail. They must be free from flaws, honeycomb, or blisters, and must be straight in all directions, free from twist of any kind. The rails shall be drilled for fish-plates and bands as per plan, and must be laid perfectly straight, level, and to gauge. They must be fastened by two $5 \times \frac{9}{16}$ in. iron spikes at each end of tie, so driven as to avoid splitting the tie. Rail joints must be carefully made, and so adjusted that the rail ends are not closer together than $\frac{1}{16}$ in., nor farther apart than $\frac{1}{8}$ in. Fish-plates must be of the standard pattern made for this particular rail, and bolted up, using thoroughly reliable lock-nuts, the same to be driven home with a hammer while the nut is being drawn up. This operation must be gone through twice on every joint. Special care is to be taken in laying curves, which must be placed accurately to plan, and spikes must be so driven that they will not draw from strain on the rail. This work must be done subject to the most rigid inspection and approval of the supervising engineer, from whom all sights and centers can be obtained.

Special Work.—The special work shall consist of three turnouts and such switches and curves as may be necessary to run cars into the car-house, or at the terminals. This work must be done in a thoroughly reliable manner, and none but first-class workmanship will be accepted.

All switches, turnouts, crossovers, station tracks, terminal tracks, etc., must be built of the standard rail, and must correspond in character with the track specified for the general line, and as little cast iron used as is consistent with first-class construction. All surfaces subject to wear must be of steel. Switch tongues must be of the standard length and size. The gauge of the track shall be 4 feet $8\frac{1}{2}$ inches, and the gauge of turnouts, switches, etc., shall be so full as to insure the operation of motors and trucks easily at high speed.

Bonding.—At each joint the rail must be bonded by two 0000 B. and S. copper rail bonds of a length of $6\frac{1}{2}$ inches between centres. These bonds must be securely riveted into each side of the lower flange of the rail, the hole in the rail to be countersunk from top side and to be brightened before the bond is driven in. The bond must have a solid shoulder against which to rivet. This work must be done in a careful and accurate manner, so that an absolutely perfect contact shall be obtained from the bond. Each bond and rivet-head must, after being placed, be painted with a coating of black asphalt. The bonds must be so placed that the fish-plates can be displaced without interfering with bonds.

Pole Line.—Side poles for span-wire construction are to be used throughout, except in such places as it may be found impracticable to set a pole at each side of the roadway, in which case a pole with a single bracket arm is to be used. All poles shall consist of carefully selected white cedar timber, straight and trimmed round. They must not be less than 7 inches diameter at the top, which must be roofed or pointed. The height of the pole shall be 20 feet above the level of the

track line, and each pole is to be set at least 6 feet in solid ground. They must be placed not less than 100 feet nor more than 125 feet apart, and there must be used at least 90 poles per mile. Poles must be set in line, with a rake or inclination in opposite direction to strain of 18 inches in 20 feet. The hole in which the pole is to be set must be dug as small as possible consistent with the proper placing of pole as specified, and the filling so well packed that the pole shall be able to withstand any ordinary strain on the span wire.

Where marshy ground is encountered, such provision must be made and material furnished as will secure a reliable setting for the pole. Cement or broken stone shall be used if required, and if necessary guy stays must be provided. All poles necessary for curves or anchor poles must be set according to plan or direction of supervising engineer. All poles shall be painted with one coat of lead-color oil-paint. Where brackets are to be used, a substantial wrought-iron arm shall be furnished. A span wire of ¼-inch galvanized steel wire shall be stretched from pole to pole, with a ratchet at one end so that slack can be readily taken up. This ratchet shall be of the best and most approved make. The span wire to be drawn tight with a slight strain.

Trolley Line.—The trolley wire shall be No. 0 B. and S. hard-drawn copper, in lengths of not less than one mile, and must be supported on straight line hangers with either Medbury, Ætna or John's insulation, using 15-inch soldered ears to support the wire. The overhead work on curves to be constructed in the most approved manner, using 9-inch slips with improved pull-offs for all spans. All switches to be of the most improved plan, and so adjusted as to allow the trolley wheel to follow accurately. Strain pull-offs are to be placed at intervals not exceeding one mile, by which the line must be anchored in both directions. The ends of the line must be securely anchored through reliable strain insulators. Standard splicing ears must be used to connect the ends of the wire, and the joints must be well soldered. This work must stand an insulation test of not less than one megohm per mile.

Feeders.—Provided the power-house is placed at a distance not exceeding 2000 feet from the terminus of the line, two feeders are to be provided for carrying the current from the power-house to the line. These must have weather-proof insulation and be placed on feed-wire insulators and supported by two pin-oak cross-arms securely bolted to the side poles. There will be three feed-wires of No. 0000 B. and S. gauge. One feed-wire will be 10,000 feet long and of an area of 4.0 circular mills, and tap at the end through a feeder ear to the trolley wire. The second feeder must be 20,000 feet long and of an area of 4.0 circular mills, and the third 30,000 feet long, tapped at the end through a feeder ear to the trolley wire. These feeders shall be put up in a substantial and reliable manner, in accordance with the best electrical practice. All joints must be well soldered.

The feeders are to be protected by a water lightning-arrester at the power-house end, and no fewer than five non-arcing lightning-arresters are to be placed along the line at intervals of one mile. These arresters must be securely fastened to the poles, and connected with No. 6 insulated wire from the rail for the ground connection and a No. 6 insulated wire from a feeder-ear on the trolley wire for the current connection.

Cars.—The contractor shall furnish two closed motor cars, 25 feet long in the body, 33 feet long over all, and 7 feet 10 inches wide, with a seating capacity for 44 people, and an approximate weight of body of about 7000 pounds; one closed motor car, 34 feet long over all, and 7 feet 4 inches wide, with seating capacity for 44 people, and weight of about 4500 pounds; also three open cars for use as trailers, having a width of 6 ft. 10 in., a total length of 34 feet, and having twelve seats, with capacity for 60 persons, and weighing about 6500 pounds. These cars are to be furnished with monitor roofs, full-line ventilators, fare registers, mirrors, gongs, radial draw-bars, ratchet brake-handles, sand-boxes, bumpers, commodious seats, etc., and shall be trimmed and finished in

such style and with such appurtenances, carpets, and trimmings as shall be required by the railway company. The painting and lettering of cars shall be as directed by the railway company.

Trucks.—There shall be furnished for each of the closed 25-feet motor cars two pivotal trucks, with four wheels to each truck; the wheels to weigh not less than 300 pounds each. The truck must be furnished with standard brakes, and with proper method of suspension for motors; the wheels for said trucks to be 36 inches diameter, and all four so connected as to receive uniformly the power transmitted to their axles from the motor; all wheels of said trucks to be of same size. Each truck shall be furnished with improved brake appliances and life-guards. For the 16-foot closed motor car a standard motor truck of some first-class reliable make shall be furnished, having a wheel-base of at least 6 ft. 6 in., provided with equalizing springs, life-guards, adjustable brake system, journal-boxes, and all parts and adjustments in consonance with the best truck construction. For the three 34-foot trail-cars pivotal four-wheel plain trucks shall be furnished, with proper springs, life-guards, and brake attachments, designed especially to secure easy riding and noiseless action.

Motors.—The motor cars shall be equipped with motors as follows: The two eight-wheel, double-truck motor-cars shall be furnished with two 30-H.P. single reduction motors, and the one four-wheel motor car (closed) shall be furnished with two 15-H.P. single motors. These motors shall be furnished with all the equipment and appliances necessary and usual for a complete motor equipment and use upon a motor car, including controller, switches, lightning-arresters, fuse-box, light circuit, etc., and all must be of the best known material and standard make. The motors must be of the latest style, either of the General Electric Co. or the Westinghouse railway types, with single reduction gearing, and guaranteed to give a speed, with 36-inch wheels, of 35 miles per hour when running to their full horse-power.

The controller must be that which is known as Type K, General Electric controller, or standard Westinghouse, and be of the latest improved form. All other apparatus, such as switches, trolleys, contacts, lightning-arresters, etc., must be of standard make from the same company furnishing the motors. Ten lights are to be placed on each motor car, two four-light clusters inside the car, placed on suitable fixtures, and one light on each platform, to be controlled on a five-light circuit by two snap switches. The trail-cars must also have light fixtures the same as the motor cars, with connections so arranged as to take light from the motor cars.

Tests.—All work shall be regularly and systematically tested while in process of construction, and any defects found shall be immediately remedied. The final test shall be made in the presence of the supervising engineer or his representative, and the right is reserved by the railway company, in case any doubt arises as to the fulfilment of the true spirit and intent of these specifications, to demand a test by expert engineers, selected as is usual in matters of arbitration, whose decision shall be final on all disputed points, the expense of such tests to be borne equally by both parties, unless the apparatus or material shall prove defective, in which case the contractor shall bear the expense and shall also remedy the defects, and he shall also be liable for any damage or loss to the said railway company resulting from conditions incident to the remedying of such defects.

During the progress of the work it shall be subject to the inspection of the supervising engineer or of his representative. The railway company will assume no liability nor responsibility for any of the installation until formally accepted in writing, and no part will be accepted until the company is satisfied that it fully complies with the spirit and intent of the specifications. The acceptance of any portion of the work shall not be construed as a final acceptance, and the failure of any part to perform its proper function shall be sufficient ground for the rejection of the whole.

The final acceptance shall be given only after the completion of the work contemplated under the specifications, according to their true spirit and intent, and after the final test as specified.—(Abstracts from the specifications of the chief engineer, Mr. Wm. Powrie, Waukesha, Wis. See *Engineering News*, Jan. 24, 1895.)

Electric Street-railway, Denver, Colo.

The Denver Consolidated Tramway Co. is now operating a system of 100 miles of electric street-railway, including some 175 crossings (many of which are railway crossings, 325 switches, and 375 curves. The track is laid entirely with T rails, many miles of which are in paved streets, and this track is found very smooth and to require but little expense for maintenance. The rail is 6 in. high, with 5 in. base, and weighs 72 pounds per yard. The curves are all "spiraled" with a system of graduated guards introduced by Mr. John A. Beeler, constructing engineer of the company.

On curves from 85 to 75 ft. radius a spiral approach of 18 ft. is used at both ends. At the point of spiral curve (P. S. C.) the radius equals infinity, thence decreasing until it corresponds with the given radius of the central curve, whatever that may be. (53 ft. is the example taken in *Engineering News*, May 10, 1894.)

From the diagram given it seems that the gauge of the track on the straight portions of the line is 3 ft. 6 in.; the half-gaugo or distance from the centre line of the track to the outer or convex rail is maintained throughout at 21 in. The distance from the centre line to the inner or concave rail is 21 in. on the straight portions of the line, until a point 8 ft. from the beginning, the P. S. C., of the spiral, where it is made $21\frac{1}{8}$ in.; at the P. S. C. it is $21\frac{3}{16}$ in.; at the middle of the 18-ft. spiral, $21\frac{5}{16}$ inches; and at the junction of the spiral with the regular curve, $21\frac{1}{2}$ in.; and continues at this to the junction of the regular curve and spiral on other side of the middle point.

Points intermediate to those given above have a proportional width. The graduated guard is a piece of flat sleigh-shoe steel bolted to the rail at intervals of 12 in., and separated from the rail by means of cast-iron fillers, which vary in thickness. On the straight track and about 8 ft. from the P. S. C. the space between the rail and guard is made $1\frac{1}{4}$ in.; then gradually increasing by the use of wider fillers to $1\frac{5}{16}$ in., $1\frac{3}{8}$ in, $1\frac{7}{16}$ in., until at the junction of the spiral and regular curve a space $1\frac{1}{2}$ in. is reached, which continues around the regular or central curve. On a curve of a shorter or longer radius than 53 ft. the fillers may be placed accordingly. By this construction the guard rail can be placed in its proper position with respect to the wheel and where it is wanted, and not at a fixed distance from the gauge, as is common, regardless of the length of the radius; consequently it does not wear out as rapidly as the grooved girder-rail, which renders the whole rail useless. When this steel guard thus spaced is worn to any extent it can be taken off, reversed, and used over again. With proper elevation of the outer rail a very high rate of speed may be maintained (from 15 to 18 miles per hour on curves having a radius of 45 ft.) without causing the usual bumping and lurching of the car. The construction gives a very smooth riding track, without danger of the car leaving it.

Abstracts from an Article on Electricity in Mining, Transactions of the American Institute of Mining Engineers, by F. O. Blackwell, Lynn, Mass.

Roughly estimated there are 300 companies in the United States engaged in mining and kindred arts, who now employ electricity. All these plants have been installed in the last few years. The great danger of accident, loss of life, destruction of property, etc., humanity, and economy demand the employment of

all means to secure greater safety. Whatever adds to the comfort and facilitates work, diminishes financial risk and increases profits, should be adopted.

Electricity has been shown to be safe, efficient, and reliable—a system of the greatest simplicity, completeness, and flexibility, permitting power from one source to be distributed in units of any desired size and for any purpose to the places where it can be employed to the greatest advantage, thereby securing the minimum consumption of power and expenditure of labor. With electricity there is neither friction, heat, nor condensation. There is no leakage or loss of power when not in use, which especially recommends it for intermittent work. It is not affected by heat or cold, and does not vitiate the air as is the case with steam or compressed air. Rapid deterioration of timbering, a source of great expense in all mines, due to bad air and heat, is to a great extent obviated. The risk of fire is greatly decreased. With proper safety devices and a system of concentric wiring it is practically impossible to start a fire from the current.

Prime Source of Power.—By conversion into electrical energy, water-power, even at considerable distance from the mine or mill, can be made valuable for all purposes, and steam-power can be so located as to secure fuel and water cheaply. In addition to this, one large engine costs less, is more evenly loaded, requires less attendance, and is necessarily more efficient than a number of small ones. The introduction of electricity has in this way made important reductions in the cost of power, and there are cases in which it would be impossible to run without its help.

At Virginius Mine at Ouray, Colo., 300 H.P. is carried at 800 volts from a water-power 4 miles distant. The cost of coal at this mine was formerly $18 per ton, which would amount, for the present plant, to about $80,000 per year, equalling the total cost of the electric plant. The item of fuel was a complete saving, the expense of operating the steam and electric plant being about the same in other respects.

Several mines in Colorado are operated by electricity that could not be by steam, on account of their situation on the face of precipitous cliffs, up which the transportation of water and fuel would be nearly impossible. The route to the Virginius plant the character of the country is very rough, a portion at an elevation of 12,000 feet above sea-level and well above snow-line.

Electric generators can now be obtained of any size and designed to run at any reasonable speed for belting or direct connection to engine or water wheels. The largest generator in the world was used in the power station of the Intramural Electric Railway at the World's Fair in Chicago. It is 2000 H.P., and directly connected to a compound-condensing engine, running at 75 revolutions. The generator is substantially the same as the motor, and both can be relied upon to give an efficiency of from 90 to 95 per cent.

Electric-power Transmission and Distribution.—The cost of copper, with any fixed loss, power, and distance, varies inversely as the square of the voltage. Economy of copper alone requires as high a voltage as possible. Cost of copper for long distances is the largest item of expense. For general distribution, especially underground, convenience of insulation and handling are of more importance and call for a low potential. Ordinary plants are supplied with current at 220 or 500 volts; sometimes voltages of 1000 and over are in use—notably at the Poorman, Comstock, Calumet and Hecla mines. When bare wires are used, as for haulage, the potential is limited to 500 volts. Direct-current generators are confined, by difficulties of construction, to potentials of less than 500 volts. The distance over which direct current can be economically carried at this potential is about 4 miles. Above this point alternating currents are used.

Long-distance Transmission.—The power of Lauffen Falls, converted into electricity, was transmitted 115 miles to the Exposition at Frankfort, where it operated a number of motors. This distance is beyond the limits of commercial success. Distances from 4 to 13 (and even 20 miles) are in full operation.

Alternating currents are desirable on account of the ease with which the electrical pressure can be changed by means of transformers, without moving parts; the high potential used for transmission is converted into low pressure for distribution. Alternating generators are wound for as high voltage as 5000, but it is probably better to limit them to 2500 volts. This potential permits of economical transmission to a distance of 7 or 8 miles. Above that distance step-up and step-down transformers are used. With 10,000 volts a limit of about 25 miles is attainable for practical work. Alternating-current motors possess nearly the same properties as those built for direct current. They have the advantage of being commutatorless, but cannot be so heavily overloaded as a series-wound direct-current motor. They are not therefore as suitable for railway or hoisting service. Direct currents can, however, be readily obtained from alternating by means of a rotary transformer, which is a combined alternating motor and direct-current generator.

Electric Haulage.—Above ground it has already supplanted animal power, and in some cases steam. For underground haulage, ready control, compactness, and freedom from smoke render it still more desirable, and in many cases electric locomotives are used in mines. The electric locomotive occupies no more space than the car. With other methods of traction greater width and height are required. The increased speed also permits a larger tonnage to be taken from the same outlet, with few turnouts, switches, and cars. Gauge of track need not be over 18 inches. The Belt Line tunnel through Baltimore is to be operated by electric locomotives, weighing 90 tons, speed 15 miles per hour, and will develop 1500 H.P. For mining haulage large sizes are not required—from 10 to 150 H.P., commonly 30 H.P. A 50-H.P. locomotive has been constructed and no part of the machine is wider or higher than the wheels. The speed under load varies from 6 to 12 miles per hour.

Electric Hoists.—Electric hoists are now largely in use, and follow as nearly as possible the methods and constructions approved for steam-hoist. The governing mechanism is similar, rheostats replacing the steam controlling gear; the brakes and clutches are the same. The rotary motion of the motor avoids the use of connecting-rods entirely. If it is desired to lower by power, the current can be admitted to the reversed motor. For small hoists used in winzes and in timber-construction work electricity is well adapted, the power connection being easily and quickly changed. Simplicity of the application of electricity and consequent economy are exemplified in many cases: in one, electric hoists have been in continuous operation for two years, with a total cost for repairs of $23.

An efficiency of 50 per cent is usually obtained between the power taken by the motor and the foot-pounds hoisted.

Electric Pumping.—A number of large pumping plants are now in operation equipped with electric motors.

Triplex single-acting and duplex and triplex double-acting all work well, and give an efficiency of over 70 per cent between the electric power taken by the motor and the water delivered by the pumps. If the lift is small and the water contains much solid matter, centrifugal pumps may be used, but their efficiency is low.

Electric Air-compressors.—The same general application exists as for hydraulic pumps. A decided saving of power can be effected by locating drill-compressors near to the drills, and thus avoiding losses by leakage and shrinkage in the pipes.

Electric Drilling.—Although it has been a matter of great difficulty to make an electric percussion-drill which would stand the continual abuse and hard usage to which all drills are subjected, they are at present in constant and successful operation. With care in designing the coils, so as to make them fireproof and rigid, they have been made to stand the strain. The drill consists of an outer iron pipe casing, inside of and attached to which are two coils and

ratchet mechanism. The plunger, equipped with the ordinary chuck and rifle-nut, is in the centre of the coils. The coils are supplied with current alternately, and draw the plunger up and down at a speed fixed by the generator which supplies the current. Electric diamond-drills are in use for drilling solid holes and for removing cores for prospecting purposes. For prospecting, both above and below ground, the electric drill is particularly available, as it is extremely light and compact, and temporary wires can be run to places where it would be difficult to carry steam pipes or boilers. A drill driven by a 2½-H.P. motor will remove a 1⅜-inch core, and can drill 600 feet deep without difficulty.

Ventilation.—In order to obtain the best results from fans and blowers, it is often desirable to locate them at the most distant points from the surface. This is done more easily in the use of electricity than of any other power, and the blowers can be advanced as the workings are pushed deeper. The electric motor is especially suited for high-speed blowers, which are efficient and occupy but small space. Much saving in piping can also be effected by using higher velocities and forcing air out from the inside rather than sucking it from the surface. Fans can be controlled by a shutter on the outlet pipe, or the speed of the motor can be varied. A motor of small H.P. is sufficient.

Electric Lighting.—It has been found that much more can be got from men by giving them better illumination, by avoiding the use of oil lamps and candles. The air is left purer, and both time and power are saved, by the use of electric lighting in doing under ground work which otherwise would have to be done on the surface.

In gaseous mines, where many explosions have resulted from the reckless use of miners' lamps, electric lighting is invaluable.

EXTRACTS FROM THE JOURNAL OF THE MILITARY SERVICE INSTITUTION. BY LT. W. L. SIMPSON, U. S. ARMY.

A project for a deep waterway to connect the great lakes with tide-water has of late been somewhat prominently before the public, and that it may at no distant day materialize seems probable.

Many years ago the Erie Canal was projected, and though a purely State undertaking, it was carried to completion. Its importance outside the State of New York was scarcely recognized in the days of its inception, and its most enthusiastic advocates underestimated the importance of the part it was to play in commercial affairs.

Notwithstanding the almost endless network of railroads reaching out and gathering in the immense products of the country and transporting them to the great commercial centres for distribution, the conditions are far from satisfactory to those interested in the production. During the summer of 1892 when wheat was quoted in New York from 80 to 85 cents per bushel, a Nebraska paper quoted wheat at 40 cents. This is a startling difference in this day and age to be due to cost of transportation only, and the financial meaning of such a difference becomes apparent with a brief comparison of shipping statistics. The immensity of the shipping of the Great Lakes is scarcely comprehended by those who have made a careful study of the subject. One-eighth of the entire commerce of the United States passes through the St. Mary's Falls Canal. The tonnage that passed through this canal in 1891 exceeded by over 2,000,000 tons the entire freight of all the nations that passed through the Suez Canal during that year.

More tonnage passes the city of Detroit than any other point in the world. In 1889 there were nearly 10,000,000 tons more than the total entrances and clearances of all United States seaports, and nearly 3,000,000 tons more than the aggregate shipping of London and Liverpool, and this latter excess increased

10,000,000 tons during the following year. A great portion of this freight is destined for eastern markets, and gains an outlet through New York or Montreal.

The writer goes on to show the inadequacy of the present means of transportation by either route; and then continues:

It is not a matter of securing deep-water transportation between two given point and along the connecting line only, but also and primarily of securing the conditions to render it practicable to load several hundred miles nearer the place of production for distant points, and to load on such transport vehicles as shall reduce the actual cost of transportation, do away with the necessity of breaking bulk or transferring in transit, and establish a wholesome competition to railroad traffic.

It is then suggested to construct a deep waterway connecting Lake Ontario with the Hudson River, and another connecting Lake Ontario with Lake Erie, both entirely in United States territory, enabling ships to load at Chicago or Milwaukee, or at any other lake port, not only for our Eastern and Southern seaports, but for any desired foreign port.

Then with a widening and deepening of the Illinois canal [the writer of this article seemingly is ignorant of the construction of the Chicago Drainage Canal, now well on the way to completion, and described elsewhere in this book.—AUTHOR] the system and its benefits will be extended to the Mississippi River. The passage from Lake Erie to Lake Huron is by the canal through the St. Clair Flats; the passage from Lake Huron to Lake Michigan is easy.

The writer then discusses the great importance of such a waterway from a military standpoint.

In the same magazine is found the following by Lt. Carl Reichman, U. S. Army:

Speaking of the waters enclosed by the West Indies and Central America, he says: This great West Indian sea-basin is open to communication with the world only to the east, i.e., towards the Atlantic Ocean; on the west it is encircled by the high-rearing isthmus of Central America. The time, however, is not far remote when a waterway will pierce this isthmus, bringing the Eastern and Western hemispheres, the Atlantic and the Pacific Oceans, in direct and rapid communication. There can be no doubt that the Nicaragua Canal will raise the West Indies to an extraordinary importance, which is now beyond calculation. A glance at the globe suffices to show that America occupies the middle of the earth's surface, its face towards Europe, its back towards Asia. What forcible changes the opening of the Nicaragua Canal will and must produce some day can be but guessed at now, though this much stands beyond cavil, that an immeasurable shifting of the relative commerce, possessions, and power will take place. From its completion will date a transformation and revivification of the great economical, political, and military relations. Because this is the straightest and the easiest, there will be but one world centre, one universal path, joining the Pacific and the Atlantic. Natural superiority is assured the possessor of the Nicaragua Canal. When we follow the maritime routes which lead from Grey Town, the eastern exit of the Nicaragua Canal, to the coasts of the territories of the Union bordering on the Gulf, and to the most important coast lines, i.e., where the chief weight of the United States lies and the continent is most accessible, we find that these routes favor North America to a high degree.

Two oceans have been united by the Suez Canal; at no remote date another continent will be pierced, and the middle Atlantic will be in interoceanic communication with the wide Pacific. Then will we travel westward to reach East India, and every space of land and every square metre of water of the West Indian land and water territories will possess commercial, political, and military importance

heretofore unsuspected. Greater battle will be waged in America for the isthmian canal than for the cut at Suez.

THE ILLINOIS AND MISSISSIPPI CANAL.

The length of this canal is 77 miles, of which 50 miles are canal and 27 miles slack-water navigation. The dimensions of the canal are 80 feet wide at the water level and 7 feet deep. The rise to the summit level in the east end is 205 feet in a distance of 20 miles, and requires 24 locks. The fall to the Mississippi River on the west is made by 13 locks—a total of 37 locks. There is one guard-lock at the mouth of the feeder.

The guard-lock consists of the lock proper, a culvert about 150 feet long and 10 feet by 6 feet by 9 inches inside, and three Taintor gates closing the sluices. The object of this lock is to regulate the admission of water into the canal from the pool above the dam at the head of the rapids. The sluices and gates are for the purpose of passing the water into the canal.

All lock masonry is made of concrete. Three brands of German Portland ce-cement were used, but in the main Alsen's Yellow Label brand was employed, 99 per cent of which passed a 2500-mesh sieve, and 87 per cent a 10,000-mesh sieve. The initial set of this cement took place in from twenty-two to thirty minutes. All the concrete was mixed by machinery, and from five to ten barrels of cement were dumped at one time into the cement box to insure a better mixing of the cement itself. One of the locks requiring 3536 cubic yards of concrete the average progress was 86.2 cubic yards per day of eight hours.

The mixer had a capacity of 4 cubic feet, and was run by a 15-H.P. engine at 9 revolutions per minute. The sand, pebbles, and broken stone were dumped into a charging-box of 45 cubic feet with the cement and water, from which it was dumped into the mixer.

Different mixtures of concrete were used for the facing and backing, but they were laid simultaneously. The backing was composed of 1 part cement, 2½ parts sand, and 5 parts of broken stone, or if pebbles were used they took the place of 1 part of the broken stone. For the facing a mortar of 1 part cement and 2 parts sand was used. The facing was 8 inches thick, and a coping 5 inches thick was also made of the same mortar. The rule followed was to use as much water in the concrete as was possible, and yet not have it quake under thorough ramming. To insure sufficient moisture to the concrete during its hardening, 12 × 12 inch wells were left at points along the centre line of the wall and filled with water. These wells were afterwards filled with concrete. The total amount of concrete used in the guard-lock, sluices, etc., was 3762 cubic yards, and the cost of laying it was as follows:

5246 bbls. Portland cement @ $2.90	$15,603.82
152 " Utica " @ 0.548	83.60
2901 cu. yds. broken stone @ 0.589	1,709.15
1970 " " sand @ 0.71	1,398.41
128 " " pebbles @ 0.881	113.30
Lumber for forms (or moulds)	2,635.96
Labor building forms, etc	2,726.72
Labor mixing and placing concrete	6,693.02
Miscellaneous bills	416.72
Total	$31,380.70

The cost per cubic yard was $8.34.

The amount of cement used per cubic yard of concrete was 1.4 barrels. It is to be noted that this included the mortar facing and coping, which made the proportion of cement large.

For lock No. 36, the only one completed, the foundation concrete, except under the lower gates, consisted of 1 part Utica cement, 1 part sand, and 3 parts of screened pebbles, while that under the lower gates consisted of 1 part Germania Portland cement, 2 parts sand, and 4½ parts screened pebbles. The Utica cement cost, delivered at Milan, Ill., 55 cents per barrel, and the Germania $3.10 per barrel. The contract for this work was $5.00 per cubic yard, the Government furnishing the cement. Utica cement concrete, consisting of 1 part cement and 3 parts gravel, was used for lock No. 37. Here the Government did the work by day labor, and the mixing, depositing, and ramming cost $1.00 per cubic yard. At both locks the material was hauled upon the foundations in wagons, and the ingredients mixed by hand and shovelled directly in place. The lock walls and all other masonry are of concrete. The lock walls are 4 feet thick at the top, and are vertical on the chamber side and with a batter on the back.

The total amount of concrete used in lock No. 36 was 2146 cubic yards, 1819.2 cubic yards of which were in the main walls. This amount was laid at the rate of 86.6 cubic yards per eight-hour day.

The proportions of the concrete were 1 part cement, 3 parts gravel, and 4 parts broken stone, and the cost was as follows:

3010 bbls. Portland cement @ $3.09	$9,057.45
1252 cu. yds. broken stone @ 1.535	1,922.95
851 " " gravel @ 0.78	664.17
500 " " sand @ 1.777	888.61
Lumber for forms, etc	281.65
Labor for building forms	1,472.09
Labor mixing and depositing concrete	3,897.19
Miscellaneous bills	253.36
Total	$18,437.47

Cost of concrete per cubic yard, $8.60. One cubic yard concrete required 1.4 barrels of cement.

The distribution of cost is approximately as follows:

Cement, 45 per cent; other ingredients, 20 per cent; lumber and iron, 3 per cent; carpenter work, 9 per cent; mixing and depositing concrete, 18 per cent; plant and miscellaneous items, 5 per cent. The average cost of facing in place was $16.00, as compared with $8.25 for the concrete proper. The weight per cubic foot of concrete was 148 pounds, and for facing 133½ pounds, as determined by weighing 6-inch cubes. The percentage of water absorbed during four days' immersion was 2½ per cent and 3.4 per cent, respectively.

It was specified for locks located in soft ground that piles capped with timber and concrete grillage should be used in the foundations, to carry the weight of the concrete walls and the weight of water necessary to fill the chamber. To provide against the pressure of a possible hydrostatic head tending to force the floor upwards, the foundations and the rear of the lock walls were well underdrained. A flooring of 2-inch plank was spiked to the timbers of the grillage.

The sluiceway for letting the water into the canal is provided with three Taintor gates. Each of these gates is 21 feet wide and 13 feet high. Their upstream faces are cylindrical, the radius of the circle being 12 feet and 3 inches. This work is done under the directions of Col. Marshall, U. S. Army. The gates are hoisted by means of a hoisting carriage running on a track on the bridge extending over the gates.

Canadian Canal System.*

This system will give the Dominion of Canada within the next few years a waterway 14 feet deep and 2384 miles long, from the Atlantic Ocean to the head

* Abstracted from *Engineering News*, March, 1895.

of Lake Superior. Of this waterway about 72 miles will be artificial, and the remainder the natural navigation of the St. Lawrence River and the Great Lakes.

The artificial part comprises the six short canals along the St. Lawrence River, between Montreal and Lake Ontario, known as the St. Lawrence canals; the Welland Canal connecting Lake Erie with Lake Ontario; and the Sault Ste. Marie Canal, connecting Lake Huron with Lake Superior. The six short canals have lengths in order, $8\frac{1}{2} + 11\frac{1}{4} + 11\frac{1}{2} + \frac{3}{4} + 4 + 7\frac{5}{8} = 43\frac{5}{8}$ miles total. The number of locks vary from 1 to 9, of which there are 27 in all. Length of each, 200 feet; width, 45 feet; rise, from 4 to 82½ feet; depth on sills, 9 feet. The distances between these canals vary from 4½ to 22¾ miles.

The old Welland Canal was 27½ miles in length; had two locks 200 feet long by 40 wide, and one 230 × 45 feet; remaining 24, 150 × 26½ feet; rise, 330 feet; depth on sills, 10¾ feet. The new Welland Canal, as remodelled from the old one, is 26¾ miles long, and has 25 locks, each 270 × 45 feet; rise, 326¾ feet; depth on sills, 14 feet.

The Sault Ste. Marie Canal is 3500 feet long; has one lock, 900 × 60 feet; rise, 18 feet; depth on sills, 21 feet.

The locks of the first Welland Canal were constructed of wood, 110 × 22 feet, 8 feet on sills, 40 locks in all, changed as first indicated and subsequently enlarged.

SAULT STE. MARIE CANAL LOCK CONSTRUCTION.

The excavation for this lock was in red a Potsdam sandstone, with layers of shale and occasionally of sand. After removing the rock to the proper depth the bottom was levelled off with Portland cement concrete, upon which the masonry was laid. There were 70,000 cubic yards of masonry. The facing was of Amherstburg limestone in 2-feet courses; mortar used, 1 Portland cement to 1 sand; the backing of Manitoulin Island limestone. This masonry was laid at the rate of 10,000 cubic yards per month.

The lock-chamber is filled and emptied through culverts running longitudinally under the floor. In the floor of the lock there are 152 openings or orifices connecting the interior of the filling culverts and the lock-chamber. There are four of these culverts, which are 8 × 8 feet inside dimensions, and these are constructed of wood. The construction is as follows: Longitudinal sills 12 × 15 inches are bolted to the bed-rock with bolts 6 feet long and 1¼ inches diameter, spaced 6 feet apart. Upon the longitudinal sills are placed 12 × 12 inch transverse timbers 6 inches apart; the interstices are then filled with Portland cement mortar; the whole covered over with two courses of plank 3 inches and 2 inches in thickness.

The walls between the four culverts are 2 feet thick, built up of timbers 12 × 12 inches, across which transverse timbers 12 × 12 inches are placed 6 inches apart, and these covered with two courses of plank3 inches and 2 inches in thickness. Through-bolts are built into the walls, gripping bottom sills and both sets of transverse timbers. These filling culverts are separated from the emptying culverts by bulkheads near the centre of the length of the lock.

The emptying culverts are constructed in a similar manner, but have inside dimensions of 8 × 10½ feet.

The lock-gates are of wood, and constructed in the usual manner. There are five sets of gates, a lock and guard gate at each end, and an extra lock-gate at the lower end.

The gates and valves are operated by electric machines.

The time required to pass a vessel is: 50 seconds closing lower gates, 9 minutes filling lock-chamber, 50 seconds opening valves, and 50 seconds opening upper gates—11½ minutes in all. A vessel can be locked down in 10 minutes.

American Types of Movable Dams.

(From an article in *Engineering News*, Feb. 7, 1895, by Lieut. Hiram M. Chittenden, U. S. Army.)

"The conditions to be fulfilled by a successful automatic water-gate have been stated to be: 'That the pressure of the water, standing or flowing, shall furnish the power by which the dam is erected or removed; and 'that, notwithstanding the requisite solidity of the structure, it must be manageable with a slight water-power, for which a few men and a short time are required.' In other words, . . . it is required so to utilize this water-power in the operation of the gates as to dispense with the plant ordinarily used for that purpose."

"There have been many attempts to devise a gate which will satisfy these conditions; as, for example, the Desfontaines and Cuvinot drum wickets, the Krantz ponton wicket, the Brunot gate, and the Petididier counterpoise gate. But the only devices which as yet promise a satisfactory solution of the problem embody in direct or modified form the principle of what is known as the American bear-trap gate.

"The essential features of this gate are two leaves, built across the sluice which the gate is to close, and fastened by horizontal hinges to the bottom of the sluice at right angles to its axis. When the sluice is open the leaves lie in a horizontal position, the up-stream leaf overlapping the other a certain portion of its length. When the gate is up and the sluice is closed the form of the gate is that of a triangle, of which the three sides are the two leaves and the bottom of the sluice between the hinges, and of which the apices are the two hinges and the point where the leaves abut against each other. The space within the triangle is a chamber, which is filled with, or emptied of, water by means of valves or wickets under control of the operator of the gates.

"The operation of the gate is extremely simple. To raise it, the outlet from the chamber below is closed and the inlet above is opened. Water fills the chamber and presses against the lower surfaces of the leaves. Since the water has access also to the upper surface of the upper leaf, the pressure from below upon that leaf is neutralized. On the lower leaf there is no such counter-pressure, and the pressure on the lower surface therefore tends to raise the lower leaf and also the upper (or overlapping leaf) which rests upon it. This it will do in a properly proportioned structure, and will carry the gate to a height limited only by the dimensions of the leaves.

"To lower the gate, the water from above is cut off and the lower valves are opened, so as to empty the chamber in the pool below. The sustaining pressure on the lower leaf is thus withdrawn, and the pressure of the pool above is thrown upon the upper leaf. This pressure forces the leaves back into their horizontal position, leaving the sluice free for the passage of water.

"Two conditions are necessary for the successful operation of the old bear-trap dam. First, there must be such a proportion between the length of leaves and the distances between the hinges that, when the gate is being raised, there shall be a preponderance of the upward over the downward hydraulic forces acting on the leaves just as they are beginning to rise from a horizontal position, no matter how small the initial head may be; second, the angle between the leaves when the gate is at its full height must be such as not to prevent the leaves from moving freely upon each other when the gate starts to fall. It will be observed that in both these conditions we provide only for the case of initial movement. When the gate is once under way, either rising or falling, the preponderance of the moving over the retarding forces rapidly increases." Any system of valves which will satisfy the first condition will satisfy the second also.

The values of the two leaves corresponding to the maximum head obtainable are, assuming the base or distance between the lower hinged edges of the leaves as 1 (unity):

The maximum head is........................0.3320
The length of upper leaf is....................0.5239
The length of lower leaf is..... ....0.6821

The relative lengths of the two leaves may vary within considerable limits without sensibly affecting the head attainable.

There were many faults and defects in the design described. The gate built on what is known as the "Carro system" was an improvement by doing away with the overlap of the up-stream leaf, and consequent sliding friction between the leaves, and in hinging the leaves at the apex or crest. In lowering, this up-stream leaf slided along its lower extremity until both leaves lay flat in the same horizontal plane; a sliding strut rested on it. The main objection to this type of gate arises from the upper leaf having to slide against the water-pressure.

The Parker gate retains the two hinges of the "bear-trap," the Carro hinge at the apex, and constructs the upper leaf in two sections joined by a fourth hinge. The upper leaf thus folds in upon itself, so that when the gate is down the order of superposition is: (1) The lower section of the upper leaf; (2) the upper section of the upper leaf; (3) the lower leaf. This improvement eliminates all the defects of the bear-trap. There is no overlap at the apex; the head obtainable for the same length of sluice is over twice as great; there is no sliding friction; the gate can never be brought to a sudden stop when it approaches its full height, but comes to rest gently; the leaves, of necessity, rise uniformly; their shorter length greatly diminishes the strain upon them and correspondingly increases the economy of construction. In the angle between the sections of the upper leaf drift is liable to accumulate and prevent the complete lowering of the gate. To avoid this an open leaf, hinged to the apex of the gate and sliding on the bottom of the sluice, is introduced. As this leaf plays no part in the operation of the gate, but acts solely to fend off drift, it is called an idler.

"The next step in the development of the automatic gate is what is known as the Lang gate, after its inventor. It undertakes to make the idler an essential part of the gate. It replaces the upper section of the upper leaf by chains or rods, and the idler of the Parker gate by a solid leaf hinged at the apex and sliding on the lower section of the lower leaf. It is thus a combination of the Carro and Parker systems. Of this latest modification it cannot be said that it marks any advance. The reintroduction of sliding friction is a distinct step backward; . . . it clearly indicates that the Lang gate is inferior to the Parker in efficiency. The principal advantages of the Lang pattern are the doing away with the idler of the Parker gate, and the increased facility of access to the gate chamber."

The author of the above article recommends the substitution of some form of automatic gate for the old and common type of lock-gates for canals.

Just at present the Lang gate is brought into prominence by the fact that Mr. Isham Randolph, Chief Engineer of the Chicago Drainage Canal, contemplates using it in connection with this work.

Reclaiming Zuider Zee.

There are about 750 square miles of good argillaceous soil to be reclaimed. The work is expected to extend over a period of thirty-three years. The estimated cost is $78,750,000, and with interest compounded $131,250,000. It is expected that this land will be sold for the sum of $135,000,000.

The work of reclamation requires the construction of a dam twenty-five miles in length, requiring nine years for its completion. The foundation of the dam will consist of willow branches, loaded down with earth, sand, mud, and stone. The embankment will be 216 feet wide at the bottom, 17.8 feet high above sea-level. Oak piles will also be used to afford greater protection. The highest part of the bank will be only 6½ feet wide on the sea side, but on the southern side will be a plateau 56 feet wide.

West Side Trunk Sewer, Rochester, N. Y.

The main trunk of this sewer is 17,700 feet in length, and has lateral extensions aggregating 13,000 feet. The trunk sewer at the beginning is 5 feet in diameter (egg-shaped), and increases to 6, 6½, 8, 8½, 9, and 9½ at the outlet. The minimum grade is 1 in 500 for about one third its length; the remaining two thirds has grades of 1 in 150 and 1 in 200. There are 1980 feet of 9½-foot tunnel. The sizes pass from the egg-shaped to circular and thence to the horse-shoe cross-section. The outer shell is concrete rubble masonry to a point 1 foot above spring of arch, lined and arched with brick. The invert has one and two rings of brick imbedded in concrete to the bed-rock. This one-sided rock excavation is one of the peculiarities of the work, and is made available to lessen the cost of the mason-work lining. The smaller sizes have a two-ring brick arch and invert, except in rock, where there is a 1-ring invert. The 6½ feet circular and egg-shaped sewer and all the larger sizes have concrete foundations resting on 1-inch boards when not in rock. These have three rings in the arch, except on rock, where only one ring is invert. In solid-rock tunnel there is no lining except one ring of brick on the invert resting on a thin layer of concrete. For some distance from the outlet the sewer traverses a deep ravine with precipitous sides from 10 to 98 feet in height. The fall is so great in this ravine, in places 1 in 23, that five drops from 6 to 18 feet each have been designed and one shaft 112 feet deep. The drops are constructed with a curve, which is a parabola reversed in a circle to connect with the lower section. The 112-foot shaft is provided with an 8-foot sump. This shaft is only intended for temporary use.

Table XCVII.

COST OF LAYING WATER-MAINS IN WASHINGTON, D. C.

Size in Inches.	Number Lineal Feet.	Cost per Foot in Cents.		
		Material.	Labor.	Total.
3	3,017	26.42	21.84	48.26
4	8,671	29.57	20.10	40.67
6	51,908	51.42	23.22	74.64
12	6,473	108.10	33.26	141.36

The figures do not include cost of relaying pavements, which ranged from 6.29 cents per lineal foot for trap-rock to 54.41 cents for sheet asphalt, and as much as 60 cents for block asphalt. Relaying brick cost from 6.36 to about 9 cents, cobble from 9 to 10 cents, and macadam some 16 cents per lineal foot. The diameter of the pipe had little to do with the cost of relaying pavements.

WENTWORTH AVENUE SEWER, CHICAGO,

now under construction, is 10½ feet in diameter, circular in cross-section, and built with four concentric rings of brickwork. The depth of cutting required varies from 20 to 47 feet. The work of excavation, driving supporting piles on each side, building brickwork, and refilling over the completed culvert are carried on simultaneously. Steam shovels, pile-driver, and earth-conveyors are used. There is nothing especially novel in these machines except the conveyor. The excavation is carried on in two lifts. The foremost steam-shovel excavates to a depth of from 25 to 30 feet above the bottom of the sewer-level. This material is emptied in cars alongside and carried away. It is not used in the refilling. Following this is the pile-driver, which drives two rows of piles 16½ feet apart. The piles in each row are spaced about 3 feet centres. The piles are from 20 to 30 feet in length, are sawed off, and capped. The second steam-shovel is carried on these, and excavates to the full depth required for

the sewer two or three feet below its intended bottom. The material excavated by this shovel is dumped on a platform constituting a part of the conveyor frame. Upon this slides a series of scrapers attached to two endless chains which pass over a pair of sprocket-wheels at the ends of the conveyor frame. The material dumped on the platform is carried along on the scrapers. At the rear end of the conveyor the material falls on a cross-conveyor, which dumps it into the trench over the completed sewer. The conveyor chains are operated from a power-car. The contractors for this work are Wilson & Jackson. The design of the conveyor is due to Mr. Jackson, and it performs its work in a very satisfactory manner. The work is done under the direction of Mr. A. W. Cooke, Chief Engineer, Department of Sewers, and under the direct charge of Mr. N. D. Pound, Assistant Engineer.

New Maas Jetties.

The north jetty is 6560 feet in length, south jetty 7550 feet. These begin at high dikes 3 feet 3 inches above mean high water, and descend in an easy curve to 3 feet 3 inches above low water. The tidal range is 5 feet 7 inches.

The jetties are of mattress of fascine work covered with stone, one half ton per cubic yard of fascine work. The top is curved to 20 inches and paved with stone in large blocks. Fascines 4 to 6 inches diameter, placed 36 to 40 inches centres, laid crosswise, form the bottom layer; over this a layer of fascines at right angles, and spaced 36 inches centres. These longitudinal fascines are as long as practicable, and are spliced by short fascines or by interlocking for 5 or 6 feet at ends. The crossing joints are bound with withes on the outside and at alternate inside joints with tarred rope. This grillage is then covered over with loose layers or bundles of osier willow covering the longitudinal fascines. A second layer is then placed crosswise, and over it another longitudinal layer, forming the upper grillage, and over this another filling layer. An upper cross layer of fascines is then laid so that their intersections with the longitudinal fascines are vertically over those of the lower grillage. These intersections are bound with the same cords, so that the filling is compressed as tightly as possible between the grillages. The other intersections are bound with withes. The three layers form a mattress 16 to 24 inches in thickness. Hurdle-work on top holds the ballast in place. A mattress contains not less than 480 square feet. The largest were 50 feet in breadth and 490 to 590 feet in length. Above low water the fascines are placed separately by hand and loaded with stone.

The jetties are consolidated by oak piles, 30 to 40 feet in length, driven through all layers and into the bottom. The fore-shore berms and slopes surrounding the pier-head are further secured by three rows of oak piles framed together. As soon as a portion of the jetty was completed above low tide ballast stone was thrown in upon the base, berms, and sides to an amount of 56 to 64 tons per lineal yard of jetty. No stone less than 110 pounds in weight was used; above low water they were carefully laid and packed with smaller stones, spawls, and quarry refuse. Stones of one half to one ton were placed around fore-shore berm of the pier-head. No artificial stone was used. Heaviest natural stones handled were 1¼ tons.

Success is due to keeping the jetties low enough to allow storm waves of high tides to go over them, and the protection of the more exposed slopes by flat stones of good dimensions placed over each other like shingles on a roof, the lines of shingles being normal to the direction of the sea as it strikes the slopes of the jetties. The width of tidal stream is 2960 feet at the sea end of the jetties, and gradually decreasing at the other end to 740 feet.

The cost of the whole improvement was $2,500,000; annual cost of maintenance, $7200.

Duty of Water for Irrigation.

The duty of water is that quantity required to irrigate a certain area of land. It is usually expressed by stating the number of acres that a continuous flow of one cubic foot per second will irrigate. Thus, if a stream discharging 40 cu. ft.

of water per second is expended in irrigating 8000 acres of land, then its duty is equivalent to 200 acres, i.e., each cubic foot per second irrigates 200 acres. The duty varies from 85 to 2200 acres per cubic foot per second. Sometimes the duty is defined as the average depth of water over the whole land, and again as the number of cubic yards per acre.

The average duty in India is 200 acres, and in the United States not over 100 acres.

For a wheat crop in India, which is grown in the cold season, four waterings are quite sufficient, and few other crops require more—notably rice and sugar-cane, which are sometimes irrigated as often as 12 times. In the month of December and February 1 cubic foot of water per second will irrigate 4.57 acres of rough uncleaned ground previous to ploughing, 5.64 acres of a well-cleaned and level field of young wheat. A safe mean in India is 5 acres in 24 hours to be watered by 1 cubic foot per second. In general the soil is light. Watering at intervals of 50 days gives $5 \times 50 = 250$ acres. This water runs more than 300 miles from head of canal to field watered; 20 per cent is deducted for filtration, evaporation, etc., *en route.* Of these 260 acres 18 per cent is rice, 18 per cent sugar-cane, each requiring much water; 50 per cent wheat and barley, and the remainder of inferior crops.

The rainfall is principally in June, July, and August, and is about 40 inches a year. In Madras 6000 cubic yards of water are usually given to irrigate an acre of rice, equivalent to a depth of 3.7 inches. Wheat in a dry season requires five waterings—one to prepare the land for ploughing—10,500 cubic feet, and four for the standing crop of 8000 cubic feet, or in all 42,500 cubic feet. This gives an average depth of less than 1 foot over the entire area.

The Henares Canal in Spain gives 12 waterings, each 916 cubic yard, or an average depth of each watering 0.57 foot, or 6.8 feet for the twelve.

In Colorado the expenditure of water for a single irrigation is generally reckoned at about 12 inches in depth, and the number of irrigations from three to four, or in all 36 to 48 inches in depth.

The machines used to measure water in irrigation canals are generally known as modules or meters. The principal objects to be sought are: (1) that they should deliver a constant quantity of water with a varying depth or head of water in the supply channel; (2) that they should expend very little head in delivering the constant quantity; (3) that it should be so free from friction as not to be easily deranged, and that sand or silt in the water will not affect its working; (4) that it should be cheap, and so simple in construction that any ordinary mechanic should be able to make or repair one.

The common forms are like weirs, either discharging under a small but constant pressure, or the common knife-edge iron weir with overflow.

Subways for Rapid Transit.

In London, even along the busy streets, unless great depths necessitated tunnelling, the subways for rapid-transit traffic were constructed by the cut-and-cover system. The cross-sections of the completed tunnels are 25 or more feet, according to the number of tracks, and height from 16 to 20 or more feet. The side walls of these tunnels are in many cases of concrete, while the arch-ring is of brick masonry. At the crown the thickness varies from 5 rings, about 22 inches, to as much as 2 feet 7½ inches. The concrete side walls are 4 feet 3 inches thick, and 11 feet 7½ inches to springing-line. For the largest arches the arch-ring increases to as much as 12 rings at the skew-back. The method of construction is to build a timber platform of sufficient strength to carry the traffic across the street. As the rail is generally below the base of the house walls, these are underpinned; then trenches are dug along the curb-lines, in which the side walls are built. Between these and from under the timber platforms enough earth is excavated and removed to enable the roof arch to be constructed. Afterwards the remaining material or core is removed and the invert laid, and finally the street surface is restored to its original condition.

The Lobnitz Rock-cutters on the Danube.

(See *Eng. News*, Dec. 20, 1894.)

As now used for the removal of the Iron Gates of the River Danube, this machine has one instead of ten cutting bars, as the rapid current and local conditions make one cutter of great weight more efficient than a battery of cutters of various weights. The barge now in use is 100 feet long, 25 feet wide, and 7 feet deep, with a draught of 3 feet. The boiler is placed near the stern, and near it are the manœuvring winches for moving the boat. Near the centre is a tripod for handling the cutter. This latter may be worked through a square well, forward of the centre of the boat, or may be swung over the bow of the boat by tilting the tripod forward.

The action of the cutter is that of a pile-driver, the cutter replacing the falling monkey. The cutter is made of wrought iron or mild steel. It is 30 feet long and is square in section, the dimensions being 7 inches square at top, increasing to 13 inches square 10 feet from the point, and diminishing again to 11 inches square at the lower end. For a length of 4 feet from the bottom a hard steel core, 11 inches by 4 inches, is welded in along the centre line of the bar, with the point chisel-shaped, and the sides rounded off to 9 inches radius. The cutter point is hardened by heating to a cherry-red and plunging it into water. This steel blade, being protected on two sides by wrought iron, can be used up to the last few inches without breaking, and always keeps sharp. It weighs 8½ tons.

The boat is advanced after 20 inches in width of the rock has been broken away by the falling chisel, but the varying hardness of the rock prevents any accurate estimate of work accomplished.

The cutter is dropped about 16½ feet, at a rate varying from 50 to 100 blows per hour. The average amount of rock removed per blow is 2 cubic feet. The rock is very hard, and is always broken sufficiently small to be removed by dredging.

The drilling of bore-holes for blasting operations is carried out on two plans—the Ingersoll-Sergeant and the Fontane systems. The first is a scow which can be lifted from the water by four legs resting on the bottom and connected with hydraulic jacks fed by a Worthington pump. Along the side of this scow are ranged 4 Ingersoll steam-drills supplied by a boiler on the scow. In the Fontane system the drills are mounted on a platform resting upon two barges. The author says, that while there has been trouble with the Ingersoll system, it has on the whole given satisfaction; the Fontane barges are too light for the work to be done, and the system is not yet looked upon with satisfaction.

The quantity of work executed up to the end of the year 1893 is given in the following table:

Class of Work.	Cubic Metres.	Per Cent of Estimated Total.
Blasting under water (with 67,586 cubic metres removed)	122,758	77.77
Blasting at the Iron Gate	307,223	90.60
Stone deposited	345,978	68.63
Various stone depositing	133,065	20.05
	Square Metres.	
Stone revetment	9,150	....
Facing up dams	56,340	....

Adhesion of Cement Mortar to Brick.

The following table shows the results of experiments made by M. Félix de Walques. (See *Engineering*, Nov. 16, 1894, and *Engineering News*, Jan. 10, 1895.)

The bricks were soaked in water before being cemented together, and were tested at the ages of 2, 4, 8½, 15, and 44 days. The results given at the ages of 2 and 4 days were very irregular, and also in a lesser degree at 8½ days. Each experiment was repeated four times, and the mean result is given in the table, which does not include the 2- and 4-day tests. It will be noticed that the smooth-pressed bricks gave better hold to the mortar than the rougher varieties.

All results are given in pounds per square inch.

TABLE XCVIII.

Cement.	Sand.	Hand-made clamp-burnt stocks. Days.			Hand-made clamp-burnt facing bricks Days.			Hand-made clamp-burnt clinkers, place bricks. Days.			Machine-made repressed-h'rd burnt brick. Days.			Machine-made facing bricks. Days.		
		8½.	15.	44.	8½.	15.	44.	8½.	15.	44.	8½.	15.	44	8½.	15.	44.
Slow-setting Portland, 1 part	½	17.7	54.5	37.1	17.1	26.8	36.0	14.2	35.2	32.7	36.0	34.6	36.3	34.7	21.3	16.2
	1	8.2	35.3	42.7	4.5	20.9	20.4	15.3	25.9	....	23.4	27.5	42.7	12.8	16.7	15.2
	2	10.9	17.8	27.0	7.5	11.2	18.4	10.5	14.2	19.2	13.5	29.7	25.7	11.9	13.4	15.3
Quick-setting Portland, 1 part	1½	16.77	54.3	68.7	13.3	32.8	43.0	19.6	50.0	54.4	49.9	44.2	91.7 ab've	13.4	17.1	20.4
	1	11.5	44.4	35.5	11.0	18.1	23.1	14.2	33.6	38.9	32.3	43.8	106.7	9.9	12.8	23.6
	2	11.0	19.2	21.2	4.5	18.1	24.2	4.9	26.1	37.5	20.3	31.3	33.9	3.3	10.7	13.7
Slow-setting slag cement, 1 part	½	7.5	42.0	38.8	10.4	14.0	29.2	14.0	28.6	33.3	20.6	34.1	45.9	10.8	16.7	19.6
	1	10.4	37.0	29.6	7.3	18.1	27.0	23.0	33.4	26.9	30.0	32.7	45.3	10.3	18.6	13.0
	2	7.1	19.2	23.4	2.9	18.1	11.8	4.5	26.1	20.1	18.1	19.5	29.2	8.6	10.6	8.7
Medium-setting slag cement, 1 part	½	9.7	39.6	39.1	15.5	28.4	29.9	26.3	38.0	47.1	17.9	33.4	41.7	9.4	23.6	16.3
	1	10.9	34.8	26.6	5.7	12.9	20.3	19.2	25.0	26.7	19.8	26.5	32.0	9.8	16.0	27.5
	2	4.9	28.9	22.4	8.2	18.3	22.0	10.5	17.8	13.7	21.7	22.0	25.9	6.3	14.4	8.0
Slow-setting Portland, 1 part	½	20.9	37.7	31.0	16.0	82.4	36.0	17.7	38.4	38.4	18.6	46.9	42.3	11.1	19.0	20.3
	1	7.3	27.7	19.9	10.3	15.6	21.7	17.8	19.0	21.3	22.8	23.9	28.4	16.3	14.9	21.3
	2	7.3	11.9	18.9	6.9	12.9	13.3	12.5	15.0	18.8	10.1	8.4	16.7	11.3	6.1	9.4

WHITAKER PORTLAND CEMENT CO.

The manufacturers of Portland cement in this country add, as a rule, a certain amount of limestone to the natural rock before burning. The Coplay, Pa., rock does not contain a sufficient proportion of lime to make Portland cement which will meet the requirements of the present day, though formerly it was obtained from the natural rock. The quarries of the Whitaker Cement Co., Easton, Pa., supply an argillaceous limestone, practically free from magnesia, and containing the right proportion of lime for a triple-silicate Portland cement. The following table shows the average analysis of the material, which is very uniform, in different parts of the quarry:

	Per Cent.
Silica	14.44
Alumina and sesquioxide of iron	5.91
Carbonate of lime	75.17
Carbonate of magnesia	.77

A full description of the operations of crushing, grinding, burning, etc., can be found in *Engineering News*, Jan. 10, 1895, from which the foregoing and following facts are abstracted:

The company guarantees that 70 per cent of the finished product will pass through a No. 200 sieve (40,000 holes per square inch). The average quality of the product is so uniform that it is seldom necessary to mix the contents of the storage bins in order to produce a cement meeting special requirements.

Samples of the cement and powdered rock are taken daily.

Briquettes of neat cement show an average tensile strength of 625 pounds per square inch in 7 days, 875 pounds in 28 days, and 913 pounds in 6 months.

Briquettes of 1 cement and 3 sand show an average of 191 pounds in 7 days, 295 pounds in 28 days, and 343 pounds in 6 months.

The cement is guaranteed to show in 7 days (1 in air and 6 in water) a minimum tensile strain of 400 pounds per square inch for neat briquettes, and 125 pounds for briquettes of 1 cement to 3 sand, standard crushed quartz being used. Boiling tests are also made, as a matter of safety, to see if there is any excess of free lime.

The analyses of the finished product average as follows:

	Per Cent.
Silica	21.70
Alumina and sesquioxide of iron	8.88
Lime	63.27
Magnesia	.55

The works have a capacity of 400 barrels per day, or an average of 100 barrels from each of the four kilns.

The cement, which is known as the Alpha Portland cement, is largely used in buildings and architectural works, for floors and pavements, etc.

In this connection the following results of tests made by Messrs. Hazlehurst & Huchel to determine the relative cost of Rosendale and Portland cement, with a view of using the cheapest in building the Odd Fellows' Temple, Philadelphia, Pa., are tabulated.

Tensile strength is given in pounds per square inch, and is the average of five briquettes of 1 square inch cross-section.

Age—Days.	Portland Cement. 1 to 3 River Sand.	Portland Cement. 1 to 4 River Sand.	Rosendale Cement. 1 to 1 Bar Sand.	Rosendale Cement. 1 to 2* Bar Sand.
7	160	105	90	45
14	200	135	109	56
21	230	146	131	78
28	231	165	145	101

The Rosendale was from one of the largest and most reliable manufacturers in Utica County, N. Y., and the Portland was manufactured at Coplay, Pa. The Portland was selected as the cheapest of the two.

Contraction and Expansion of Masonry.

As there is but little information available as regards the effects on masonry of contraction and expansion, the following facts given in *Engineering News*, Oct. 18, 1894, will be interesting. A masonry dam 460 feet in length arched with a radius 420 feet in length on the side exposed to the sun moved to and fro $\frac{7}{8}$ inch in the course of the year, on the other side only $\frac{1}{8}$ inch, the crest expanding $\frac{1}{8000}$ of its length, or $\frac{5}{8}$ inch.

The Temperature.—No harm may be caused in a dam thus arched by such movements, as a curved dam can readily adjust itself to them without cracking, but not so with a straight dam, and serious injury may result. Taking Adie's coefficient of contraction and reduction in temperature of 10° C., the contraction will be $\frac{1}{10000}$ of its length. The modulus of elasticity for stone varies between 1,400,000 and 2,800,000 pounds per square inch, while that of mortar, according to Hartig, is higher, and if the temperature is lowered 10° C., and the masonry is not free to contract, a tension varying from 140 to 280 pounds per square inch may set up, which is greater than mortar can stand, resulting in cracks or fissures.

The great length of dams as compared with their thickness and the high compressive strength of building materials will prevent fracture from expansion, the dam forming a compressed beam.

Many dams have developed cracks in cold weather. In a dam 1346 feet in length, 100 feet in height, with a temperature ranging from $-10°$ C. to $-20°$ C., seven vertical cracks appeared, about 100 feet apart, widest at top, and vanishing 87 feet below normal water-level, the water surface being at the time 10 feet 8 inches below normal. The aggregate breadth of these cracks was $2\frac{1}{4}$ inches; they either wholly or in great part closed as the temperature increased.

A long quay wall at Bremen showed cracks from $\frac{1}{8}$ to $\frac{1}{4}$ inch in winter, closing to fine hair-cracks during summer.

A concrete coping 285.66 feet in length, divided into four straight reaches connected by curves with radii of 20 feet, on top of a reservoir wall showed 15 cracks extending across the wall at irregular intervals. These cracks were from $\frac{1}{16}$ to $\frac{3}{16}$ inch each, and they were attributed to contraction from change of temperature. The cement used was American Portland, Atlas Brand.

TUNNELS.

BALTIMORE AND OHIO RAILWAY TUNNEL, HARPER'S FERRY.

This tunnel is for a double track. Its length is 812 feet, 28 feet in width at springing, which is 10 feet 6 inches above subgrade, with semicircular arch of 14 feet radius. A top heading was excavated 8 × 25 feet, the full top section of the tunnel, and it was kept 15 feet in advance of the bench. No timbering was required. Rate of progress at east end 18 feet and at west end 19 feet per week. The total amount of excavation was 50,000 cubic yards, which made into an embankment showed an increase in volume of 80 per cent.

TUNNEL, COLORADO MIDLAND RAILWAY.

This tunnel, recently completed, is 9894.7 feet in length, for single track, with cross-section 15 feet in width and 21 feet in height. The amount of excavation per lineal foot was 10.19 cubic yards, and where enlarged for timbering 13.79 cubic yards. The lining was similar to that shown in Fig. 292, except that the vertical posts were in sets of three, 8 inches apart in each set, and 18 inches between the sets. Sills, posts, and arch segments were 12 × 12inches, and lagging 4 × 6 inches on outside of posts, 6 × 6 inches over arch segments. It was necessary to double the lining in some places.

The heading was 7 feet high and full width of tunnel. In driving the heading two sets of holes were drilled. The first set of eight holes was drilled in two rows from top to bottom. These holes were 2 feet apart on the face of the heading and converged towards the axis of the tunnel, depth of holes, 12 feet, so that the action of the blast was to drive out a V-shaped opening in the rock. The holes of the second set were drilled at the sides and parallel to the axis of the tunnel, by means of which the remaining rock was blown into the V-shaped opening. The bench below was excavated in a similar manner. Progress was as follows:

Driving the heading	1,118	days.
Average daily progress, two headings	8.4	feet.
Greatest progress, 1 month	337.0	"
Average daily progress 1 month, 31 days	10.87	"
Greatest progress, 1 month, one end, 28 days	202.5	"
Average	7.23	"
Greatest monthly progress on bench	218.00	"
Daily average on bench	7.79	"

The material tunnelled through was gray granite. Owing to disintegration on exposure, fissures, faults, etc., and large cavities filled with liquid sand, much trouble and inconvenience were experienced, and about 78 per cent had to be timbered.

The elevation of grade at the east end of tunnel was 10,810.72 feet above sea-level and at west end 10,943.18 feet. The grade was continuous and ascending from

east to west through the 132.36 feet difference of level at a uniform rate of 1.41 per cent.

Good ventilation was easily maintained owing to the continuous ascent and great difference of level between the two ends.

Machinery employed at the west end: Three 100-H.P. boilers, two 20 × 24-inch Ingersoll compressors; one 20 × 24 inch Norwalk compressors; one 20-H.P. engine used to drive a No. 6 Baker blower, forcing fresh air into the tunnel through 14-inch pipes; one No. 7 and one No. 9 Cameron pump, and a Dean Duplex pump, with 14-inch steam cylinders, 10-inch water cylinders, and 10-inch stroke. These were required as the water followed the excavation from this end.

At the east end: Three 80-H.P. boilers, two 20 × 24 inch Ingersoll compressors, and a 10 and 20 H.P. engine, driving dynamos and blower. No pumps for water were required, as there was good natural drainage at this end. Four 3½-inch Ingersoll eclipse drills were used in each heading, and two on the bench, that is, 6 in all at each end. A small traction engine was constructed to run on a track with 20-inch gauge, and used to haul the material. Using coke for fuel in these engines no inconvenience was caused to the men in the tunnel. Estimate of cost:

9,393.66 lin. ft. @ $62.50	$587,103.75
Enlargement for timbering 32,575 cu. yds. @ $2.50	81,437.50
Cost of timber	81,600.00
Cost of labor on timbering, 2,723,000 ft. B. M. @ $12	32,676.00
Total	$782,817.25

DRAINAGE-WORKS OF THE ONTARIO SILVER MINE, UTAH.

(See *Engineering News*, Nov. 29, 1894.)

The clear section of the tunnel is 8′ 8″ high, 4′ 6″ wide at the top, and 5′ 6″ to 6′ at the bottom. The floor of the tram track is of 3″ plank, resting on 6″ × 10″ sills. Gauge of track 18 inches. The drainage channel for the water has an average width of 5½ feet in the clear and an average of 1½ feet deep below the floor of the tramway. A timber bent consisted of the sill, two batter posts, and the cap, all 10″ × 12″ or 12″ × 12″. The section made for timbering usually averaged 11 feet in height and 9 feet wide, a good layer of lagging being placed behind the posts, and poling-boards driven over the caps. While this was the standard bent, no set rules could be followed, and the method of timbering was varied to suit the material excavated. In some places where the pressure overhead was particularly great the roof was torn down so as to put in a second bent on the top of the first, using an inverted V bent, with the feet of the batter posts on the cap of the first bent. The distance apart of the bents was usually 4 to 5 feet, with bracing between them. In soft materials the timbering was kept close up to the face and 8 feet by 3 × 6 inches poling-boards were driven over the caps of each forward bent and driven forward with an inclination upward so as to hold up the roof until the next bent could be placed.

Immense quantities of water were encountered, which, rushing in from fissures in the rock, flooded the tunnel and caused much damage to the lining. One body of water was tapped, discharging 13,000 gallons per minute. After waiting for five weeks for this to exhaust itself, rubber suits—costing $13 each—were purchased. Dressed in these the men waded in against the strong current and raised the track 18 inches, to get it above the water. All this caused a delay of 70 days. Side drifts curving out from the main tunnel kept about 50 feet from it, and returning into it well to the front, were driven. These served to divert the flow of water from the main tunnel. It was found that wood was the poorest backing that could be placed behind the lagging and over the poling-

boards, but old clothes, sacks, and similar materials answered the purpose admirably, as they caused puddling of the material, were elastic, and gave the ground room to swell. The enormous pressure on some of the timbers reduced them from 12″ to 4″ in thickness.

"Swelling ground" sometimes forced the timbers in from all sides, leaving an opening not large enough to crawl through. There were not only torrents of water and falling rocks to be watched for, but timbers were cracking with a great noise as the swelling ground gradually forced them in. With all the dangers encountered in the six years of work, but one man was killed while working in the tunnel. For this excellent record much credit is due to Mr. John H. Keetly, the superintendent of construction. Ample boiler capacity and engine power were provided. A No. 6 Root blower (run at 150 revolutions per minute) was used as an exhaustion to suck foul air out of the tunnel after blasts, through a 20-inch pipe made of No. 6 galvanized sheet iron. A No. 4 Burleigh and a 14 × 22 inch Rand air-compressor were used.

The compressed air was conveyed into the tunnel through a 6-inch kalsomined pipe. This supplied fresh air to the men after use in the drills. In drilling at the face a horizontal bar was used, extending across the tunnel and wedged at the ends, with two 3¼-inch Ingersoll drills mounted on it. The blasting was done in sections, beginning at the bottom, and was all done by fuse, as battery blasting gave too much shock and did not suit the work. For each blast 8 to 16 holes 1¼ inches in diameter were drilled 3 to 5 feet deep, and each was loaded with 1½ to 5 sticks of dynamite, making a total charge of 40 to 50 pounds. Some rock was so extremely hard that steel would stand but a few minutes, and 20 feet per week was good progress.

Eight men constituted a shift at the face, four at the drills, and four muckers, who loaded the tram-cars and also took them out to the dump. These cars held 20 cubic feet each (1500 to 1700 lbs.), and were made into trains of 10 to 15 cars, which were hauled by one mule. Other gangs did the timbering, laid the tracks, etc. Altogether it took 30 to 50 men to keep the work going day and night. In good ground 10 to 15 feet per day was the average progress, but for weeks at a time no headway whatever was made. Four single delays aggregated 286 days. The total stoppage and delays were about 1½ years. The greatest progress in any one month was 430 feet. The greatest in one day, 21 feet. The time from commencement to completion was 6 years, 8 months, and 12 days. The line of the tunnel was located by triangulation, but the centre line was run over to insure its correct alignment. The total amount expended up to December 31, 1893, was $384,064, or an average of $27.75 per lineal foot. Total distance then completed 13,840 lineal feet. The total cost will probably be $425,000. The average flow of water during the time of construction was from 6000 to 8000 gallons per minute.

Street Railways.

The following facts and tables are taken from *Engineering*, Jan. 4, 1895. Beginning with a single line, constructed in New York about 1850, American

TABLE XCIX.

RATIO OF STREET-RAILWAY MILEAGE TO THE POPULATION OF SEVEN AMERICAN CITIES, IN THE YEAR 1893.

Name of City.	Population.	Track Miles.	Ratio of Mileage to Population.
Seattle	60,000	102	1 to 588
Denver	100,000	278	1 " 720
San Francisco	297,000	244	1 " 1,221
Boston	446,500	279	1 " 1,600
Baltimore	434,100	222	1 " 1,955
Chicago	1,098,500	513	1 " 2,141
New York	1,513,500	294	1 " 5,180

street railroads show a practically unbroken record of financial success. Only six or eight lines were built prior to 1855, about 30 in the next five years, over 80 in the succeeding decade, and so on in rapidly increasing ratio.

To the comparative small mileage of street railways of New York should really be added the great system of elevated railways, which run over more than 50 miles of its principal thoroughfares, carrying more than 221 million passengers annually.

TABLE C.

RATIO OF STREET-RAILWAY MILEAGE TO THE POPULATION OF FIVE ENGLISH CITIES, IN THE YEAR 1891.

Name of City.	Population.	Miles of Tramway.	Ratio.
Northampton	70.872	6.0	1 to 11,812
Blackburn	120,064	8.5	1 " 14,125
Leeds	367,566	23.0	1 " 15,978
Liverpool	517,980	61.5	1 " 8,422
London	5,633,806	250.0	1 " 22,523

In 1873 the Hallidie cable system was first introduced in San Francisco, and its pre-eminent value where heavy grades had to be encountered was fully demonstrated. Within the next twelve years important lines in San Francisco, Chicago, New York, and Philadelphia were equipped with cable-traction plant.

In 1883 the first electric line actually doing business was opened at the Chicago Exposition, and at the close of 1889 the entire success of the electric system had been demonstrated beyond question.

The following table shows some estimates of cost and efficiency:

COMPARISON OF COST AND EFFICIENCY OF CABLE AND ELECTRIC STREET-CAR LINES.

	£	$
Cost per track mile of cable and conduit..	10,000 to 30,000	48,400 to 145,200
" " " " " electrical conductors	500 " 2,000	2,420 " 9,680
" " " " " complete cable equipment	18,000 " 50,000	87,120 " 242,000
" " " " " complete electrical equipment	1,500 " 7,000	7,260 " 33,880

Efficiency.	Cable.	Electric.
Average effective horse-power applied to axle of each car on the line	3 to 5	4 to 5
Average indicated horse-power at engine per car on the line	4 " 10	6 " 15
Friction load in per cent of total load	50 " 65	40 " 60
Coal consumption per car, mile pounds	5 " 8	5 " 10

The path to the introduction of electric traction in America was undoubtedly smoothed by the valuable results attained by the cable system. Formerly bitter contention existed between the adherents of the two systems. It may be fairly said to-day that they do not compete, and that each has found its peculiar and appropriate place.

For great and constant passenger traffic, at stated speeds, in broad and straight thoroughfares, and where the conditions are such as to induce the investment of large capital upon ordinary commercial terms, the cable system has no equal, and the same is true where long and steep gradients are encountered. In Chicago, New York, and San Francisco the cable system is at its best.

In smaller towns, where the traffic is not so great, where curves and branches are of constant occurrence, where suburban routes are in question, or where the cost of road-bed and power-plant must be kept within reasonable bounds, the electric system found a field that the cable system could never satisfactorily fill.

The following is abstracted from the *Engineering Magazine :*

"The finest example of modern electric street-railway engineering, in this country, or in the world for that matter, is undoubtedly that of the Philadelphia Traction Company.

The company operates 119 miles single-track road by electricity, 34 miles by cable, and 25 by horse-power, and an extension is contemplated to a total of 800 miles operated by electricity.

There are four power-houses, with a capacity of 11,550 H.P. and ultimate of 19,800 H.P. The equipment of one power-station is as follows: Five Westinghouse vertical compound engines, each coupled directly to Westinghouse 4-pole generators of 600 H.P. each. To these are to be shortly added four immense railway generators, 1500 H.P. capacity each, driven direct by Wetherill-Corliss twin-tandem compound engines of equivalent capacity. These generators weigh, exclusive of shaft, 130,000 pounds, make 80 revolutions per minute, and give 2250 amperes at 500 volts. Length of each detached section of trolley wire varies from 1000 to 4000 feet. From 3 to 5 feeder connections are made with each section. A complete system of return feeders is installed in the conduits and connected with the rails at every manhole. One of the longest is 16,000 feet, having a cross-section of 700,000 circular mils. The return feeders are of twisted tinned copper, 1,000,000 circular mils in cross-section. There are 52 miles of return-feeder cable. The conduits are of tough iron shell, lined with concrete to about 3 in. in diameter, laid in bunches with about 3 in. of concrete between them, and 2 feet below the surface. Manholes are from 200 to 500 feet apart. Length of single conduit aggregates 800 miles. Street poles are of wrought-iron tubing, 28 feet in length, of varying thicknesses, and set 5½ feet deep in concrete; there are 15,000 poles in use. The trolley wires are of No. 0 hard-drawn copper; guard-wires of No. 6 silicon-bronze wire, supported 18 inches above trolley wire and 3 feet apart. The track joints are lace-bonded with No. 0 tinned copper wire, fastened with channel-pins and cross-bonded every 50 to 90 feet. The joint next to manholes is tied to return feeder.

There are 400 cars in use, and these will be doubled in the near future. The average round-trip speed is 12 miles per hour; each car travels 130 miles per day. The station pressure is 515 volts; average current per car 10 amperes, or an equivalent of 7 H.P.

ELEVATED RAILWAYS IN CITIES.

The elevated railway has become an important and almost necessary mode of rapid transit in large cities. It may be defined as a special design of iron viaduct adapted to construction along streets, with a minimum obstruction to the use of streets for ordinary wheel traffic. It consists essentially of vertical steel columns, two to each bent, these columns being set along the curb-lines, one on each side of the street, the bents being placed from 40 to 100 feet centres. Strong cross-girders, commonly solid-built beams, span the street and rest on the columns. Longitudinal stringers are supported by these transverse girders; for a double-track railway there are four stringers, or in general two stringers for each track. The stringers vary in length from 40 to 100 ft., and may be either solid-built beams, or open lattice-work of angle-irons riveted together. The clear height above the street-level is regulated by law—commonly from 18 to 20 feet. Cross-ties are placed over the stringers, and to these the rails are spiked. The only lateral bracing permitted is between the stringers, as the spaces between the columns both transversely and longitudinally must be unobstructed and open for the passage of vehicles. The cross-girders and columns are connected by stiffening brackets or arched knees, and the stringers and columns are connected by short diagonal struts, giving additional transverse and longitudinal stiffness. In very wide streets the columns may be near the centre line, leaving passage way on both sides as well as beneath the structure. The girders,

columns, and stringers are usually of steel. The construction in all respects is similar to that of any iron trestle. The columns usually rest on concrete or masonry pedestals, the upper surfaces of which are kept a little below the street surface. The area of base required is determined in the usual manner, depending upon the total load on each column and the character of the material forming the foundation-bed. In Chicago a pit is excavated from 8 to 10 ft. in depth and 8 to 10 ft. square. In this is placed a layer of concrete 1½ to 2 ft. in thickness, a similar layer of less area on the bottom one, and so on in smaller layers until near the street surface, the offsets on the layers being regulated so that the area of the top surface will be from 4 to 5 ft. square, and upon this the column rests, and is anchored by 2 or 4 bolts built into the concrete mass. The concrete is composed commonly of 1 Portland cement (Empire brand), 3 sand, and 6 broken stone.

As seen in a report in the *Engineering News* of Oct. 18, 1894, made by Mr. Parsons on the subways for rapid transit in European countries, the following conclusions were reached:

(1) That an underground railway operated by steam, even with the most approved system of mechanical ventilation, would be intolerable to the people of New York.

(2) That a railway with a steady and frequent service can be operated successfully and economically by electricity.

(3) That an underground railway operated by electricity has a comfortable atmosphere, and that it can be arranged so as to avoid great changes in temperature.

(4) The advice and experience of foreign engineers lean toward keeping the rail as close to the surface as possible, and that excavating from the surface is cheaper and safer than tunneling; but

(5) If the conditions demand, a deep tunnel can be constructed, for which the circular form is best.

(6) That an underground road can be so designed as to be attractive in appearance.

(7) That the work can be carried on through a busy street without endangering the houses and without seriously impeding travel.

BROADWAY CABLE ROAD, N. Y. CITY.

The cables are 1¼ inches in diameter. Those on the lower section are 4 miles long. At a speed of C miles an hour and 40 seconds headway, as many as 60 cars may sometimes be hauled at one time. A 1¼-inch cable would doubtless have been strong enough, cheaper, and lighter, but would have stretched 50 per cent more under varying loads, giving a more irregular motion to the cars and increasing the travel of the tension weights and the wear on the cable drivers. The large cables weigh 40 tons each.

TYPES OF RAILS FOR STREET-RAILWAY TRACKS.

(Extracts from a report of a committee of experts.)

Port Huron, Mich.—All street-car tracks are laid with a 45-lb. rail, ordinary section, laid on a 5-in. by 5-in. longitudinal stringer resting on ties spaced 3 feet centres. The space on the outside of the rail between head and flange is filled with an 2-in. by 2-in. pine strip against which the pavement is laid, the top flush with the top of the rail. On the inner side of the rail the space is filled with a 2-in. by 2-in. oak strip, set about 1 inch below the top of the rail, against which the pavement is laid, and crowning to the centre between the rails to the level of the tops of the rail. When laid concurrently with new pavements no obstruction is caused to the free use of vehicles.

Detroit, Mich.—The 77½-lb. grooved rail has been laid in parts on several avenues, in conjunction with the paving of these streets and avenues with asphalt, brick, granite and cedar blocks, and without special care to fit the paving close against the web of the rail. The pavement is laid close against the head or thread of the rail, and enclosed space filled with cement grouting. The rail is 6¼ inches in height and 4 inches wide, laid on tie-plates spiked to white oak ties, which are laid about 3 feet centres. The rail is connected every 8 feet with cross-rods 1 in. by 1½ in. in cross-section, to preserve the gauge of the track, which is very important with this section of rail. Little obstruction is offered to the free use of the streets for vehicles.

Toledo, O.—The rails used in this city for street-car lines are the common flat-top rail with a train 1½ inches wide. The pavement is generally laid even with the top of the rail, which forms a groove on the tram of the rail 1½ inches wide and 1 inch deep, which makes it very objectionable for public traffic.

Columbus, O.—All the street car tracks in this city are laid with the flat-top rail with narrow tram, except a short piece of grooved girder-rail 9 inches in height and weighing 90 lbs. per yard. The flat-top rail generally in use is a girder section, weighing 46 lbs. per yard, laid on chairs to elevate the rail above the pavement foundation. This section is too light to stand up under electric-motor cars, and is being replaced by a 70-lb. rail of the same section, except that it is 6¼ inches deep. An ordinance has recently been passed by the city council authorizing the use of a T rail, the space on the outside of the rail to be paved closely with paving brick.

Springfield, O.—In this city all the street-car lines are laid with the T rail, except a few blocks of the old flat rail originally constructed for horse-car lines. The ordinary railway section is used, weighing 50 lbs. per yard. One street which is now being paved simultaneously with the laying of the car track is to have the electric-welded T rail known as the Johnson Street Railway T rail weighing 62 lbs. per yard. The rail tracks are laid in either the paved or dirt streets. When laid in brick pavements the bricks are laid close to the head of the rail on the outside of the track and level with the top of the rail. Between the rails they are ½ inch below the top of the rail and crown to the centre of the track level with the top of the rail. On dirt and gravel streets the space between the rails and along the outside of the rails is filled with dirt and gravel, the same as the rest of the street. There is no obstruction to traffic.

Chicago, Ill.—All the street-car tracks in this city are laid with the girder-rail 5 to 6 inches high, with a 3-inch wheel tram. This width of train is adopted to prevent the wheels of heavily loaded wagons from cutting a rut along the inside of the rail. These rails are laid on cross-ties, longitudinal stringers, or cast-iron yokes when used in the cable tracks. The committee reported that only two sections of rails should be permitted in our wide-paved streets, namely, the street-railway T rail and the full-grooved or English-groove girder-rail. The T rail should not be less than 60 to 70 lbs. per yard, 5½ inches high, with the paving brick or blocks shaped to fit close to the web of the rail and ½ inch below the top of the rail on the inside, and level with the top of the rail on the outside. If the grooved rail is used it should weigh not less than 70 to 80 lbs. per yard, and the paving should be fitted close around the head and web of the rail.

The following is taken from *Engineering News* of Oct. 25, 1894, and selected from a list of 26 cities using the T rail. Modern street-railway construction and street paving imply a broken-stone, concrete, or other solid foundation, a high girder or T rail, and a brick, asphalt, or granite surface to the streets in the larger cities, and cedar blocks, cobble, or macadam in the smaller ones.

Asphalt or macadam can be paved as easily to a T rail as to any other. They should be laid flush, and room should be made for the flange by running a railway freight car over the track, or by some similar apparatus, before it is opened for traffic. Whether it is more expensive to chip the corners of granite or medina

blocks, or to leave them intact a short distance from the head of the rail and to fill the space thus made with asphalt, creosoted wood, or concrete, is open to question; but in either case a first-class job can be made.

The following table needs no explanation :

TABLE CI.

Height of Rail.	Foundation.	Pavement.	
		Inside.	Outside.
3½-inch rail......	Ties 3 ft. centres and steel chairs........	Brick.......	8 in. brick, then asphalt
1½ to 9 inch rail	Largely 6″ rail on stringers, and 9″ rail on ties with tie plates	Granite block	" " " "
6 " "	Ties 2 ft. centres; broken-stone ballast...............	Brick.......	Brick
4 " "	Cable, yokes 4 ft. apart.............	Asphalt with stone toothing.........	Same
8½ to 6 " "	6-inch rail, concrete under and between ties	Stone blocks and asphalt;	Stone blocks
6 " "	Spiked to ties........	Brick.......	Brick
4¾ " "	Ties, chairs and stringers................	Brick, gravel, cypress, bois d'arc, rock	
5½ " "	Ties, 2 ft. centres....	Brick, cedar	
4 " "	Cedar ties, gravel, and ballast............	Granite......	Macadam
4½ to 4¾ " "	Chairs and ties.......	Half block... " cobble...	Same
6 " "	Three braced chairs to each rail...........		
4 " "	Chairs and stringers..		
4 and 4½ " "	Stringers, ties, and chairs............	Granite and cobble......	Granite and Macadam
6 to 7 " "	Spiked to ties; 2 ft. centres............	Belgn. blocks for 7-inch rail; cobble for 6-inch...	Belgian blocks
4½ " "	Steel chairs, ties bedded in gravel, then inch board and block paving......	Cedar blocks.	Cedar blocks
4¾ " "	On broken stone.....	Macadam....	Macadam

The filling between the rail and pavement varied : in some cases none at all was used, in others any of the following materials : Oak strip, gravel, etc.; hard wood, covered with coal-tar; oak strip, bevelled to fit; block, cut or notched to fit; concrete; sand, with brick or wooden strip ; pine or oak strip ; oak strip inside, pine strip outside ; sand ; asphalt and brick; and so on.

RAIL JOINTS WITH BASE SUPPORT.

In order to prevent deflection of the rail ends, various designs of angle bars have been proposed and discussed. The main points to be considered are stiff-

ness and elasticity. In order to secure stiffness, the moment of inertia of the section should be as great as possible. This requires that the web should be made as thin as is consistent with strength, and that the metal should be concentrated in the head and base, as far as practicable from the neutral axis. As to elasticity, it is stated on good authority that the metal in angle-plates is inferior to that used in the rails, as seen in the following table, taken from a number of tests made at Scranton, Pa., in 1894:

	Tensile Strength. Pounds.	Elastic Limit. Pounds.	Behavior under Impact.
Rail steel........	120,000	60,000	Stood 2000 lbs. falling 20 ft.
Angle-plate steel.	57,000	30,000	Broke under 2000 lbs. falling 6 ft.

The moment of resistance at the elastic limit is the measure of the ability of the joint to stand up in the track under impact or steady loads without taking a set. This moment is increased in the manner above described.

In addition to the above the tendency is to design the angle-bars so as to give a base support; that is, to extend the angle-plates under the rail. Several forms of joints embodying this principle of construction have been suggested. The Heath rail-joint consists of a plate 11x24 inches and $\frac{3}{8}$ in. thick, bent to form an angle-bar and base-plate, extending under the full width of the rail base, the ends of the plate resting upon the joint ties, while the middle of the plate is dished so as to form a pocket, which stiffens the joint under the rail ends. On the inner side of the rail is a plain splice-bar or fish-plate, and four ordinary track-bolts pass through the bars and rails. The combined angle-bar and base-plate is made of $\frac{3}{8}$-in. tank steel, of low carbon (0.1 per cent), and is heated in oil ready for being shaped under a steam-hammer, and then punched for the bolts, a double punching machine being used. The joint has been applied, with satisfactory results, to old rails, and is in use on over 50 railways. The cost is said to be from 57 cents to $1.15, according to size of rail.

The continuous rail-joint consists of two angle-bars, with wide flanges bent round to fit under the rail and form the base support, each plate extending under approximately half the width of the rail-base. The ends of the plates rest upon the joint ties, and the edges are slotted for the spikes. For 67-pound rails the plates are $\frac{9}{16}$ in. thick in the vertical web, $\frac{5}{8}$ in. in the flange, and $\frac{7}{16}$ in. in the base. The bars are made of low-carbon steel (0.1 per cent). They may be made of any desired length, and are secured by four or six ordinary track-bolts, but shorter bars and four bolts are preferred. This joint is in use on some 50 railways, and has been used under old and new rails, while it has also been used to stiffen and strengthen rails at points where interlocking apparatus has made it necessary to place ties four feet apart. It is used on the new standard 75-pound rails, as well as on the old 72-pound rails, and is found to materially reduce the cost of surfacing.

ST. GOTHARD RAILWAY.

The following article is abstracted from *Engineering*, Jan. 4, 1895:

"It is now more than ten years since the St. Gothard Railway was opened for traffic, and it is not too much to say that in that time this great Alpine highway has proved an immense boon not only to the three countries directly concerned, viz., Switzerland, Germany, and Italy, but to international passenger and goods traffic generally. While on the rival route by the Mont Cenis traffic has been dwindling away, so that this great and costly railway is at present worked at serious loss to both the Paris, Lyons, and Mediterranean Company of France, and the Mediterranean Company of Italy. The traffic on the St. Gothard Railway has steadily increased and developed from year to year. Nor is this fact traceable only to the commercial rupture between France and Italy, or to the superior natural advantages, such as the grander scenery, the great tourist centres, and

the more numerous and attractive points of interest along the St. Gothard route: it is very largely due to the vastly superior accommodations and facilities, as well as to the far better management of the St. Gothard Railway as compared with its western rival. Many travellers by the latter, viz. the Mont Cenis route, still have a ghastly recollection of being packed like so many parcels or herrings into the indifferent first-class through carriages from Paris to Turin, and *vice versa*; and although the opening of the St. Gothard route at last put an end to that monopoly, and there is now too much instead of too little space in passenger trains, yet, beyond running sleeping-cars and one or two inferior second-class carriages in the express trains, the Paris, Lyons and Mediterranean Company has done little or nothing to improve the accommodations and increase the comfort of passengers on the Mont Cenis Railway. Indeed, more especially as regards first and second-class accommodation and comfort, the carriages of the Mont Cenis line are not to be compared with those of the St. Gothard, nor with those of the Arlberg and Brenner Railways. As has been said, the great success which has attended the St. Gothard Railway, alike from an engineering and financial point of view, is in a great measure due to excellent management, and this is the more commendable when the extraordinary care required in working and maintaining a mountain railway of such magnitude is taken into account." The St. Gothard system, properly speaking, has a length of 165 miles (265 kilometres). The total distance of the through communication from Bâle, by the St. Gothard to Milan, is 236 miles (378 kilometres), within which no less than six different main water-sheds are crossed in succession. The valley sections, with gradients up to 1 per cent, or 1 in 100, represent 45 per cent; the mountain sections, with gradients up to 2.7 per cent, or 1 in 37, represent 55 per cent of the total length. The alignment and gradients may be summed up as follows:

Total sections in straight	95 miles.
" " " curves	70 "
Total of level sections	35 "
" " grade "	130 "

The minimum radius of curvature is 200 metres or 10 chains (660 feet), the mean radius 408 metres or 20 chains, and the sum of all curve angles is 15,624 degrees.

The sum total of all differences of level, viz., of all rises and falls, 2500 metres, or 8200 feet; hence the mean gradient is 1.2 per cent., or 1 in 83. The steepest grade on the northern ascent is 2.5, that on the south side 2.7 per cent.

EARTHWORKS, WORKS OF ART, AND STATIONS.

The total lengths of these are:

Length of embankments	85 miles.
" " cuttings	36 "
" " 65 tunnels	26 "
" " 687 culverts, 360 bridges and viaducts	3 "
" " 41 stations	15 "

In the 65 tunnels is included the summit tunnel, 9.6 miles (15 kilometres); also the seven helicoidal or "corkscrew" tunnels, characteristic of the St. Gothard Railway, viz., three on the north side, by means of which a vertical rise of about 1000 feet is effected in a length of 7.5 miles, and the four similar tunnels on the south side, two to effect a drop of 187 metres or 618 feet in 5 miles, and two 548 feet in the same distance. All of these helicoidal tunnels have a uniform radius of 990 feet.

Of the 25 viaducts, the longest is 846 feet. The greatest number of openings is seven. The maximum length of a single iron span is 247 feet, at an elevation of 260 feet above river level, and the maximum width of masonry arches 40 feet, the greatest depth of stone piers being 174 feet.

Great difficulties were encountered in driving the summit tunnel, owing to faults, fissures, and intermediate soft strata in the gneisso-granitic rock formation. The stopping and the deviation of the enormous inpour of water, together with the propping of soft superincumbent strata, "may be described as a truly gigantic task," which not only delayed the completion of the tunnel, but increased the cost 7,000,000 francs over the contract price of 50,000,000 francs. The rising grade from the northern end of the tunnel to the summit in it is 0.85 per cent, or 1 in 118; and the fall from that point to the southern end is 0.2 per cent, or 1 in 500. The summit level of the tunnel is 5600 feet below the summit peak of the St. Gothard "massif." Assuming that the temperature below the surface of the earth increases at the rate of 1 degree cent. or 1.8 degree Fahr. every 100 feet, and assuming the temperature on the summit peak to be zero, the maximum temperature at the depth of the tunnel corresponds to about 53° cent. or 127° Fahr. By the use of the ventilating machinery the temperature during the work did not exceed 30° cent., or 86° Fahr., whilst in the completed tunnel the maximum temperature, even in summer, does not exceed 22° cent., or 72° Fahr.

The average cost of great Alpine tunnels has been gradually reduced from £143 to £85 per lineal yard, while, inversely, the rate of perforation and masonry work has increased from 0.5 to 1.6 miles per annum.

The following table gives the comparative length and cost, and the rock formations of the principal tunnels, all of which have a double line of rails, and are lined throughout with masonry:

TABLE CII.

Name.	Summit Level, Feet.	Length, Miles.	Period of Construction.	Years.	Total Cost.		Per Kilometre.	Per Mile.	Rock Formation.
					Fr.	£	Fr.	£	
Mont Cenis.	4,250	7.6	1857 to 1871	15	75,000,000	3,000,000	6,150,000	393,600	Limestone.
St. Gothard.	3,785	9.4	1871 to 1881	11	57,000,000	2,280,000	3,800,000	243,000	Gneiss, granite.
Arlberg.....	4,297	6.4	1880 to 1883	4	24,000,000	900,000	2,330,000	149,100	Mica slate and schist.

Permanent Way, Pennsylvania Railroad.

The ballast is placed, before laying the ties, to the depth of from 8 to 12 inches. The quantity of ballast required per mile of track is as follows:

	Gravel.	Broken Stone.
Single lines, cu. yds................	1,900	2,315
Double " " "	4,075	5,300
Three " " "	6,958	8,500
Four " " "	10,185	12,200

These include the filling between the ties up to their top surfaces. The lengths of rails used are 25, 27½, and 30 feet. Weight of rails, from 70 to 100 pounds per yard.

On the main line 14 first-class ties are used to each 30 linear feet of track. The track is laid with the joints between the ties, and broken; that is, the joint in one rail is opposite the centre of the other rail in the same track. In winter 5-$\frac{1}{16}$-inch, and in summer $\frac{1}{16}$-inch, must be left between the ends of rails to allow for expansion.

HINGED CONCRETE ARCH BRIDGE OVER DANUBE AT MUNDERKINGEN IN WURTEMBURG (see paragraph 707 of text).

The construction of this arch is similar in many respects to the one described in paragraph 694.

It has a clear span of 164 feet, and a rise of 16.48 feet.

The foundation-bed on the right bank is the natural rock which crops out at the surface; on the left side the rock was found at considerable depth below the surface, and it was necessary to drive piles to support the side of the arch. There were driven for this abutment 145 piles, inclined at an angle of 15° in order to place their longitudinal axes in the direction of the thrust at the springing.

The pressure on the rock abutment is 218 pounds per square inch, about 15 tons per square foot, and the shearing 148 pounds. The pressure on the gravel averaged 29 pounds, and a maximum of 43 pounds per square inch, about 3 tons per square foot; the horizontal shear is 50 pounds. The allowed pressure on each of the 145 piles is 35 tons in the direction of their lengths. The pile-heads, which are embedded in concrete, have a pressure across the fibre of 250 pounds each, corresponding to a shear of 500 pounds, or, allowing for friction of the concrete and gravel, 150 pounds per square inch.

The piles were driven to a depth of 13 to 20 feet in the gravel. The concrete and pile foundation is 46.6 feet wide by 31.2 feet long; the width of the arch is 24.8 feet.

The concrete arch is 3.28 feet in thickness at the crown, and has a pressure of 500 pounds to the square inch; the thickness at the springing is 3.61 feet, with a somewhat higher intensity of pressure than at the crown. The pressure was assumed to be uniform from crown to springing. In the so-called "breaking sections" in any cross-section of the arch the line of pressure was found to approach the intrados or extrados, as one or the other half of the arch was fully loaded. At these sections the thickness was increased, so that the maximum pressure would not exceed 500 pounds per square inch. The actual maximum pressure in the left section is 540 pounds, and in the right 560 pounds.

The cement used in the construction of the arch proper had to be of sufficient fineness to pass through a sieve of 5800 meshes per square inch without leaving any residue, and not over 15 per cent residue on a sieve of 32,200 meshes. For other portions of the structure a residue of 1½ per cent on the first and 24 per cent on the second sieve was allowed. Soundness was examined into by placing pats on glass plates, and no cracks or warping was allowed. In tensile tests specimens of 0.78 square inch, composed of 1 cement, 3 standard sand, were tested after being 1 day in air and 6 days in water.

The following table gives some of the tests:

TABLE CIII.

Grade of Cement.	Time of Setting in Hours.	Number of Tests.	Strength in Pounds per sq.in.	Number of Tests.	Strength in Pounds per sq.in.
Quick-setting Portland, slow-setting..	¼	45	150 to 297		
Ordinary fineness....................	4	10	250 to 260	95	241 to 400
Fine gray port......................	4	10	234 to 386	20	278 to 480
" red "	4			9	291 to 485
" green "	4			6	311 to 326
" yellow "	4			3	294 to 390

Chemical analysis showed not over 1.1 per cent of magnesia.

All materials for concrete were required to be perfectly clean.

Broken limestone and gravel was not over 1¼ inches in diameter. The concrete was machine-mixed. The mixer was an iron cylinder 4.9 feet in diameter and 3.3 feet in length, revolving on a horizontal axis. In this cylinder were 40 balls, 4.8 inches in diameter, weighing about 660 pounds. In the circumference of the cylinder was a door, through which the stone, cement, and sand was put in and the mixed concrete taken out; a grating with bars about 0.4 foot apart prevented the balls from falling out. During mixing the door is closed. The materials were mixed dry for 2 minutes; then water from a reservoir at one end of the mixer was sprayed through the hollow axis, and the mixing continued for 3 minutes longer. The proper quantity of water was regulated by means of a float and gauge. The drum held 0.8 cubic yard of concrete (about 21.5 cubic feet). A batch was made in 10 minutes; one machine furnished 48 cubic yards (1296 cubic feet) in 10 working hours.

The effect of mixing the concrete in this manner is not merely to reduce the sizes of the stone and gravel by grinding, *but principally to cause the cement to firmly adhere to the surfaces of the other materials.* [Italics by author. It is a matter almost universally ignored and neglected in mixing concrete.]

Samples 4 inches long made from 1, 2, 3, and 5 concrete, for the arch, showed after seven days in air from 2070 to 3840 pounds per square inch, the average of 10 tests being 2970 pounds. Ten tests with blocks 28 days old ranged from 2950 to 4650 pounds—average 3730 pounds per square inch—while the maximum working pressure is 500 pounds. The arch was jointed or hinged at the crown and springing, making it a statically determinate structure, no arbitrary assumptions as to the course of the line of pressure being necessary, settlement of the foundations or deflections of the arch were also thereby compensated for. There are 12 such hinges at the crown and 12 at each abutment. Each hinge is made of two steel boxes, built of I beams and plates 2 inches wide, ¼ inch deep, and 3¼ inches high; the boxes are free to turn, one on the other, through a ball-joint placed between the boxes at the centre; or, as described by another writer, the bearings are 1.64 feet long, and have two steel bars 2.8 inches wide and 1 inch thick, machined so as to fit close together on surfaces curved to a 6-inch radius, and subjected to a maximum pressure of 9700 pounds per square inch from the actual abutting supports. This is distributed by wrought-iron boxes over a sufficient area on the concrete. The boxes are 1.64 feet long, 32 inches wide, and 8 inches thick. Each is formed of two I beams, to which are riveted .6-inch iron plates. They were tested to a pressure of 1240 pounds per square inch, but transmit only 840 pounds to the concrete.

The backing is formed of hollow arches above the haunches, and the spandrel is faced with concrete. The arch ring is faced with limestone projecting about 3¼ inches. The roadway is macadamized; the sidewalks on each side are paved with concrete blocks and have an iron railing at the edge. The moving load is taken at 82 pounds per square foot, the weight of concrete at 144 to 150 pounds per cubic foot.

The soffit on left half of arch, where the pile foundation is employed, is curved to a radius of 213 feet;—two thirds of the other half have a radius of 230 feet, and the remaining one third 151 feet radius. The approximate line of pressure under full load was determined experimentally.

The false-work was erected on piles, and consisted of 12 bents supporting the arched logitudinal stringers on double wedges. The flooring (lagging) resting on these stringers was made of 4 by 4 in. planks laid close together, but not nailed. The faces of the arch were covered with boards. The concreting was done in layers 11¾ inches thick, these commencing at the abutments, where the metal hinges were first put in place, and working alternately from either end towards the centre for a distance of 26.25 feet. The work then proceeded from the neigh-

borhood of the crown, and finally the remaining openings were filled in. The hinges at the crown were adjusted and concreted 19 days after commencing work; 28 days after closing the arch the planking was removed. The deflection was then observed for a long period of time and amounted to $4\frac{3}{16}$ inches. This deflection, as well as the settlement and compression at the left abutment, had all been provided for by raising the false-work a sufficient amount. The construction lasted in all seven months, after which the bridge was open to traffic. It has not so far shown any defects.

The cost was as follows (1 mark = 23.8 cents):

Foundations	\$3,332
False-work	1,600
Superstructure	9,615
Superintendence, etc	2,261
Total	\$16,898

or, for a clear span of 164 and a width between railings of 26.25 feet, the price is about \$3.90 per square foot of traffic surface. The total expenditure, including paving and railing, amounted to \$21,420. Descriptions and drawings of this structure found in *Engineering News* of December 20, 1894; *Engineering Record*, and *Engineering*, vary in some of the details.

Methods of Overcoming Steep Grades.

(See *Engineering Magazine*, May, 1894.)

The common methods of climbing steep grades are:

(1) *Grip-wheels or Drums Operated by Steam or Electric Power.*—The drums are placed at the top of the incline, and cables, attached to the cars at the foot and the drum at the top, are used to haul the cars to the top, the cables winding around the drum as the car ascends. The track upon which the cars run is often composed of three rails, the middle one being common to both tracks. The descending and ascending cars pass by means of a turn-out near the middle of the incline, or two separate tracks may be used. This has proved an economical means of hauling loads up steep grades. Of this type a notable example is the steam-cable incline at the Catskill Mountains. This cable incline was built in 1892. The length of the incline is 6780 feet, and the rise in this distance is 1600 feet, or 1 vertical in about 4.25 horizontal. The first departure from the common mode of construction consists in adopting a grade on a vertical curve, so graduated as to result in compensation for the shifting of the great weight of the cable from one side of the road to the other, which would of course occur at every trip. When starting, the resistances to be overcome is not only the weight of the lower load, but also the entire weight of the cables, and these wire ropes weigh 35,000 pounds. As the lower load ascends this weight is constantly decreasing, for the cables, passing around the friction-drums, follow the descending car and add their weight to its gravity-power. Consequently, in order that the engine shall only be called upon to exert a uniform power at all stages of the trip, and perform the most economical work, the grade is made lightest at the bottom, but constantly increasing with an upward curve in exact proportion as the weight of the cable is transferred from an upward resistance to a downward pull. The steepest grade is near the top of the incline. The descent is regulated by the use of an automatic clutch.

(2) *Rack-and-pinion Gearing.*—The common construction of this type of railway consists in laying a centre rail midway between the track rails. The middle rail has a series of teeth, into which gear the teeth of a wheel attached to the car and turned by steam or electric power. In this manner the loaded car climbs, as it were, the incline.

In the earliest track of this kind—the Mount Washington Railway, New

Hampshire—the centre or rack-rail was built of two angle-irons with wrought-iron rungs between them, with which the teeth of the pinion-wheel engaged. The grade surmounted was at the rate of 1 foot vertical to 2¾ horizontal.

The next improvement consisted in the substitution of trapezoidal teeth in the rack with a system of involute gearing. Then a double rack and pinion was used, the teeth in one rack being opposite the open spaces. By this construction the power was applied more regularly and continuously, and the motion was easier. This is known as the Abt system. It is the one almost universally adopted in Europe. The Pike's Peak road in this country is worked on this system, on a grade of 1 in 4.

In the Mount Pelatis railroad, instead of teeth on the upper surface of the centre rail and pinion-wheel turning a horizontal axis, teeth were placed on both sides of the centre rail, and horizontal pinion-wheels turning on vertical axes, are used, on grades of 48 in 100, or 1 vertical in 2.08 feet horizontal. In these systems no great speed is attainable.

(3) *The Water-balance Cable Incline.*—This is perhaps the simplest, and where water is or can be stored at the top of the incline, also the most economical. The construction is simple, and as follows: A cable passes around a sheave at the top of the incline, each end of which is attached to a car. The cars are provided with water-tanks, which can be filled or discharged rapidly. The only resistances to be overcome are the difference between the weights of the ascending and descending cars and friction of moving parts. The car at the foot of the incline is discharged of its water and loaded with its weight to be carried to the top of the incline; the car at the top is filled with water and discharged when it reaches the foot of the incline. When necessary, the descent is regulated by rack and pinion. The Glion railway in Switzerland is a water-balance incline. Rate of grade, 57 feet vertical in 100 feet horizontal; total rise, 997 feet.

From Bulletin of the American Society of Civil Engineers, April 4, 1894.

COMBINED IRON AND CONCRETE ARCHES.

Pont d'Alma and Pont Napoleon railway viaducts have spans about 140 feet in length, and rise of $\frac{1}{10}$ of the span. Erlack, Germany: arch length of span 105 feet, and rise 13 feet, with thickness at crown of 20 inches. At Ulm, Germany, an arch is built with span of 150 feet, and thickness at crown of 6¾ inches.

A stone arch is constructed for pressure only. In concrete arches $\frac{1}{10}$ of the pressure at least may be taken up as tension. The use of iron with concrete is to take up the tension, and in building flat arches there is sometimes a saving of over 50 per cent in the material used as compared with stone. The Monier system at first used only one wire netting on the intrados. A second is advisable in the extrados near the haunches. At Bremen, Germany, a bridge is designed for a live load of 200 lbs. per sq. ft., with a span of 132 feet, rise of 14¾ feet, and thickness at crown of 9¾ inches. In Switzerland, in 1891, three bridges were built, having spans of 128 feet, rise 11 feet, and thickness at crown 6¾ inches, increasing to 10 inches at the abutments. The more recent systems adopt rolled shapes for the iron instead of wire netting. A span of 16.6 feet, designed for a load of 80 lbs. per sq. ft., was tested up to 565 lbs. per sq. ft. before rupture occurred. In flat arches it is claimed that the horizontal thrust at the abutments is almost entirely done away with. The Melan system consists of bent I beams with concrete bedded between them, and no cross-rods used. The bending is done at the mill at a cost of half a cent per pound. Only a light centring is required to support the ring until the concrete is set. Concrete is laid in rings between the beams in layers of about 6 inches in thickness, and well rammed.

A series of tests were made by the Austrian Society of Engineers and Architects, at an expense of $36,000, on arches from 7 to 77 feet span, and loaded to breaking. An arch of brick and cement mortar, 13½ feet span, 15¾ inches rise, thickness at crown 6 inches, broke with 321 lbs. per sq. ft. On the Monier

system an arch with $13\frac{1}{8}$ feet span, $15\frac{3}{4}$ inches rise, and thickness at crown $2\frac{5}{8}$ inches required 840 lbs. per sq. ft. to break it. An arch on the Melan system, span $13\frac{1}{8}$ feet, rise 11 inches, and thickness at crown $3\frac{1}{8}$ inches, broke at 3360 lbs. per sq. ft.

The best practice in mixing materials for such work is to make the concrete at the crown of 1 part cement and 5 parts of other materials, in the haunches 1 to 7, and in the spandrels 1 to 9. The common practice is to use less cement. The centring can be removed after two weeks, but the arch should not be subjected to severe tests before three months have elapsed. After a test on a bridge of 6 spans, at the expiration of thirty-one days the deflection was 0.59 inch, and after forty-three days 0.08 inch.

The modulus of elasticity of concrete is about $\frac{1}{10}$ of that of mild steel. The adhesion between iron and concrete after hardening would be about 600 lbs. per sq. in., acting like a glue after the concrete has once hardened. Concrete is considered the best conservator of iron. Rods bedded in concrete for 400 years below low-water level were found to be free from rust. Expansion from heat is the same in both concrete and iron.

The advantages are: (1) No expense for maintenance, no vibration, and practically no noise; (2) not affected by a change of live load; (3) they are tornado and high-water proof; (4) have a solid appearance, which can be made to harmonize with architectural surroundings; (5) their construction is cheap where sand and gravel are to be had.

The average span in Europe is 50 feet, with a maximum of 150 feet, and there is no reason why this should not be exceeded.

The above is from a paper written by Fr. Von Emperger, C.E.

Solid Iron Floor-systems.

Ben Venue Bridge, Pittsburg, Pa. — Solid concrete and corrugated steel plates were used. A uniform live load of 125 pounds per square foot, and excess loads arising from electric cars and a 15-ton Aveling & Porter steam road-roller. The solid floor consists of Carnegie's corrugated steel-plate sections; to these the 9-inch girder rails for the street railway are attached directly by hook-bolts. Upon the corrugated plates is a bed of elastic "binder," in two layers, used to insulate the ironwork and prevent its use as a ground for the return electric current of the street railways. To further the same end the electric connection between the rails is insulated by being surrounded by asphalt. On the top of the binder is placed a layer of concrete, carrying the pavement of Neuchatel asphalt. To provide for contraction and expansion, joints were made in the concrete at intervals of 25 feet. The asphalt is, of course, continuous over these joints.

Other systems consist of some kind of trough-shaped section. These are built up with plates and bolted together with rivets. The sections are of the ∨, ⊔ or ╲_╱ shapes. The plates either overlap and are riveted together, or cover-plates, either plain or bent, are riveted to the main plates. Taking the trapezoidal section, the dimensions are as follows: The vertical depth of the trough is 10 inches, clear width at top $15\frac{1}{2}$ inches, at bottom 5 inches. Sides and bottom overlap, and are held by one row of rivets on each side. The top cover-plate between the troughs proper is $5\frac{1}{2}$ inches, covering the portions of the side plates of two adjacent troughs, bent to a horizontal, and held by one row of rivets to each side plate. Bottom plate is $\frac{3}{4}$ inch in thickness, side plates $\frac{3}{8}$ inch, and top plate $\frac{7}{16}$ inch. The bottom plate is bent at its sides to fit the side plates, which splay outwards from the bottom plate through 123° of arc. This construction gives a water-tight-bottom. The other types, of the triangular and rectangular sections, do not, as usually constructed, give a positive water-tight bottom.

The solid iron floor of the Archer-avenue subway, Chicago, is made of plates and angles. Its depth is $10\frac{1}{4}$ inches, and the troughs are spaced 21 inches apart, centre to centre. The troughs are made up as follows: Top plate, $10 \times \frac{7}{16}$ inches;

side plates, $9\frac{5}{8} \times \frac{5}{8}$ inches; bottom plate, $11\frac{3}{4} \times \frac{7}{16}$ inches; angles, $3 \times 3 \times \frac{5}{8}$ inches. The flooring fits between the lower-chord flanges of the girders, with a clearance of 1 inch on each side for drainage, and its under side is on a level with the under side of the lowest-chord cover-plate.

TESTS OF CONCRETE AND IRON-FLOOR ARCHES ON THE MELAN SYSTEM.

Tests were made on arches 7 feet span, 6 feet in width, with three 4-inch I beams, spaced 3 feet apart. Two other arches of 6 feet span, also two of 6 feet span, 4 feet wide, with tee-bars 3 in. by 3 in., spaced 2 feet apart.

The arch beams rest between 12-inch I beams, held together by angle-irons 3 in. by 2 in., connected to top and bottom flanges at each end; bottom flanges were also connected by two flat bars $\frac{3}{4}$ by 2 in., having the ends bent around the flanges. The concrete arch was built upon wooden centres, the thickness of the arch 4 inches, composed of 1 part Manheim Portland cement, 2 parts sand, and 4 of broken stone of $\frac{3}{4}$-inch size, well rammed against the sides of arch and backed up to the under side of the top flanges of the 12-inch beams. The tests upon these arches have not yet been reported.

The tests of flat fire-proof floors of concrete with wire and iron rods embedded therein have been recently made. The wire is stretched across between the top flanges of I beams; it was covered by and bedded in a mixture of plaster of Paris, coarse sawdust, and cement, forming a flat arch or pavement, the ends carried down so as to cover the webs and lower flanges of the beams. An arch span 4 feet 6 inches centre to centre of bearings, 4 feet $0\frac{1}{4}$ inch clear between flanges, surface area of 10.105 square feet, deflected $\frac{1}{16}$ inch under 348 pounds per square foot, and $1\frac{1}{8}$ inch under 894 pounds, and gave way under 1551 pounds by the breaking of some of the wires on one side.

In a hollow segmental arch, span 5 feet $2\frac{5}{8}$ inches, the lower joints commenced to open before a load of 300 pounds per square foot was reached, and failed suddenly under 411 pounds, breaking at the skew-backs.

Impact Tests.—A 205-pound cylindrical weight $9\frac{1}{4}$ inches in diameter was dropped from a height of 5 feet, and striking with its edge on the arch, cutting into it and spreading, but not breaking the wires until it was dropped five times in one place, when it broke or cut them and dropped through to the floor below, making a clean-cut hole. Length of span, 3 feet.

In the fire-test proof a hardwood fire was kept burning under an arch of 5 feet 6 inches span for three hours, when it was extinguished by water discharged from a hose, water being also thrown upon the floor without causing the composition to crack, splinter, or show signs of disintegration. The beams and the arch surface not exposed to the flame remained cool. The surface exposed was affected for a depth of about $\frac{1}{4}$ inch.

CHAMBER-MINE FIRING—"IRON GATES," GREBEN POINT, DANUBE RIVER.

A heading 3 feet wide, 4 feet high, is driven about 80 feet in length. At its end a chamber 6 ft. × 6 ft. × 6 ft. is excavated at right angles to the heading. This is charged in the usual way, and the heading closed by brickwork laid in cement and dry-stone packing. At first "carboazotine" was used. This is a low explosive of the nature of blasting-powder, composed of 74 per cent potassium nitrate, 12 of sulphur, 8 of soot, and 6 of bran, prepared by the wet process. In one chamber-mine with a breast of 60 feet and a height of 99 feet 3.885 tons brought down 700,000 cubic feet of rock.

Five tons of second-grade new dynamite, containing 45 per cent of blasting gelatine, were used in the chamber. This second-grade dynamite, in charges of 13 tons, brought down 2,100,000 cubic feet of rock, in each case about 80 cubic feet per pound of explosive. The explosive effects of both the dynamite and "carboazotine" were the same under similar conditions.

The formula used for determining the proper quantity of explosive was

$$L = 3(v^3 + 5h)q.$$

L = weight of charge in kilograms;
v = line of least resistance in metres;
h = weight of rock above in metres;
q = coefficient depending on explosive used.

The height of rock above the charge seemed to be of little importance. Omitting the term $5h$ from the formula only made a difference of 100 kilograms in a charge of 4000 kilograms. Thus the formula practically resolves itself into the cube of the line of least resistance multiplied by a coefficient. This is almost identical with the formula obtained by considering the explosive to act in spherical waves in an unlimited homogeneous solid, namely,

$$L = 4.1888r^3c.$$

This is the formula used in the harbor at Fiume, where the ratio of height to line of least resistance was kept constant at 3 to 2. Mr. Guttmann says that both of these give the quantity of charge too high, since they can only be true in a perfectly confined space, a condition never existing in practice, for there is always at least one free surface, consequently the formula based on the formation of a conical crater must be more accurate:

$$L = k(w + r)^3,$$

where L is the weight of the charge in kilograms, w is the longest line of least resistance at right angles to and towards the free surface in metres, and r is the radius of the crater in metres, and should not be greater than $3.2w$. As an additional quantity of charge is desirable to insure success, the spherical wave formula may be used, though at some extra expense.

The constant q or c may be taken equal to from 0.18 to 0.23, or on an average at about 0.22 for carboatzotine. For other explosives the constant must be determined by experiment. With the second-grade dynamite as given above it would be about the same as for carboatzotine.

Sheet-asphalt Paving Practice in Twelve American Cities.

Thickness of the concrete base varies from 4 to 6 inches. All except two use broken stone, sand, and cement in making the concrete; one, Washington, uses half broken stone and half gravel; and Pittsburg uses broken stone and screenings, with sand and cement. Six of them use no cushion coat on the concrete, while the remaining six use a half-inch cushion. Four lay no binder coat, six use one of 1½ inches in thickness; the remaining two cities one of one inch and the other of one-half inch in thickness. In six of the cities using binders the material is broken stone. In Cleveland it is sand and gravel. The thickness of the surface after final compression is as follows: Washington, 1½ inches, or 2½ inches loose; Harrisburg, Chicago, Cincinnati, Detroit, Columbus, Cleveland, and Syracuse, 2½ inches, including the half-inch cushion; Pittsburg, 1½ to 2 inches; Kansas City and Buffalo, 2 inches. The average price paid per square yard: Washington, base, binder, and surface, for the 6-inch base, \$1.68, and \$1.53 for the 4-inch base; Harrisburg, grading and foundation, \$2.75; Scranton, all but curbing, including grading, foundation, remodelling street basins, raising or lowering manholes, crosswalks, and gratings, \$2.58; Pittsburg, removal of the subgrade, 4 and 5-inch base, no binder, \$2.49; the 6-inch base, with binder, \$3.06; Kansas City. everything but curbing, if the street has been graded; if not graded, this is done first by special contract, \$2.58½; Chicago, grading and foundation, \$2.83 to \$2.93; Cincinnati, grading, rolling subgrade, and pavement complete, no curbing, \$2.75; Columbus, grading and foundation, curbing 40 cents per lineal foot

extra, $2.65; Detroit, concrete, $2.50; Cleveland, concrete base only, $2.85 to $2.95; Syracuse, including the 6-inch concrete foundation, $2.43. In Washington, Harrisburg, Scranton, Pittsburg, Kansas City, Chicago, Cincinnati, Columbus, Detroit, Cleveland, and Syracuse the price does not include grading, foundation, and curbing. In Buffalo the price does include these items, but only grading exceeding one foot is paid for by the cubic yard. Buffalo and Erie are the only cities including curbing in the paving price. The terms of guarantee for sheet-asphalt pavements are as follows: Five years in all the cities except Cleveland, where it is 10 years. The average price in 11 cities is $2.605. Of course the sheet-asphalt pavement or covering is included in all of the foregoing prices.

Tar-McAdam street-paving is used extensively at Withington, England. It is constructed as follows:

A 10-inch bed of hard clinkers and broken stone is well rolled by a 12-ton steam-roller, and covered with 4 inches of 2½-in. broken stone, which is also well rolled. Upon this is laid a 3-in. layer of tar-macadam, consisting of one ton of 1½-in. granite to 12 gallons of tar, 28 pounds of pitch, and 2 gallons of creosote oil. This coat is well rolled, and is covered with 1 in. of limestone screenings, mixed with the same cementing materials, then covered with a blending of dry screenings and finally rolled. The work should only be done in fine weather; the rolling should average one day's work for each 100 sq. yds., and the traffic should not be let on the road directly after it is completed. The cost is from 84 cts. to $1.00 per square yard.

In Boston for the granite-block pavement large blocks have been preferred—namely, in width 3½ to 4½ inches, in length 9 to 14 inches, and usually not less than 11½ inches, and in depth 7½ to 8 inches. These blocks delivered cost $73.50 per thousand. Boston has relatively a small percentage of paved streets. There are 33 per cent of gravel, 37 per cent of macadam, 10 per cent of telford, 1.41 per cent asphalt. Buffalo has 151 miles of asphalt and 120 miles of block-stone out of a total of 396 miles, or 38 per cent asphalt and 30 per cent. of block, against 1.44 and 17.38 per cent, respectively in Boston.

Kentucky Rock Asphalt.

The table opposite shows the valuable qualities of this rock asphalt, for which the author is indebted to Mr. Marshall Morris, of Louisville, Ky. After comparisons made with Trinidad asphalt a contract for paving with this material in Buffalo, N. Y., has been awarded at $1.65 per square yard, with $2.00 for cuts and openings, 60 cents per foot for limestone curbing, and 30 cents per foot for redressing and setting curbstones.

Bearing-power of Pavements.

A 38-ton bridge girder was hauled through the streets of Troy, N. Y., upon two 4-wheeled trucks, with tires 5 inches wide. The weight caused an indentation of about 1 inch in passing over a granite pavement, possibly without a concrete base. It passed over the whole length of a brick-paved street with a concrete base without making indentations or even marks. In Philadelphia a 50-ton girder was hauled on two 2-wheeled trucks, with 12-inch tires, over a brick pavement on a concrete base. The wheels broke through the base, and sank up to the hubs in the material below. The load per wheel was 4.8 tons in the first and 12 tons in the second case; the load per inch of tire was about the same. The concentrated load was greater in the second case.

Kosmocrete Concrete Sidewalks.

Pavements made of this concrete are found in several cities, notably in Brooklyn, New York, where 200,000 square feet have been laid. They are constructed in the following manner:

Table CIV.

KENTUCKY BITUMINOUS-ROCK PAVEMENTS LAID IN BUFFALO N. Y., 1890–93.

Street.	Between—		Blocks.	Square Yards.	Repairs.	Opened to Traffic.
Clinton...	Niagara	N. Y. C. & H. R. R.	One	385	None	November 1, 1890
Cleveland....................	Albany	Breckenridge	"	1543	In gutters	July 30, 1891
Barton.....	"	"	Four	4572	None	August 21, 1891
Delaware Avenue...........	Utica	Ferry	Three	7413	"	" 31, 1891
Emily.....	Delevan	Clinton	One	2330	Over sewer	September 15, 1891
Meyers..........	Genesee	Best	"	1695	None	October 5, 1891
Sweeney.....................	"	"	"	2329	"	" 15, 1891
Rich.........................	"	"	"	3037	In gutter	" 25, 1891
Guilford.	"	"	"	3225	"	July 9, 1692
Mathews....	Jefferson	Mortimer	"	1006	None	" 13, 1892
Clark........................	Broadway	Lovejoy	Two	3787	"	August 3, 1892
Beck.........................	"	Stanislaus	One	3976	"	September 7, 1892
Loefiere...	"	Waldron Avenue	Three	10628	"	October 18, 1892
Jones........................	Clinton	Lyman	"	4465	"	November 3, 1892
Glore.......	Military Road	Austin	One	2464	"	June 24, 1893
Kehr.........................	Genesee	Fougerou	Two	4508	"	July 14, 1893
Huron.......................	Delaware	Oak	Six	5397	At railroad crossing	" 25, 1893
Brighton....................	Broadway	W. S. R. R.	One	1617	None	" 27, 1893
North Central...	"	N. Y. C. & H. R. R.	"	2240	"	" 30, 1893
Lewis................	Howard	Clinton	Four	6580	In gutter	August 30, 1893
Playter......................	Broadway	Lovejoy	Two	4881	None	September 9, 1893
Grimes......................	Playter	Clark	"	1324	"	" 11, 1893
Mills.	Broadway	Sycamore	"	5076	"	" 26, 1893
Peter........................	Amherst	Grote	One	4066	In gutter	October 5, 1893
Auchinwohl.................	Grant	Herkimer	"	1531	None	" 31, 1893

A bottom course of dry cinders, about 12 inches thick, is laid. Upon this is placed a 4-inch layer of concrete, composed of 3 parts of granulated granite, or sharp gravel, 4 parts of 1½-inch broken stone, and 1 part of Portland cement; on this is worked a facing course, 1 inch in thickness, composed of granulated granite, a small percentage of silicious grit, Portland cement, and carbon. The purpose of the granulated granite mixed with silicious grit is to prevent the surface from becoming slippery. It is used for sidewalks, stable flooring, and paving in public buildings. It has been in use in the New York custom-house for about 4 years, and is to be employed in the new terminal stations of the Brooklyn Bridge. The cost ranges from 25 to 35 cents per square foot. An experimental section is in fair condition after 6 years' service under heavy traffic in Brooklyn. In this section the concrete layer is 6 inches in thickness. At the middle of the street the surface is divided into rectangular blocks by deep grooves. Time must be given to allow the cement to set and become hard before opening the street to traffic. This material is also suitable for making sewer-pipe. (See *Engineering News*, Jan. 8, 1895.)

FORMULÆ FOR DETERMINING THE HEIGHT TO WHICH A JET OF WATER WILL RISE.

H = head on the jet in feet;
h' = difference between the height of head and height of jet;
d = diameter of jet in $\frac{1}{8}$ inch.

Then

$$h' = \frac{H^2}{d} \times 0.0125.$$

Discharge by the jet in gallons per minute,

$$G = \sqrt{H} \times d^2 \times 0.24.$$

Example.—Let d = 1 inch = $\frac{8}{8}$ inch, or $d = 8$; $H = 445$ feet. Then

$$h' = \frac{(445)^2}{8} \times 0.0125 = 309 \text{ feet};$$

and the height to which the jet will reach = 445 = 309 = 136 feet.

$$G = \sqrt{445} \times 64 \times 0.24 = 324 \text{ gallons per minute.}$$

NICKEL.

This metal has a specific gravity of 8.8, a little in excess of iron, namely, 7.86. It has greater strength than iron, and is practically non-corrodible. It can be welded to iron and the two metals then rolled out into thin sheets. It is used to a large extent for plating purposes. The Canadian Copper Company exhibited a block of practically pure nickel weighing 4500 pounds. It forms alloys with other substances, such as copper, zinc, iron, etc., known commercially as German silver, argentan, tutenag, etc., but its most important alloy is nickel steel. Meteoric iron contains nickel, is perfectly malleable, and may be easily forged into cutting instruments. Nickel is very widely distributed over the earth's surface. The two most important deposits of nickel ore at present known to exist are found on the Island of New Caledonia and in the Province of Ontario, Canada. Large deposits of less importance occur in nearly all European countries, and in the United States important deposits are found in Oregon and Nevada, and are known to exist in many other States—California, Connecticut, New York, North Carolina, Michigan, Missouri, and others. The New Caledonia ores are what are known as hydrated silicates of magnesia and nickel, from 6 to 8 per cent, and are intimately associated with serpentine. They are free from copper, sulphur, and arsenic. The Canadian ores consist of iron sulphides, with which 2 to 3 per cent of nickel is associated. The nickel is accompanied by copper. Steel alloyed with a small percentage of nickel (3 to 4 per cent) possesses great tensile strength with a corresponding elastic limit.

CONSTRUCTION OF STAND-PIPES.

Stand-pipes are now constructed to a large extent of steel plates. The thickness of these plates decreases from the bottom toward the top of the stand-pipe. As a good example we will take the stand-pipe at East Providence, R. I. The completed height was to have been 125 feet, with a diameter of 40 feet. There were 25 courses of plates, each 5 feet in height.

TABLE CV.

Course.	Thickness in Inches.	Course.	Thickness in Inches.
1	$1\frac{9}{32}$	9	$\frac{3}{4}$
2	$1\frac{3}{16}$	10	$\frac{11}{16}$
3	$1\frac{5}{32}$	11	$\frac{5}{8}$
4	$1\frac{1}{16}$	12	$\frac{9}{16}$
5	1	13 to 14	$\frac{1}{2}$
6	$\frac{15}{16}$	15 " 16	$\frac{7}{16}$
7	$\frac{7}{8}$	17 " 20	$\frac{3}{8}$
8	$\frac{13}{16}$	21 " 25	$\frac{5}{16}$

The bed-plate was $\frac{3}{4}$ inch in thickness and $41\frac{1}{4}$ feet in diameter, formed of radial strips. The $1\frac{9}{32}$-inch bottom course was secured to the bed-plate by means of a 6 × 6-inch angle, double riveted with 1-inch rivets, spaced $2\frac{3}{4}$ inches between rivets and $2\frac{1}{2}$ inches between rows. In the plates of the pipes triple riveting with $1\frac{1}{8}$-inch rivets and 8-inch laps was used in the first five horizontal and vertical seams; double riveting in the next ten horizontal and in the remaining vertical seams; and the last five horizontal seams were single riveted. The stand-pipe was erected by using inside staging. Its diameter was unusually large, considering its height. It was constructed of excellent material and with good workmanship. Its collapse is considered as due to the force of a severe wind-storm. At the time of failure the self-registering anemometer indicated a maximum velocity of about 38 miles per hour. Several days preceding the collapse the stand-pipe was slightly lifted on the windward side by a pressure from the wind, having a velocity of 60 miles per hour. At the time of the accident the stand-pipe had not been fully completed. The topmost course of plates with its stiffening angle was not in place. It had been pumped full of water, without showing leaks or other defects.

The stand-pipe at Maryville, Mo., was 135 feet in height and $18\frac{1}{2}$ feet in diameter. It was constructed of the best quality of steel plates, varying in thickness from $\frac{13}{16}$ to $\frac{5}{16}$ inch. It was anchored to a substantial foundation by means of eight pairs of 2-inch bolts attached to plate brackets, which reached to the top of the second course of plates. This structure also collapsed. For full particulars of these stand-pipes and the causes of collapse see *Engineering News*, May 10, 1894.

Another water-tower, designed by Mr. J. Todd for the new water-works system of Parkersburg, Iowa, is constructed as follows: Larimer steel columns are used for tower legs, twelve in number; the four inside legs are vertical and placed at the corners of a square $6\frac{1}{2}$ feet length of sides; the eight outside legs are in pairs and equidistant from the centre of the tower and at 14 feet 7 inches from it; these have a batter of 1 inch in 1 foot. The entire system of columns is braced with 5-in. steel I beam struts and from $\frac{7}{8}$- to $\frac{3}{4}$-in. round steel diagonal rods. Beams for the tank platform are 12-in. I beams with riveted angle connections to the tops of the tower legs; at the lower extremities the columns are anchored to masonry pedestals. The tank itself is made of 3-in. pine and consists of twenty staves and is 24 ft. in diameter at bottom. There are sixteen hoops of steel, varying from $6 \times \frac{3}{16}$-in. at bottom of tank, to $3 \times \frac{1}{8}$-in. at top; these have a tensile strength of 50,000 pounds per square inch. The roof of the tank is conical, built of 3 × 4-in. rafters covered with 1-in. boards and shingles. Each hoop has three sets of lugs and bolts for tightening; the bottom one has four sets.

Water is pumped from six artesian wells 126 feet deep and located about 1500 feet from the tower. It is conducted to consumers through about 2 miles of 8-in., 6-in., and 4-in. cast-iron mains. The power plant consists of a vertical pump, with 10 × 36-in. steam-cylinders and a 4-in. water-cylinder. Capacity is estimated at 3300 gallons per hour. The tower proper is 58 feet high, located on an elevation of 30 feet. Pressure afforded, approximately 50 pounds per square inch.

Track-laying by Machinery.

Strictly speaking, track-laying by machinery simply means a rapid and convenient method of delivering the materials, rails, ties, angle-bars, and bolts and spikes, to the front, where the track is laid in ordinary manner by the track gangs. In the ordinary method of track-laying the ties are hauled to the front on wagons drawn by horses, or are pushed by hand on light cars or trucks to the end of the front rail; in either case they are unloaded and distributed on the road-bed. The rails are brought forward and distributed in like manner. The advantage of this system is that, as the loads to be hauled are light, it is only nnecessary to use a small portion of the full complement of ties upon which to lay the rails, and corresponding numbers of spikes and bolts are used, with little attention given to accurate gauging and aligning the rails; but what is thus left undone must be completed by other gangs following close behind the front one, who place the requisite number of ties under the rails and do the spiking, bolting, aligning, gauging, and expansion-spacing at the ends of the rails. So that at the end of the day or week only a limited actual progress has been made and at a maximum cost.

Where long stretches of track have to be laid, or the country is rough and rugged, or hauling either difficult or impracticable, it has been found to be more economical and rapid to deliver all materials at the head of the track on construction trains, and, to facilitate the handling of material on tramways with narrow gauge, rollers and other fixtures are attached to the sides of the cars or on the cars, by means of which rails and ties can be conveniently and rapidly transferred to the front or pioneer car, to the end of which projecting platforms or chutes are attached, and down which the materials are delivered to the trackmen and distributed and placed by them. A full description of tramways, chutes, and general arrangements required would occupy too much space in this work. Suffice it to say that there are three track-laying machines in common use, namely, Holman's, Harris's, and Roberts's. The first two employ hand-power in transferring the material to the head of the train; the last, steam-power.

A full description of these machines, with methods of working, daily progress, actual and relative costs, number of men required, etc., can be found in *Engineering News*, Jan. 3, 1895, from which is also taken the following statement: The track in advance of the train can be either fully bolted, spiked to the full number of ties, or can be left in a partly completed condition and completed by gangs following the train. As to the speed of the work, the average is from 1¼ to 2½ miles per day, varying according to the nature of the ground, the weight of the rails and number of ties, and the general methods of working, while in exceptionally fast work a mile has been laid in 2 hrs. 50 min. with a gang of 80 men, or with a much larger gang from 1300 feet in 30 min. to 11,000 feet in 8 hrs. 30 min. On the extension of the Burlington & Missouri River R. R., from Sheridan, Wyo., to Billings, Mont., built in 1894, machine track-laying progressed at an average rate of 1¼ miles per day with a force of 85 men, at a cost of about $100 per mile. Track-laying by hand, on the same road, was let by contract about eight years ago at $300 per mile. This, however, included loading the material, and higher wages were then paid than those now ruling. Machine track-laying on another road, about seven years ago, cost about $150 per mile, which may perhaps be properly compared with the $300 per mile above. On another road, where machine work cost $140 to $170 per mile, hand work cost $250 per mile.

The following particulars of rate and cost of track-laying in the usual way may be of interest, but of course variations in price of materials and labor will preclude any direct comparisons. In 1882, on a Texas railway, a track-laying gang of 164 men in all, with 18 teams, could lay 1¼ miles of rails per day, but could not keep up the back work and lay much more than 1 mile; the cost per working day averaged $292, which probably represented the average cost per mile. In 1883, on the Western Division of the Canadian Pacific Ry., track-laying with a force of 300 men and 35 teams (about 180 men in the track gang proper) averaged 2½ miles per day, the highest records being half a mile in 35 minutes, 6.02 miles in one day (6 miles 100 ft. of main track and 1800 ft. of side track, all full tied, full bolted, and full spiked), and 25.86 miles in one week (4.31 miles per day). It has been stated that the cost of track-laying by hand costs from $292 to $350 per mile, but on the Atchison, Topeka & Santa Fé Ry. track-laying with a force of 164 men for laying two miles of track per day cost $170 per mile for labor only, or $247 for the labor of track-laying and surfacing.

With machine track-laying some records are as follows: Fargo & Southern R. R., 1884, 1½ miles per day with 40 men; Northern Pacific R. R., 1886, 8400 feet in 8 hours with 67 men (fully tied, bolted, and spiked); Northern Pacific R. R., 1889, 4000 to 5000 feet per half-day (work delayed by bridging, etc.), and 9000 feet in 6 hours (full tied, half-bolted, and quarter-spiked); Lake Shore & Eastern R. R., 1890, 1 mile in 3 hours; Great Northern Ry., 1891, 1¼ miles per day for 82 days and 50 miles in 25 days; Nelson & Fort Shepard Ry., 1894, an average of 2 miles per day. In one case 13,200 feet were laid in 9 hrs. 45 min. with a crew of 35 men.

The Great Blast at Hell Gate.

The following facts are taken from *Van Nostrand's Magazine*, Nov., 1876:

The work for the purpose of removing this obstruction to navigation was commenced in July, 1869, by building a coffer-dam between high- and low-water marks. This dam was finished in October, and the sinking of a huge shaft immediately ensued. This shaft or pit measured 117 ft. in its greatest length and 62 ft. in its greatest width; the depth below low water was 33 feet. From the bottom of the shaft 10 headings were driven; these radiated under the entire reef; they were from 10 to 22 feet high, with an average length of 270 feet. These were connected by cross-galleries at 30-foot intervals; they were roughly of the same section as the main headings. The whole space was thus converted into a vast honeycomb, the roof of solid rock under the river being held up by the columns left to support it. These columns were finally reduced as much as was deemed prudent by minor headings, until 173 piers were formed, each averaging 10 feet in thickness. The entire reef, covering an area of three acres, was thus undermined. The aggregate length of tunnels and galleries driven under the bed of the river was 7425 feet.

From the excavation 47,461 cubic yards of rock were removed by drilling and blasting; this operation required 208,174 lineal feet or 39½ miles of drill-holes, of which 90,107 feet in depth were drilled by hand and 118,077 feet by various kinds of machine drills worked by compressed air. The following quantities and kinds of explosives were used in blasting the rocks: blasting-powder, 24,431 pounds; nitro-glycerine, 26,471 pounds; giant powder, 1932 pounds; mica powder, 600 pounds; vulcan powder, 4017 pounds; rendrock, 1500 pounds—total, 58,951 pounds. In exploding these compounds 63,756 exploders and 331,516 feet or 62¼ miles of Bickford's safety-fuse were used. In all about 75,000 blasts were fired.

The usual method of driving a tunnel was as follows: The face of the rock was pierced obliquely with as many drill-holes, from three to four feet in depth, as were deemed necessary; the charges were then prepared by placing the explosive in water-tight paper cartridges from 8 to 12 inches in length, and containing from 8 to 12 ounces of explosive; into each of these cartridges was inserted a

copper cap containing mercury fulminate fastened to the end of a piece of safety-fuse, generally about 5 feet in length. A cartridge was then pushed to the bottom of each hole, which was filled with water. The end of the fuse hanging from the hole was ignited. The broken rock was carried in cars holding about one third cubic yard each, pulled by mules to the floor of the shaft, where a derrick worked by steam power hoisted the loaded car to the surface, and thence hauled to the spoil-bank. The excavation was completed in the year 1875.

Also holes were drilled in the supporting columns and the roof above preparatory to bringing down the roof. The holes were bored in the columns at vertical intervals of about 2 feet, the depths of the holes averaging 10 feet, and in the roof near the tops of the columns.

There were in all 4462 charged holes containing 52,206 pounds of explosives. Three different kinds of explosives were used, namely, dynamite, rendrock, and giant powder. The cartridges were metallic and all of the same length, about 1 foot and 11 inches, but the diameters varied, four different sizes being used, 2¼, 2, 1¾, and 1½ inches. The smallest, containing 10 pounds of explosive, was placed in the bottom of the hole, over which in succession three other cartridges were placed. Special precautions were taken to mark the charged holes and to insure electrical connection with each and every hole. In each hole an exploding cartridge about one foot in length and lined with vulcanite was placed. The explosion of the first cartridge was relied upon to cause the explosion of others in the same hole.

Electrical connection was then made between the exploding cartridges and the batteries, and all charges fired simultaneously. No tamping of any kind was used, but before the charges were exploded water was admitted to fill the workings, and proved sufficient for the purpose.

The charges were arranged in battery groups, and there were 184 of these groups. The wires of a group were connected each to each. There were 960 bichromate-of-potash cells, divided into 23 batteries of about 42 cells each, connected for intensity. Each battery exploded 8 groups of holes or 160 fuses in continuous circuit. The wires from the batteries to the charges were insulated by a gutta-percha coating and placed in wooden boxes. The circuit-closer consisted of 23 brass pins which were to drop simultaneously into 23 mercury cups, thus closing each battery circuit at the same instant.

The entire work was under the control of Gen. Newton, who reported, after sounding over the area, that the results were entirely satisfactory.

Concrete and Steel Foundations.

Given a load uniformly distributed over a certain length of the top of a beam, what is the safe projection of the beam beyond this distributed load, assuming that the beam distributes the same total load uniformly over its bottom surface? In the following analysis the weight of the beam itself is neglected.

Let the intensity of the load over a length l_1 of the top of the beam, p. 457, corresponding to AD in Fig. 169, be w_1, and that over the entire length l_2 of the bottom EF in Fig. 169 be w_2; then in order for the loads to be the same, and neglecting the weight of the beam itself, we must have $l_1w_1 = l_2w_2$.

Let M be the bending moment at any point within the area of the base of the column or pier, and distant $x = AH$ from its outer edge. Then

$$M_x = \frac{w_2}{2}\left(\frac{l_2 - l_1}{2} + x\right)^2 - \frac{w_1x^2}{2} \quad \text{. . . . . (1)}$$

Since the entire upward pressure on the portion of the beam between A and H is

$$w_2\left(\frac{l_2 - l_1}{2} + x\right) \text{ and its lever-arm is } \frac{1}{2}\left(\frac{l_2 - l_1}{2} + x\right),$$

the downward pressure of the load on that portion of the beam $= AH = x = w_1x$ and its lever-arm $= \frac{1}{2}x$. In both cases the pressure being uniformly distributed.

M_x in eq. (1) is a maximum when $x = \dfrac{w_2(l_2 - l_1)}{2(w_1 - w_2)}$.

Since $\dfrac{dM_x}{dx} = w_2\left(\dfrac{l_2 - l_1}{2} + x\right) - w_1 x$, and for a maximum $\dfrac{dM_x}{dx} = 0$,

$$\therefore\ w_2\left(\frac{l_2 - l_1}{2} + x\right) - w_1 x = 0; \quad \therefore\ x = \frac{w_2(l_2 - l_1)}{2(w_1 - w_2)}.$$

Substituting this value of x in eq. (1), there results after reduction

$$\text{Max. } M = \frac{w_2(l_2 - l_1)^2}{8} \cdot \frac{w_1}{w_1 - w_2} = \frac{w_2(l_2 - l_1)^2}{8} \cdot \frac{l_2}{l_2 - l_1}, \quad . \quad . \quad . \quad (2)$$

since $w_1 l_1 = w_2 l_2$,

$$\text{Max. } M = \frac{w_2 l_2 (l_2 - l_1)}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let n be the number of beams and $W =$ the total load on the foundation; then $nw_2l_2 = W$, and $w_2l_2 = \dfrac{W}{n}$, which substituted in eq. (3) gives

$$\text{Max. } M = \frac{W(l_2 - l_1)}{8n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If in eq. (1) $x = 0$, we have

$$M_0 = \frac{w_2(l_2 - l_1)^2}{8} = \frac{w_2(l_2 - l_1)(l_2 - l_1)}{8} \quad \text{and} \quad \frac{w_2(l_2 - l_1)}{8} = \frac{M_0}{l_2 - l_1},$$

which substituted in eq. (3) gives

$$\text{Max. } M = \frac{l_2}{l_2 - l_1} M_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Noting that M_0 is the bending moment at the outer edge A of the base of the load or column, and that it has been usual to take M_0 for determining the safe projection of the beam, and from eq. (5) maximum bending moment M is always greater than M_0, therefore the beams may be greatly overstrained considered simply as beams unsupported laterally by the concrete.

Applying the above equations, see *Engineering News*, Nov. 8, 1894.

A foundation consisting of a top course of nine 15-inch I beams, 50 pounds per foot, supporting a column with a base 5 feet square. The length of the beam $l_2 = 15$ feet 8 inches, and $l_1 = 5$ feet. The length of the projection was determined from M_0, and extreme fibre strains of 20,000 pounds per square inch were used. Referring to eq. (5) it is seen that whereas a maximum fibre strain was supposed to be 20,000 pounds, the actual maximum fibre strain was $= 20{,}000 \times \dfrac{l_2}{l_2 - 1} = 20{,}000 \times \dfrac{15'\,8''}{10'\,8''} = 29{,}375$ pounds per square inch.

The second course consists of 31 steel rails 9 feet 10 inches long; the third course is of 19 rails 21 feet 4 inches long, and the bottom course is of 30 rails 15 feet 11 inches long. These rails weigh 75 pounds per yard and measure 4¾ inches in height and width of base. An extreme fibre strain of 16,000 pounds per square inch was used. If these rails have an extreme fibre strain of 16,000 pounds where the moment was taken, they have an extreme fibre strain at the point where the moment is a maximum, as indicated in eq. (5), in the three courses of 32,200 pounds, 60,000 pounds, and 41,000 pounds per square inch, respectively, found by multiplying 16,000 by $\dfrac{l_2}{l_2 - l_1}$, after giving l_1 and l_2 their respective values in the three courses. The second course from the bottom seems to be subjected to a stress dangerously near its ultimate tensile strength. The load on the column is 1,166,000 pounds, from which and the relative lengths of its base and the lengths of the beams and rails M_0 was calculated.

By referring to the table in the front portion of this volume (page 965) it will be seen that beams supported laterally are very much stronger than those unsupported; so while theoretically some of the layers of beams in the example are overstrained, practically they may not be, as no consideration was taken of this fact. "Of course it is admitted that nothing like exact figures are available for computing the strength of a composite mass of concrete and iron, while the strains which the beams will stand if the concrete were absent can be very closely calculated. The real question is, Which method of computation will give the best idea of the actual strength? And on this question there appears to be plenty of room for honest difference of opinion."

LIME-MORTAR.

Mortar made of lime and ordinary sand which contains little soluble silica, in its hardening process may continue for centuries, while with pozzolanas which are rich in soluble silica the hardening is of sufficient quickness to warrant the use of these mortars in damp localities and even under water.

The santonin earths and trass confer hydraulicity upon fat limes. Both of these are richer in silica, alumina, and alkalies than the Roman pozzolana, while they contain neither sand nor water. The pozzolana of Rome contains, in 100 parts, 48 parts of salica, 14.3 of alumina, 3.9 of magnesia, 10 of oxide of iron, 7.7 of lime, 4 of alkalies, 3 of sand, and 9.1 of water.

These substances, mixed with lime in the proper proportions and in the presence of water, combine and harden in every respect similarly to Portland cement.

THE TEHUANTEPEC SHIP-RAILWAY.

The length of this railway will be 154 miles. Its estimated cost is, including improvements of terminal harbors, $8,000,000. The through traffic is estimated at 5,288,037 tons, and the gross receipts from all kinds of traffic will be over $10,000,000.

	Steam.	Sail.
Register tonnage......................	2,535,202	3,958,891
Equal to cargo tonnage....	3,250,000	6,400,000

By whomsoever operated, this route is certain to effect a revolution more far-reaching and more important to the commerce and industry of the world than that which followed the construction of the Suez Canal.—*Mr. E. L. Corthell* in *Engineering Magazine*, Nov. 1894.

PRESERVATION OF STONE.

PARAFFINE melted and heated in a painter's furnace is the best material for rendering natural stones, concrete, and brickwork impervious to water. If dissolved in the proportion of ¼ paraffine and ¾ kerosene it remains soft longer and penetrates the stone further. Paraffine is unaltered by weather or acids. If well melted in, it does not change the color of the stone, it simply deepens the color like water. It is very cheap, easily applied, and efficacious. It is most easily applied in hot weather.

RELIANCE BUILDING, CHICAGO—ABSTRACTS FROM SPECIFICATIONS.

The material employed shall be open-hearth or Bessemer steel containing not over 0.1 per cent of phosphorus, ultimate net strength in tension 60,000 pounds per square inch, elastic limit 30,000 pounds, elongation 25 per cent in 8 inches, reduction of area at point of fracture 45 per cent.

A duplicate test from each blow or cast and furnace heat will be required. It must stand bending through 180° over a mandrel, whose diameter is 1½ times the original thickness of specimen without showing signs of rupture either on convex or concave side of curve. After being heated to a dark cherry and quenched in water, 100° Fahr. must stand bending as before.

No steel beam or angle shall be heated in a forge or other fire after being rolled, but shall be worked cold unless subsequently annealed.

Rivet-steel shall have an ultimate tensile strength not less than 56,000 pounds nor more than 62,000 pounds per square inch, elastic limit 30,000 pounds, elongation 25 per cent in 8 inches, and reduction of area of 50 per cent.

Specimens of original bar must bend 180° close down on itself (as before described), and must stand cold hammering to one third its original thickness without flaying or cracking, and stand quenching as described.

Wrought iron shall be tough, fibrous, and uniform in quality, elastic limit not less than 26,000 pounds per square inch. It should be thoroughly welded during rolling, free from injurious seams, blisters, buckles, cinders, or imperfect edges. Ultimate strength 50,000 pounds per square inch, elongation 18 per cent in 8 inches. Specimens taken from angle and other shaped iron shall have an ultimate strength of 50,000 pounds, elongation 15 per cent in 8 inches. All iron and specimens from plate, angle, and shape iron must bend cold through 90° to a curve having a diameter not over twice the thickness of piece without fracture. When nicked on one side and bent by a blow from a sledge, the fracture must be nearly all fibrous, showing but few crystalline specks.

Cast iron to be of the best quality castings, clear and free from defects of every kind, boldly filleted at the angles, and arrises short and perfect. A bar 1 inch square and 5 feet long, 4½ feet between bearings, shall support a centre load of 550 pounds.

The columns for all sizes consist of a number of angles riveted together; safe load per sq. in. $= 17,100 - 57\frac{l}{r}$, ends of columns assumed to be fixed.

PILES.

Piles should be stripped of the bark before driving. It is not necessary nor desirable to sharpen their small ends when driving into soft materials. When necessary they may be shod with some form of iron point. Most iron shoes, however, are very imperfect and do little good. For full discussion of these matters see work on Foundations by the author.

Piles are now sometimes encased in iron or clay pipes between mud line and low water; the space between the two is then filled with cement-mortar. This has been found to give protection against the attacks of sea-worms.

ALUMINUM AND ALUMINUM BRONZE.

In the *Engineering Magazine*, December, 1894, is found an interesting and instructive article headed, "Aluminum, the Superabundant Metal." "Aluminum is destined in the near future to enter abundantly into the affairs of men, to reach a production comparable with those of tin or copper, and rapidly to become as cheap as those metals, at least for equal bulk."

"So far as we know, the solid portions of our planet is more than one-half oxygen. This no chemist believes to possess a metallic form. Of silicon, the next element in order of abundance, we cannot say the same, as it undoubtedly forms many metallic alloys and may yet be discovered in the metallic form. The element next in abundance, aluminum, forms certainly as much as 8 (possibly 10) per cent of the shell of the earth of the sixteen miles in thickness that we can explore. It is therefore by far the most abundant of the known metals. The chemical history of the formation of the earth's shows that aluminum is exceptional in several respects. Though not difficult to dissolve in water by the aid of alkalies, yet natural and even alkaline waters contain none. Hence it does not, like other abundant elements, flow from springs, pass through rivers, and diffuse throughout the ocean. Though abounding in soils, it is not absorbed by plant excretions and products of decay, which dissolve other mineral matters. Hence, during the vast erosions, degradations, and sifting of the ma-

terials of continents, aluminum has remained in concentrated forms, and in the débris is possibly as abundant as the all-pervading element silicon. The commonest mica muscovite (potash-mica) is its abundant ore, containing 20 per cent of aluminum. Clay contains 21 per cent; dehydrated clay, 24¼ per cent; emery, 31½ per cent; corundum, 52¼ per cent; and bauxite (now the favorite ore), 28 to 31¼ per cent. Bauxite was formerly found only in Europe, but was recently discovered in Georgia, Alabama, and Arkansas. It is hydrate of alumina, with a little iron oxide and silica. It has to be converted into pure alumina, and therefore has but small advantage over clays, which in time, on a larger scale, will supplant it. But now bauxite is the only important ore." The various attempts and processes of obtaining the metal aluminum is fully traced.

The French chemist Deville, in 1854, worked first with potassium, then with sodium, and finally by electrolyses, as also did Bunsen, in Germany. The discovery of electrolytic aluminum is due to one of the two. In America, Castner, in 1886 brought out his sodium process, making aluminum at a cost of 18 cents a pound. "We should ere this have had cheap aluminum by sodium but for the amazing advancement of electro-magnetic machinery. This gave birth to the American Cowles process for alloys, and the American Hall and French Heroult processes for rich metal, which now control the aluminum industry.

In this article the processes of getting aluminum will not be described. Some of the more important physical and chemical properties of commercially pure aluminum—not containing over 2 per cent of iron and silicon—are: Color, "silver white," with a shade of violet or lavender; when "matted" by an alkali, followed by an acid, it exhibits a beautiful dead-white surface. It takes a high polish and burnish. Its specific gravity ranges from 2.6 for ingots, to 2.61 for wrought and rolled, and 2.67 to 2.8075 for foil. While a cubic foot averages 165 pounds, one of silver weighs 656 pounds. At present prices of the two aluminum 70 cents per pound avoirdupois, and silver 60 cents per ounce troy—spoons, forks, cups, etc., of silver should cost 12¼ times, and with aluminum at 50 cents per pound (as in Europe) 19 times, as much as those of aluminum."

It is about as malleable as gold and silver, can be drawn into fine wire and tubes, forged, stamped, pressed, and spun into all shapes, and beaten as thin as gold, but does not become translucent. It resists oxidation, but loses its lustre, which can be restored by rubbing. It is not corroded by sulphur compounds. Alkalies and hydrochloric acid dissolve it. "Mercury, made to enfilm it, causes it to oxidize energetically." It melts at some temperature between the melting-points of silver and zinc. It volatilizes in the electric arc, but not in any furnace fire. The specific heat is twice that of iron, quadruple that of silver, and nearly seven times that of gold. Aluminum foil rolled to $\frac{1}{1000}$ inch is superseding silver for decoration.

"The uses for the lower grades of aluminum are as yet comparatively restricted. The veneering of non-metallic surfaces and the plating of metallic surfaces will be important also for protective purposes. Here its persistence, resistance to corrosion, color, softness, and pliability are paramount to its lightness.

The uses of the commercially pure metal, say 94 to 95 per cent, must soon become more numerous. From a possible mode of coating iron with such metal, as now with tin or zinc, a very important industry would rapidly rise, in which alloys would also be employed. The principal uses for these grades are now for making alloys, and as an addition to molten steel and iron. In the order of their importance are: alloys with copper (bronzes), copper and zinc (brasses), iron and steel, zinc (for galvanizing), lead and antimony (type-metals), manganese, silver, nickel, tin. Cowles' "A1" bronze, made from corundum, tests from 95,000 to 128,000 pounds per square inch. Heroult-process bronze has stood from 50,000 to 90,000 pounds. Unlike steel, these bronzes are softened by chilling. In electric conduction they are close to copper. Alloys with nickel possess a rigidity equal to that of the best steel.

ALUMINUM BRONZE.

(*Engineering News*, Oct. 11, 1894.)

In the same sense that steel is a carbide of iron, aluminum bronze is an aluminide of copper. It is a metal showing the fracture-qualities of the highest grades of steel, and equalling steel in elongation, elastic limit, and tensile strength, and possessing also a different class of properties. This series of metals is non-magnetic. They are very permanent. The oftener they are remelted the better, providing they are kept free from oxygen, iron, and silicon. The value of the metal is just as great after the tool has outlived its usefulness as it was at the beginning, and for all cases in which non-magnetic properties, or high conductivity, or extremely uniform and permanent behavior are required, you have the ideal metal. One of the chief reasons why steel and iron hold their high value is the fact that they are commercially workable, and the metal does not lose its mechanical properties throughout the range of the temperatures of ordinary use. That is not true of any of the alloys of copper made with tin or zinc, but it is true of aluminum-bronze.

The hold-down bolts for the plates of mortar-batteries along the coast have been made of this material. It requires a submerged bolt to take a strain of 200,000 pounds per bolt, to remain years in place, and yet to afford a guaranty that at the end of years and after any number of firing-shocks that bolt shall be unchanged.

The metal will far surpass the present requirements for cartridge-shells, which are made from flat disks, spun and swedged, and thus gives, perhaps, the severest test that we have of the complete working qualities of sheet metal. The already great value of aluminum and its alloys, the many uses to which it can be applied and the many articles manufactured of it or of its alloys, the important part it plays in the manufacture of other substances, especially its influence over iron and steel, render it one of the most important metals.

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